Status and prospects of parton shower accuracy

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within PanScales:
Mrinal Dasgupta, Frederic Dreyer, Basem El Menoufi, Silvia Ferrario Ravasio, Keith Hamilton, Alexander Karlberg, Rok Medves, Pier Monni, Gavin Salam, Ludovic Scyboz, Alba Soto-Ontoso, Gregory Soyez, Melissa van Beekveld, Rob Verheyen

June 2023

Some directions of parton shower development

- Assessing logarithmic accuracy:
  Herwig [1904.11866, 2107.04051], Deductor [2011.04777], Forshaw, Holguin, Plätzer [2003.06400], PanScales [1805.09327, 2002.11114,2205.02237,2305.08645], Alaric [2110.05964], ...

- Fixed order matching:
  NLO; i.e. Frixione & Webber [0204244], Nason [0409146], PanScales[2301.09645] ... NNLO; i.e. UNNLOPS [1407.3773], MiNNLOps [1908.06987], Vincia [2108.07133], ... NNNLO; Prestel [2106.03206], Bertone, Prestel [2202.01082]

- Triple collinear and double soft splittings:
  Dulat, Höche, Krauss, Gellersen, Prestel [1705.00982, 1705.00742, 1805.03757, 2110.05964] Li & Skands [1611.00013], Löschner, Plätzer, Simpson Dore [2112.14454],...

- Colour and spin correlations:
  Forshaw, Holguin, Plätzer, Sjödahl [1201.0260, 1808.00332, 1905.08686, 2007.09648, 2011.15087] Deductor [0706.0017, 1401.6364, 1501.00778, 1902.02105], Herwig [1807.01955], Plätzer & Ruffa [2012.15215] PanScales [2011.10054, 2103.16526, 2111.01161] ...

- Electroweak corrections:
  Vincia [2002.09248, 2108.10786], Pythia [1401.5238], Herwig [2108.10817], ...

Not exhaustive!

Some directions of parton shower development

Talks at the recent CERN workshop on parton showers

- **Assessing logarithmic accuracy:**
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- **Electroweak corrections:**
  Vincia [2002.09248, 2108.10786], Pythia [1401.5238], Herwig [2108.10817], ...

Plus more...

Will mainly focus on logarithmic accuracy

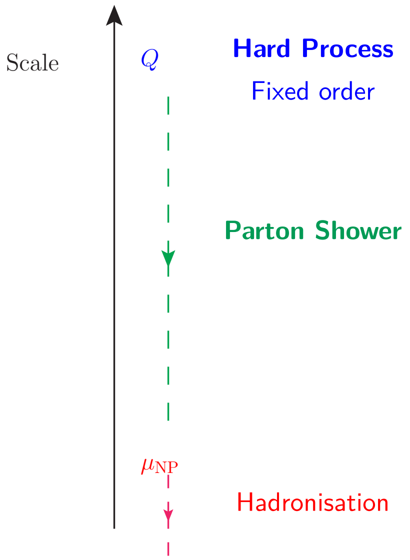
But will mention quite a few of the other headings in the context of log accuracy

Selection of closely related topics

Also discussed at the CERN parton showers workshop

- Hadronisation:
  Plätzer [2204.06956], Bierlich, Chakraborty, Gustafson, Lonnblad [1901.07447,2205.11170,2202.12783], Gieseke, Kirchgaeßer, Plätzer, Siodmok[1808.06770], Bellm, Duncan, Gieseke, Myska [2012.02070]
- Tuning:
  (Leif's talk)
  Skands, Carrazza, Rojo [1404.5630], Gieseke, Rohr,Siodmok [1206.0041], Bellm, Gellersen [1908.10811], Krishnamoorthy, Schulz, Ju, Wang, Leyffer, Marshall, Mrenna, Müller, Kowalkowski [2103.05748]
- Shower uncertainties:
  Les Houches [1803.07977], Snowmass [2203.11110], Hoche, Reichelt, Siegert [1711.03497], Hamilton, Karlberg, Salam, Scyboz, Verheyen [2301.09645],Amoroso, Caron, Jueid, Austrie, Skands [1812.07424]
- Non-perturbative power corrections:
  Luisoni, Monni, Salam [2012.00622], Caola, Ferrario-Ravasio, Limatola, Melnikov, Nason, Ozcelik [2204.02247,2108.08897] Nason, Zanderighi [2301.03607]
- QED corrections:
  (Stefano and Bennie's talks)
  Bertone, Cacciari,Frixione, Mattelaer,Stagnitto, Zaro, Zhao[2108.10261,2207.03265],Jadach,Ward,Was [1307.4037,2303.14260], Krauss, Price, Schonherr[2203.10948.],Snowmass [2203.12557]
- ....

Plus more...


Scale
Q
Hard Process
Fixed order
Parton Shower
μ_{NP}
Hadronisation

Scale

Q

Hard Process

Fixed order

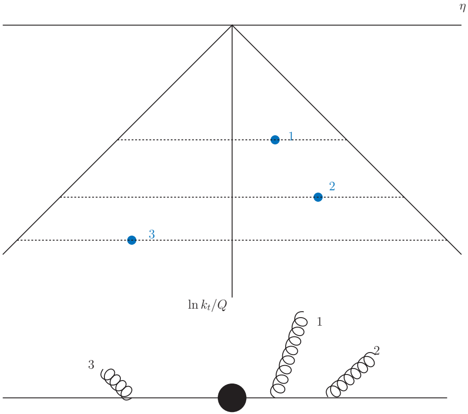
Parton Shower

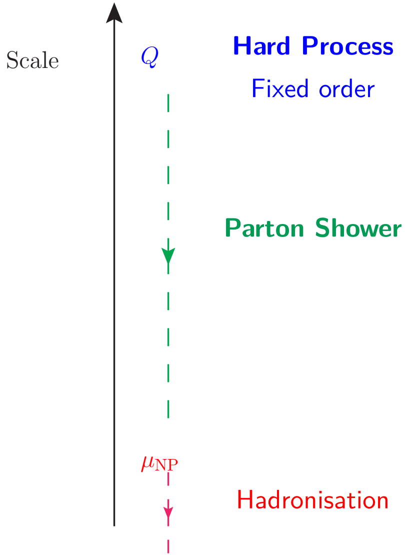
Dipole shower iterates $2 \to 3$ splittings ordered in $v \simeq k_t e^{\beta|\eta|}$.

$\mu_{\rm NP}$

Hadronisation

k_t ordering $(\beta = 0)$





Expect logs between disparate scales:

$$\alpha_s \ln^2 Q/\mu_{NP}$$
$$\alpha_s \ln Q/\mu_{NP}$$

...

Would like to re-sum these logs

$\rightarrow$ **Logarithmic accuracy:**

Well defined

Systematically improvable

Exponentiating observables:

$$\Sigma^{\mathrm{N}^k\mathrm{LL}}(\lambda = \alpha_s L) = (\underbrace{1}_{\mathrm{LL,NLL}} + \underbrace{\alpha_s c_1}_{\mathrm{NNLL}} + \ldots)\exp[\underbrace{\alpha_s^{-1} g_1(\lambda)}_{\mathrm{LL}} + \underbrace{g_2(\lambda)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\lambda)}_{\mathrm{NNLL}} + \ldots]$$

Other observables:

$$\Sigma^{\mathrm{N}^k\mathrm{DL}}(\xi = \alpha_s L^2) = \underbrace{h_1(\xi)}_{\mathrm{DL}} + \underbrace{\sqrt{\alpha_s} h_2(\xi)}_{\mathrm{NDL}} + \underbrace{\alpha_s h_3(\xi)}_{\mathrm{NNDL}} + \ldots$$

Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

At NLL the shower should reproduce known resummations of:

- global event shapes (at NLL $= \alpha_s^n L^n$)
- pdf/fragmentation function evolution (at $\alpha_s^n L^n$)
- non-global observables (at $\alpha_s^n L^n$)
- (subjet) multiplicity (at NDL $= \alpha_s^n L^{2n-1}$)

Matrix Element Requirement:

NLL - shower reproduces correct matrix element when emissions are well separated in the Lund plane

NNLL - shower reproduces correct matrix element for pairs of emissions close in the Lund plane which are well separated from other emissions/pairs

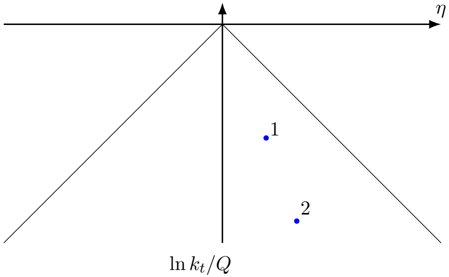
⋮

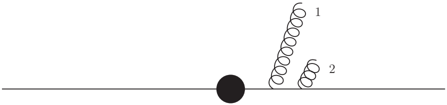
Logarithmic accuracy of parton showers

Matrix Element Requirement:

- Matrix elements for well separated emissions factorise (e.g emission 1's matrix element is factorised from that of emission 2)
- At NLL a shower should reproduce the correct matrix element for the configuration shown

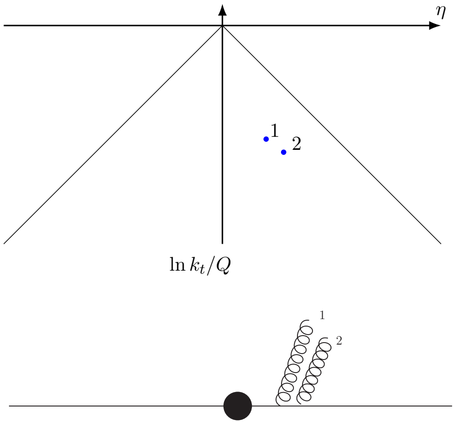
→ Emissions should not be influenced by subsequent emissions that are far away in the Lund plane





Matrix Element Requirement:

- An NLL shower may make a mistake for this configuration (two emissions close in the Lund plane)
- Would require higher order splitting functions to correctly describe this configuration



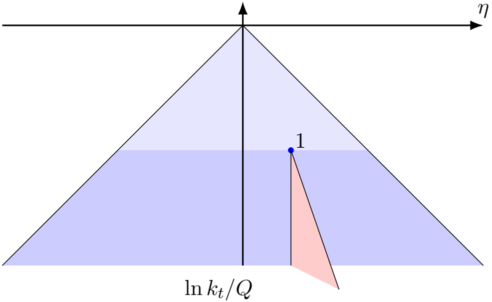
Recoil

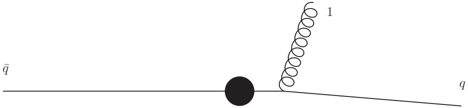
- How recoil is distributed is a key factor in determining the logarithmic accuracy of a parton shower

- Shower conserves momentum at each splitting

→ Need to distribute recoil amongst other particles

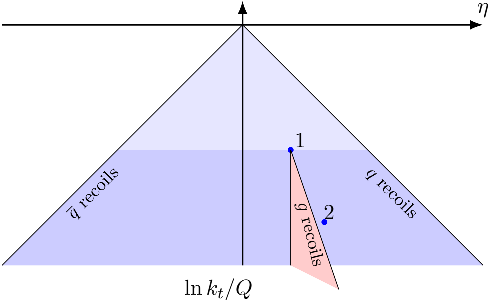
- The red leaf of the Lund plane is the phase space associated with emissions from gluon 1

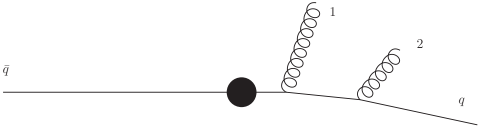




Expected assignment of recoil

- In line with NLL accuracy requirement, emission of gluon 2 should not influence emission 1 (see diagram)
- Recoil assigned to the quark





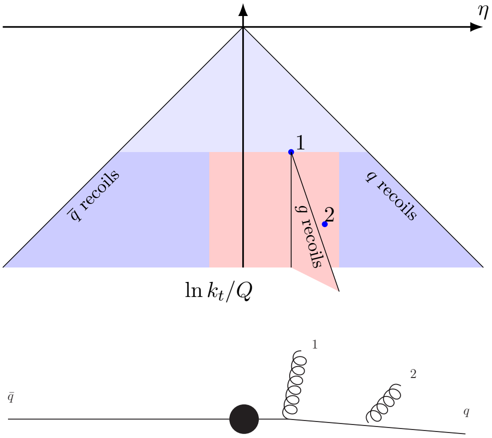
- In many standard showers with dipole local recoil, the emission of gluon 2 modifies the kinematics of emission 1

→ Kinematics of 1 are now different to what the emission was accepted with. Matrix element is now incorrect wrt final momentum of 1

- Not consistent with NLL

- Solutions available with local and global recoil schemes

Incorrect assignment of recoil



Possible NLL shower solutions for e^+e^-

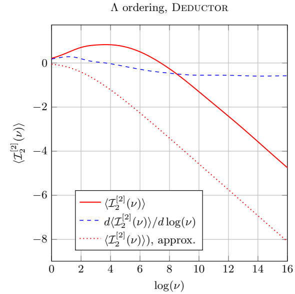
Shower	Ordering	NLL Validation
PanScales [2002.11114]	[1] $0 \leq \beta < 1$	Fixed and all order numerical tests for a range of observables
Alaric [2208.06057]	k_t $(\beta = 0)$	Analytical, numerical tests for global event shapes
Deductor [2011.04777]	k_t, Λ $(\beta = 0, 1)$	Analytical and numerical tests for thrust
Manchester-Vienna [2003.06400]	k_t $(\beta = 0)$	Analytical for thrust and multiplicity

- Differences between showers including: implementation of splitting functions, ordering variables available, kinematic map (distribution of recoil) ...
- Also differing approaches to testing NLL accuracy

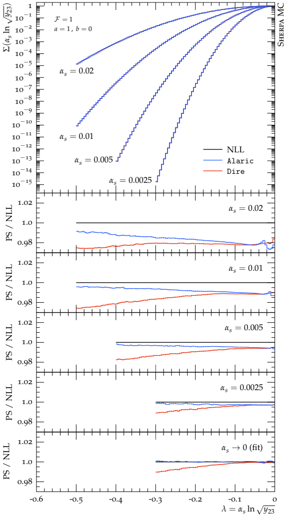
[1] β=0 only for Global recoil scheme

Numerical tests of NLL accuracy

- PanScales and Alaric have similar approaches to all orders tests (see right for example (y_{23}) of NLL test from Alaric)
- Idea is to eliminate terms beyond NLL accuracy by taking $\lim_{\alpha_s \to 0}$ while keeping $\alpha_s L$ fixed
- Deductor tests shower operator for thrust distribution (see below)



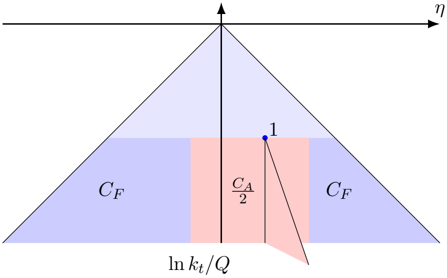
Nagy, Soper [2011.04777]



Herren, Hoche, Krauss, Reichelt, Schoenherr [2208.06057]

Can also get issues with the way colour factors are assigned

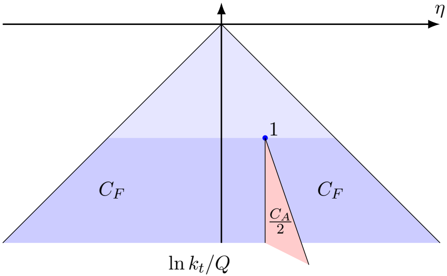
Sub-leading colour issues at LL $\mathcal{O}(1/N_c^2)$



Full colour (FC) at LL

basis of FC for global observables at NLL

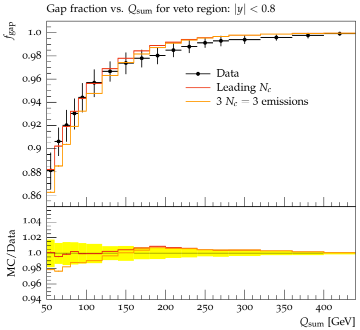
[2011.10054]



- Both recoil and colour issues stem from how dipoles are partitioned into an emitter and spectator

Colour

- Work by several groups on improvements in handling of sub-leading colour correlations (Both *Traditional parton shower* and *Amplitude evolution* approaches):
  Platzer,Sjodahl,Thorén[1808.00332, 1201.0260], Forshaw, De Angelis, Holguin, Platzer, [2007.09648, 1905.08686, 2003.06399] , Deductor [1202.4496],Hoche, Reichelt[2001.11492], PanScales[2011.10054]
- Important e.g for inclusion of sub-leading colour corrections for non-global logs (see plot)
- Can hadronisation models make use of more colour information if parton showers include it, e.g colour reconnection (Discussed at CERN workshop in talks by Gieseke and Platzer)
  Gieseke, Kirchgaeßer, Platzer,Siodmok [1808.06770]



Platzer, Sjodahl, Thoren [1808.00332]

Matching

- NLO matching has been available for a long time (Frixione, Webber [0204244], Nason [0409146]). De-facto tool for particle physics simulation

- NNLO and even N^{3}LO matching is available for certain processes (UNNLOPS [1407.3773], MiNNLOps [1908.06987], Vincia [2108.07133], Prestel [2106.03206], Bertone, Prestel [2202.01082])

- Can think about NLO matching in context of logarithmic accuracy Hamilton, Karlberg, Salam, Scyboz, Verheyan[2301.09645]

Interplay of matching with logarithmic accuracy

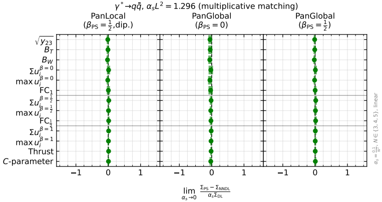
$$\Sigma^{\mathrm{N}^k\mathrm{LL}}(\lambda = \alpha_s L) = (\underbrace{1}_{\mathrm{LL,NLL}} + \underbrace{\alpha_s c_1}_{\mathrm{NNLL/NNDL}} + \ldots) \exp[\underbrace{\alpha_s^{-1} g_1(\lambda)}_{\mathrm{LL}} + \underbrace{g_2(\lambda)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\lambda)}_{\mathrm{NNLL}}]$$

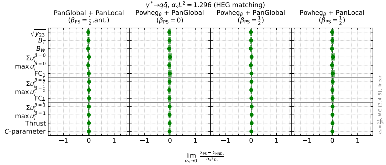
- c_1 can be obtained through NLO matching
- Including c_1 promotes global event shapes from NLL to NNDL accuracy
- Also a step towards NNLL accuracy more generally

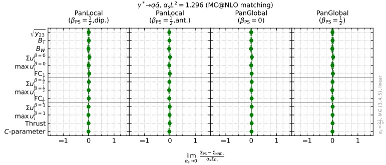
Interplay of matching with logarithmic accuracy

Hamilton, Karlberg, Salam, Scyboz, Verheyan[2301.09645]

- Standard NLO matching procedures do not break NLL accuracy
- They can augment an NLL shower to NNDL for global event shapes
- There are some subtleties with HEG-matching (but not prohibitive)





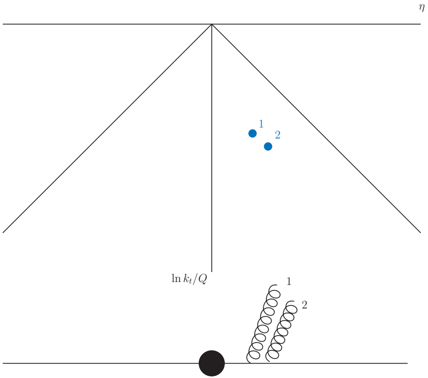


Towards higher log accuracy

- A NNLL shower should reproduce the correct matrix element for configurations where 2 emissions are close in the Lund plane

→ Requires higher order splitting functions

- NNLL will also require new ingredients to account for virtual corrections



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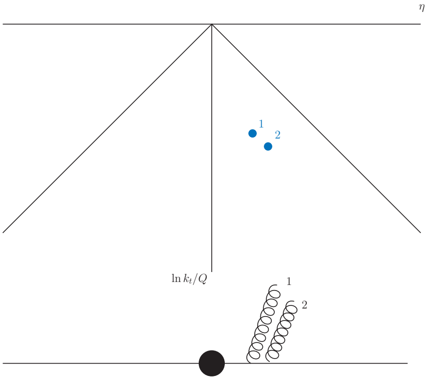


Structure of Sudakov factor:

$$S = \exp\left[-\int_{b_0^2/b^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2}\Big(A(\alpha_s(q^2))\ln(Q^2/q^2) + B(\alpha_s(q^2))\Big)\right]$$

$$A(\alpha_s(q^2)) = \sum_{n=1}\left(\frac{\alpha_s(q^2)}{2\pi}\right)^n A_n \qquad B(\alpha_s(q^2)) = \sum_{n}\left(\frac{\alpha_s(q^2)}{2\pi}\right)^n B_n$$

Collins, Soper, Sterman, 1981

NLL:

$A_1 = 2C_F,\ A_2 = C_F K_{\mathrm{CMW}}$

$B_1 = -3C_F$

NNLL:

need B_2 and A_3

B_2 is observable dependent.

For quarks:

$$B_2 = -2\gamma_q^{(2)} + C_F b_0 X_v, \qquad b_0 = \frac{11C_A - 4T_R n_f}{6}$$

$$\gamma_q^{(2)} = -C_F\left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3)\right) - C_A\left(\frac{17}{24} + \frac{11\pi^2}{18} - 3\zeta(3)\right) + T_R n_f\left(\frac{1}{6} + \frac{2\pi^2}{9}\right)$$

X_v depends on the observable

Davies and Stirling 1984; Catani, de FLorian, Grazzini, 2001; Banfi et al 2019

(Similar for gluons)

In a shower could use a differential $B_2(z)$ which depends on emission's kinematics.

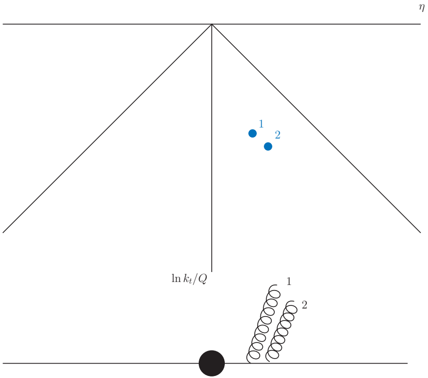
Involves triple collinear splitting functions integrated over second emission. Combined with virtual corrections. NNLL hard collinear terms isolated.

Dasgupta, El-Menoufi [2109.07496]

Towards higher log accuracy

- an NNLL shower should reproduce the correct matrix element for configurations where 2 emissions are close in the Lund plane

→ **Requires higher order splitting functions**



- NNLL will also require new ingredients to account for virtual corrections

Higher order splitting kernels

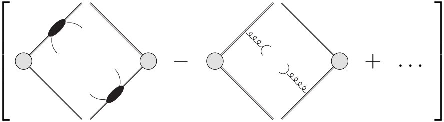
Dulat, Gellersen, Hoeche, Prestel[1705.00742, 1805.03757, 2110.05964]

Method for including Double soft and triple collinear corrections in parton showers

Consider emission of a quark pair:

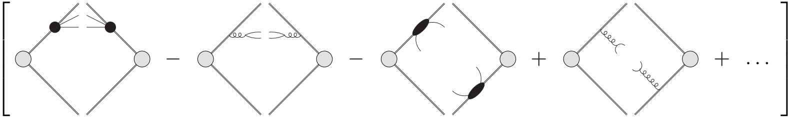
Double soft correction with iterated LO removed:

$P^{(\mathrm{ds})} \sim$



Triple collinear correction with iterated LO and overlap with double soft removed:

$P^{(\mathrm{tc-ds})} \sim$



Note that this is not an NNLL shower as it has the recoil problem discussed earlier

Summary

- Very quick tour of some elements of parton shower accuracy
- Lots of recent developments in parton shower accuracy
- NLL accurate showers now available from several groups
- Work ongoing for further improvements, e.g towards NNLL, sub-leading colour

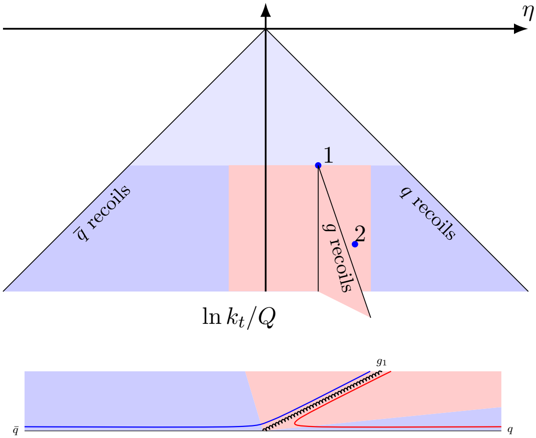
Backup

More details on recoil issues at NLL

Dipole partitioned in dipole center of mass frame

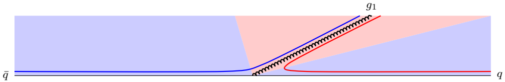
Boost back to event frame

More details on recoil issues at NLL



Local kinematic Map:

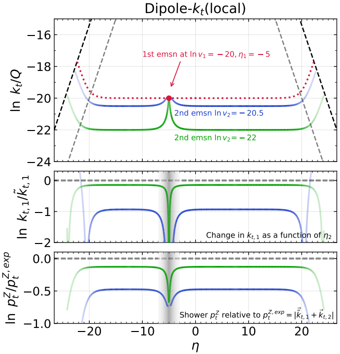
- Partition dipole in event COM frame
- Choose ordering variable with angular dependence so as not to emit in wrong region ($\beta > 0$)



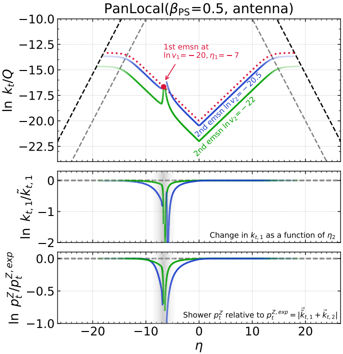
Or Choose a global map

PanScales tests of recoil

Not consistent with NLL



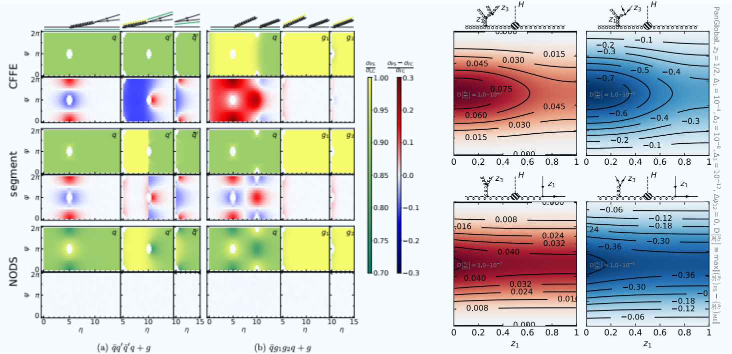
Consistent with NLL



[2002.11114, 2103.16526, 2011.10054, 2111.01161, 2205.02237, 2207.09467]

Fixed order tests

check that the shower will reproduce the correct matrix element in well separated configurations



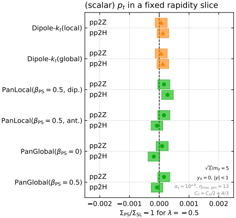
(a) $\bar{q}q'\bar{q}'q+g$ (b) $\bar{q}g_1g_2q+g$

PanScales Log Accuracy Tests

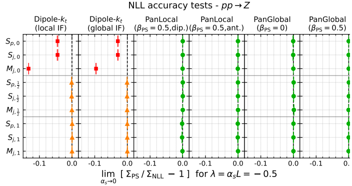
[2002.11114, 2103.16526, 2011.10054,2111.01161, 2205.02237, 2207.09467]

All orders tests

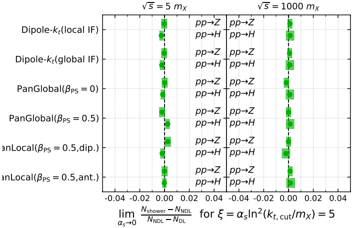
non-global logs



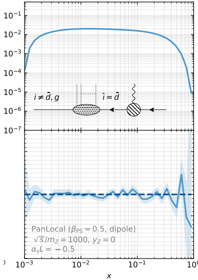
global observables



multiplicity



DGLAP



Subtleties with HEG matching

[2301.09645]

- Contours of shower and HEG need to match
- Solution to this already known and applied in typical POWHEG use.
- After HEG emission, shower should start from max scale and veto all emissions above the HEG contour.
- Also need to make sure the shower and HEG handle partitioning of $g \to gg(q\overline{q})$ in the same way