Tools in Effective Field Theories of (Massive) Gravity

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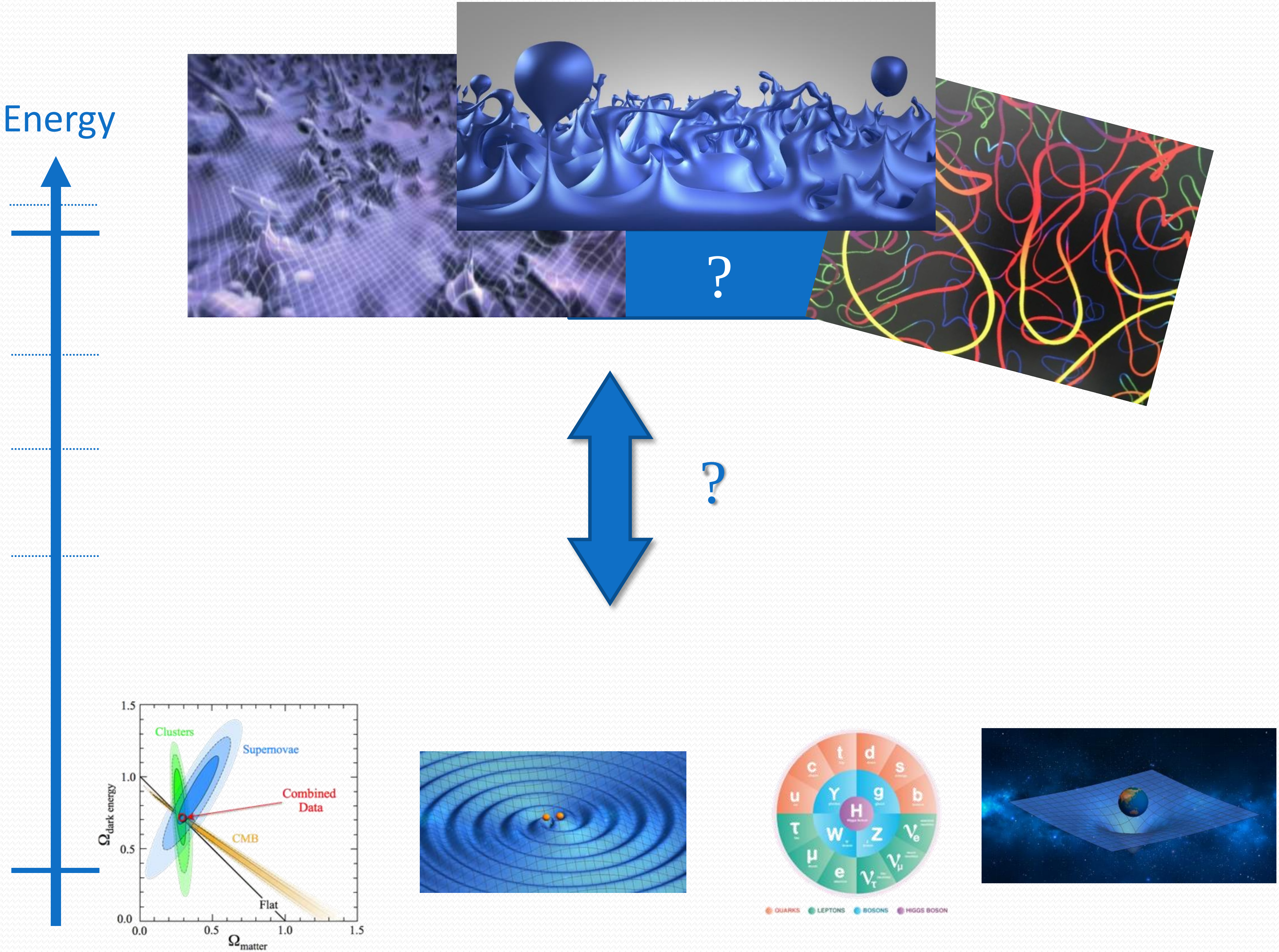
Île de Noirmoutier
3 Juin 2024



Energy
?
?
Clusters
Supernovae
Combined Data
CMB
Flat
Ωdark energy
Ωmatter
0.0
0.5
1.0
1.5
QUARKS
LEPTONS
BOSONS
HIGGS BOSON

Effective Field Theory

Energy

UV completion of Quantum Gravity

Low-energy EFT

E.g.: GR + irrelevant operators
+ (B)SM fields
+ light degrees of freedom

Join Noirmoutier Sandcastle Competition

Effective Field Theory

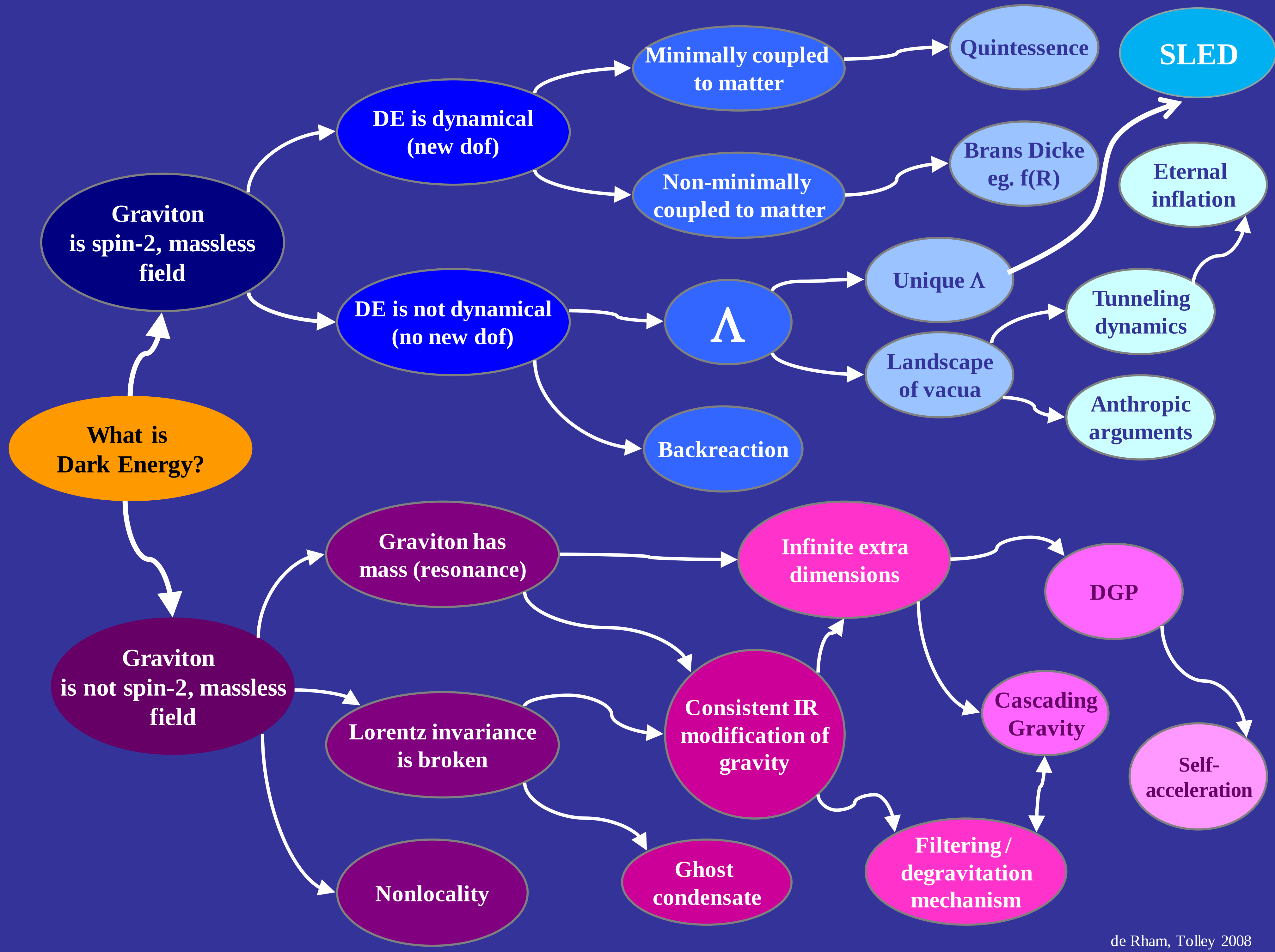
Energy

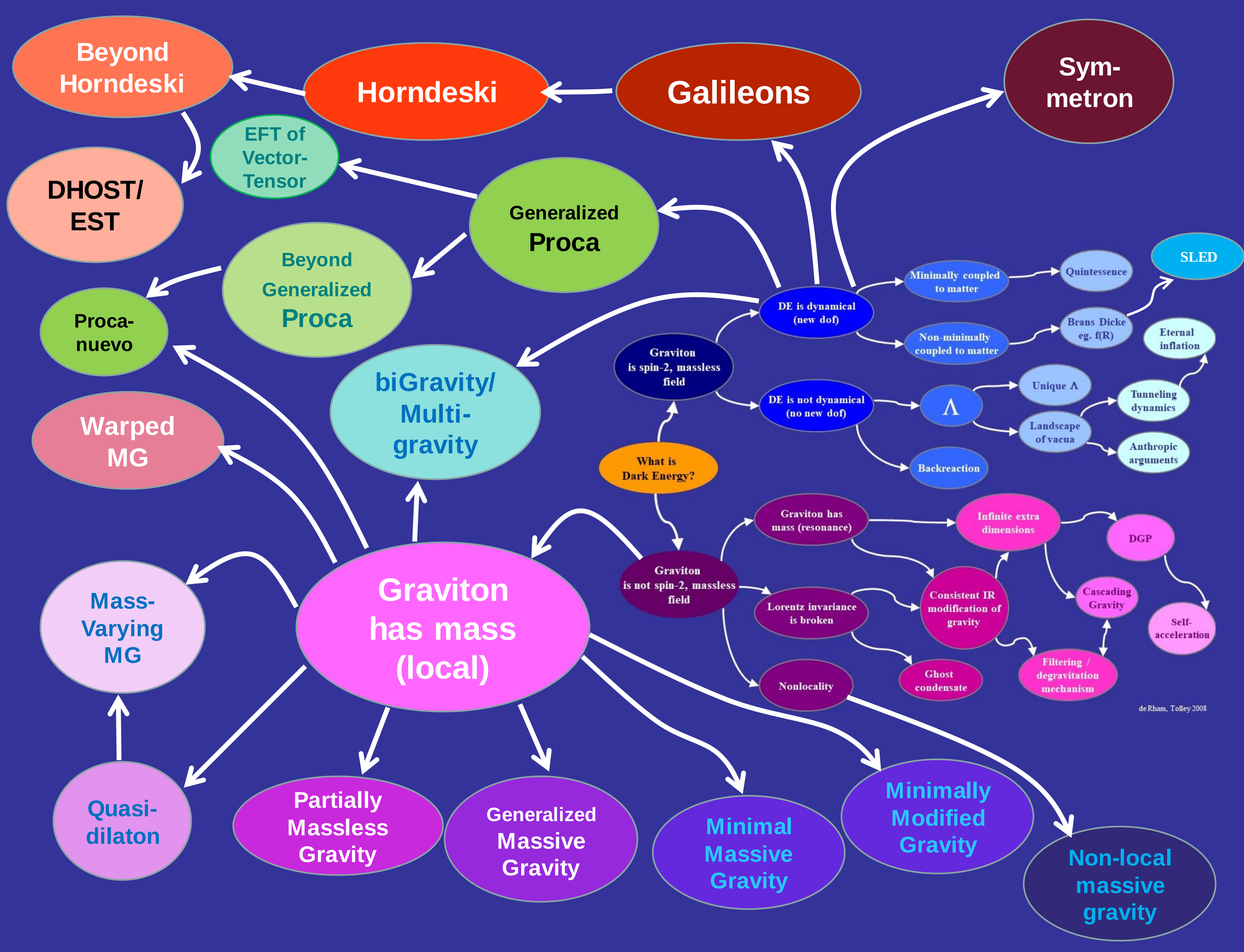
UV completion of Quantum Gravity

???

???

Join Noirmoutier Sandcastle Competition


What is Dark Energy?
Graviton is spin-2, massless field
DE is dynamical (new dof)
Minimally coupled to matter
Quintessence
SLED
Non-minimally coupled to matter
Brans Dicke eg. f(R)
Eternal inflation
DE is not dynamical (no new dof)
Λ
Unique Λ
Landscape of vacua
Tunneling dynamics
Anthropic arguments
Backreaction
Graviton is not spin-2, massless field
Graviton has mass (resonance)
Infinite extra dimensions
DGP
Lorentz invariance is broken
Consistent IR modification of gravity
Cascading Gravity
Self-acceleration
Nonlocality
Ghost condensate
Filtering / degravitation mechanism
de Rham, Tolley 2008



Beyond Horndeski
Horndeski
Galileons
Sym-metron
DHOST/ EST
EFT of Vector-Tensor
Generalized Proca
Beyond Generalized Proca
Proca-nuevo
biGravity/ Multi-gravity
Warped MG
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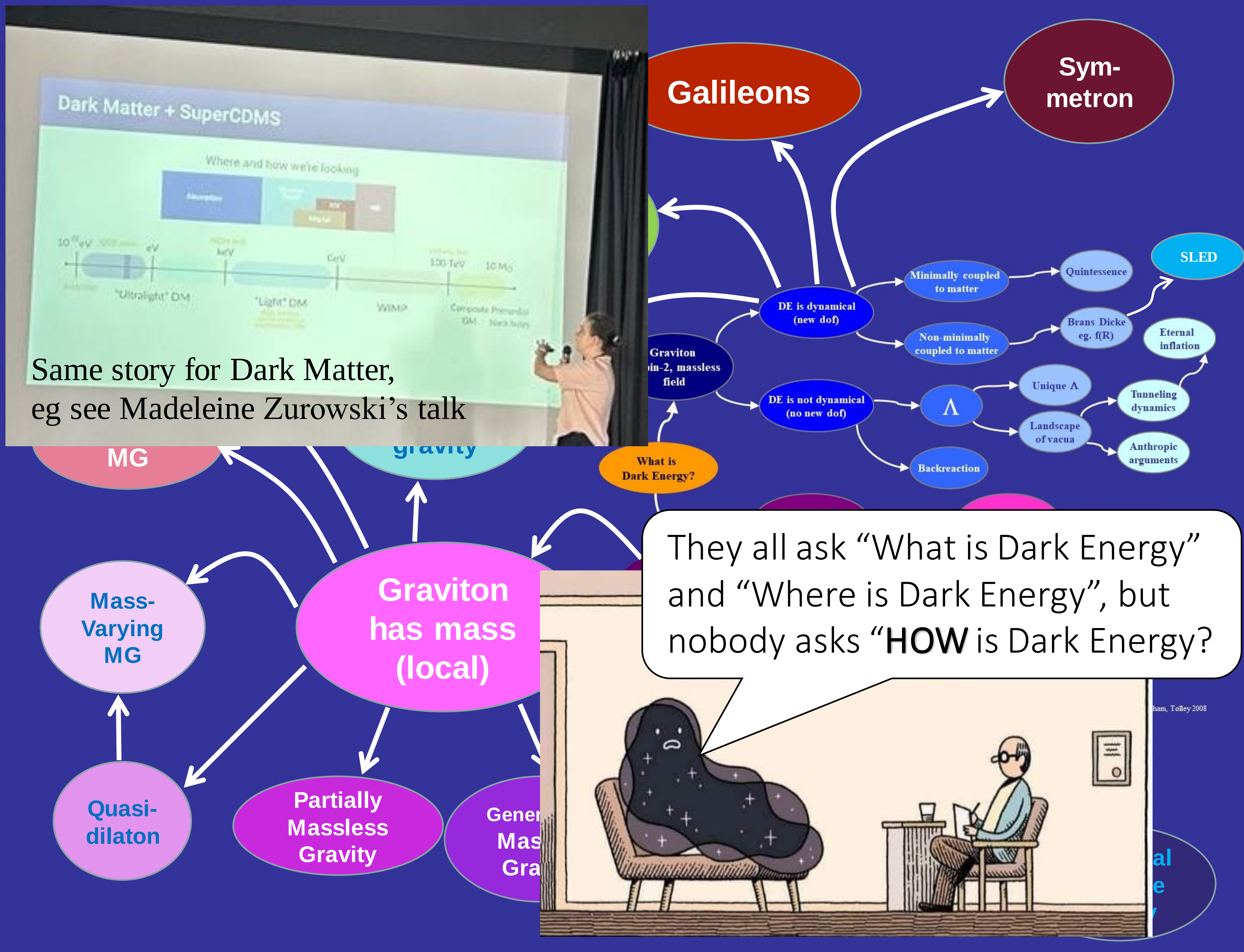
de Rham, Tolley 2008
Quasi-dilaton
Partially Massless Gravity
Generalized Massive Gravity
Minimal Massive Gravity
Minimally Modified Gravity
Non-local massive gravity


Dark Matter + SuperCDMS
Where and how we're looking
"Ultralight" DM
"Light" DM
WIMP
Same story for Dark Matter,
eg see Madeleine Zurowski's talk
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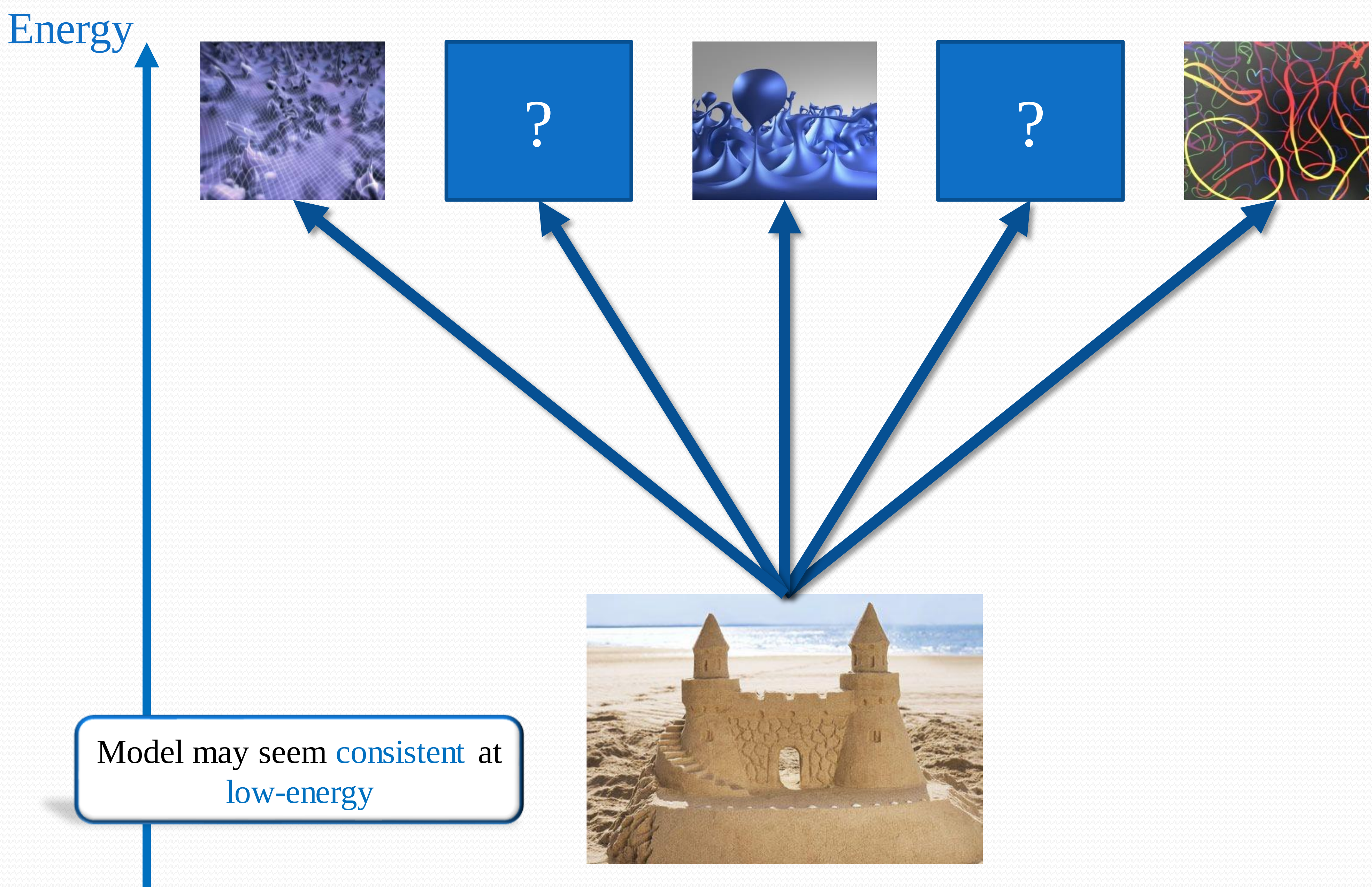
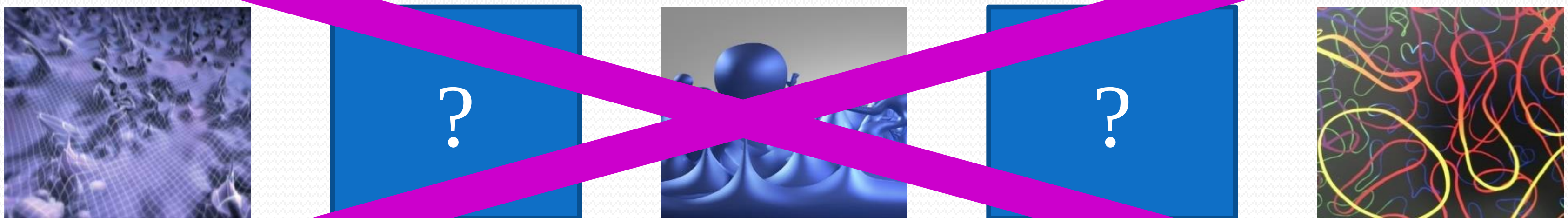
ham, Tolley 2008
They all ask "What is Dark Energy" and "Where is Dark Energy", but nobody asks "**HOW** is Dark Energy?

High-energy completion



High-energy completion

Energy



Yet we know that some models can never enjoy a standard UV completion

60 orders of magnitude

Model may seem consistent at low-energy

Its **not** that a theory with parameter $\alpha = 0$ is ruled out but $|\alpha| > 10^{-60}$ is allowed

Constraints from UV Completion

Starting with the assumption of a healthy Wilsonian UV completion
(string theory or other)

Set up classes of IR constraints

Positivity Bounds

Precise statements at the level
of 2 $\rightarrow$ 2 scattering amplitude
(complement S-matrix bootstrap program)

Causality Bounds

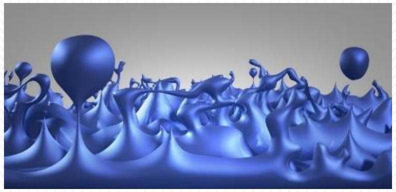
Unresolvable support outside
front velocity light cone

Alberte, Carrillo-Gonzalez, Chen, Franckfort, Heisenberg, Held, Jaitly, Klapanek, Kozuszek, Margalit

Melville, Momeni, Noller, Pozsgay, Rumbutis, Tokareva, Tolley, Wiseman, Zhang, Zhou

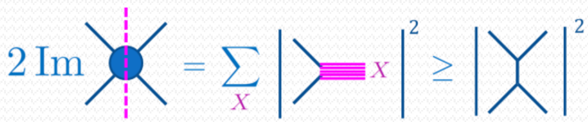
CoM Energy

Low-energy EFT



- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)
- ✓ Local (Froissart Bound)

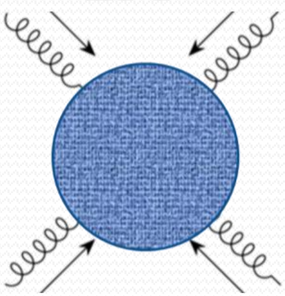
$\mathcal{A}$: 2 – 2 elastic amplitude



Positivity bounds

(applied to low-energy scattering amplitude)

Low-energy EFT



$$\left.\frac{\mathrm{d}^2\mathcal{A}(s,t)}{\mathrm{d}s^2}\right|_{t=0} > 0 \qquad s < 4m^2$$

Pham and Truong 1985

Ananthanarayan, Toublan and Wanders, 1994

Adams et. al. 2006

Applications to SMEFT

New Physics Beyond the Standard Model

$$\mathcal{L}_{\rm BSM} = \mathcal{L}_{\rm SM}^{(4)} + \text{New Physics} \equiv \text{new modes with mass } M \gtrsim \Lambda \gtrsim \text{TeV}$$

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda}\sum_n c_n^{(5)}\mathcal{O}_n^{(5)} + \frac{1}{\Lambda^2}\sum_n c_n^{(6)}\mathcal{O}_n^{(6)} + \cdots$$

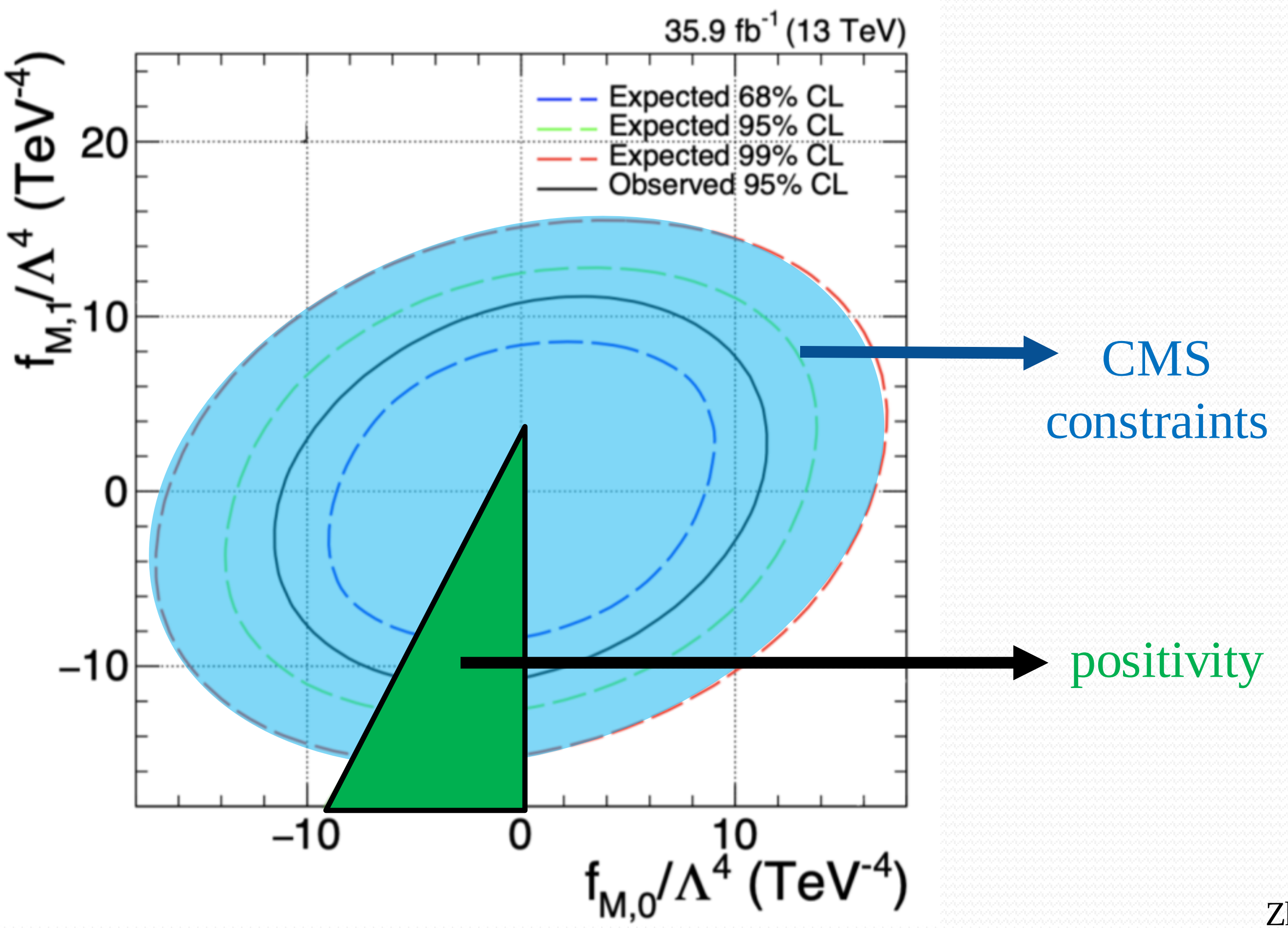
Quickly leads to a huge parameter space
e.g. SMEFT up to dim-6 operators given in 1008.4884 includes 59 operators+ B-violating

Becomes even more challenging at higher order
(44,807 dim-8 operators…)

From Grzadkowski, Iskrzyński, Misiak&Rosiek 1008.4884

SMEFT

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda}\sum_n c_n^{(5)}\mathcal{O}_n^{(5)} + \frac{1}{\Lambda^2}\sum_n c_n^{(6)}\mathcal{O}_n^{(6)} + \cdots$$



Zhang & Zhou, 2020

Adding Gravity

Causality ⟷ analyticity

Locality, Froissart Bound with Gravity???

- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)
- ✓ Local (Froissart Bound)

positivity bounds

More subtle for gravitational EFTs

(sub)luminal sound speed

Applications to EFT of Cosmology/Gravity

Direct application to cosmology is more subtle due to:

- absence of S-matrix
- lack of Lorentz invariance/crossing symmetry
- ambiguous connection with analyticity
- gravitational exchange pole

Directions in progress:

- Cosmological bootstrap on correlators (only perturbative)

  No constraints from UV completion (analyticity, non-perturbative UV unitarity still need to be formulated)

  Arkani-Hamed, Baumann, Lee & Pimentel, 1811.00024
  Baumann, Duaso Pueyo, Joyce, Lee & Pimentel, 1910.14051, 2005.04234

  see Sleight & Taronna for Bootstrapping Inflationary Correlators in Mellin Space, 1907.01143, 2007.09993

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is more subtle due to:

- absence of S-matrix
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Directions in progress:

- Cosmological bootstrap on correlators (only perturbative)
- Alternative approach is to focus on **approximate S-matrix with broken boosts**
  Full formalism still under development

CdR & Melville 1703.00025
Grall & Melville 2102.05683+2022+

See also recent work by Creminelli, Janssen & Senatore, 2207.14224
assuming conformal invariance in 3d

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Directions in progress:

- Cosmological bootstrap on correlators (only perturbative)
- Approximate S-matrix with broken boosts
- Constrain Wilson coefficients defined **around Minkowski vacuum** and translate into constraints around cosmological backgrounds (requires adiabatic assumptions)

1904.05874, 1905.08816, 1906.11840, 1908.08644, 2103.06855, 2103.11195, …

Will show a quick example

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quick example

- Use Infrared causality as a proxy for positivity

Worked developed with Carrillo Gonzalez, Chen, Jaitly, Margalit, Pozsgay, Tolley & Zhang

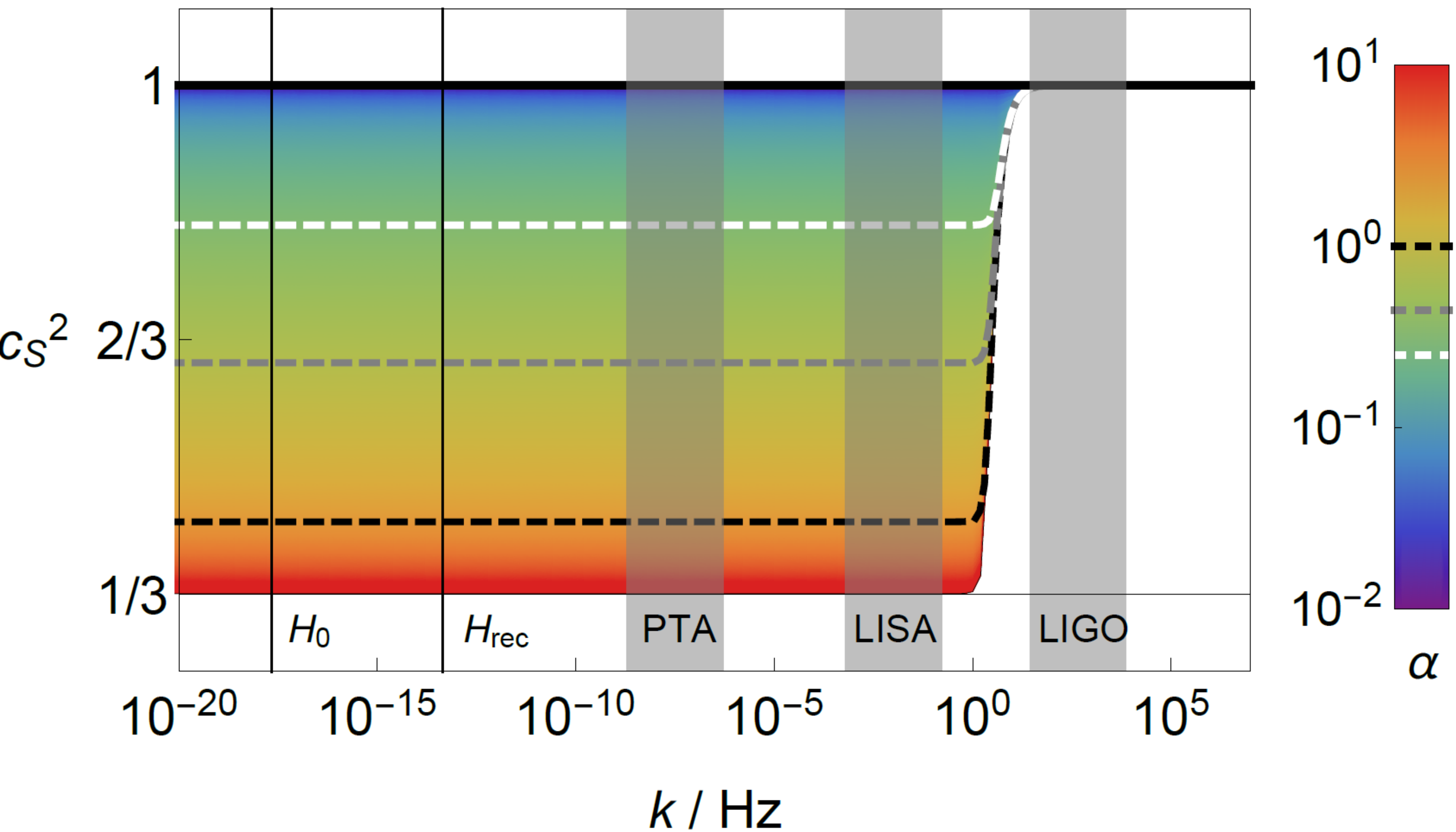
Constraining Models DE/MG

Example of DE/MG: quartic Horndeski with parameters $c_{B,M,T}$

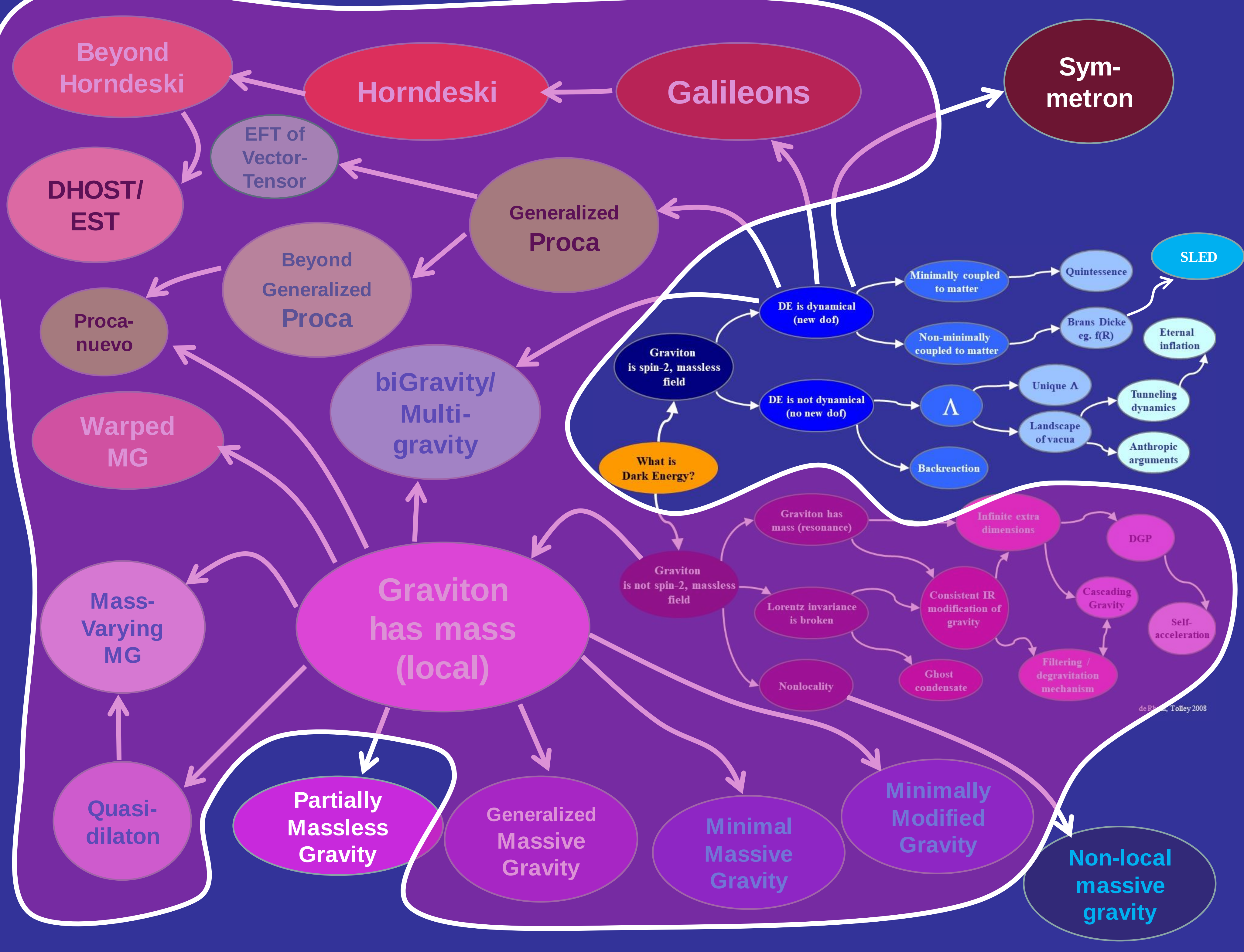
$$\mathcal{L} = M_{\rm Pl}^2 G_4(X) R + \Lambda^4 G_2(X) + \frac{1}{H_0^2} G_{4,X}(X) \left((\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right) + \mathcal{L}_{\rm matter}(\psi, g)$$

$$\begin{aligned} c_M &= -\frac{2\dot{X}}{H M^2 \Omega_{\rm DE}} \left(G_{4,X} + 2X G_{4,XX} \right), \quad \text{Effective Planck Mass} \\ c_B &= \frac{8X}{M^2 \Omega_{\rm DE}} \left(G_{4,X} + 2X G_{4,XX} \right), \quad \text{``Braiding'' (scalar/tensor mixing)} \\ c_T &= \frac{4X}{M^2 \Omega_{\rm DE}} G_{4,X}, \quad \text{Speed of GWs} - 1 \end{aligned}$$

$$M^2 = 2 \left(G_4 - 2X G_{4,X} \right)$$



1
2/3
1/3
$c_S{}^2$
H_0
H_{rec}
PTA
LISA
LIGO
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}
k / Hz
10^{1}
10^{0}
10^{-1}
10^{-2}
α



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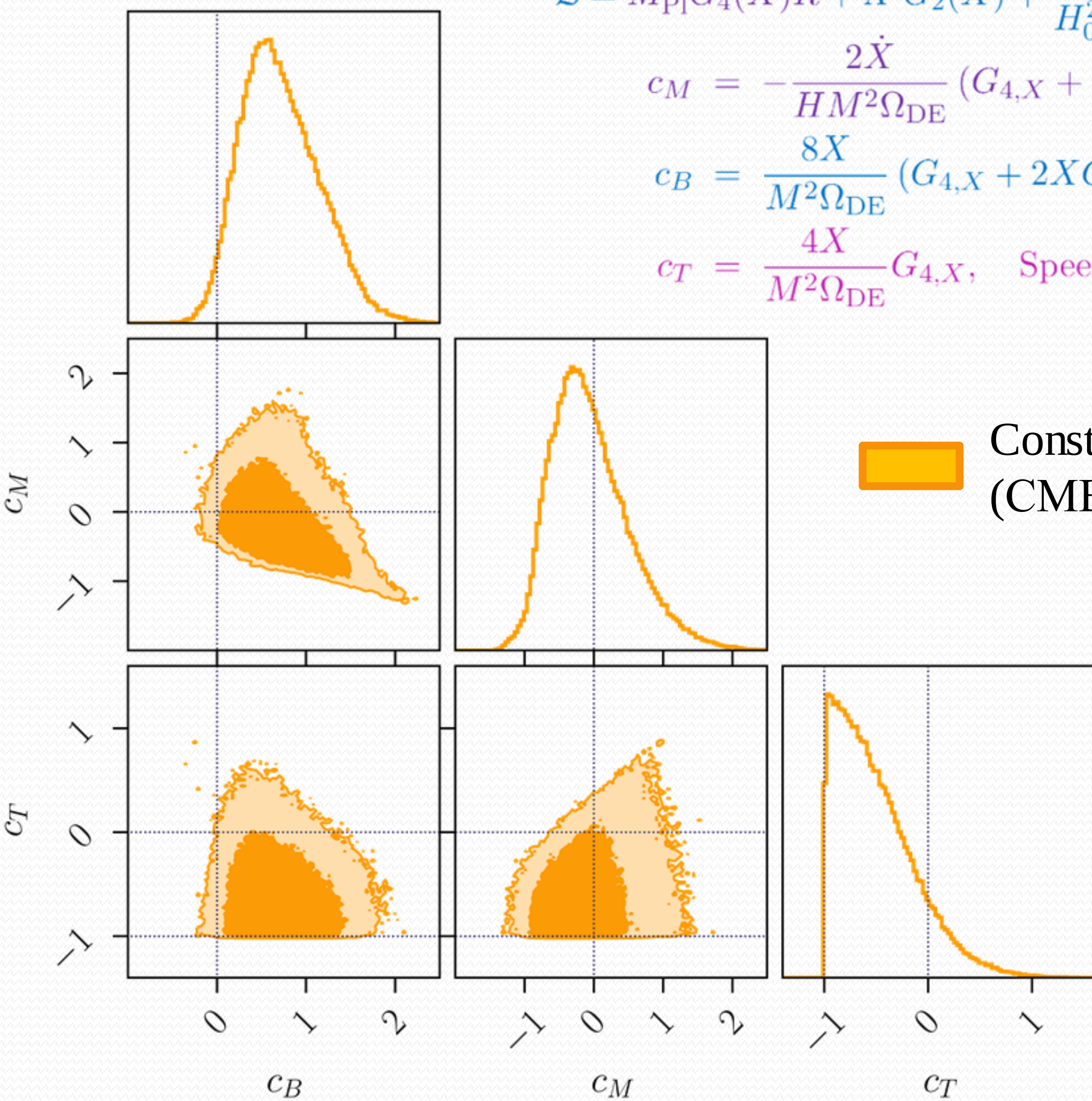

Constraints from observations (CMB, BAO, redshift space distortion,…)

CdR, Melville and Noller 2103.06855

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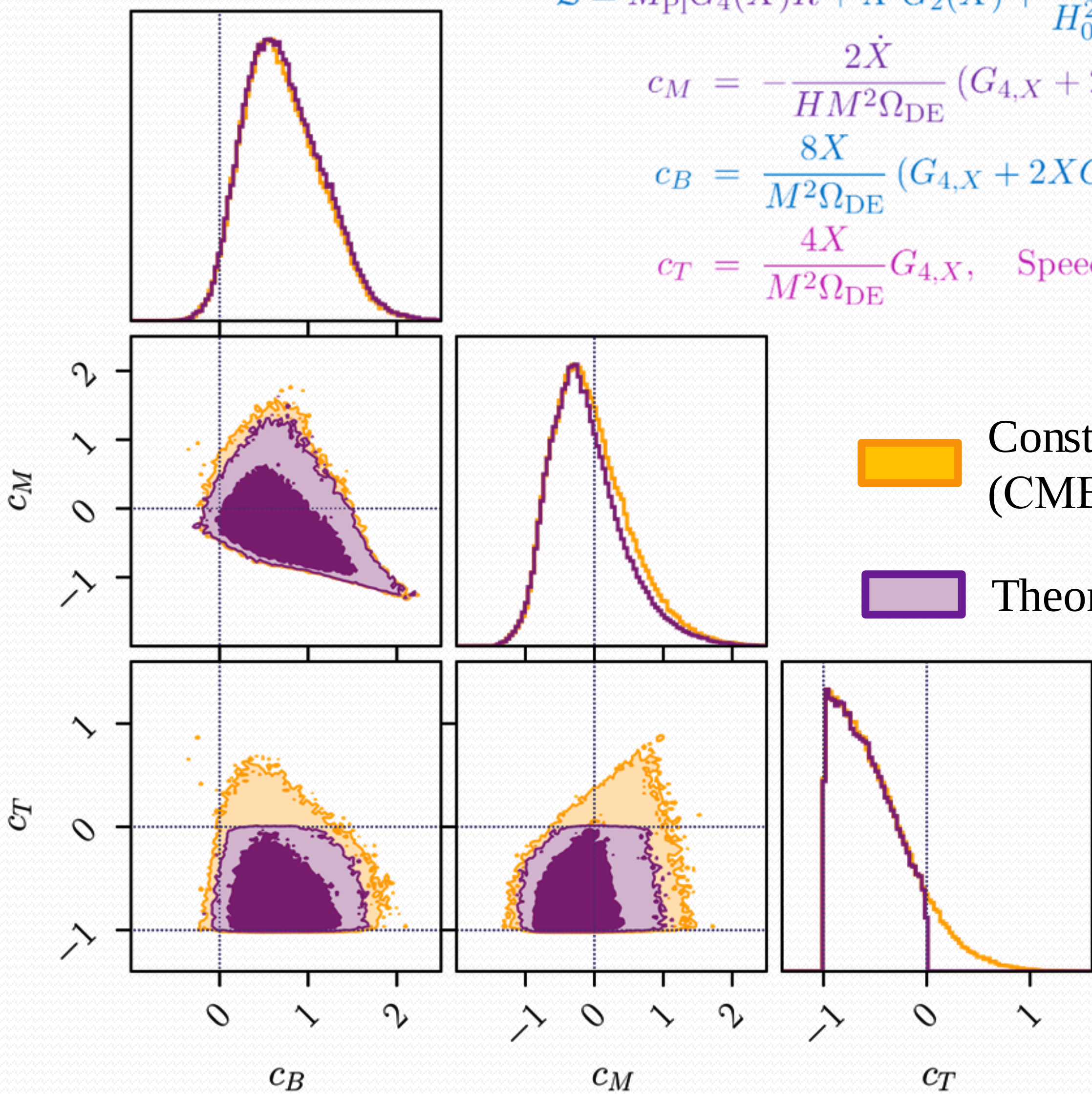


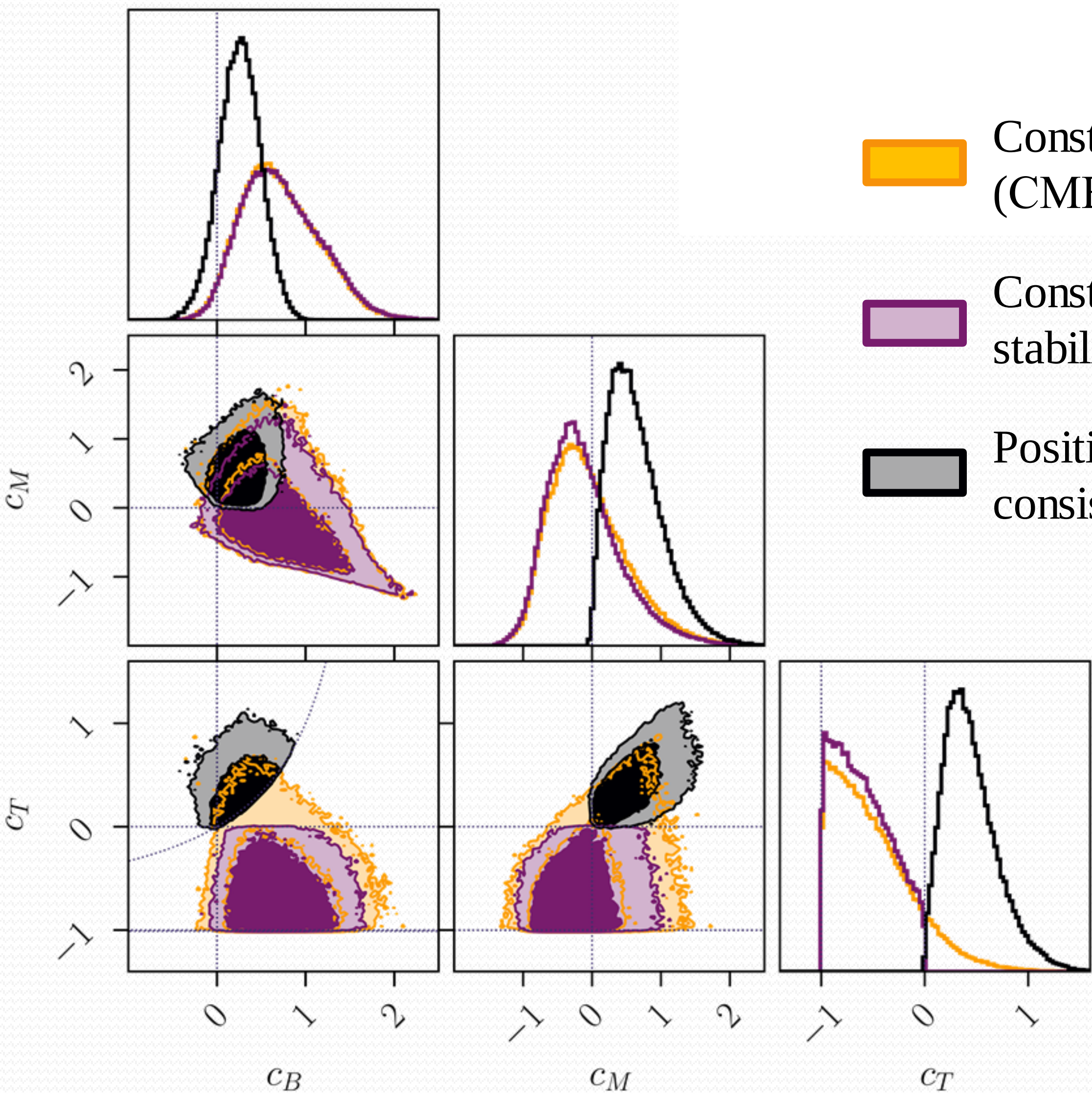
Constraints from observations (CMB, BAO, redshift space distortion,…)

Theoretical constraints

CdR, Melville and Noller 2103.06855

Constraints on Low-energy Models

Example of Dark Energy model (quartic Horndeski) with parameters $c_{B,M,T}$



Constraints from observations (CMB, BAO, redshift space distortion,…)

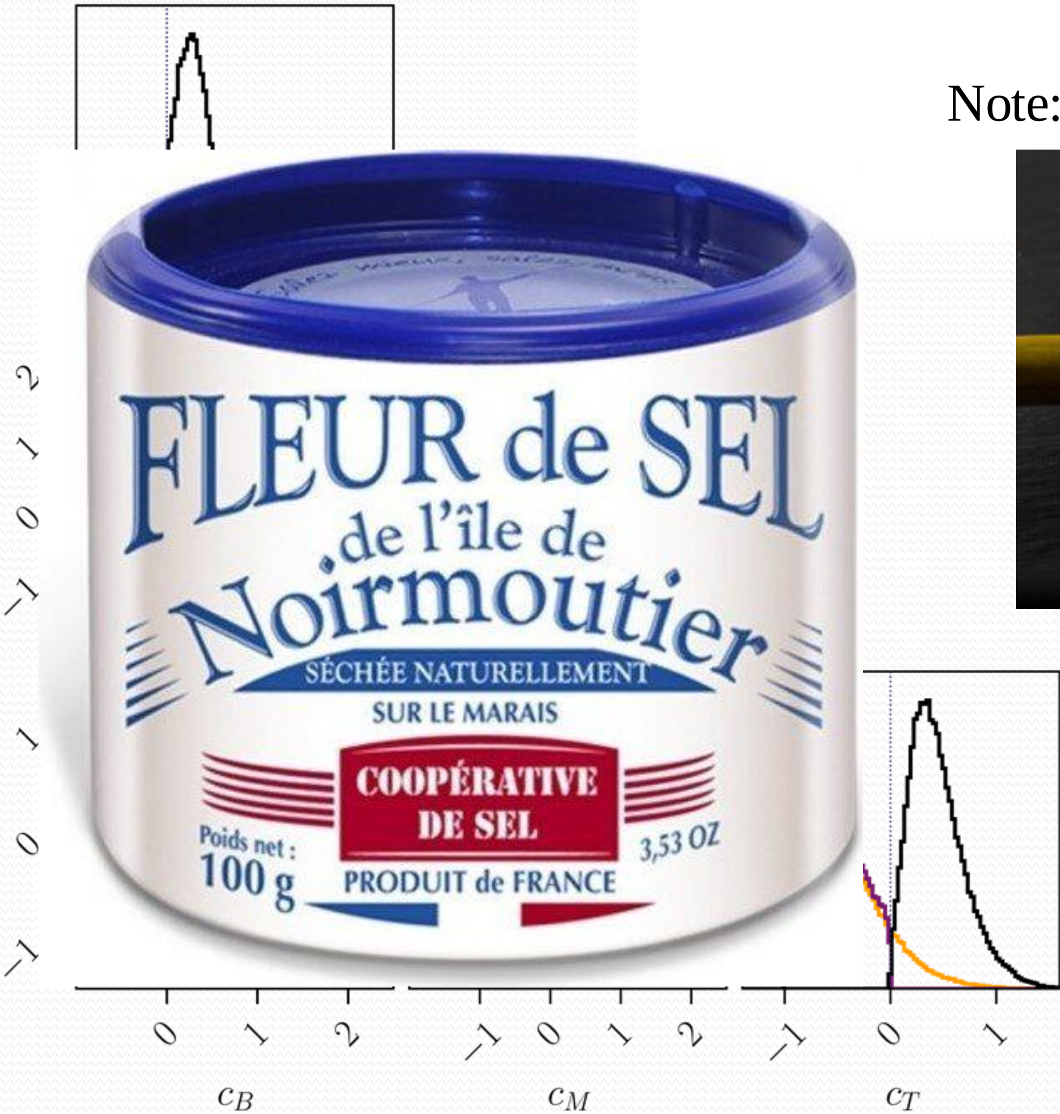
Constraints we would have imposed from stability and subluminality at low-energy

Positivity Priors (constraints from causality and consistent high energy completion)

with Melville and Noller, 2103.06855

Constraints on Low-energy Models

Example of Dark Energy model (quartic Horndeski) with parameters $c_{B,M,T}$



Note: To be taken with a pinch of salt

Only makes sense if assume can connect back to Minkowski

with Melville and Noller, 2103.06855

Applications to EFT of Cosmology

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- ambiguous connection with analyticity
- gravitational exchange pole

Directions in progress:

- Cosmological bootstrap on correlators (only perturbative)
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- Use Infrared causality as a proxy for positivity


Carrillo-Gonzalez


Chen


Jaitly


Margalit

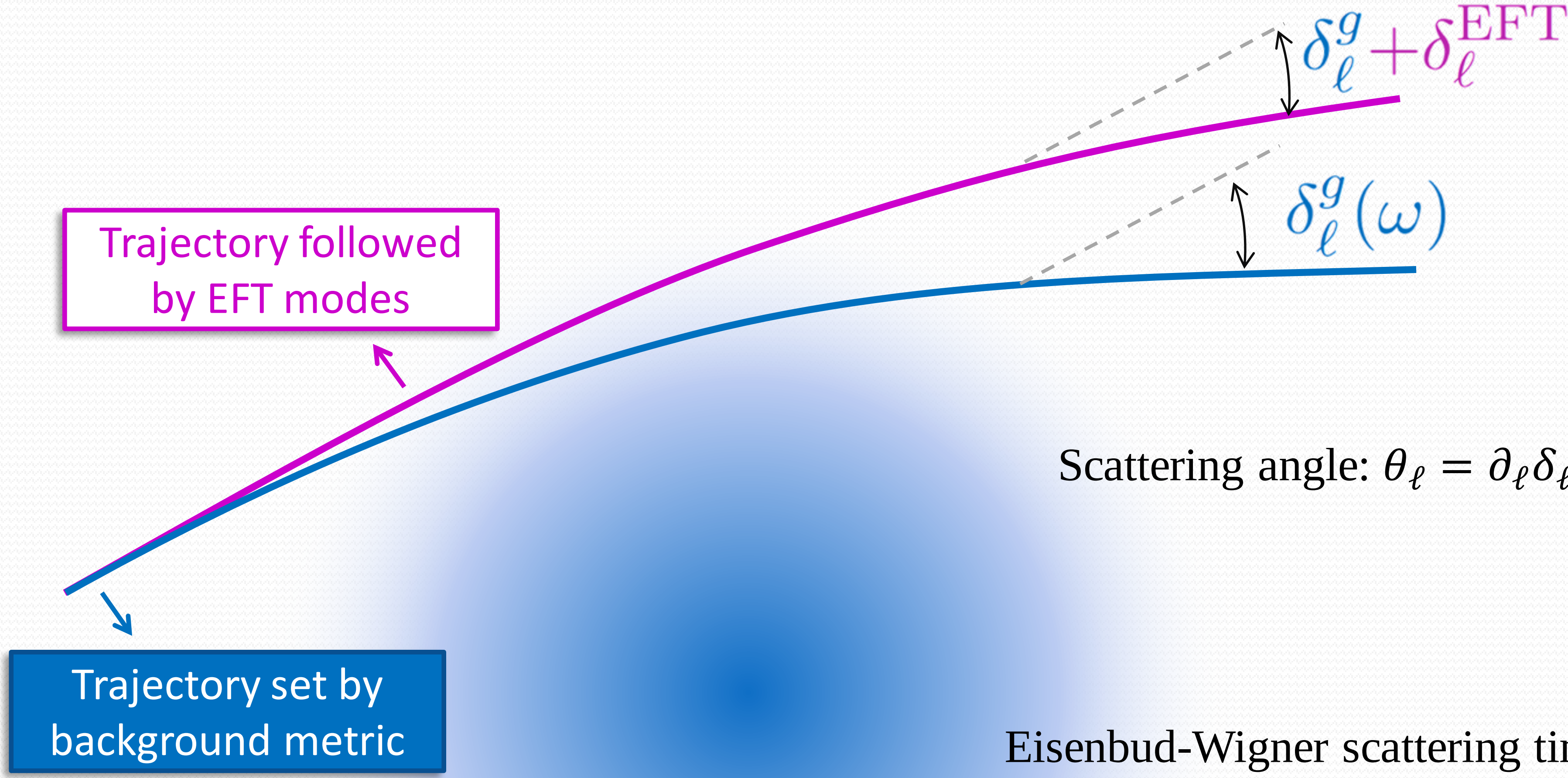

Pozsgay


Tokareva


Tolley


Zhang

Scattering



Scattering angle: $\theta_\ell = \partial_\ell \delta_\ell$

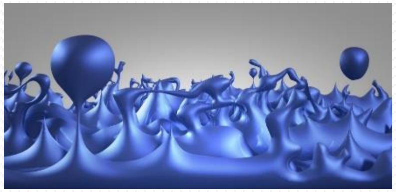
Eisenbud-Wigner scattering time-delay:

$$\Delta T_\ell = 2 \frac{\mathrm{d}}{\mathrm{d}\omega} \delta_\ell(\omega)$$

$$\Delta T = \Delta T^g + \Delta T^{\mathrm{EFT}}$$

CoM Energy

Gravitational EFTs



✓ Unitary (optical theorem)

✓ Lorentz invariant (crossing symmetry)

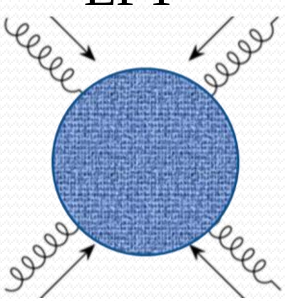
✓ CAUSAL (analyticity)

✓ Local (Froissart Bound)

positivity bounds

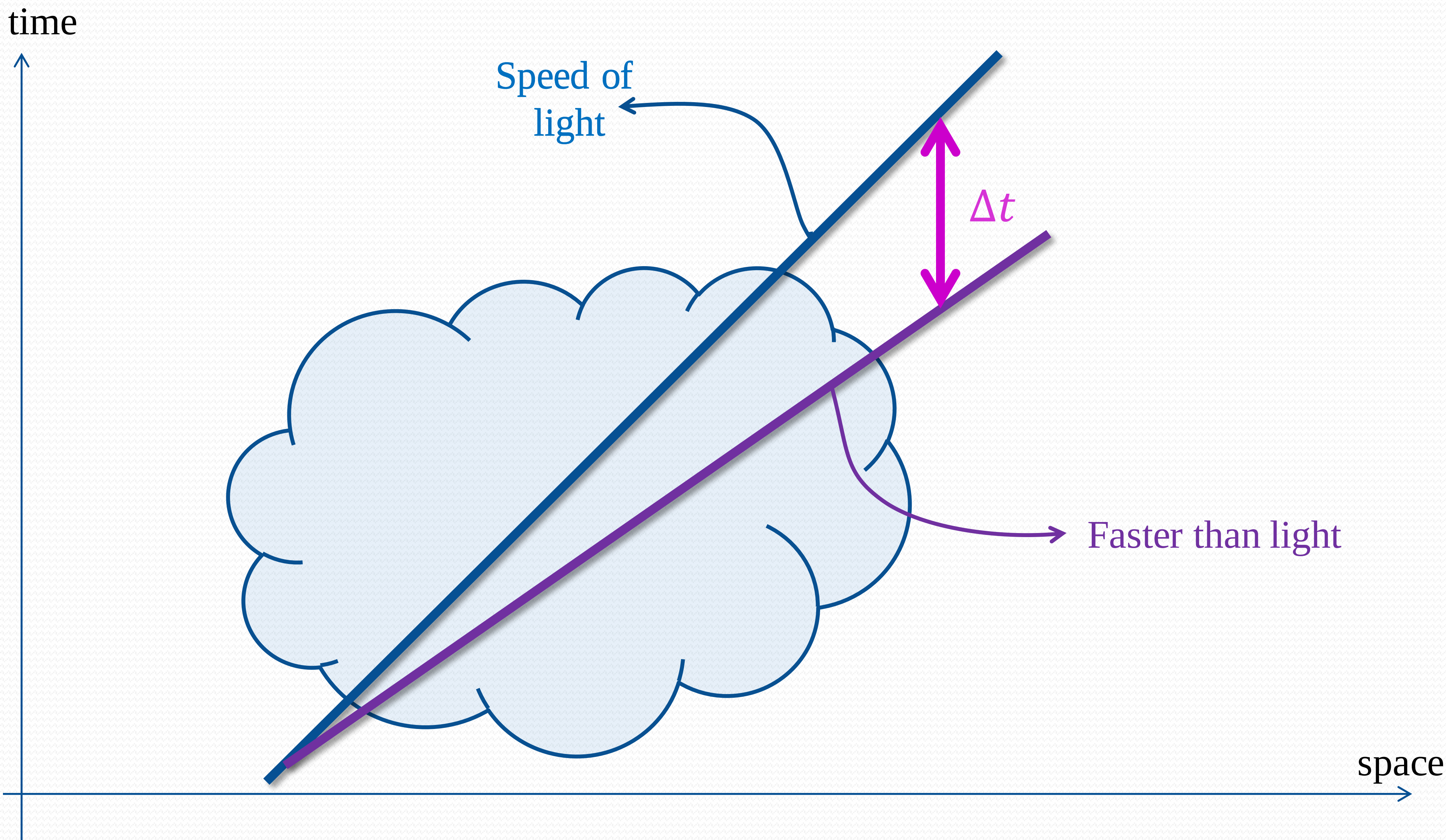
(sub)luminal sound speed

Low-energy EFT



Small Superluminality – without gravity

As soon as a "*substance*" allows the tiniest superluminal speed,
nothing prevents us from stacking it so as to end up with a significant/observable "time advance"



No Gravity

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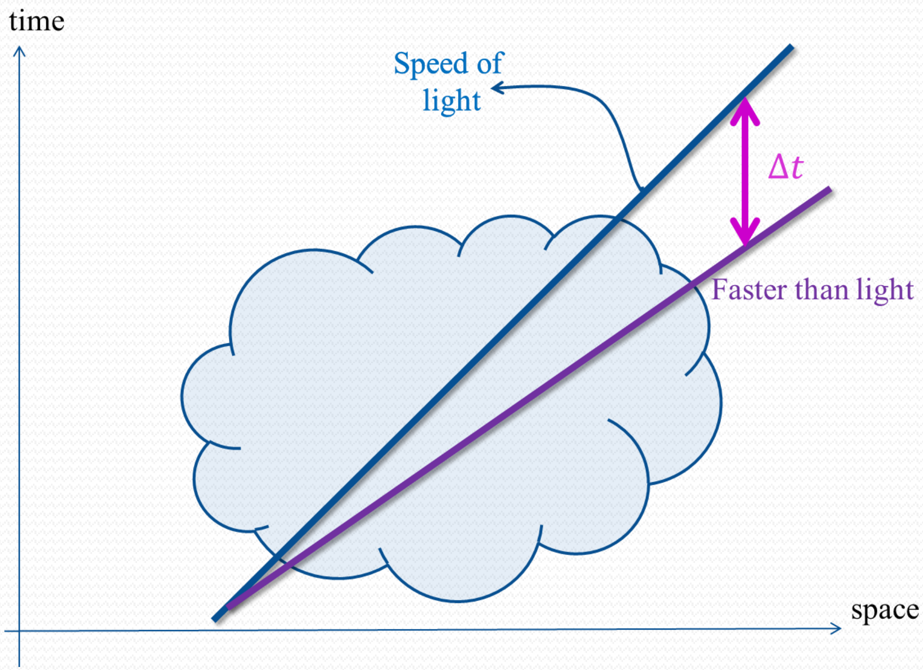
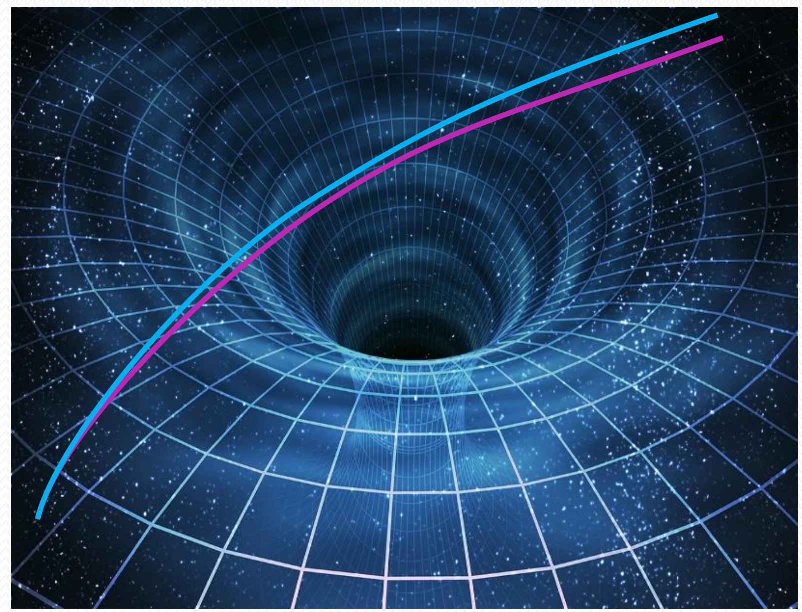


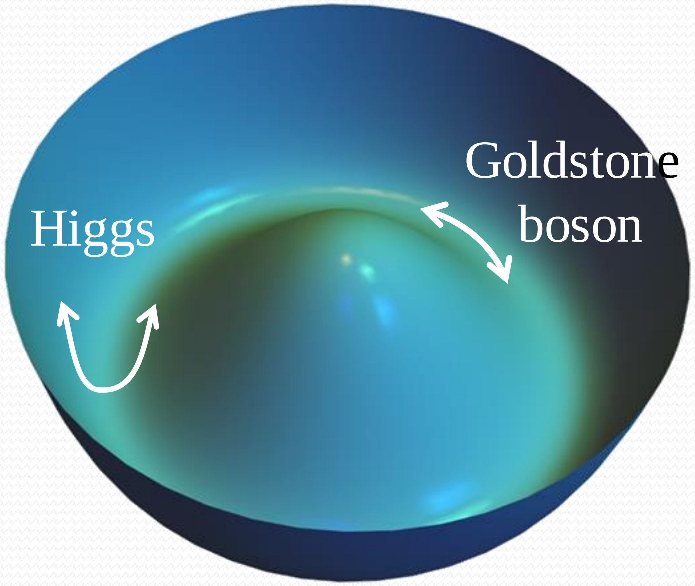
With Gravity

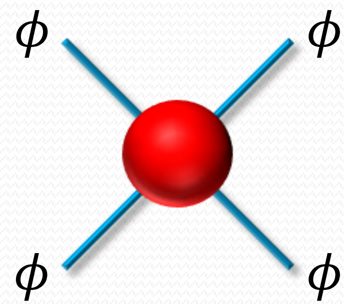
Anything living on the spacetime *inexorably* curves the geometry.
There is a limit to "stacking"

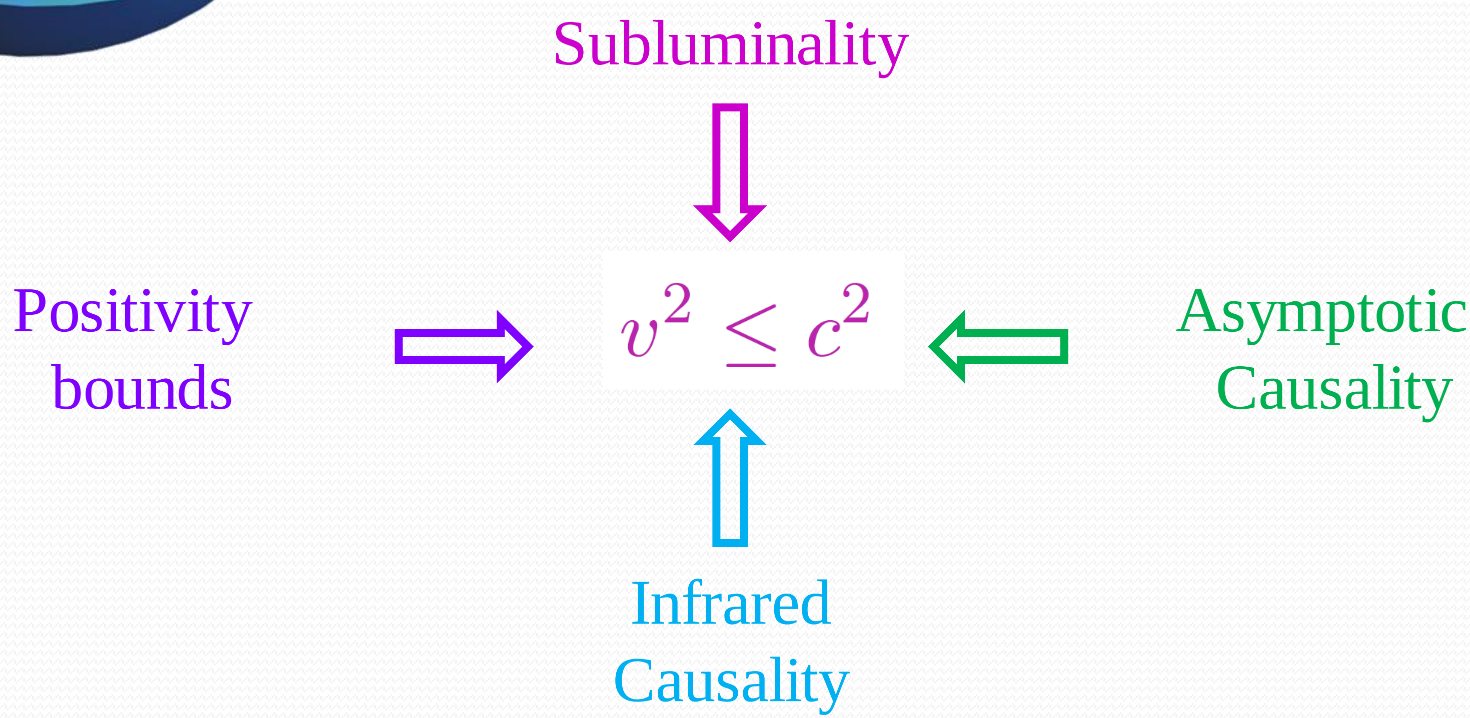


Not always meaningful violation of causality

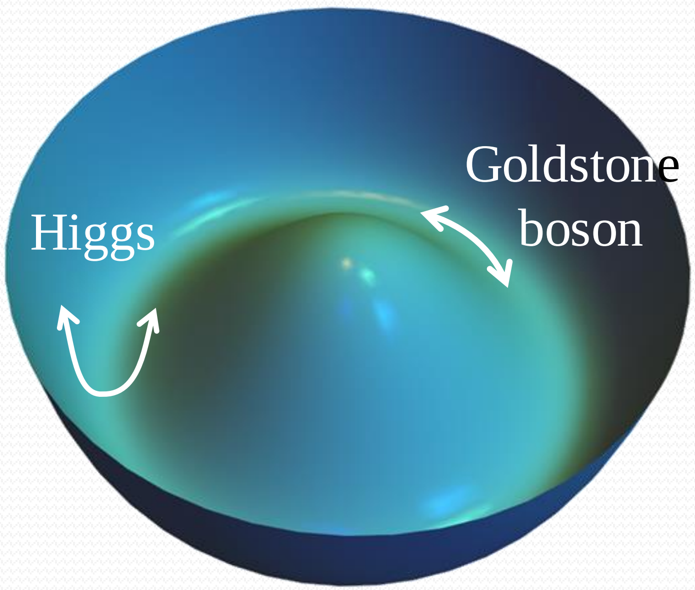
Goldstone Boson without Gravity

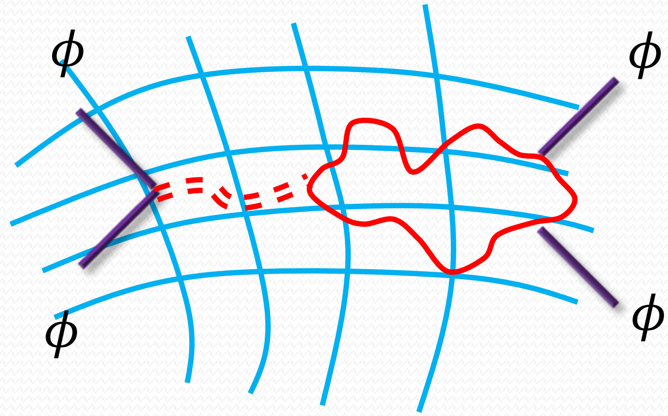






Goldstone Boson with Gravity





Subluminality
(in Einstein frame)

Positivity bounds

$$v^2 \leq c^2 + \frac{\Lambda^2}{M_{\rm Pl}^2}$$

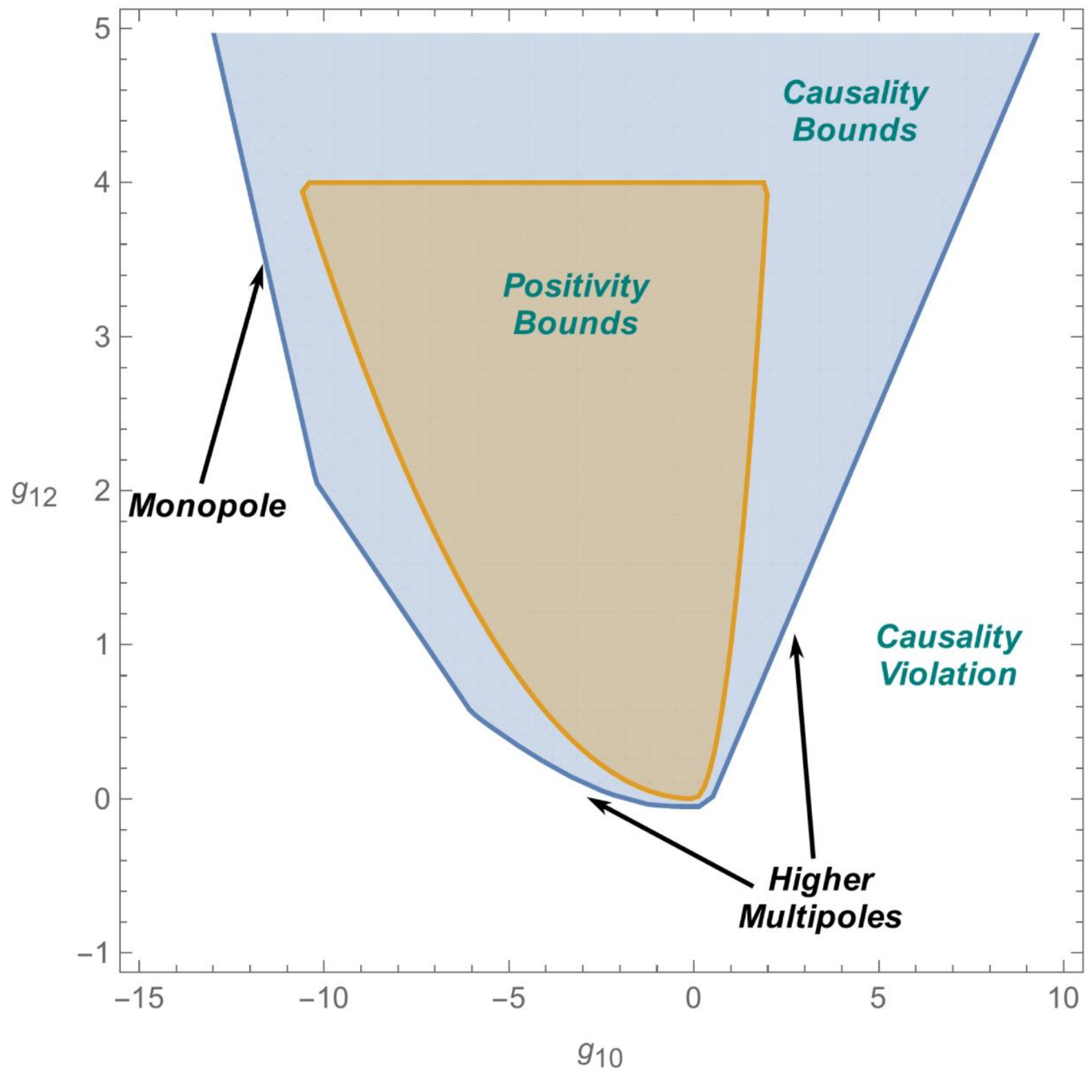
Asymptotic Causality

$$v^2 \leq c^2 + \frac{\Lambda}{M_{\rm Pl}}$$

Infrared Causality

Positivity vs Causality

$$\mathcal{L}_{\rm EFT} = -\frac{1}{2}(\partial\phi)^2 + \frac{c^2}{\Lambda^4}(\partial\phi)^4 + \frac{g_{10}}{\Lambda^6}(\partial\phi)^2(\phi_{,\mu\nu})^2 + \frac{g_{12}}{\Lambda^8}((\phi_{,\mu\nu})^2)^2 + \cdots$$



Carrillo Gonzalez, CdR, Pozsgay, Tolley, 2207.03491

$c = 1$

Tools for EFT of (massive) Gravity

- Positivity bounds applied on low-energy EFTs can probe unknown UV contributions. Connection between UV and IR is made thanks to analyticity demanded from causality & non-perturbative unitarity.

- Application to **SMEFT** already proven powerful.
  → Generalization to bounds beyond 2-2 would be instrumental

- Applications to **gravitational EFTs & EFT of Cosmology** is more subtle

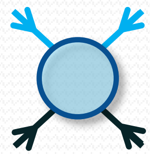
- **Causality bounds** applied directly at the level of the low-energy EFT can provide powerful complementary bounds

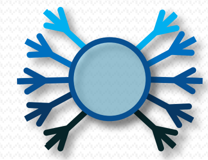
- Applicable to **EFT of Gravity**, **Cosmology** and **generalizable beyond 2-2**

Causality: Towards new Opportunities

While powerful, current positivity bounds have not yet reached their full potential

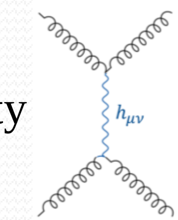
1. Positivity Limited to 2→2 scattering statements





$\partial^m \phi^n$

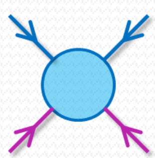
2. Gravitational pole spoils full applicability

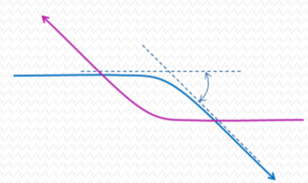


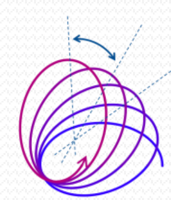
Current causality can go beyond

3. Need of more precise dictionary with actual (gravitational) observables

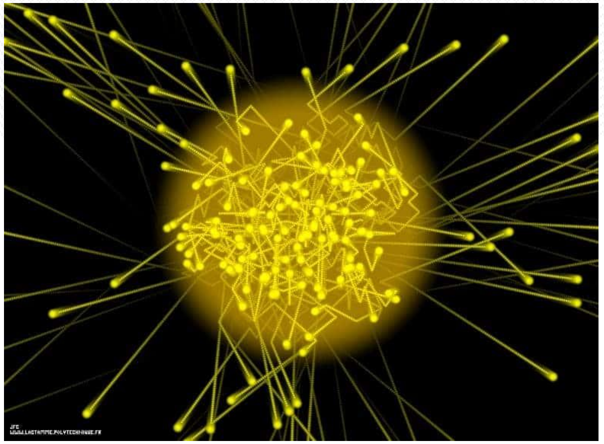
real world, not *empty* & *Minkowski as required by S-matrix bootstrap bounds*







EFT of photons



$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{c_1}{\Lambda^4}F^{\mu\nu}F_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} + \frac{c_2}{\Lambda^4}F^{\mu\nu}F^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$$

$$+ \frac{c_3}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\mu F_{\beta\gamma}\partial_\nu F_\alpha{}^\gamma + \frac{c_4}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\beta F_{\mu\gamma}\partial^\gamma F_{\alpha\nu} + \frac{c_5}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\beta$$

+ many higher order operators

From integrating out heavy charged fields (electron,…)

4-point tree-level scattering amplitude arising from this theory can be parame

$$\mathcal{A}_{++++} = f_2\left(s^2+t^2+u^2\right) + f_3 stu + f_4\left(s^2+t^2+u^2\right)^2$$

$$\mathcal{A}_{++--} = g_2 s^2 + g_3 s^3 + g_4 s^4 + g_4' s^2 tu$$

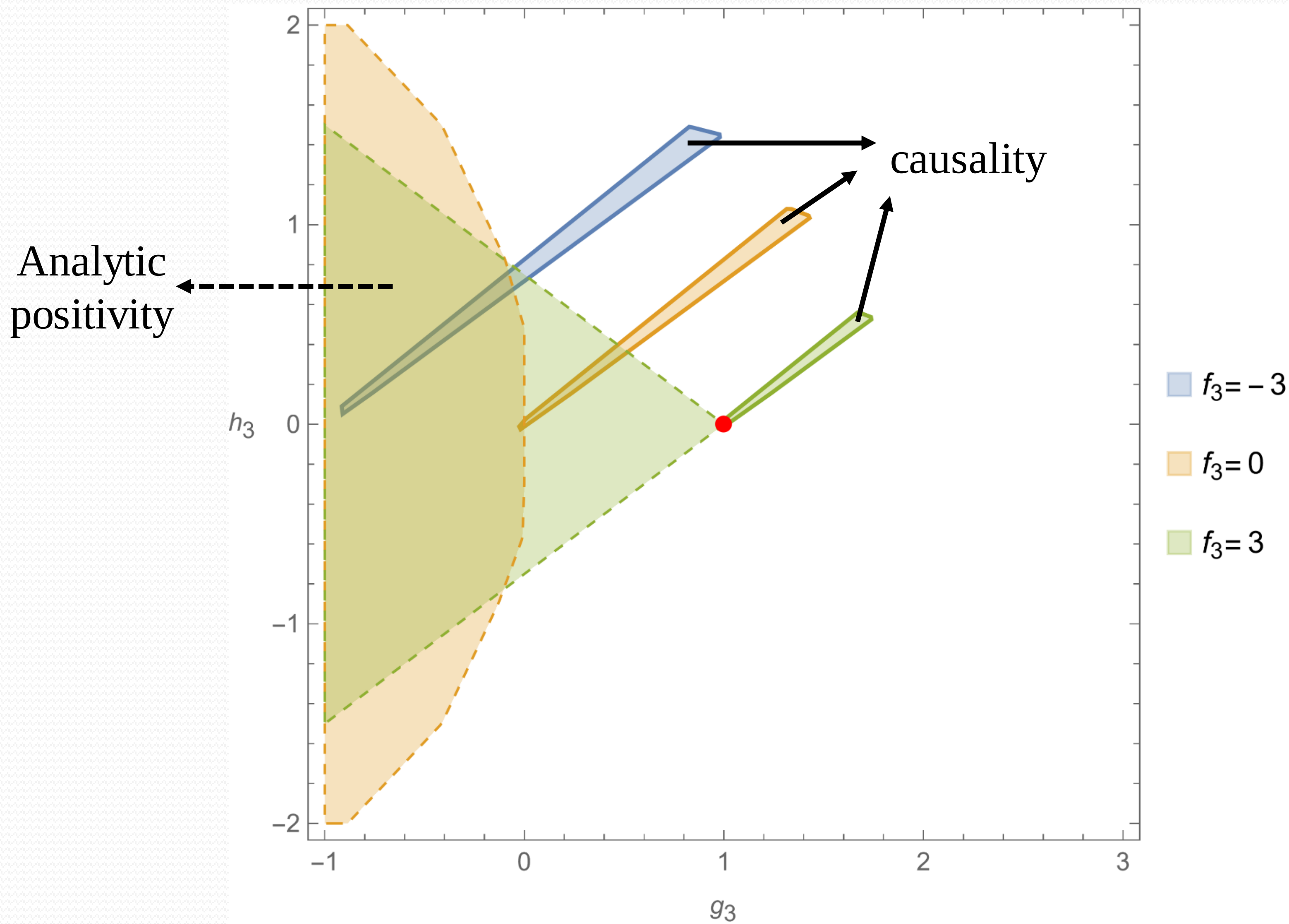
$$\mathcal{A}_{+++-} = h_3 stu$$

$$f_2 = 2\left(4c_1 + c_2\right)\ , \quad g_2 = 2\left(4c_1 + 3c_2\right)$$

$$f_3 = -3\left(c_3 + c_4 + c_5\right)\ , \quad g_3 = -c_5\ , \quad h_3 = -\frac{3}{2}c_3\ ,$$

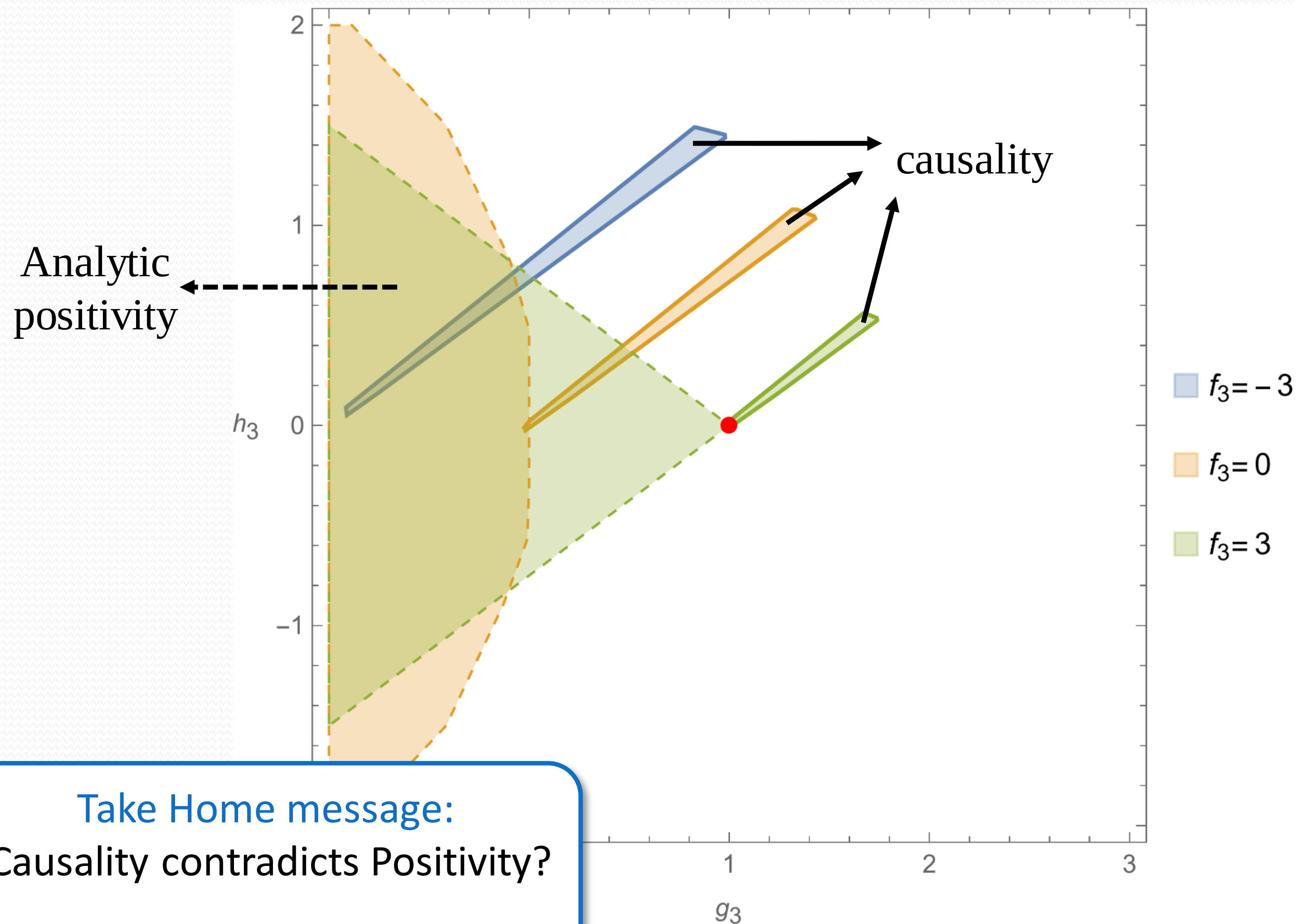
$$f_4 = \frac{1}{4}c_6\ , \quad g_4 = \frac{1}{2}(c_6 - c_8) + c_7\ , \quad g_4' = -\frac{1}{2}(c_7 + c_8)\ .$$

Positivity vs Causality in EFT of photons



With Carrillo Gonzalez, Jaitly, Pozsgay, Tokareva, 2208.12631

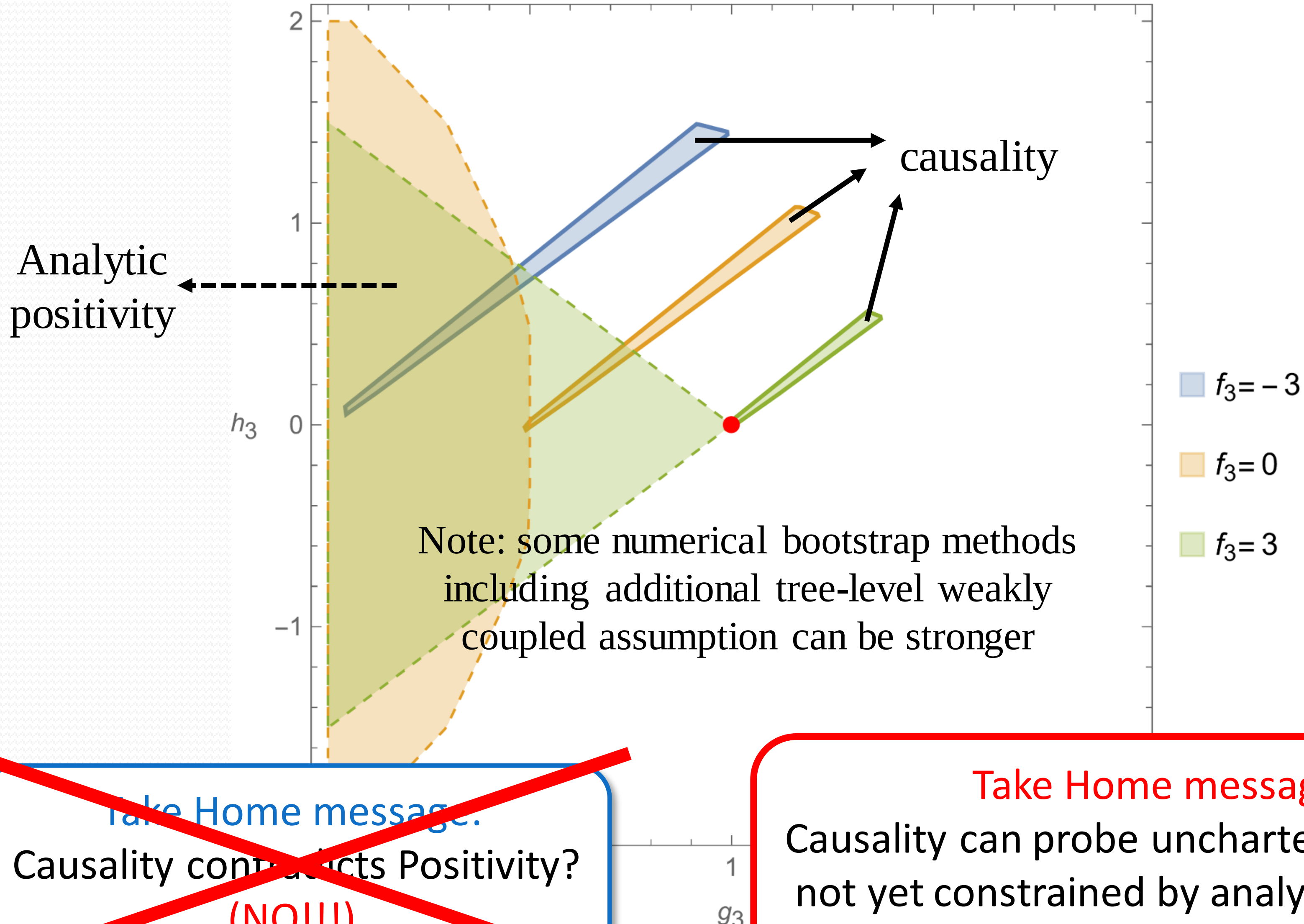
Positivity vs Causality in EFT of photons



Take Home message:
Causality contradicts Positivity?

With Carrillo Gonzalez, Jaitly, Pozsgay, Tokareva, 2208.12631

Positivity vs Causality in EFT of photons



~~Take Home message:~~
~~Causality contradicts Positivity?~~
(NO!!!)

Take Home message:
Causality can probe uncharted territories, not yet constrained by analytic positivity! Combining both is powerful!

With Carrillo Gonzalez, Jaitly, Pozsgay, Tokareva, 2208.12631

Constraints on Dark Energy

Example of Dark Energy model (quartic Horndeski)

$$\mathcal{L} = M_{\rm Pl}^2 G_4(X) R + \Lambda^4 G_2(X) + \frac{1}{H_0^2} G_{4,X}(X) \left((\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right) + \mathcal{L}_{\rm matter}(\psi, g)$$

$$X = -\frac{1}{2\Lambda^4} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\Lambda^2 = H_0^2 M_{\rm Pl}^2 \sim 10^{-3}{\rm eV}$$

$$c_M = -\frac{2\dot{X}}{H M^2 \Omega_{\rm DE}} \left(G_{4,X} + 2X G_{4,XX} \right), \quad \text{Effective Planck Mass}$$

$$c_B = \frac{8X}{M^2 \Omega_{\rm DE}} \left(G_{4,X} + 2X G_{4,XX} \right), \quad \text{"Braiding" (scalar/tensor mixing)}$$

$$c_T = \frac{4X}{M^2 \Omega_{\rm DE}} G_{4,X}, \quad \text{Speed of GWs} - 1$$

$$M^2 = 2\left(G_4 - 2X G_{4,X} \right)$$

with Melville and Noller, 2103.06855

with Melville and Noller, 2103.06855

Constraints on Dark Energy

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Assume a ΛCDM background.
Linear perturbations controlled by 4 background functions
(including kineticity related to G_2)

$$c_M = -\frac{2\dot{X}}{HM^2\Omega_{\rm DE}} \left(G_{4,X} + 2XG_{4,XX}\right), \quad \text{Effective Planck Mass}$$

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Constrained performing a Markov chain Monte Carlo analysis, using:

- Planck 2015,
- CMB lensing and low-ℓ polarisation data,
- SDSS/BOSS BAO
- SDSS DR4 LRG matter power spectrum shape
- redshift space distortion constraints from BOSS and 6dF

with Melville and Noller, 2103.06855

Constraints on Dark Energy

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In addition, these parameters can be bounded using positivity bounds

with Melville and Noller, 2103.06855

Constraints on Dark Energy

$$\mathcal{L} = M_{\rm Pl}^2 G_4(X) R + \Lambda^4 G_2(X) + \frac{1}{H_0^2} G_{4,X}(X)\left((\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2\right) + \mathcal{L}_{\rm matter}(\psi, g)$$

$$c_M = -\frac{2\dot{X}}{HM^2\Omega_{\rm DE}}\left(G_{4,X} + 2XG_{4,XX}\right), \quad \text{Effective Planck Mass}$$

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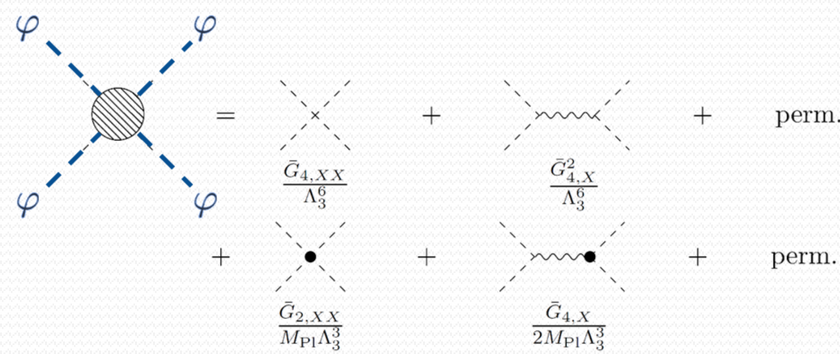
$$c_T = \frac{4X}{M^2\Omega_{\rm DE}} G_{4,X}, \quad \text{Speed of GWs} - 1$$

Positivity Bounds from

$$\varphi\varphi \to \varphi\varphi$$



$$\mathcal{A}^{\varphi\varphi\to\varphi\varphi} = \frac{\bar{G}_{2,XX}}{\Lambda^4}\left(s^2 + t^2 + u^2\right) - \frac{\bar{G}_{4,X}}{\bar{G}_4 \Lambda^4}\left(su + st + ut\right) + 3!\,\frac{\bar{G}_{4,XX} + \bar{G}_{4,X}^2/\bar{G}_4}{H_0^2\Lambda^4}\, stu$$

$$\bar{G}_{2,XX} \geq -\bar{G}_{4,X}\frac{\bar{G}_{2,X}}{\bar{G}_4}, \qquad \bar{G}_{4,XX} + \frac{\bar{G}_{4,X}^2}{\bar{G}_4} \leq 0 \qquad \varphi\varphi \to \varphi\varphi$$

with Melville and Noller, 2103.06855

Constraints on Dark Energy

$$\mathcal{L} = M_{\rm Pl}^2 G_4(X) R + \Lambda^4 G_2(X) + \frac{1}{H_0^2} G_{4,X}(X) \left((\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right) + \mathcal{L}_{\rm matter}(\psi, g)$$

Standard (minimal) coupling to matter fields ψ

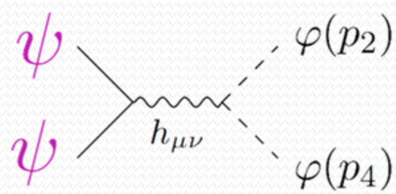
$$c_M = -\frac{2\dot{X}}{HM^2\Omega_{\rm DE}} \left(G_{4,X} + 2X G_{4,XX} \right), \quad \text{Effective Planck Mass}$$

$$c_B = \frac{8X}{M^2\Omega_{\rm DE}} \left(G_{4,X} + 2X G_{4,XX} \right), \quad \text{“Braiding” (scalar/tensor mixing)}$$

$$c_T = \frac{4X}{M^2\Omega_{\rm DE}} G_{4,X}, \quad \text{Speed of GWs} - 1$$

Positivity Bounds from

$$\varphi\psi \to \varphi\psi$$



$$= \frac{\bar{G}_{4,X}}{2\Lambda_2^4}(s^2 + u^2 - t^2)$$

$$\bar{G}_{4,X} \geq 0 \qquad \varphi\psi \to \varphi\psi$$

$$\bar{G}_{2,XX} \geq -\bar{G}_{4,X} \frac{\bar{G}_{2,X}}{\bar{G}_4}, \qquad \bar{G}_{4,XX} + \frac{\bar{G}_{4,X}^2}{\bar{G}_4} \leq 0 \qquad \varphi\varphi \to \varphi\varphi$$