The QCD axion sum rule

Belén Gavela, Pablo Quílez, Maria Ramos

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Hunting Invisibles: Dark sectors, Dark matter and Neutrinos
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The standard QCD axion

In the Standard Model: $\mathcal{L} \supset \frac{\alpha_s}{8\pi}\bar{\theta} G\widetilde{G}$

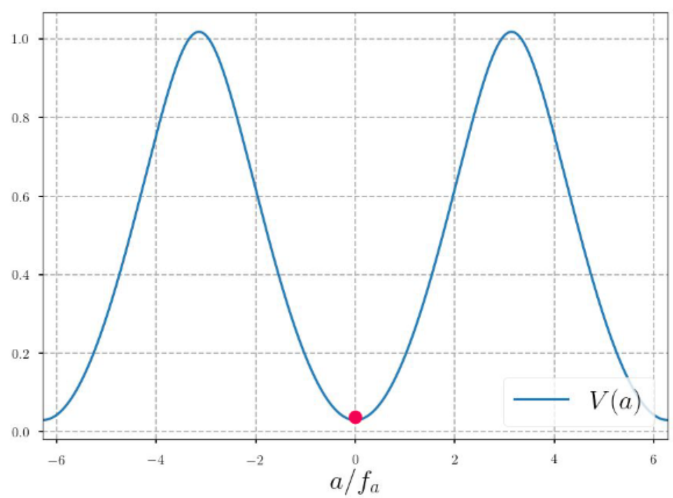
A dynamical $U(1)_A = U(1)_{PQ}$ solution:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\underbrace{\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}}_{a/f_a}\right) G\widetilde{G}$$

The standard QCD axion

In the Standard Model: $\mathcal{L} \supset \frac{\alpha_s}{8\pi} \bar{\theta} G\widetilde{G}$ $\quad d = u = d \quad \lesssim 10^{-10}$

A dynamical $U(1)_A = U(1)_{PQ}$ solution: $\mathcal{L} = \frac{\alpha_s}{8\pi} \underbrace{\left(\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta} \right)}_{a/f_a} G\widetilde{G}$



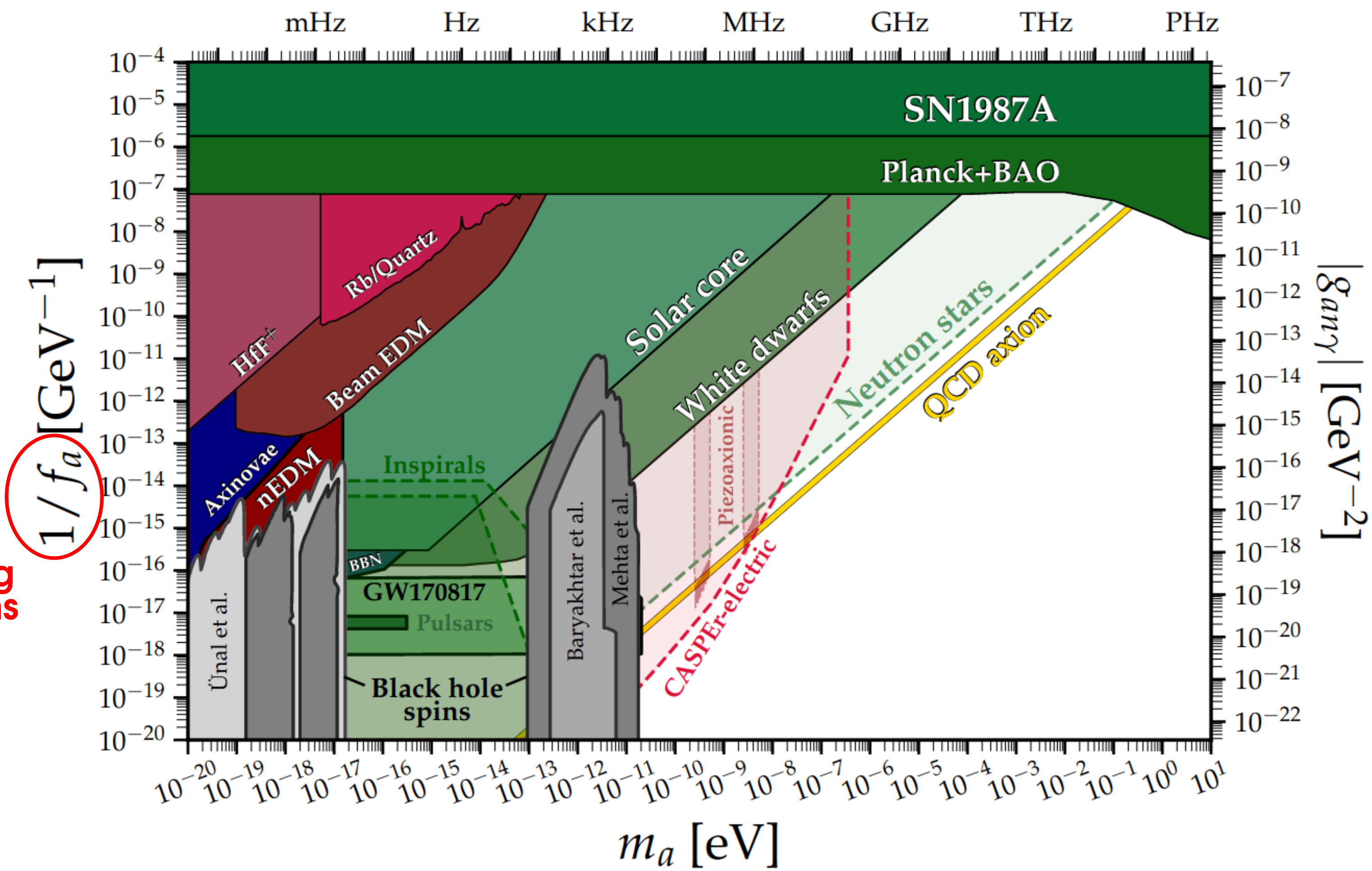
$$\Rightarrow V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{1}{2}\frac{a}{f_a}\right)}$$

standard QCD axion

$$m_a^2 f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

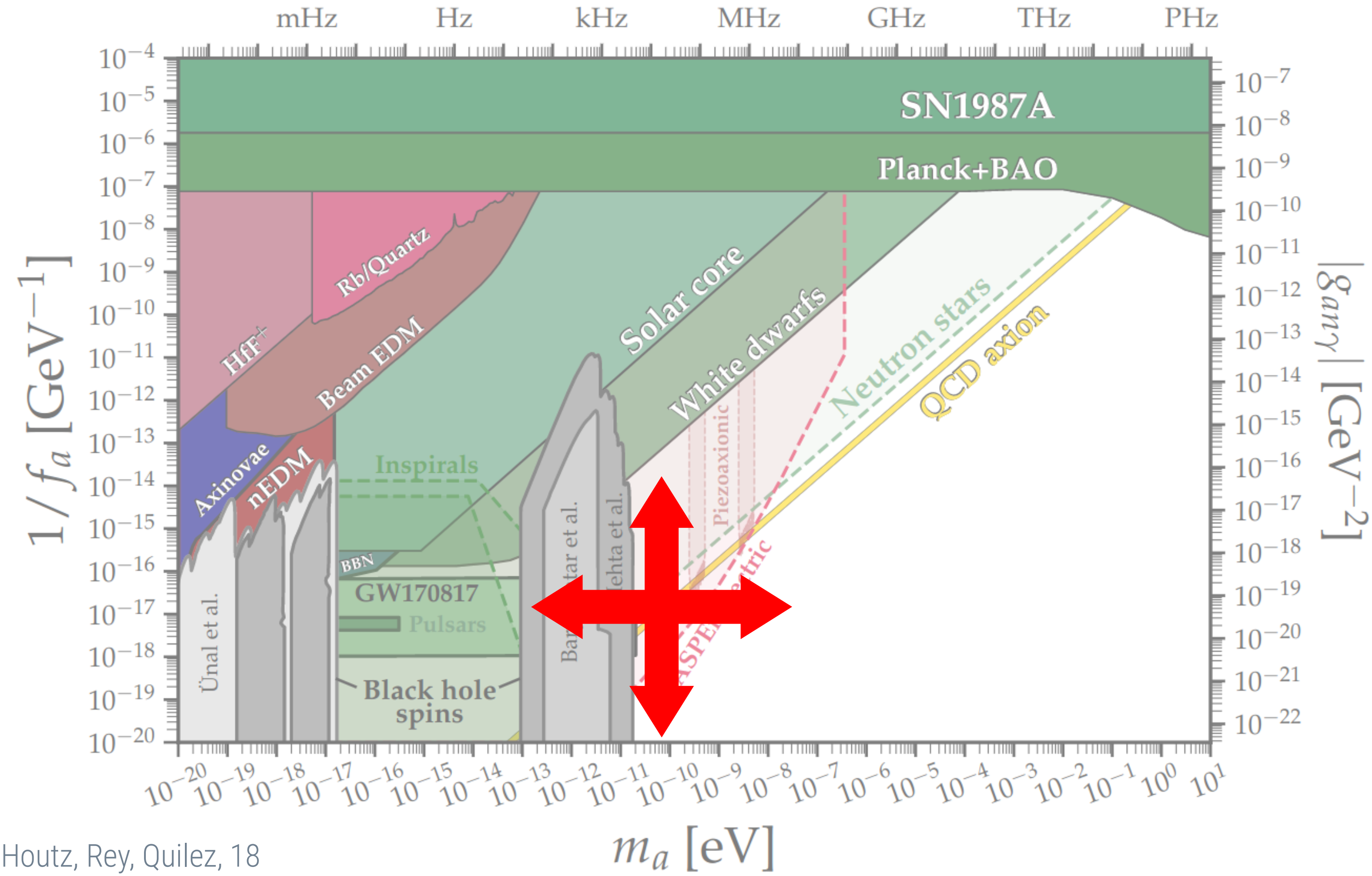
The standard QCD axion

Also a dark matter candidate!



coupling to gluons

The non-standard axion



[many others]
Gaillard, Gavela, Houtz, Rey, Quilez, 18
Csaki et al, 19
Gherghetta et al, 20
Hook, 18
Luzio, Gavela, Quilez, Ringwald 21
Valenti, Vecchi, Xu 22

Challenging the standard assumptions

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}\right) G\widetilde{G}$$

$$\Rightarrow m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2\, \frac{m_u\, m_d}{(m_u + m_d)^2}$$

interaction basis = mass basis

But the axion may not be the only singlet scalar in Nature.

e.g. "String axiverse"

Arvanitakia, Dimopoulos, Dubovskyc, Kalopere, Russell 09

Challenging the standard assumptions

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$$\Rightarrow m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2\, \frac{m_u\, m_d}{(m_u + m_d)^2}$$

interaction basis ≠ mass basis

So instead, one can have:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V'(\hat{a}_{G\widetilde{G}}, \ldots, \hat{a}_N)$$

$$\Rightarrow m_i^2\, f_i^2 = g_i \chi_{\rm QCD}$$ **within QCD**

Toy model: 2 axions

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \bar{\theta}\right) G\widetilde{G} - \frac{\mu^2}{2}\hat{a}_2^2$$

Equivalently,

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{a_{G\widetilde{G}}}{F} + \bar{\theta}\right) G\widetilde{G} - \frac{\mu^2}{4}(\underbrace{a_{G\widetilde{G}} - a_\perp}_{\text{mixes with N-1 fields}})^2$$

Toy model: 2 axions

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \bar{\theta}\right) G\widetilde{G} - \frac{\mu^2}{2}\hat{a}_2^2$$

Below confinement:

$$V_{N=2} \supset \frac{\chi_{\rm QCD}}{2}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2}\right)^2 - V(\hat{a}_2)$$

Minima

$$\frac{\chi_{\rm QCD}}{\hat{f}_1}\sin\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2}\right) = 0 \quad \text{and} \quad \frac{\chi_{\rm QCD}}{\hat{f}_2}\sin\underbrace{\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2}\right)}_{v_1+v_2=0} - \underbrace{\frac{\partial V(\hat{a}_2)}{\partial \hat{a}_2}}_{v_2=0} = 0$$

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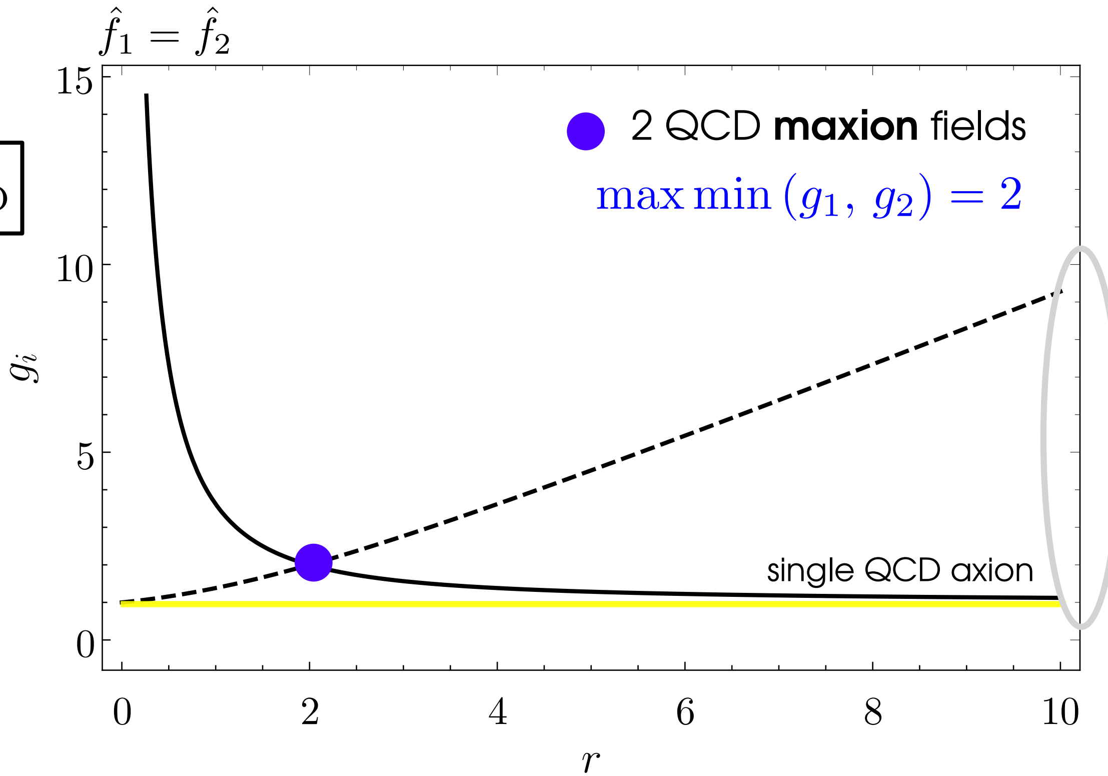
$$\mathbf{M^2} = \chi_{\rm QCD}\begin{pmatrix} 1/\hat{f}_1^2 & 1/(\hat{f}_1\hat{f}_2) \\ 1/(\hat{f}_1\hat{f}_2) & (1+r)/\hat{f}_2^2 \end{pmatrix}$$

$$r \equiv \mu^2\frac{\hat{f}_2^2}{\chi_{\rm QCD}}$$

Toy model: 2 axions

$$m_i^2\, f_i^2 = g_i \chi_{\rm QCD}$$



Decoupling limit assumed in other constructions

Kim et al, 04
Agrawal et al, 17 and 22
Farina et al, 17
Coy, Frigerio, 17
Choe et al, 14 and 16
Kaplan et al, 16
Giudice et al, 16

$$\mathbf{M^2} = \chi_{\rm QCD} \begin{pmatrix} 1/\hat{f}_1^2 & 1/(\hat{f}_1 \hat{f}_2) \\ 1/(\hat{f}_1 \hat{f}_2) & (1+r)/\hat{f}_2^2 \end{pmatrix}$$

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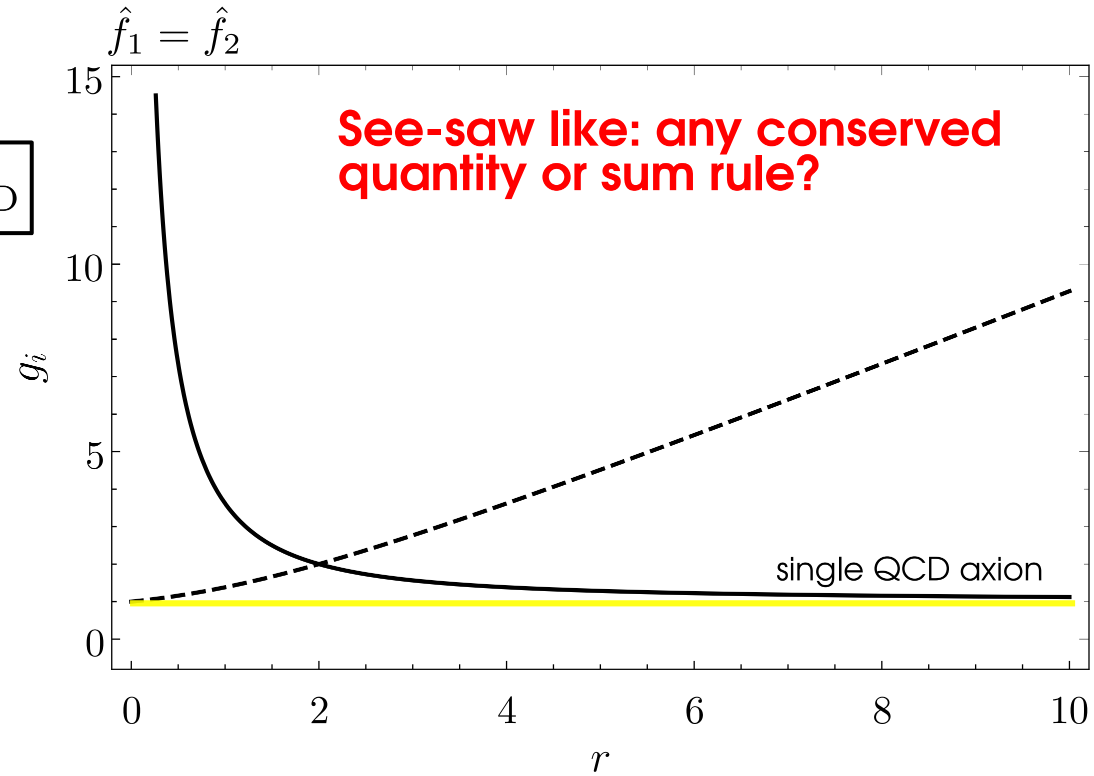


$$g_1 = \frac{2\hat{f}_2^2 h(r)}{\hat{f}_2^2 h(r) - \hat{f}_2^2 + \hat{f}_1^2\,(r-1)}$$

$$g_2 = \frac{2\hat{f}_2^2 h(r)}{\hat{f}_2^2 h(r) + \hat{f}_2^2 - \hat{f}_1^2\,(r-1)}$$

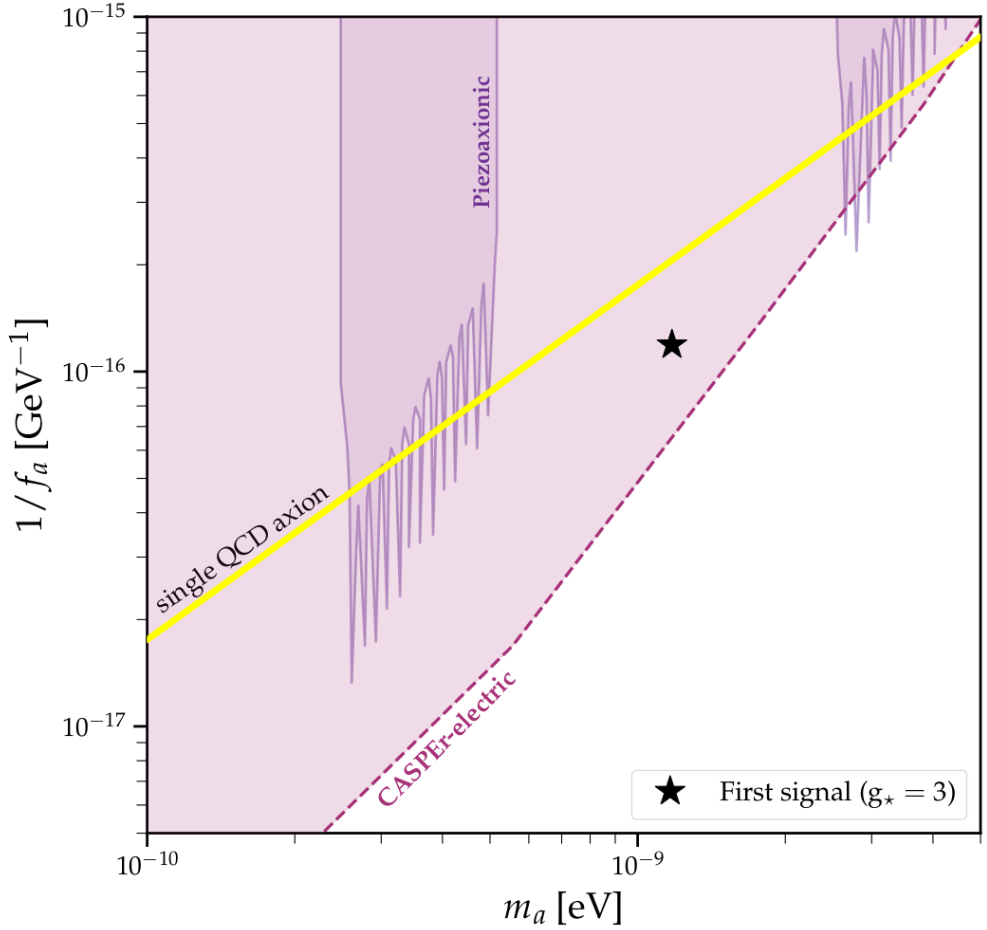
$$\beta_1 + \beta_2 = 1$$

$$\beta_i \equiv 1/g_i$$

Strong constraint on the system!

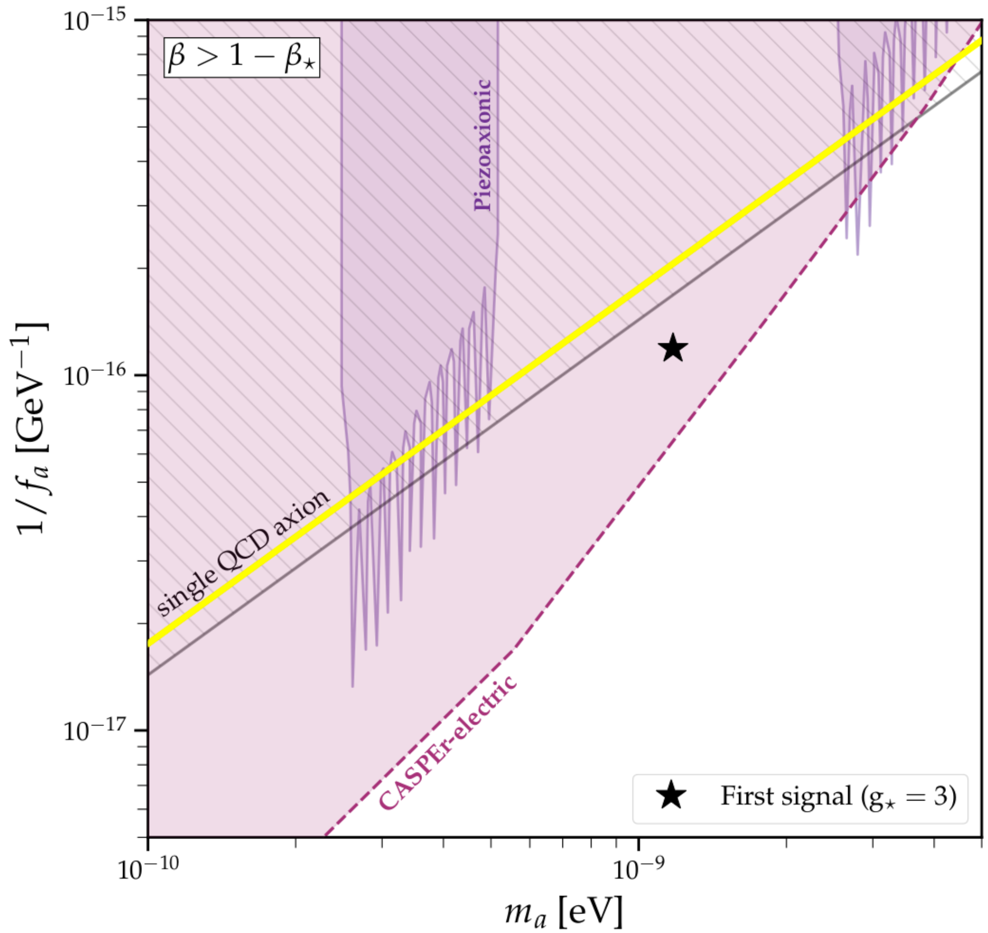
$$h(r) \equiv \sqrt{1 - 2\frac{\hat{f}_1^2}{\hat{f}_2^2}\,(r-1) + \frac{\hat{f}_1^4}{\hat{f}_2^4}\,(r+1)^2}$$

An ALP or a true QCD axion?



Assume a first signal is measured in the star location

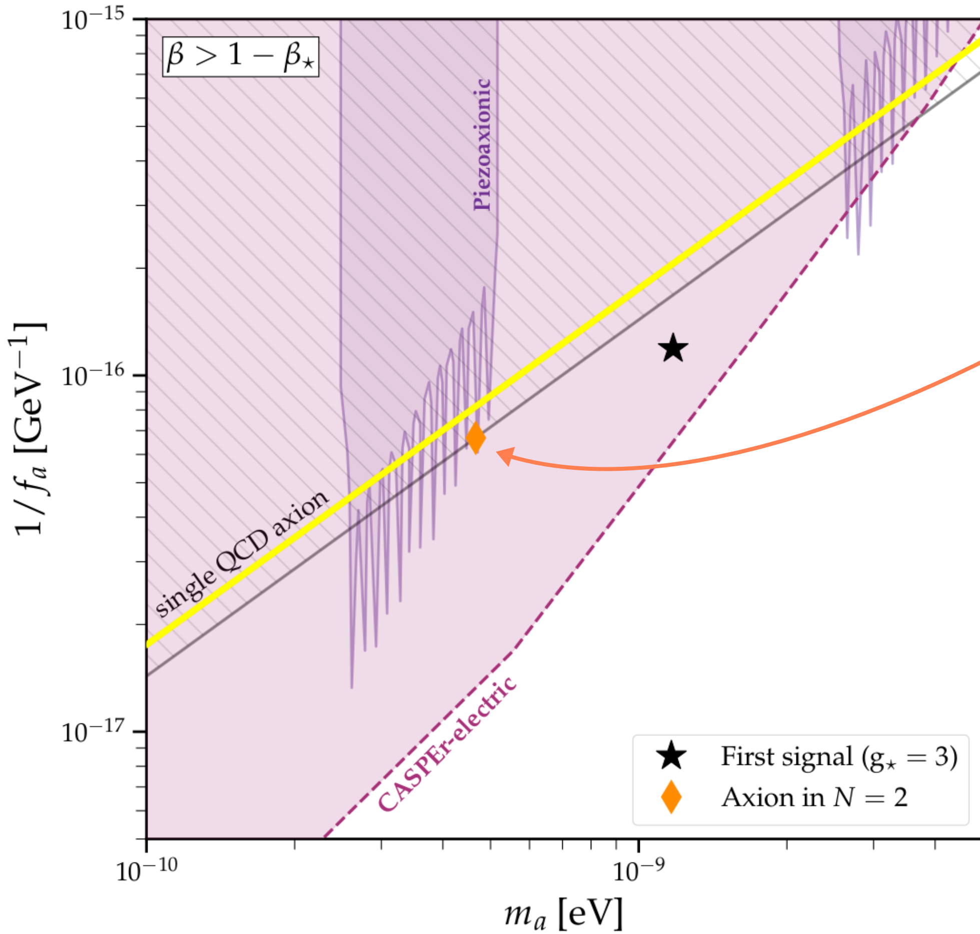
An ALP or a true QCD axion?



Have to wait for another signal outside the dashed region

An ALP or a true QCD axion?

$$\sum_{i=1}^{2} \beta_i = 1$$

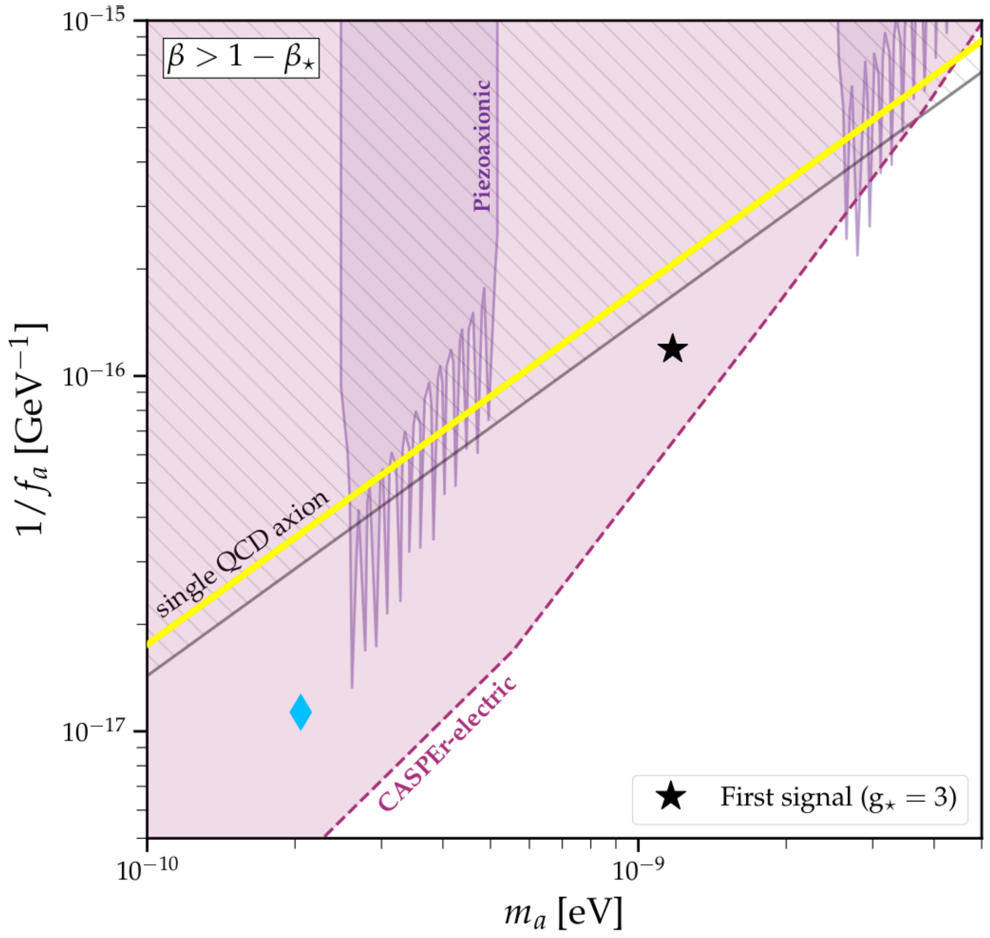


Then we have found all axions in Nature!

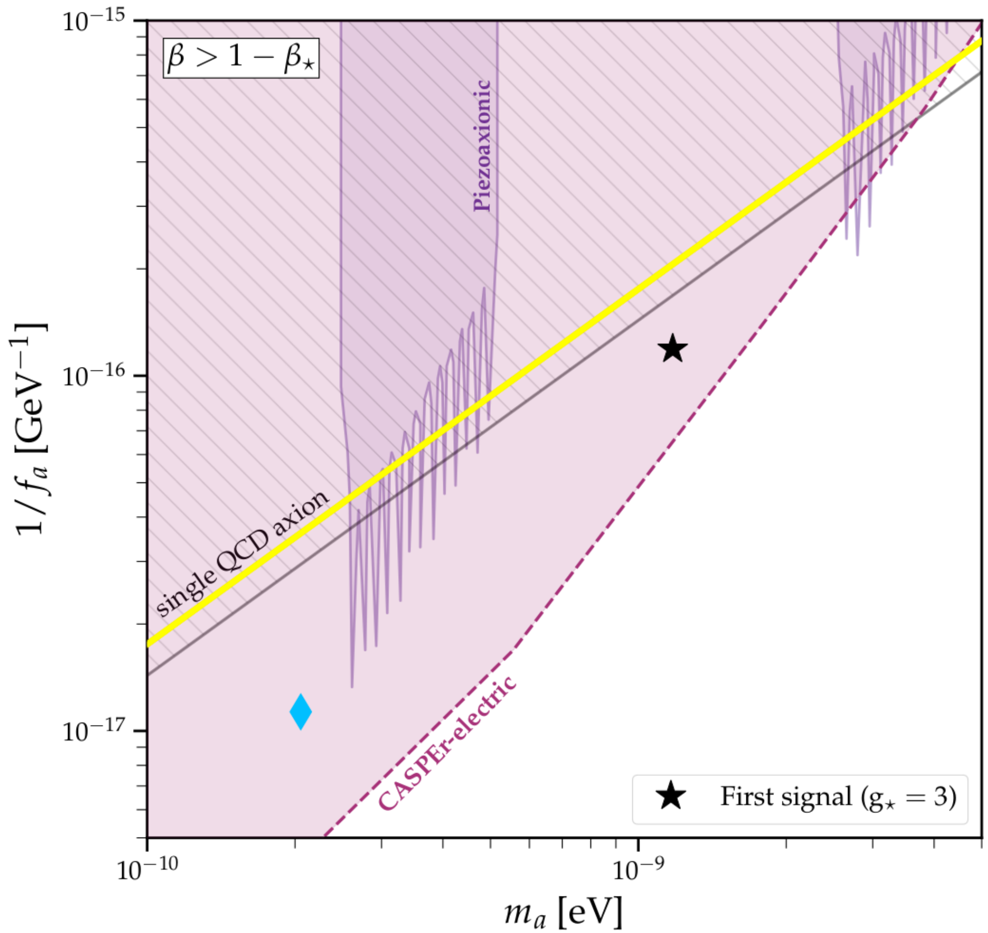
Have to wait for another signal outside the dashed region

What if?

$$\sum_{i=1}^{2} \beta_i < 1$$



General potential for arbitrary N scalars



Exact results and sum rules

PQ condition

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1,\, \hat{a}_2, \dots, \hat{a}_N) \to \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V_B^{\mathrm{R}}(\hat{a}_{G\widetilde{G}}, \dots)$$

$$\frac{1}{F^2} = \sum_{k=1}^{N}\frac{1}{\hat{f}_k^2}$$

A preferred basis.

$$\boxed{\mathbf{M}^2 \equiv \mathbf{R}\,\hat{\mathbf{M}}^2\mathbf{R}^T}$$

$$\mathbf{M}^2 = \mathbf{M}_A^2 + \mathbf{M}_B^2 = \begin{pmatrix} b_{11} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix} = \frac{\chi_{\mathrm{QCD}}}{F^2}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{0} \end{pmatrix} + \begin{pmatrix} b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix},$$

PQ condition

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$$\boxed{\exists\, U(1)_{PQ} \implies \lim_{\chi_{\rm QCD}\to 0} \det \mathbf{M}^2 = 0 \implies \det \mathbf{M}_B^2 = 0} \quad \langle \hat{a}_0 | a_{G\widetilde{G}} \rangle \neq 0$$

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Applying Schur's formula.

$$\det \mathbf{M}_1^2 \left(b_{11} - \frac{\chi_{\rm QCD}}{F^2} - \mathbf{X}^\dagger \mathbf{M}_1^{-2}\mathbf{X}\right) = 0$$

$$\Rightarrow \boxed{\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2}\mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}}$$

Generalized sum-rule

$$\Rightarrow \quad \frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2} \mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}$$

Moving to the physical basis.

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \frac{a_i}{f_i} G\widetilde{G} \qquad \text{with} \qquad \frac{1}{f_i} = \frac{\langle \hat{a}_{G\widetilde{G}} | a_i \rangle}{F} \equiv \frac{v_{i1}}{F} \implies \sum_{i=1}^{N} \frac{1}{f_i^2} = \frac{1}{F^2}$$

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PETER B. DENTON, STEPHEN J. PARKE, TERENCE TAO, AND XINING ZHANG

ABSTRACT. If A is an $n \times n$ Hermitian matrix with eigenvalues $\lambda_1(A), \ldots, \lambda_n(A)$ and $i, j = 1, \ldots, n$, then the $j^{\rm th}$ component $v_{i,j}$ of a unit eigenvector v_i associated to the eigenvalue $\lambda_i(A)$ is related to the eigenvalues $\lambda_1(M_j), \ldots, \lambda_{n-1}(M_j)$ of the minor M_j of A formed by removing the $j^{\rm th}$ row and column by the formula

$$|v_{i,j}|^2 \prod_{k=1; k\neq i}^{n} (\lambda_i(A) - \lambda_k(A)) = \prod_{k=1}^{n-1} (\lambda_i(A) - \lambda_k(M_j)) \ .$$

We refer to this identity as the *eigenvector-eigenvalue identity*

Generalized sum-rule

$$\Rightarrow \frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2} \mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}$$

Moving to the physical basis.

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \frac{a_i}{f_i} G\widetilde{G} \qquad \text{with} \qquad \frac{1}{f_i} = \frac{\langle \hat{a}_{G\widetilde{G}} | a_i \rangle}{F} \equiv \frac{v_{i1}}{F} \Longrightarrow \sum_{i=1}^{N} \frac{1}{f_i^2} = \frac{1}{F^2}$$

A corollary.
(generic A matrix)

$$\frac{\det\left(\lambda \mathbb{I}_{N-1} - M_j\right)}{\det\left(\lambda \mathbb{I}_N - A\right)} = \sum_{i=1}^{N} \frac{|v_{ij}|^2}{\lambda(A) - \lambda_i(A)}$$

$$\frac{\det \mathbf{M}_1^2}{\det \mathbf{M}^2} = \sum_{i=1}^{N} \frac{|v_{1i}|^2}{m_i^2} = \frac{F^2}{\chi_{\rm QCD}} \sum_{i=1}^{N} \frac{1}{g_i} \xrightarrow{\exists\, U(1)_{\rm PQ}} \boxed{\sum_{i=1}^{N} \beta_i = 1, \;\; \beta_i \equiv \frac{1}{g_i}}$$

axionness is shared

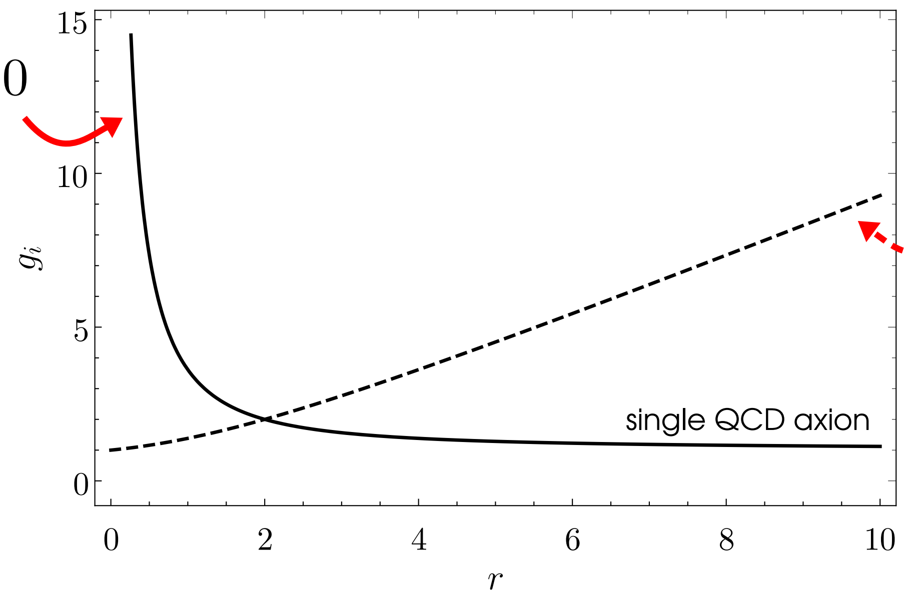
A more physical interpretation

$$\beta_i = \frac{\langle \hat{a}_{\rm PQ} \mid a_i \rangle \langle a_i \mid \hat{a}_{G\widetilde{G}} \rangle}{\langle \hat{a}_{\rm PQ} \mid \hat{a}_{G\widetilde{G}} \rangle}$$

Recall the toy example:

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \bar{\theta}\right) G\widetilde{G} - \frac{\mu^2}{2}\hat{a}_2^2 \implies \quad \hat{a}_{GG} = \frac{1}{2}(\hat{a}_1 + \hat{a}_2) \quad \text{and} \quad \hat{a}_{\rm PQ} = \hat{a}_1$$

$$\beta_1 = 1 \times 0$$



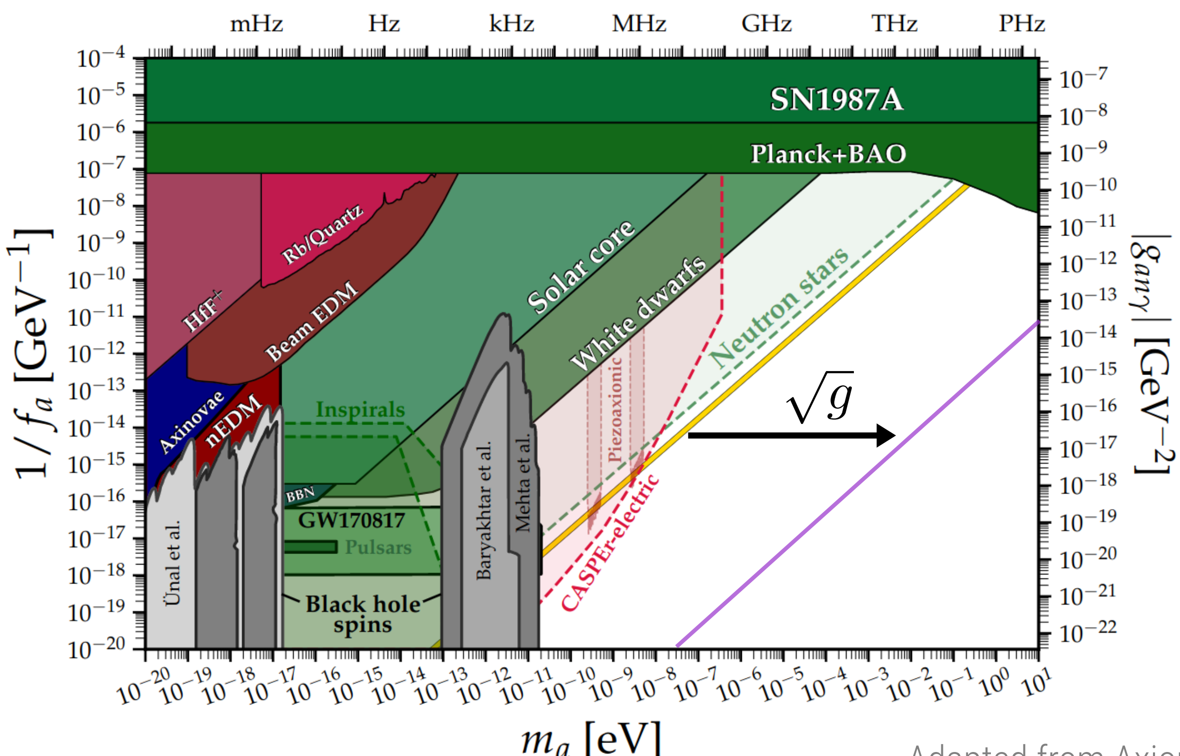
$$\beta_2 = 0 \times 1$$

$$r \equiv \mu^2 \frac{\hat{f}_2^2}{\chi_{\rm QCD}}$$

Experimental consequences

$$\mathbf{1.}\ g_i \geq 1 \quad \left(= 1 + \frac{F^2}{\chi_{\rm QCD}} \frac{\langle a_i | \mathbf{M}_B^2 | a_i \rangle}{\left| \langle a_i | a_{G\widetilde{G}} \rangle \right|^2} \right)$$

$$g_i = \frac{m_i^2\, f_i^2}{\chi_{\rm QCD}}$$

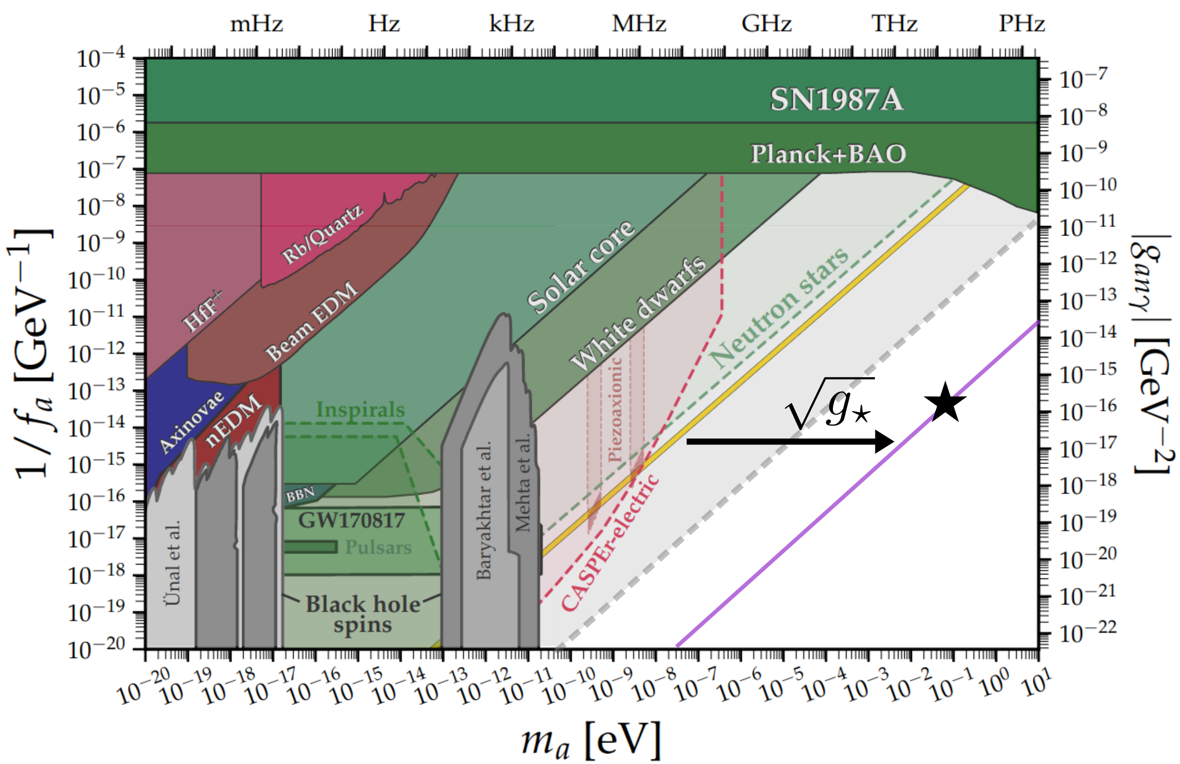


Adapted from AxionLimits

Experimental consequences

$$\mathbf{2}.\ g_j \geq \frac{1}{1 - 1/g_\star}$$

$$g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}$$



Experimental consequences

$$\mathbf{2.}\ g_j \geq \frac{1}{1 - 1/g_\star}$$

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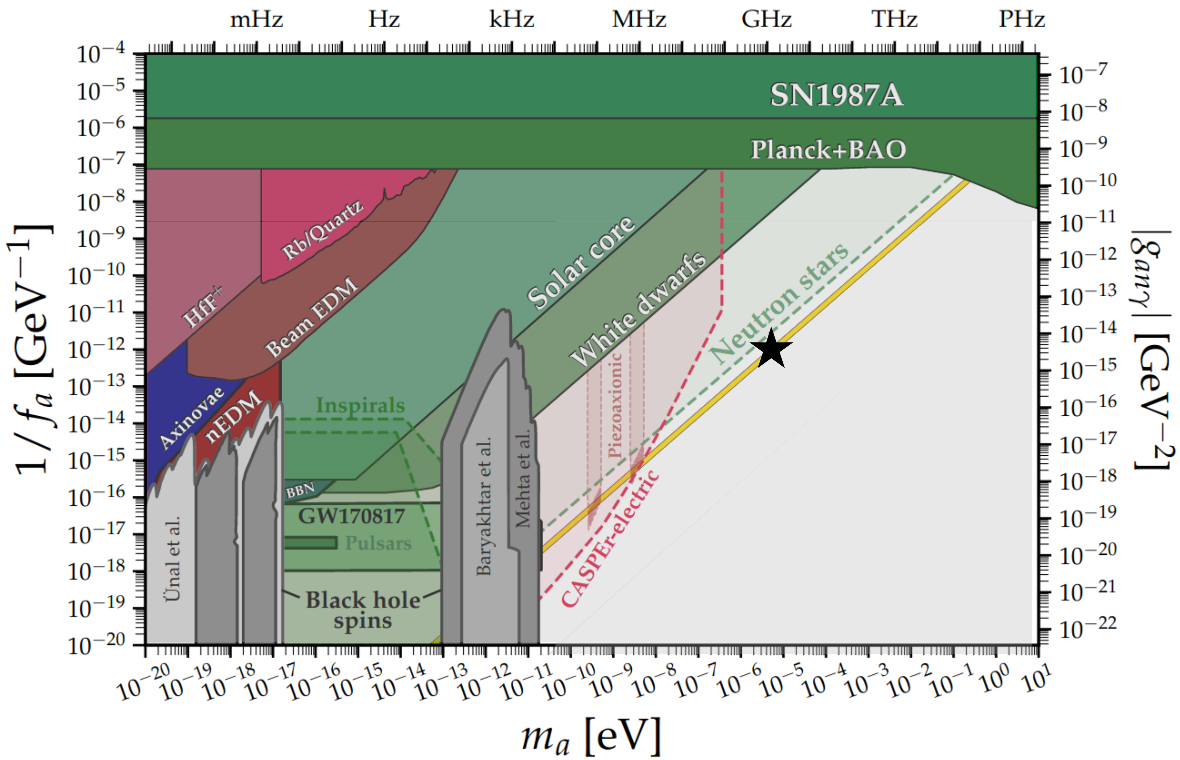
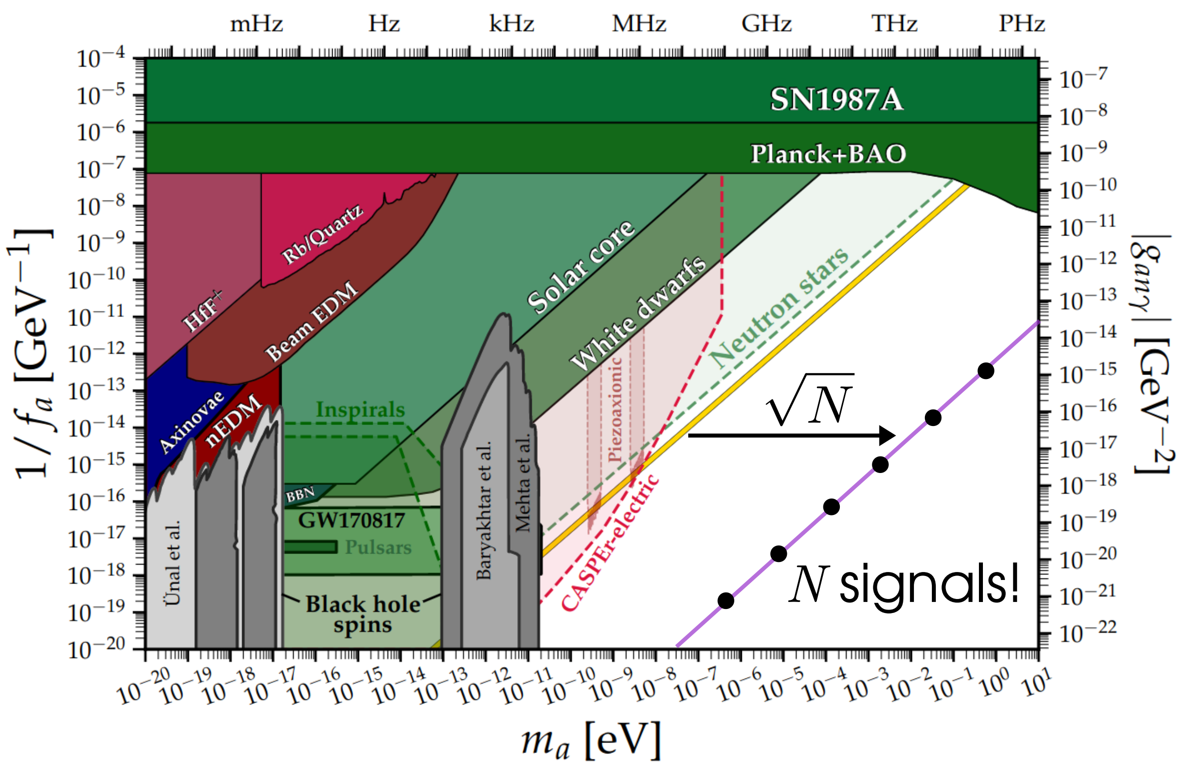


Experimental consequences

$$\mathbf{3}.\ \max\left\{\min_i\{g_i\}\right\} = N \quad \Longrightarrow \quad g_i = N,\ \forall\, i$$



QCD Maxions

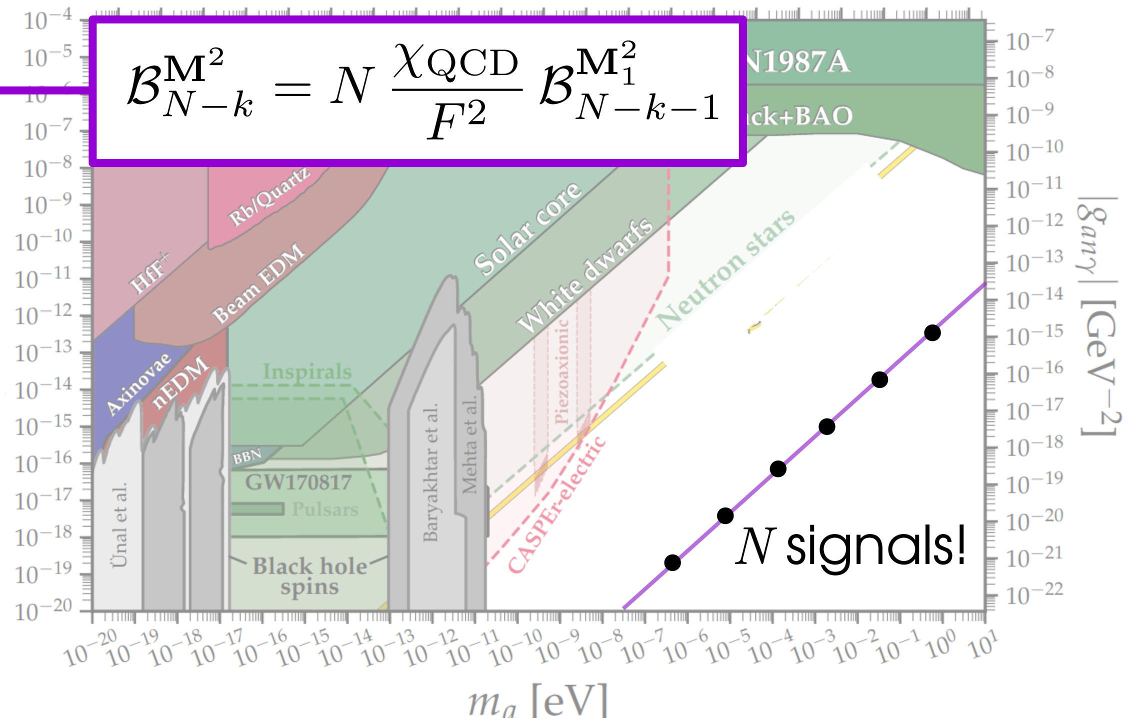
$$\mathbf{3.}\quad \max\left\{\min_i\{g_i\}\right\} = N \qquad \Longrightarrow \quad g_i = N,\ \forall\, i$$

m-parameter family of maxions: $m = N(N+1)/2$

$$\mathrm{tr}\,\mathbf{M}^2 = N\frac{\chi_{\rm QCD}}{F^2}$$

$$\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{\chi_{\rm QCD}}{F^2}$$

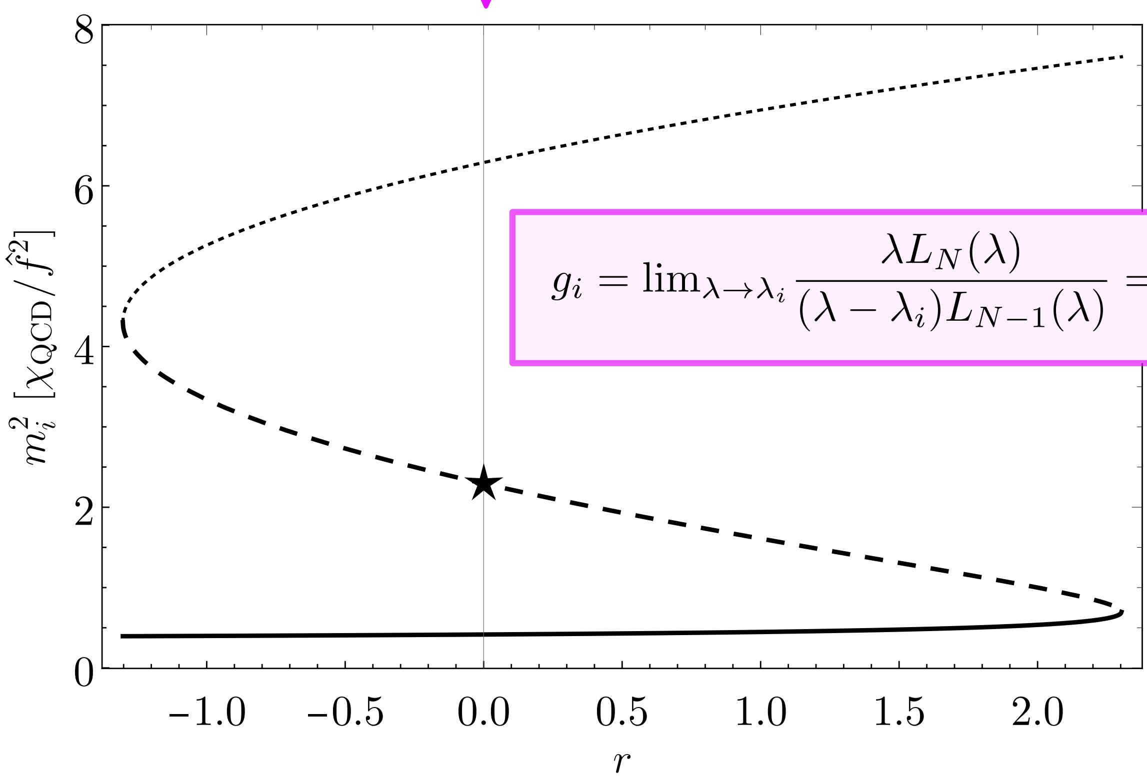
$$\mathcal{B}^{\mathbf{M}^2}_{N-k} = N\,\frac{\chi_{\rm QCD}}{F^2}\,\mathcal{B}^{\mathbf{M}_1^2}_{N-k-1}$$



$$p_{\mathbf{M}^2}(\lambda) = \sum_{k=0}^{N}\frac{(-1)^{N-k}}{(N-k)!}\,\mathcal{B}^{\mathbf{M}^2}_{N-k}\,\lambda^k$$

e.g. Laguerre maxions

$$\hat{\mathbf{M}}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4-\sqrt{3+r-r^2} & 1+r \\ 1 & 1+r & 4+\sqrt{3+r-r^2} \end{pmatrix}$$



$$g_i = \lim_{\lambda\to\lambda_i} \frac{\lambda L_N(\lambda)}{(\lambda-\lambda_i)L_{N-1}(\lambda)} = N$$

Coupling to photons

Assuming universal anomaly factors,

$$\mathcal{L} \supset \frac{\alpha_{em}}{8\pi} \sum_{k=1}^{N} \frac{E_k}{\mathcal{N}_k} \frac{\hat{a}_k}{\hat{f}_k} F\widetilde{F} \implies \frac{\alpha_{em}}{8\pi} \frac{E}{\mathcal{N}} \frac{a_{G\tilde{G}}}{F} F\widetilde{F}$$

Making an axion-dependent rotation, $q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 a_{G\tilde{G}}/(2F) Q_a} \begin{pmatrix} u \\ d \end{pmatrix}$:

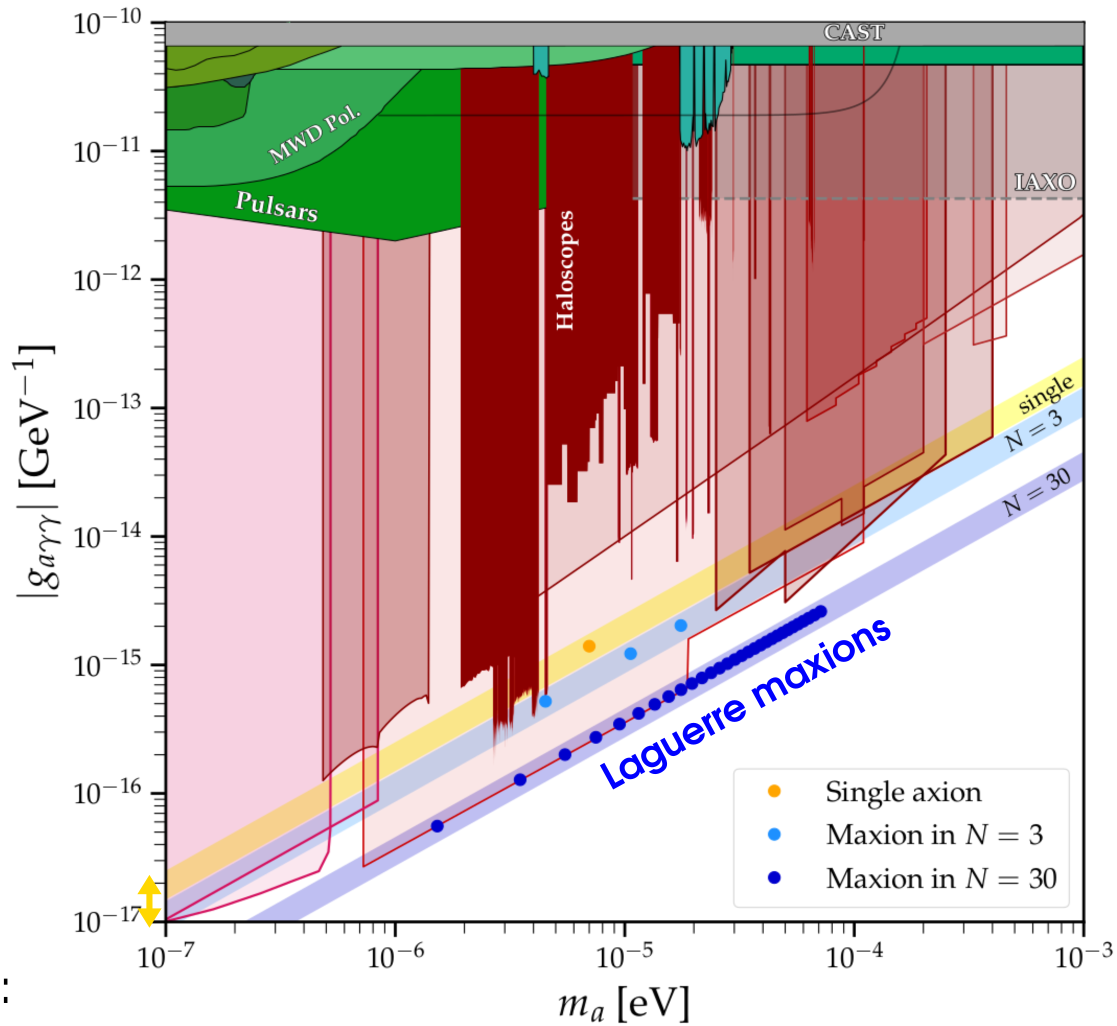
G. di Cortona, E. Hardy, J. Vega, G. Villadoro 15

$$\mathcal{L} \supset \frac{\alpha_{em}}{2\pi} \left[\frac{E}{\mathcal{N}} - 1.92 \right] \sum_i \frac{a_i}{f_i} F\widetilde{F}$$

$$\boxed{\frac{m_i^2}{g_{a_i\gamma\gamma}^2} = \left.\frac{m_a^2}{g_{a\gamma\gamma}^2}\right|_{\text{single QCD axion}} \times g_i}$$

$$\frac{(2\pi)^2}{\alpha_{em}^2} \left[\frac{E}{\mathcal{N}} - 1.92 \right]^{-2} \sum_{i=1}^{N} \frac{g_{a_i\gamma\gamma}^2}{m_i^2} = 1$$

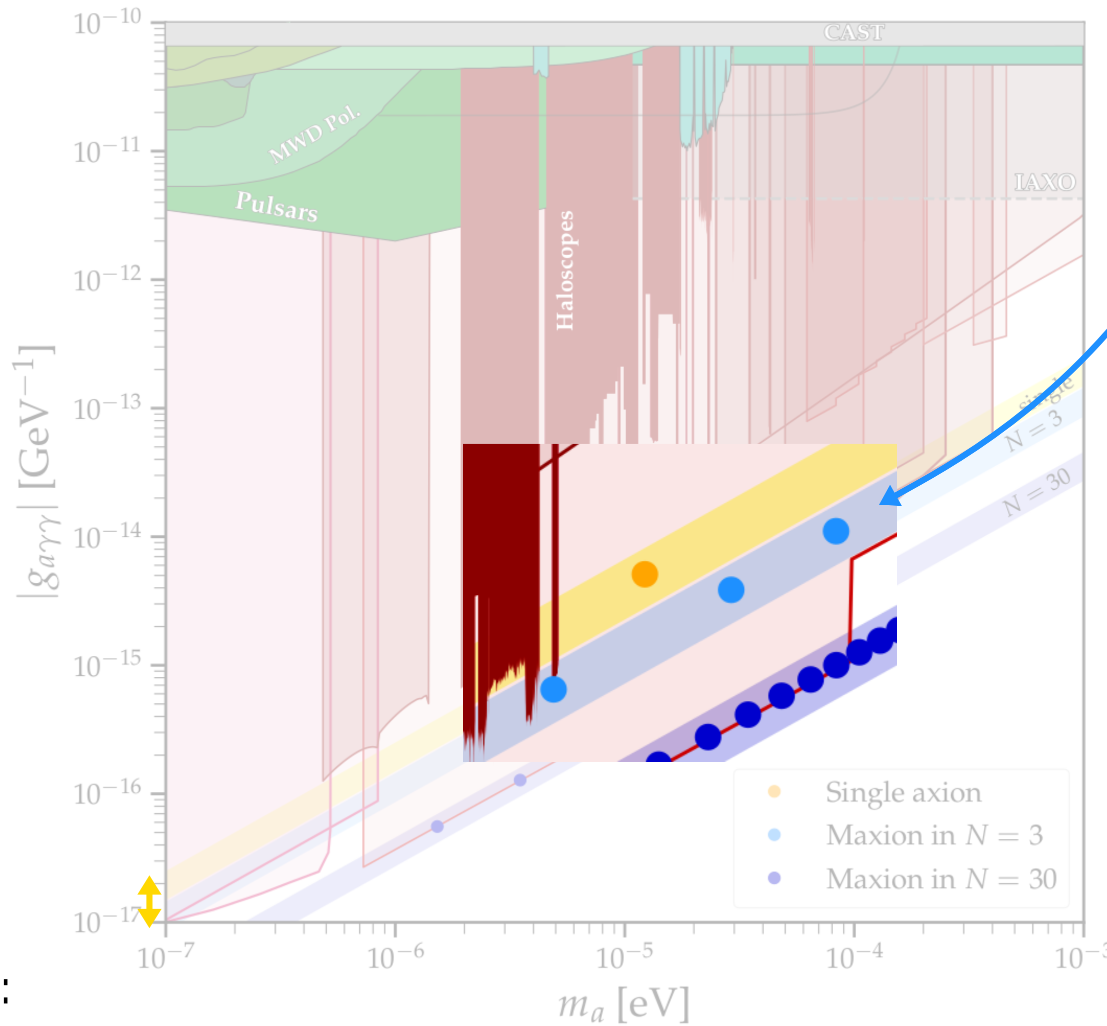
Coupling to photons



↕ Band width spans:

$$E/\mathcal{N} \in [2/3, 8/3]$$

Coupling to photons

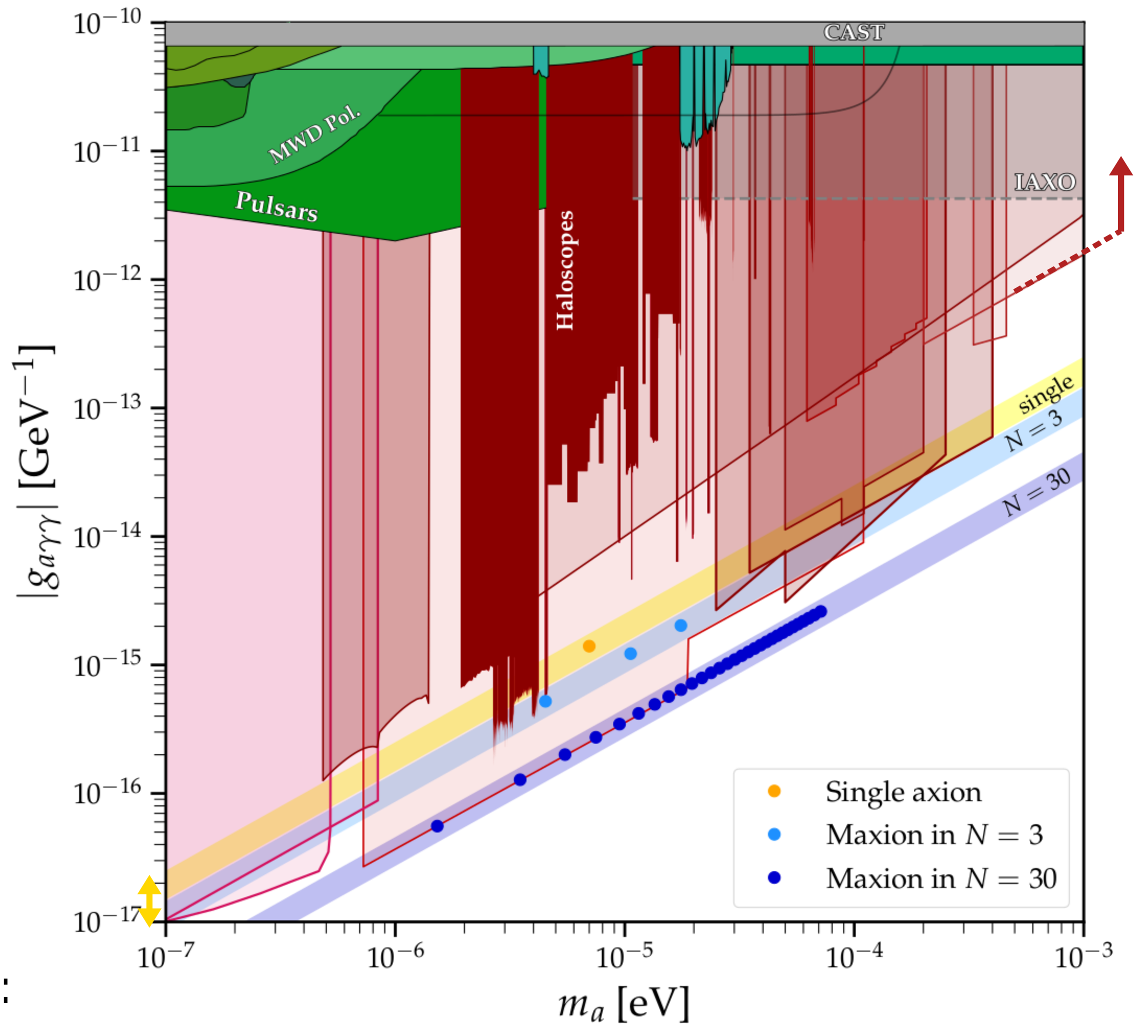


Multiplicity of signals might be the smoking gun

Band width spans:

$$E/\mathcal{N} \in [2/3, 8/3]$$

Coupling to photons



$\sqrt{N}$

if $\rho_i = \rho_{\rm DM}/N$

work in progress

Pablo Quílez, David Dunsky, Claudio Manzari, MR, Philip Soerensen

Band width spans:

$E/\mathcal{N} \in [2/3, 8/3]$

In summary

1. Any **signal to the right** of the canonical axion band can indicate a **multiple QCD axion** solution to the strong CP problem!

2. Our sum rule links the possible mass-scale values of the different axions, and allows us to count how many axions may exist in Nature

3. All axions can be maximally deviated from the QCD line, by a factor of $\sqrt{N}$.

4. Sizable effects require that the contribution from the extra potential is of the order of the QCD contribution

backup

UV completion

$$\mathcal{L}_{\rm UV} = |\partial_\mu S_1|^2 + |\partial_\mu S_2|^2 + \overline{\Psi}_1 {\rm i} D \Psi_1 + \overline{\Psi}_2 {\rm i} D \Psi_2 - \left[y_1 \overline{\Psi}_1 \Psi_1 S_1 + y_2 \overline{\Psi}_2 \Psi_2 S_2 + {\rm h.c.}\right] - V(S_{1,2})$$

$$U(1)_{\rm PQ}: \quad \Psi_{j,L} \to e^{{\rm i}\alpha_j/2} \Psi_{j,L}\,, \; \Psi_{j,R} \to e^{-{\rm i}\alpha_j/2} \Psi_{j,R}\,, \; S_j \to e^{{\rm i}\alpha_j} S$$

After SSB, $$S_{1,2} = \frac{1}{\sqrt{2}} \left(\hat{f}_{1,2} + \rho_{1,2}\right) e^{{\rm i}\hat{a}_{1,2}/\hat{f}_{1,2}}\,.$$

$V(S_{1,2}) = \lambda S_1^3 S_2 + {\rm h.c.}$ reduces the symmetry to just one PQ

After QCD confinement,

$$V_{\rm eff} = \frac{1}{2}\chi_{\rm QCD} \left(\frac{\hat{a}_1 + \hat{a}_2}{\hat{f}} - \bar{\theta}\right)^2 + \frac{\lambda}{4} \hat{f}^4 \left(\frac{3\hat{a}_1 + \hat{a}_2}{\hat{f}}\right)^2$$

$$r = \lambda \frac{\hat{f}^4}{2\chi_{\rm QCD}} \qquad \mathbf{M}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2} \begin{pmatrix} 2+8r & -4r \\ -4r & 2r \end{pmatrix}, \quad \text{with } 1/F^2 = 2/\hat{f}^2$$

which contains maxion solutions (r = 1/5)!

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Counterexample

$$\hat{\mathbf{M}}^2 = \frac{\chi_{\text{QCD}}}{\hat{f}^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r\frac{\chi_{\text{QCD}}}{\hat{f}^2}\begin{pmatrix} 1 & -q & 0 \\ -q & 1+q^2 & -q \\ 0 & -q & q^2 \end{pmatrix}$$

Correspondingly, $v_{j0} \propto \frac{1}{q^j}$ *leads* to decay constant exponentially enhanced

PQ: $\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{\chi_{\text{QCD}}}{F^2}$ ✔

Maxions:

$$\begin{cases} \operatorname{tr}\mathbf{M}^2 = N\frac{\chi_{\text{QCD}}}{F^2} \Leftrightarrow r = \frac{1}{10} \\ \operatorname{tr}^2\mathbf{M}^2 - \operatorname{tr}\mathbf{M}^2\cdot\mathbf{M}^2 = N\frac{\chi_{\text{QCD}}}{F^2}\operatorname{tr}\mathbf{M}_1^2 \Leftrightarrow r = 0 \vee r = \frac{11}{182} \end{cases}$$

✗

for $q = 3$ Farina, Pappadopulo, Rompineve, Tesi 17 [result is more general]

backup

Potential scales

In the basis where the extra potential is diagonal, $\mathbf{M}_B^2 = \mathrm{diag}(\tilde\lambda_1, \ldots, \tilde\lambda_N)$

$$g_i = \frac{m_i^2\, F^2}{|\langle a_{G\tilde G}|a_i\rangle|^2\, \chi_{\rm QCD}} = \frac{m_i^2}{\left|\langle a_{\rm PQ}|a_i\rangle/f_{\rm PQ} + \sum_j^{N-1}\langle\tilde a_j|a_i\rangle/\tilde f_j\right|^2 \chi_{\rm QCD}}$$

For $\tilde\lambda_j \gg \chi_{\rm QCD}/F^2$:

$$\frac{1}{g_j} \sim \frac{|\langle a_{G\tilde G}|\tilde a_j\rangle|^2\, \chi_{\rm QCD}}{\tilde\lambda_j\, F^2} = \frac{(F/\tilde f_j)^2\, \chi_{\rm QCD}}{\tilde\lambda_j\, F^2} \leq \frac{\chi_{\rm QCD}}{\tilde\lambda_j\, F^2} \longrightarrow 0$$

For $\tilde\lambda_j \ll \chi_{\rm QCD}/F^2$:

$$a_\varepsilon = \frac{a_{\rm PQ}}{f_{\rm PQ}} - \frac{\tilde a_j}{\tilde f_j} + \mathcal{O}(\varepsilon), \quad m_\epsilon^2 \sim \tilde\lambda_j = \varepsilon\, \chi_{\rm QCD}/F^2$$

$$\frac{1}{g_j} \sim \frac{\left|\langle a_{G\tilde G}|\tilde a_\varepsilon\rangle\right|^2 \chi_{\rm QCD}}{\tilde\lambda_j\, F^2} \sim \frac{\varepsilon^2}{\varepsilon} \longrightarrow 0$$

Whenever one scale is very different from the QCD induced mass, one state decouples.

backup

Mixing effects in DM abundance

$$\mathcal{L} \supset \frac{1}{2}(\partial\phi_a)^2 + \frac{1}{2}(\partial\phi_S)^2$$
$$- m_a^2(T) f_a^2 \left[1 - \cos\left(\frac{\phi_a}{f_a} + \frac{\phi_S}{f_S}\right)\right]$$
$$- m_S^2 f_S^2 \left[1 - \cos\left(\frac{\phi_S}{f_S}\right)\right]$$

$$m_a^2(T) = m_{a,0}^2 \max\left\{1, \left(\frac{T}{T_{\rm QCD}}\right)^{-n}\right\}$$

$$V \approx \begin{pmatrix} \phi_a & \phi_S \end{pmatrix} \begin{pmatrix} m_a^2 & \frac{f_a}{f_S} m_a^2 \\ \frac{f_a}{f_S} m_a^2 & m_S^2 + \frac{f_a^2}{f_S^2} m_a^2 \end{pmatrix} \begin{pmatrix} \phi_a \\ \phi_S \end{pmatrix}$$

Would typically dominate the late-time energy density

Assume $f_s \gg f_a$:

At early times,

$$m_a(T) \ll m_S \to \phi_H \sim \phi_S\,,\ \ \phi_L \sim \phi_a$$

At late times,

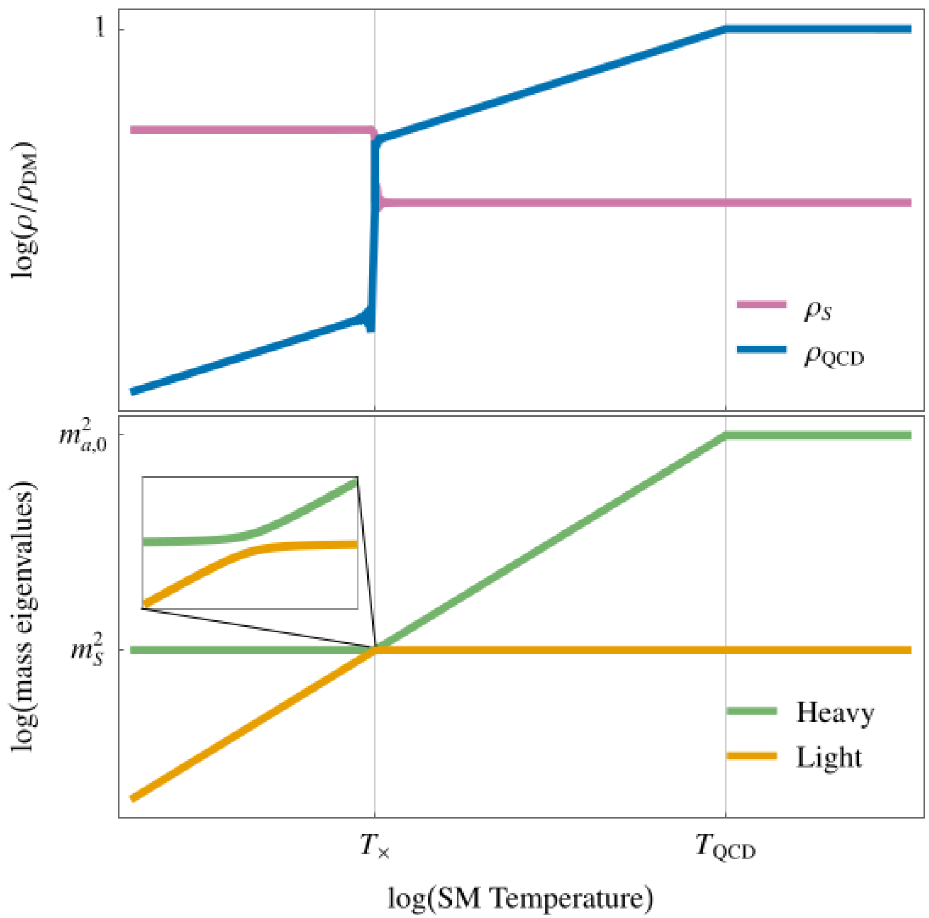
$$m_a(T) \gg m_S \to \phi_H \sim \phi_a\,,\ \ \phi_L \sim \phi_S$$

so energy is transferred into the QCD axion...

Cyncynates, Thompson 23
[other examples from Takahashi et al]

backup

Mixing effects in DM abundance



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