The Dark Stodolsky Effect

Constraining effective dark matter operators with spin-dependent interactions

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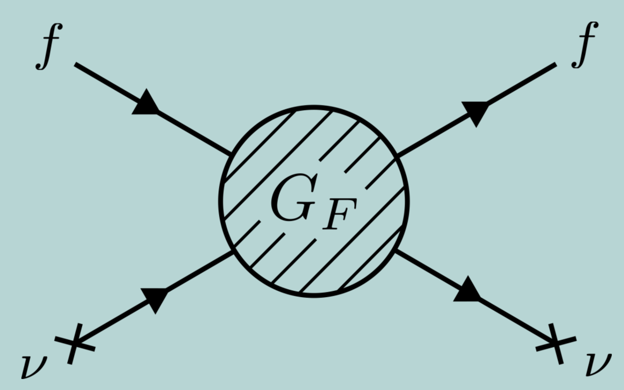
We present a discussion of the Stodolsky effect for dark matter and apply it to dark matter candidates from spin-0 to spin-3/2, considering all effective operators up to mass dimension 6. In all cases, the effect causes energy shifts which scale inversely with the candidate mass and require an asymmetric background. We compute the energy shift for a model of scalar dark matter and demonstrate that the Stodolsky effect can be used to constrain regions of parameter space that are not presently excluded.

The Stodolsky effect

The *Stodolsky effect*[1] is the spin-dependent shift in the energy of a Standard Model fermion sitting in a bath of neutrinos. This effect has historically been proposed as a method of detecting the Cosmic Neutrino Background.



For an electron scattering with a CνB neutrino this looks like

$$\Delta E \propto \sum_{\nu_i} \overbrace{G_F}^{\text{Linear in } G_F} \beta_\oplus A_{ii} \left[\overbrace{\sum_s \left(n(\nu_i^s) - n(\bar{\nu}_i^s)\right)}^{\nu\text{-anti-}\nu\text{ asymmetry}} + \frac{1}{\beta_{\nu_i}} \underbrace{\left(n(\nu_i^L) - n(\nu_i^R) + n(\bar{\nu}_i^R) - n(\bar{\nu}_i^L)\right)}_{\text{Helicity asymmetry}} \right]$$

The *dark* Stodolsky effect

The "dark" Stodolsky effect (DSE) is the equivalent of the effect explained above, where we now consider a fermion in a bath of dark matter particles.

The DSE will have features similar to the neutrino Stodolsky effect:

- It is *linear* in the DM-electron coupling;
- It is proportional to the number density $n = \rho_{\rm DM}/m_{\rm DM}$ and therefore *inversely proportional to the candidate mass*;
- It depends on one/several *asymmetry numbers*: matter-antimatter, helicity, polarisation...

Dark matter operators

Rather than restricting ourselves to a few dark matter models, we use an effective theory approach where we write down a basis for *all the individual operators that can contribute to the DSE*, up to mass dimension 6.

We categorise DM candidates by spin and find in total

- Spin-0: 1 operator
- Spin-1/2: 4 operators
- Spin-1: 8 operators
- Spin-3/2: 6 operators

The Dark Stodolsky Recipe

Or how to calculate the energy shift caused by the DSE for a dark matter model of your choice (shown here for a scalar model with a heavy Z' mediator).

1. Start with the Lagrangian for your favorite dark matter model

$$\mathcal{L}_{Z'} = -ig_\phi Z'^\mu \phi^* \overleftrightarrow{\partial}_\mu \phi - Z'_\mu \bar{\ell}(g_L P_L + g_R P_R)\ell$$

2. Integrate out heavy D.o.F.s to obtain a low-energy effective Lagrangian

$$\mathcal{L}_{\rm eff} = -i\frac{g_\phi(g_R - g_L)}{2m_{Z'}^2}\left(\phi^* \overleftrightarrow{\partial}_\mu \phi\right) \bar{e}\gamma^\mu\gamma^5 e$$

3. Perform a Legendre transform, yielding the Hamiltonian

$$\mathcal{H} = i\frac{g_\phi(g_R - g_L)}{2m_{Z'}^2}\left(\phi^*(\vec{\nabla}\phi) - (\vec{\nabla}\phi^*)\phi\right) \cdot (\bar{e}\vec{\gamma}\gamma^5 e)$$

4. Compute the *expectation value of the Hamiltonian* in a bath of DM particles (and antiparticles)

$$\langle \phi | \mathcal{H} | \phi \rangle = -\frac{2g_\phi(g_R - g_L)}{m_{Z'}^2} m_e h_e (\vec{p}_\phi \cdot \vec{S}_e)$$

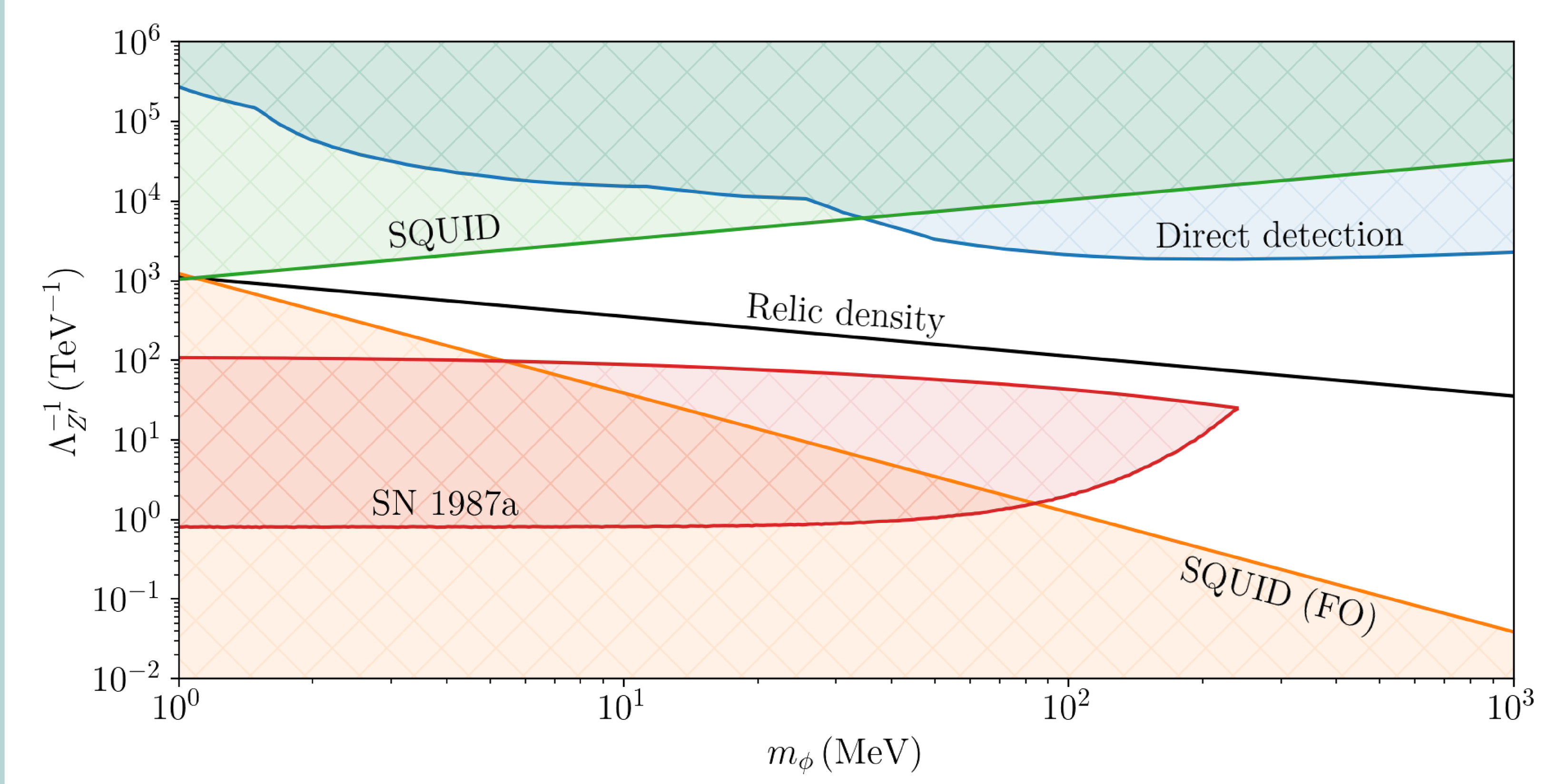
5. For your experimental setup, *average over the DM flux* at Earth, sum over particles and antiparticles, and over helicities or polarisations to obtain the total energy shift

$$\Delta E = -\frac{2g_\phi(g_R - g_L)}{m_{Z'}^2}\beta_\oplus \left(n(\phi) - n(\phi^*)\right)$$

Experimental feasibility

Energy shifts caused by the DSE manifest as a spin precession that can be measured as a torque with a *torsion balance*[2], or as a transverse magnetisation with a *SQUID magnetometer*[3].

The latter can achieve sensitivities of 10^{-32} eV, which for the model described above yields the following constraints:



Further reading:
[1] L. Stodolsky, *Speculations on Detection of the "Neutrino Sea"*, PRL **34** (1975) 110
[2] V. Domcke and M. Spinrath, *Detection Prospects for the Cosmic Neutrino Background using Laser Interferometers*, JCAP **06** (2017) 055
[3] D. Budker et al., *Proposal for a Cosmic Axion Spin Precession Experiment (CASPEr)*, PRX **4** (2014) 021030