Stefano Frixione

A few thoughts on ISR simulations at e^+e^- colliders

See: 1909.03886 (SF), 1911.12040 (Bertone, Cacciari, SF, Stagnitto)
2105.06688 (SF), 2108.10261 (SF, Mattelaer, Zaro, Zhao)
2207.03265 (Bertone, Cacciari, SF, Stagnitto, Zaro, Zhao)

ECFA Higgs factories 2^{nd} generators meeting, Brussels, 21/6/2023

- Cross sections stemming from e^+e^- collisions are plagued by large logs that must be resummed

- One way to do that is by means of collinear factorisation; another, with YFS

- Either way, the so-called precision tools currently available are not sufficiently accurate to meet the expected precision targets

Consider a generic cross section, sufficiently inclusive:

$$\sigma = \alpha^b \sum_{n=0}^{\infty} \alpha^n \sum_{i=0}^{n} \sum_{j=0}^{n} \varsigma_{n,i,j} L^i \ell^j$$

This is symbolic, and only useful to expose the presence of:

$$\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}\,, \qquad L = \log \frac{Q^2}{m^2}$$

Numerology: consider the production of $Z \to ll$ at:

- $\sqrt{Q^2} = m_Z$

$$\begin{aligned} L = 24.18 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} L = 0.06 \\ 0 \le m_{ll} \le m_Z\,, \quad \ell = 6.89 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.017 \\ m_Z - 1\ \mathrm{GeV} \le m_{ll} \le m_Z\,, \quad \ell = 10.60 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.026 \end{aligned}$$

Consider a generic cross section, sufficiently inclusive:

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Numerology: consider the production of $Z \to ll$ at:

- $\sqrt{Q^2} = 500$ GeV

$$\begin{aligned} L = 27.59 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} L = 0.069 \\ 0 \le m_{ll} \le m_Z\,, \quad \ell = 1.449 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.0036 \\ m_Z - 1 \text{ GeV} \le m_{ll} \le m_Z\,, \quad \ell = 1.453 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.0036 \end{aligned}$$

It takes a lot of brute force (i.e. fixed-order results to some $\mathcal{O}(\alpha^n)$) to overcome the enhancements due to L and ℓ.

It is always convenient to first improve by means of factorisation formulae:

$$\begin{aligned} d\sigma(L,\ell) &= \mathcal{K}_{soft}(\ell;L)\beta(L)d\mu && (1) \\ &= \mathcal{K}_{coll}(L;\ell)\otimes d\hat{\sigma}(\ell) && (2) \end{aligned}$$

Use of:

(1) YFS (resummation of ℓ)

(2) collinear factorisation (resummation of L)

Common features: $\mathcal{K}$ is an *all-order* universal factor; β and $d\hat{\sigma}$ are process-specific and computed order by order

(still brute force, but to a lesser extent)

YFS

Aim: soft resummation for:

$$\left\{ e^+(p_1) + e^-(p_2) \longrightarrow X(p_X) + \sum_{i=0}^{n} \gamma(k_n) \right\}_{n=0}^{\infty}$$

Achieved with:

$$\begin{aligned} d\sigma(L,\ell) &= \mathcal{K}_{soft}(\ell; L)\beta(L) d\mu \\ &= e^{Y(p_1,p_2,p_X)} \sum_{n=0}^{\infty} \beta_n \left(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}p_X; \{k_i\}_{i=0}^{n}\right) d\mu_{X+n\gamma} \end{aligned}$$

This is symbolic, and stands for both the EEX and CEEX approaches [hep-ph/0006359 Jadach, Ward, Was] that build upon the original YFS work [Ann.Phys.13(61)379]

EEX: exclusive (in the photons) exponentiation, matrix element level

CEEX: coherent exclusive (in the photons) exponentiation, amplitude level, including interference

YFS

Aim: soft resummation for:

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Achieved with:

$$d\sigma(L,\ell) \quad = \quad e^{Y(p_1,p_2,p_X)} \sum_{n=0}^{\infty} \beta_n \left(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}p_X; \{k_i\}_{i=0}^{n}\right) d\mu_{X+n\gamma}$$

- Y essentially universal (process dependence only through kinematics); resums ℓ
- The soft-finite β_n are process-specific, and are constructed by means of local subtractions involving matrix elements and eikonals (i.e. *not* BN)

$$\beta_n = \alpha^b \sum_{i=0}^{n} \alpha^i \sum_{j=0}^{i} c_{n,i,j} L^j$$

- For a given n, matrix elements have different multiplicities, hence the need for the kinematic mapping $\mathcal{R}$

Collinear factorisation

Aim: collinear resummation for:

$$\left\{ k(p_k) + l(p_l) \longrightarrow X(p_X) + \sum_{i=0}^{n} a_i(k_n) \right\}_{n=0}^{\infty} \qquad a_i = e^{\pm}, \gamma \ldots$$

with initial-state particles stemming from beams:

$$(k,l) = (e^+, e^-)\,, \quad (k,l) = (e^+, \gamma)\,, \quad (k,l) = (\gamma, e^-)\,, \quad (k,l) = (\gamma, \gamma)\,, \ldots$$

Master formula:

$$\begin{aligned} d\sigma(L,\ell) &= \mathcal{K}_{coll}(L;\ell) \otimes d\hat{\sigma}(\ell) \\ \longrightarrow \quad d\sigma_{kl} &= \sum_{ij} \int dz_+ dz_- \, \Gamma_{i/k}(z_+, \mu^2, m^2) \, \Gamma_{j/l}(z_-, \mu^2, m^2) \\ & \qquad \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2; p_X, \{k_i\}_{i=0}^{n}) \end{aligned}$$

- $\Gamma_{\alpha/\beta}$ universal (the PDF); resums L
- The collinear-finite $d\hat{\sigma}_{ij}$ are process-specific, and are the standard short-distance matrix elements, constructed order by order (*with* BN). May or may not include resummation of other large logs (including ℓ)

YFS vs collinear factorisation

Both are systematically improvable in perturbation theory: in YFS the β_n's (fixed-order), in collinear factorisation both the PDFs (logarithmic accuracy) and the $d\hat{\sigma}$'s (fixed-order, resummation)

+ YFS: very little room for systematics. Exceptions are the kinematic mapping $\mathcal{R}$, and the quark masses (when the quarks are radiators). Renormalisation schemes??

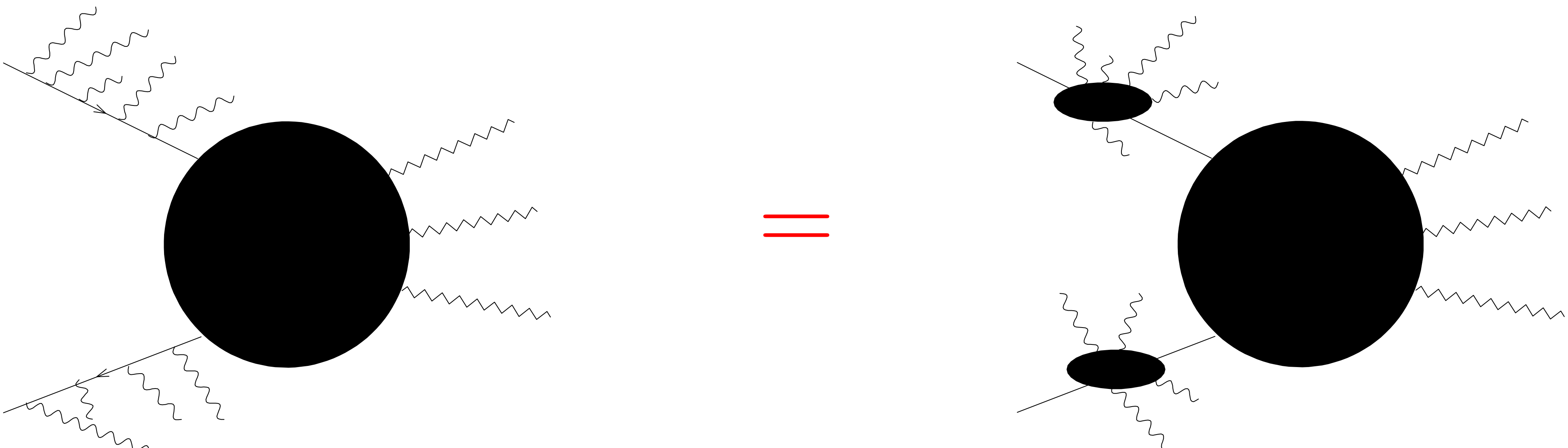
− Collinear factorisation: systematic variations much larger. At the LL (used in phenomenology so far) a rigorous definition of uncertainties is impossible (parameters are arbitrary), and comparisons with YFS are largely fine tuned

− YFS: the computations of β_n are not standard (EEX) and highly non-trivial (CEEX)

+ Collinear factorisation: the computations of $d\hat{\sigma}_{ij}$ are standard

COLLINEAR FACTORISATION

Collinear factorisation



$$d\sigma = \mathrm{PDF} \star \mathrm{PDF} \star d\hat{\sigma}$$

PDFs collect (universal) small-angle dynamics

All physics simulations based on collinear factorisation done so far are based on a LL-accurate picture

This is not tenable at high energies/high statistics:

- accuracy is insufficient (see e.g. W^+W^- production)
- systematics not well defined

Step 0 was to upgrade PDFs from LL to NLL accuracy: increase of precision, and meaningful systematics, in particular factorisation-scheme dependence

z-space LO+LL PDFs $(\alpha \log(Q^2/m^2))^k$:

$\sim$ 1992

- $0 \leq k \leq \infty$ for $z \simeq 1$ (Gribov, Lipatov)
- $0 \leq k \leq 3$ for $z < 1$ (Skrzypek, Jadach; Cacciari, Deandrea, Montagna, Nicrosini; Skrzypek)
- matching between these two regimes
- for e^-

z-space NLO+NLL PDFs $(\alpha \log(Q^2/m^2))^k + \alpha\,(\alpha \log(Q^2/m^2))^{k-1}$:

$\longrightarrow$ 1909.03886, 1911.12040, 2105.06688, 2207.03265 (Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao)

- $0 \leq k \leq \infty$ for $z \simeq 1$
- $0 \leq k \leq 3$ for $z < 1 \iff \mathcal{O}(\alpha^3)$
- matching between these two regimes
- for e^+, e^-, γ, and light quarks
- both numerical and analytical
- factorisation schemes: $\overline{\rm MS}$ and Δ (that has DIS-like features)

Bear in mind that PDFs are fully defined only after adopting a definite *factorisation scheme*, which is the choice of the finite terms associated with the subtraction of the collinear poles

- 1911.12040 $\longrightarrow$ $\overline{\mathrm{MS}}$
- 2105.06688 $\longrightarrow$ a DIS-like scheme (called Δ)

At variance with the QCD case, there is also an interesting *renormalisation-scheme* dependence of QED PDFs

Asymptotic $\overline{\rm MS}$ solution

Non-singlet $\equiv$ singlet; photon is more complicated

$$\Gamma_{\rm NLL}(z,\mu^2) \stackrel{z\to 1}{\longrightarrow} \frac{e^{-\gamma_{\rm E}\xi_1} e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)} \xi_1 (1-z)^{-1+\xi_1}$$
$$\times \left\{ 1 + \frac{\alpha(\mu_0)}{\pi} \left[\Big(L_0 - 1\Big) \left(A(\xi_1) + \frac{3}{4} \right) - 2B(\xi_1) + \frac{7}{4} \right.\right.$$
$$\left.\left. + \Big(L_0 - 1 - 2A(\xi_1)\Big) \log(1-z) - \log^2(1-z) \right] \right\}$$

where $L_0 = \log \mu_0^2/m^2$, and:

$$\begin{aligned} A(\kappa) &= -\gamma_{\rm E} - \psi_0(\kappa) \\ B(\kappa) &= \frac{1}{2}\gamma_{\rm E}^2 + \frac{\pi^2}{12} + \gamma_{\rm E}\psi_0(\kappa) + \frac{1}{2}\psi_0(\kappa)^2 - \frac{1}{2}\psi_1(\kappa) \end{aligned}$$

with:

$$
\begin{aligned}
\xi_1 &= 2t - \frac{\alpha(\mu)}{4\pi^2 b_0}\left(1 - e^{-2\pi b_0 t}\right)\left(\frac{20}{9} n_F + \frac{4\pi b_1}{b_0}\right) \\
&= 2t + \mathcal{O}(\alpha t) = \eta_0 + \ldots \\
\hat{\xi}_1 &= \frac{3}{2}\, t + \frac{\alpha(\mu)}{4\pi^2 b_0}\left(1 - e^{-2\pi b_0 t}\right)\left(\lambda_1 - \frac{3\pi b_1}{b_0}\right) \\
&= \frac{3}{2}\, t + \mathcal{O}(\alpha t) = \lambda_0 \eta_0 + \ldots \\
\lambda_1 &= \frac{3}{8} - \frac{\pi^2}{2} + 6\zeta_3 - \frac{n_F}{18}(3 + 4\pi^2)
\end{aligned}
$$

and:

$$
\begin{aligned}
t &= \frac{1}{2\pi b_0} \log \frac{\alpha(\mu)}{\alpha(\mu_0)} \\
&= \frac{\alpha(\mu)}{2\pi}\, L - \frac{\alpha^2(\mu)}{4\pi}\left(b_0 L^2 - \frac{2b_1}{b_0} L\right) + \mathcal{O}(\alpha^3)\,, \qquad L = \log \frac{\mu^2}{\mu_0^2}\,.
\end{aligned}
$$

Asymptotic Δ solution

Non-singlet $\equiv$ singlet; photon is trivial

$$\Gamma_{\rm NLL}(z,\mu^2) \quad \overset{z\to 1}{\longrightarrow} \quad \frac{e^{-\gamma_{\rm E}\xi_1} e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)}\, \xi_1 (1-z)^{-1+\xi_1} \times \left[\left(1+\frac{3\alpha(\mu_0)}{4\pi} L_0\right) \sum_{p=0}^{\infty} \mathcal{S}_{1,p}(z) - \frac{\alpha(\mu_0)}{\pi} L_0 \sum_{p=0}^{\infty} \mathcal{S}_{2,p}(z) \right]$$

The $\mathcal{S}_{i,p}(z)$ functions are increasingly suppressed at $z \to 1$ with growing p.
The dominant behaviour is:

$$\Gamma_{\rm NLL}(z,\mu^2) \quad \overset{z\to 1}{\longrightarrow} \quad \frac{e^{-\gamma_{\rm E}\xi_1} e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)}\, \xi_1 (1-z)^{-1+\xi_1} \times \left[\frac{\alpha(\mu)}{\alpha(\mu_0)} + \frac{\alpha(\mu)}{\pi} L_0 \left(A(\xi_1) + \log(1-z) + \frac{3}{4} \right) \right]$$

- A vastly different logarithmic behaviour w.r.t. the $\overline{\rm MS}$ case
  However, $\Gamma_{\rm NLL}^{(\overline{\rm MS})} - \Gamma_{\rm NLL}^{(\Delta)} = \mathcal{O}(\alpha^2)$

Key facts

- Both $\overline{\mathrm{MS}}$ and Δ results feature an integrable singularity at $z \to 1$, basically identical to the LL one

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- Both $\overline{\mathrm{MS}}$ and Δ results feature an integrable singularity at $z \to 1$, basically identical to the LL one

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- We believe that the Δ scheme resums also soft logs
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- Owing to the integrable singularity, it is essential to have large-z analytical results: the PDFs convoluted with cross sections are obtained by matching the small- and intermediate-z numerical solution with the large-z analytical one

On top of increased precision, for sensible phenomenology we need:

[2207.03265; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao]

- evolution with all fermion families (leptons and quarks), including their respective mass thresholds
- renormalisation schemes other than $\overline{\mathrm{MS}}$: $\alpha(m_Z)$ and G_μ
- assess implications by studying realistic observables in physical processes

Sample results for:

$$
\begin{aligned}
e^+e^- &\longrightarrow q\bar{q} \\
e^+e^- &\longrightarrow t\bar{t} \\
e^+e^- &\longrightarrow W^+W^-
\end{aligned}
$$

with $q\bar{q}$ production (massless quarks) restricted to ISR QED radiation.
The other two are in the SM

NLO accuracy, automated generation with MG5_aMC@NLO

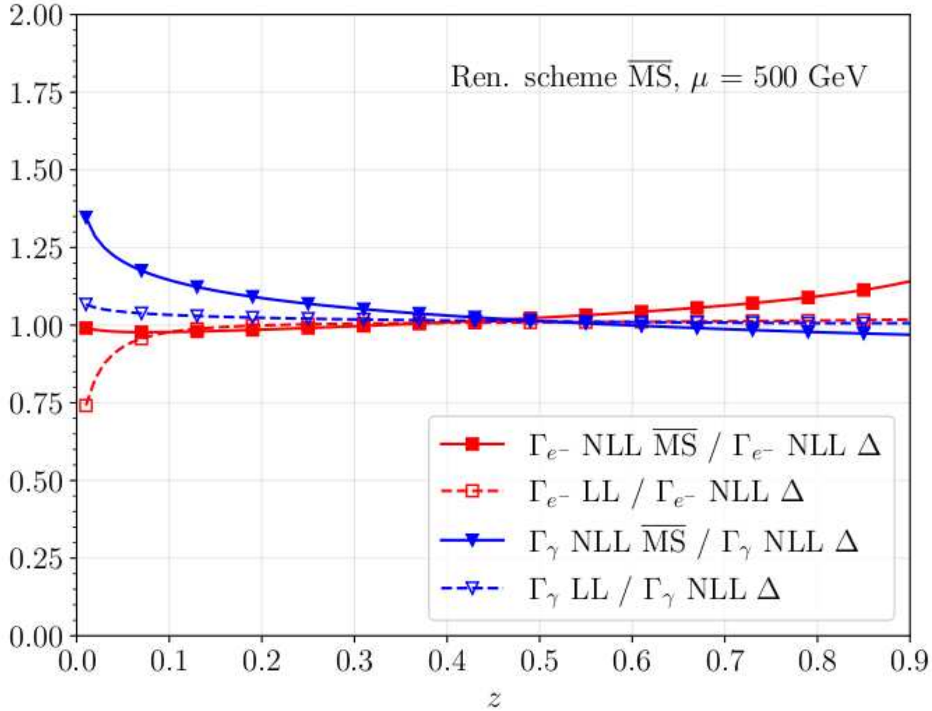
(this version is now public, v3.5.0) [2108.10261; Frixione, Mattelaer, Zaro, Zhao]

What is plotted:

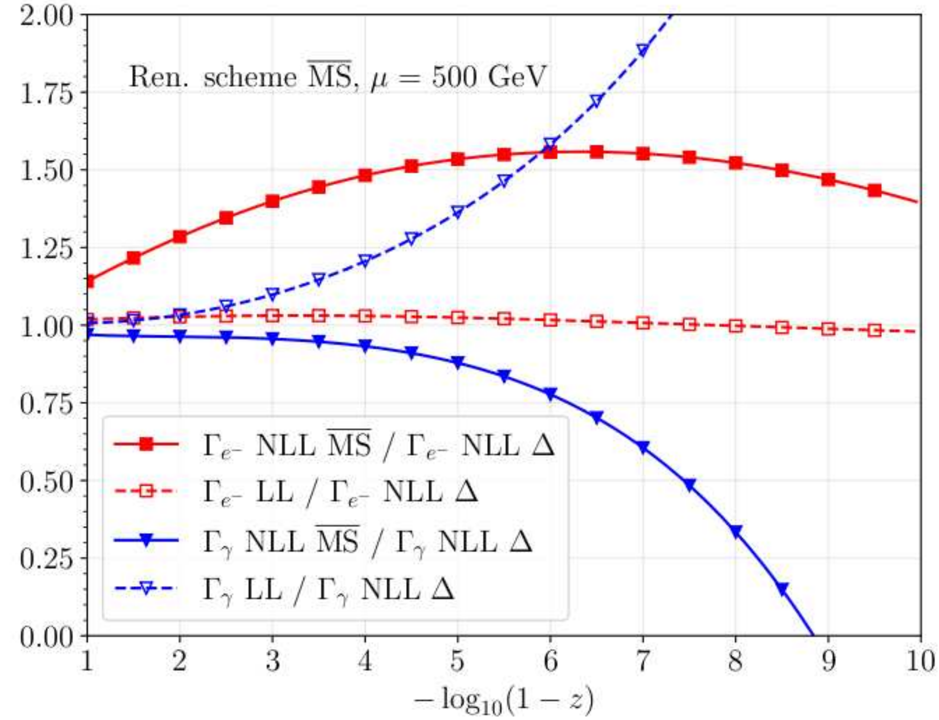
$$\sigma(\tau_{min}) = \int d\sigma\, \Theta\left(\tau_{min} \le \frac{M^2_{p\bar{p}}}{s}\right), \qquad p = q\,, t\,, W^+$$

$\tau_{min} \sim 1$ is sensitive to soft emissions (not resummed)

Dependence of PDFs on factorisation scheme



$z < 1$

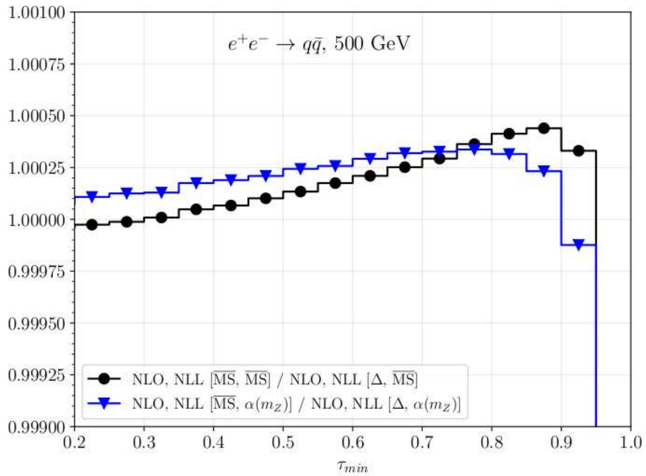


$z \simeq 1$

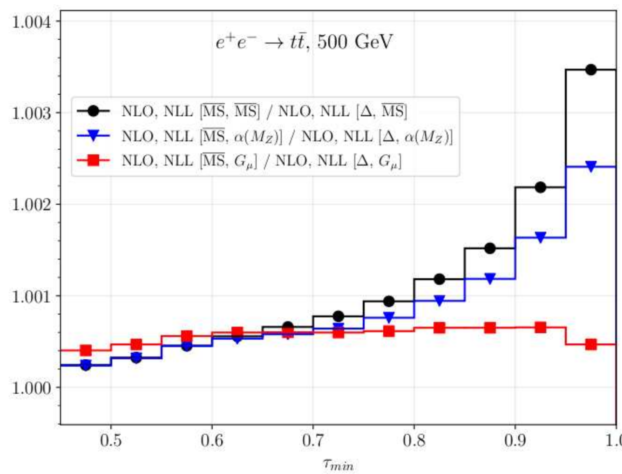
Very large dependence at the NLL at $z \to 1$ ($\mathcal{O}(1)$); this is particularly significant *(but unphysical!)* since the electron has an integrable divergence there

Electron at NLL in the Delta scheme close to the LL result (differences of $\mathcal{O}(5\%)$)

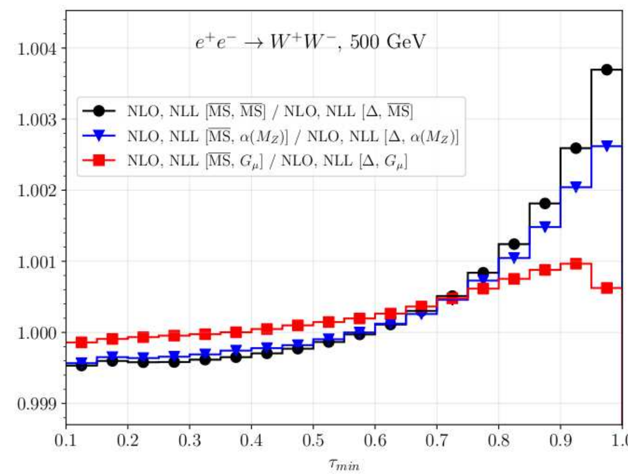
Dependence of observables on factorisation scheme



$q\bar{q}$



$t\bar{t}$



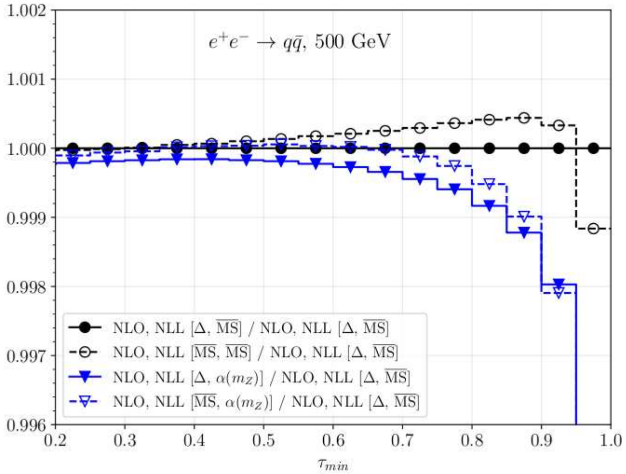
W^+W^-

$\mathcal{O}(1)$ differences for PDFs down to $\mathcal{O}(10^{-4} - 10^{-3})$ for observables

In the $\overline{\text{MS}}$ scheme, huge cancellations between PDFs and short-distance cross sections

Behaviour qualitatively similar for different renormalisation schemes

Factorisation vs renormalisation scheme dependence



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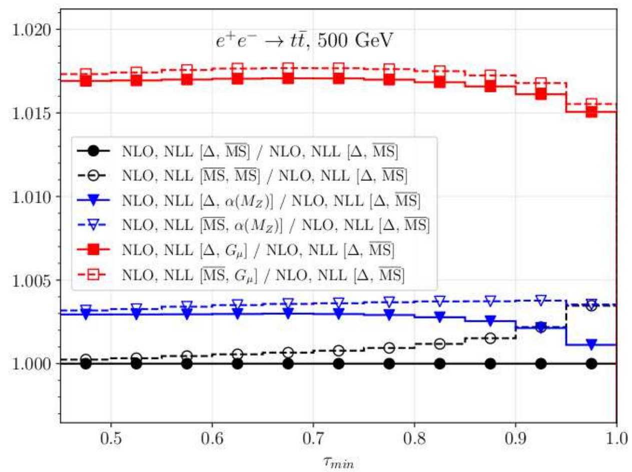
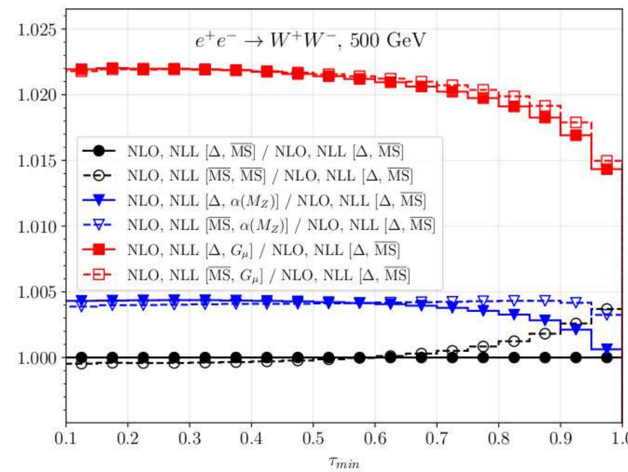


$t\bar{t}$

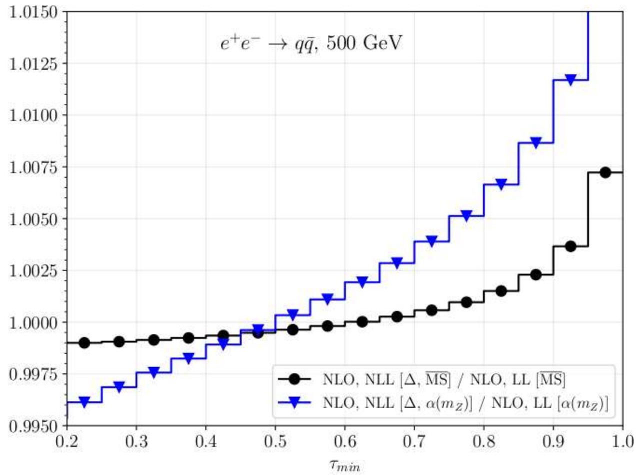


W^+W^-

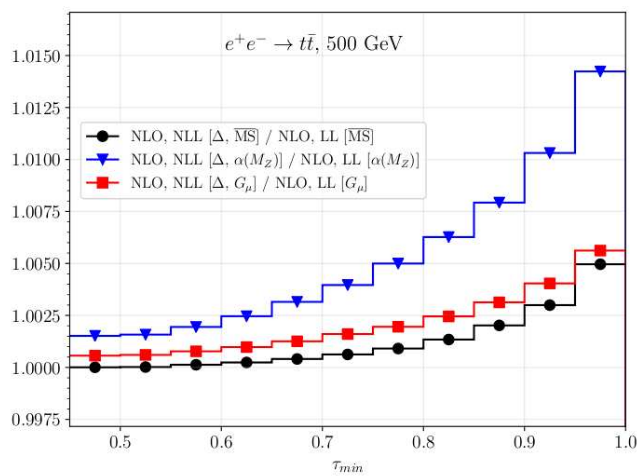
Renormalisation-scheme dependence much larger than factorisation-scheme dependence, with process-dependent pattern

Depending on the precision, renormalisation scheme is an informed choice; factorisation scheme always induces a systematic

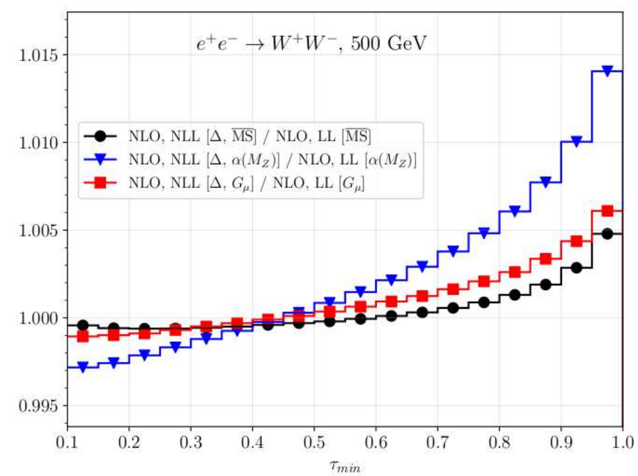
NLL vs LL



$q\bar{q}$



$t\bar{t}$

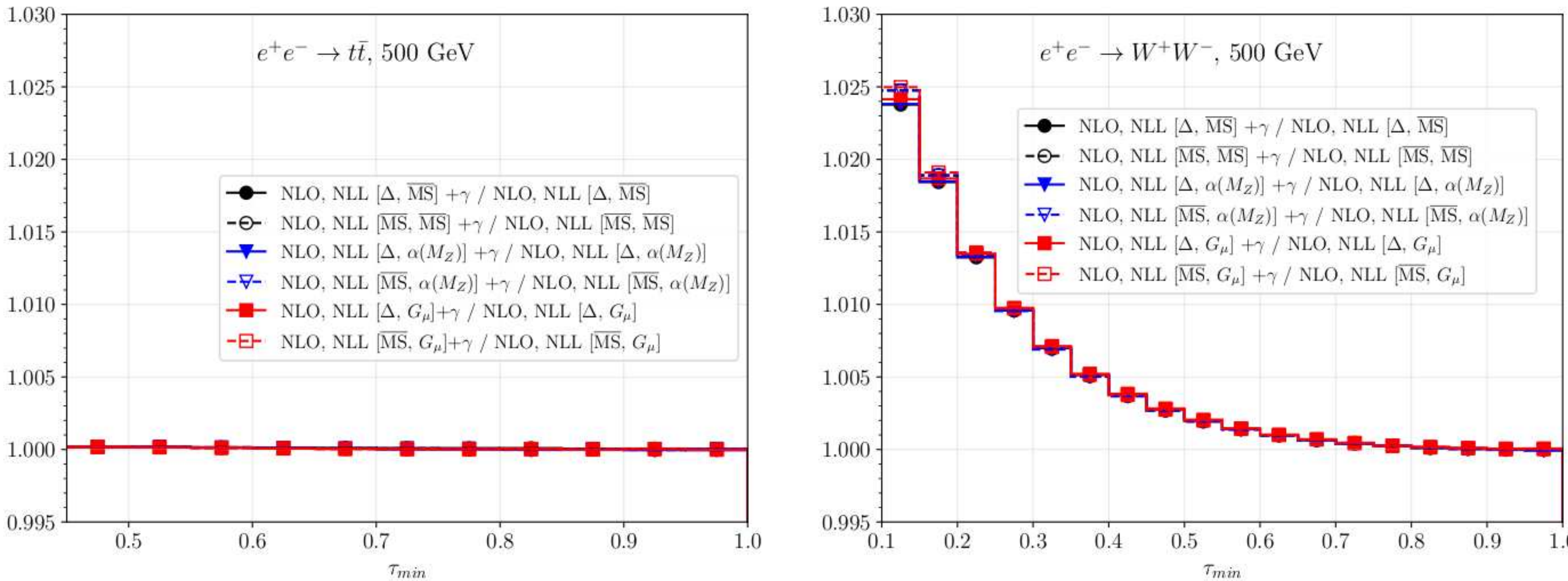


W^+W^-

Effects are non trivial

Pattern dependent on the process (and on the observable) as well as on the renormalisation scheme

Impact of $\gamma\gamma$ channel



$t\bar{t}$ W^+W^-

Essentially independent of factorisation and renormalisation schemes: a genuine physical effect

Utterly negligible for $t\bar{t}$, significant for W^+W^- – process dependence is not surprising

Thus:

- The inclusion of NLL contributions into the electron PDF has an impact of $\mathcal{O}(1\%)$ (precise figures are observable and renormalisation-scheme dependent)
- This estimate does not include the effects of the photon PDF
- The comparison between $\overline{\mathrm{MS}}$- and Δ-based results shows differences compatible with non-zero $\mathcal{O}(\alpha^2)$ effects, as expected
  (but: these are potentially *large in the soft region*)
- Renormalisation-scheme dependence is of $\mathcal{O}(0.5\%)$

If the target is a $10^{-\mathrm{some\ large\ number}}$ relative precision, these effects must be taken into account

The power of automation

$\sqrt{s}$ [GeV]	$\sigma(e^+e^- \to q\bar{q})$ [pb]	$\sigma(e^+e^- \to W^+W^-)$ [pb]	$\sigma(e^+e^- \to ZH)$ [pb]	$\sigma(W^+W^- \to H)$ [pb]	$\sigma(e^+e^- \to t\bar{t})$ [pb]
20.0	898.8	-	-	-	-
30.0	434.6	-	-	-	-
40.0	259.9	-	-	-	-
50.0	182.1	-	-	-	-
60.0	153.0	-	-	-	-
70.0	177.7	-	-	-	-
80.0	423.9	-	-	-	-
88.0	3891.0	-	-	-	-
91.2	29250.0	-	-	-	-
94.0	8953.0	-	-	-	-
125.0	417.9	-	-	-	-
157.5	177.4	-	-	-	-
162.5	162.0	-	-	-	-
165.0	155.2	8.773	-	0.00021	-
217.0	-	17.63	0.04278	0.004497	-
240.0	-	16.62	0.1998	0.005859	-
350.0	-	11.57	0.1306	0.024613	0.3771
360.0	-	11.22	0.1236	0.027064	0.5534

Cross-sections have been computed with MADGRAPH5_AMC@NLOv3.5.0 [1, 6], exploiting the recent developments for lepton colliders [9, 3]. In particular, ISR partonic densities with NLL-accurate evolution [7, 2, 3] have been employed, using the so-called Δ factorisation scheme [8]. All cross-sections include NLO EW and QCD corrections (the latter only when relevant), with the exception of $e^+e^- \to q\bar{q}$ where only QCD corrections are computed. NLO EW corrections are computed in the G_μ scheme; all fermions, with the exception of the top quark, are considered massless. Contributions from photons in the initial state are included whenever NLO EW corrections are computed. It is worth to note that the only processes where initial-state photons contribute at the LO are $e^+e^- \to t\bar{t}$ and $e^+e^- \to W^+W^-$. The following parameters have been employed:

$$m_t = 173.33\,\mathrm{GeV}\,,\ \mathrm{m_W} = 80.419\,\mathrm{GeV}\,,\ \mathrm{m_Z} = 91.189\,\mathrm{GeV}\,,\ \mathrm{m_H} = 125\,\mathrm{GeV}\,,\ \mathrm{G}_\mu = 1.166\times 10^{-5}\,\mathrm{GeV}^{-2}\,. \quad (1)$$

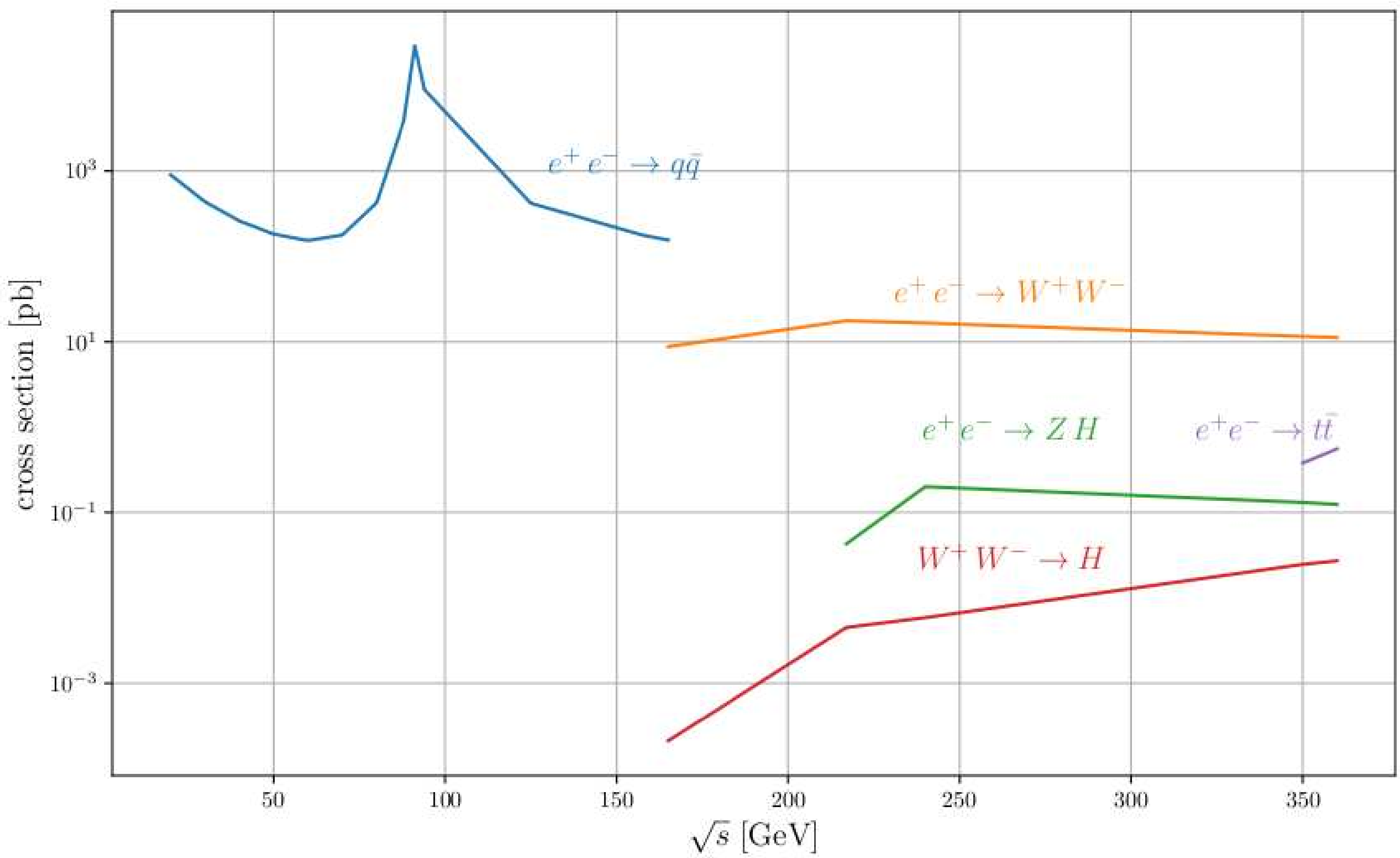
The cross section for Higgs production in VBF has been obtained as the difference of the cross sections for the processes $e^+e^- \to H\nu_e\bar{\nu}_e$ and $e^+e^- \to H\nu_\mu\bar{\nu}_\mu$, both computed at NLO EW accuracy in the complex mass scheme [4, 5]. For these cross sections, the following non-zero widths have been employed:

$$\Gamma_t = 1.3776\,\mathrm{GeV}\,,\quad \Gamma_\mathrm{W} = 2.093\,\mathrm{GeV}\,,\quad \Gamma_\mathrm{Z} = 2.499\,\mathrm{GeV}\,, \quad (2)$$

MG5_aMC@NLO, EW(+QCD) NLO accurate results, NLL PDFs

A few days of work (Selvaggi, Zaro)

The power of automation



MG5_aMC@NLO, EW(+QCD) NLO accurate results, NLL PDFs

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Are we done?

Not quite

- What was done at the NLL gives one a blueprint to go to NNLL, if need be. Most of the ingredients are available from QCD, but one still has to figure out the $z \to 1$ behaviour analytically

- In an orthogonal direction, one must achieve an exclusive generation, at the desired logarithmic accuracy

Exclusive means the ability to retain the information on the dof's of the particles stemming from the (ISR) branchings that do not enter the hard process

- Well established within YFS; not so much within collinear factorisation

- We cannot blindly apply MC@NLO or Powheg: hadron and lepton PDFs have dramatically different behaviours

- Besides, there is currently no NLL-accurate ISR hadronic shower

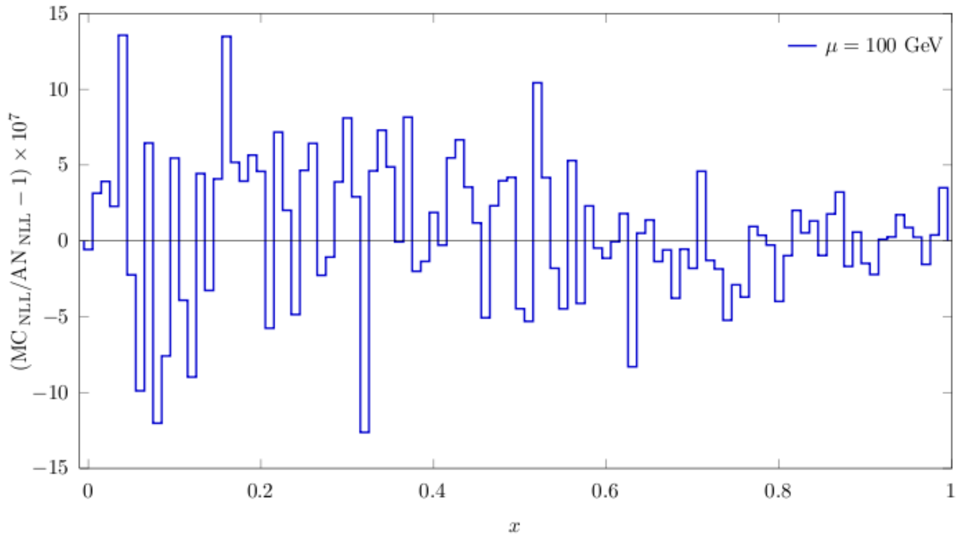
A possible approach: follow BabaYaga (Carloni Calame, Montagna, Nicrosini, Piccinini)

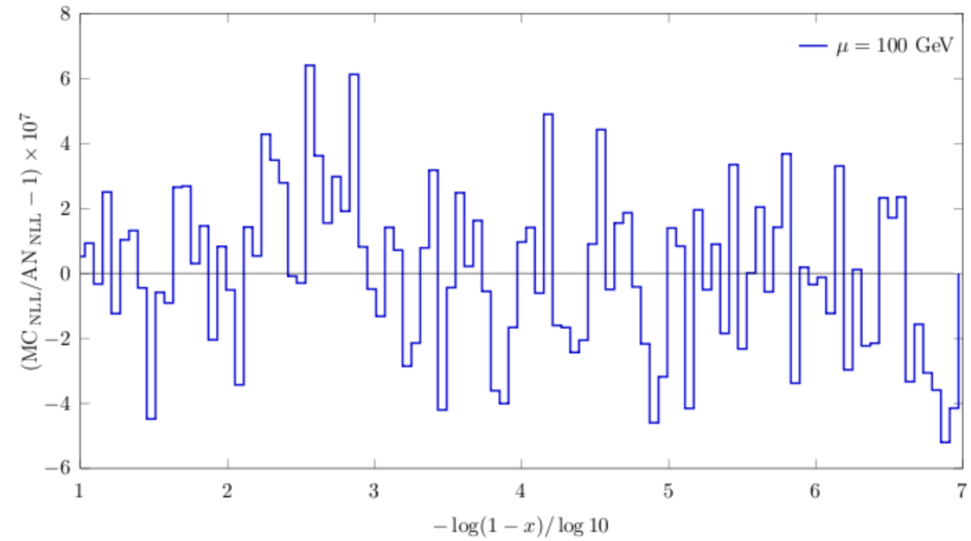
- α is small
- Thus, resumming to all orders is not that different w.r.t. to summing to a fairly high order (say, ~ 15)
- First step: write the PDFs as recursive, MC-compatible, solutions of the evolution equations, whose individual contributions can be associated with events (i.e. with given number and types of branchings)

This now works for the *non-singlet* at the *NLL* accuracy

(Carloni Calame, Frixione, Montagna, Piccinini, Stagnitto)

MC vs analytical





This is the fractional difference between the known PDFs and those generated exclusively

Agreement of $\mathcal{O}(10^{-7})$ up to $z \simeq 1 - 10^{-10}$ (cutoff $\epsilon = 10^{-14}$)

This is NLL Δ; NLL $\overline{\mathrm{MS}}$ and LL are analogous

Optimistic conclusions

There has been significant progress recently towards increasing the precision of factorisation-based simulations

A lot remains to be done (e.g. exclusive simulations), but we are a generation away: there is plenty of time

Pessimistic conclusions

There has been significant progress recently towards increasing the precision of factorisation-based simulations

A lot remains to be done (e.g. exclusive simulations), but we are a generation away: there is ~~plenty of~~ too much time