Updates on the Exploration of the Possible Spin Matching Methods used in the FCC-ee

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$\delta \hat{n}_0(s)$ correction

- $\hat{n}_0(s)$: Periodic and stable spin direction on the closed orbit
- Misalignments in the flat ring $\rightarrow$ vertical misalignments of quadrupoles $\rightarrow$ $\hat{n}_0(s)$ deviation from vertical $\rightarrow$ stronger spin diffusion
- Vertical quadrupole misalignments are difficult to control
- After conventional orbit correction, the spin matching is needed to correct the $\hat{n}_0(s)$ deviation and reduce the polarization loss due to spin diffusion

D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

Three correction schemes

EPFL

1. DESY formalism

   D. P. Barber, et al. A general harmonic spin matching formalism for the suppression of depolarisation caused by closed orbit distortion in electron storage rings. No. DESY–85-044. DESY, 1985.

2. Rossmanith-Schmidt scheme

   R. Rossmanith and R. Schmidt, Compensation of depolarizing effects in electron-positron storage rings." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 236.2 (1985): 231-248.

3. LEP method (Deterministic)

   R. W. Assmann, Optimierung der transversalen Spin-Polarisation im LEP-Speicherring und Anwendung für Präzisionsmessungen am Z-Boson. Diss. Munich U., 1994.

DESY formalism

$$\delta\hat{n}_0 = \alpha\hat{m}_0 + \beta\hat{l}_0$$

Expand α and β to Fourier series

$$(\alpha - i\beta)(s) = -i\frac{C}{2\pi}\sum_k \frac{f_k}{k-\tilde{\nu}} e^{i2\pi ks/C}$$

f_k: Fourier harmonics, related to the closed orbit and perturbing fields

Make additional orbit corrections using vertical correction magnets, and reduce the rms tilt by minimizing the Fourier coefficients

D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

Previous results using simplified DESY formalism

- Extract n_0 direction at the end of all elements
- Expand $n_{0x}(s) + i n_{0z}(s)$ into Fourier series
- Minimize harmonics 0 and 1 using bumps

Each closed bump can be represented using a single variable, and each bump has an independent and linear contribution to the Fourier harmonics.

$$\mathbf{MK} = \mathbf{C}$$

K: amplitudes (the first kick value) of the bumps
C: real and imaginary parts of the required harmonics coefficients $[c_{0\mathrm{real}}, c_{0\mathrm{imag}}, c_{1\mathrm{real}}, c_{1\mathrm{imag}}]$

If the harmonics 0 and 1 of a misaligned lattice is **A**, the bumps should generate $-\mathbf{A}$, and bump amplitudes can be estimated via

$$\mathbf{K} = \mathbf{M}^{-1}(-\mathbf{A})$$

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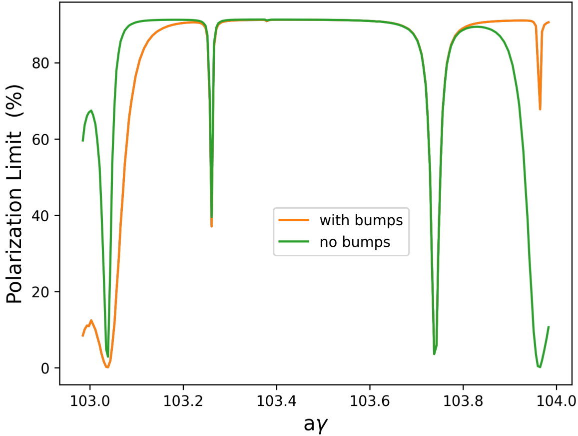
At 45.82 GeV ($a\gamma = 103.983$)

- Without correction
  $(\Delta y)_{\mathrm{rms}} = 71.96\,\mu\mathrm{m}$, $\delta n_0 = 2.28\,\mathrm{mrad}$, $P_{DK} = 10.68\%$
- Four random bumps
  $(\Delta y)_{\mathrm{rms}} = 72.09\,\mu\mathrm{m}$, $\delta n_0 = 0.912\,\mathrm{mrad}$, $P_{DK} = 90.58\%$
- Optimized four bumps
  $(\Delta y)_{\mathrm{rms}} = 71.30\,\mu\mathrm{m}$, $\delta n_0 = 0.9\,\mathrm{mrad}$, $P_{DK} = 90.96\%$
- Eight random bumps
  $(\Delta y)_{\mathrm{rms}} = 70.80\,\mu\mathrm{m}$, $\delta n_0 = 0.903\,\mathrm{mrad}$, $P_{DK} = 90.79\%$

Energy scan comparison

Using 4 bumps which are optimized at 45.82 GeV ($a\gamma = 103.983$)



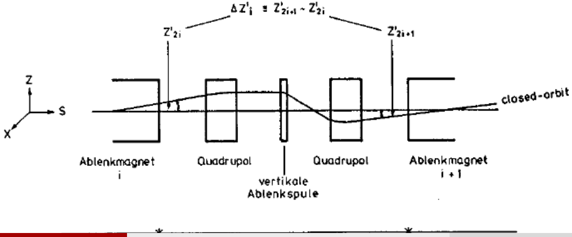
Rossmanith-Schmidt scheme

$$|\delta \vec{n}_0(s)| = \frac{1/c^2}{2(1-\cos 2\pi\nu)} \left[\left(\int_s^{s+L} \delta\Omega_x \cos\phi \mathrm{d}s \right)^2 + \left(\int_s^{s+L} \delta\Omega_x \sin\phi \mathrm{d}s \right)^2 \right]$$

$$\delta\Omega_x = \frac{e}{m_0 c\gamma}(1 + a\gamma)B_x \text{ and } \phi_x = \gamma a\alpha$$

$$\text{also } \frac{e}{m_0 c\gamma} \int_{s_{2i}}^{s_{2i+1}} B_x(s)\mathrm{d}s = -\Delta y_i^{'}$$

$$|\delta \vec{n}_0| = \frac{1/c^2}{2(1-\cos 2\pi\nu)}(1 + \gamma a) \left[\left(\sum_{i=1}^{N} \sin(\gamma a\alpha_i)\Delta y_i^{'} \right)^2 + \left(\sum_{i=1}^{N} \cos(\gamma a\alpha_i)\Delta y_i^{'} \right)^2 \right]$$



Rossmanith-Schmidt scheme

Expand $\Delta y'(\alpha)$ into Fourier series

$$\Delta y'(\alpha) = \sum_{n=1}^{\infty} (a_n \cos n\alpha + b_n \sin n\alpha)$$

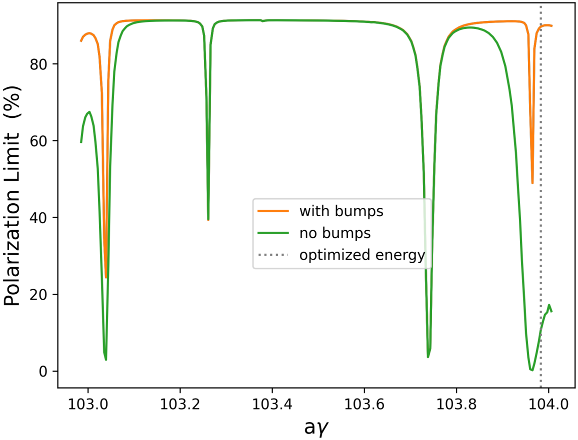
$$\begin{matrix} a_n \\ b_n \end{matrix} = \frac{1}{N} \sum \Delta y_i'(\alpha_i) \begin{matrix} \cos n\alpha_i \\ \sin n\alpha_i \end{matrix}$$

The n which is adjacent to $a\gamma$ contributes most to the sum. FCC-ee (Z) operates between $a\gamma$ 103 and 104, so that a/b_{103} and a/b_{104} are to be suppressed using four closed bumps.

R. Schmidt, Polarisationsuntersuchungen am Speicherring PETRA. No. DESY-M-82-22. DESY, 1982.

$$45.82\,\text{GeV}\ (a\gamma = 103.983)$$

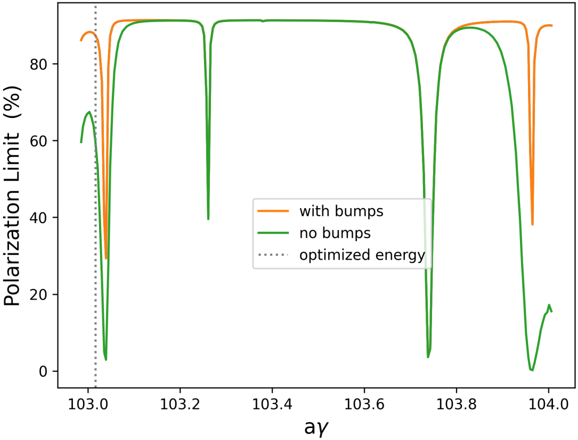
$$\delta n_0 : 2.28\,\mathrm{mrad} \Rightarrow 0.90\,\mathrm{mrad}\ ,\ P_{DK} : 10.68\% \Rightarrow 89.65\%$$



Rossmanith-Schmidt scheme

$$45.394\,\text{GeV}\ (a\gamma = 103.016)$$

$$\delta n_0 : 1.14\,\mathrm{mrad} \Rightarrow 0.98\,\mathrm{mrad}\ ,\ P_{DK} : 59.66\% \Rightarrow 87.35\%$$



LEP method

Extract vertical orbits at BPMs and minimize critical Fourier harmonics

$$a_k = \frac{1}{\pi} \sum_{i=1}^{N_{BPM}} y_i \cdot \Delta\theta_i \cdot \cos\left(k \cdot \theta_i\right)$$

$$b_k = \frac{1}{\pi} \sum_{i=1}^{N_{BPM}} y_i \cdot \Delta\theta_i \cdot \sin\left(k \cdot \theta_i\right)$$

If there is a BPM next to each quadrupole and functions properly

$$45.82\,\mathrm{GeV}\ (a\gamma = 103.983)$$

$$\delta n_0 : 2.28\,\mathrm{mrad} \Rightarrow 2.03\,\mathrm{mrad}\ ,\ P_{DK} : 10.68\% \Rightarrow 13.72\%$$

F. Sonnemann, Increase of spin polarization for energy calibration at LEP. Diss. Aachen, Tech. Hochsch., 1998.

Comparison

EPFL

At 45.82 GeV ($a\gamma = 103.983$) with polarization 10.68%

Method	$(\delta n_0)_{rms}$	Polarization
DESY formalism	$0.90\,\mathrm{mrad}$	90.96%
Rossmanith-Schmidt scheme	$0.90\,\mathrm{mrad}$	89.65%
LEP method	$2.03\,\mathrm{mrad}$	13.72%

Comparison

1. DESY formalism
   Rigorous mathematical derivation, a systematic way of analyzing δn_0. But empirically setting the bumps will be inevitable. Turning on wigglers would make manual adjustment practical.
2. Rossmanith-Schmidt scheme
   Need to know the change of the angle of the vertical closed orbit between two bending magnets $\Delta y^{'}$. Could install BPMs at both ends of each dipole. Need to further consider the influence of BPM misalignments and calibration errors.
3. LEP method (Deterministic)
   Has not been successfully tested in simulation, need further investigation. Also influenced by BPM misalignments and calibration errors.

Thank you!