Dark matter in the exponential growth scenarios arXiv:2308.09801 [hep-ph]

Disha Bhatia
Instituto de Fisica, Universidade de São Paulo

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Dark matter observed density

- Information about DM as of today:
  - It's present day energy density (which is 5 times the visible matter).
  - It is cold(for structure formation), collisionless(assumption that may be lifted).
  - It interacts atleast gravitationally and possibly could have another form of interaction with normal matter which could be of the order of weak or smaller.
- Using the above, the most pressing questions in DM research are:
  - How DM can be produced in the early universe?
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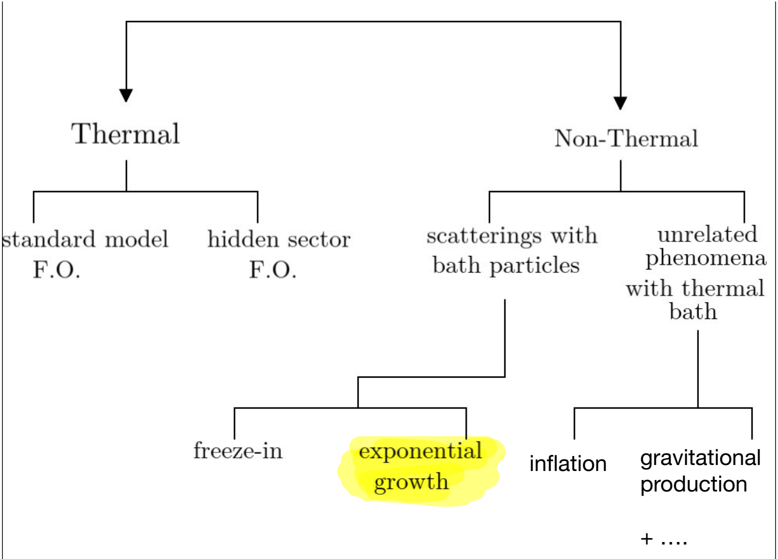
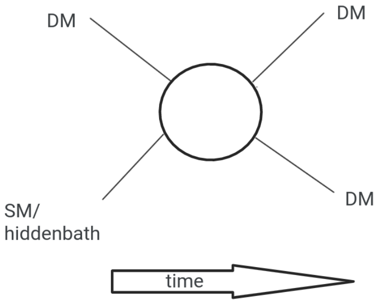
Various production mechanisms of dark matter



Exponential growth mechanism (2103.16572, 2104.05684)



This is opposite to the semi-annihilations
Hence also been named as semi-production

What kind of models can give rise to these phenomena?

- Basically all models explaining semi-annhililations in the low coupling regions.
- A Z_3 symmetric dark matter interaction with a singlet scalar $\Rightarrow \chi^3\phi +$ h.c.
- A mass-mixed fermion having self interactions via scalar/gauge boson $\Rightarrow \Delta m\overline{\chi_1}\chi_2$ with $\overline{\chi_1}\chi_1\phi$ or $\overline{\chi_1}\gamma^\mu\chi_1 Z'_\mu$
- As the process requires some initial density of dark matter, some mechanism beyond the above must be present to generate the initial abundance.

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Exponential growth mechanism

- In literature, a simplifying assumption is made to solve the BE i.e.

$$f_\chi(p,T) = A(T) f_\chi^{\rm eq}(p,T)\ , \quad \text{where} \quad A(T) = n_\chi(T)/n_\chi^{\rm eq}(T)\ .$$

i.e. dark matter traces the (*kinetic*) equilibrium distribution
A standard assumption while determining abundance for the thermal scenarios

- With this, the dark matter density can be estimated by considering the zeroth moment of the Boltzmann equation:

$$\frac{1}{a^3}\frac{d}{dt}(n_\chi a^3) \;=\; \langle\sigma v\rangle \left(n_\phi^{\rm eq} n_\chi - \cancel{n_\chi^2}\right)$$

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Exponential growth mechanism continued...

Ques: Can we make the equilibrium assumption?

- Usually the assumption works well for thermal or nearly thermal scenarios.
- Production of dark matter in our case non-thermal.
- We want to solve this for generic case and see how different is the obtained distribution function from the equilibrium case.

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Boltzmann equation for dark matter abundance

- B.E. is an integrodifferential equation and solving it for generic cases is highly non-trivial
- Two ways to solve BE: either in operator form or by taking moments: $\sum_{\text{spins}} \int d^3p\, p^n \times (\mathcal{L}[f_\chi] = \mathcal{C}[f_\chi])$
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Generalised solution to exponential growth scenarios

- We assume some process generated the negligible DM initial density to onset the exponential growth which contributes maximally to the relic.
- The Boltzmann equation

$$\mathcal{L}[f_\chi(p,t)] = \left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f_\chi(p,t)\,,$$

$$\mathcal{C}[f_\chi(p,t)] = \frac{1}{2E_p\, S}\int \frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{d^3k}{(2\pi)^3 2E_k}\frac{d^3k'}{(2\pi)^3 2E_{k'}} \times (2\pi)^4\delta^4(\vec{P}_i - \vec{P}_f)$$
$$\times |M|^2 \Big[f_\chi(p,t) f_\phi^{\rm eq}(p',t)\,(1 \pm \cancel{f_\chi(k,t)})\,(1 \pm \cancel{f_\chi(k',t)})$$
$$- \cancel{f_\chi(k,t) f_\chi(k',t)}\,(1 \pm f_\chi(p,t))\left(1 \pm f_\phi^{\rm eq}(p',t)\right)\Big]\,.$$

$\Rightarrow$ BE is a linear partial differential equation in p and t.

- Model dependence encoded in S and $|M|^2$ and in the initial condition.

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Generalized solution to exponential growth scenarios

- The solution simplifies further in the comoving frame as $p \to q \equiv p/[s(x)]^{1/3}$

$$\mathcal{L}[f_\chi(q,t)] = \frac{d}{dt} f_\chi(q,t) = \frac{\partial}{\partial t} f_\chi(q,t) + \frac{\partial}{\partial q} f_\chi(q,t) \cancelto{0}{\frac{dq}{dt}}$$

For each q, we have an ordinary differential equation.

- The solution to Boltzmann equation is given as:

$$f_\chi(q,x) = f_\chi(q, x_{\rm init}) \exp\left(\int_{x_{\rm init}}^{x} dx\, P'(q,x)\right) \quad \text{where },$$

$$P'(q,x) = \frac{h_{\rm eff}(x) g_\chi g_\phi}{S\, H(x)\, x} \int \frac{d^3p'}{(2\pi)^3} (\sigma v_{\rm mol}(q,p'))\; f_\phi^{\rm eq}(p',x)\,.$$

- The above equation is similar to any growth/decay equation $\Rightarrow P'(q,x)$ is a growth function.

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Generalised solution to Boltzmann equation

For dark matter production in the universe:

- There should be some initial dark matter abundance in the universe i.e. $f_\chi(q, x_{\rm init}) \neq 0$.
- The growth function:

$$\begin{aligned} P'(q,x) &> 0 \quad \text{for} \quad x_{\rm init} \leq x \leq x_{\rm end} \quad \text{and} \\ P'(q,x) &\to 0 \quad \text{for} \quad x > x_{\rm end} \, . \end{aligned}$$

- Since the growth function depends on q, not all momentum modes grow similarly with the expansion of the universe.
- DM growth in the universe is purely exponential if $P'(q,x)$ is constant in x.
- In general the growth function can be complicated function in x and it is hard to parameterise the growth as simple exponential.
- The growth of the distribution in general can scale as a factor of exponential $\exp[A(x)]$.
- We still call growth as exponential to avoid new names.

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Comparison between the generalised approach and the equilibrium assumption:

Generalised approach for distribution function:

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\begin{aligned}
Y_\chi(x) &= g_\chi \int \frac{d^3q}{(2\pi)^3} \left[f_\chi(q, x_{\rm init}) \exp\left(\int_{x_{\rm init}}^{x} dx \; P'(q,x) \right) \right] \quad \text{where} \; , \\
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Simplified approach for distribution function:

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Y_\chi(x) &= Y_\chi(x = x_{\rm init}) \exp\left(\int_{x_{\rm init}}^{x} dx \; P(x) \right) \; , \quad \text{where} \\
P(x) &= \frac{h_{\rm eff}(x) \; n_\phi^{\rm eq} \langle \sigma v \rangle}{\mathrm{S} \; x \; H(x)}
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Comparison between the generalized approach and the equilibrium assumption:

- Simplified approach:
  - The dependence on initial conditions is through appearance of $Y_\chi(x = x_{\text{init}})$.
  - Do not need to specify the process of generation of the initial density.
- Generalised approach:
  - The dependence on initial conditions is through appearance of $f_\chi(q, x = x_{\text{init}})$
  - The final relic intrinsically depends on the initial process populating dark matter and the momentum modes.

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Model dependent approach:

- Toy model for $\chi\phi \to \chi\chi$ production.

- Toy model for generating the initial χ abundance.

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First: Toy model for exponential growth mechanism

- The models with fermionic mass mixings/ or Z_3 symmetric dark matter can give rise to scatterings of the form: $\chi + \phi \to \chi + \chi$.
- We consider a scalar DM coupled with a bath particle ϕ.
- For simplicity, we assume ϕ to be coupled with the SM bath.
- We consider the case where $m_\phi > m_\chi$ but $m_\phi < 3m_\chi$ to avoid $\phi \to \chi\chi\chi$.
- The growth function in this case:

$$P'(q,x) = \frac{|\lambda_{\rm tr}|^2 h_{\rm eff}(x)}{32\pi E_q \, x \, H(x)} \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{\sqrt{s - 4m_\chi^2}}{\sqrt{s}} f_\phi^{\rm eq}(p', x)$$

- $P'(q,x) \propto 1/E_q \Rightarrow$ low momentum modes grow faster than high momentum modes.

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- We consider a scalar DM coupled with a bath particle ϕ.
- For simplicity, we assume ϕ to be coupled with the SM bath.
- We consider the case where $m_\phi > m_\chi$ but $m_\phi < 3m_\chi$ to avoid $\phi \to \chi\chi\chi$.
- The growth function in this case:

$$P'(q,x) = \frac{|\lambda_{\rm tr}|^2 h_{\rm eff}(x)}{32\pi E_q\, x\, H(x)} \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{\sqrt{s-4m_\chi^2}}{\sqrt{s}} f_\phi^{\rm eq}(p',x)$$

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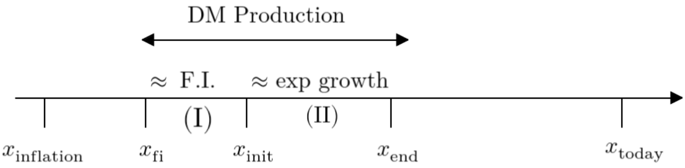
Two initial conditions example:

Freeze-in: $\phi\phi \to \chi\chi*$



- The collision operator in region (I) can be approximated as:

$$\mathcal{C}^{(\mathrm{I})}[f_\chi(q,x)] \approx \frac{1}{2E_q\, S}\int \frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{d^3k}{(2\pi)^3 2E_k}\frac{d^3k'}{(2\pi)^3 2E_{k'}}(2\pi)^4\delta^4(P_i - P_f) \times f_\chi^{\mathrm{eq}}(q,x) f_\chi^{\mathrm{eq}}(p',x)\, |M|^2_{\phi\phi\to\chi\chi}$$

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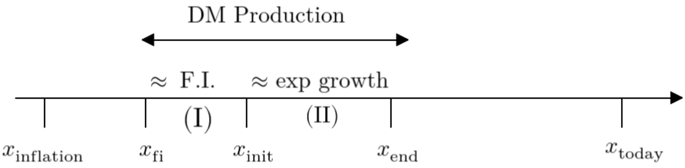

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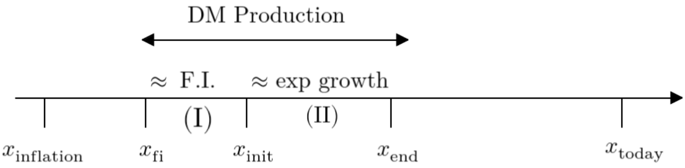

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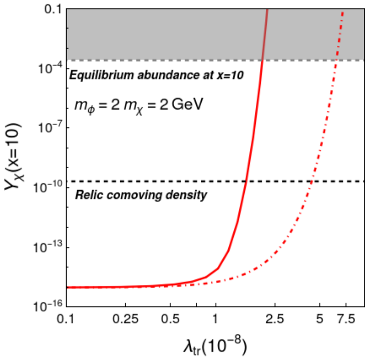

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Equilibrium abundance at x=10
$m_\phi = 2\, m_\chi = 2\,\text{GeV}$
Relic comoving density
$Y_\chi(x\!=\!10)$
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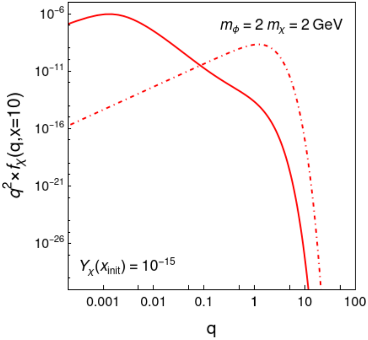

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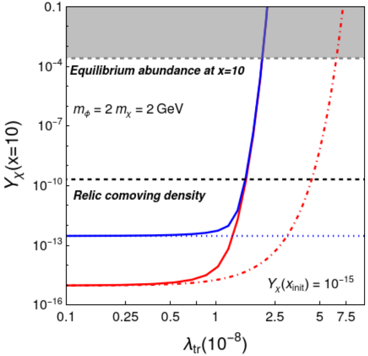

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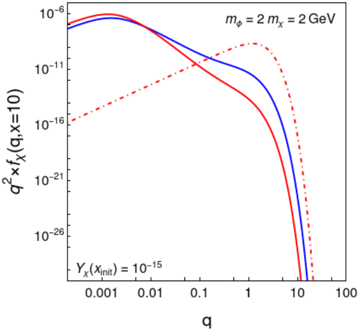

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Out-of-equilibrium decay of a heavy non-relativistic particle

- The initial distribution function can be solved by considering: $\Phi \to \chi\chi$
- Ideally it depends on the f_Φ.
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$$f_\chi(q,x) = A\,\delta(|q| - q_{\rm init})\,\Theta\left(x - x_{\rm init}\right)\ .$$

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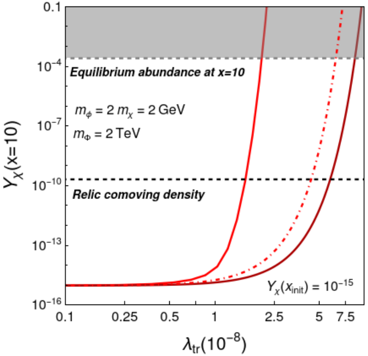
Out-of-equilibrium decay of a heavy non-relativistic particle continued...



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