Superradiance

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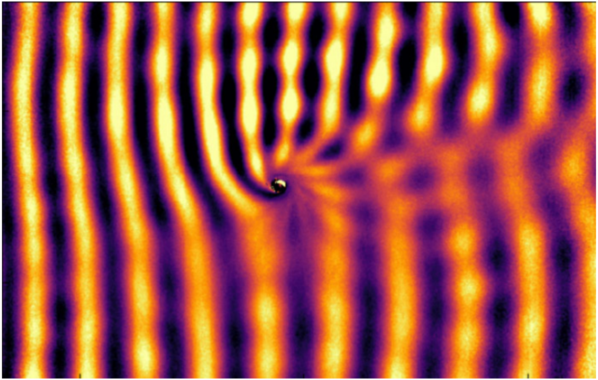
Invisibles Workshop
Göttingen University
August 2023

Outline

Superradiance

Superradiance is the amplification or enhancement of radiation in a dissipative system.



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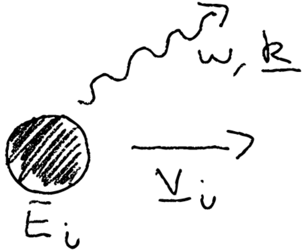
Radiation from a moving particle



$$E_f = E_i - \omega, \, \mathbf{p}_f = \mathbf{p}_i - \mathbf{k}$$

Find the particle's rest mass by moving to comoving frame:

$$m_i = \gamma_i (E_i - \mathbf{v}_i \cdot \mathbf{p_i}), \; m_f = \gamma_f (E_f - \mathbf{v}_f \cdot \mathbf{p_f})$$

$$\Delta m = -\gamma_i (\omega - \mathbf{v_i} \cdot \mathbf{k}) + \mathcal{O}(\delta \mathbf{v}).$$

Brito, Cardosa & Pani, 1501.06570
Bekenstein & Schiffer, gr-qc/9803033

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- This can occur with tachyons or from medium effects giving $\omega(k) < k$.

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- Cerenkov radiation: $v_{\rm ph} - v_i \cos(\theta) = 0$, $m_i = m_f$.

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- If the particle can *absorb* photons, we can also have spontaneous radiation with $m_i < m_f$.
- When $v_{\rm ph} > v_i$, an absorption effect can become a spontaneous radiation effect, taking energy from the particle's kinetic energy.

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- Superradiance requires that the rotating body be dissipative.

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- The superradiant instability is effective for *light bosons*.

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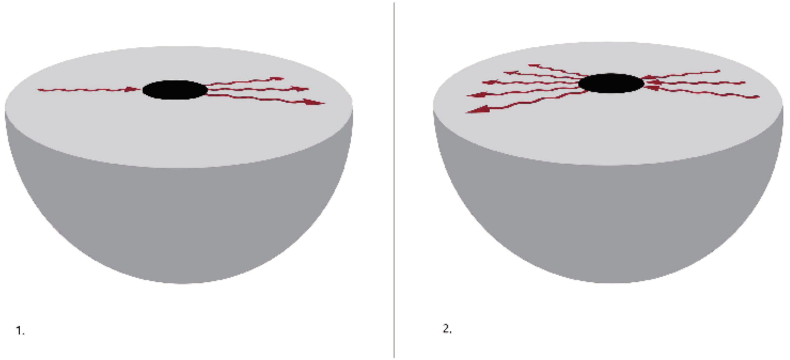

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- Similar to Hydrogen atom wavefunctions $\psi_{nlm}(r)$.
- The eigen-energies will have an imaginary component, corresponding to ϕ being eaten by the black hole, or to superradiant amplification of ϕ.

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- The instability is less efficient for higher l and m modes.

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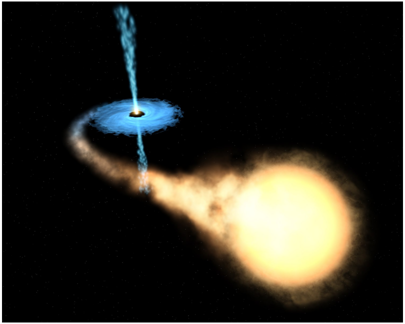
Black hole spin depletion

We can measure black hole spins:

- X-ray spectra of black hole X-ray binaries
- Gravitational wave emission from mergers



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- Advanced Ligo will be sensitive to $m_a \lesssim 10^{-10}\,\mathrm{eV}$. (Arvanitaki *et al*, 1604.03958).

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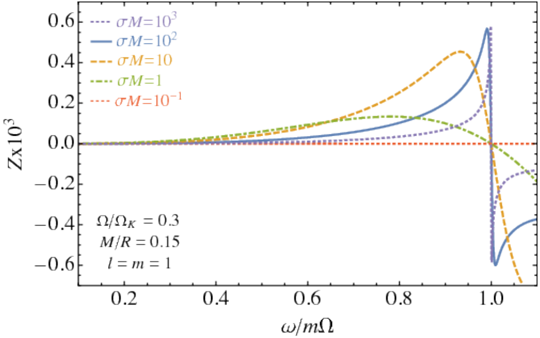
- Axion annihilations could decrease the superradiance rate.
- For large initial seeds, if both superradiant and non-superradiant modes are populated, the instability may not occur (Ficarra, Pani & Witek, 1812.02758.).

Superradiance in Stars

No horizion - superradiance in stars relies on non-gravitational dissipative dynamics, which become amplifying due to the star's rotation (Zel'dovich, 1971).

Example: Dark photons in neutron stars

Massive dark photons with dissipation from a hidden sector conductivity (V Cardoso, P Pani and T Yu, 1704.06151).



$$Z := \frac{|A_{\text{out}}|^2}{|A_{\text{in}}|^2} - 1.$$

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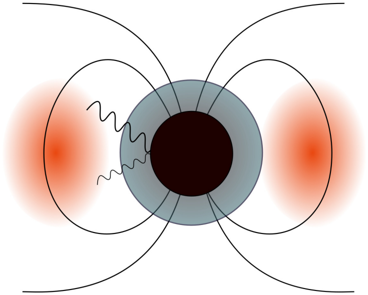
- This gives the required dissipative interaction.

Kaplan & Rajendran, 1908.10440

Axion-photon superradiance in neutron stars



The axion boundstate (orange) mixes with a photon mode which is then amplified by scattering off the rotating magnetosphere (grey). The photon energy is then deposited back into the axion sector.

FCD & McDonald, 1904.08341

Stellar superradiance: A general approach

- Many different Beyond the Standard Model interactions could lead to stellar superradiance.
- Stellar environments are complex, with many more degrees of freedom than black holes.
- Spin down from stellar superradiance can be observed directly.
- Can we find a general method for computing stellar superradiance rates from a BSM Lagrangian?

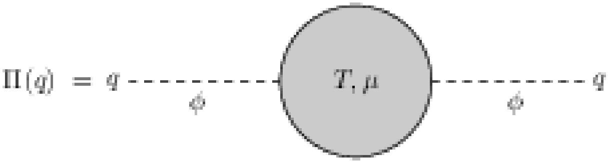
See FCD, Garbrecht & McDonald, 2207.07662.

Stellar superradiance: Damping

Stellar superradiance depends on the **damping rate** of the field into the star. We can find this with thermal field theory.

$$\partial^2\phi + \mu^2\phi + \Gamma_\phi\dot{\phi} = 0,$$

$$\Gamma_\phi = -\lim_{p\to 0} \mathrm{Im}\,\Pi(p)/p^0.$$



Stellar superradiance: Worldline Effective Field Theory

- Describe interaction of a field ϕ and the star by expanding in $\frac{R}{\lambda}$.
- The extended nature of the star is described by an infinite series of interactions between ϕ and a point-like object.

$$H_{\rm int}(t, \mathbf{x}) = \partial^I \phi(x) \mathcal{O}_I^{(1)}(x) \delta^{(3)}(\mathbf{x} - \mathbf{y}(t)) + \partial^I \partial^J \phi(x) \mathcal{O}_{IJ}^{(2)}(x) \delta^{(3)}(\mathbf{x} - \mathbf{y}(t)) + \ldots$$

Stellar superradiance: Worldline Effective Field Theory

If the star is rotating:

$$H_{\rm int}(t) = \partial^I \phi(t) R_I^J(t) \mathcal{O}_J^{(1)}(t) + \partial^I \partial^J \phi(t) R_I^K(t) R_J^L(t) \mathcal{O}_{KL}^{(2)}(t) + \dots$$

See S. Endlich and R. Penco, 1609.06723

Superradiant scattering in the worldline EFT

Superradiant scattering from a rotating star:

$$P_{\rm abs} = \sum_{X_f} \frac{|\langle X_f; 0|S|X_i; \omega, \ell, m\rangle|^2}{\langle \omega, \ell, m \mid \omega, \ell, m\rangle},$$

$$S = T \exp\left\{-i \int_0^{t'} dt H_{\rm int}(t)\right\}.$$

Amplification factor:

$$Z_{\ell m} = \frac{\Phi_{\rm out} - \Phi_{\rm in}}{\Phi_{\rm in}} = \frac{\ell! q^{2\ell+2}}{4\pi(2\ell+1)!! v\omega}\rho_\ell(m\Omega - \omega),$$

$\rho_\ell(m\Omega - \omega)$ is related to the worldline EFT operators and must be found with a matching calculation.

Superradiant instabilities in the worldline EFT

Superradiant instabilities arise from *bound states*:

$$P_{\rm abs} = \sum_{X_f} \left|\langle X_f; 0|S|X_i; n\ell m\rangle\right|^2 .$$

The superradiance rate is:

$$\Gamma_{n\ell m} = \Gamma_{\rm em} - \Gamma_{\rm abs} = \frac{A_{n\ell m}}{2\omega_{\ell n}} \left(\frac{1}{r_{n\ell}}\right)^{2\ell+3} \rho_\ell \left(m\Omega - \omega_{\ell n}\right),$$

where $r_{n\ell} = (n+\ell+1)/(2GM\mu^2)$ and $\omega_{\ell n}$.

Matching the worldline EFT

- We can also find the amplification factor $Z_{\ell m}$ directly from the equation of motion $\partial^2\phi + \mu^2\phi + \Gamma_\phi\dot{\phi} = 0$.
- Matching these results we can obtain $\rho_\ell\left(m\Omega - \omega_{\ell n}\right)$ (or equivalently the EFT coefficients).

This gives a superradiance rate:

$$\Gamma_{n\ell m} = C_{n\ell m}\left(\frac{R}{r_{n\ell}}\right)^{(2\ell+3)}\frac{(m\Omega - \omega)}{\omega}\Gamma_\phi,$$

where the damping Γ_ϕ of the field ϕ into the star can be calculated in thermal field theory.

Stellar superradiance: Axion Example

Consider an axion damping into a neutron star via the interactions

$$\mathcal{L}_{aNN} = \frac{g_{an}}{2m}\partial_\mu a\bar{N}\gamma^\mu\gamma_5 N,$$

$$\mathcal{L}_{\pi NN} = \frac{2m}{m_\pi} f\pi_0\bar{N}\gamma^5 N,$$

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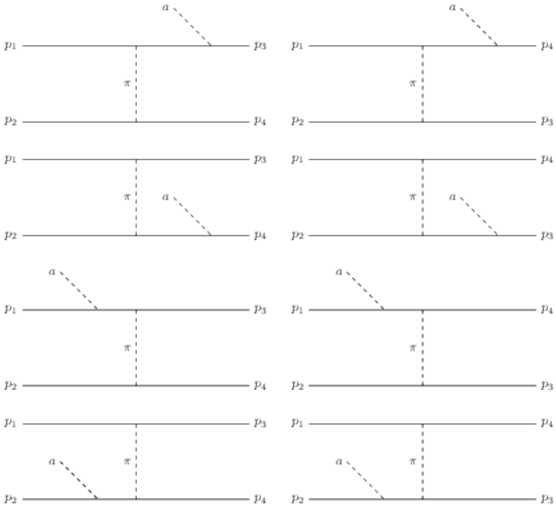
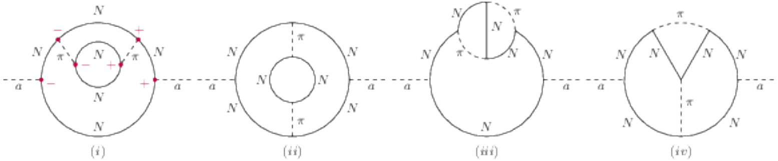


Stellar superradiance: Axion Example

To find the superradiance rate, we must compute Γ_ϕ from the self-energy:



The self-energy is related to the axion mean free path

$$\lambda^{-1} = \frac{\mathrm{Im}\Pi}{\omega}$$

calculated in e.g. S. Harris, 2005.09618 - we find a superradiance time somewhat larger than the age of the universe!

Stellar superradiance: A general approach

- We find the damping rate from the **in-medium self energy**.
- We find the superradiant instability rate from the damping rate using **worldline effective field theory** (S Endlich & R Penco, 1609.06723).
- Match calculations at low energy to obtain the superradiant instability rate:

$$\Gamma_{n\ell m} = C_{n\ell m} \left(\frac{R}{r_{n\ell}} \right)^{(2\ell+3)} \frac{(m\Omega - \omega)}{\omega} \Gamma_\phi$$

This allows us to calculate the stellar superradiance rate for any interaction between a bosonic field and a star.
See FCD, Garbrecht & McDonald, 2207.07662.

Conclusions

- Black hole superradiance offers a purely gravitational probe of Beyond the Standard Model bosons.
- Stellar superradiance can probe additional interactions between new bosons and the Standard Model.