HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

University of Southampton

Tri-hypercharge: a path to the origin of flavour

Mario Fernández Navarro†

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in collaboration with Stephen F. King

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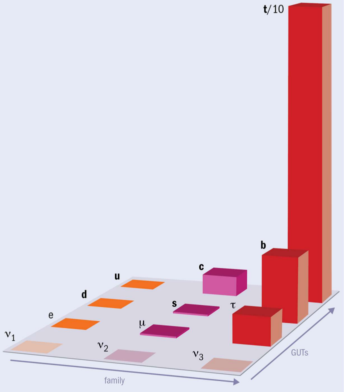
The flavour puzzle



$$m_t \sim \frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_c \sim \lambda^{3.3}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_u \sim \lambda^{7.5}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

$$m_b \sim \lambda^{2.5}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_s \sim \lambda^{5.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_d \sim \lambda^{7.0}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

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$$V_{us} \sim \lambda\,, \qquad V_{cb} \sim \lambda^2\,, \qquad V_{ub} \sim \lambda^3\,,$$

$$\tan\theta^\nu_{23} \sim 1\,, \qquad \tan\theta^\nu_{12} \sim \frac{1}{\sqrt{2}}\,, \qquad \sin\theta^\nu_{13} \sim \frac{\lambda}{\sqrt{2}}\,,$$

where $v_{\rm SM} \simeq 246\,\text{GeV}$ and $\lambda = \sin\theta_C \simeq 0.224$

- Why three families?
- Why the three families interact so differently with the Higgs?
- What is the origin of very small neutrino masses, and why the PMNS mixing is so different from the CKM mixing?

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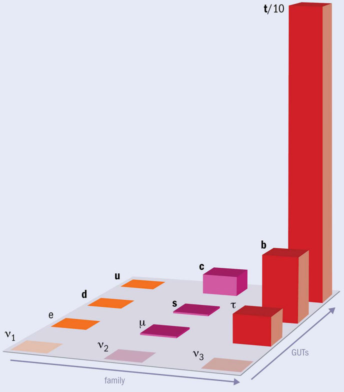

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- Why the three families interact so differently with the Higgs?
- What is the origin of very small neutrino masses, and why the PMNS mixing is so different from the CKM mixing?
  $\Rightarrow$ A theory of flavour is needed!

Tri-hypercharge in a nutshell

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$
$$\to SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3}$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$
Q_1	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0	0
u_1^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	0	0
d_1^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	0	0
L_1	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0	0
e_1^c	$\mathbf{1}$	$\mathbf{1}$	1	0	0
Q_2	$\mathbf{3}$	$\mathbf{2}$	0	$1/6$	0
u_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$-2/3$	0
d_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$1/3$	0
L_2	$\mathbf{1}$	$\mathbf{2}$	0	$-1/2$	0
e_2^c	$\mathbf{1}$	$\mathbf{1}$	0	1	0
Q_3	$\mathbf{3}$	$\mathbf{2}$	0	0	$1/6$
u_3^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	0	$-2/3$
d_3^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	0	$1/3$
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H	$\mathbf{1}$	$\mathbf{2}$	0	0	$-1/2$

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- If $H(\mathbf{1},\mathbf{2})_{(0,0,-1/2)}$, then only third family Yukawa couplings are allowed at renormalisable level (two doublets H_u, H_d explain $m_{b,\tau}/m_t$ hierarchy if large $\tan\beta$).

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- Light charged fermion masses and CKM mixing are naturally small because they arise from non-renormalisable operators (involving the scalar SM singlets breaking $U(1)_Y^3$ down to SM hypercharge).
- Very interesting and non-standard neutrino sector; very rich phenomenology if NP scales are low.

Tri-hypercharge gauge theory

Minimal set of scalars and work in EFT framework ($\mathcal{O}(1)$ coefficients implicit)

$$\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\,, \qquad \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}\,, \qquad \phi_{q12}^{(-\frac{1}{6},\frac{1}{6},0)}\,.$$

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$$\mathcal{L} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}^3_{q12}\phi_{\ell 23}/\Lambda^4 & \phi_{q12}\phi_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \tilde{\phi}^4_{q12}\phi_{\ell 23}/\Lambda^5 & \phi_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \tilde{\phi}^4_{q12}\phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^6 & \phi_{\ell 23}\tilde{\phi}_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} u^c_1 \\ u^c_2 \\ u^c_3 \end{pmatrix} H_u$$

$$+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi^3_{q12}\tilde{\phi}_{\ell 23}/\Lambda^4 & \phi_{q12}\tilde{\phi}_{\ell 23}/\Lambda^2 & \phi_{q12}\phi_{q23}/\Lambda^2 \\ \phi^2_{q12}\tilde{\phi}_{\ell 23}/\Lambda^3 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{q23}/\Lambda \\ \phi^2_{q12}\phi^2_{q23}/\Lambda^4 & \phi^2_{q23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \end{pmatrix} H_d$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \phi^3_{q12}\tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}^3_{q12}\tilde{\phi}_{\ell 23}/\Lambda^4 & \tilde{\phi}^3_{q12}\phi_{\ell 23}/\Lambda^4 \\ \phi^6_{q12}\tilde{\phi}_{\ell 23}/\Lambda^7 & \tilde{\phi}_{\ell 23}/\Lambda & \phi_{\ell 23}/\Lambda \\ \phi^6_{q12}\tilde{\phi}^2_{\ell 23}/\Lambda^8 & \tilde{\phi}^2_{\ell 23}/\Lambda^2 & 1 \end{pmatrix} \begin{pmatrix} e^c_1 \\ e^c_2 \\ e^c_3 \end{pmatrix} H_d$$

- Naive power counting in the EFT already motivates natural mass hierarchies.

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- Naive power counting in the EFT already motivates natural mass hierarchies.
- $\langle \phi_{\ell 23} \rangle$ and $\langle \phi_{q23} \rangle$ explain m_2/m_3 and small CKM mixing
- $\langle \phi_{q12} \rangle$ crucial to explain m_1/m_2 and Cabibbo angle.

$$\frac{\langle \phi_{\ell 23} \rangle}{\Lambda} \sim \frac{m_c}{m_t} \simeq \lambda^3\,, \qquad \frac{\langle \phi_{q23} \rangle}{\Lambda} \sim V_{cb} \simeq \lambda^2\,, \qquad \frac{\langle \phi_{q12} \rangle}{\Lambda} \sim V_{us} \simeq \lambda,$$

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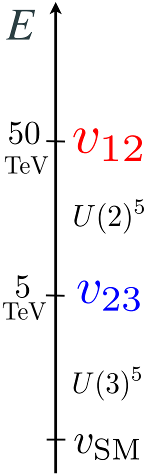

- Largest VEV is $\langle \phi_{q12} \rangle \approx v_{12}$ triggering $U(1)_{Y_1} \times U(1)_{Y_2} \xrightarrow{v_{12}} U(1)_{Y_1+Y_2}$

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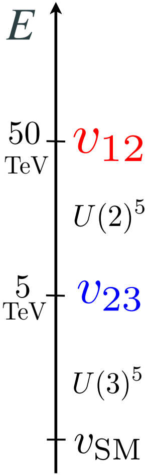

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- Both 23-breaking VEVs trigger $U(1)_{Y_1+Y_2} \times U(1)_{Y_3} \xrightarrow{v_{23}} U(1)_{Y_1+Y_2+Y_3}$ at the scale v_{23}, mild hierarchy $v_{23}/v_{12} \approx \lambda$.

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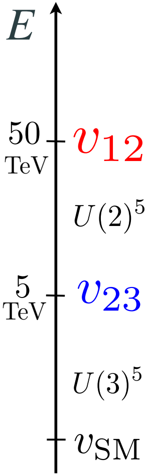

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- Natural explanation of fermion mass hierarchies and CKM mixing!
- Spurion analysis, texture zeros and more models in the paper/poster!

Neutrinos: Seesaw mechanism *à la tri-hypercharge*

Seesaw type I via full singlet neutrinos unnatural in the $U(1)^3_Y$ framework:

- e.g. introduce $U(1)^3_Y$ singlet $N(\mathbf{1},\mathbf{1})_{(0,0,0)}$

  $\Rightarrow \mathcal{L}_N \supset L_3 H_u N + m_N NN$

  with $L_2 H_u N$ and $L_1 H_u N$ forbidden by $U(1)^3_Y$! (they could originate from higher dimensional operators, but then neutrino mixing would be naturally small)

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  $\Rightarrow$ unsuccessful PMNS mixing

- Solution $\Rightarrow$ Add RH neutrinos *à la tri-hypercharge* $N(\mathbf{1},\mathbf{1})_{(Y_1,Y_2,Y_3)}$ (with vanishing SM hypercharge $Y_1+Y_2+Y_3=0$).

Seesaw mechanism *à la tri-hypercharge*

- Consider adding $N_{\mathrm{atm}}^{(0,\frac{1}{4},-\frac{1}{4})}$ and $\phi_{\mathrm{atm}}^{(0,\frac{1}{4},-\frac{1}{4})}$, then (remember $\phi_{\ell 23}^{(0,1/2,-1/2)}$)

$$\mathcal{L}_{N_{\mathrm{atm}}} \supset \frac{1}{\Lambda_{\mathrm{atm}}}(\phi_{\mathrm{atm}} L_2 + \tilde{\phi}_{\mathrm{atm}} L_3) H_u N_{\mathrm{atm}} + \tilde{\phi}_{\ell 23} N_{\mathrm{atm}} N_{\mathrm{atm}} \,,$$

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- But gauge anomalies via $N_{\rm atm}^{(0,\frac{1}{4},-\frac{1}{4})}$ $\Rightarrow$ add conjugate neutrino $\overline{N}_{\rm atm}^{(0,-1/4,1/4)}$

$$\begin{aligned}\mathcal{L}_{N_{\rm atm}} =& \frac{1}{\Lambda_{\rm atm}}(\phi_{\rm atm}L_2 + \tilde{\phi}_{\rm atm}L_3)H_u N_{\rm atm} + \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}}L_3 H_u \overline{N}_{\rm atm} \\ &+ \phi_{\ell 23}N_{\rm atm}N_{\rm atm} + \tilde{\phi}_{\ell 23}\overline{N}_{\rm atm}\overline{N}_{\rm atm} + M_{N_{\rm atm}}\overline{N}_{\rm atm}N_{\rm atm}\,,\end{aligned}$$

Notice $L_3 H_u = (0,0,0)$.

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- Add another RH neutrino and play the same game to obtain solar mixing.

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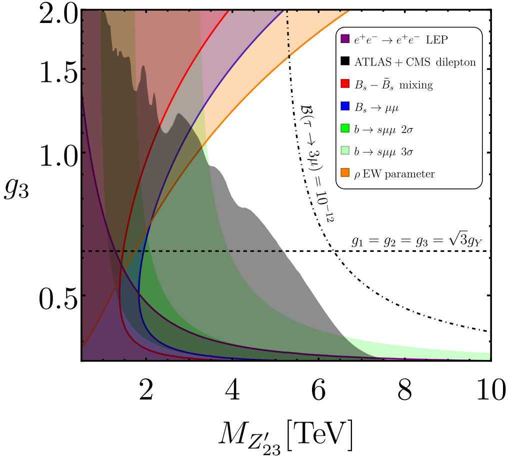
  Notice $L_3 H_u = (0,0,0)$.

- Add another RH neutrino and play the same game to obtain solar mixing.
- Successful seesaw only if $M_{\rm VL} \lesssim \langle \phi_{\ell 23} \rangle \Rightarrow$ RH neutrinos at scale $\langle \phi_{\ell 23} \rangle$, which can be as low as TeV!

$$m_\nu \simeq \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} v_{23} \frac{v_{\rm SM}^2}{\Lambda_{\rm atm}^2} \,.$$

Phenomenology: Z'_{23}



Very rich phenomenology if NP scales are low

- Flavour observables: meson-antimeson mixing, LFV, B-physics...
- LHC dilepton searches $pp \to Z'_{23} \to e^+e^-, \mu^+\mu^-$
- Inevitable $Z - Z'_{23}$ mixing $\Rightarrow$ breaking of custodial symmetry $\Rightarrow$ EWPOs

Take home messages

$$\boxed{SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}}$$

- Tri-hypercharge gauge group might be the first step towards understanding the origin of three flavours, the hierarchical charged fermion masses and CKM mixing.

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- Tri-hypercharge gauge group might be the first step towards understanding the origin of three flavours, the hierarchical charged fermion masses and CKM mixing.
- If seesaw mechanism is implemented via adding RH neutrinos, the $U(1)_Y^3$ setup leads to a low scale seesaw where RH neutrinos might be as light as a few TeV.

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- Tri-hypercharge gauge group might be the first step towards understanding the origin of three flavours, the hierarchical charged fermion masses and CKM mixing.
- If seesaw mechanism is implemented via adding RH neutrinos, the $U(1)_Y^3$ setup leads to a low scale seesaw where RH neutrinos might be as light as a few TeV.
- Rich phenomenology via Z' bosons if NP scales are low: from flavour-violating observables to LHC physics and EW precision physics.

Take home messages

$$\boxed{SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}}$$

Several questions still open:

- Gauge unification?
- Phase transitions in the early Universe? (multi-peaked GW?)
- Family decomposition of the remaining gauge symmetry: $SU(3)_c^3$ and $SU(2)_L^3$?
- Why is the Higgs a third family particle? (is there a symmetry relating the three families?)
- ...

If you are interested on the origin of flavour, come to my poster!

Tri-hypercharge: a path to the origin of flavour

HIDDeN — Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Mario Fernández Navarro[†] and Stephen F. King

University of Southampton

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arXiv: **2305.07690**

Flavour puzzle

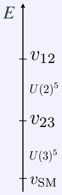
Flavour sector can be parameterised in terms of $\lambda = 0.225$ as

$$m_t \sim \frac{v_{\rm SM}}{\sqrt{2}}\,, \quad m_c \sim \lambda^{3.3}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad m_u \sim \lambda^{7.5}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad V_{us} \sim \lambda\,, \quad \tan\theta^{\nu}_{23} \sim 1\,,$$
$$m_b \sim \lambda^{2.5}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad m_s \sim \lambda^{5.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad m_d \sim \lambda^{7.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad V_{cb} \sim \lambda^2\,, \quad \tan\theta^{\nu}_{12} \sim \frac{1}{\sqrt{2}}\,,$$
$$m_\tau \sim \lambda^{3.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad m_\mu \sim \lambda^{4.9}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad m_e \sim \lambda^{8.4}\frac{v_{\rm SM}}{\sqrt{2}}\,, \quad V_{ub} \sim \lambda^3\,, \quad \sin\theta^{\nu}_{13} \sim \frac{\lambda}{\sqrt{2}}\,.$$

Why three families? Why the three families interact so differently with the Higgs? What is the origin of very small neutrino masses, and why the PMNS mixing is so different from the CKM mixing?

Tri-hypercharge in a nutshell

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$
Q_1	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0	0
u^c_1	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	0	0
d^c_1	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	0	0
L_1	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0	0
e^c_1	$\mathbf{1}$	$\mathbf{1}$	1	0	0
Q_2	$\mathbf{3}$	$\mathbf{2}$	0	$1/6$	0
u^c_2	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$-2/3$	0
d^c_2	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$1/3$	0
L_2	$\mathbf{1}$	$\mathbf{2}$	0	$-1/2$	0
e^c_2	$\mathbf{1}$	$\mathbf{1}$	0	1	0
Q_3	$\mathbf{3}$	$\mathbf{2}$	0	0	$1/6$
u^c_3	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	0	$-2/3$
d^c_3	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	0	$1/3$
L_3	$\mathbf{1}$	$\mathbf{2}$	0	0	$-1/2$
e^c_3	$\mathbf{1}$	$\mathbf{1}$	0	0	1
H_u	$\mathbf{1}$	$\mathbf{2}$	0	0	$1/2$
H_d	$\mathbf{1}$	$\mathbf{2}$	0	0	$-1/2$



$$\boxed{SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}}$$
$$\overset{v_{12}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$
$$\overset{v_{23}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

- Gauge anomalies cancel separately for each family, as in the SM, but without family replication.
- If Higgs doublets only carry third family hypercharge, then only third family Yukawa couplings allowed at renormalisable level. Type II 2HDM helps with hierarchies between charged sectors via $\tan\beta \approx 20$.
- Light charged fermion masses and CKM mixing are naturally small because they arise from

Neutrino masses and mixing

- $U(2)^5$ also in the neutrino sector $\Rightarrow$ naive expectation is a heavier active neutrino with tiny mixing.
- Solution $\Rightarrow$ Add SM singlet neutrinos which carry $U(1)^3_Y$ charges (with vanishing SM hypercharge).
- If the singlet neutrinos carry $U(1)^3_Y$ charges, gauge anomalies are an issue $\Rightarrow$ vector-like neutrinos

$$N^{(0,\frac{1}{4},-\frac{1}{4})}_{\rm atm}\,, \qquad \overline{N}^{(0,-\frac{1}{4},\frac{1}{4})}_{\rm atm}\,, \qquad \phi^{(0,\frac{1}{4},-\frac{1}{4})}_{\rm atm}$$

$$\mathcal{L}_{N_{\rm atm}} \supset \frac{1}{\Lambda_{\rm atm}}(\phi_{\rm atm}L_2 + \tilde{\phi}_{\rm atm}L_3)H_u N_{\rm atm} + \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}}L_3 H_u \overline{N}_{\rm atm}$$
$$+ \phi_{\ell 23}N_{\rm atm}N_{\rm atm} + \tilde{\phi}_{\ell 23}\overline{N}_{\rm atm}\overline{N}_{\rm atm} + M_{N_{\rm sol}}\overline{N}_{\rm atm}N_{\rm atm}\,,$$

$$N^{(\frac{1}{4},\frac{1}{4},-\frac{1}{2})}_{\rm sol}\,, \qquad \overline{N}^{(-\frac{1}{4},-\frac{1}{4},\frac{1}{2})}_{\rm sol}\,, \qquad \phi^{(-\frac{1}{2},-\frac{1}{2},1)}_{\rm sol}\,, \qquad \phi^{(\frac{1}{4},-\frac{1}{4},0)}_{e12}\,, \qquad \phi^{(-\frac{1}{4},-\frac{1}{4},\frac{1}{2})}_{\nu 13}\,,$$

$$\mathcal{L}_{N_{\rm sol}} \supset \frac{1}{\Lambda_{\rm sol}}(\phi_{e12}L_1 + \tilde{\phi}_{e12}L_2 + \phi_{\nu 13}L_3)H_u N_{\rm sol} + \frac{\phi_{\nu 13}}{\Lambda_{\rm sol}}L_3 H_u \overline{N}_{\rm sol}$$
$$+ \phi_{\rm sol}N_{\rm sol}N_{\rm sol} + \tilde{\phi}_{\rm sol}\overline{N}_{\rm sol}\overline{N}_{\rm sol} + M_{N_{\rm atm}}\overline{N}_{\rm sol}N_{\rm sol}\,,$$

Assume $M_{N_{\rm sol}} \approx M_{N_{\rm atm}} \equiv M_{\rm VL}$ for simplicity, $\langle\phi_{e12}\rangle \approx \mathcal{O}(v_{12})$ and the rest $\langle\phi\rangle \approx \mathcal{O}(v_{23})$, apply seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\rm T} = \left[\begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} M_{\rm VL} + \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} v_{23}\right]\frac{1}{v_{23}^2 - M_{\rm VL}^2}\,.$$

$\Rightarrow M_{\rm VL} \lesssim v_{23}$ required to describe PMNS mixing (singlet neutrinos at low scale v_{23}!), then

$$m_\nu \approx \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} v_{23}^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} v_{23}\frac{H_u H_u}{\Lambda^2_{\rm atm}} + \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & \lambda \\ \lambda & \lambda & \lambda^2 \end{pmatrix} v_{23}\frac{H_u H_u}{\lambda^2\Lambda^2_{\rm sol}}\,.$$

Mild hierarchy $v_{23}/v_{12} \approx \lambda$ has introduced a small hierarchy in m_ν. If $\Lambda_{\rm atm}/\Lambda_{\rm sol} \simeq \lambda$ then

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HIDDeN
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Neutrinos: Example of seesaw mechanism

Getting ready to apply the seesaw formula (neglecting $\mathcal{O}(1)$ coefficients)

$$m_{D_L} = \begin{pmatrix} & \overline{N}_{\rm sol} & \overline{N}_{\rm atm} \\ \hline L_1| & 0 & 0 \\ L_2| & 0 & 0 \\ L_3| & \frac{\tilde{\phi}_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \end{pmatrix} H_u \,, \qquad m_{D_R} = \begin{pmatrix} & N_{\rm sol} & N_{\rm atm} \\ \hline L_1| & \frac{\phi_{e12}}{\Lambda_{\rm sol}} & 0 \\ L_2| & \frac{\tilde{\phi}_{e12}}{\Lambda_{\rm sol}} & \frac{\phi_{\rm atm}}{\Lambda_{\rm atm}} \\ L_3| & \frac{\phi_{\nu 13}}{\Lambda_{\rm sol}} & \frac{\tilde{\phi}_{\rm atm}}{\Lambda_{\rm atm}} \end{pmatrix} H_u \,,$$

$$M_L = \begin{pmatrix} & \overline{N}_{\rm sol} & \overline{N}_{\rm atm} \\ \hline \overline{N}_{\rm sol}| & \tilde{\phi}_{\rm sol} & 0 \\ \overline{N}_{\rm atm}| & 0 & \tilde{\phi}_{\ell 23} \end{pmatrix} \approx v_{23}\mathbb{I}_{2\times 2} \,, \quad M_R \approx \begin{pmatrix} & N_{\rm sol} & N_{\rm atm} \\ \hline N_{\rm sol}| & \phi_{\rm sol} & 0 \\ N_{\rm atm}| & 0 & \phi_{\ell 23} \end{pmatrix} \approx v_{23}\mathbb{I}_{2\times 2} \,,$$

$$M_{LR} = \begin{pmatrix} & N_{\rm sol} & N_{\rm atm} \\ \hline \overline{N}_{\rm sol}| & M_{N_{\rm sol}} & 0 \\ \overline{N}_{\rm atm}| & 0 & M_{N_{\rm atm}} \end{pmatrix} \approx M_{\rm VL}\mathbb{I}_{2\times 2} \,,$$

Backup: Neutrinos

Full neutrino mass matrix (remember $M_L \approx M_R \approx v_{23}\mathbb{I}_{2\times 2}\,, \quad M_{LR} \approx M_{\mathrm{VL}}\mathbb{I}_{2\times 2}$)

$$M_\nu = \begin{pmatrix} & \nu & \overline{N} & N \\ \hline \nu| & 0 & m_{D_L} & m_{D_R} \\ \overline{N}| & m_{D_L}^{\mathrm{T}} & M_L & M_{LR} \\ N| & m_{D_R}^{\mathrm{T}} & M_{LR}^{\mathrm{T}} & M_R \end{pmatrix} \equiv \begin{pmatrix} 0 & m_D \\ m_D^{\mathrm{T}} & M_N \end{pmatrix} .$$

$$m_{D_L} = \begin{pmatrix} & \overline{N}_{\mathrm{sol}} & \overline{N}_{\mathrm{atm}} \\ \hline L_1| & 0 & 0 \\ L_2| & 0 & 0 \\ L_3| & \frac{\tilde{\phi}_{\nu 13}}{\Lambda_{\mathrm{sol}}} & \frac{\phi_{\mathrm{atm}}}{\Lambda_{\mathrm{atm}}} \end{pmatrix} H_u\,, \qquad m_{D_R} = \begin{pmatrix} & N_{\mathrm{sol}} & N_{\mathrm{atm}} \\ \hline L_1| & \frac{\phi_{e12}}{\Lambda_{\mathrm{sol}}} & 0 \\ L_2| & \frac{\tilde{\phi}_{e12}}{\Lambda_{\mathrm{sol}}} & \frac{\phi_{\mathrm{atm}}}{\Lambda_{\mathrm{atm}}} \\ L_3| & \frac{\phi_{\nu 13}}{\Lambda_{\mathrm{sol}}} & \frac{\tilde{\phi}_{\mathrm{atm}}}{\Lambda_{\mathrm{atm}}} \end{pmatrix} H_u\,,$$

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$$M_\nu = \left(\begin{array}{c|ccc} & \nu & \overline{N} & N \\ \hline \nu & 0 & m_{D_L} & m_{D_R} \\ \overline{N} & m_{D_L}^{\mathrm{T}} & M_L & M_{LR} \\ N & m_{D_R}^{\mathrm{T}} & M_{LR}^{\mathrm{T}} & M_R \end{array}\right) \equiv \begin{pmatrix} 0 & m_D \\ m_D^{\mathrm{T}} & M_N \end{pmatrix} .$$

$$m_{D_L} = \left(\begin{array}{c|cc} & \overline{N}_{\mathrm{sol}} & \overline{N}_{\mathrm{atm}} \\ \hline L_1 & 0 & 0 \\ L_2 & 0 & 0 \\ L_3 & \frac{\tilde{\phi}_{\nu 13}}{\Lambda_{\mathrm{sol}}} & \frac{\phi_{\mathrm{atm}}}{\Lambda_{\mathrm{atm}}} \end{array}\right) H_u\,, \qquad m_{D_R} = \left(\begin{array}{c|cc} & N_{\mathrm{sol}} & N_{\mathrm{atm}} \\ \hline L_1 & \frac{\phi_{e12}}{\Lambda_{\mathrm{sol}}} & 0 \\ L_2 & \frac{\tilde{\phi}_{e12}}{\Lambda_{\mathrm{sol}}} & \frac{\phi_{\mathrm{atm}}}{\Lambda_{\mathrm{atm}}} \\ L_3 & \frac{\phi_{\nu 13}}{\Lambda_{\mathrm{sol}}} & \frac{\tilde{\phi}_{\mathrm{atm}}}{\Lambda_{\mathrm{atm}}} \end{array}\right) H_u\,,$$

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\mathrm{VL}} \\ -M_{\mathrm{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\mathrm{T}} \\ m_{D_R}^{\mathrm{T}} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2}$$

$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} .$$

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$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} .$$

- Contributions proportional to M_{VL} do not provide a successful m_ν

$$m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} M_{\mathrm{VL}} ,$$

$$m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} + m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} v_{23} .$$

Neutrinos: Example of seesaw mechanism

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\mathrm{VL}} \\ -M_{\mathrm{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\mathrm{T}} \\ m_{D_R}^{\mathrm{T}} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2}$$

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- Contributions proportional to M_{VL} do not provide a successful m_ν

$$m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} M_{\mathrm{VL}} \,,$$

$$m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} + m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} v_{23} \,.$$

$M_{\mathrm{VL}} \lesssim v_{23}$ required to describe PMNS mixing $\Rightarrow$ SM singlet neutrinos with mass at the lower scale v_{23}

Neutrinos: Example of seesaw mechanism

Provided that $m_D \ll M_N$, we can apply the seesaw formula

$$m_\nu = m_D M_N^{-1} m_D^{\mathrm{T}} = \begin{pmatrix} m_{D_L} & m_{D_R} \end{pmatrix} \begin{pmatrix} v_{23} & -M_{\mathrm{VL}} \\ -M_{\mathrm{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^{\mathrm{T}} \\ m_{D_R}^{\mathrm{T}} \end{pmatrix} \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2}$$

$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} .$$

- $M_{\mathrm{VL}} \lesssim v_{23}$ required to describe oscillation data. For simplicity we consider $M_{\mathrm{VL}} \ll v_{23}$ (and $v_{23}/v_{12} \approx \lambda$ obtained in the charged fermion sector) and obtain (neglecting $\mathcal{O}(1)$ coefficients)

$$m_\nu \approx m_{D_R} m_{D_R}^{\mathrm{T}} v_{23}^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} v_{23} \frac{H_u H_u}{\Lambda_{\mathrm{atm}}^2} + \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & \lambda \\ \lambda & \lambda & \lambda^2 \end{pmatrix} v_{23} \frac{H_u H_u}{\lambda^2 \Lambda_{\mathrm{sol}}^2} .$$

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$$= \left[m_{D_L} m_{D_L}^{\mathrm{T}} v_{23} - m_{D_L} m_{D_R}^{\mathrm{T}} M_{\mathrm{VL}} - m_{D_R} m_{D_L}^{\mathrm{T}} M_{\mathrm{VL}} + m_{D_R} m_{D_R}^{\mathrm{T}} v_{23} \right] \frac{1}{v_{23}^2 - M_{\mathrm{VL}}^2} .$$

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- If $\Lambda_{\mathrm{atm}}/\Lambda_{\mathrm{sol}} \simeq \lambda$ then (remember $\langle H_u \rangle = v_{\mathrm{SM}}/\sqrt{2}$)

$$\boxed{m_\nu \simeq \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} v_{23} \frac{v_{\mathrm{SM}}^2}{\Lambda_{\mathrm{atm}}^2} ,}$$

scales $\Lambda_{\mathrm{atm}} \approx 10^6\,\mathrm{TeV}$ and $v_{23} \approx \mathcal{O}(1\,\mathrm{TeV})$ provide enough suppression for $m_\nu \approx \mathcal{O}(0.05\,\mathrm{eV})$. SM singlet neutrinos with mass at scale v_{23}.

Model 2: RH mixing suppressed

$$\phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\,, \qquad \phi_{q23}^{(0,-\frac{1}{6},\frac{1}{6})}\,, \qquad \phi_{q13}^{(-\frac{1}{6},0,\frac{1}{6})}\,, \qquad \phi_{d12}^{(-\frac{1}{6},-\frac{1}{3},\frac{1}{2})}\,, \qquad \phi_{e12}^{(\frac{1}{4},-\frac{1}{4},0)}\,.$$

$$\begin{aligned}
\mathcal{L} = &\begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{q13}\tilde{\phi}_{q23}\phi_{\ell 23} & \phi_{q13} \\ \phi_{e12}^2\tilde{\phi}_{q13}\phi_{q23}\phi_{\ell 23} & \phi_{\ell 23} & \phi_{q23} \\ \phi_{e12}^2\tilde{\phi}_{q13}\phi_{\ell 23} & \phi_{\ell 23}\tilde{\phi}_{q23} & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u \\
&+ \begin{pmatrix} Q_1 & Q_2 & Q_3 \end{pmatrix} \begin{pmatrix} \phi_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{d12} & \phi_{q13} \\ \phi_{q13}^2\phi_{q23} & \tilde{\phi}_{\ell 23} & \phi_{q23} \\ \phi_{q13}^2 & \phi_{q23}^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d \\
&+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{e12}^2\tilde{\phi}_{\ell 23} & \phi_{e12}^2\phi_{\ell 23} \\ \tilde{\phi}_{e12}^4\tilde{\phi}_{\ell 23} & \tilde{\phi}_{\ell 23} & \phi_{\ell 23} \\ \tilde{\phi}_{e12}^4\tilde{\phi}_{\ell 23}^2 & \tilde{\phi}_{\ell 23}^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d\,.
\end{aligned}$$

$$\frac{\langle\phi_{\ell 23}\rangle}{\Lambda} = \frac{\langle\phi_{q13}\rangle}{\Lambda} \simeq \lambda^3\,, \quad \frac{\langle\phi_{q23}\rangle}{\Lambda} \simeq \lambda^2\,, \quad \frac{\langle\phi_{d12}\rangle}{\Lambda} \simeq \lambda^4\,, \quad \frac{\langle\phi_{e12}\rangle}{\Lambda} \simeq \lambda\,.$$

$$\begin{aligned}
\mathcal{L} = &\begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^8 & \lambda^3 \\ \lambda^{10} & \lambda^3 & \lambda^2 \\ \lambda^7 & \lambda^5 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\mathrm{SM}}}{\sqrt{2}} \\
&+ \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^3 \\ \lambda^8 & \lambda^3 & \lambda^2 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}} \\
&+ \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^5 & \lambda^5 \\ \lambda^7 & \lambda^3 & \lambda^3 \\ \lambda^{10} & \lambda^6 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\mathrm{SM}}}{\sqrt{2}}\,.
\end{aligned}$$