Collinearly Enhanced Realizations of the YFS MC Approach to Precision Resummation Theory

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OUTLINE

- Introduction
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- Improving the Collinear Limit in YFS Theory
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Introduction

- The Future of Precision Theory: Dictated by Future Accelerators – FCC, CLIC, ILC, CEPC, CPPC, $\cdots$
- Using FCC as an example, factors of improvement from ~ 5 to ~ 100 are needed from Theory
- Resummation is a key to such improvements in many cases: Today, we discuss amplitude-based resummation following the YFS methodology
- YFS $\rightarrow$ 'no limit to precision'
- See 1989 CERN Yellow Book article by Berends *et al.*

Introduction

- YFS methods are exact in the infrared but treat the collinear logs perturbatively in the $\bar{\beta}_n$ residuals
- DGLAP-based collinear factorization treats the collinear logs to all orders but has a non-exact IR limit - see Stefano's talk and references therein ('all roads lead to Rome')
- Today, we investigate improving the collinear limit of YFS theory
- A Key Point: Exact Amplitude-Based Resummation Realized on Evt-by-Evt Basis – Enhanced Precision for a Given Level of Exactness: LO, NLO, NNLO, ...

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$$d\bar{\sigma}_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}} \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}, \tag{1}$$

where *new* (YFS-style) *non-Abelian* residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons.

Review of Exact Amplitude-Based Resummation Theory

Here,

$$\begin{aligned} \mathrm{SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED} \\ D_{\mathrm{QCED}} &= \int \frac{d^3k}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right) \tilde{S}^{nls}_{QCED} \end{aligned} \tag{2}$$

where K_{max} is "dummy" and

$$\begin{aligned} B^{nls}_{QCED} &\equiv B^{nls}_{QCD} + \frac{\alpha}{\alpha_s} B^{nls}_{QED}, \\ \tilde{B}^{nls}_{QCED} &\equiv \tilde{B}^{nls}_{QCD} + \frac{\alpha}{\alpha_s} \tilde{B}^{nls}_{QED}, \\ \tilde{S}^{nls}_{QCED} &\equiv \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED}. \end{aligned} \tag{3}$$

"nls"≡ DGLAP-CS synthesization.

Shower/ME Matching: $\tilde{\bar{\beta}}_{n,m} \rightarrow \hat{\tilde{\bar{\beta}}}_{n,m}$ --*KKMCee, KKMChh, Herwiri,...*

Improving the Collinear Limit in YFS Theory

- Basic Formula for CEEX/EEX realization of the YFS resummation of

  $e^+e^- \to f\bar{f} + n\gamma,\ f = \ell, q,\ \ell = e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau,\ q = u, d, s, c, b, t$:

$$\sigma = \frac{1}{\text{flux}} \sum_{n=0}^{\infty} \int d\text{LIPS}_{n+2}\, \rho_A^{(n)}(\{p\},\{k\}), \tag{4}$$

-

$$\rho^{(n)}_{\text{CEEX}}(\{p\},\{k\}) = \frac{1}{n!} e^{Y(\Omega;\{p\})} \bar{\Theta}(\Omega) \frac{1}{4} \sum_{\text{helicities } \{\lambda\},\{\mu\}} \left| \mathcal{M} \begin{pmatrix} \{p\}\,\{k\} \\ \{\lambda\}\,\{\mu\} \end{pmatrix} \right|^2. \tag{5}$$

By definition, $\Theta(\Omega, k) = 1$ for $k \in \Omega$ and $\Theta(\Omega, k) = 0$ for $k \notin \Omega$, with

$\bar{\Theta}(\Omega; k) = 1 - \Theta(\Omega, k)$ and

$$\bar{\Theta}(\Omega) = \prod_{i=1}^{n} \bar{\Theta}(\Omega, k_i).$$

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Improving the Collinear Limit in YFS Theory

- For Ω defined with the condition $k^0 < E_{\text{min}}$, the YFS infrared exponent reads

$$
\begin{aligned}
Y(\Omega; p_a, \ldots, p_d) = {} & Q_e^2 Y_\Omega(p_a, p_b) + Q_f^2 Y_\Omega(p_c, p_d) \\
& + Q_e Q_f Y_\Omega(p_a, p_c) + Q_e Q_f Y_\Omega(p_b, p_d) \qquad (6) \\
& - Q_e Q_f Y_\Omega(p_a, p_d) - Q_e Q_f Y_\Omega(p_b, p_c).
\end{aligned}
$$

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Improving the Collinear Limit in YFS Theory

- Here

$$
\begin{aligned}
Y_\Omega(p,q) \equiv & 2\alpha\tilde{B}(\Omega,p,q) + 2\alpha\Re B(p,q) \\
\equiv & -2\alpha\,\frac{1}{8\pi^2}\int\frac{d^3k}{k^0}\Theta(\Omega;k)\left(\frac{p}{kp}-\frac{q}{kq}\right)^2 \\
& + 2\alpha\Re\int\frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left(\frac{2p-k}{2kp-k^2}-\frac{2q+k}{2kq+k^2}\right)^2 .
\end{aligned}
\tag{7}
$$

- Fundamental Idea of YFS: isolate and resum to all orders in α the infrared singularities so that these singularities are canceled to all such orders between real and virtual corrections.

  What collinear singularities are also resummed in the YFS resummation algebra?

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Improving the Collinear Limit in YFS Theory

- Focusing on the s-channel and s'-channel contributions, we have

$$Y_e(\Omega_I; p_1, p_2) = \gamma_e \ln \frac{2E_{min}}{\sqrt{2p_1p_2}} + \frac{1}{4}\gamma_e + Q_e^2 \frac{\alpha}{\pi}\left(-\frac{1}{2} + \frac{\pi^2}{3}\right),$$
$$Y_f(\Omega_F; q_1, q_2) = \gamma_f \ln \frac{2E_{min}}{\sqrt{2q_1q_2}} + \frac{1}{4}\gamma_f + Q_f^2 \frac{\alpha}{\pi}\left(-\frac{1}{2} + \frac{\pi^2}{3}\right), \tag{8}$$

where

$$\gamma_e = 2Q_e^2 \frac{\alpha}{\pi}\left(\ln \frac{2p_1p_2}{m_e^2} - 1\right), \quad \gamma_f = 2Q_f^2 \frac{\alpha}{\pi}\left(\ln \frac{2q_1q_2}{m_f^2} - 1\right), \tag{9}$$

$\Rightarrow$ The YFS exponent resums the collinear big log term $\frac{1}{2}Q^2\frac{\alpha}{\pi}L$ to the infinite order in both the ISR and FSR contributions.

- Can this be improved to the result of Gribov and Lipatov to exponentiate $\frac{3}{2}\frac{\alpha}{\pi}L$ via the QED form-factor?

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Improving the Collinear Limit in YFS Theory

- The YFS form factor derivation illustrated in Fig. 4

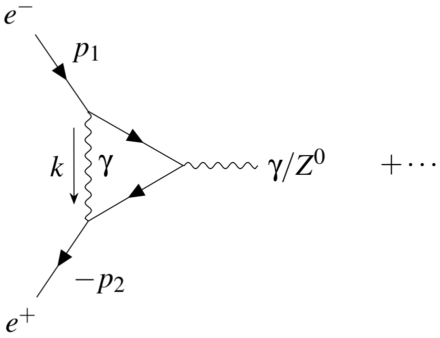

Figure: Virtual corrections which generate the YFS infrared function *B*. Self-energy contributions are not shown.

Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ the amplitude factor

$$\mathcal{M}_\mu = \frac{\int d^4k}{(2\pi)^4}\frac{-i}{k^2+i\varepsilon}\bar{v}(p_2)(-iQ_e e)\gamma^\alpha \frac{i}{-\not{p}_2-\not{k}-m+i\varepsilon}(-ie)\gamma_\mu(v_A-a_A\gamma_5)$$
$$\frac{i}{\not{p}_1-\not{k}-m+i\varepsilon}(-iQ_e e)\gamma_\alpha u(p_1) \qquad (10)$$

where $A=\gamma$ or Z.

Improving the Collinear Limit in YFS Theory

- Scalarising the fermion propagator denominators $\Rightarrow$

$$\mathcal{M}_\mu = -ie\frac{\int d^4k(-iQ_e{}^2e^2)}{(2\pi)^4}\frac{1}{k^2+i\varepsilon}\bar{v}(p_2)\gamma^\alpha\frac{-\not{p}_2-\not{k}+m}{k^2+2kp_2+i\varepsilon}\gamma_\mu(v_A-a_A\gamma_5)\frac{\not{p}_1-\not{k}-m}{k^2-2kp_1+i\varepsilon}\gamma_\alpha u(p_1). \quad (11)$$

- Using the equations of motion

$$(\not{p}_1-\not{k}-m)\gamma_\alpha u(p_1) = \{(2p_1-k)_\alpha - \frac{1}{2}[\not{k},\gamma_\alpha]\}u(p_1), \quad (a)$$

$$\bar{v}(p_2)\gamma^\alpha(-\not{p}_2-\not{k}+m) = \bar{v}(p_2)\{-(2p_2+k)^\alpha + \frac{1}{2}[\not{k},\gamma^\alpha]\}, \quad (b).$$

(12)

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- $\Rightarrow$ Contribution to $2Q_e^2\alpha B(p_1,p_2)$ corresponding to the cross-term in the virtual IR function on the RHS of eq.(7):

$$2Q_e^2\alpha B(p_1,p_2)|_{\text{cross-term}} = \int d^4k \frac{(iQ_e{}^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon} \frac{(2p_1-k)(2p_2+k)}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)}. \quad (13)$$

This term, together with the two squared terms in $2\alpha Q_e^2 B(p_1,p_2)$, leads to the exponentiation of $\frac{1}{2}Q_e^2\frac{\alpha}{\pi}L$.

Improving the Collinear Limit in YFS Theory

- The two commutator terms on the RHS of eq.(12), usually dropped, can be analyzed further: possible IR finite collinearly enhanced improvement of the YFS virtual IR function B.
- Isolate the collinear parts of k via the change of variables

$$k = c_1 p_1 + c_2 p_2 + k_\perp \tag{14}$$

where $p_1 k_\perp = 0 = p_2 k_\perp$, $\Rightarrow$ we have the relations

$$\begin{aligned} c_1 &= \frac{p_1 p_2}{(p_1 p_2)^2 - m^4} p_2 k - \frac{m^2}{(p_1 p_2)^2 - m^4} p_1 k \xrightarrow[CL]{} \frac{p_2 k}{p_1 p_2} \\ c_2 &= \frac{p_1 p_2}{(p_1 p_2)^2 - m^4} p_1 k - \frac{m^2}{(p_1 p_2)^2 - m^4} p_2 k \xrightarrow[CL]{} \frac{p_1 k}{p_1 p_2}, \end{aligned} \tag{15}$$

CL denotes the collinear limit $\equiv \mathcal{O}(m^2/s)$ dropped.

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- $\Rightarrow$ $(2p_1 - k)^\alpha$ in eq.(12(a)) combines with the commutator term in eq.(12(b)) to produce

$$\begin{aligned}
\bar{v}(p_2)\{(2p_1-k)_\alpha \tfrac{1}{2}[\not k, \gamma^\alpha]\}\gamma_\mu(v_A - a_A\gamma_5)u(p_1)\\
= \bar{v}(p_2)[\not k, \not p_1]\gamma_\mu(v_A - a_A\gamma_5)u(p_1)\\
\xrightarrow[CL]{} \bar{v}(p_2)[c_2\ \not p_2, \not p_1]\gamma_\mu(v_A - a_A\gamma_5)u(p_1)\\
\xrightarrow[CL]{} \bar{v}(p_2)(-2c_2 p_1 p_2)\gamma_\mu(v_A - a_A\gamma_5)u(p_1)\\
\xrightarrow[CL]{} \bar{v}(p_2)(-2p_1 k)\gamma_\mu(v_A - a_A\gamma_5)u(p_1).
\end{aligned} \tag{16}$$

- Similarly, $-(2p_2 + k)^\alpha$ in eq.(12 (b)) combines with the commutator term in eq.(12(a)) to produce

$$\begin{aligned}
\bar{v}(p_2)\gamma_\mu(v_A - a_A\gamma_5)\{-(2p_2+k)^\alpha(-\tfrac{1}{2}[\not k, \gamma_\alpha])\}u(p_1)\\
= \bar{v}(p_2)\gamma_\mu(v_A - a_A\gamma_5)[\not k, \not p_2]u(p_1)\\
\xrightarrow[CL]{} \bar{v}(p_2)\gamma_\mu(v_A - a_A\gamma_5)[c_1\ \not p_1, \not p_2]u(p_1)\\
\xrightarrow[CL]{} \bar{v}(p_2)\gamma_\mu(v_A - a_A\gamma_5)(2c_1 p_1 p_2)u(p_1)\\
\xrightarrow[CL]{} \bar{v}(p_2)\gamma_\mu(v_A - a_A\gamma_5)(2p_2 k)u(p_1).
\end{aligned} \tag{17}$$

Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ Shift of the factor $(2p_1 - k)(2p_2 + k)$ on the RHS of eq.(13) as

$$(2p_1 - k)(2p_2 + k) \underset{CL}{\longrightarrow} (2p_1 - k)(2p_2 + k) + 2p_1 k - 2p_2 k. \quad (18)$$

Improving the Collinear Limit in YFS Theory

- What does the term quadratic in the commutator (C^2) contribute?
- Superficial UV divergence $\Rightarrow$ Cannot naively drop $k_\perp$
- Proceed directly: we need

$$2Q_e^2\alpha B(p_1,p_2)|_{C^2}\mathcal{M}_{B\mu} \equiv \frac{\int d^4k(iQ_e{}^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon} \left.\frac{\frac{1}{4}\bar{v}(p_2)[\not{k},\gamma^\alpha]\gamma_\mu[\not{k},\gamma_\alpha](-ie)(v_A-a_A\gamma_5)u(p_1)}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)}\right|_{CL}, \quad (19)$$

where we define

$$\mathcal{M}_{B\mu} = -ie\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)u(p_1). \quad (20)$$

- *CL* now further restricted to contributions singular as $m^2/s \to 0$.

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- Four terms in the numerator of eq.(19) from the respective sum of gamma matrix products $\{\not{k}\gamma^{\alpha}\gamma_{\mu}\not{k}\gamma_{\alpha} - \not{k}\gamma^{\alpha}\gamma_{\mu}\gamma_{\alpha}\not{k} - \gamma^{\alpha}\not{k}\gamma_{\mu}\not{k}\gamma_{\alpha} + \gamma^{\alpha}\not{k}\gamma_{\mu}\gamma_{\alpha}\not{k}\} = \{\gamma^{\lambda}\gamma^{\alpha}\gamma_{\mu}\gamma^{\lambda'}\gamma_{\alpha} - \gamma^{\lambda}\gamma^{\alpha}\gamma_{\mu}\gamma_{\alpha}\gamma^{\lambda'} - \gamma^{\alpha}\gamma^{\lambda}\gamma_{\mu}\gamma^{\lambda'}\gamma_{\alpha} + \gamma^{\alpha}\gamma^{\lambda}\gamma_{\mu}\gamma_{\alpha}\gamma^{\lambda'}\}k_{\lambda}k_{\lambda'} \equiv N_{\mu}^{\lambda\lambda'}k_{\lambda}k_{\lambda'}$
- This defines $N_{\mu}^{\lambda\lambda'}$.

Improving the Collinear Limit in YFS Theory

- Using standard methods, we need

$$I_\mu = 2\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{\int d^n k' (i {Q_e}^2 e^2)}{8\pi^4} \left.\frac{\frac{1}{4}\bar{v}(p_2) N_\mu^{\lambda\lambda'} [\frac{k'^2}{n} g_{\lambda\lambda'} + \Delta_\lambda \Delta_{\lambda'}](-ie)(v_A - a_A\gamma_5) u(p_1)}{[k'^2 - \Delta^2 + i\varepsilon]^3}\right|_{CL}, \qquad (21)$$

  where $\Delta = \alpha_1 p_1 - \alpha_2 p_2$.
- Equations of motion $\Rightarrow$ term involving Δ is not collinearly enhanced.

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Improving the Collinear Limit in YFS Theory

- The term contracted with $g_{\lambda\lambda'}$ gives us

$$I_\mu = \left\{ \frac{-3Q_e^2\alpha}{4\pi} \mathcal{M}_{B\mu} \right\}\Bigg|_{CL} \equiv 0 \tag{22}$$

- $\Rightarrow$ No collinearly enhanced contribution from I_μ.
- Eq.(18) gives the complete collinear enhancement of B.
- Change in B does not affect its IR behavior – shift terms are IR finite $\Rightarrow$ Entire YFS IR resummation is unaffected.
- Shifted terms can be seen to extend the YFS IR exponentiation to obtain the entire exponentiated $\frac{3}{2}Q_e^2\alpha L$.

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Improving the Collinear Limit in YFS Theory

- We have

$$
\begin{aligned}
2\alpha Q_e^2 \Delta B(p_1,p_2) &= \frac{\int d^4k(iQ_e^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon}\frac{2p_1k-2p_2k}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)} \\
&= 2\int_{x_i\geq 0, i=1,2,3} d^3x\,\delta(1-x_1-x_2-x_3)\frac{\int d^4k'(iQ_e^2e^2)}{8\pi^4} \qquad (23)\\
&\quad \frac{2(p_1-p_2)p_x}{(k'^2-d+i\varepsilon)^3}
\end{aligned}
$$

where $d=p_x^2$ with $p_x=x_1p_1-x_2p_2$.

- $\Rightarrow$ We get

$$2Q_e^2\alpha\Re\Delta B(p_1,p_2)=Q_e^2\frac{\alpha}{\pi}L. \qquad (24)$$

- We see that indeed the entire term $\frac{3}{2}Q_e^2\frac{\alpha}{\pi}L$ is now exponentiated by our collinearly improved YFS virtual IR function B_{CL}

$$
\begin{aligned}
B_{CL} &= B+\Delta B \\
&= \int\frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left[\left(\frac{2p-k}{2kp-k^2}-\frac{2q+k}{2kq+k^2}\right)^2-\frac{4pk-4qk}{(2pk-k^2)(2qk+k^2)}\right]. \qquad (25)
\end{aligned}
$$

See S. Jadach, Durham talk, 2002, for integrated form of B_{CL}.

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Improving the Collinear Limit in YFS Theory

- What about the real YFS IR algebra? Collinear enhancement desired in some applications
- $\Rightarrow$ Recall the original YFS EEX formulation of the respective algebra $\Rightarrow$ the formula for the YFS IR function $\tilde{B}$ given above in eq.(7).
- See Fig. 5.

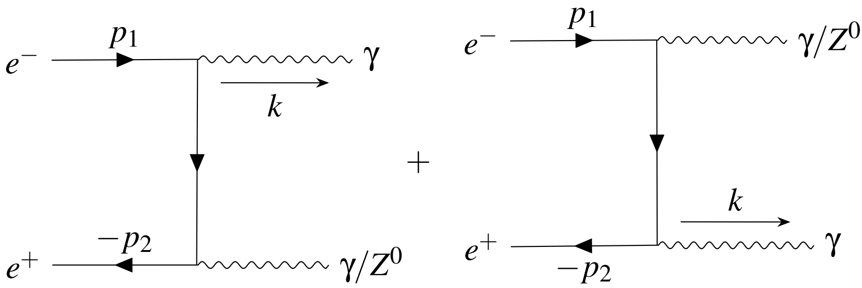

Figure: Real corrections which generate the YFS infrared function $\tilde{B}$.

Figure 2: Real corrections which generate the YFS infrared functi

Improving the Collinear Limit in YFS Theory

- Following the steps in the usual YFS algebra for real emission $\Rightarrow$

$$2\alpha Q_e^2\tilde{B}\mathcal{M}_{B\mu}^{\dagger}\mathcal{M}_{B\mu'} = \frac{\int d^3k(-1)e^2Q_e^2}{2k_0(2\pi)^3}\Bigg[\frac{\bar{u}(p_1)(2p_1^\lambda - k^\lambda + \frac{1}{2}[\not{k},\gamma^\lambda])\gamma_\mu(v_A - a_A\gamma_5)v(p_2)}{k^2-2kp_1}$$
$$+\frac{\bar{u}(p_1)\gamma_\mu(v_A - a_A\gamma_5)(-2p_2^\lambda + k^\lambda + \frac{1}{2}[\not{k},\gamma^\lambda])v(p_2)}{k^2-2kp_2}\Bigg]$$
$$\Bigg[\frac{\bar{v}(p_2)\gamma_{\mu'}(v_A - a_A\gamma_5)(2p_{1\lambda} - k_\lambda - \frac{1}{2}[\not{k},\gamma_\lambda])u(p_1)}{k^2-2kp_1}$$
$$+\frac{\bar{v}(p_2)(-2p_{2\lambda} + k_\lambda - \frac{1}{2}[\not{k},\gamma_\lambda])\gamma_{\mu'}(v_A - a_A\gamma_5)u(p_1)}{k^2-2kp_2}\Bigg]\Bigg|_{k^2=0} + K_{\mu\mu'} \quad (26)$$

where $K_{\mu\mu'}$ is infrared finite,

- $$\mathcal{M}_{B\mu} = \bar{v}(p_2)\gamma_\mu(v_A - a_A\gamma_5)u(p_1) \quad (27)$$

Improving the Collinear Limit in YFS Theory

- If we drop the commutator terms on the RHS of eq.(26) we recover the usual YFS formula for $2\alpha Q_e^2 \tilde{B}$.
- We again isolate collinearly enhanced contributions by using the representation in eq.(14) for k, respecting the condition $k^2 = 0$. $\Rightarrow$ Maintain $0 = (c_1^2 + c_2^2)m^2 + 2c_1 c_2 p_1 p_2 - |k_\perp|^2$.
- $\Rightarrow$ Collinear enhancement of $\tilde{B}$:

$$2\alpha Q_e^2 \tilde{B}_{CL} = \frac{-\alpha Q_e^2}{4\pi^2} \int \frac{d^3k}{k_0} \Big\{ \left(\frac{p_1}{kp_1} - \frac{p_2}{kp_2}\right)^2 + \frac{1}{kp_1}\left(2 - \frac{kp_2}{p_1 p_2}\right) + \frac{1}{kp_2}\left(2 - \frac{kp_1}{p_1 p_2}\right) \Big\}. \quad (28)$$

- Agreement with Berends *et al.*

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Improving the Collinear Limit in YFS Theory

- What about CEEX?
- In Fig. 5, use of amplitude-level isolation of real IR divergences, K-S photon polarization vectors $\Rightarrow$

$$\mathcal{M}_\mu = \mathcal{M}_{B\mu}\mathfrak{s}_{CL,\sigma}(k), \tag{29}$$

with

$$\mathfrak{s}_{CL,\sigma}(k) = \sqrt{2}Q_e e\Big[-\sqrt{\frac{p_1\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_1 - \sigma >}{2p_1 k} + \delta_{\lambda\ -\sigma}\sqrt{\frac{k\zeta}{p_1\zeta}}\frac{< k\sigma|\hat{p}_1\lambda >}{2p_1 k}$$
$$+\sqrt{\frac{p_2\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_2 - \sigma >}{2p_2 k} + \delta_{\lambda\sigma}\sqrt{\frac{k\zeta}{p_2\zeta}}\frac{< \hat{p}_2\lambda|k - \sigma >}{2p_2 k}\Big]. \tag{30}$$

Here, $\zeta \equiv (1,1,0,0)$ and $\hat{p} = p - \zeta m^2/(2\zeta p)$.

- Upon taking the modulus squared of $\mathfrak{s}_{CL,\sigma}(k)$ we see that the extra non-IR divergent contributions reproduce the known collinear big log contribution which is missed by the usual YFS algebra.

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SUMMARY REMARKS

- Extended the original YFS algebra to include previously missed collinear non-IR big logs
- New, collinearly enhanced soft functions $\Leftrightarrow$ Higher level of accuracy for a given level of exactness in the IR-finite YFS hard photon residuals.
- Enhanced the toolbox available to extend the CEEX YFS MC method to the other important processes at present and future colliders.
- Some New Physics may hang in the balance at both LHC and FCC!