Imprints of ΔN_{eff} in Dirac-Scotogenic framework

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Introduction

- We study a Dirac Scotogenic model. This allow us to connect
  - Neutrino mass generation at one loop level
  - A viable Dark matter candidate
- Dirac nature allows right-handed neutrinos (ν_R) to be very light.
- Thermalization of ν_R lead to additional contribution to dark radiation.
- The effective number of neutrino species is defined as:

$$N_{\text{eff}} = \left(\frac{7}{8}\right)\left(\frac{11}{4}\right)^{4/3}\left[\frac{\rho_{\text{rad}} - \rho_\gamma}{\rho_\gamma}\right]$$

- For instantaneous neutrino decoupling, $N_{\text{eff}} = 3$, else $N_{\text{eff}} = 3.046$[1]
- We will be looking for the parameter $\Delta N_{\text{eff}} (\equiv N_{\text{eff}}^{\text{new}} - 3.046)$
- Current CMB bound at 2σ CL: $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$

[1] *Nucl. Phys. B 729, pp. 221–234*

Components of the model

We have three sets of VLFs ($N_{1,2,3}$), three $\nu_R s$, one doublet scalar (ϕ) and a singlet scalar (χ).

	L	H	ν_R	N	ϕ	χ
$SU(2)$	2	2	1	1	2	1
$U(1)_Y$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0
Z_3	0	0	ω	ω	ω	0
Z_2	+	+	+	-	-	-

$$-\mathcal{L}_{\text{Yukawa}} \supset (y_\phi)_{ij}\overline{L_i}\tilde{\phi}N_j + (y_\chi)_{ij}\overline{\nu_R}_i N_j \chi + (M_N)_{ij}\bar{N}_i N_j + \text{h.c.} \quad (1)$$

The scalar potential of the model can be written as follows,

$$V = -\mu_H^2 H^\dagger H + \mu_\phi^2 \phi^\dagger \phi + \frac{1}{2}\mu_\chi^2 \chi^2 + \frac{1}{2}\lambda_1 (H^\dagger H)^2 + \frac{1}{2}\lambda_2(\phi^\dagger\phi)^2 + \frac{1}{4!}\lambda_3\chi^4 + \lambda_4(H^\dagger H)(\phi^\dagger\phi) + \frac{1}{2}\lambda_5(H^\dagger H)\chi^2 + \frac{1}{2}\lambda_6(\phi^\dagger\phi)\chi^2 + \lambda_7(H^\dagger\phi)(\phi^\dagger H) + \mu(\phi^\dagger H + H^\dagger\phi)\chi. \quad (2)$$

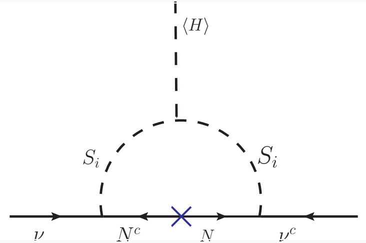
Mixing between ϕ_R and χ:

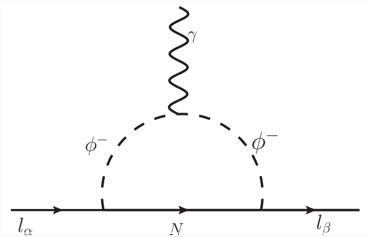
$$\begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi \\ \phi_R \end{pmatrix} \quad \text{where,} \quad \theta = \tan^{-1}\left[\frac{2\sqrt{2}\mu v}{\mu_\phi^2 - \mu_\chi^2 + (\lambda_4 - \lambda_5)v^2}\right].$$

Neutrino mass & LFV

The one-loop Dirac neutrino mass *[Phys. Rev.D86(2012) 033007]*:

$$(M_\nu)_{\alpha\beta} = \frac{\sin 2\theta}{32\pi^2\sqrt{2}} \sum_{k=1}^{3} (y_\phi)_{\alpha k}(y_\chi^*)_{\beta k} M_{N_k} \left(\frac{M_{S_1}^2}{M_{S_1}^2 - M_{N_k}^2} \ln \frac{M_{S_1}^2}{M_{N_k}^2} - \frac{M_{S_2}^2}{M_{S_2}^2 - M_{N_k}^2} \ln \frac{M_{S_2}^2}{M_{N_k}^2} \right).$$





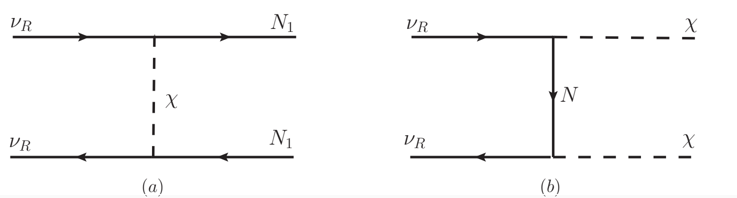
The decay branching ratio for $\mu \to e\gamma$ is given by:

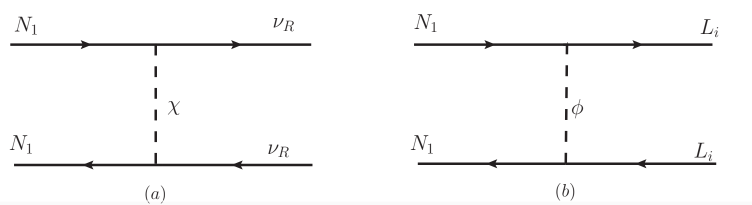
$$\mathrm{Br}(\mu \to e\gamma) = \mathrm{Br}(\mu \to e\nu_\mu\bar{\nu}_e) \times \frac{3\alpha_{\mathrm{EM}}}{16\pi G_F^2} \mathrm{Abs}\left[\sum_i \frac{(y_\phi)_{\mu i}(y_\phi^*)_{ei}}{M_{\phi^\pm}^2} f\left(\frac{M_{N_i}^2}{M_{\phi^\pm}^2}\right) \right]^2.$$

$$\text{with, } f(x) = \frac{1 - 6x + 2x^3 + 3x^2(1 - \ln x)}{12(1-x)^2}.$$

We have studied three cases:

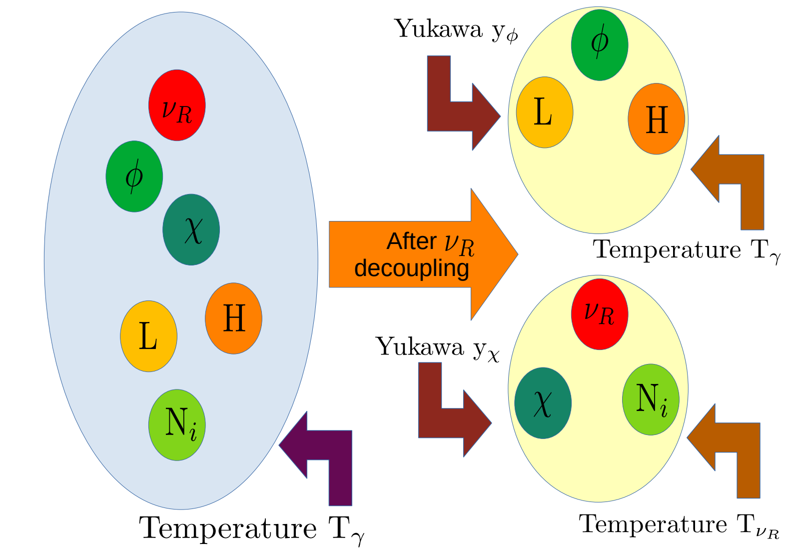
- Case-I: $y_\chi >> y_\phi$ and the mixing angle is tiny ($\sin\theta \leq 10^{-4}$).
- Case-II: Similar to previous case however, the mixing angle is large ($\sin\theta \sim 0.7$).
- Case-III: $y_\chi \simeq y_\phi$ and the mixing angle is fixed from neutrino mass bound.





Scenario for Case-I and Case-II ($y_\chi >> y_\phi$)

$$\mathcal{L} \supset (y_\phi)_{ij}\overline{L_i}\tilde{\phi}N_j + (y_\chi)_{ij}\overline{\nu_R}_i N_j \chi + ...$$



Working formulas

- **Dark matter**: There are two regions separated by $T_{\nu_R}^{\rm Dec}$.
  We defined a quantity $\xi = \frac{T_{\nu_R}}{T_\gamma}$ and the coupled Boltzmann equations as follows *[JCAP 10 (2021) 002]*

$$\frac{dY}{dx} = -\frac{1}{2}\frac{\beta s}{\mathbf{H} x}\langle\sigma v\rangle_{\rm eff}\Big[Y^2 - Y_{\rm eq}^2\Big], \quad (4)$$

$$x\frac{d\xi}{dx} + (\beta - 1)\xi = \frac{1}{2}\frac{\beta x^4 s^2}{4\alpha\xi^3\mathbf{H}M_0^4}\langle E\sigma v\rangle_{\rm eff}\Big[Y^2 - Y_{\rm eq}^2\Big]. \quad (5)$$

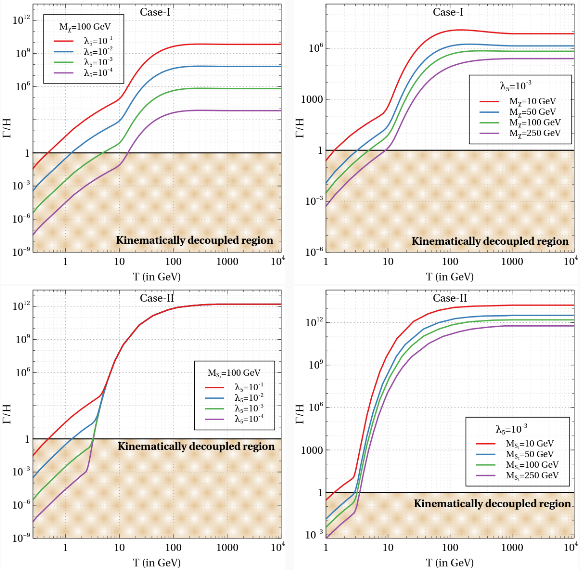
  The respective parameters are well defined.

- **For** $\mathbf{N_{\rm eff}}$**:**
  $N_{\rm eff} = \frac{\rho_{\rm rad} - \rho_\gamma}{\rho_{\nu_L}}$

$$\Delta N_{\rm eff} = \frac{\sum_\alpha \rho_{\nu_R^\alpha}}{\rho_{\nu_L}} = 3\left(\frac{\rho_{\nu_R}}{\rho_{\nu_L}}\right)\Big|_{T > T_{\nu_L}^{\rm Dec}} = 3 \times \left(\frac{T_{\nu_R}}{T_\gamma}\right)^4\Big|_{T > T_{\nu_L}^{\rm Dec}}$$

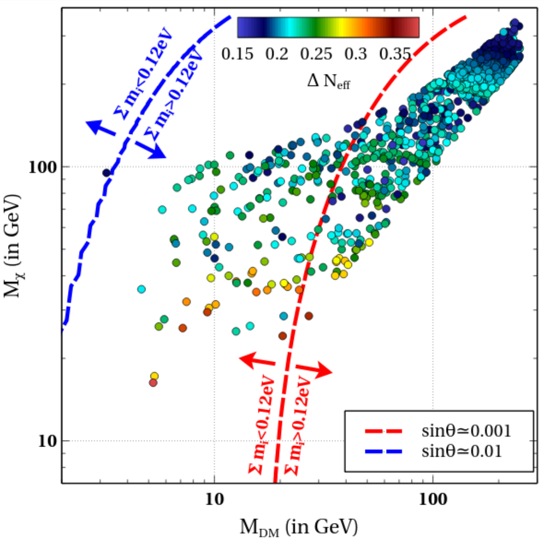
$$\implies \Delta N_{\rm eff} = 3 \times \xi^4. \qquad [\text{since}, \quad \rho \propto T^4] \quad (6)$$

Thermalization of ν_R



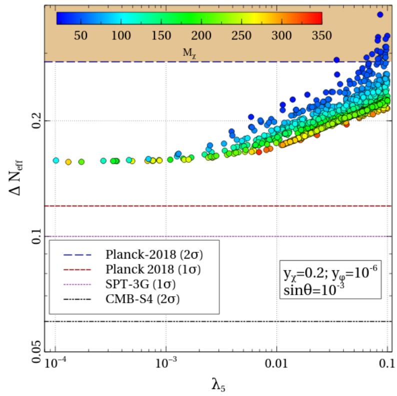
Thermalization profile for **case-I** is more prominent

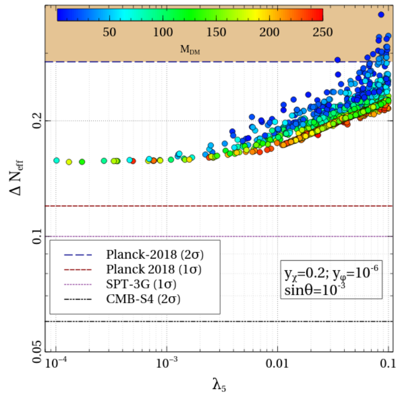
Results - Dark matter



Neutrino mass and dark matter co-relation in Case-I

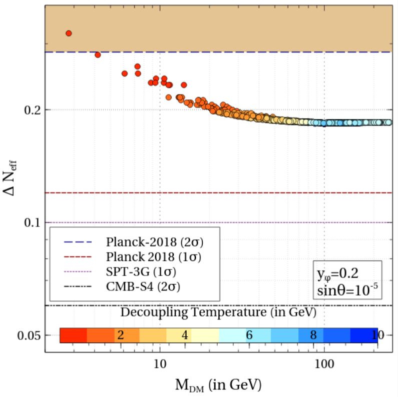
Results: ΔN_{eff}

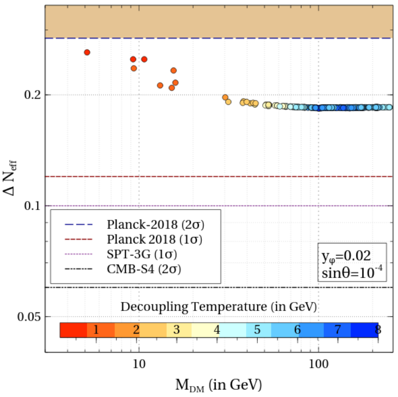




Cosmological signature of Case-I

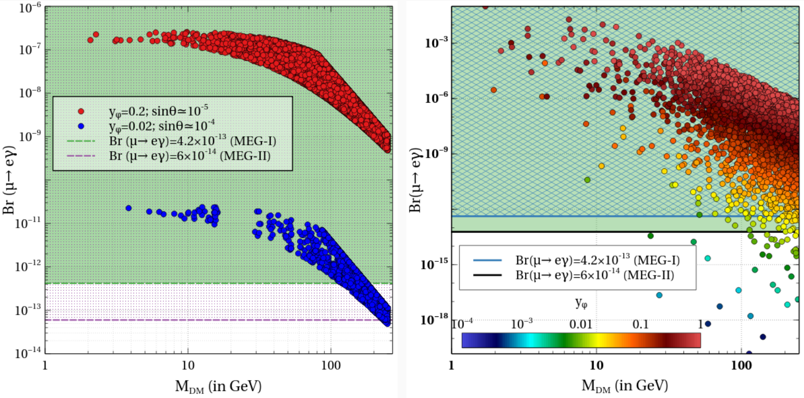
Results: $\Delta N_{\rm eff}$





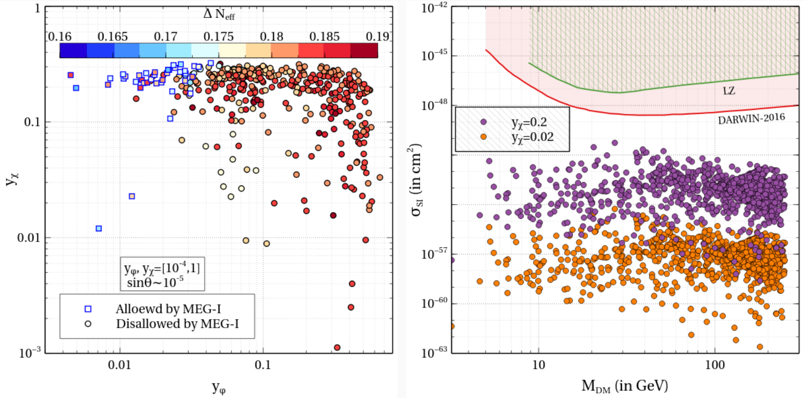
Cosmological signature of Case-III

Results: Lepton Flavour Violation



LFV restricts larger y_ϕ

Results: $\Delta N_{\rm eff}$ Direct Detection



Allowed parameter space from $\Delta N_{\rm eff}$ and direct detection bounds

Conclusion

- A minimal Dirac Scotogenic model was studied with a singlet scalar (χ), a doublet scalar (ϕ) and three massless right handed neutrinos (ν_R).
- The study was divided into three categories depending on Yukawa couplings and the mixing angle, consistent with neutrino mass.
- In every case, we discuss and show the detection prospects of the model while being consistent with the desired DM phenomenology and neutrino mass constraints.
- While direct detection prospects remain low for such fermion singlet DM due to radiative suppression of DM-nucleon scattering cross-section, some part of the parameter space is already ruled out by constraints from charged lepton flavour violation.

Thank you slide is under construction

Hope you enjoyed the talk.

The Model-II

After electroweak symmetry breaking, can be obtained as follows:

$$M_h^2 = 2\lambda_1 v^2; \quad (7)$$

$$M_{\phi^\pm}^2 = \mu_\phi^2 + \lambda_4 v^2; \quad (8)$$

$$M_{\phi_I}^2 = \mu_\phi^2 + (\lambda_4 + \lambda_7) v^2; \quad (9)$$

$$M_{\chi,\phi_R}^2 = \begin{pmatrix} \mu_\chi^2 + \lambda_5 v^2 & \sqrt{2}\mu v \\ \sqrt{2}\mu v & \mu_\phi^2 + (\lambda_4 + \lambda_7) v^2 \end{pmatrix}; \quad (10)$$

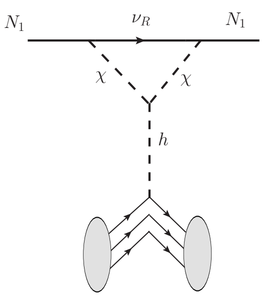
The contribution to the spin-independent scattering cross-section for the dark matter-nucleon scattering is given by

$$\sigma_{\rm SI} = \left(\frac{m_n}{v}\right)^2 \frac{\mu_{DMn}^2 g_{\overline{\psi}\psi h}^2}{\pi M_h^4} f_n^2 \qquad (11)$$

where $f_n = 0.3$ depends on the quark content within a nucleon for each quark flavour, $\mu_{\rm DMn} = \frac{M_{\rm DM} m_n}{M_{\rm DM}+m_n}$.

The effective coupling between DM and Higgs can be written as

$$g_{\overline{N}Nh} = \frac{i}{16\pi^2} \times \frac{y_\chi \lambda_5 v}{M_{\rm DM}} \times \left[1 + \left(\frac{M_\chi^2}{M_{\rm DM}^2} - 1\right) \ln\left(1 - \frac{M_{\rm DM}^2}{M_\phi^2}\right)\right]. \qquad (12)$$



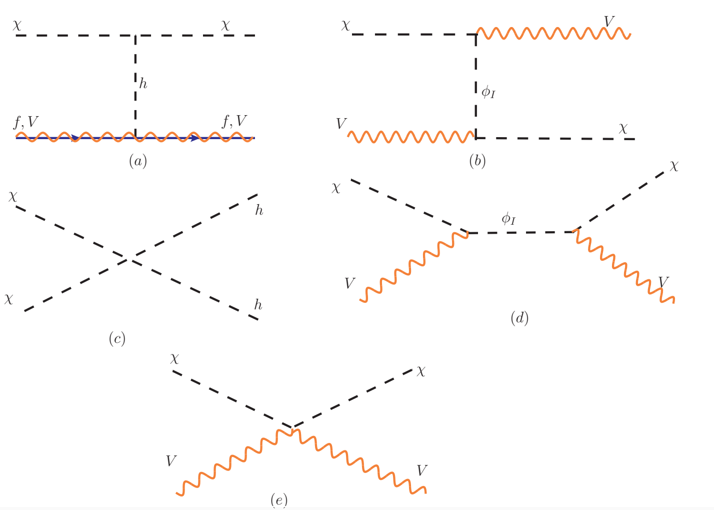


Figure 1: Scattering processes associated with thermalisation of χ with the SM bath.

Neutrino cosmology

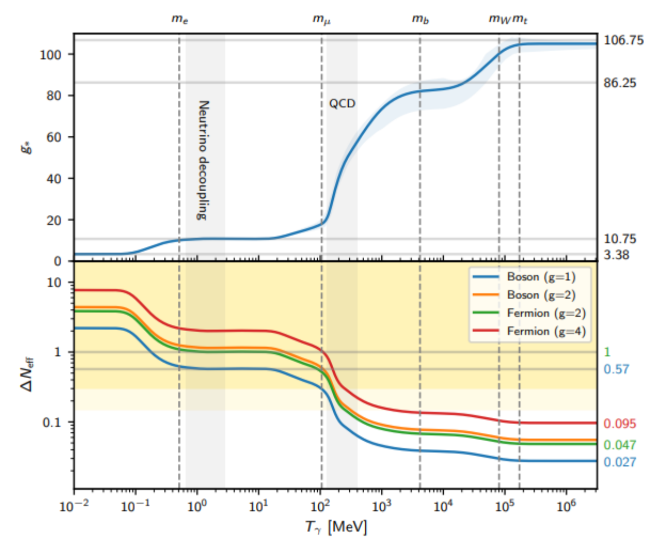
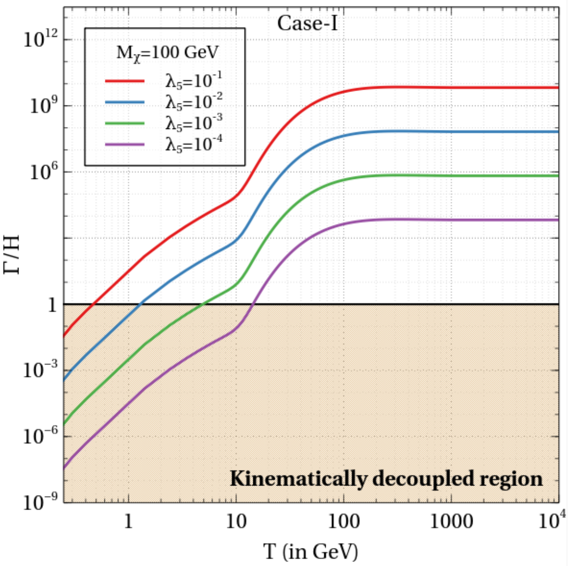


Image: Pritam, IITG

Planck 2018



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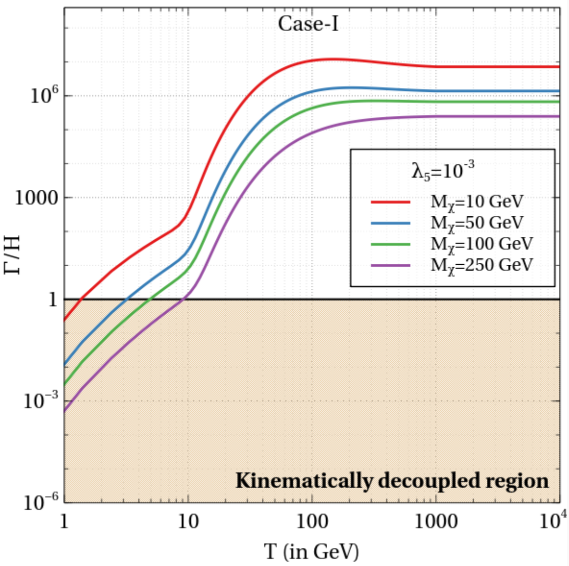

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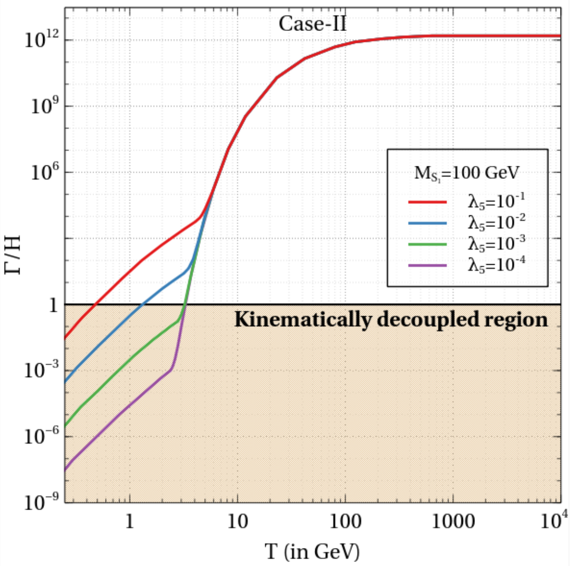

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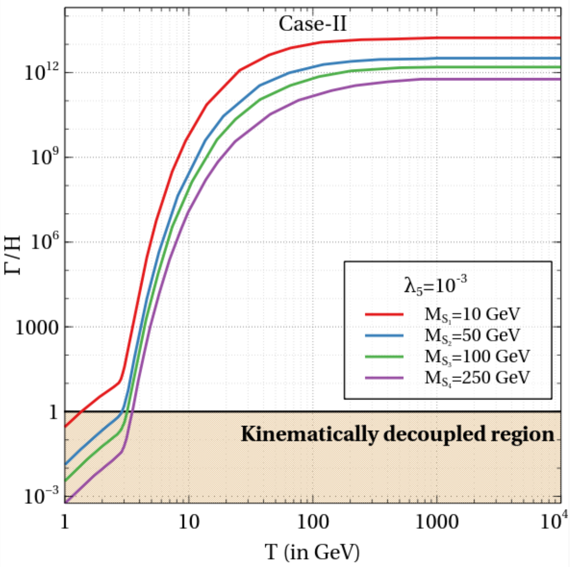

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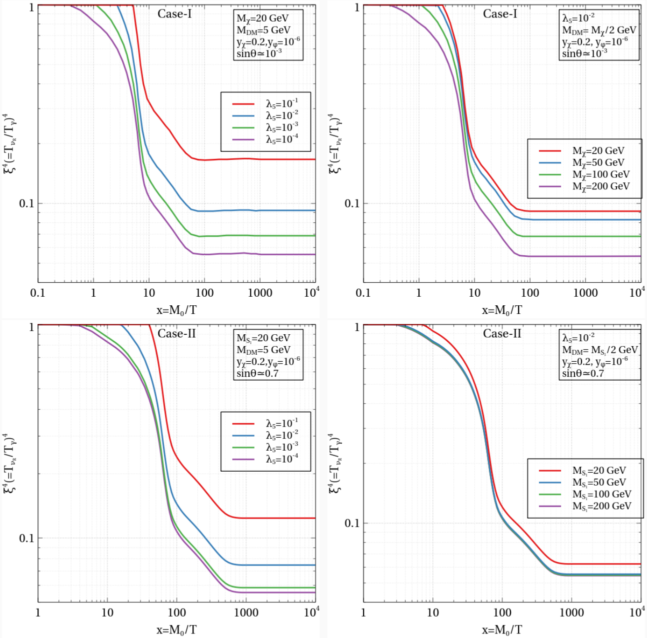
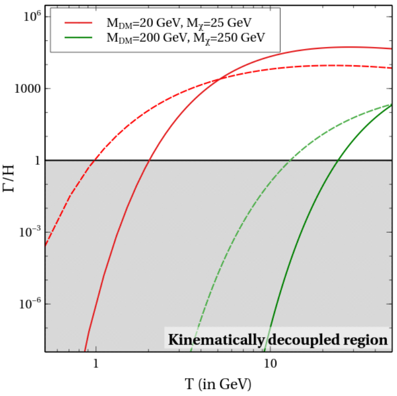
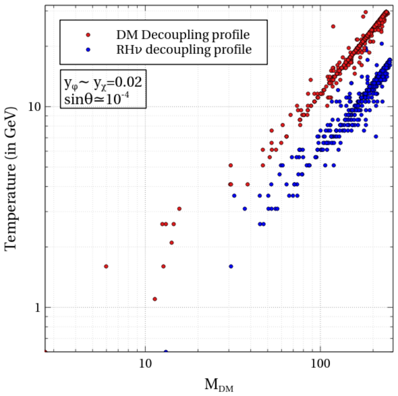

Case-I
M_χ=20 GeV
M_{DM}=5 GeV
y_χ=0.2, y_φ=10^{-6}
sinθ≃10^{-3}
λ_5=10^{-1}
λ_5=10^{-2}
λ_5=10^{-3}
λ_5=10^{-4}
$\xi^4(=T_{\nu_R}/T_\gamma)^4$
x=M_0/T
Case-I
λ_5=10^{-2}
M_{DM}= M_χ/2 GeV
y_χ=0.2, y_φ=10^{-6}
sinθ≃10^{-3}
M_χ=20 GeV
M_χ=50 GeV
M_χ=100 GeV
M_χ=200 GeV
Case-II
M_{S_1}=20 GeV
M_{DM}=5 GeV
y_χ=0.2, y_φ=10^{-6}
sinθ≃0.7
Case-II
λ_5=10^{-2}
M_{DM}= M_{S_1}/2 GeV
y_χ=0.2, y_φ=10^{-6}
sinθ≃0.7
M_{S_1}=20 GeV
M_{S_1}=50 GeV
M_{S_1}=100 GeV
M_{S_1}=200 GeV

Backup slides-Thermalization in case-III





Backup Slides- parameters in BE

We have defined the effective thermal averaged cross-section as

$$\langle E\sigma v\rangle_{\rm eff} = \frac{\langle E\sigma v\rangle'_{\nu_R\bar{\nu_R}\to {\rm DM}\overline{\rm DM}}(Y^{\rm eq}_{\rm DM})^2 + \langle E\sigma v\rangle'_{\nu_R\bar{\nu_R}\to\chi\chi}(Y^{\rm eq}_{\chi})^2}{(Y^{\rm eq}_{\rm DM}+Y^{\rm eq}_{\chi})^2}, \qquad (13)$$

where, $\langle E\sigma v\rangle'_{x\bar{x}\to y\bar{y}}$ is the thermal average of $E\times\sigma v_{x\bar{x}\to y\bar{y}}$ normalized by the product of equilibrium number densities of the final state particles *i.e.*, $n^{eq}_y n^{eq}_{\bar{y}}$.

$\alpha = g_i\frac{7}{8}\frac{\pi^2}{30}$; $s(T) = g_*(T)\frac{2\pi^2 T^3}{45}$; $\mathbf{H}(T) = \sqrt{\frac{8g_*(T)}{\pi}}\frac{T^2}{M_{\rm Pl}}$;

$\beta(T) = \frac{g_*^{1/2}(T)\sqrt{g_\rho(T)}}{g_s(T)}$;

$g_*^{1/2} = \frac{g_s}{\sqrt{g_\rho}}\left(1+\frac{1}{3}\frac{T}{g_s}\frac{dg_s}{dT}\right)$.

The effective annihilation cross-section for the combined processes are given by *[Phys. Rev. D 43 (1991) 3191]*

$$\langle\sigma v\rangle_{\rm eff} = \frac{\langle\sigma v\rangle_{{\rm DM}\bar{\rm DM}\to\nu_{\rm R}\bar{\nu_{\rm R}}}(Y^{eq}_{\rm DM})^2 + \langle\sigma v\rangle_{\chi\chi\to X\bar{X},\nu_R\bar{\nu_R}}(Y^{\rm eq}_{\chi})^2}{(Y^{\rm eq}_{\rm DM}+Y^{\rm eq}_{\chi})^2}. \qquad (14)$$

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Cases	Parameters	Chosen range
All	Dark matter mass	$M_{\rm DM} = [1, 250]$ GeV
	Singlet scalar mass	$M_\chi = [1, 350]$GeV
	Doublet masses (For $\phi^\pm, \phi^0$)	$M_\phi = [1, 350]$ GeV
	Mass separation ($M_i - M_{\rm DM}$)	$\Delta M = [1, 100]$ GeV
	Higgs portal coupling	$\lambda_5 = [10^{-4}, 10^{-1}]$
Case-I &Case-II	Yukawa coupling with χ	$y_\chi = 0.2$ (fixed)
	Yukawa coupling with ϕ	$y_\phi \simeq 10^{-6}$ (fixed)
	Mixing angle (Case-I)	$\sin\theta \sim 10^{-6}$
	Mixing angle (Case-II)	$\sin\theta \sim 10^{-2}$
Case-III	1. Yukawa coupling with χ & ϕ	$y_\chi \sim y_\phi = 0.2$ (fixed)
	Mixing angle	$\sin\theta \sim 10^{-5}$
	2. Yukawa coupling with χ & ϕ	$y_\chi \sim y_\phi = 0.02$ (fixed)
	Mixing angle	$\sin\theta \sim 10^{-4}$

Table 1: The choice of parameter ranges for all three cases in our study.