

UV/IR flavour connection in axion models

Invisibles23 Workshop

Xavier Ponce Díaz

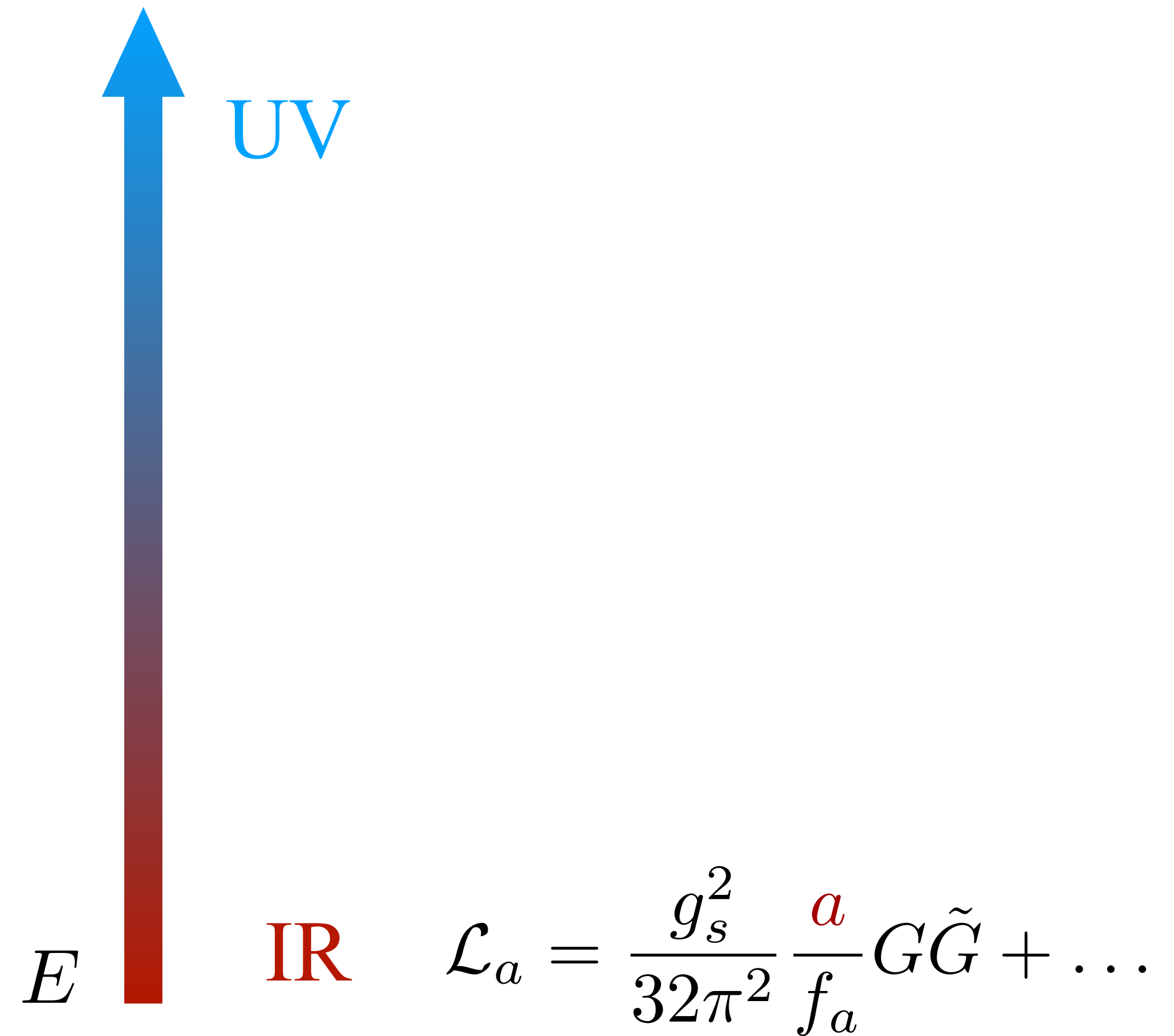
based on: JHEP 06 (2023) 046, [arXiv:2304.04643](https://arxiv.org/abs/2304.04643)
with Luca Di Luzio, Alfredo Guertera, Stefano Rigolin



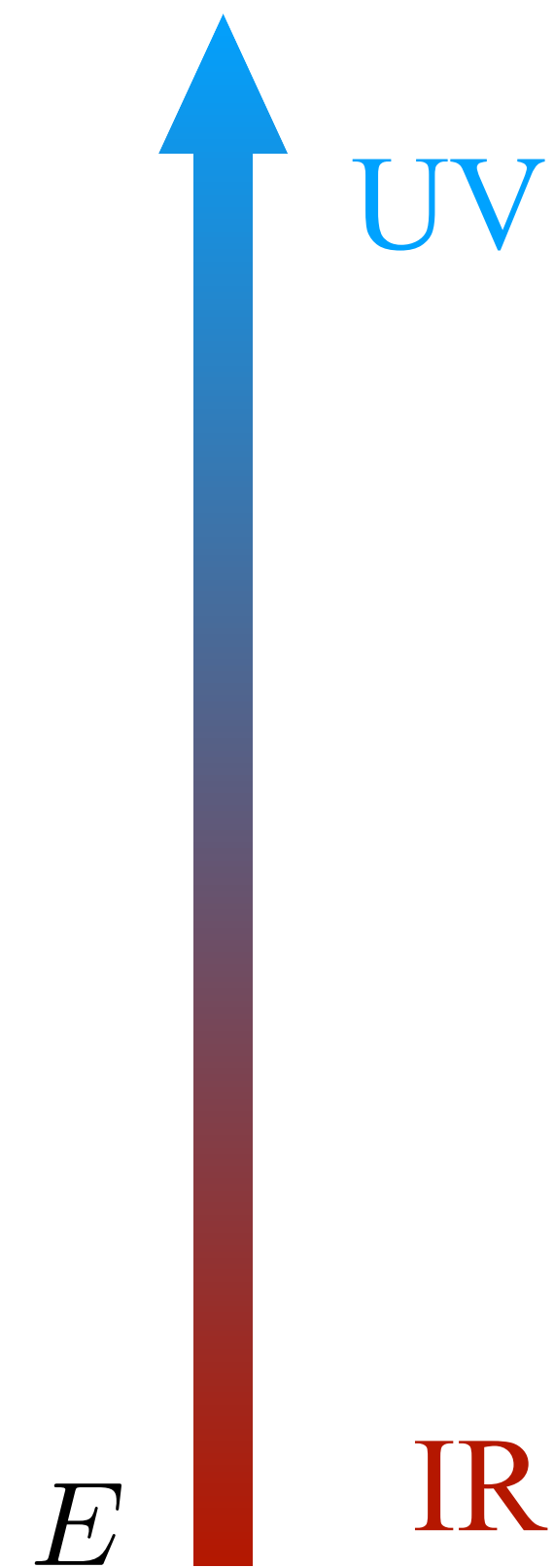
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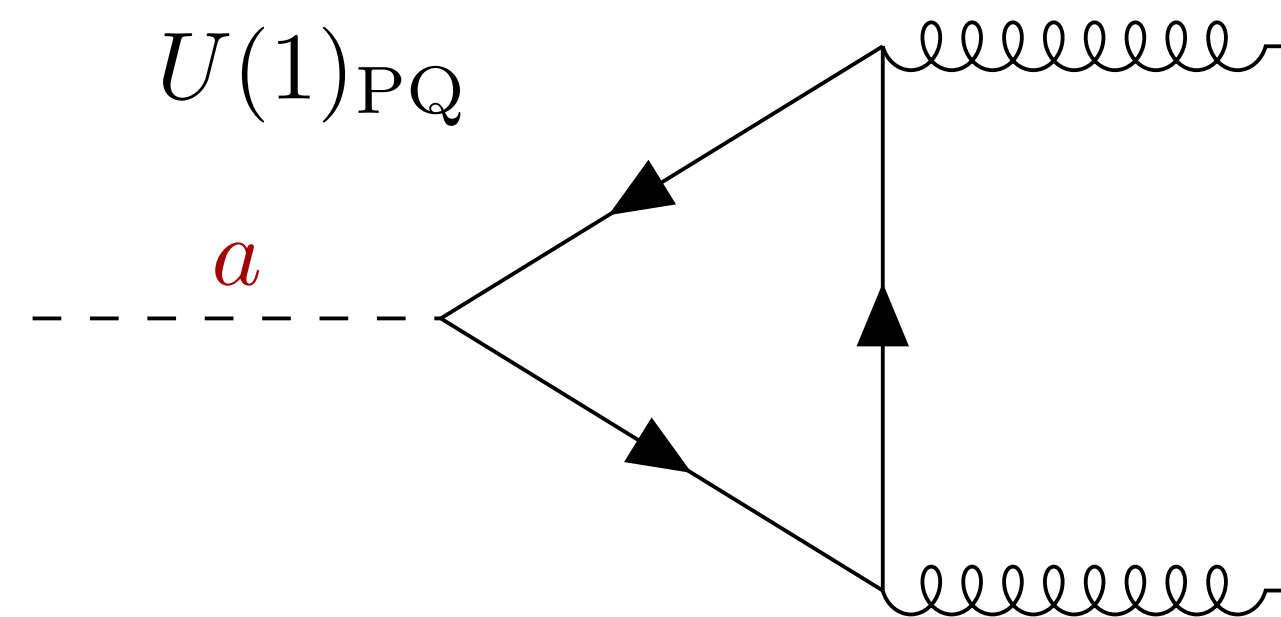


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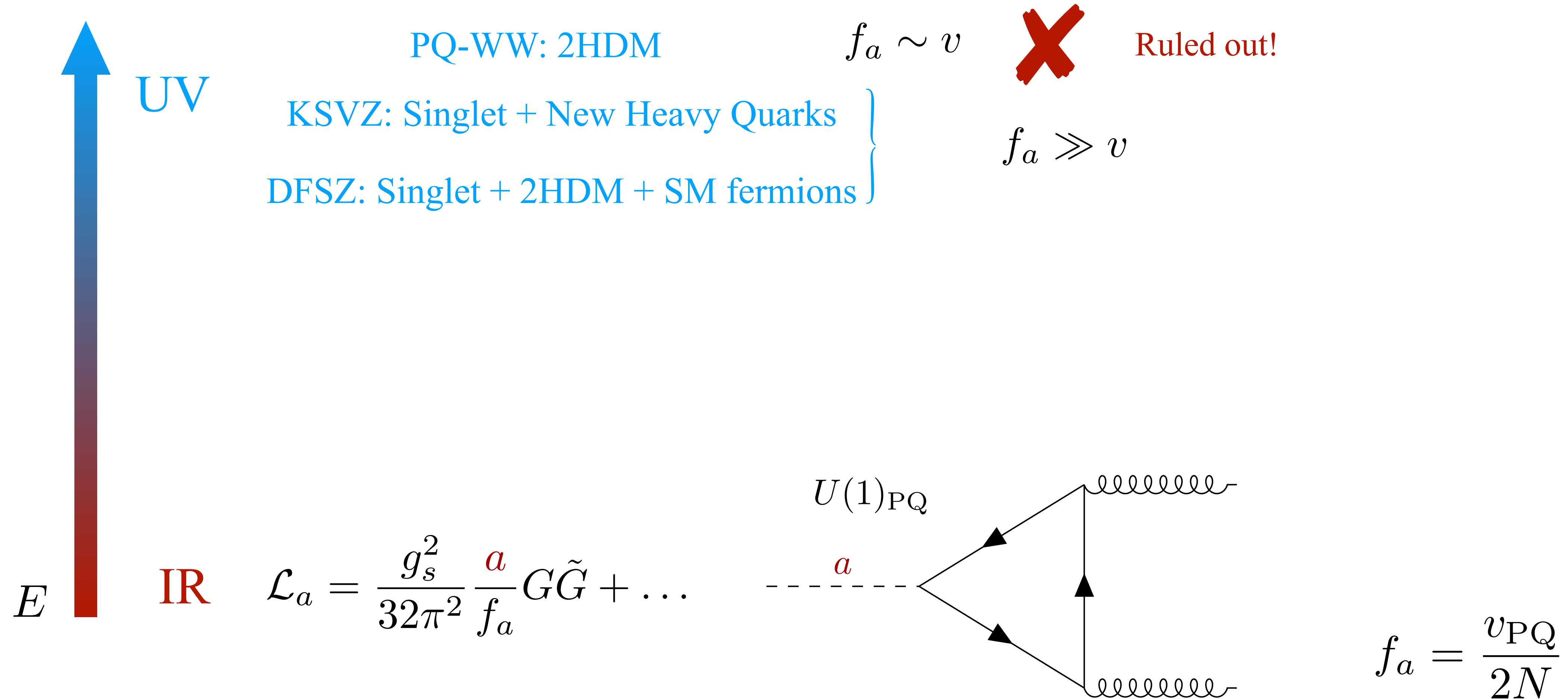
IR

$$\mathcal{L}_a = \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G\tilde{G} + \dots$$

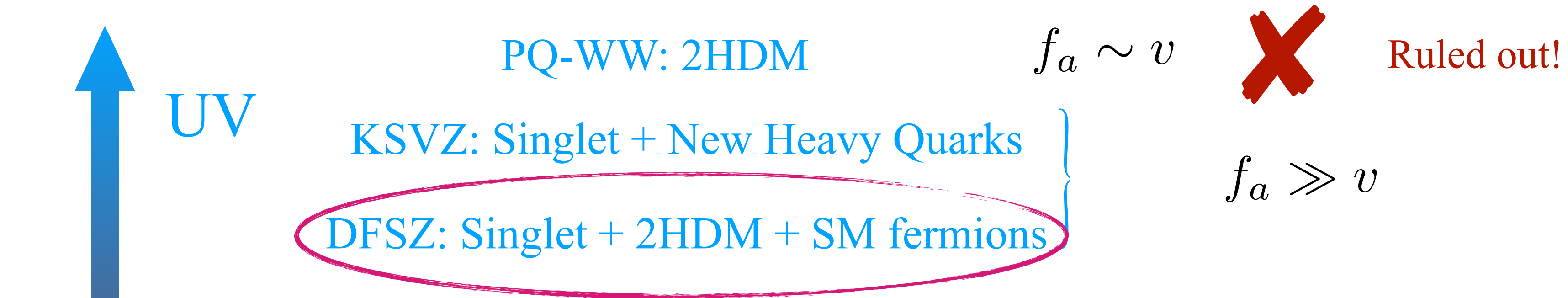


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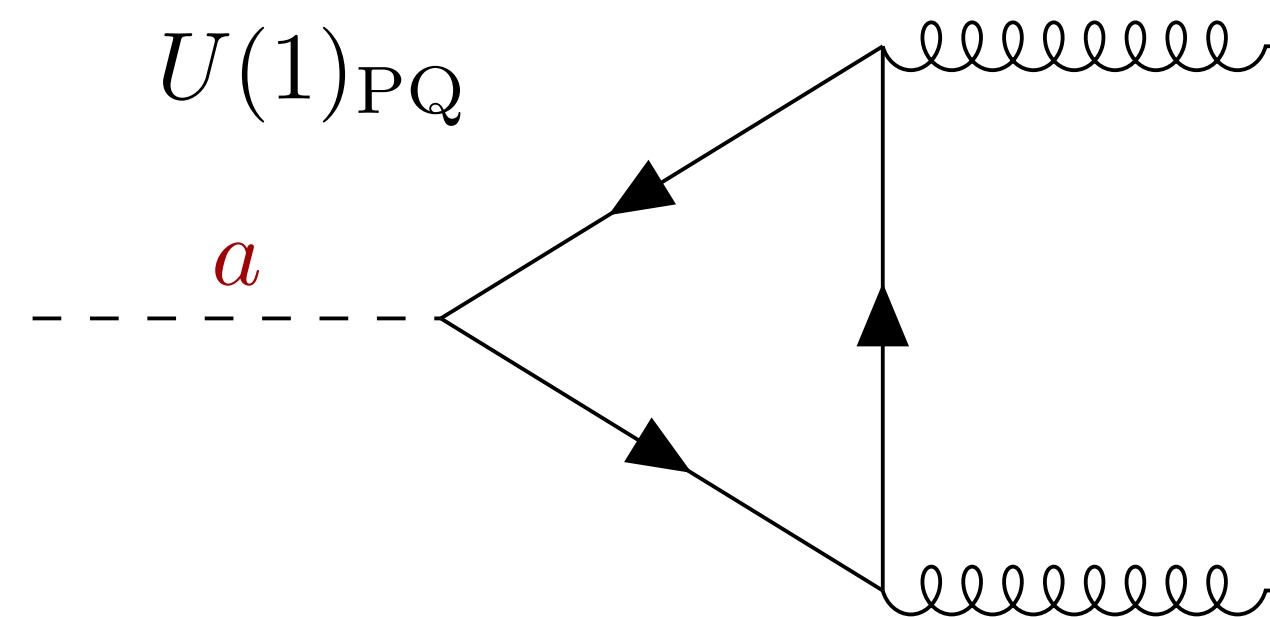


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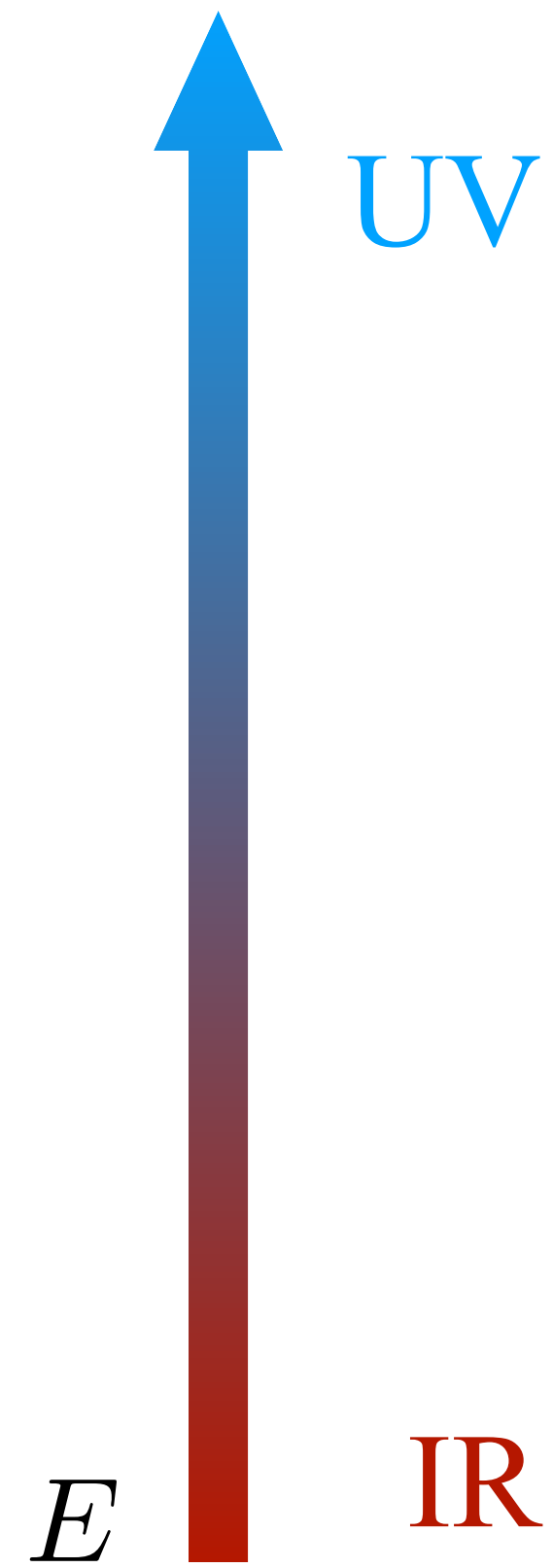
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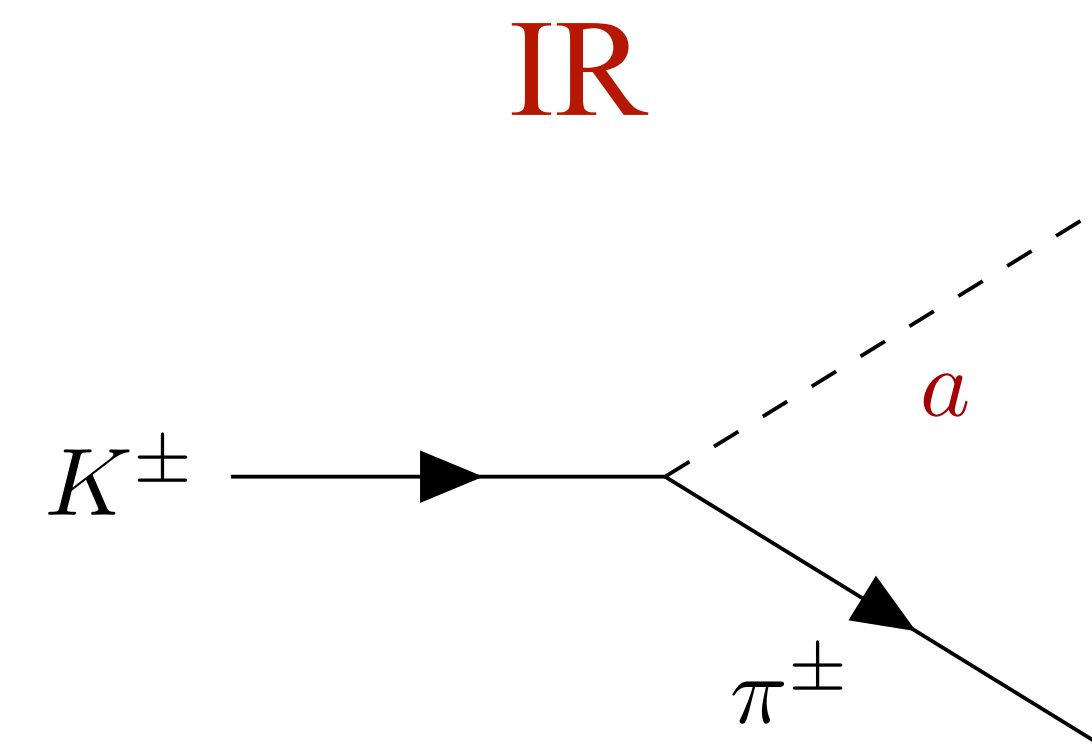
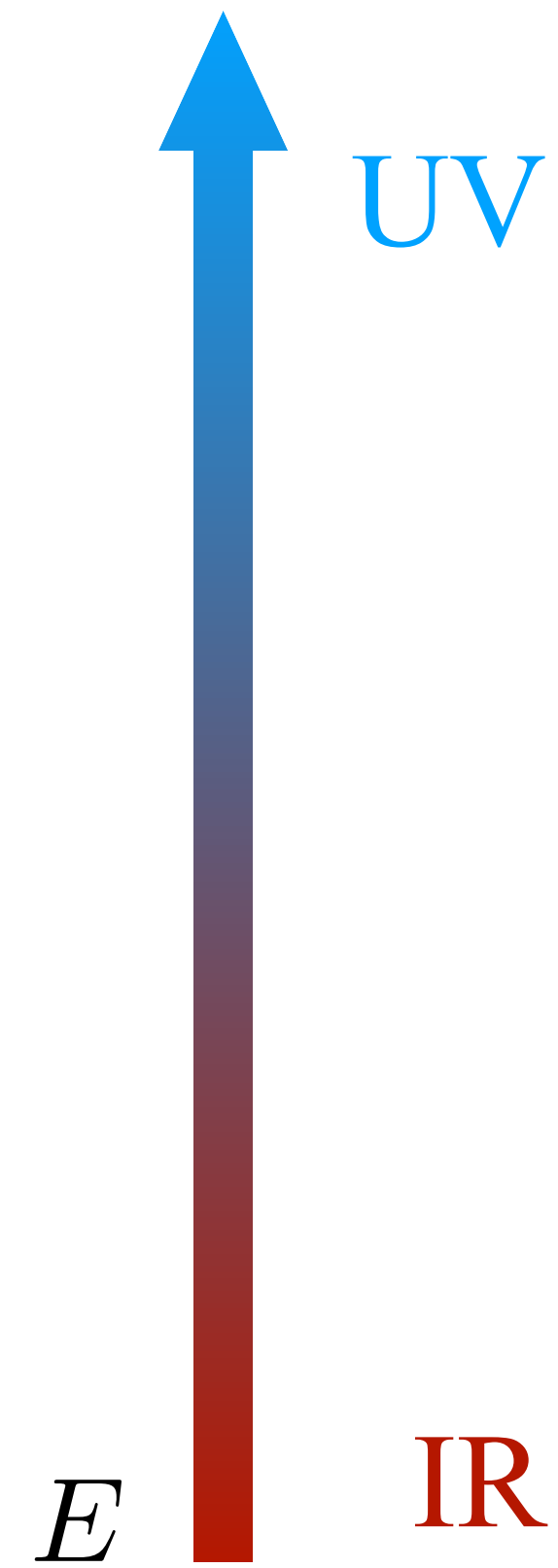


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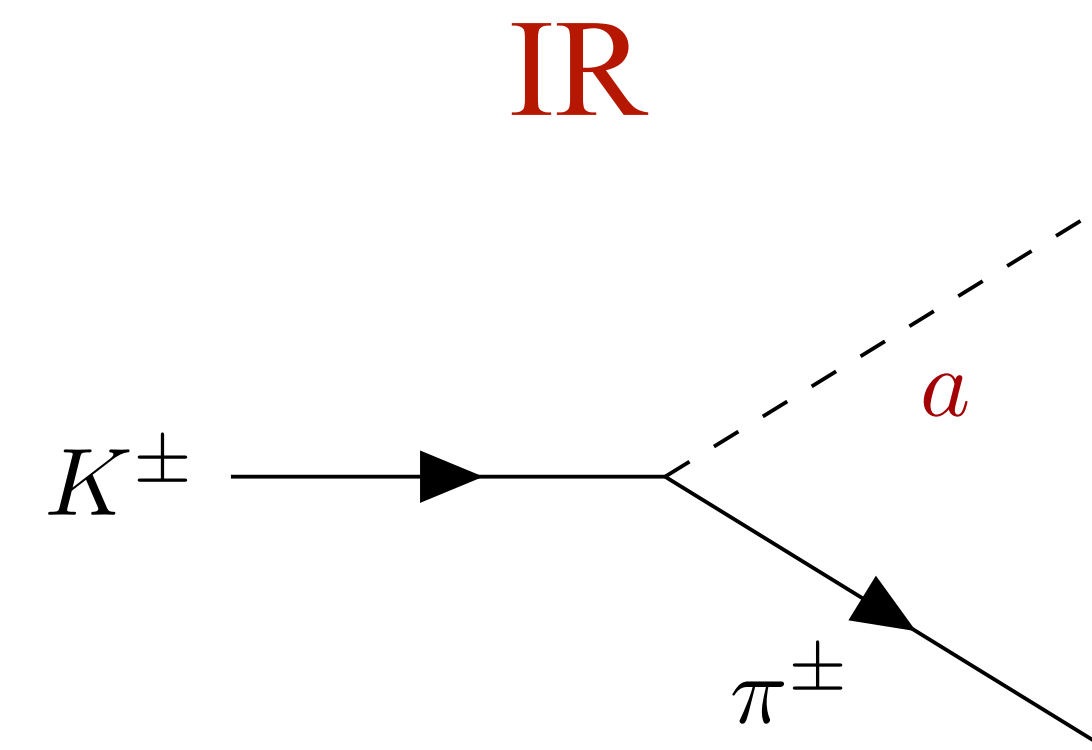
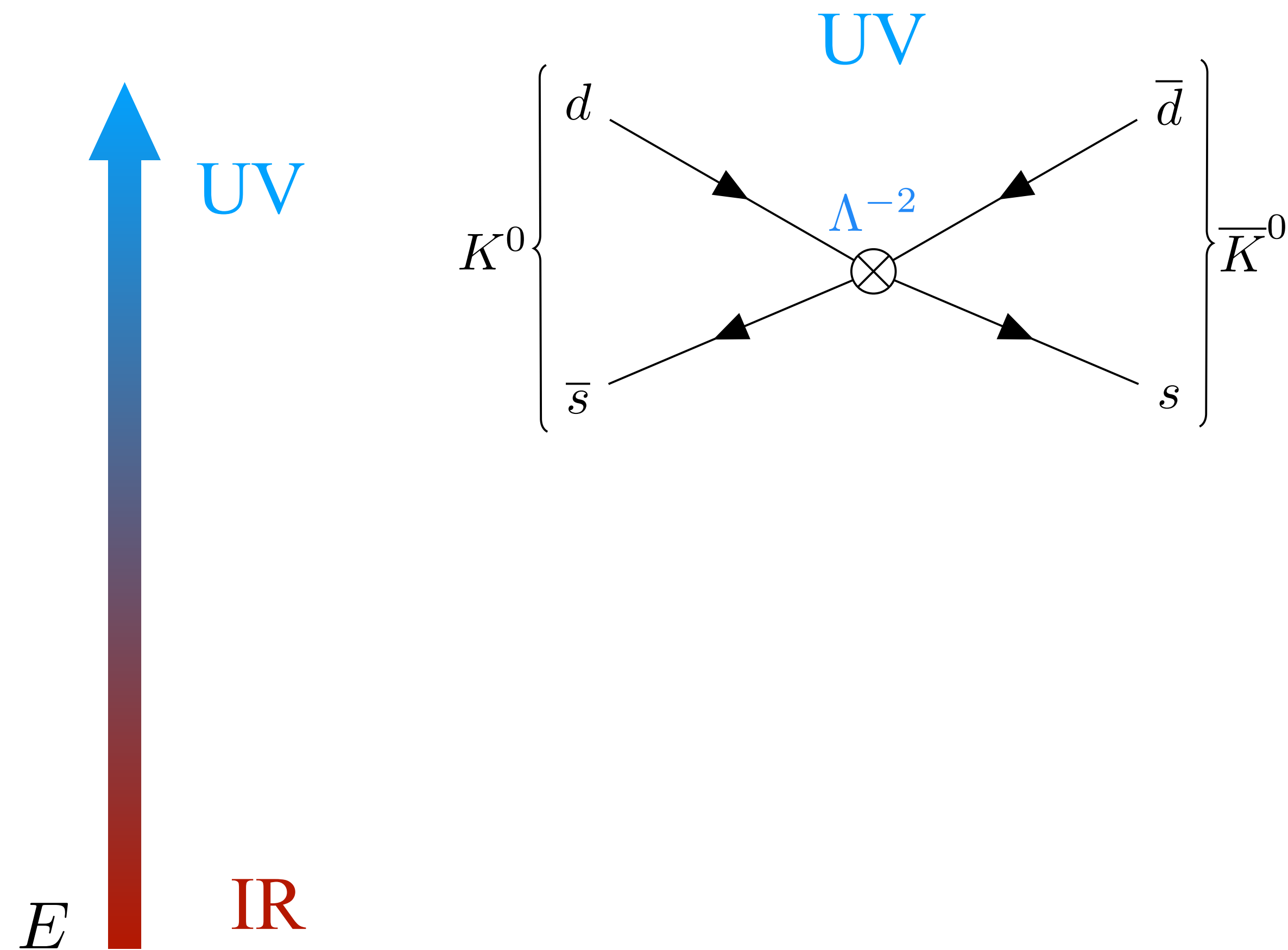
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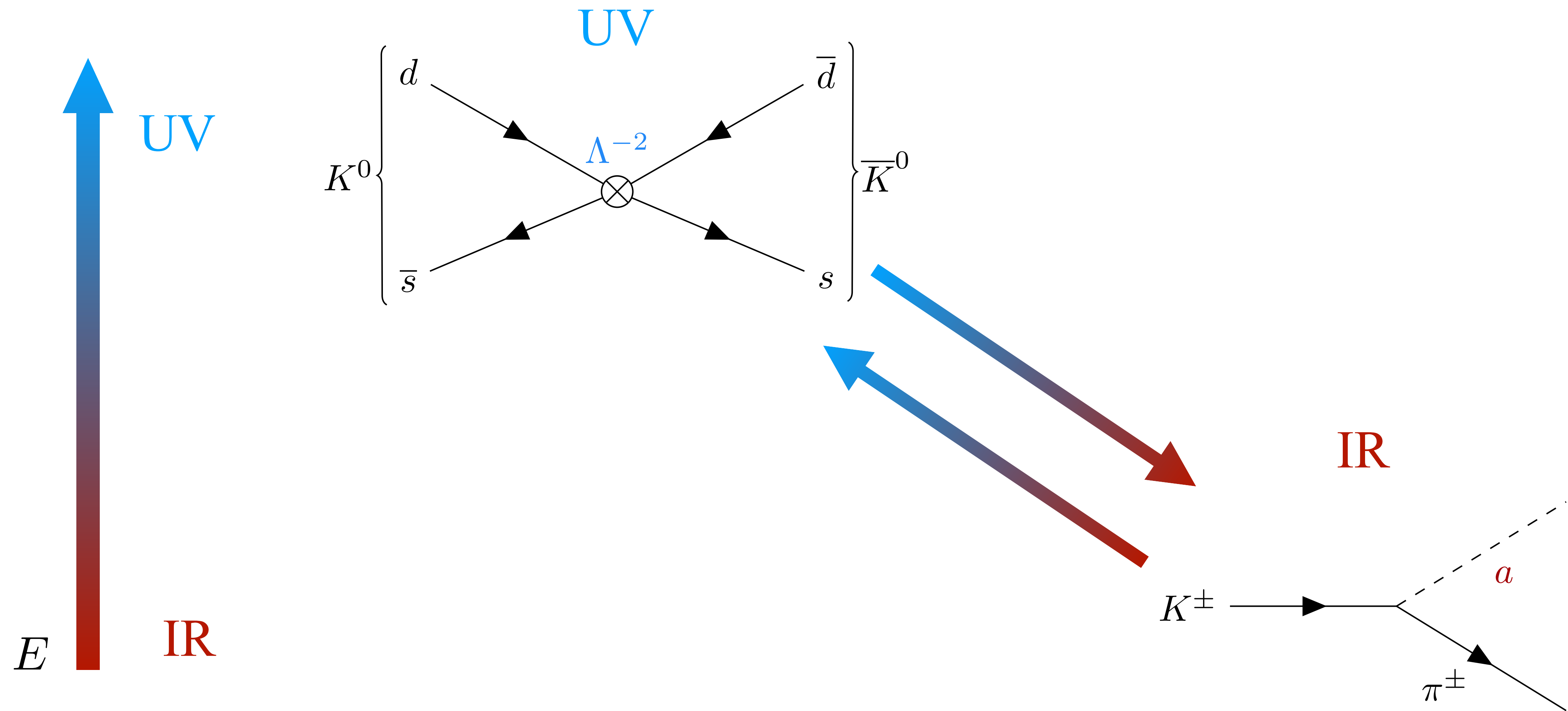
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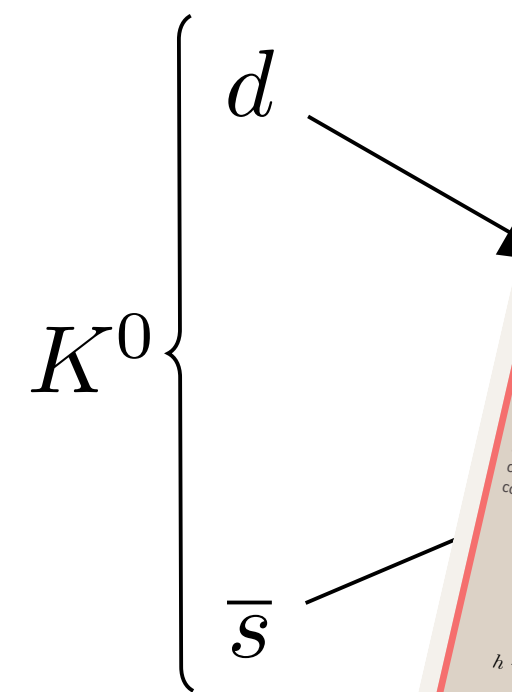
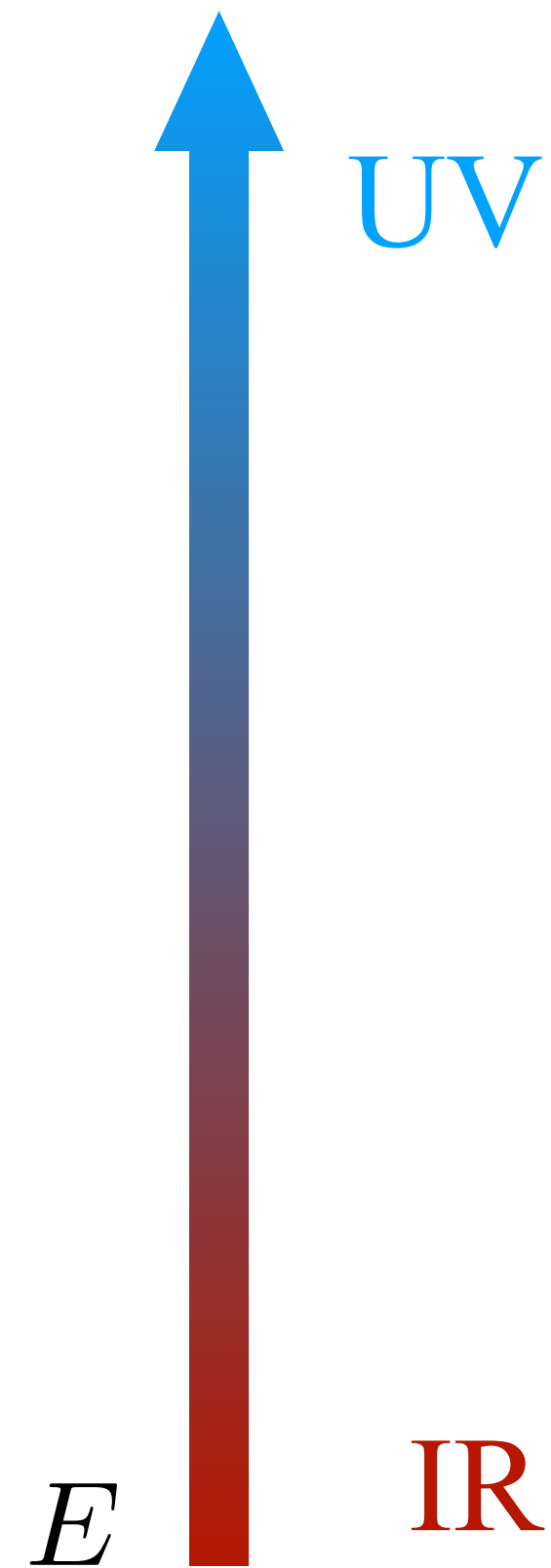
What is this talk about?



What is this talk about?



What is this talk



UV-IR Interplay in Flavoured Axion Models
 Ponce Díaz Xavier | INFN Padova & University of Padova
 Based on JHEP 06 (2023) 046 with Luca Di Luzio, Alfredo Guerrero and Stefano Rigolin

I. Introduction
 Among the unanswered questions of the Standard Model (SM), the strong CP problem has become one of the most studied. This is due to the popularity of the Peccei-Quinn (PQ) solution, in form of an axion in the low energy theory. This pseudo-Goldstone Boson, that obtains its mass due to the low energy theory, not only solves the strong CP problem but also makes for a good dark matter candidate [1]. In order to generate such a particle we need to extend the SM by adding new heavy degrees of freedom:

KSVZ: new heavy quarks + Singlet
 $C_{KSVZ} \supset - \sum_i m_{Q_i} \bar{Q}_i Q_i + \dots$

DFSZ: SM fermions charged + 2HDM + Singlet
 $C_{DFSZ} \supset - \sum_f \bar{\psi}_f \psi_f + \dots$

UV: $C_{UV}^{Q,2HDM} = - \bar{q}_L (Y_1^q H_1 + Y_2^q H_2) u_R + \bar{q}_L (Y_1^q H_1 + Y_2^q H_2) d_R + \bar{l}_L (Y_1^l H_1 + Y_2^l H_2) e_R + \text{h.c.}$

IR: $C_{IR} \supset \frac{g_a^2}{32\pi^2} a G\tilde{G} + \sum_{I=A,B} \frac{g_a^2}{2f_a} \bar{\psi}_I \gamma^5 \psi_I + (C_{ij}^A)_{ij} \bar{\psi}_i \psi_j + (C_{ij}^B)_{ij} \bar{\psi}_i \psi_j$

II. Flavour Connection
 As an example we will take a look at the lepton sector. The quark sector will be presented at the Workshop Invisibles 23 or can be found in the paper.

Flavour Violation in the IR:
 The flavour violation in the axion Lagrangian is achieved if the PQ-charges are non-universal:
 $C_{e\alpha} = \frac{1}{2N} V_{e\alpha}^* X_{e\alpha} V_{e\alpha}, C_{e\alpha} = \frac{1}{2N} V_{e\alpha}^* X_{e\alpha} V_{e\alpha}$

Condition for axion flavour violation:
 $X_{e\alpha} \neq X_{e\beta}, X_{e\alpha} \neq X_{e\beta}$

Flavour Violation in the UV:
 In a 2HDM diagonalising the mass matrix does not always imply diagonalising also the interactions. This is due to the mass matrix and the couplings being different combinations of the Yukawas:
 $C_{ij}^{Y_1} = \bar{l}_i (Y_1^l H_1 + Y_2^l H_2) e_{Rj} = -\bar{l}_i X_{ij} (M_{e\alpha} \delta_{ij} + C_{ij}^H H + C_{ij}^A A) e_{Rj} + \text{h.c.}$

Flavour Connection:
 However, the shape of the Yukawas is given by the charges. Imposing PQ-charge conservation we obtain the following condition: $-X_{12}^* X_{12} + Y_{12}^* X_{12} - X_{12} Y_{12} = 0$. If we combine it with the definition of the mass matrix we obtain:
 $V_{e\alpha}^* V_{e\alpha} = \frac{2\sqrt{2}N}{v_{\beta\gamma}} (C_{e\alpha} M_e - M_e C_{e\alpha} + X_{e\alpha} M_e), V_{e\alpha}^* Y_{e\alpha} = \frac{\sqrt{2}}{v_{\beta\gamma}} (-C_{e\alpha} M_e + M_e C_{e\alpha} - X_{e\alpha} M_e)$

III. An example with Lepton Flavour Violation
 Now the idea is to link lepton flavour violating (LFV) observables of the axion with UV observables related to the 2HDM. In Ref. [2] this was also done but in a model dependent way, here we have shown there is a model independent way of doing it!

Processes involving LFV are great proofs of new physics, in 2HDM there are many however, keeping at tree level to have a direct one-to-one correspondence, the Higgs LFV decays are currently leading in the bounds. Fig. 1 (left). Using the connection with the Yukawas obtained before we can write the branching ratio corresponding to the process of Fig. 1 (left) as

$BR(h \rightarrow l_i l_j) \approx \frac{m_h}{10\pi^2 f_a} \left(\frac{c_{\alpha-\beta}}{s_{\beta\gamma}} \right)^2 \frac{m_l^2}{v^2} \frac{1}{\alpha} \left((C_{ij}^L R)_{ij} \right)^2$

The analog process would be a lepton, decaying into an axion and a different flavour lepton, as depicted in Fig. 1 (right). The branching ratio of such a process can be written as

$BR(l_i \rightarrow l_j a) \approx \frac{m_l^2}{16\pi^2 f_a} \frac{1}{2f_a^2} \left((C_{ij}^L R)_{ij} \right)^2$

It is straightforward to link both observables getting rid of the unknown couplings:

$BR(h \rightarrow l_i l_j) \approx BR(h \rightarrow a l_j) \frac{2m_h v_l f_a^2}{m_l v^2} \left(\frac{c_{\alpha-\beta}}{c_{\beta\gamma}} \right)^2$

Some recent very small deviations have been observed by ATLAS [4], and so far compatible with CMS.

$BR(h \rightarrow \tau\mu) = 0.11^{+0.05}_{-0.04} \%$ $BR(h \rightarrow \tau e) = 0.09 \pm 0.06 \%$

while the current leading bounds at 2σ for leptonic flavour violating decays into invisible particles are given by Belle II [5]

$BR(\tau \rightarrow \mu a) < 5.9 \times 10^{-4}$ $BR(\tau \rightarrow e a) < 9.4 \times 10^{-4}$

The combination of all the bounds shows the interplay between astrophysical observations and collider bounds. Thanks to the link between the UV and IR flavour violation we see that these results do not depend on any particular assumption on the Yukawas and fermion mixing matrices.

This can be seen in Fig. 2, where we take the the anomalies seen by ATLAS [4] and impose the bounds coming from Belle II [5]. On top of these we impose the astrophysical and 2HDM bounds. These bounds are model dependent, and that's why we take different β cases that are assumed to be limiting: in axion models one can obtain so called astrophysical axions by choosing certain set of charges and some tuning [6]; in the 2HDM we take the typical types I and II models which give an idea on the current bounds for the mixing angle $\cos(\alpha - \beta)$.

This is just an example of how it would look combining several measurements, using these anomalies seen by ATLAS as benchmarks. However, the main point is that with these relations, the UV and IR sectors can talk to each other and share information, identify whether it could correspond to an axion model of the DFSZ type, and help narrowing down the parameter space.

IV. Conclusions
 The axion and 2HDM particles both theoretically and experimentally. This work shares some of the same properties with LFV, but on an example in the quark sector can be found in [7].

ther an axion model is compatible with this work. Here in examples of how this could be done with LFV, but on an example in the quark sector can be found in [7].

Outlook
 In the future we may explore this connection in more generality including one loop observables which are important, such as $\mu \rightarrow e \gamma$ and $\mu \rightarrow e \tau$ leptons. Moreover we would like to consider contributions from quarks and leptons. Moreover we would like to implement this connection into a global fit taking into account axion and 2HDM observables.

Flavour connection
 This flavour connection is useful in the current state of experiments, both for looking for axions and new heavy physics, these connections might be useful in an eventual discovery, helping in discerning whether the axion model is compatible with this work.

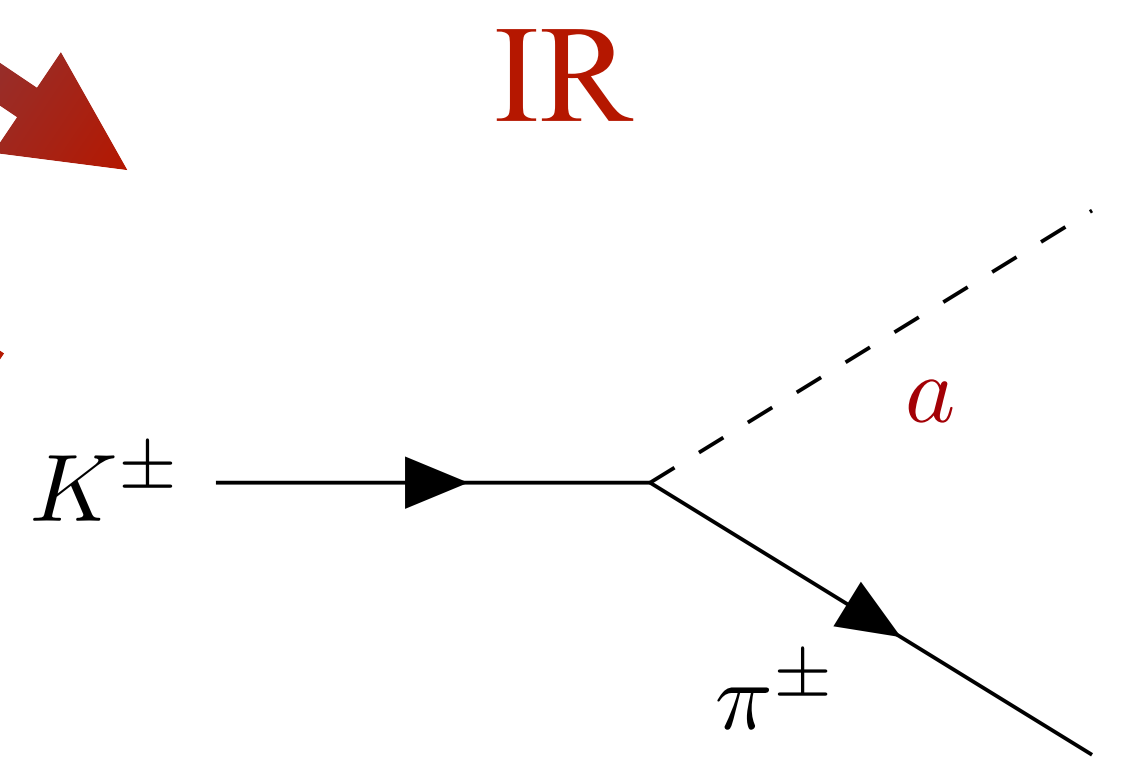
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 [1] L. Di Luzio, M. Giannotti, E. Nardi, L. Visinelli, Phys. Rept. 879 (2020) 1-117, arXiv:2003.01100 [hep-th].
 [2] M. Bzdak, G. Grilli, M. Trott, and R. Ziegler, JHEP 10 (2021) 161, arXiv:2107.09708 [hep-ph].
 [3] A. Celis, A. Koku, and G. Grilli, Phys. Rev. D 103 (2021) 075011, arXiv:2003.08777 [hep-ph].
 [4] ATLAS, JHEP 07 (2022) 189, arXiv:2202.06225 [hep-ex].
 [5] Belle II, Phys. Rev. Lett. 130, 181803 (2023), arXiv:2212.03634 [hep-ex].
 [6] L. Di Luzio et al. Phys. Rev. Lett. 120, 261803 (2018), arXiv:1712.04940 [hep-ph].
 [7] L. Di Luzio, A. Guerrero, X. Ponce Díaz, S. Rigolin, JHEP 06 (2023) 046, arXiv:2304.04843 [hep-ph].

Take home messages:
 • No assumption on the charges is needed.
 • Before we had $V_{e\alpha}^* V_{e\alpha}, C_{ij}^H$ & C_{ij}^A now we see that the Yukawas can be written in terms of the axion couplings, independently of any assumption on the Yukawas, valid for any 2HDM with a global $U(1)$.

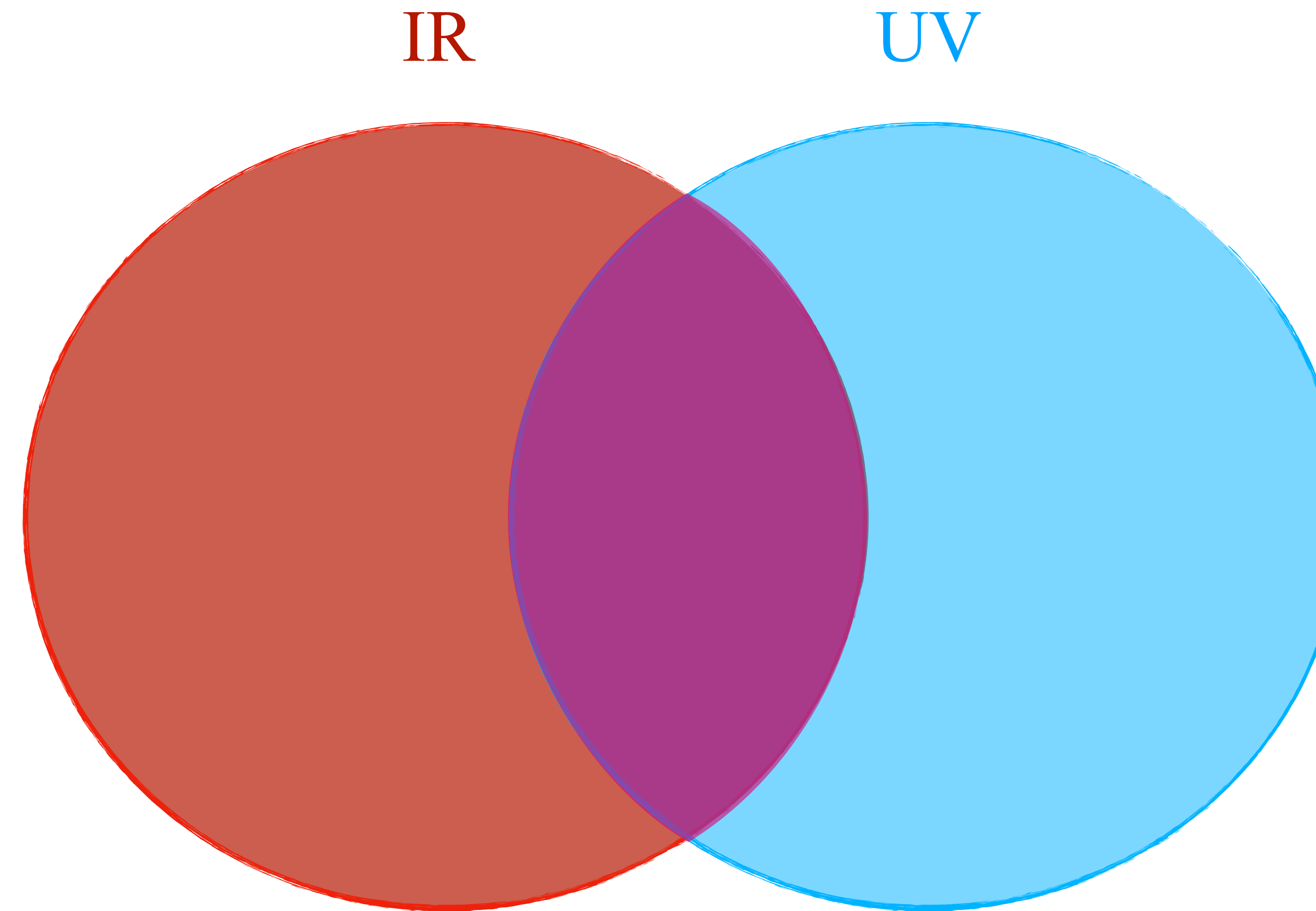
Figure 1: Diagrams involving lepton flavour violating decays. (left) Higgs decaying into a pair of different flavour leptons and (right) a lepton decaying into an axion and a lepton with different flavour.

Figure 2: Parameter space of combining observables of Higgs LFV decays level leptons decaying into a different flavour and an axion. (top) Using observables corresponding to the 3:1 transition and (bottom) using the 3:2 transition.

Come to the poster to see the Lepton Flavour violation part!



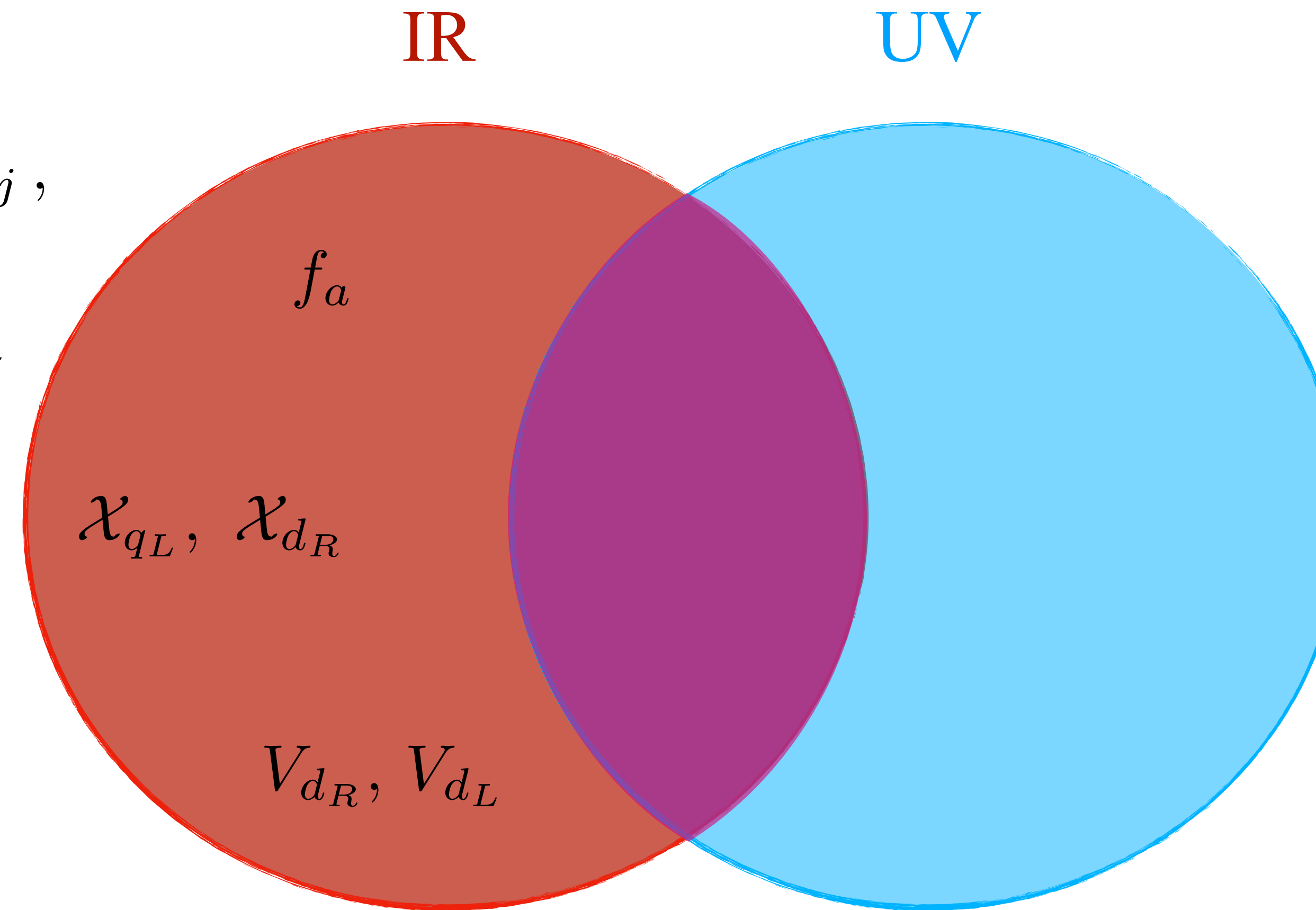
Flavour-violating axion



Flavour-violating axion

In the IR:

$$\mathcal{L}_a \supset \frac{\partial_\mu a}{f_a} \bar{d}_i \gamma^\mu \left((C_{d_L})_{ij} \gamma^\mu P_L + (C_{d_R})_{ij} \gamma^\mu P_R \right) d_j ,$$
$$C_{d_L} = \frac{1}{2N} V_{d_L}^\dagger \mathcal{X}_{q_L} V_{d_L} , \quad C_{d_R} = \frac{1}{2N} V_{d_R}^\dagger \mathcal{X}_{d_R} V_{d_R}$$



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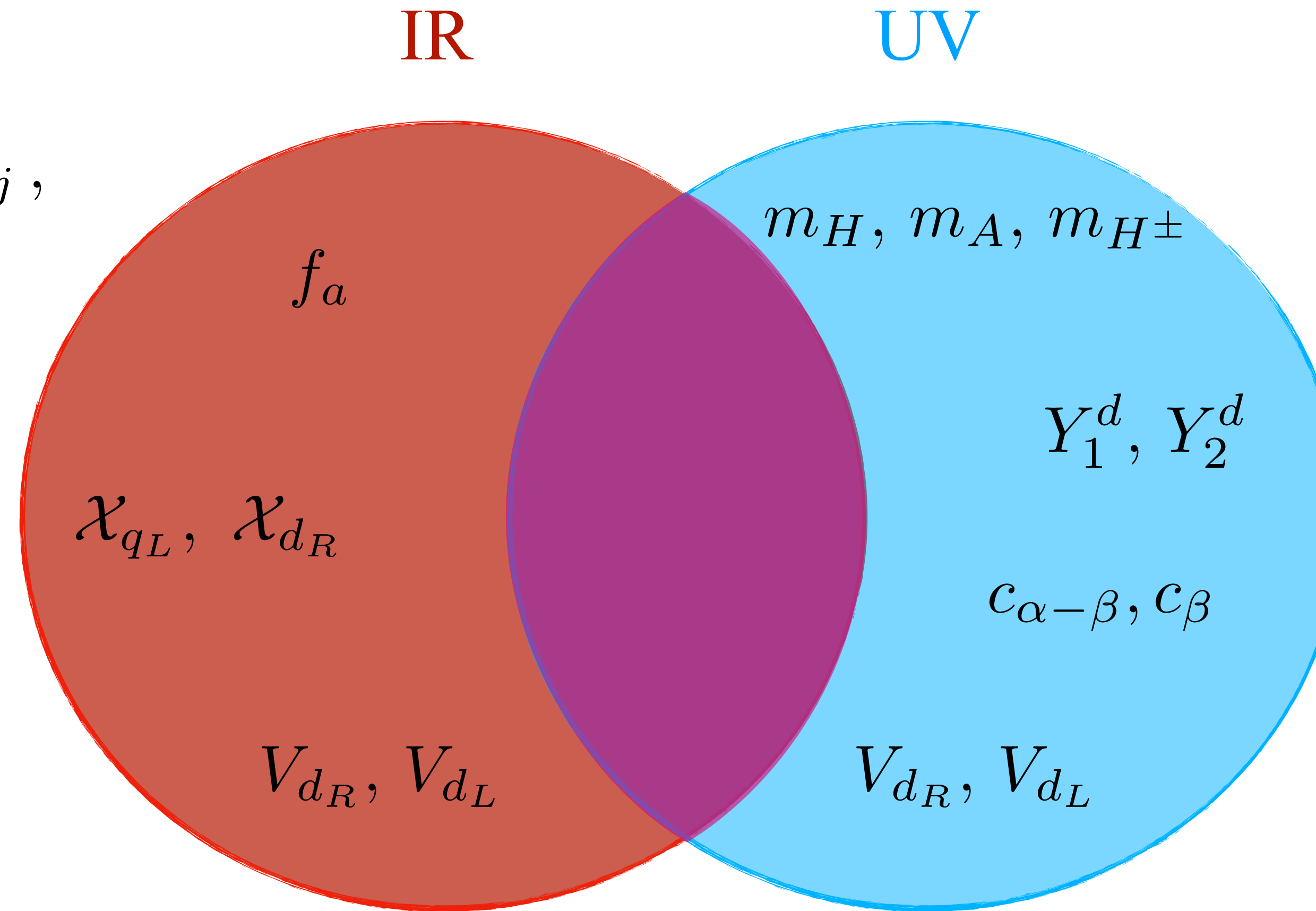
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In the **UV**:

$$\mathcal{L}_Y^{\text{PQ-2HDM}} = \bar{q}_L (Y_1^d \tilde{H}_1 + Y_2^d \tilde{H}_2^d) d_R + \dots$$

$$M_d = \frac{v}{\sqrt{2}} (c_\beta Y_1^d + s_\beta Y_2^d)$$

$$C_{ij}^{X_d} \sim (V_{d_L}^\dagger Y_2^e V_{d_R})_{ij} \equiv \epsilon_{ij}^d$$

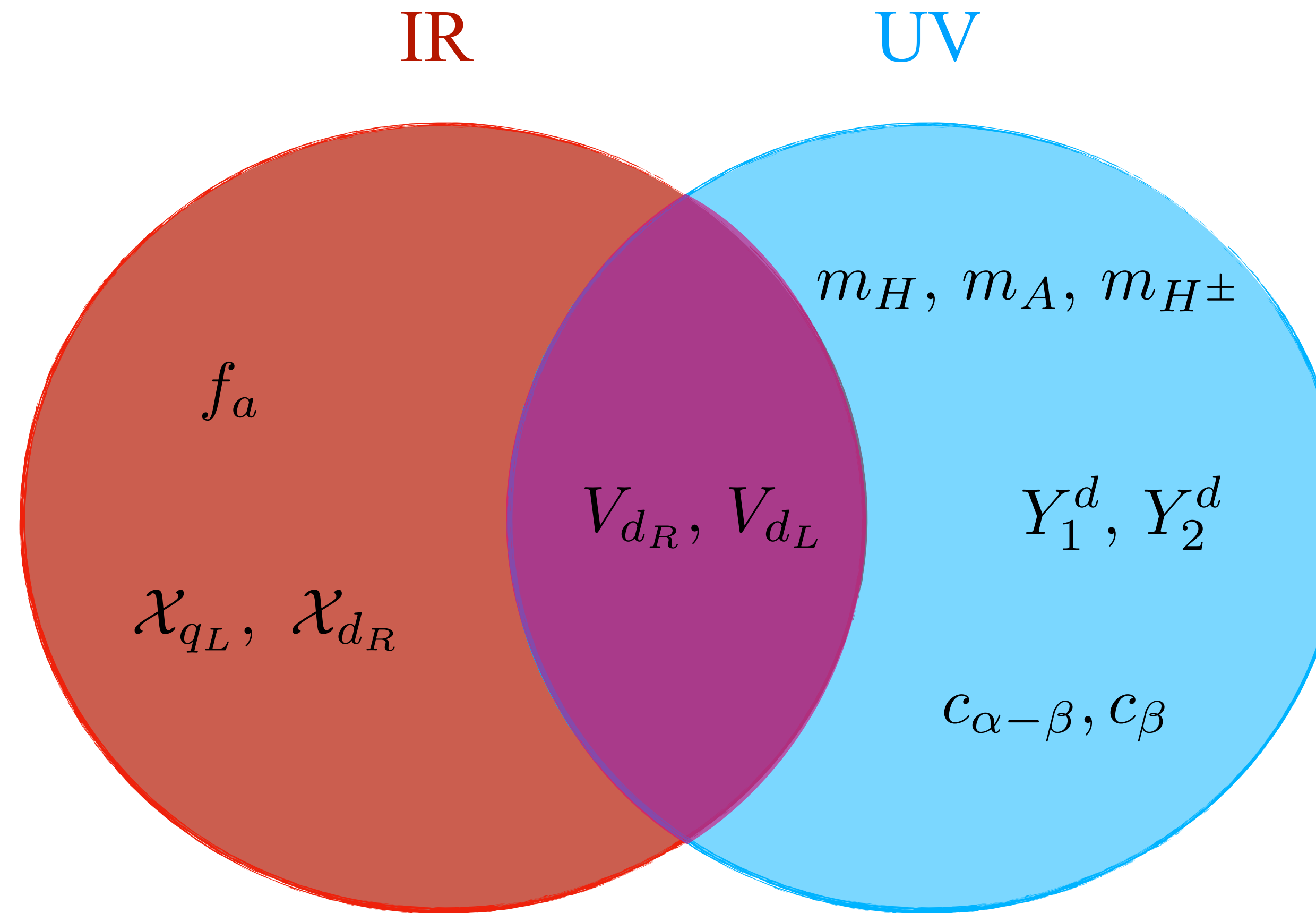


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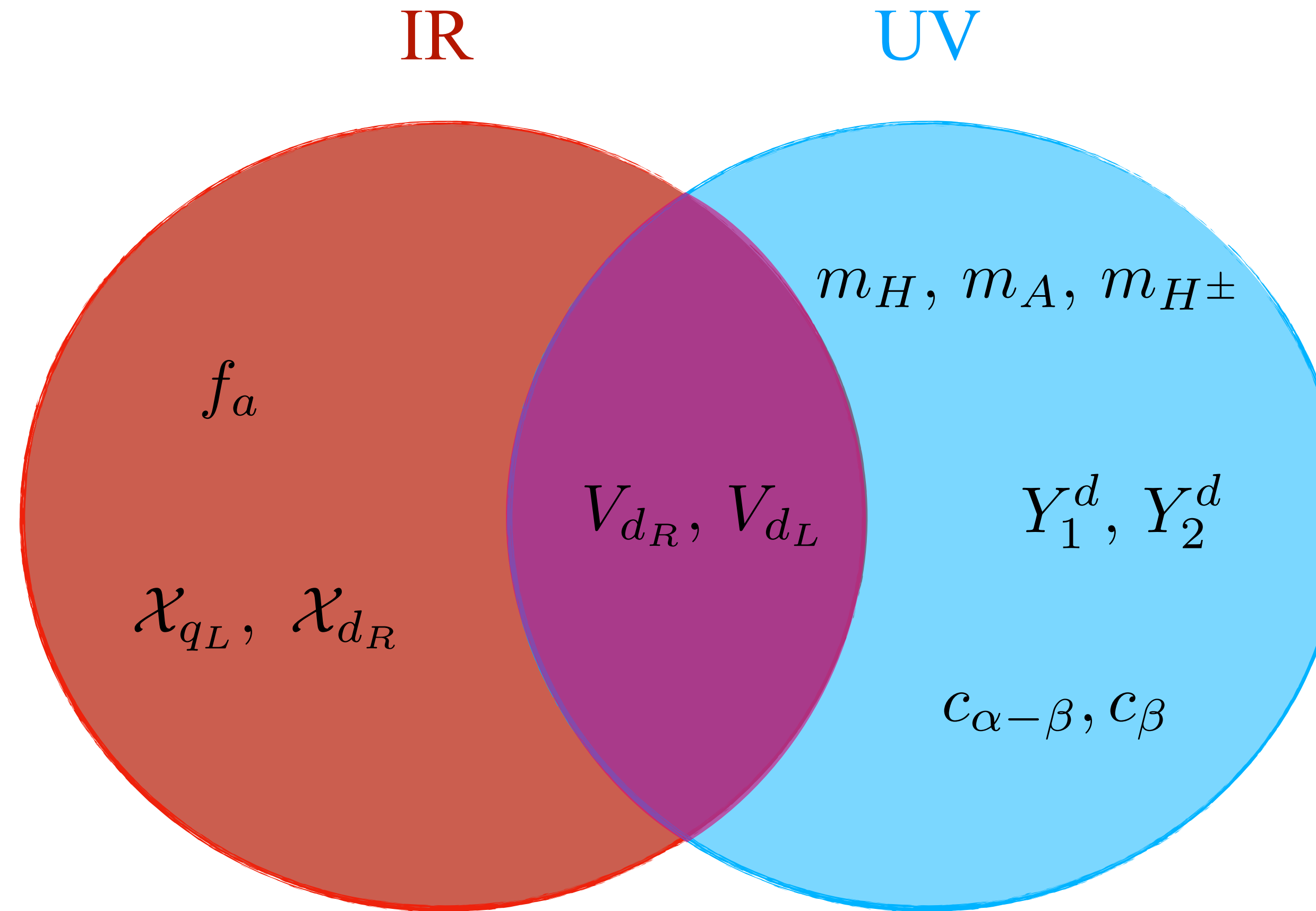
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Imposing PQ-invariance:

$$-\mathcal{X}_q Y_{1,2}^d + Y_{1,2}^d \mathcal{X}_d - \mathcal{X}_{1,2} Y_{1,2}^d = 0$$



Flavour-violating axion

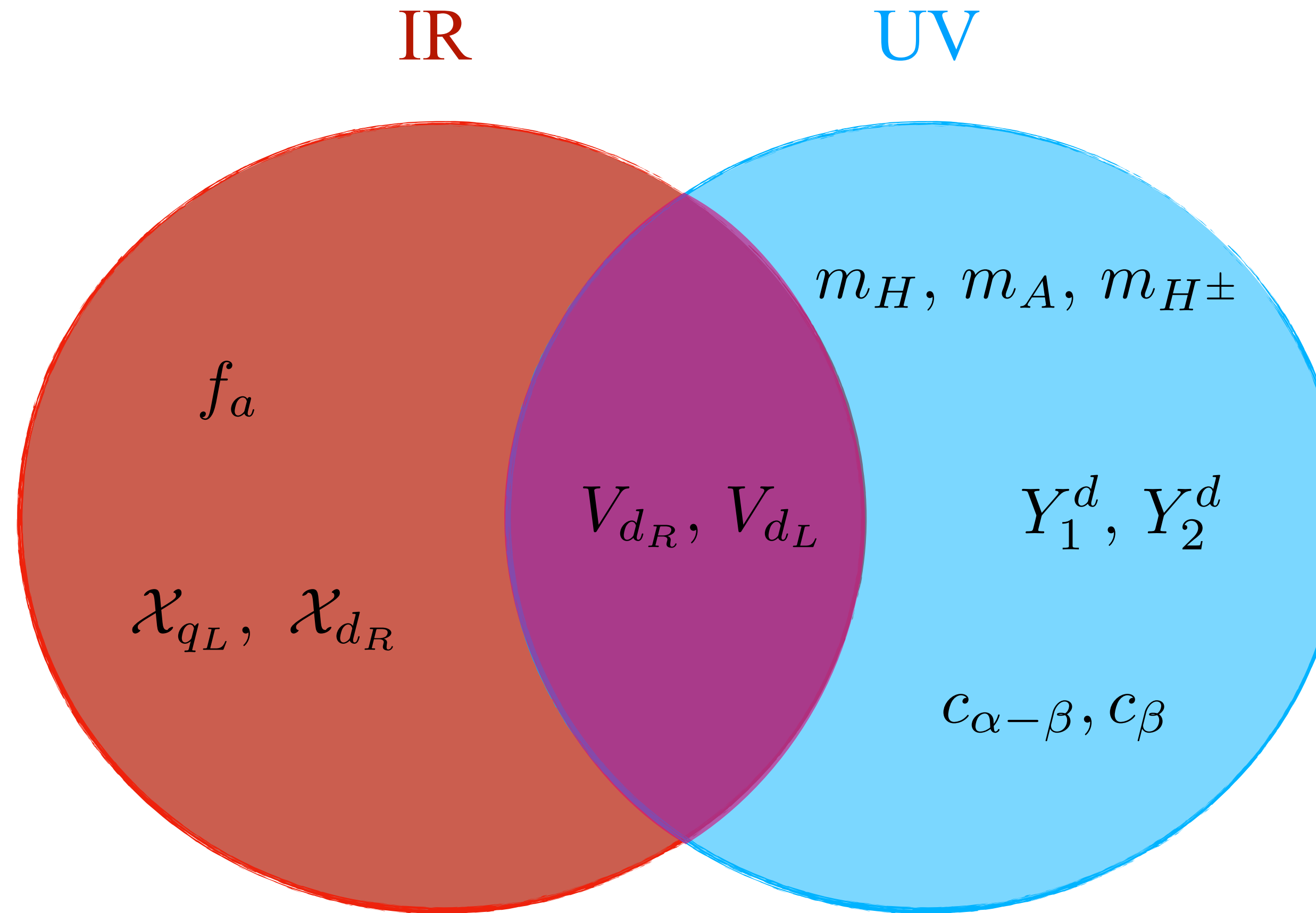
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$$\epsilon_{ij}^d = \frac{2\sqrt{2}}{v s_\beta} \left(-C_{d_L} \hat{M}_d + \hat{M}_d C_{d_R} \right)_{ij}$$

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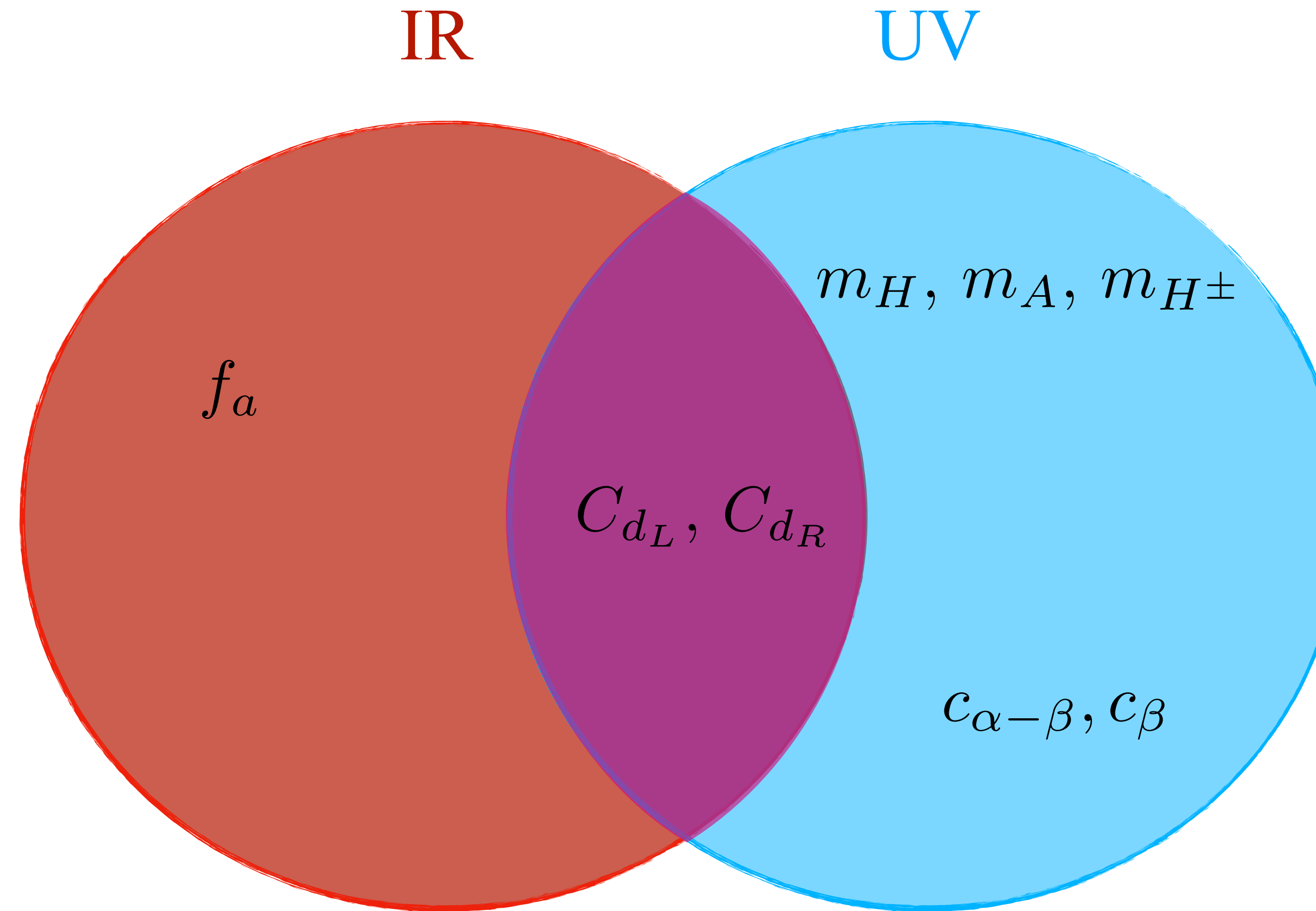
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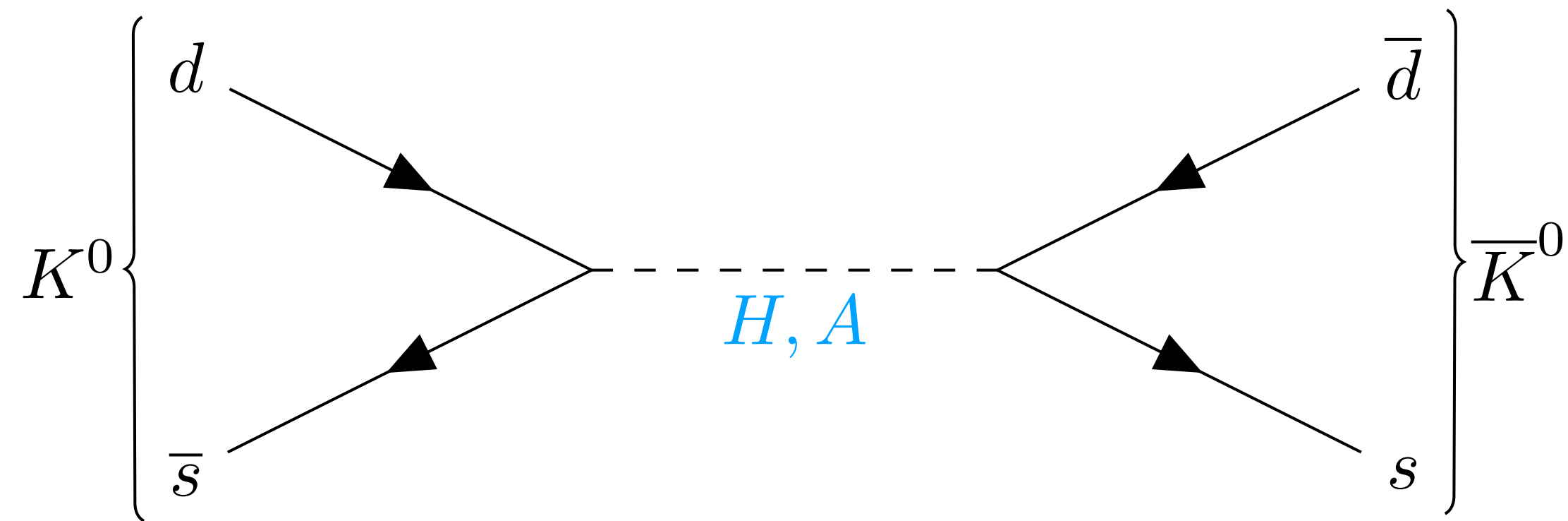
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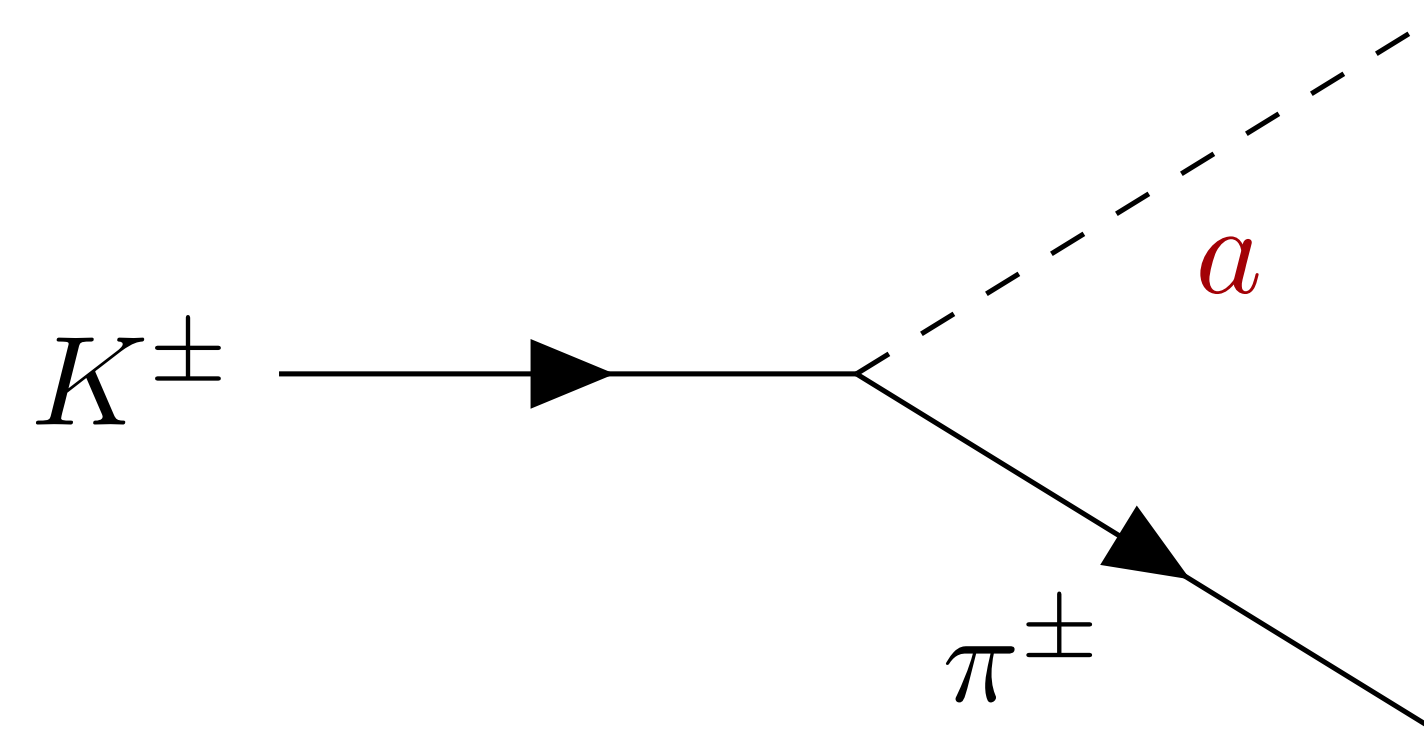
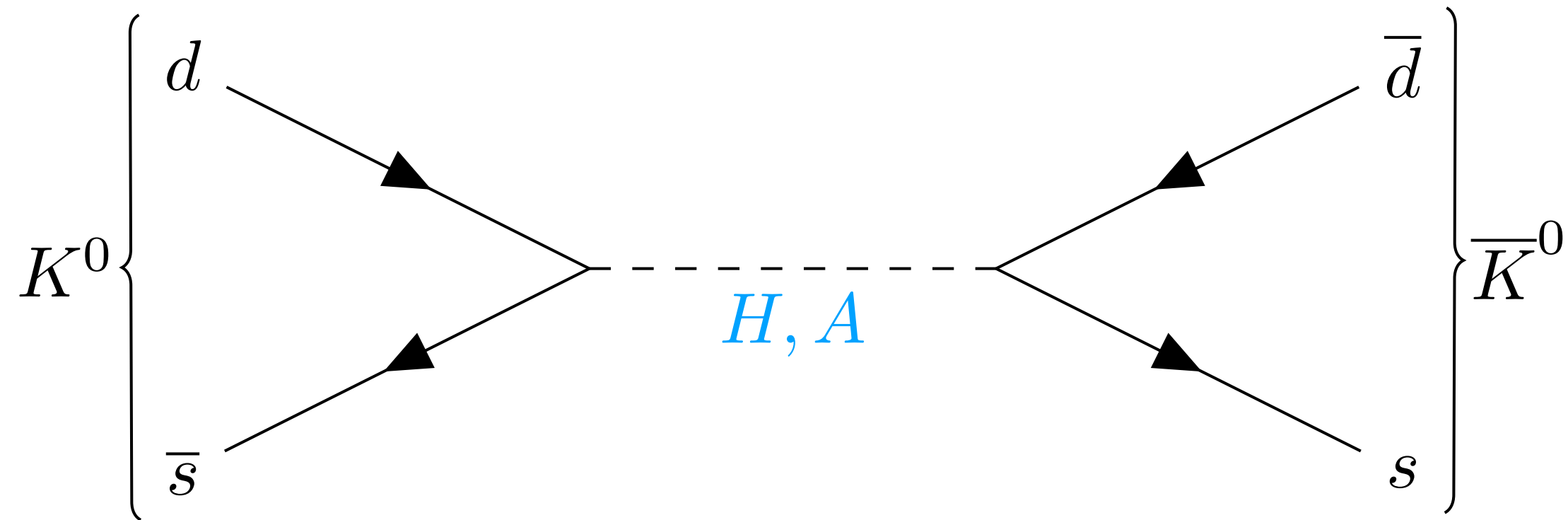


Quark flavour connection



$$\frac{2|M_{12}^{\text{NP}}|}{3.5 \cdot 10^{-15} \text{ GeV}} \sim \left(\frac{4 \text{ TeV}}{s_{2\beta} m_H} \right)^2 \left| \frac{\epsilon_{12}^d \epsilon_{21}^{d*}}{y_s^2 \lambda^2} \right|,$$

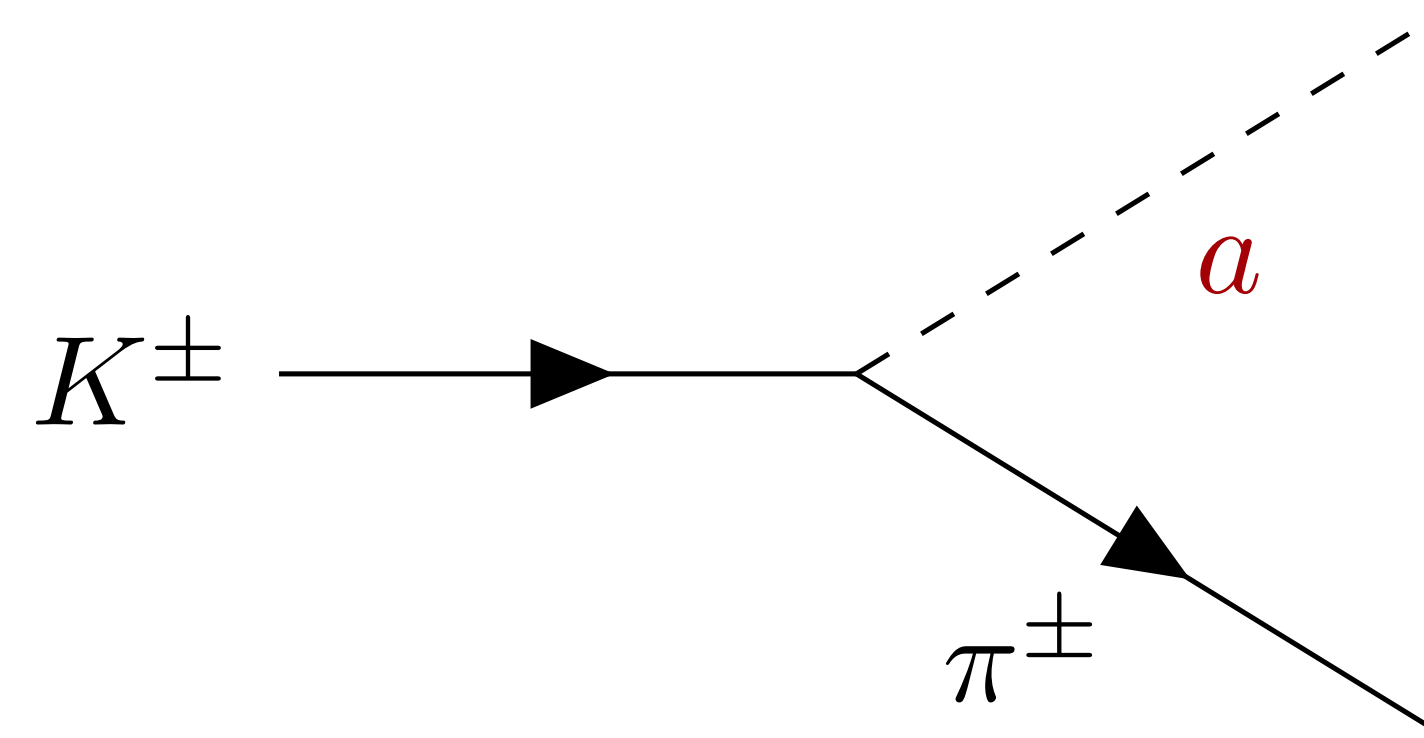
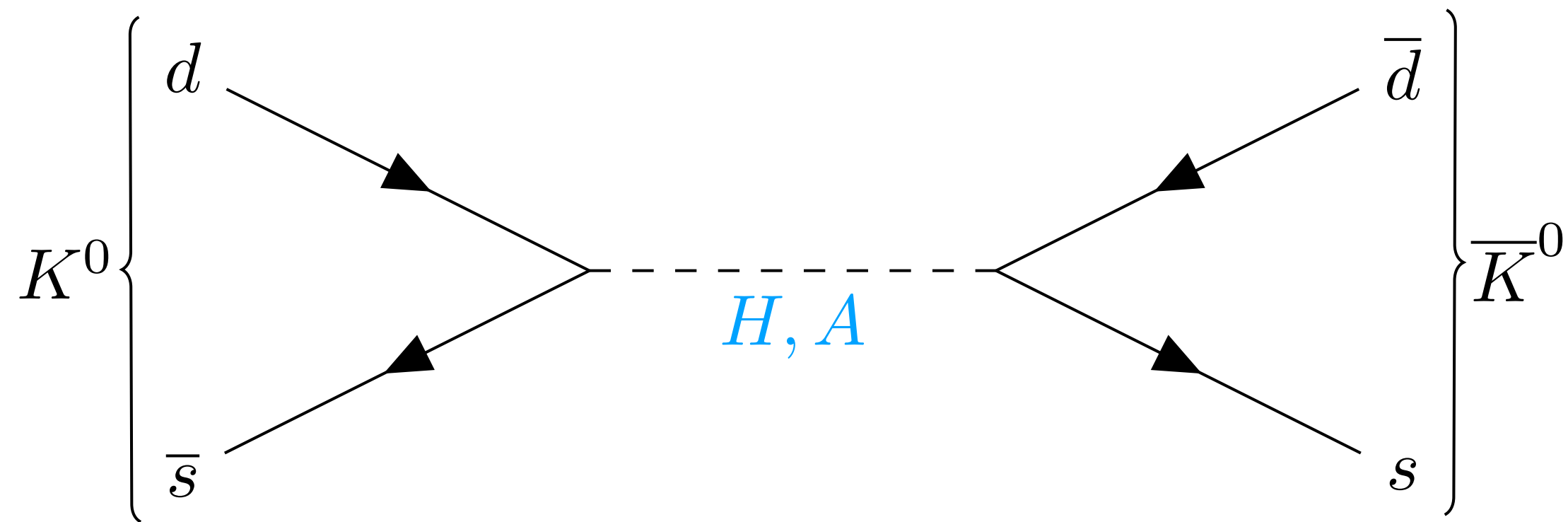
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$$\text{Br}(K^+ \rightarrow \pi^+ a) = (G_F f_K |V_{us}|^2)^{-2} \frac{m_K^2}{f_a^2} |C_{sd}^V|^2$$

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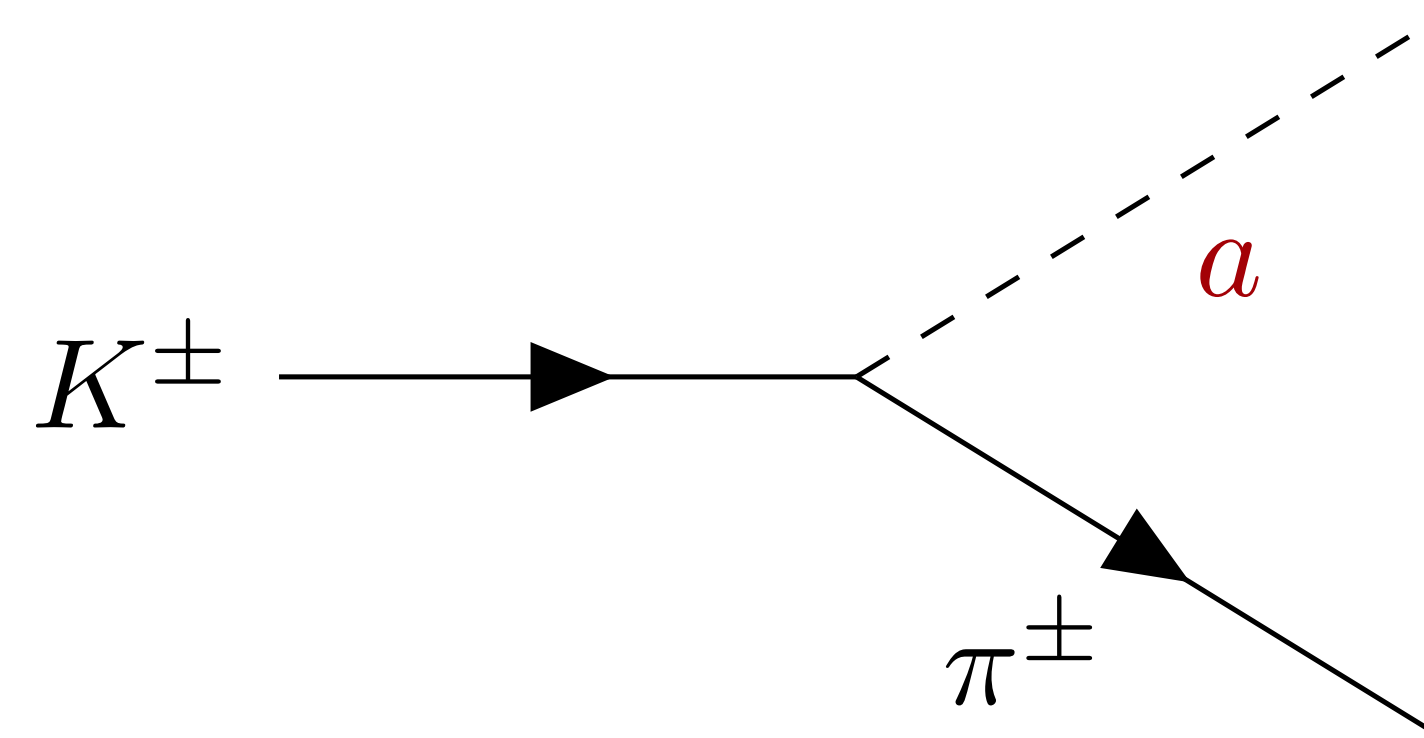
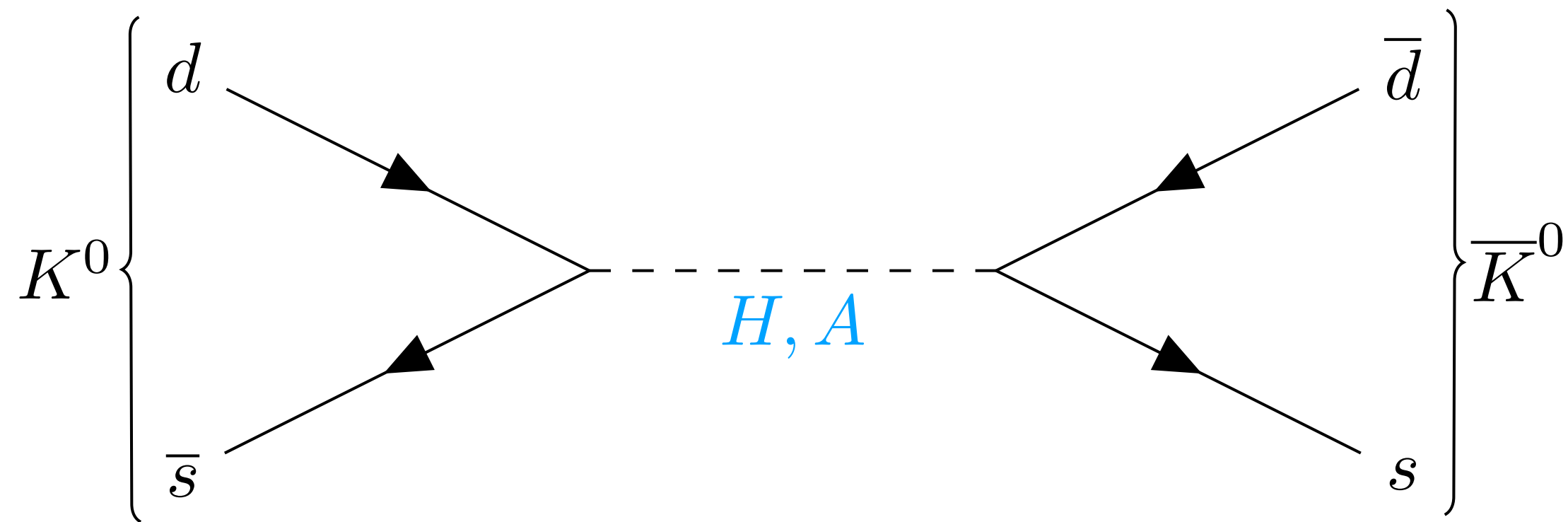


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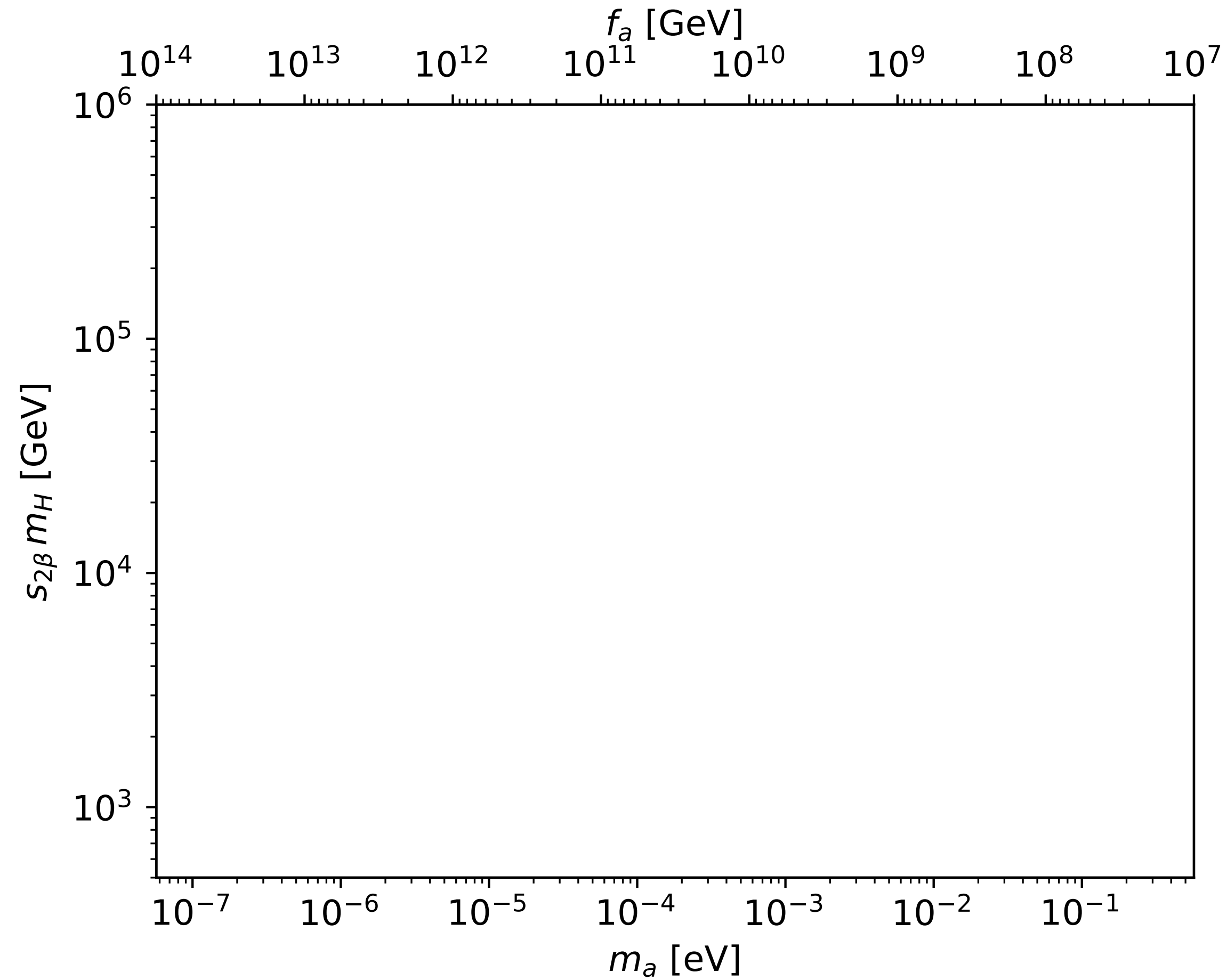
Two red arrows point from the terms \$-C_{dL} \hat{M}_d\$ and \$\hat{M}_d C_{dR}\$ in the equation above to the corresponding terms in the numerator of the first equation.

Quark flavour connection



$$\left(\frac{f_a}{10^{11} \text{ GeV}} \right)^2 \left(\frac{1 \text{ TeV}}{s_{2\beta} m_H} \right)^2 \left(\frac{\text{Br}(K \rightarrow \pi a)}{7.3 \cdot 10^{-11}} \right) = \frac{2 |M_{12}^{\text{NP}}|}{3.5 \cdot 10^{-15} \text{ GeV}}$$

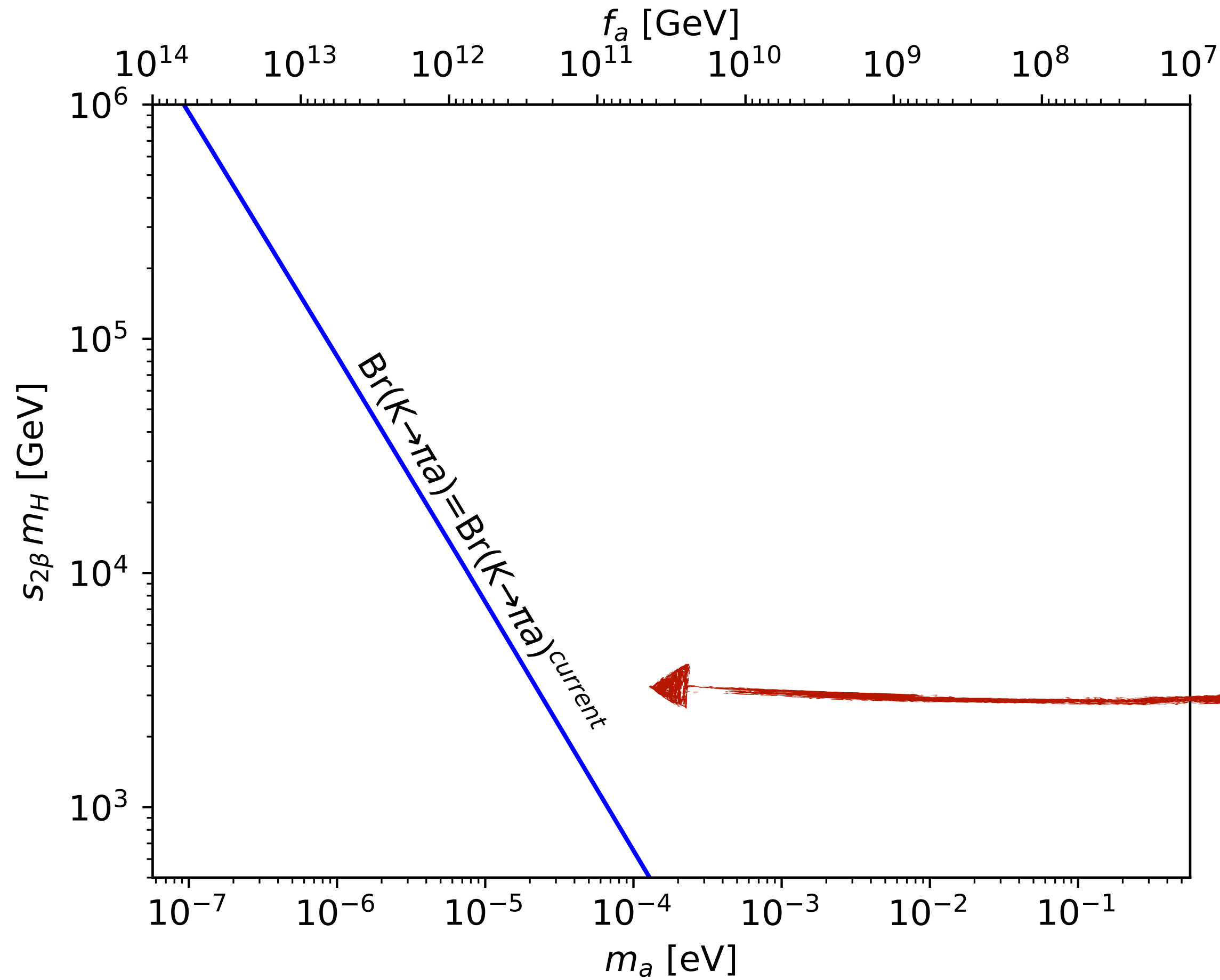
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Flavour violation in the $s \rightarrow d$ transition

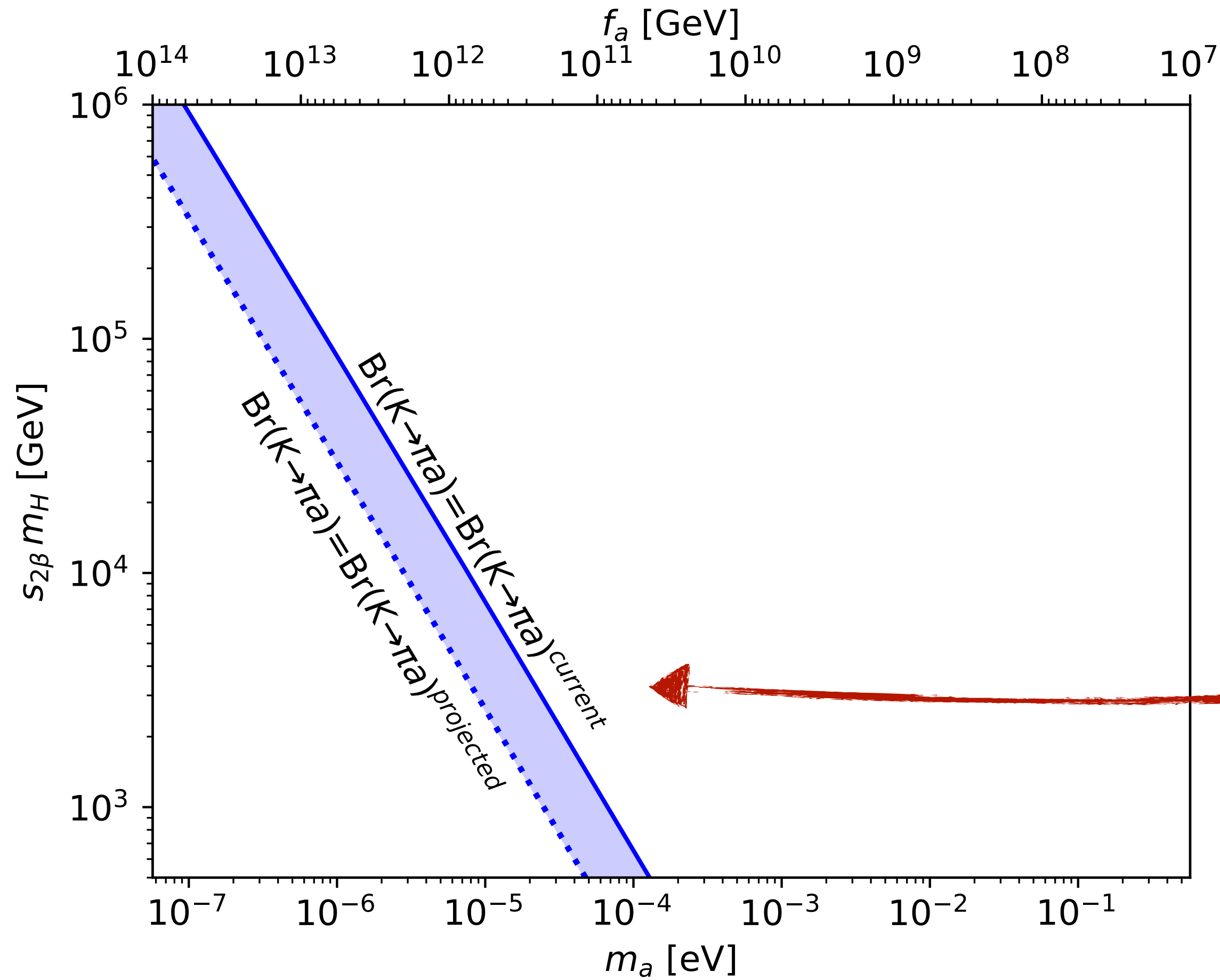
Quark flavour connection



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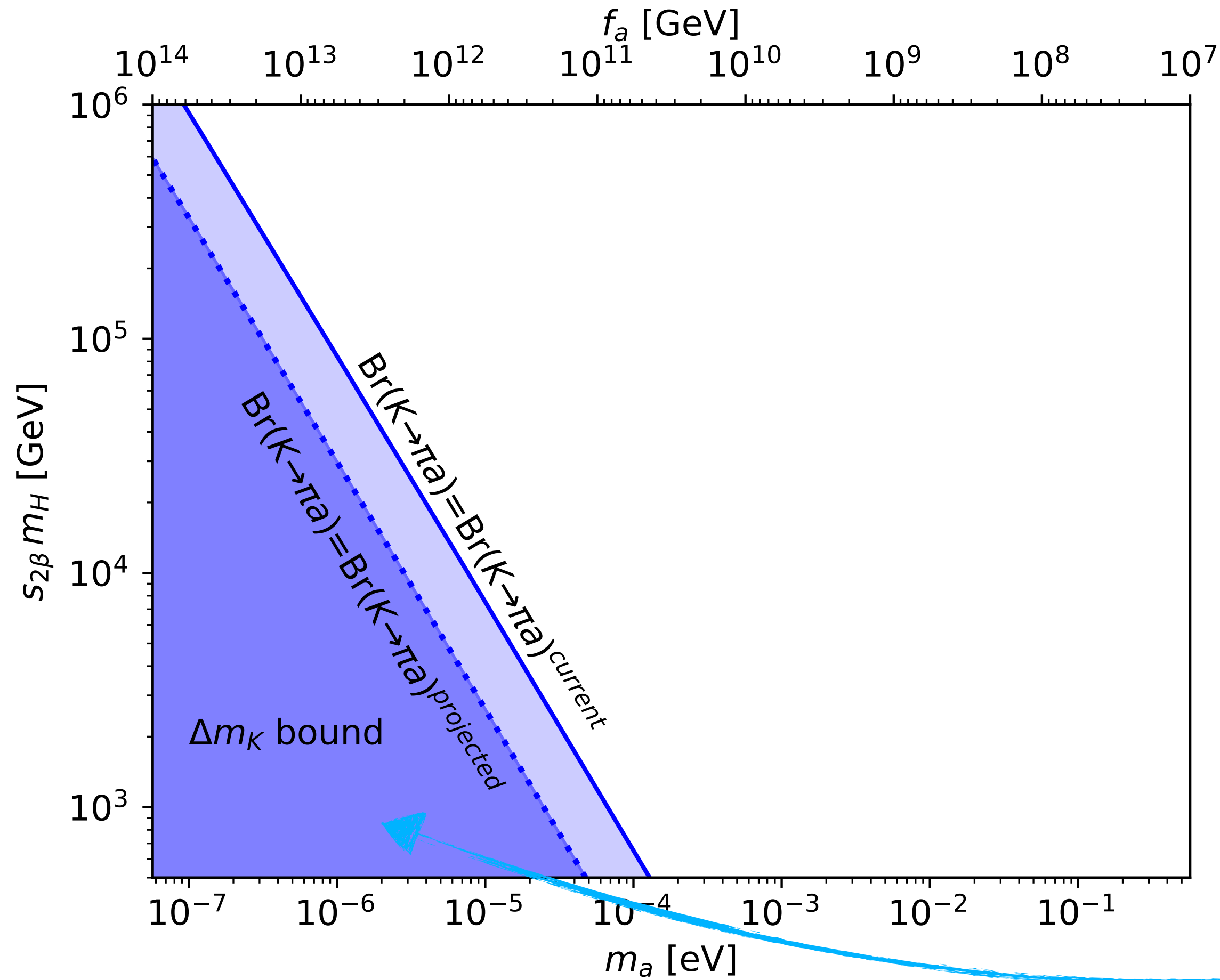
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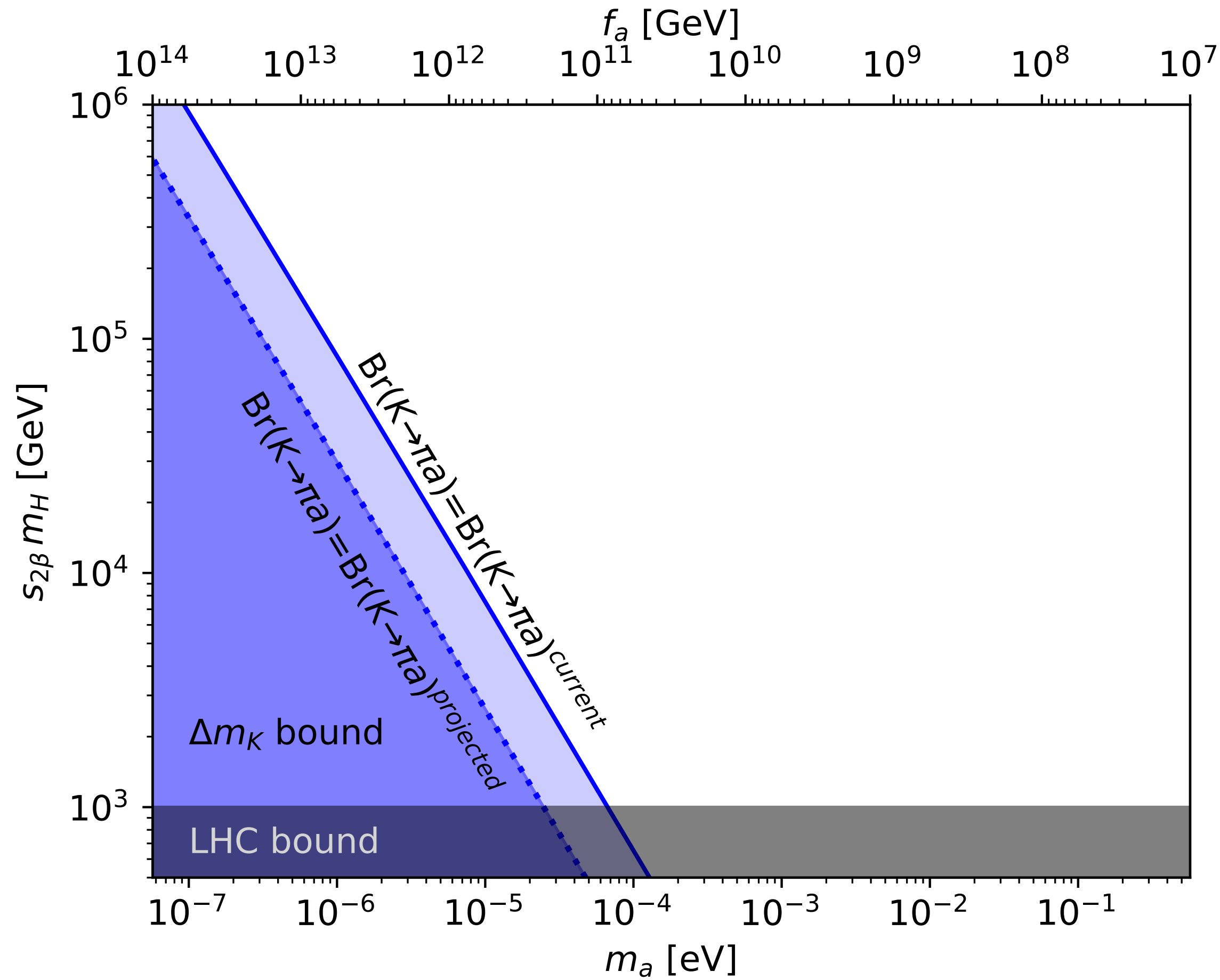
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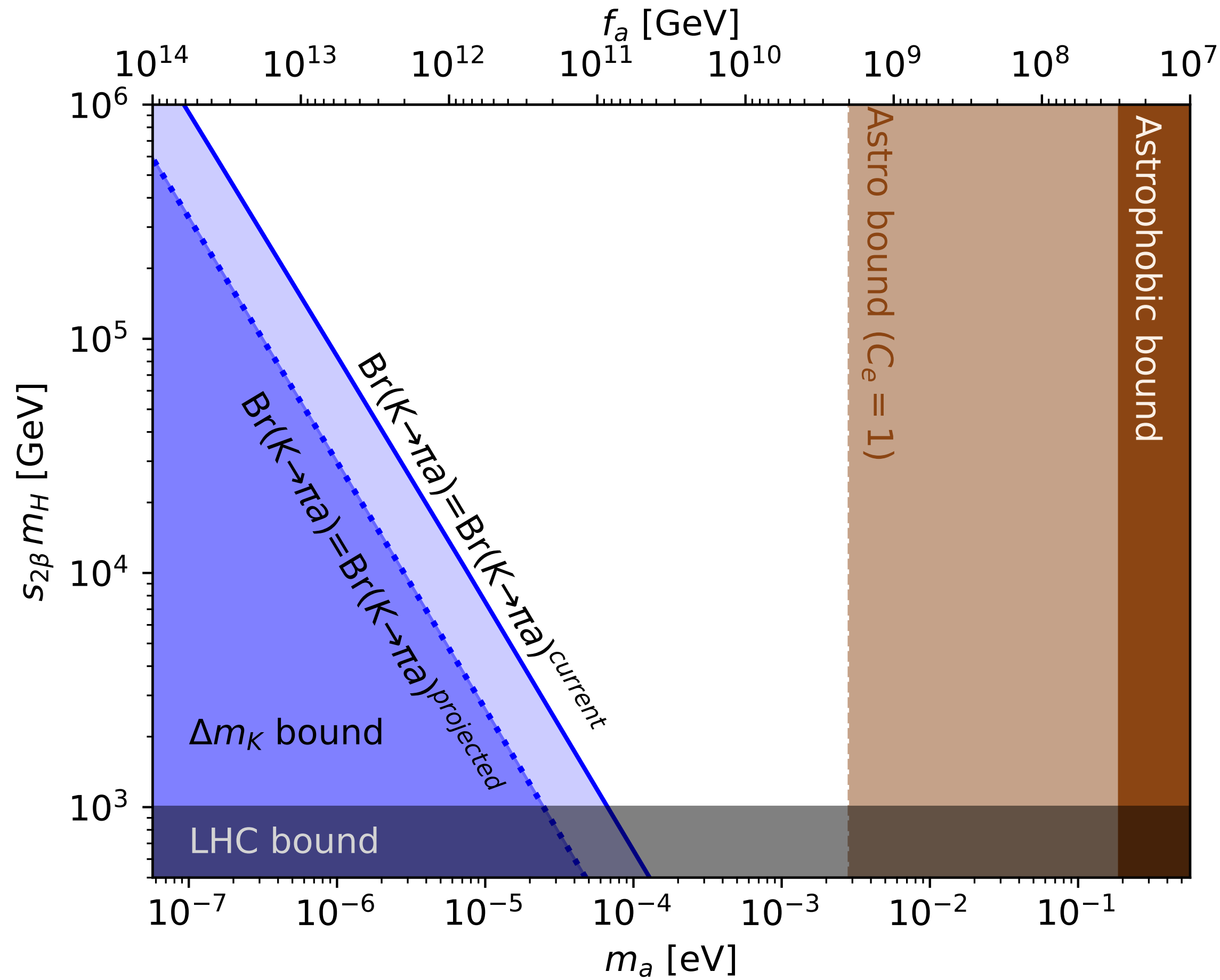
Quark flavour connection



$$\left(\frac{f_a}{10^{11} \text{ GeV}} \right)^2 \left(\frac{1 \text{ TeV}}{s_{2\beta} m_H} \right)^2 \left(\frac{\text{Br}(K \rightarrow \pi a)}{7.3 \cdot 10^{-11}} \right) = \frac{2 |M_{12}^{\text{NP}}|}{3.5 \cdot 10^{-15} \text{ GeV}}$$

Flavour violation in the $s \rightarrow d$ transition

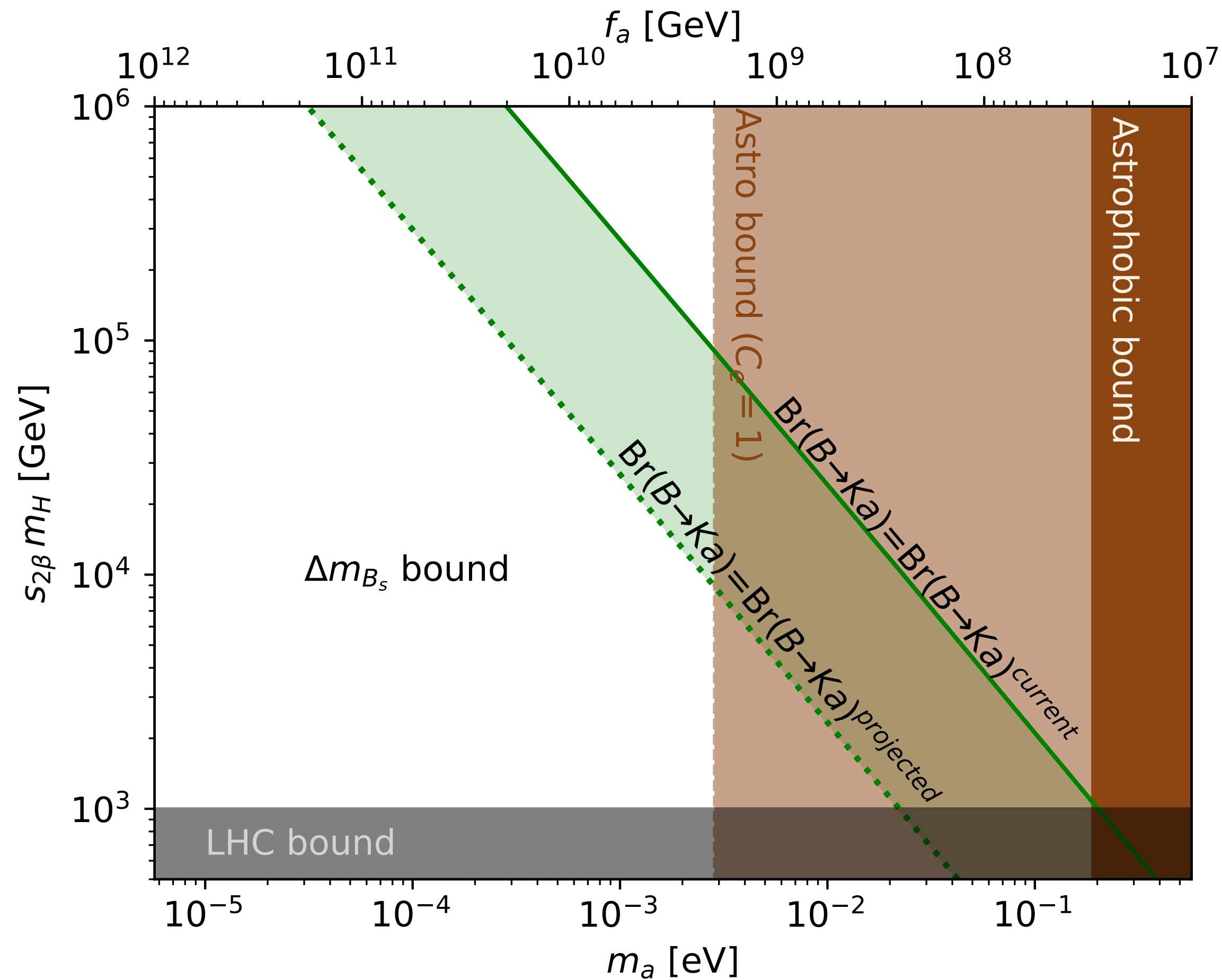
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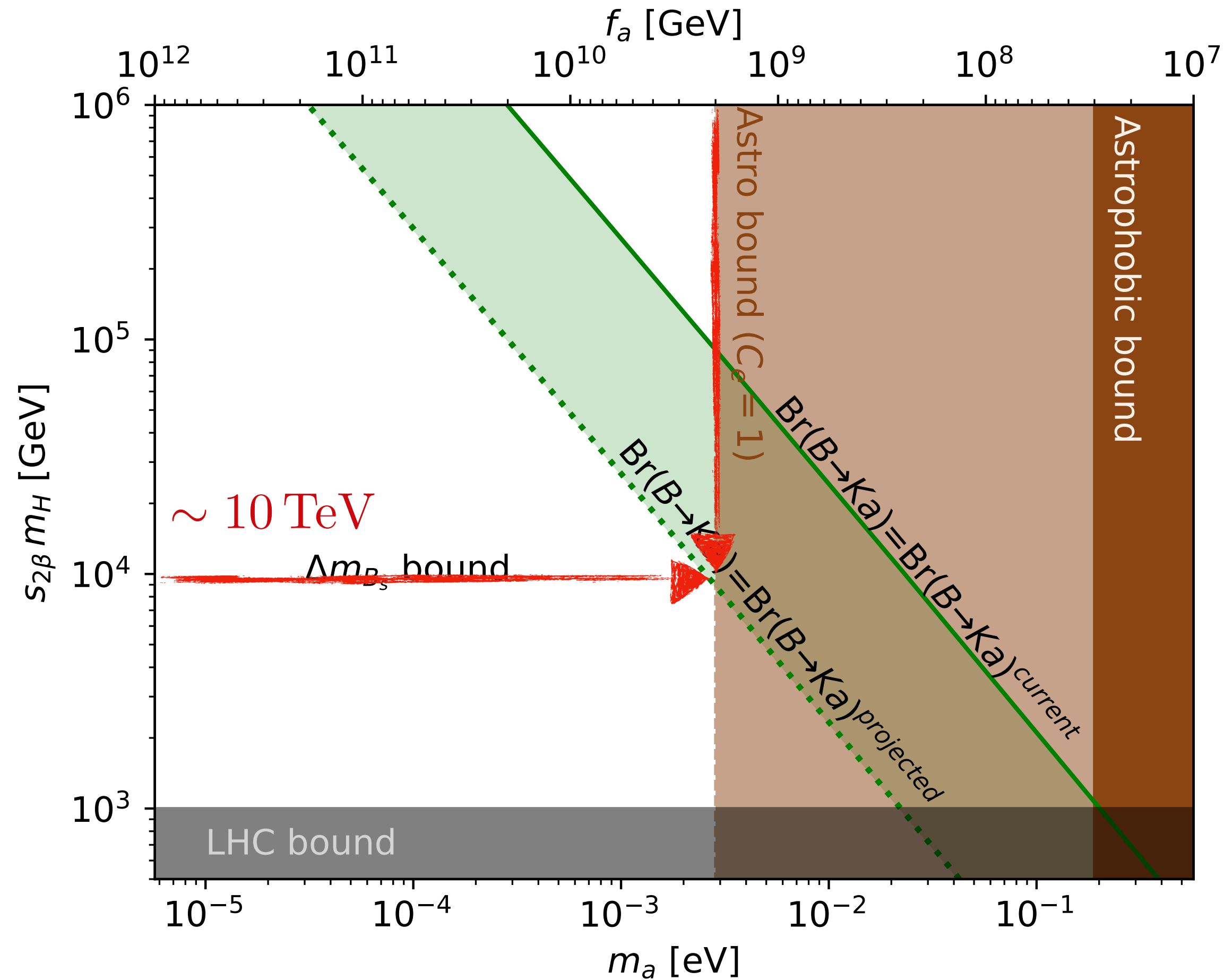
Quark flavour connection



$$\left(\frac{f_a}{8.8 \cdot 10^7 \text{ GeV}} \right)^2 \left(\frac{1 \text{ TeV}}{s_{2\beta} m_H} \right)^2 \left(\frac{\text{Br}(B \rightarrow Ka)}{7.1 \cdot 10^{-6}} \right) = \frac{2 |M_{12}^{\text{NP}}|}{1.2 \cdot 10^{-11} \text{ GeV}}$$

Flavour violation in the $b \rightarrow s$ transition

Quark flavour connection

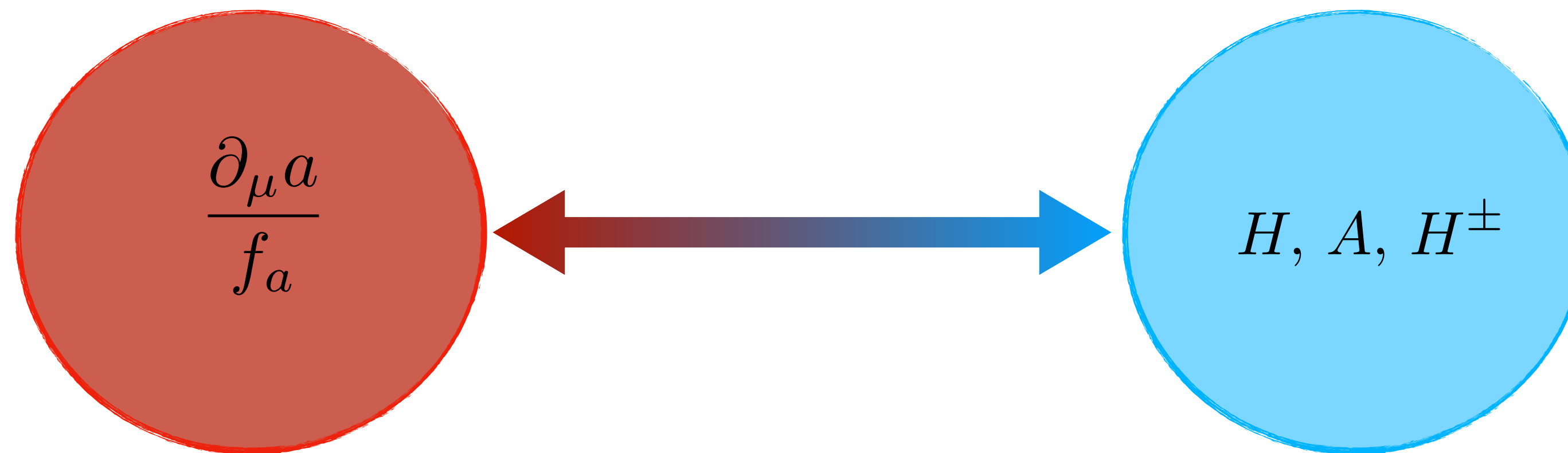


$$\left(\frac{f_a}{8.8 \cdot 10^7 \text{ GeV}} \right)^2 \left(\frac{1 \text{ TeV}}{s_{2\beta} m_H} \right)^2 \left(\frac{\text{Br}(B \rightarrow Ka)}{7.1 \cdot 10^{-6}} \right) = \frac{2 |M_{12}^{\text{NP}}|}{1.2 \cdot 10^{-11} \text{ GeV}}$$

Flavour violation in the $b \rightarrow s$ transition

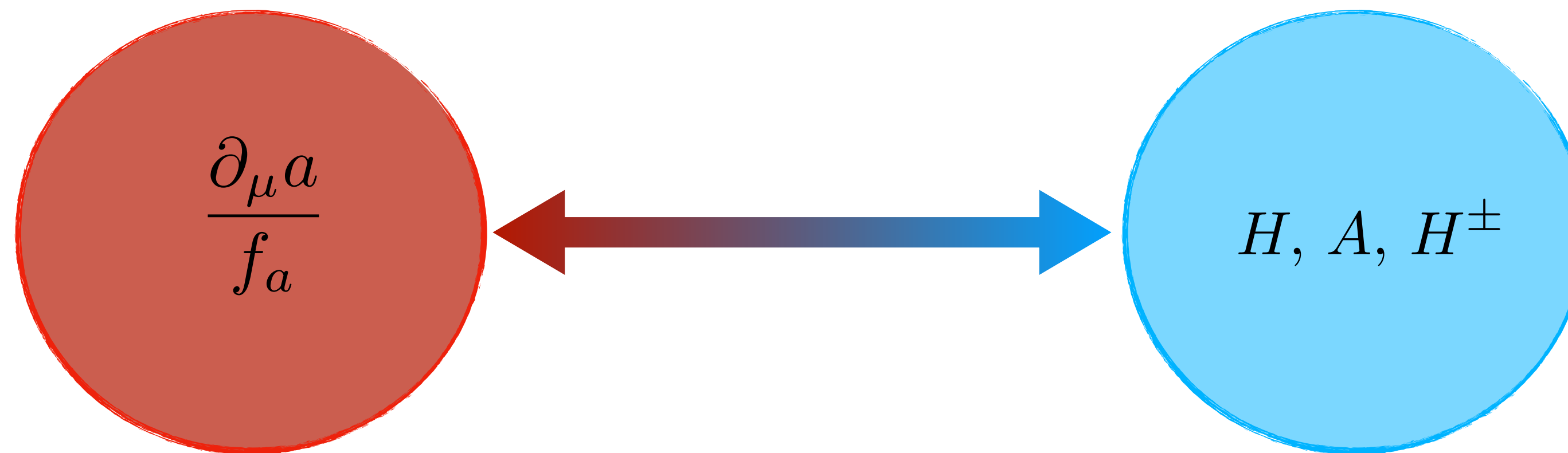
Take-home message

- Model independent 1-to-1 correspondence of FV observables between the UV and IR
- No assumption on the Yukawas or charges is needed
- Information on one sector is relevant to the other!



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Thanks for your attention!

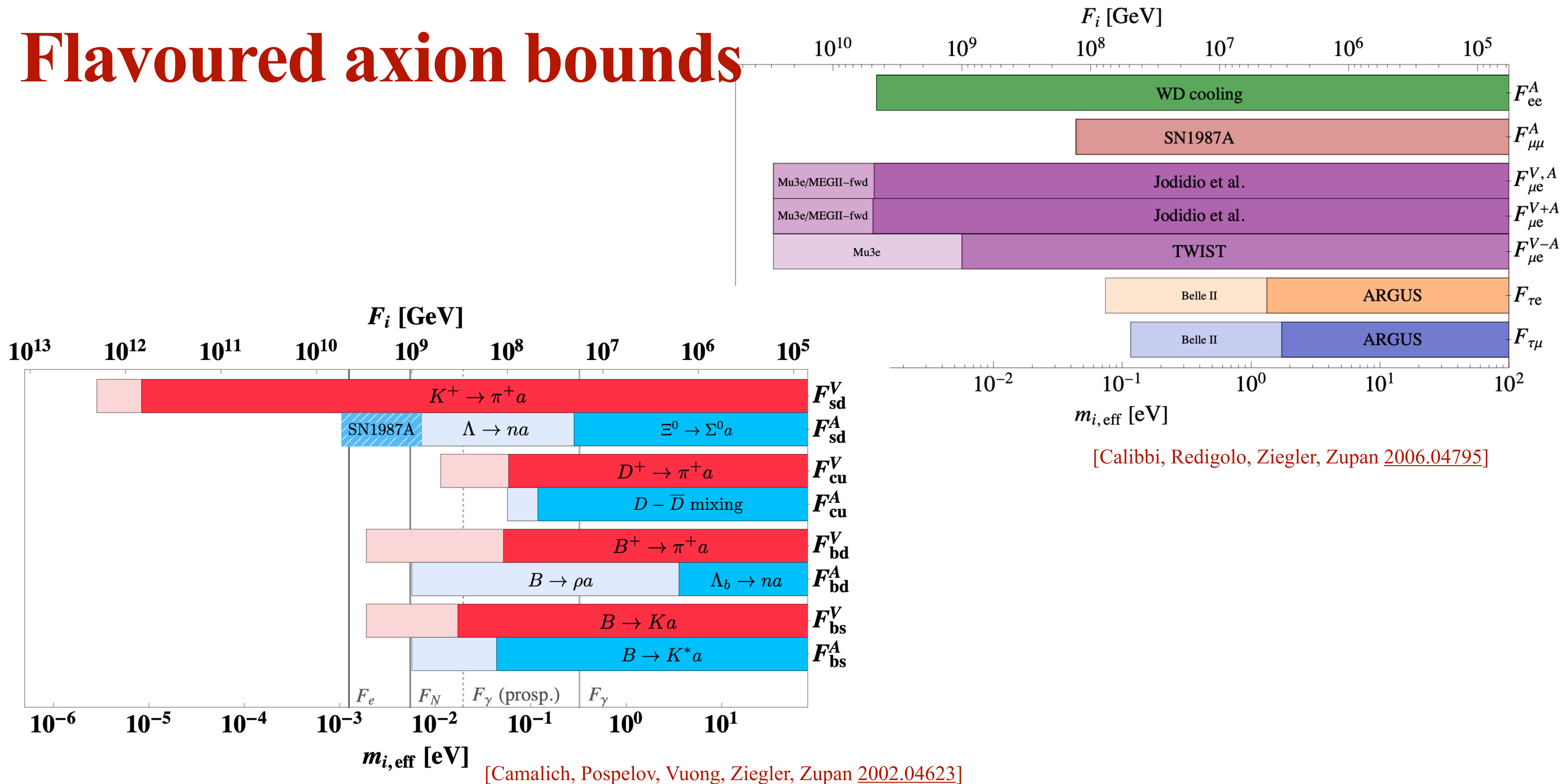
Acknowledgements



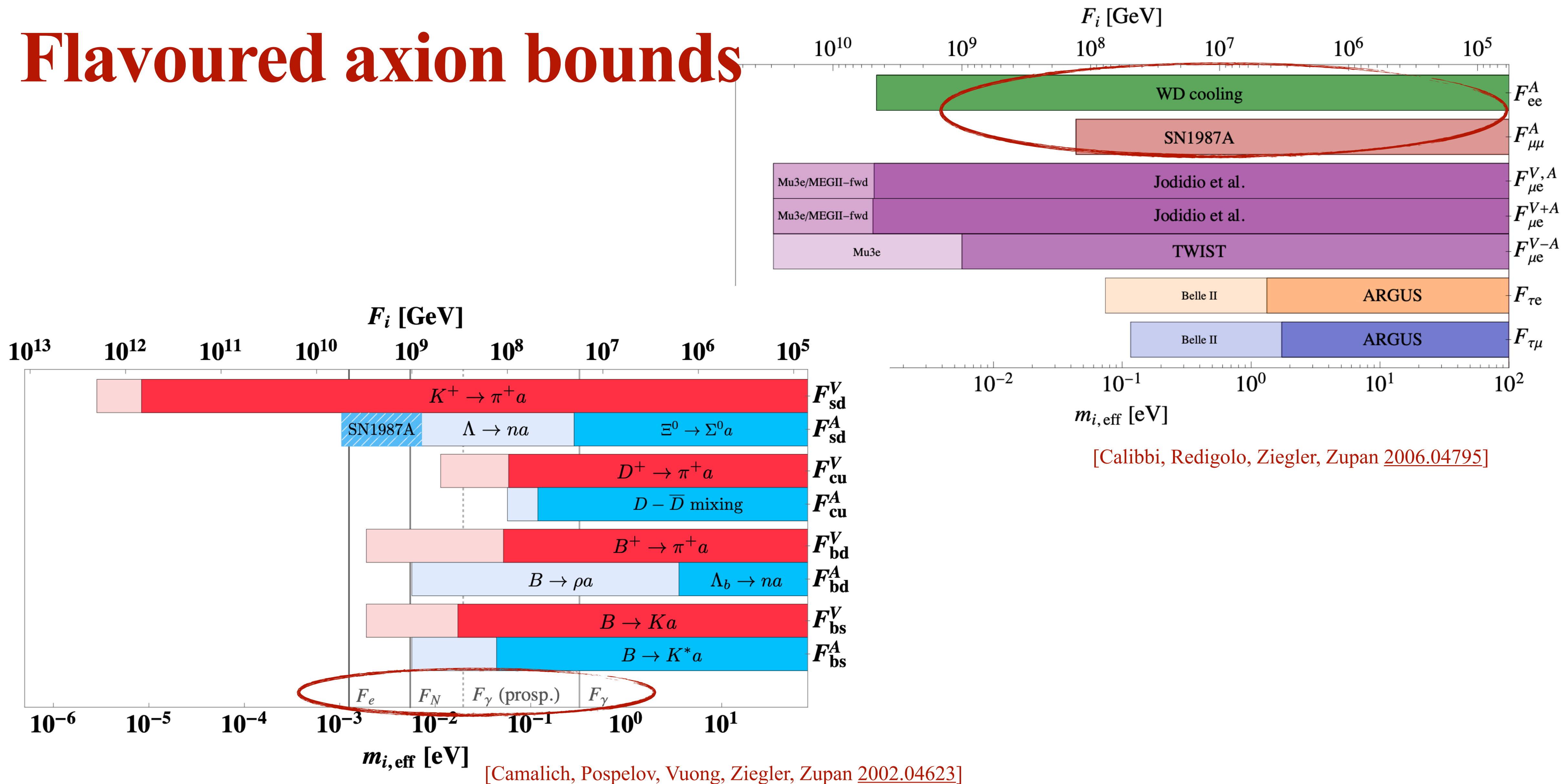
This project has received funding from the European Union's **Horizon 2020** research and innovation programme under the **Marie Skłodowska-Curie** grant agreement **No 860881**.

Backup

Flavoured axion bounds



Flavoured axion bounds



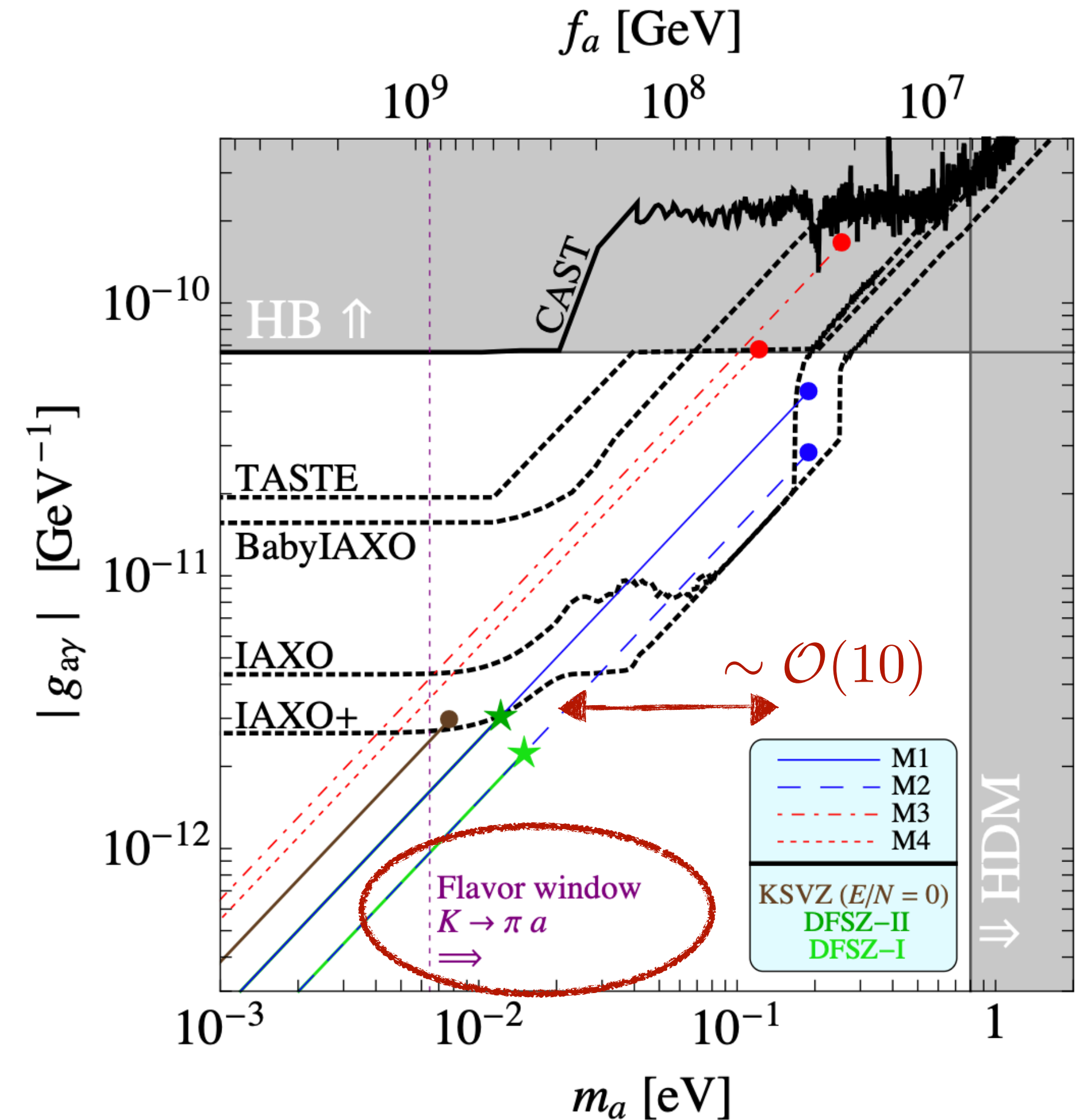
Astrophobic Models

Astrophobic models

→ Astrophobia: nucleophobia + electrophobia

- Can saturate the best fit: cooling anomalies + astrophysical bounds*
- Improved sensitivity for IAXO
- Flavour violation is a consequence!

* [Di Luzio, Fedele, Giannotti, Mescia, Nardi [2109.10368](#)]



[Di Luzio, Mescia, Nardi, Panci, Ziegler [1712.04940](#)]

[Björkeröth, Di Luzio, Mescia, Nardi, Panci, Ziegler [1907.06575](#)]

[Di Luzio, Mescia, Nardi, Okawa [2205.15326](#)]

Astrophobic axions

- Astrophobic scenario: nucleophobia + electrophobia [Di Luzio, Mescia, Nardi, Panci, Ziegler [1712.04940](#)]

Implementing nucleophobia:

$$C_p + C_n \simeq 0.50(5)(C_u + C_d - 1) \longrightarrow C_u + C_d = \frac{\mathcal{X}_{u_R} + \mathcal{X}_{d_R} - 2\mathcal{X}_{q_L}}{2N} = \frac{N_1}{N} = 1$$
$$C_p - C_n \simeq 1.273(2)(C_u - C_d - \frac{1}{3}) \longrightarrow \frac{\mathcal{X}_{u_R} - \mathcal{X}_{d_R}}{N} = \frac{1}{3}$$

Non-universal charges required
 $N_2 = N_3 = 0$
 $N_2 = -N_3$

Implementing electrophobia:

- Tuning: large mixing angles in the leptonic sector can cancel the electron coupling.
- 3HDM: see [Björkeröth, Di Luzio, Mescia, Nardi, Panci, Ziegler [1907.06575](#)]

The Scalar Sector

$$\begin{aligned}
 V(H_1, H_2, \phi) = & \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \frac{\lambda_1}{2} |H_1|^4 + \frac{\lambda_2}{2} |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 \\
 & + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{\lambda_1^\phi}{2} |\phi|^2 |H_1|^2 + \frac{\lambda_2^\phi}{2} |\phi|^2 |H_2|^2 + \lambda_3^\phi \left(|\phi|^2 - \frac{v_\phi^2}{2} \right)^2 \\
 & - \left(\mu_\phi H_2^\dagger H_1 \phi + \text{h.c.} \right)
 \end{aligned}$$

Common problem to invisible axion models.

Possible solution: use “ultra-weak” couplings which allows to separate the scales in a “technically natural way” [Volkas, Davies, Joshi [PLB 215 1988](#)]

[Foot, Kobakhidze, McDonald, Volkas [1310.0223](#)]

The Scalar Sector

$$\begin{aligned}
 V(\Phi_1, \Phi_2) = & M_{11}^2 \Phi_1^\dagger \Phi_1 + M_{22}^2 \Phi_2^\dagger \Phi_2 - \left(M_{12} \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) \\
 & + \frac{\Lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\Lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \Lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \Lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \left\{ \frac{1}{2} \Lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\Lambda_6 (\Phi_1^\dagger \Phi_1) + \Lambda_7 (\Phi_2^\dagger \Phi_2) \right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\},
 \end{aligned}$$

$$\Phi_1 = \begin{pmatrix} G^+ \\ -\frac{1}{\sqrt{2}}(v + c_{\alpha-\beta}H - s_{\alpha-\beta}h - iG^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ -\frac{1}{\sqrt{2}}(c_{\alpha-\beta}h + s_{\alpha-\beta}H - iA) \end{pmatrix}$$

$$\begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} = \begin{pmatrix} c_{\alpha-\beta} & s_{\alpha-\beta} \\ -s_{\alpha-\beta} & c_{\alpha-\beta} \end{pmatrix} \begin{pmatrix} \Lambda_1 v^2 & \Lambda_6 v^2 \\ \Lambda_6 v^2 & m_A^2 + \Lambda_5 v^2 \end{pmatrix} \begin{pmatrix} c_{\alpha-\beta} & -s_{\alpha-\beta} \\ s_{\alpha-\beta} & c_{\alpha-\beta} \end{pmatrix},$$

The Scalar Sector: Higgs alignment

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$$c_{\alpha-\beta} = 0 \quad \Phi_1 = \begin{pmatrix} G^+ \\ -\frac{1}{\sqrt{2}}(v - h - iG^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ -\frac{1}{\sqrt{2}}(H - iA) \end{pmatrix}$$

SM-like Higgs properties

$$c_{\alpha-\beta}^2 = \frac{\Lambda_6^2 v^4}{(m_H^2 - m_h^2)(m_H^2 - \Lambda_1 v^2)}$$

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Two ways of achieving alignment:

Decoupling:

$$m_H^2 \simeq m_A^2 \gg v^2$$

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Two ways of achieving alignment:

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non-decoupling: $\Lambda_6 \rightarrow 0$

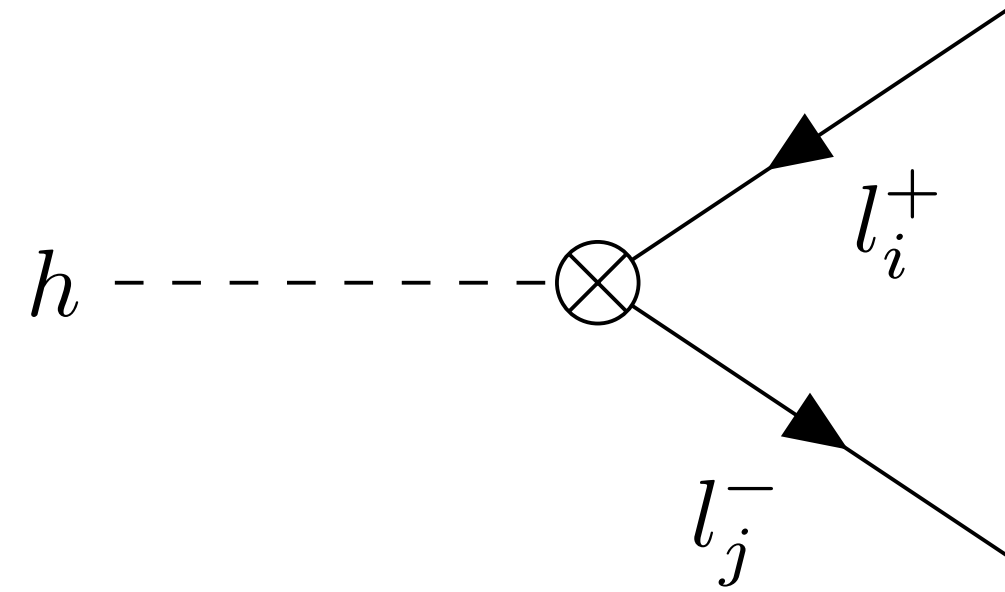
$$m_A^2 + \Lambda_5 v^2 \simeq m_H^2$$

$$m_h^2 \simeq \Lambda_1 \frac{v^2}{2}$$

The new scalars can be at the EW scale

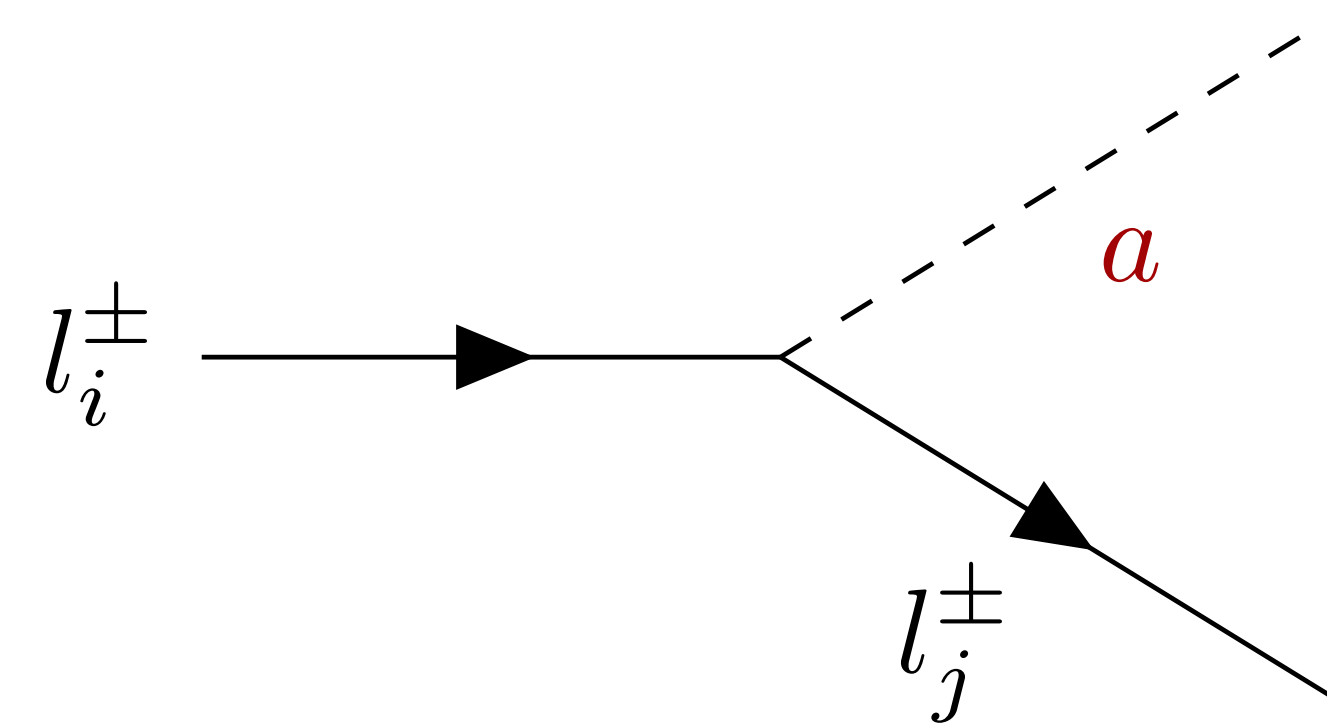
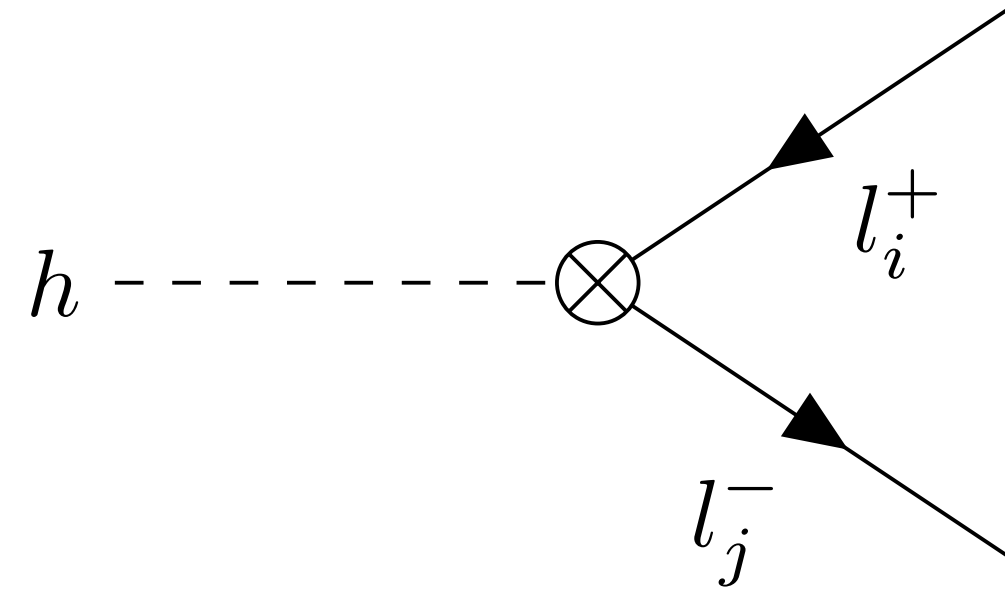
Lepton flavour connection

Lepton flavour connection



$$\text{BR}(h \rightarrow l_i l_j) \simeq \frac{m_h}{16\pi\Gamma_h} \left(\frac{c_{\alpha-\beta}}{s_\beta c_\beta} \right)^2 \frac{m_{l_i}^2}{v^2} |(C_e^{L,R})_{ij}|^2,$$

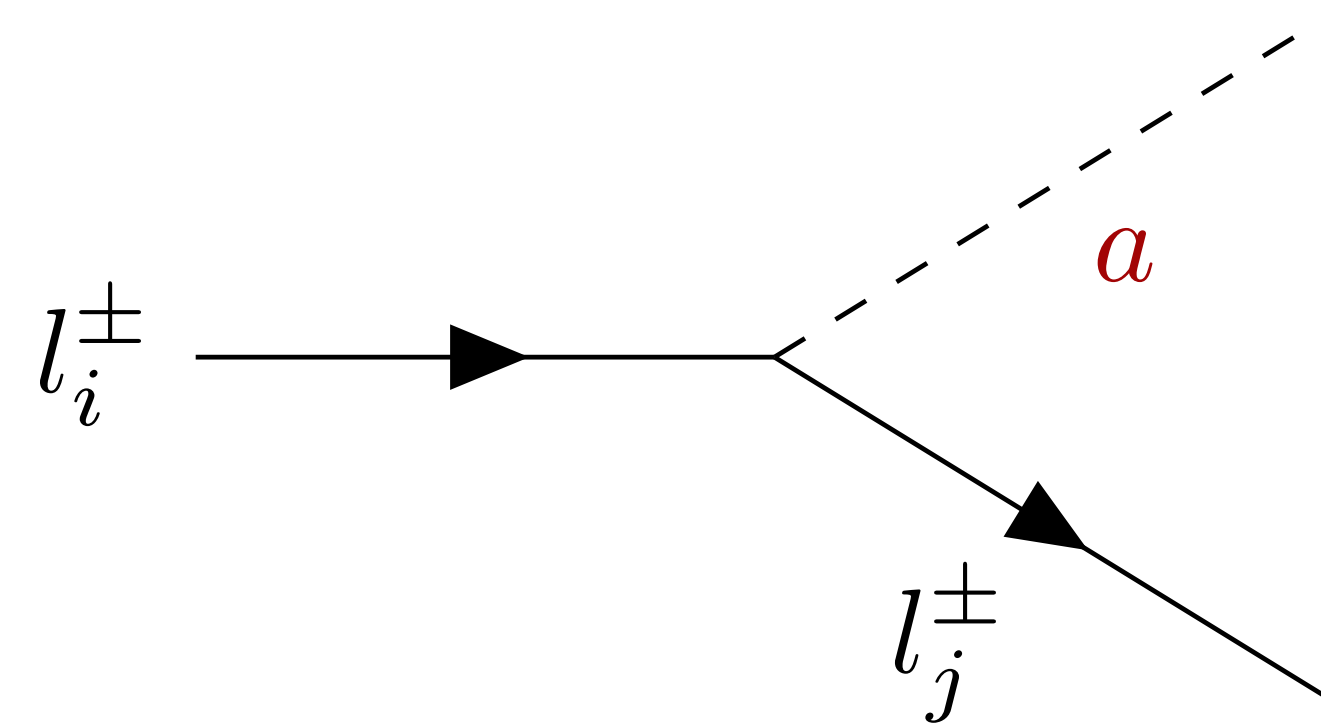
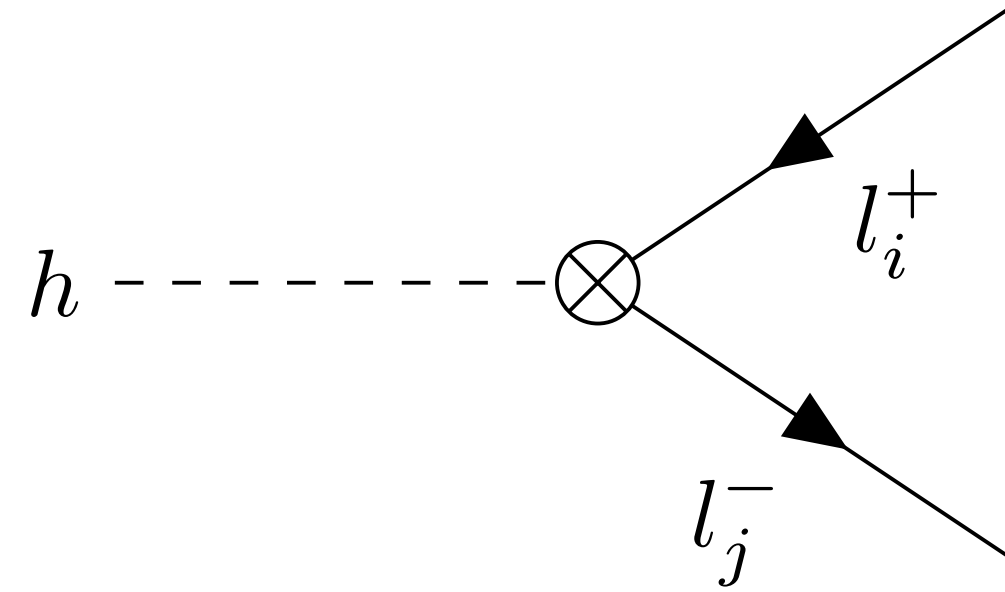
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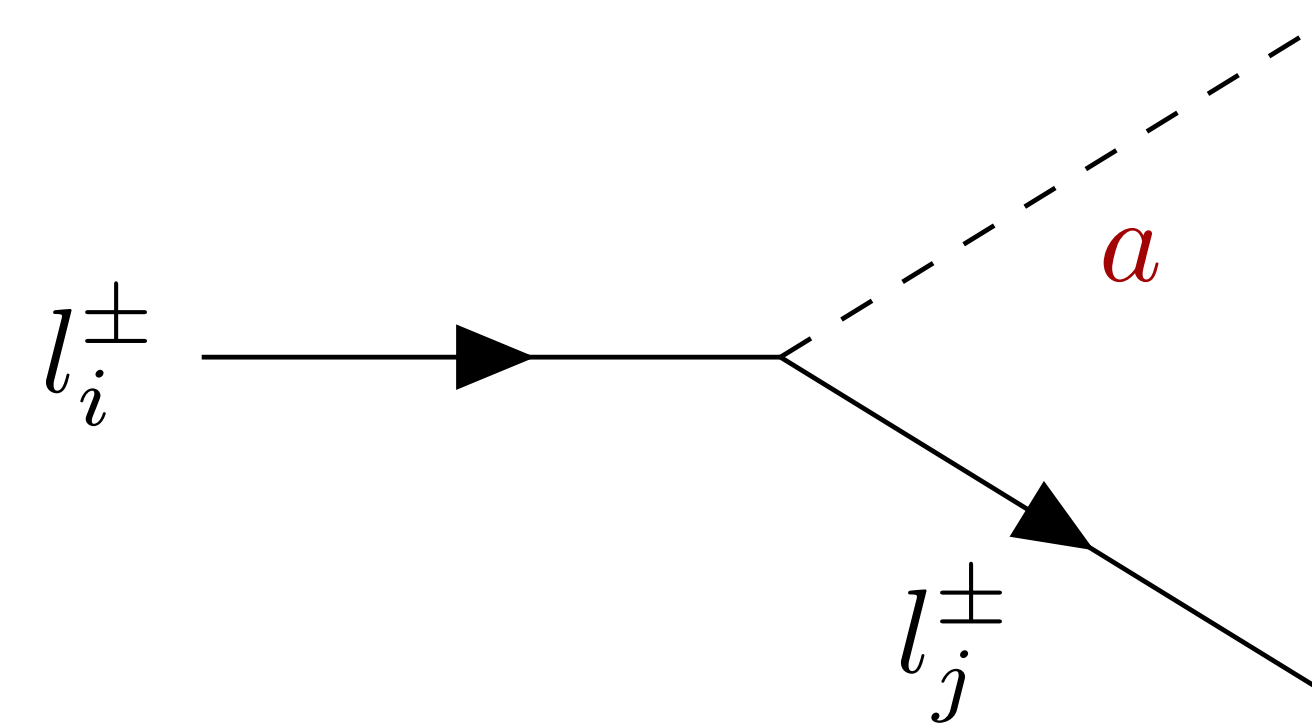
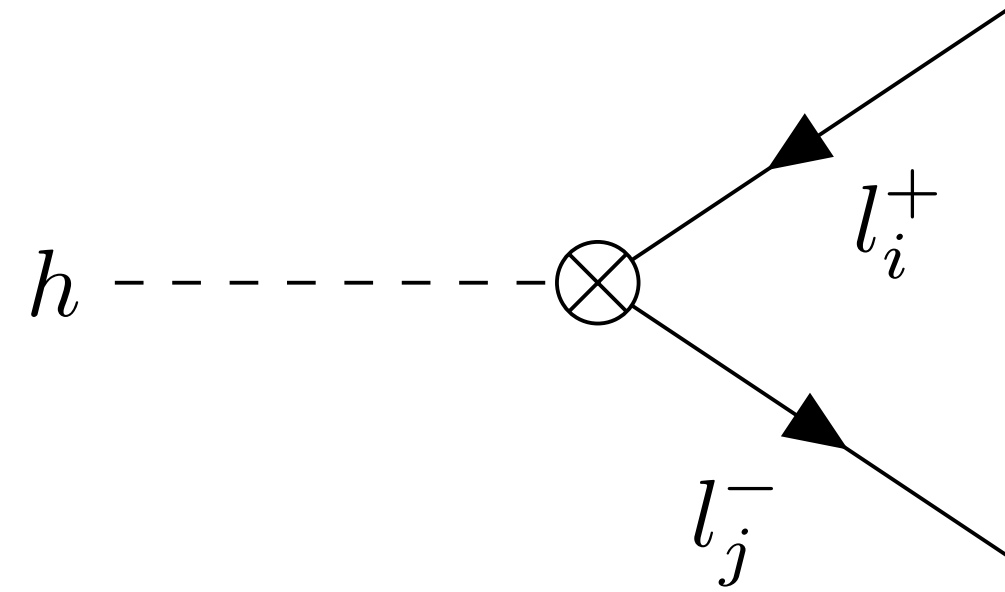


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$$\text{BR}(h \rightarrow l_i l_j) \simeq \text{BR}(l_i \rightarrow a l_j) \frac{2m_h}{m_{l_i}} \frac{\Gamma_{l_i}}{\Gamma_h} \frac{f_a^2}{v^2} \left(\frac{c_{\alpha-\beta}}{c_\beta s_\beta} \right)^2$$

Lepton flavour connection



Some mild deviations at ATLAS

$$\text{BR}(h \rightarrow \tau e) = 0.09 \pm 0.06 \%$$

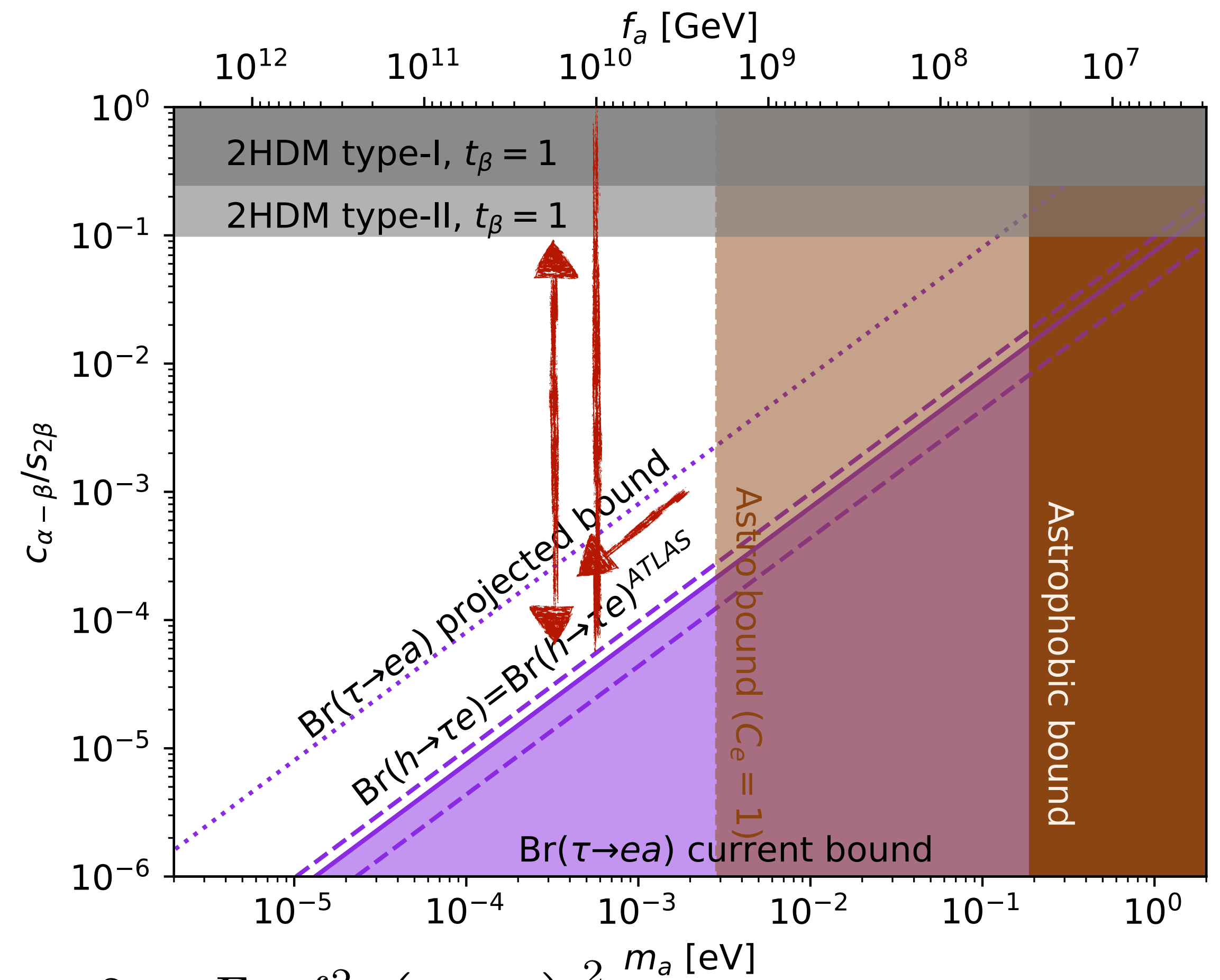
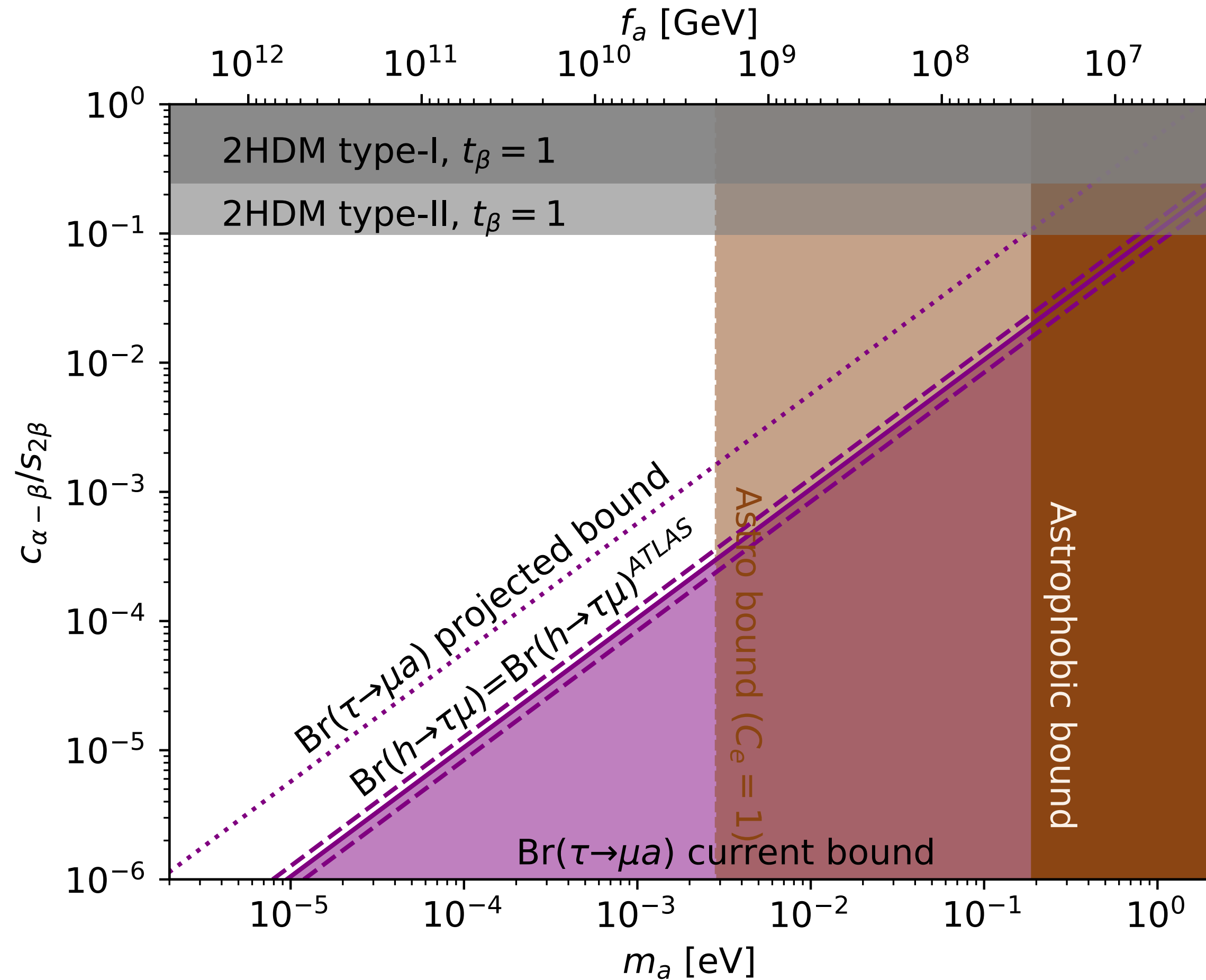
$$\text{BR}(h \rightarrow \tau \mu) = 0.11^{+0.05}_{-0.04} \%$$

[ATLAS 2302.05225]

$$\text{BR}(h \rightarrow l_i l_j) \simeq \text{BR}(l_i \rightarrow a l_j) \frac{2m_h}{m_{l_i}} \frac{\Gamma_{l_i}}{\Gamma_h} \frac{f_a^2}{v^2} \left(\frac{c_{\alpha-\beta}}{c_\beta s_\beta} \right)^2$$

$$\text{BR}(l_i \rightarrow l_j a) \simeq \frac{m_{l_i}^3}{16\pi\Gamma_{l_i}} \frac{|(C_e^{L,R})_{ij}|^2}{2f_a^2}$$

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$$BR(h \rightarrow l_i l_j) \simeq BR(l_i \rightarrow a l_j) \frac{2m_h}{m_{l_i}} \frac{\Gamma_{l_i}}{\Gamma_h} \frac{f_a^2}{v^2} \left(\frac{C_{\alpha-\beta}}{C_\beta S_\beta} \right)^2 m_a \text{ [eV]}$$

Yukawa textures

Astrophobic M1 model:

$$\mathcal{X}_{qL} = \text{diag}(0, 0, 1), \quad \mathcal{X}_{uR} = \text{diag}(s_\beta^2, s_\beta^2, s_\beta^2)$$

$$\mathcal{X}_{dR} = \text{diag}(c_\beta^2, c_\beta^2, c_\beta^2)$$

$$Y_1^u = \begin{pmatrix} y_{11}^u & y_{12}^u & y_{13}^u \\ y_{21}^u & y_{22}^u & y_{23}^u \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_2^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_{31}^u & y_{32}^u & y_{33}^u \end{pmatrix},$$

$$Y_1^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_{31}^d & y_{32}^d & y_{33}^d \end{pmatrix}, \quad Y_2^d = \begin{pmatrix} y_{11}^d & y_{12}^d & y_{13}^d \\ y_{21}^d & y_{22}^d & y_{23}^d \\ 0 & 0 & 0 \end{pmatrix}.$$

Astrophobic M4 model:

$$\mathcal{X}_{qL} = \text{diag}(0, 0, 0), \quad \mathcal{X}_{uR} = \text{diag}(s_\beta^2, s_\beta^2, s_\beta^2)$$

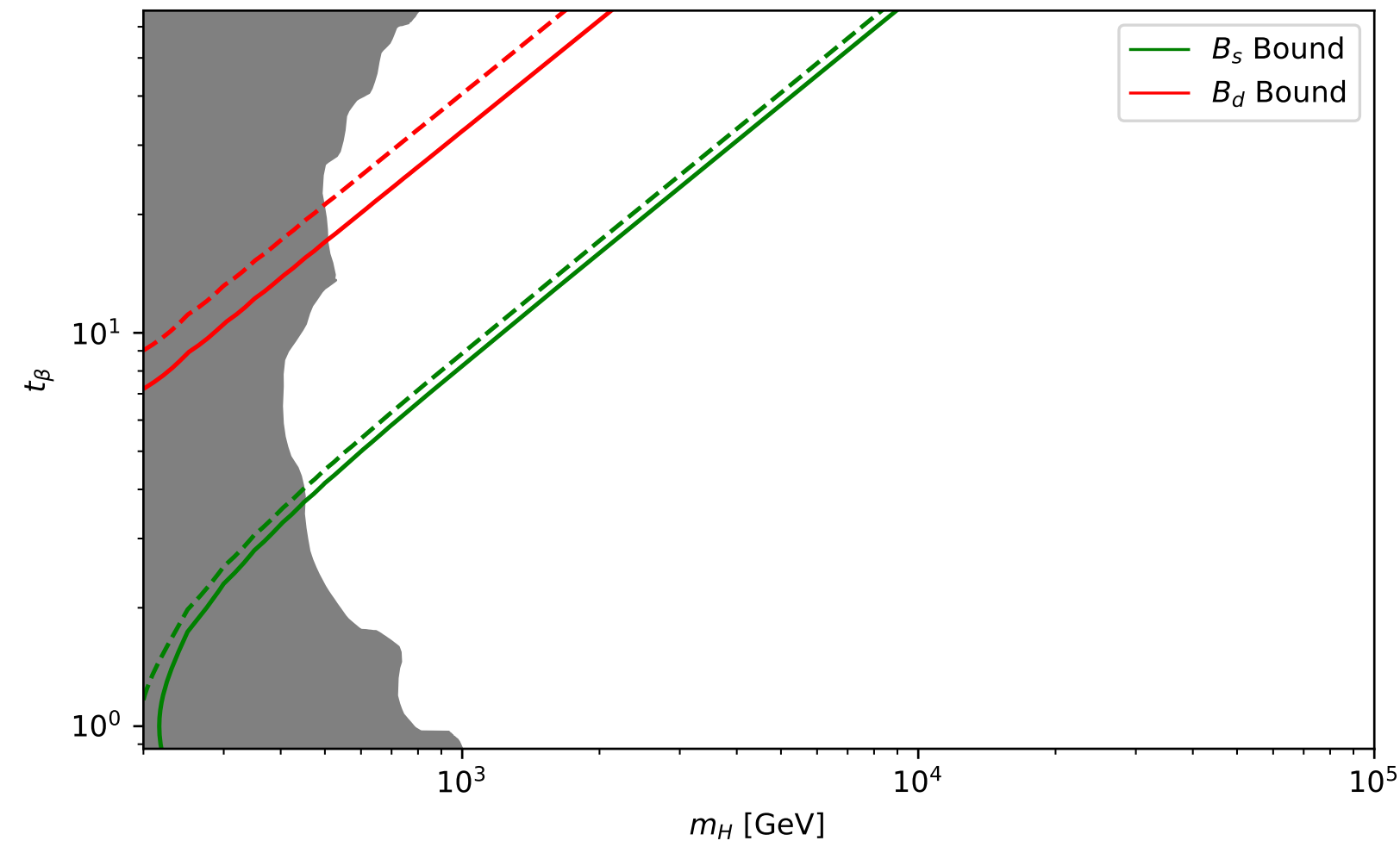
$$\mathcal{X}_{dR} = \text{diag}(c_\beta^2, -s_\beta^2, -s_\beta^2)$$

$$Y_1^u = \begin{pmatrix} y_{11}^u & y_{12}^u & y_{13}^u \\ y_{21}^u & y_{22}^u & y_{23}^u \\ y_{31}^u & y_{32}^u & y_{33}^u \end{pmatrix}, \quad Y_2^u = 0,$$

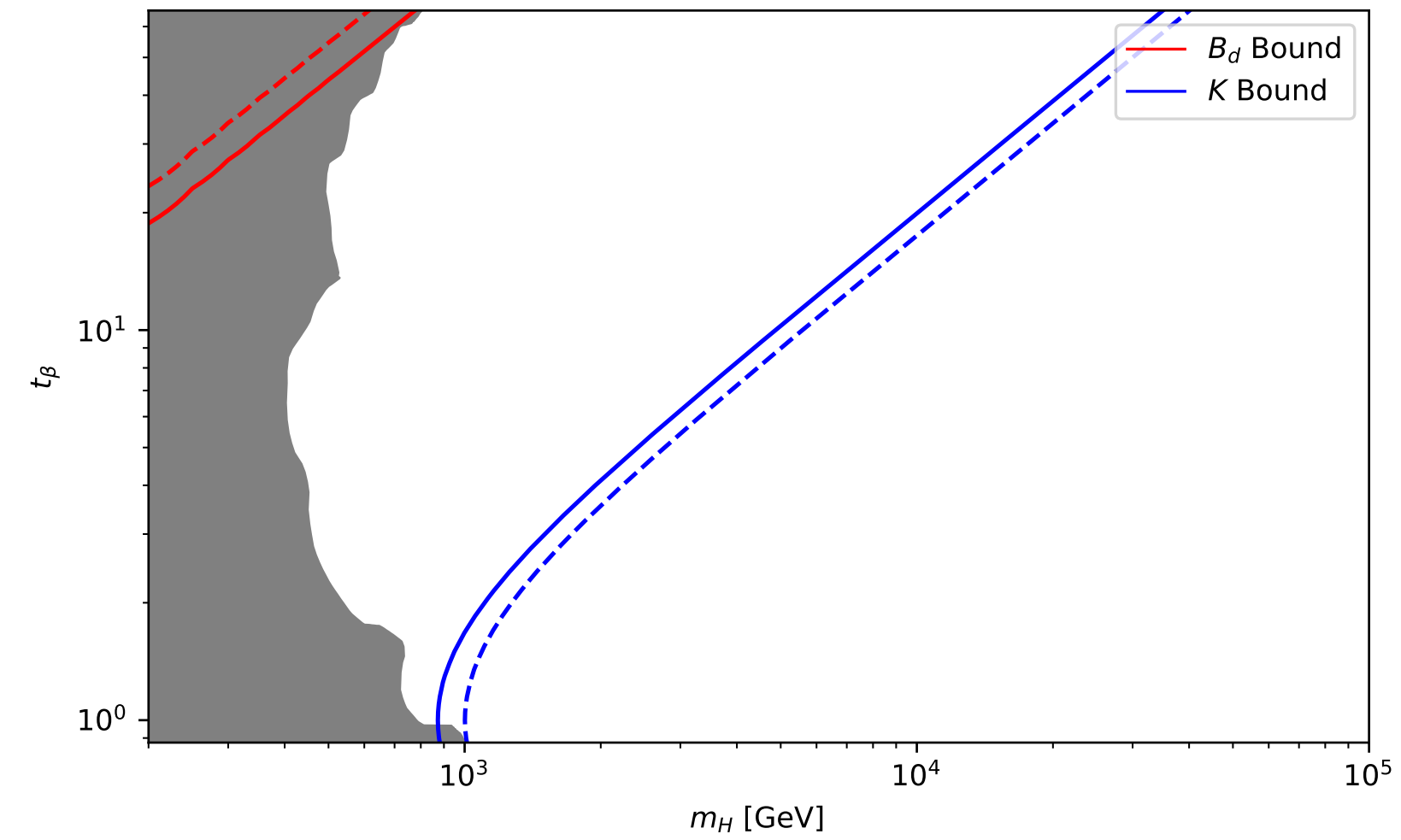
$$Y_1^d = \begin{pmatrix} 0 & y_{12}^d & y_{13}^d \\ 0 & y_{22}^d & y_{23}^d \\ 0 & y_{32}^d & y_{33}^d \end{pmatrix}, \quad Y_2^d = \begin{pmatrix} y_{11}^d & 0 & 0 \\ y_{21}^d & 0 & 0 \\ y_{31}^d & 0 & 0 \end{pmatrix}.$$

Specific Models

M4 Model

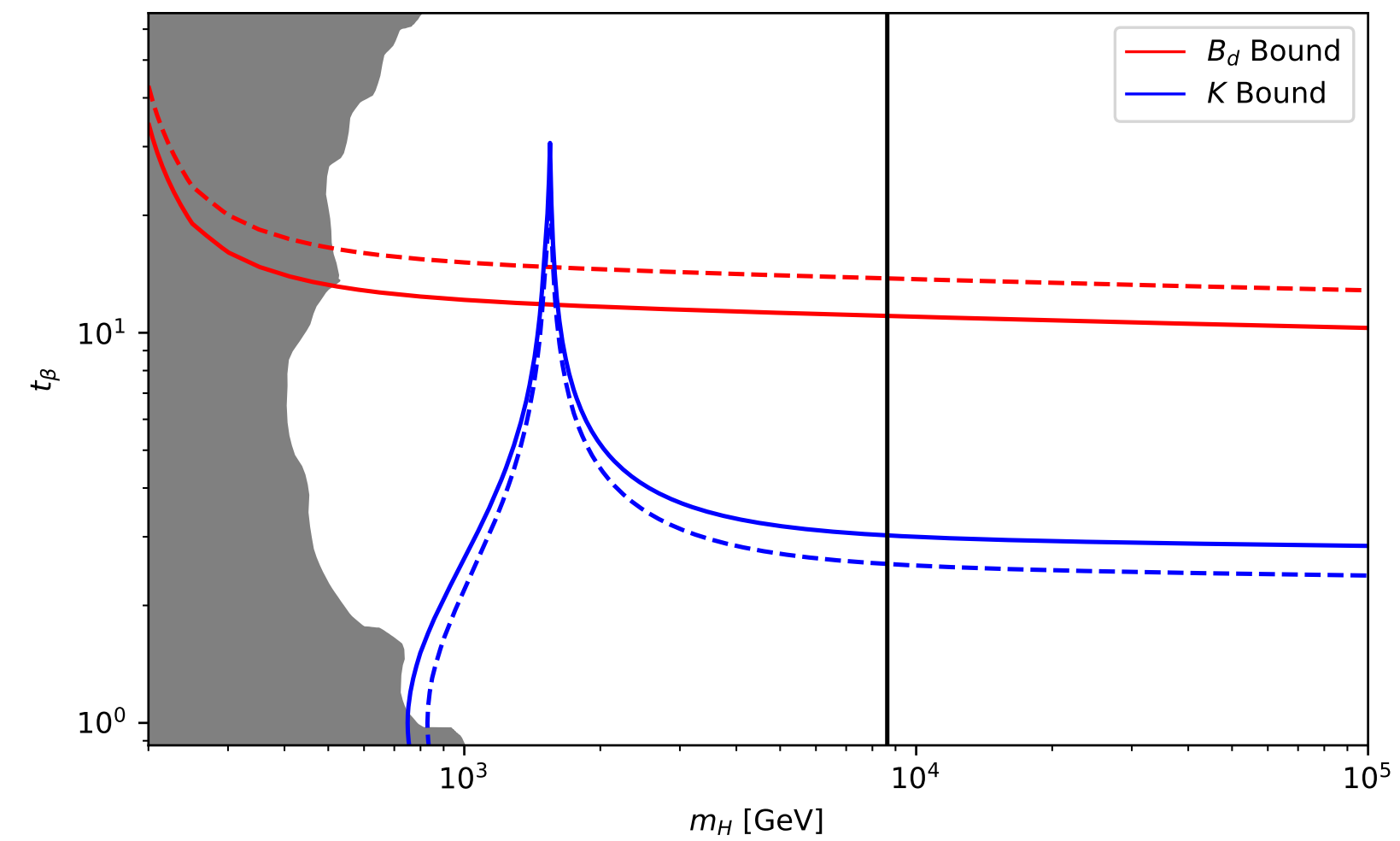
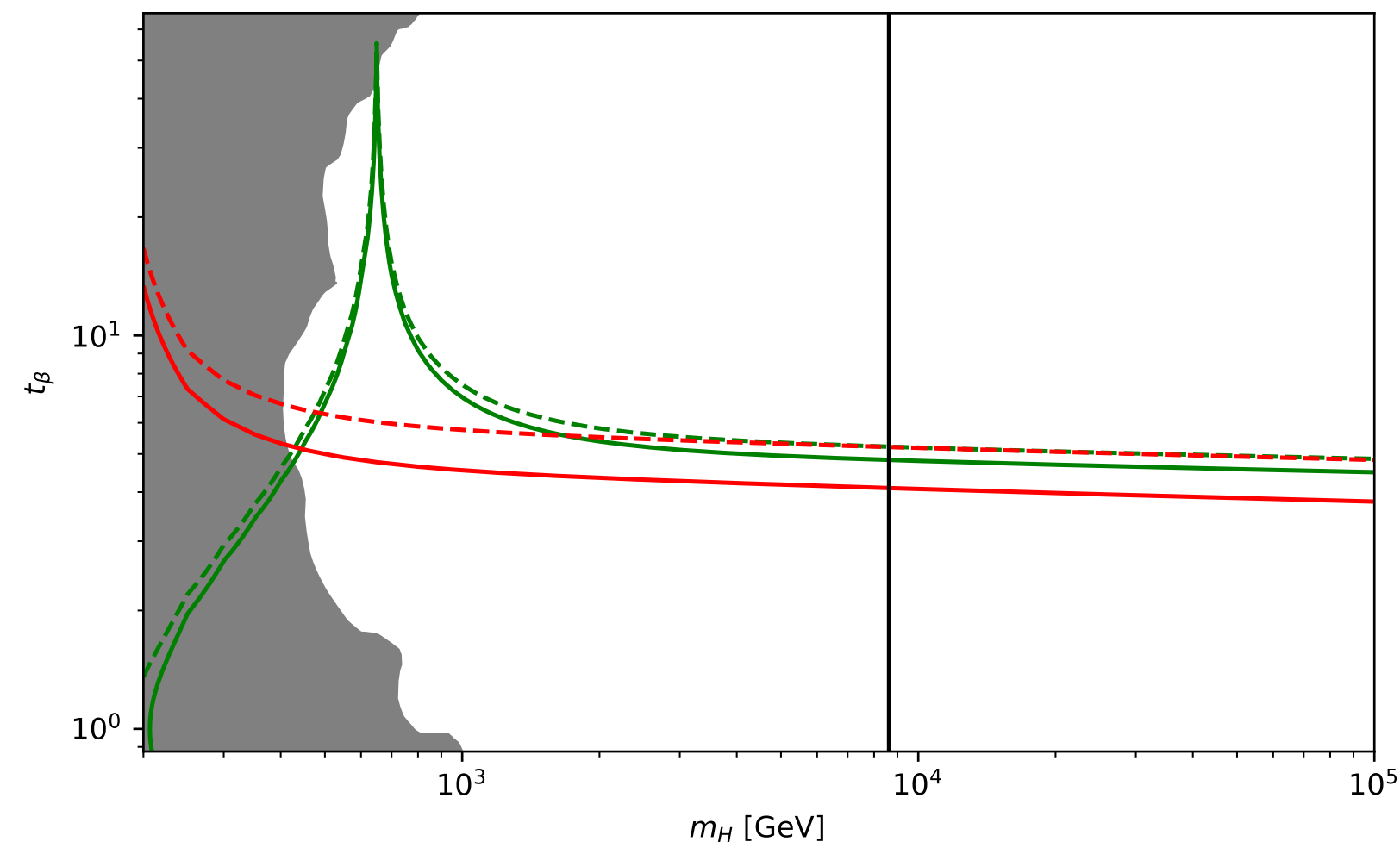


M1 Model



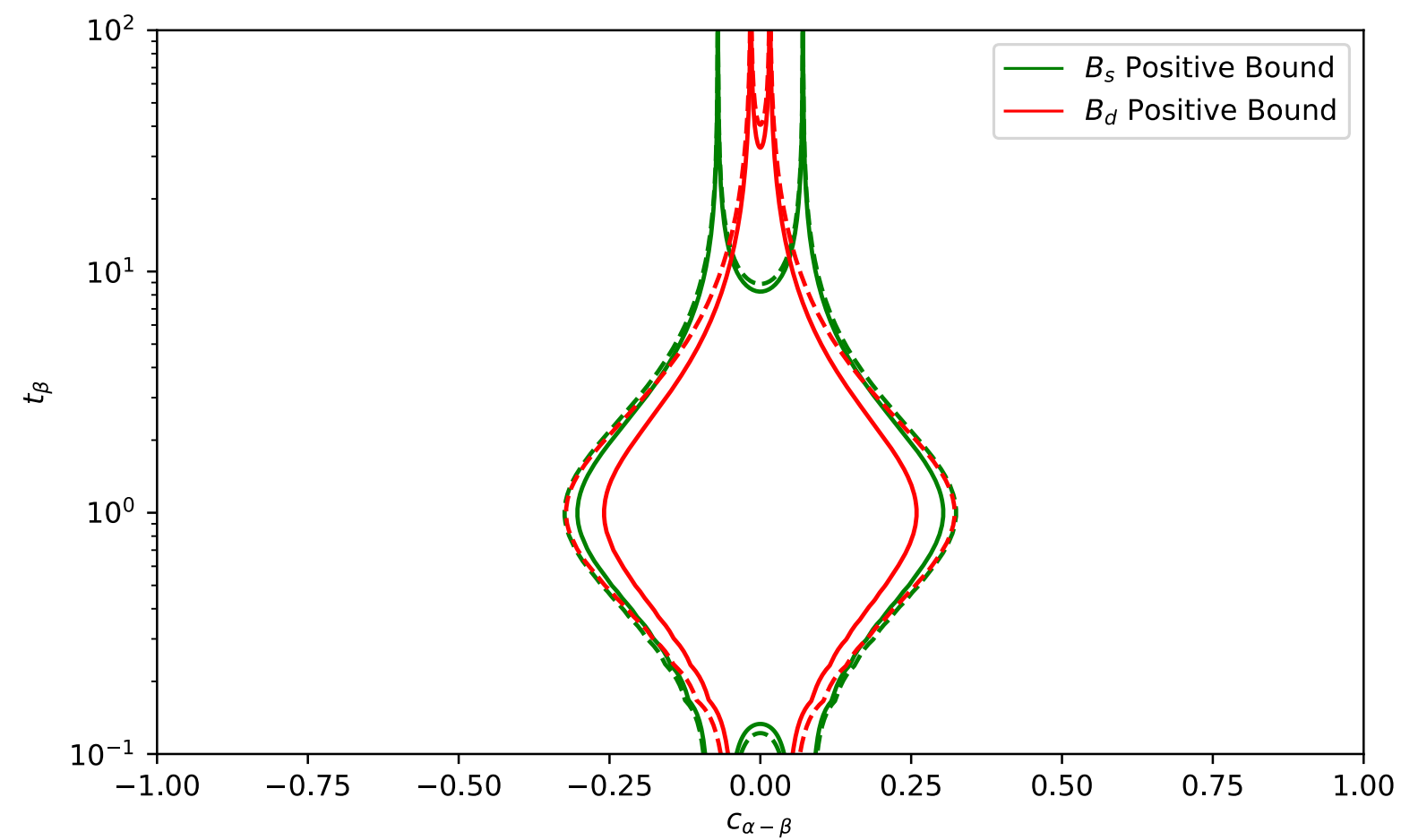
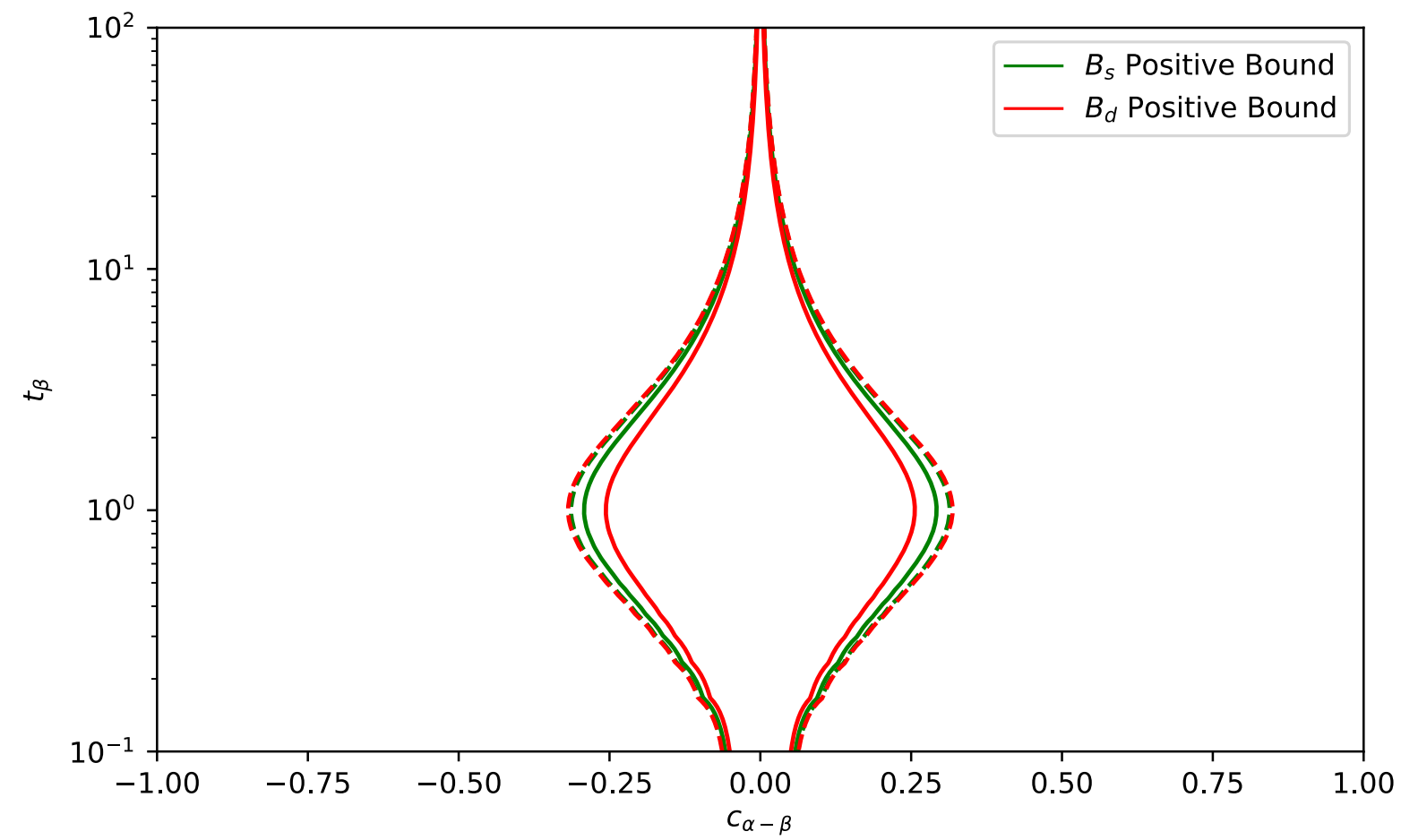
Assumption

$$V_{d_L, d_R} = V_{\text{CKM}}$$

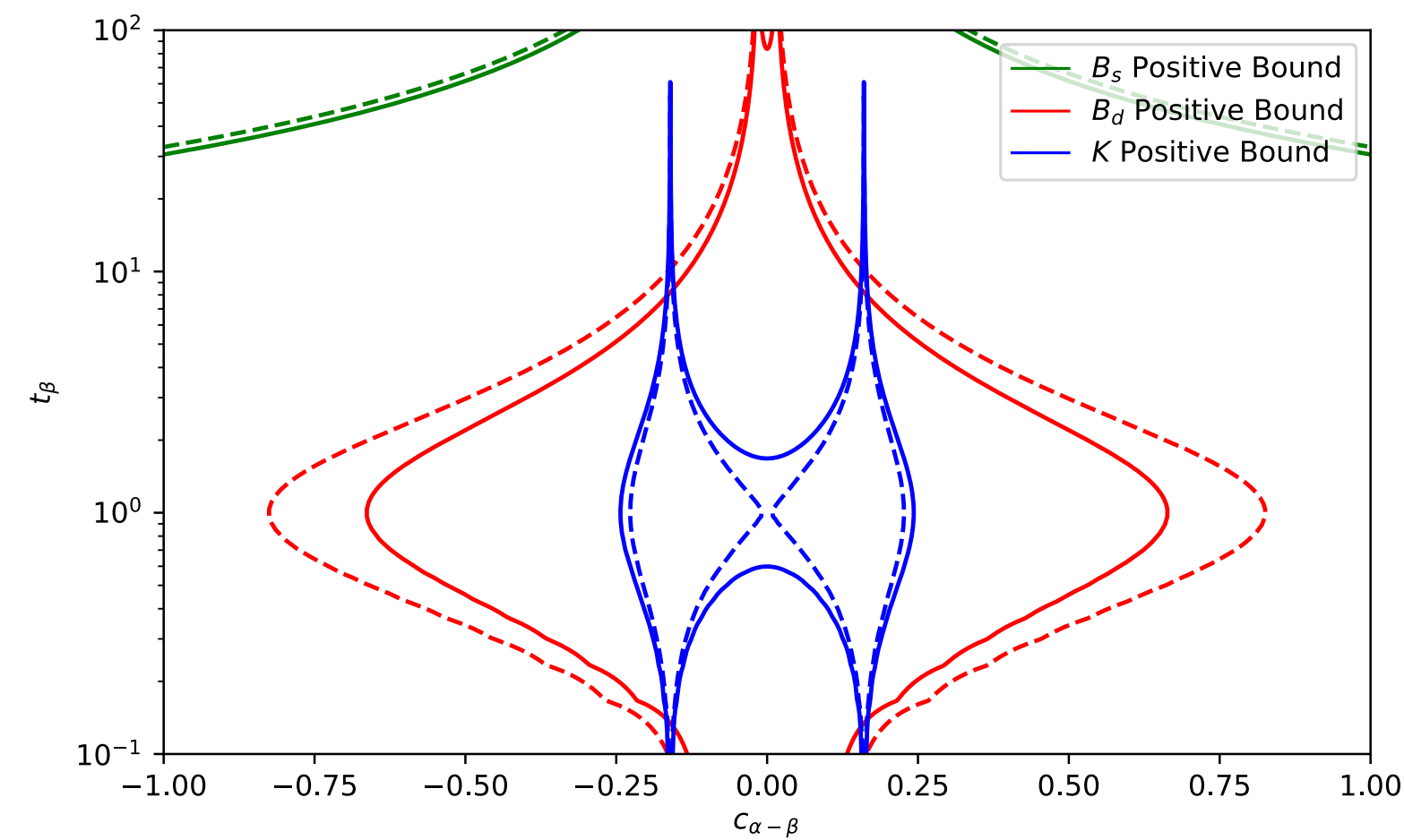
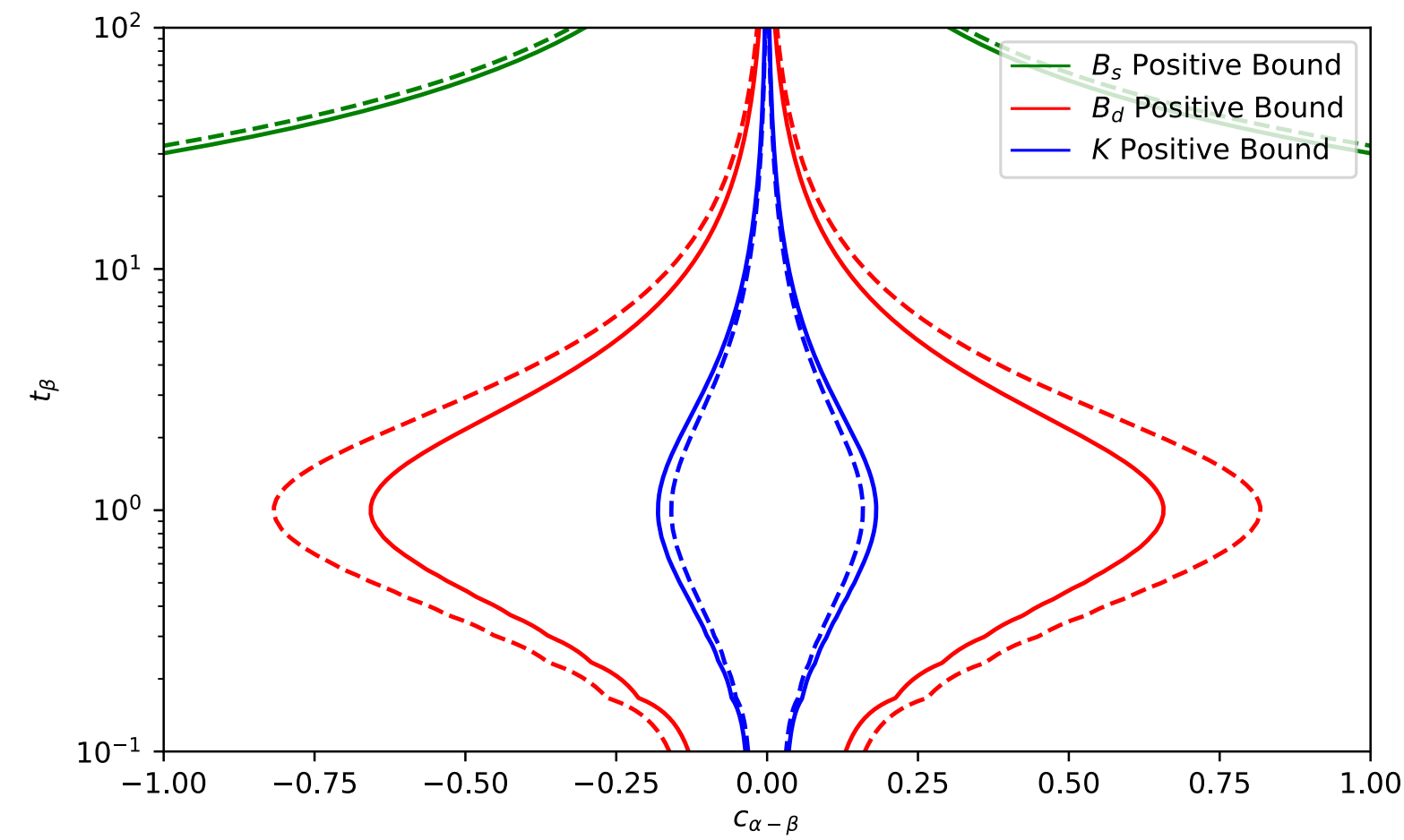


Specific Models

M4 Model



M1 Model



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