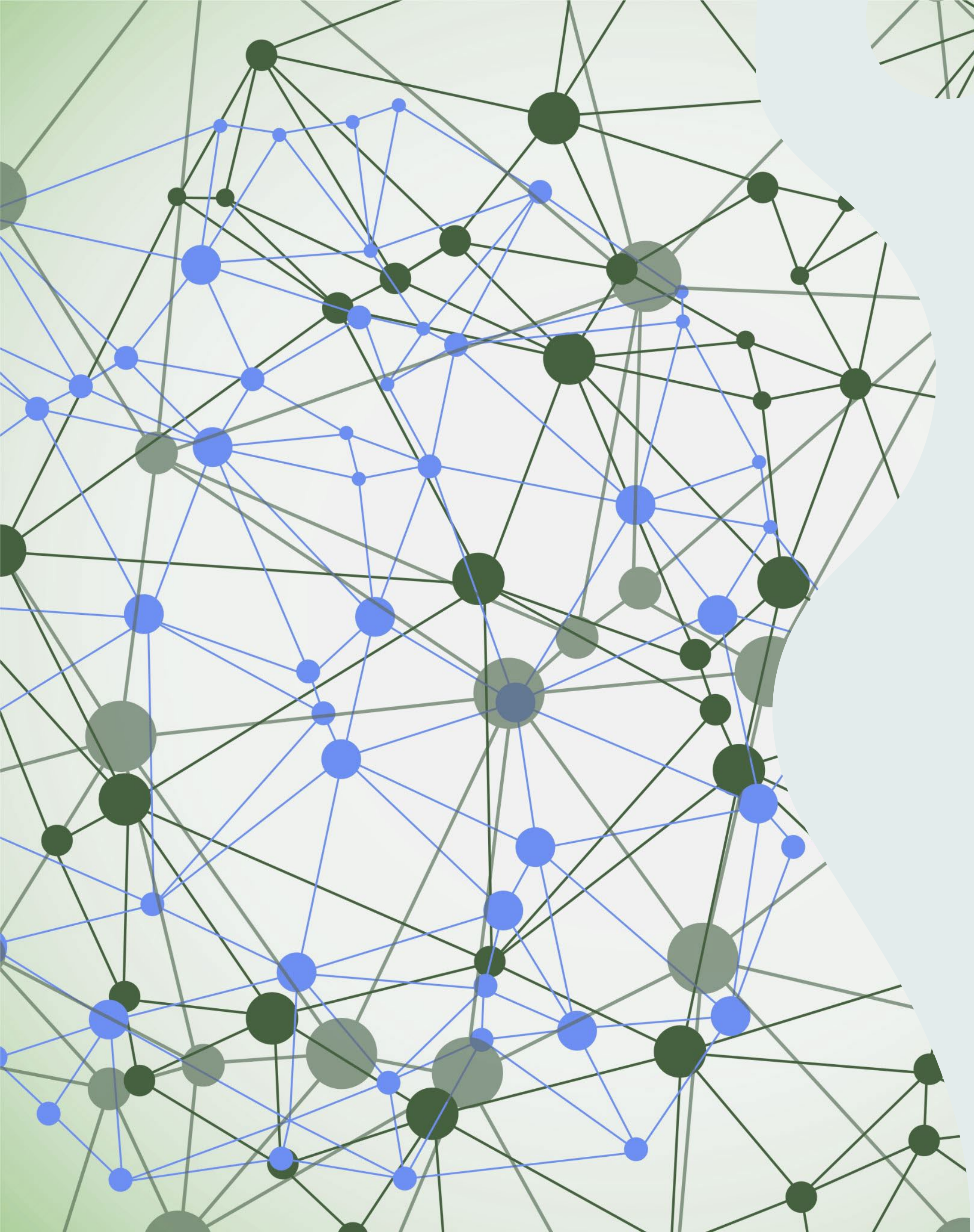

Spontaneous symmetry breaking

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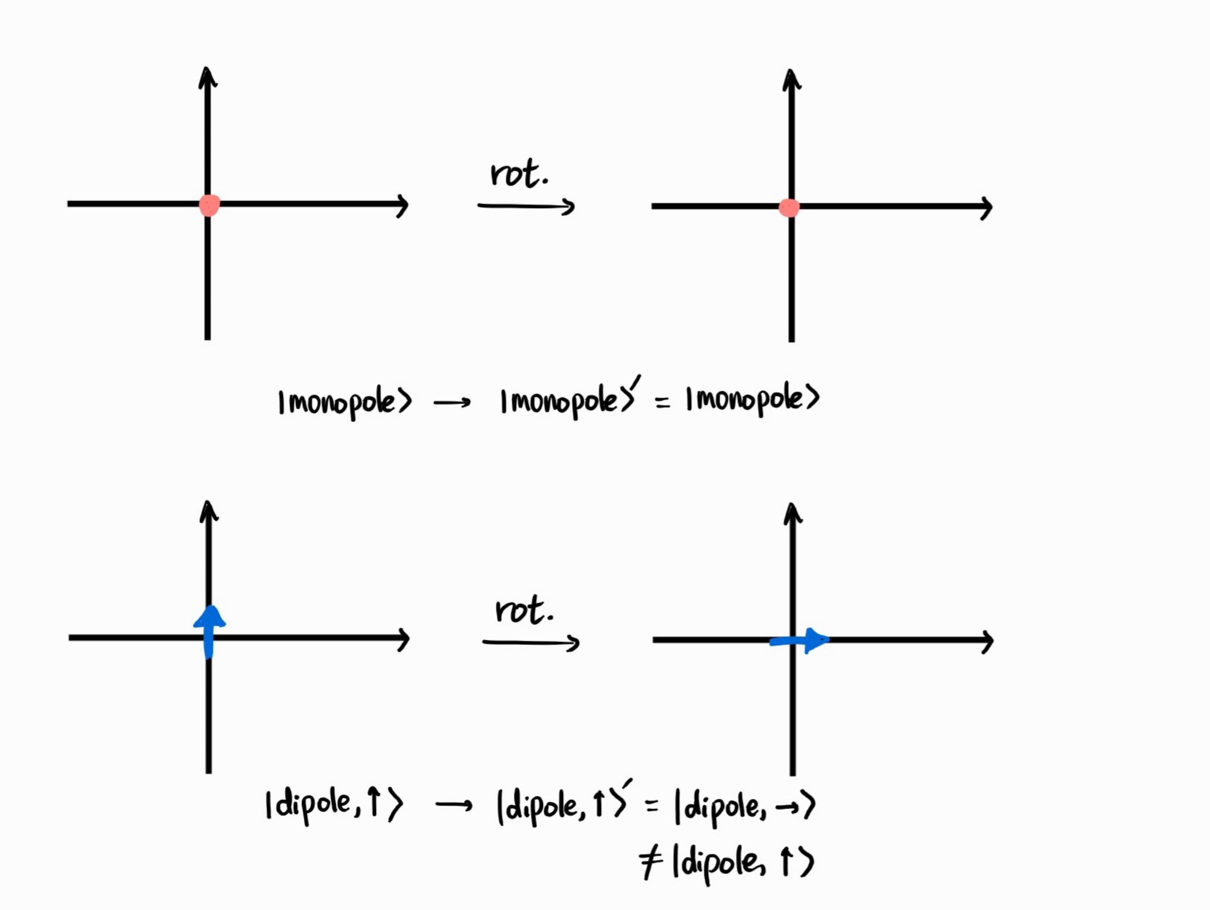
Contents

- Symmetry is important!
- Phase transition
- Landau theory
- Higgs mechanism

Symmetry is important!

- States with different symmetry properties are <u>different</u>
- Ex) monopole vs dipole

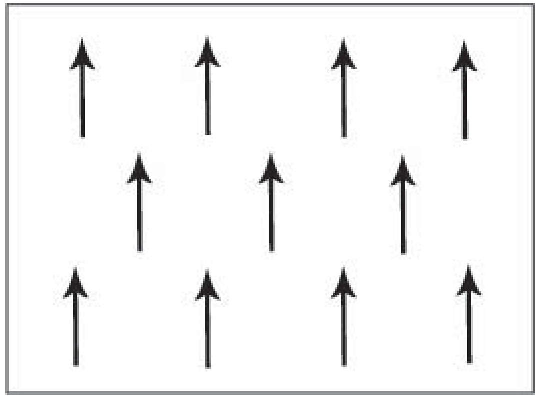


<u>Trivial</u> under rotational symmetry

<u>Non-trivial</u> under rotational symmetry

Phase transition

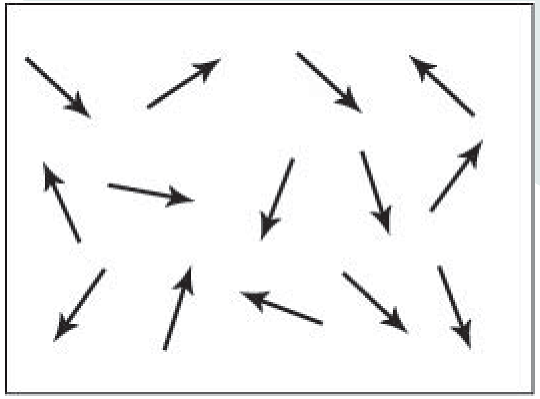
- Physical process of transition between one state and another
- 'Symmetry breaking' can be understood as phase transition
- Ex) ferromagnet vs paramagnet under $\mathbb{Z}_2 : m \rightarrow -m$



$T < T_C$

$m = \langle \sigma_j \rangle \neq 0$

symmetry broken



$T > T_C$

$m = \langle \sigma_j \rangle = 0$

symmetric

Landau theory

- In equilibrium, systems minimize the free energy!
- Landau's suggestions:

1) Free energy is invariant under symmetry transformations
2) Free energy is analytic

Under these conditions,

one can construct a free energy expression w.r.t. the order parameter

Landau theory

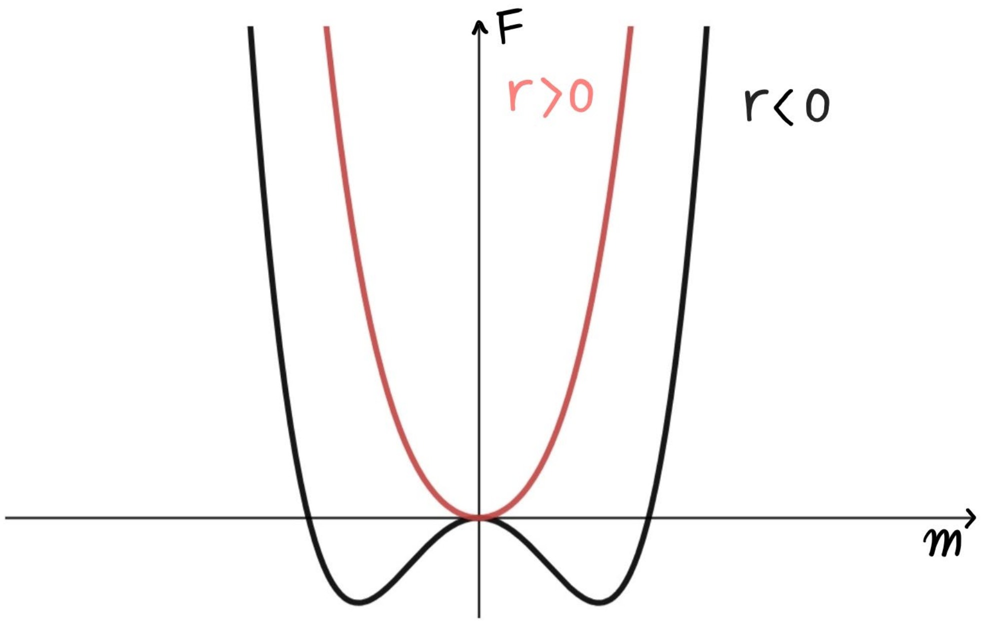
- Consider the previous case $\mathbb{Z}_2 : m \to -m$

For the free energy to be invariant under $\mathbb{Z}_2$ symmetry,

$$F(m) = a_0 + a_2 m^2 + a_4 m^4 + \cdots$$

- How can we construct free energy for the ferromagnetic system?

$$F(m) = \frac{r}{2} m^2 + \frac{u}{4} m^4$$



$$\frac{\partial F}{\partial m} = rm + um^3 = 0$$

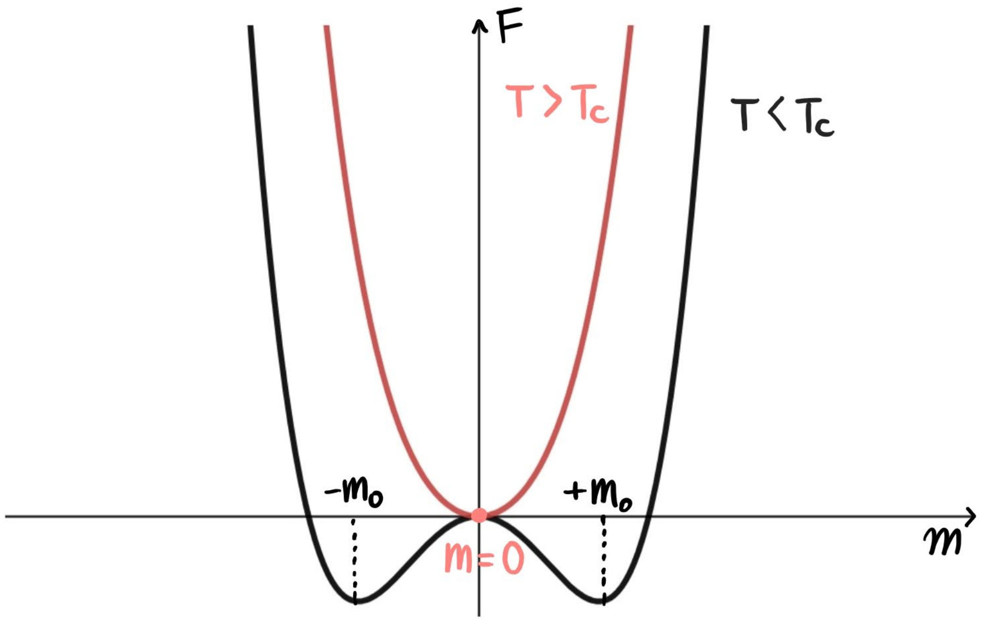
$$r > 0: \ m = 0$$

$$r < 0: \ m_0 = \pm\sqrt{\frac{|r|}{u}} \neq 0$$

Landau theory

- Ferromagnet: $T < T_C,\ m \neq 0$
- Paramagnet: $T > T_C,\ m = 0$

$$F(T,m) = \frac{\alpha(T - T_C)}{2} m^2 + \frac{u}{4} m^4$$



As the temperature cools down, the system breaks the $\mathbb{Z}_2$ symmetry ($m_0 \neq 0$) even though the free energy is still invariant

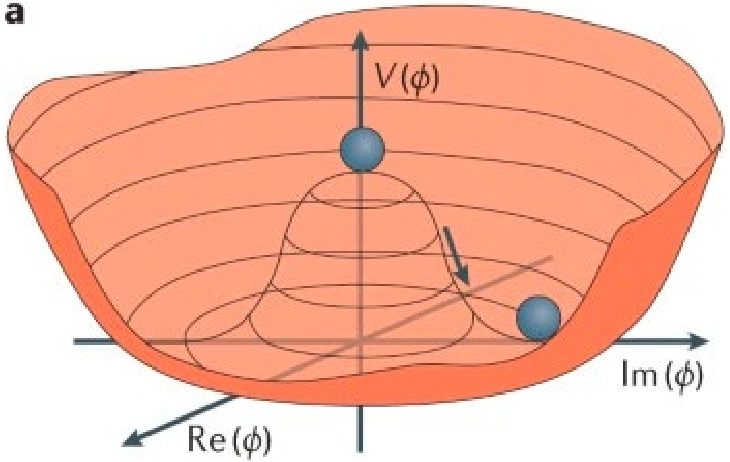
→ **Spontaneous symmetry breaking**

Higgs mechanism

- The Lagrangian is <u>invariant</u> under electroweak symmetry $SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{Higgs} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

a



Somehow nature chose $\mu^2 > 0$ → $V(\phi)$ minimum at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$

We may choose a direction: $\phi_0 = \frac{1}{\sqrt{2}}\binom{0}{v}$

→ Only one component of the doublet gains nonzero value

→ **SU(2) broken**

→ Higgs boson has nonzero hypercharge ($Y_H = \frac{1}{2}$) → **$U(1)_Y$ broken**

Higgs mechanism

- The Lagrangian is <u>invariant</u> under electroweak symmetry $SU(2)_L \times U(1)_Y$

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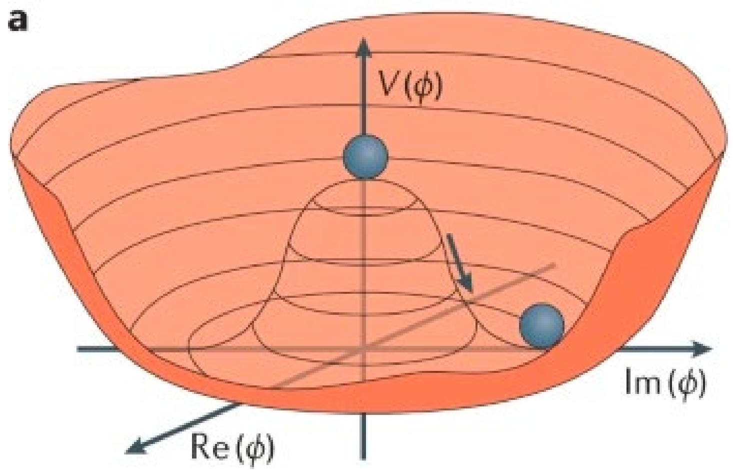

Electric charge: $Q = T^3 + Y_H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$U(1)_{EM}: \quad \phi_0 \;\rightarrow\; \phi_0' = e^{iQ\theta}\phi_0 = \phi_0$

$U(1)_{EM}$ survives!

→ **Spontaneous symmetry breaking** $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

Higgs mechanism

- Generating masses to gauge bosons

$$D_\mu\phi = \left(\partial_\mu + igT^i W_\mu^i + ig'Y_H B_\mu\right)\phi$$

$$(D^\mu\phi_0)^\dagger(D_\mu\phi_0) = \left|\left(\partial_\mu + igT^i W_\mu^i + ig'Y_H B_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}\right|^2 = \cdots$$

$$= \frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right) + \left(gW_\mu^3 - g'B_\mu\right)^2\right] = \frac{1}{2}\left(\frac{gv}{2}\right)^2 W_\mu^+ W^{-\mu} + \frac{1}{2}\left(\sqrt{g^2+g'^2}\,\frac{v}{2}\right)^2 Z_\mu Z^\mu$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right), \qquad m_W = \frac{gv}{2}$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right), \qquad m_Z = \sqrt{g^2+g'^2}\,\frac{v}{2}$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'W_\mu^3 + gB_\mu\right), \qquad m_A = 0$$

References

- Contents until Landau theory is based on the lectures by E.G.Moon, PH211 2022 Fall, KAIST Physics

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THANK YOU