Causal Set Theory
as a tool for new discoveries

Stav Zalel

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Imperial College London
UKRI Science and Technology Facilities Council

Causal Set Theory

- An approach to quantum gravity in which spacetime is fundamentally discrete

- Consistent with stringent experimental bounds on local Lorentz invariance

- Historically, geared towards phenomenology

- Now, it’s a new tool for searching for fundamental spacetime discreteness in the Early Universe

Plan

- What's a causal set?

- Spacetime as a causal set

Lorentz invariance and non-locality

- Causal sets as a tool for new discoveries

- The discrete cosmological collider

Computing cosmological correlators on a causal set

A causal set is a locally finite partial order, or an irregular Lorentzian lattice.

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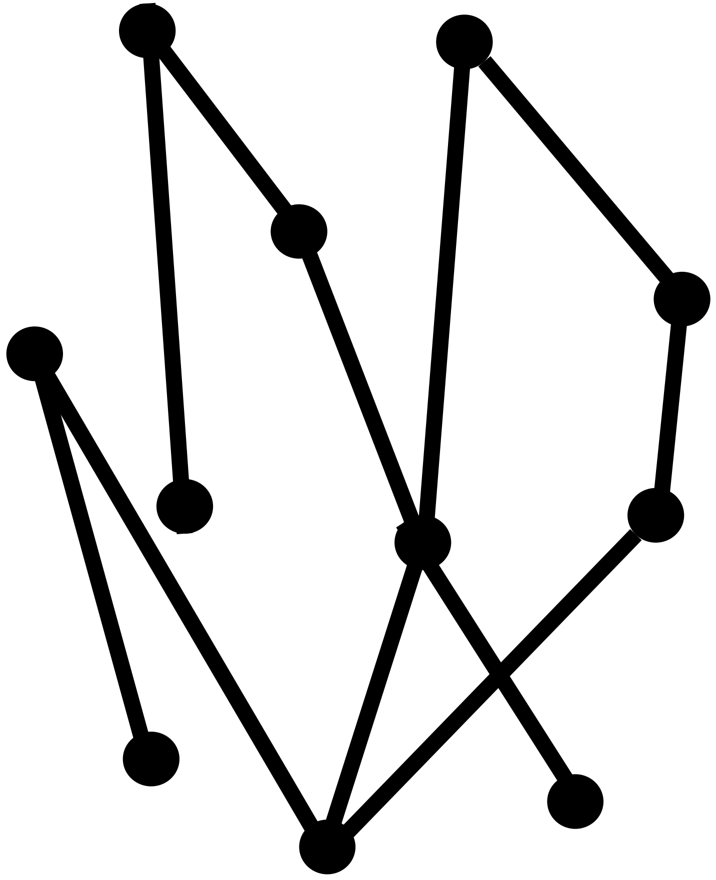
- Lattice **sites** connected by **edges**, implicitly directed up.

  If there is a path upwards from x to y we write $x \prec y$.



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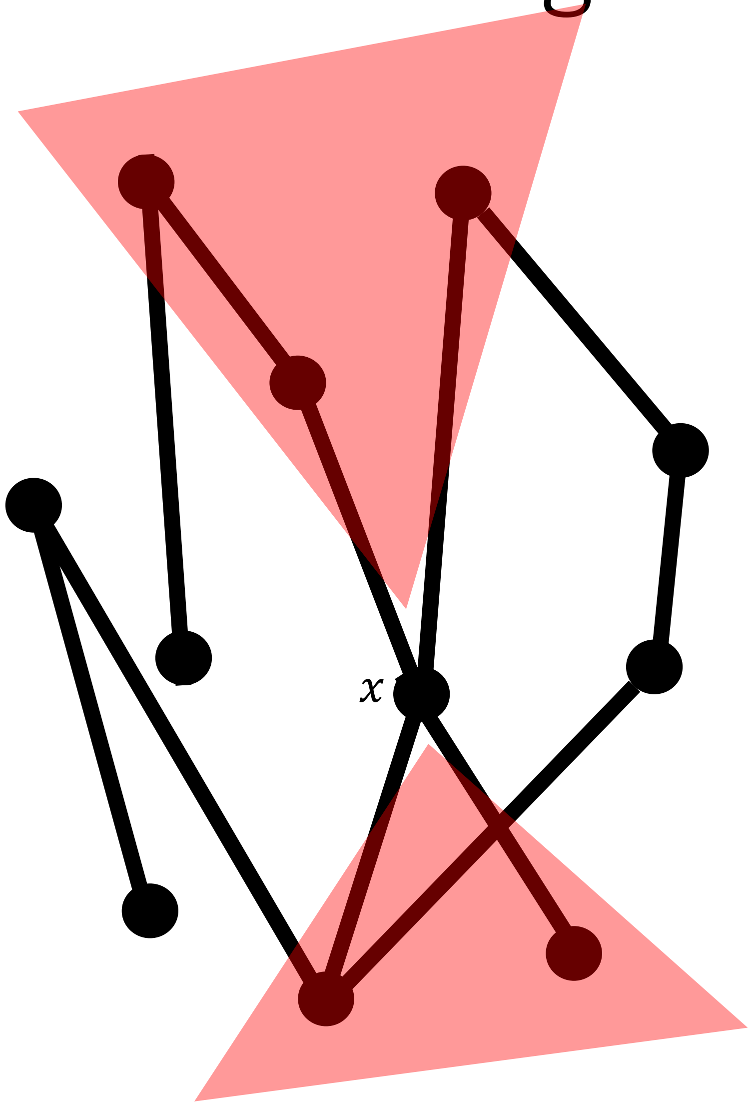

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- **Lorentzian causal structure**:

The past of a lattice site x are all the points y such that $y \prec x$.

The future of a lattice site x are all the points y such that $y \succ x$.

If a pair of points x and y are such that there is no directed path from one to the other then x and y are spacelike to each other.

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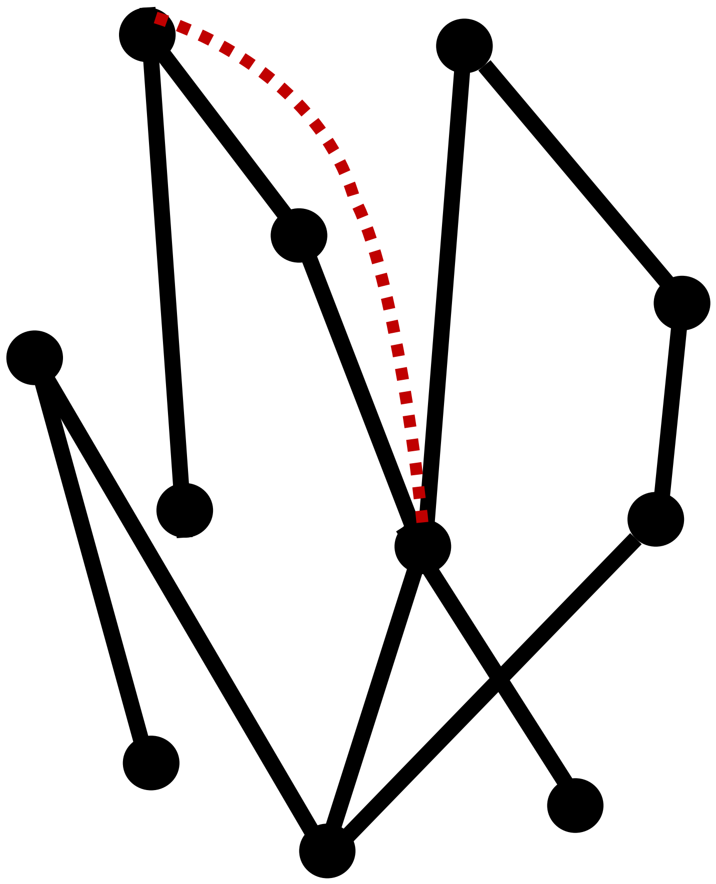

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- **Hasse diagram**: only the minimal number of edges required to encode the causal structure are drawn. **Nearest neighbours** are those directly connected by an edge.

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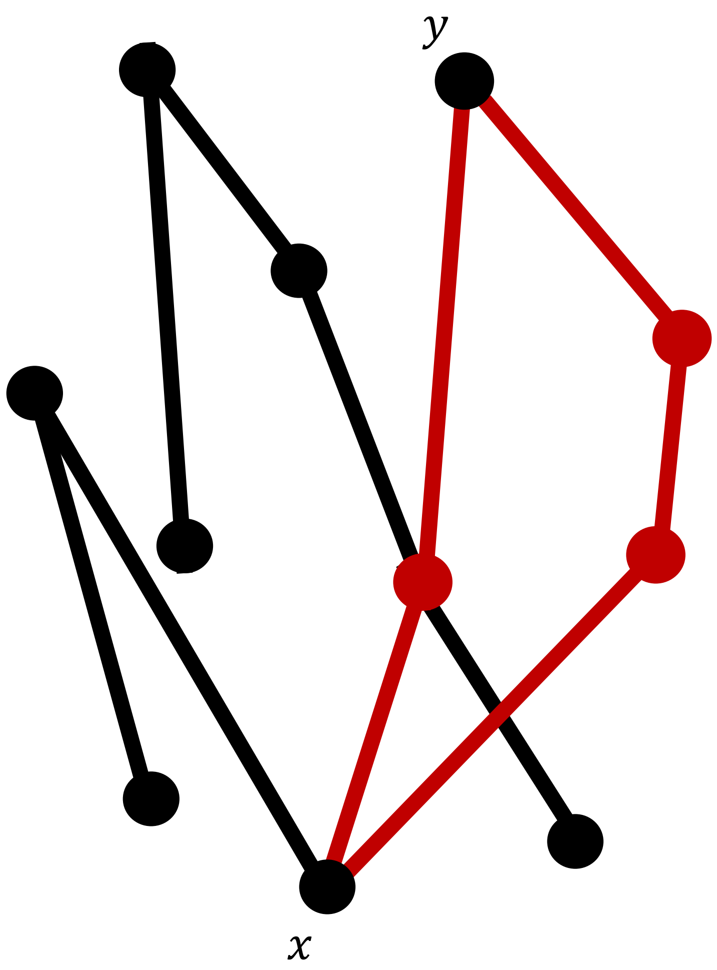

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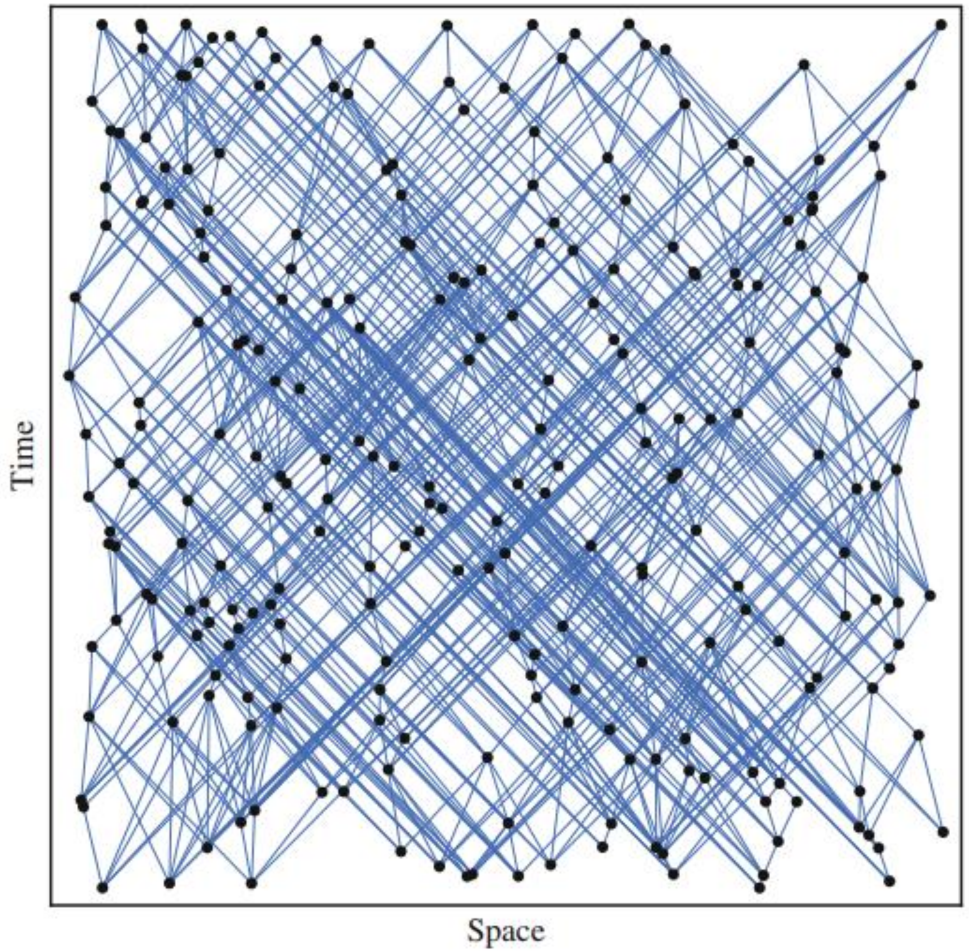
- **Hasse diagram**: only the minimal number of edges required to encode the causal structure are drawn. **Nearest neighbours** are those directly connected by an edge.

- **Locally finiteness / discreteness**:

given a pair of points x and y, the number of points z such that $x \prec z \prec y$ is finite, i.e. the cardinality of all intervals is finite.

Spacetime as a causal set

- Spacetime is fundamentally discrete and takes the form of a causal set.
- The partial order encodes the causal structure.
- **Number-volume correspondence**: the spacetime volume in a spacetime region is proportional to the number of elements the region contains.
- Continuum spacetime emerges as large-scale approximation / through coarse-graining.



A causal set which can be approximated by a portion of 1+1 Minkowski

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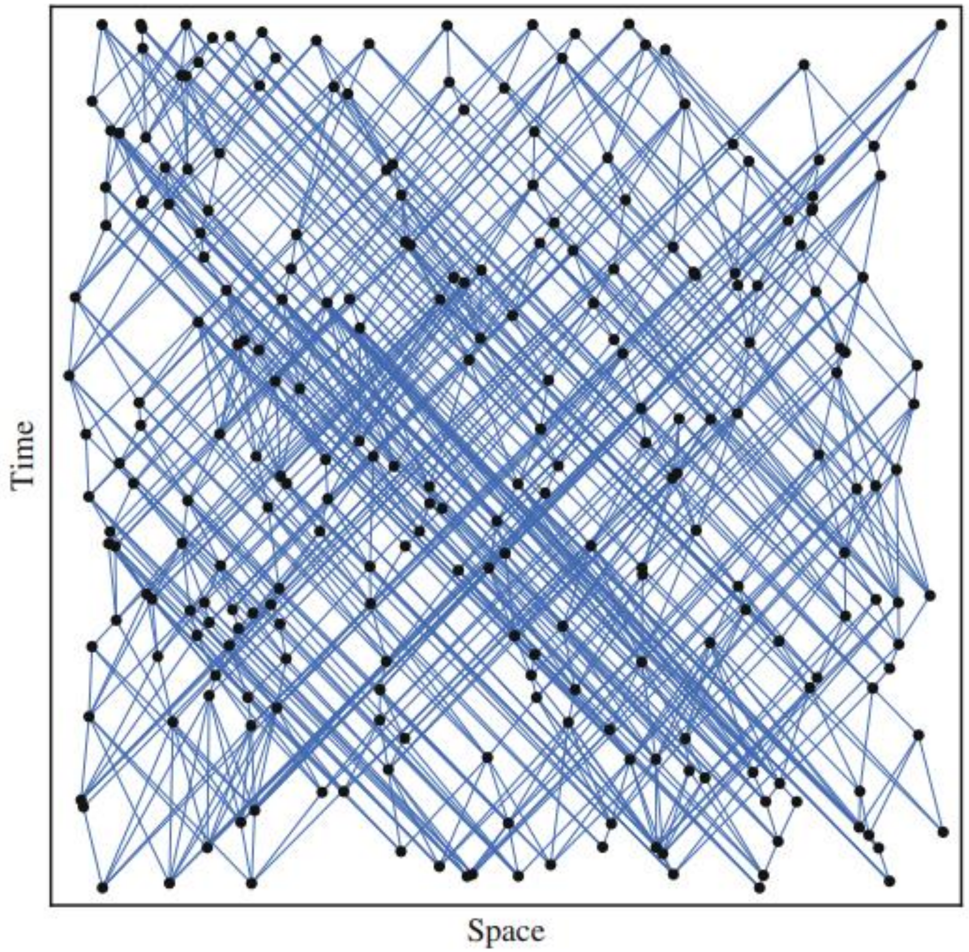
How?



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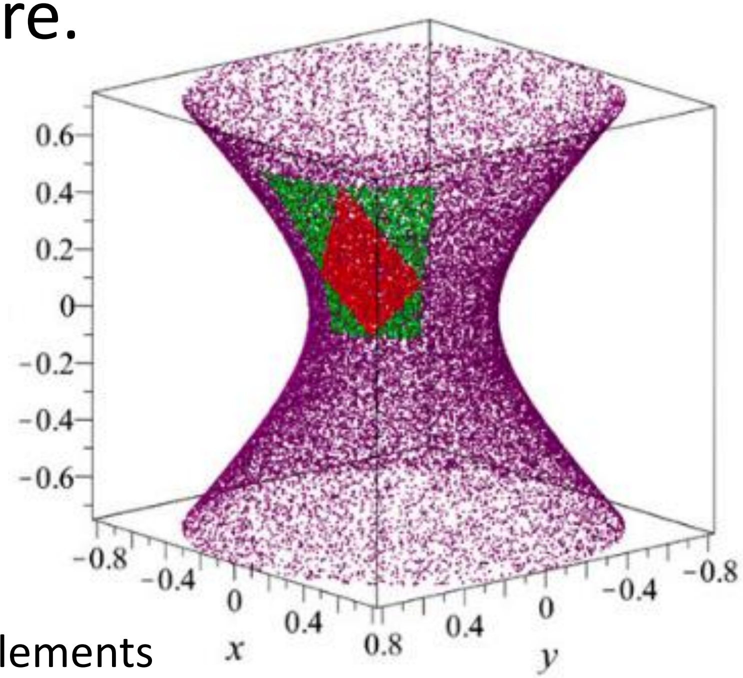
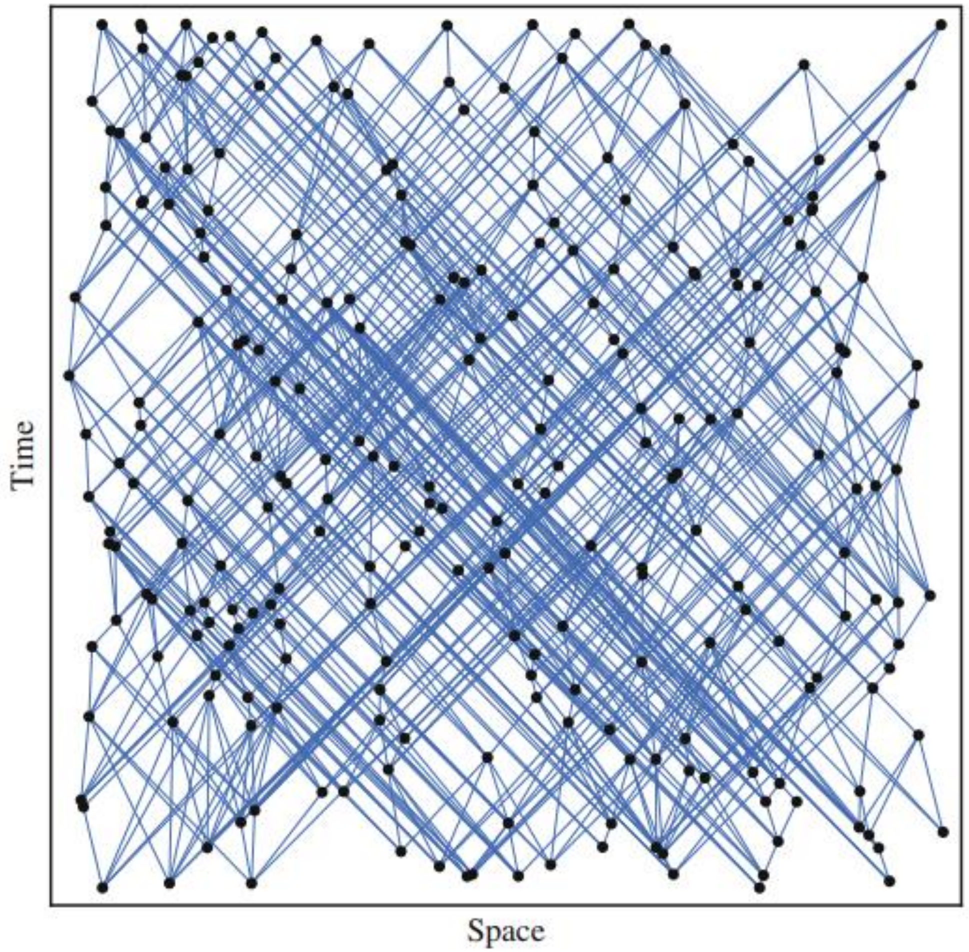
Spacetime as a causal set

A causal set is approximated by a continuum through an embedding which distributes points evenly and preserves the causal structure.



A causal set with 3200 elements approximated by dS_2. 2007.03835



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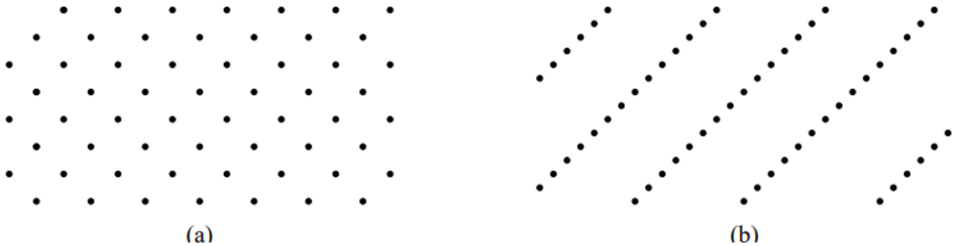
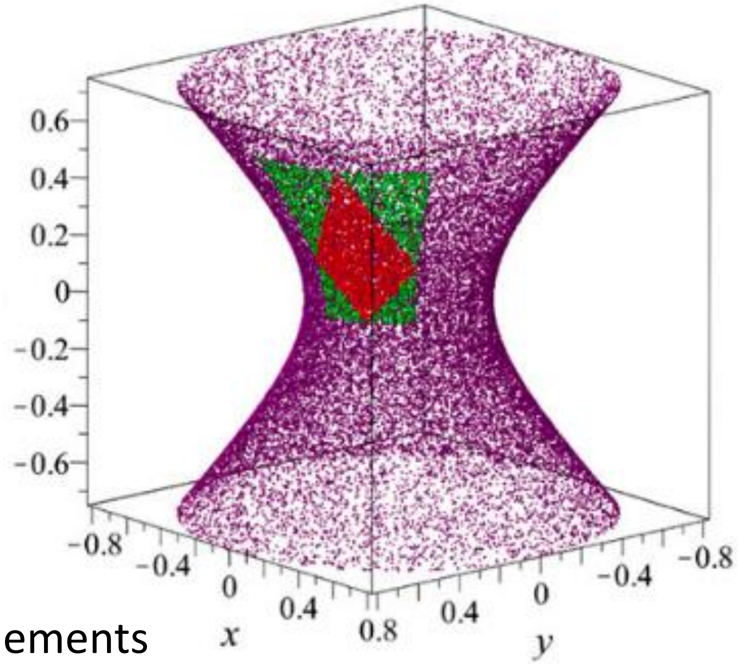
Spacetime as a causal set

A regular lattice picks a preferred frame





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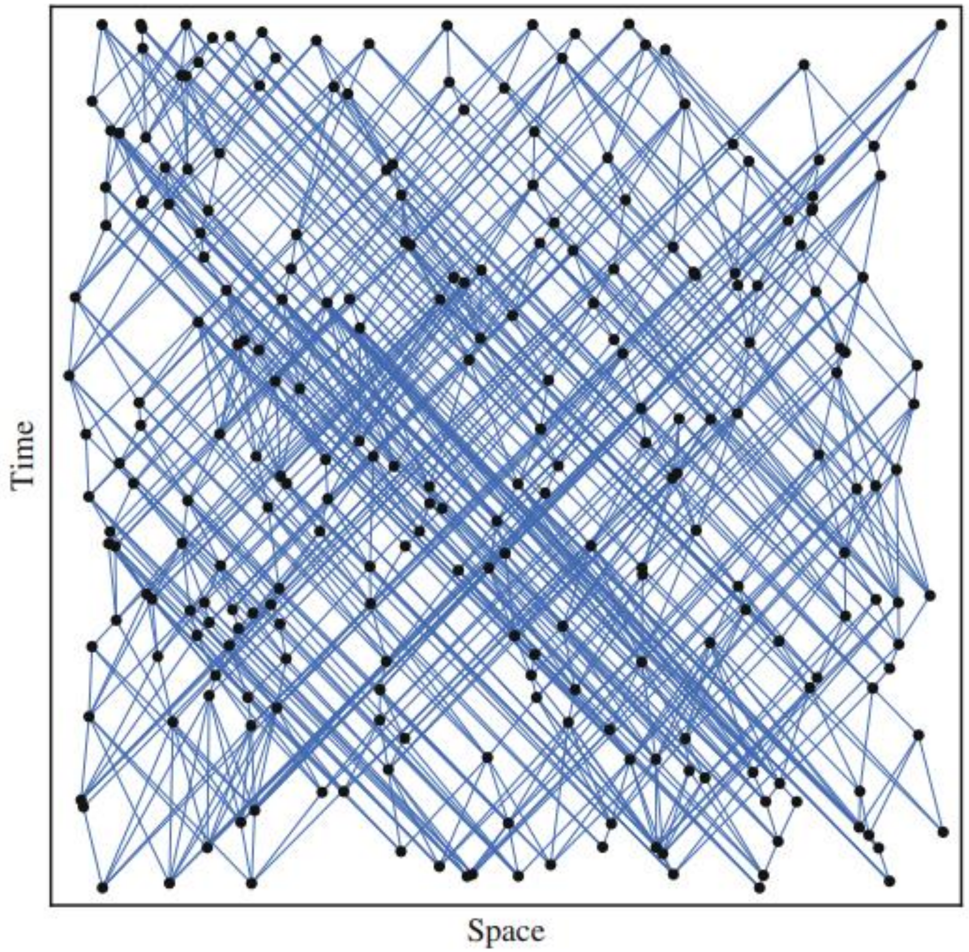

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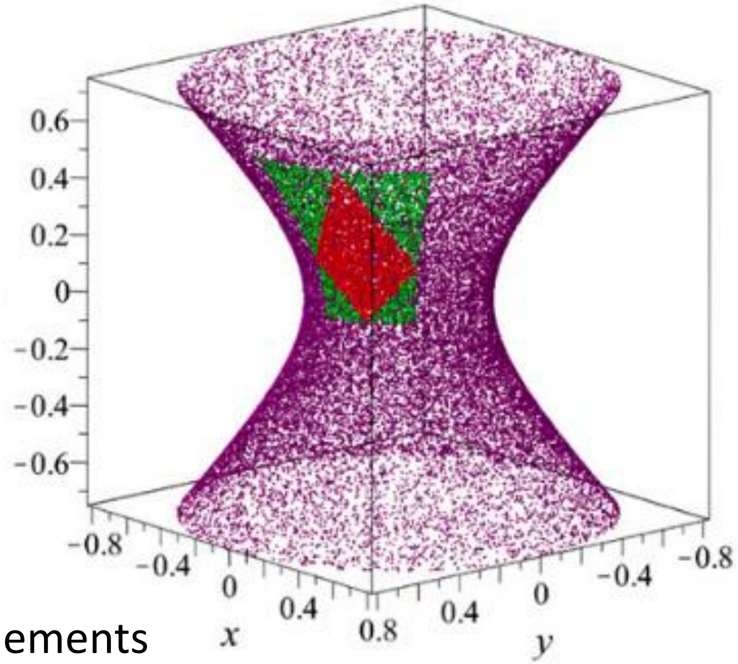
Spacetime as a causal set

A regular lattice picks a preferred frame

- Random
- Coordinate independent
- Preserves number-volume correspondence



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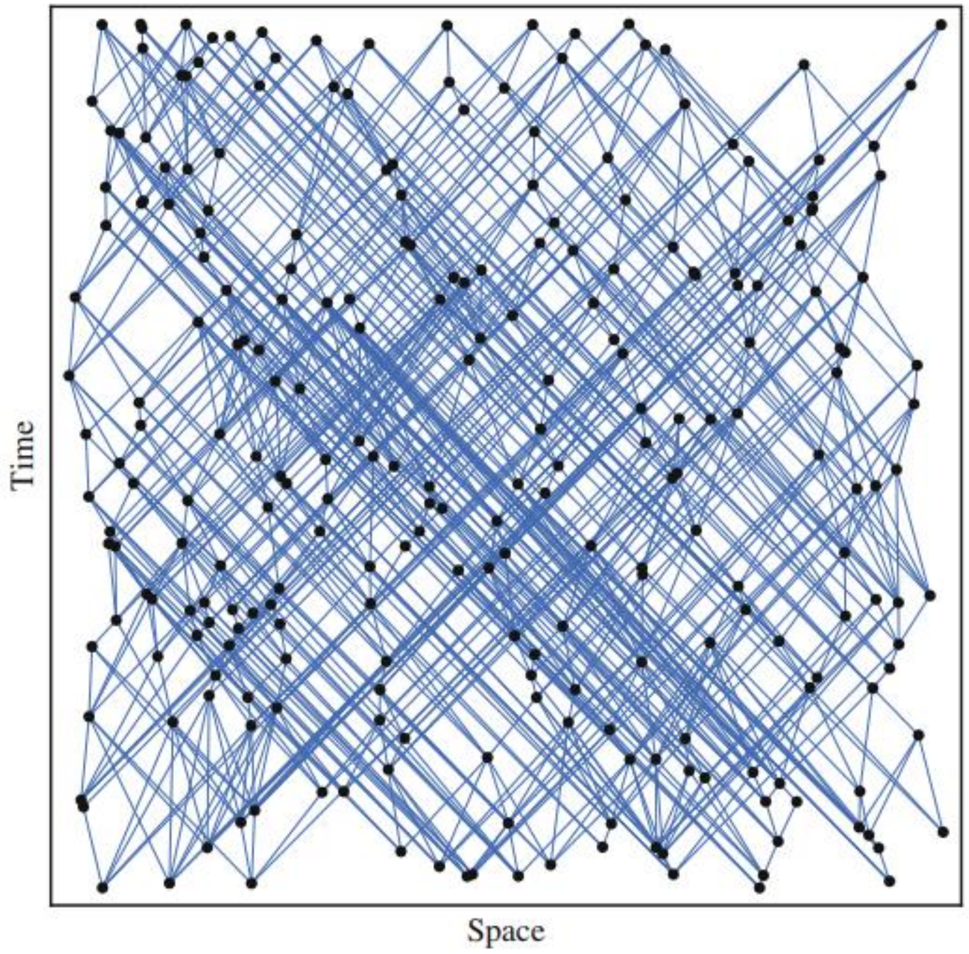

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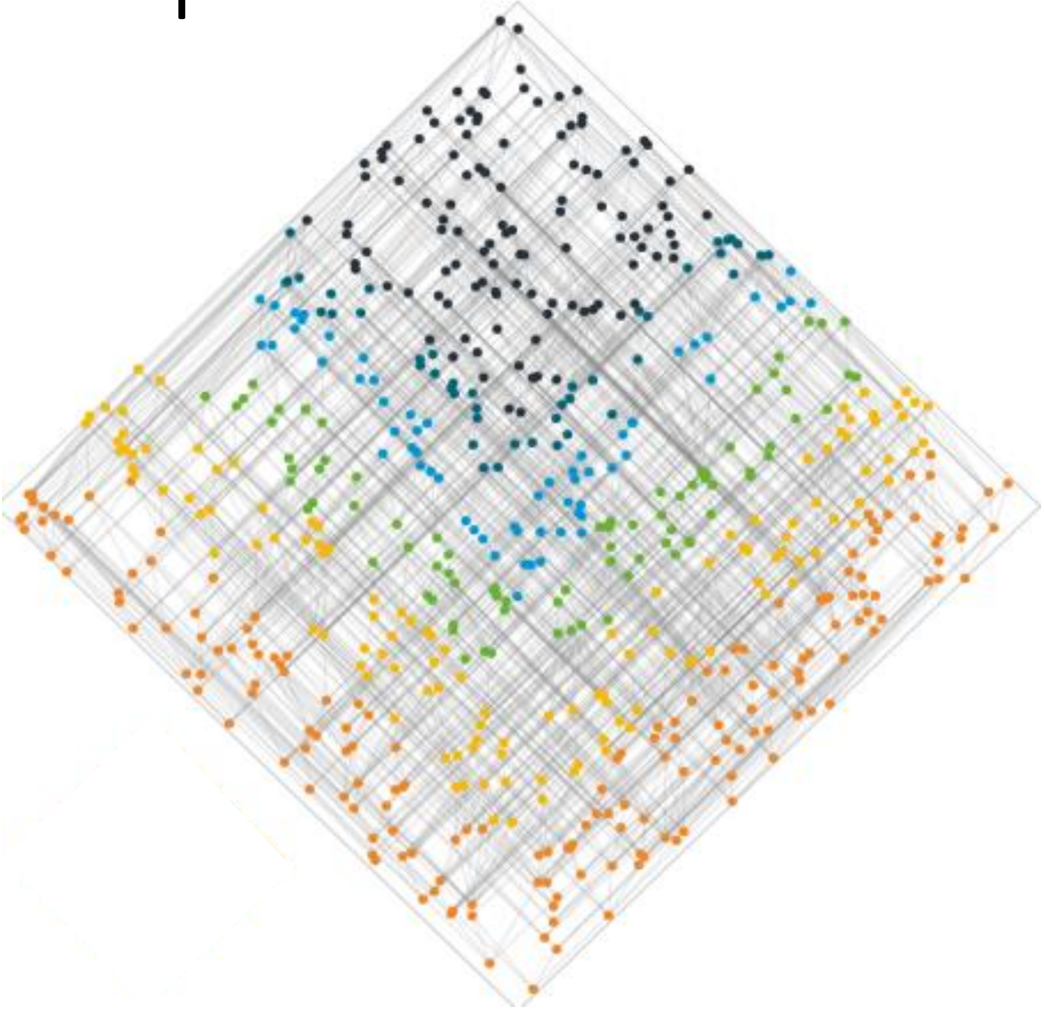
The discrete-continuum correspondence

A causal set $(C, \prec)$ is well-approximated by a continuum (M, g) if there exists a **faithful embedding** of C in M, i.e. there exists a map $f : C \to M$ which,

1. preserves the causal order: $a \prec b \Leftrightarrow f(a) \in J^{-}(f(b))$,
2. the points $f(C)$ are distributed in M according to the Poisson distribution at some fixed density ρ,
3. the discreteness scale, $l = \rho^{-\frac{1}{d}}$, is small compared to any curvature length scale in M.

The discrete-continuum correspondence

- **Sprinkling**: the process of generating a causal set from a continuum.
- This *process* is Lorentz invariant, only uses the invariant volume measure.



600 points sprinkled into 2d Minkowski diamond. Minz, 2021

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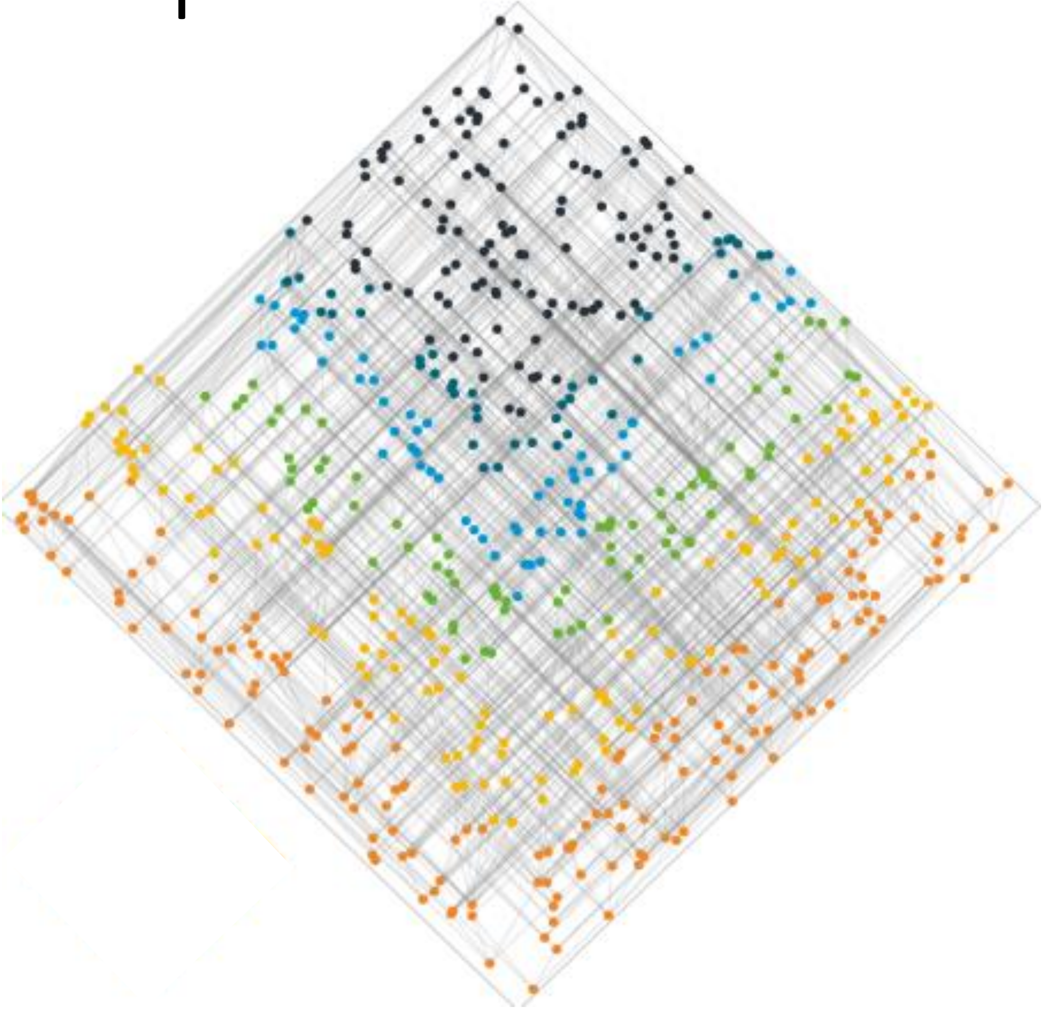
- An *individual causal set* produced by sprinkling is Lorentz invariant, i.e. a sprinkled causal set cannot be used to pick a preferred direction.

Dowker, Henson and Sorkin, gr-qc/0311055
Bombelli, Henson and Sorkin, gr-qc/0605006



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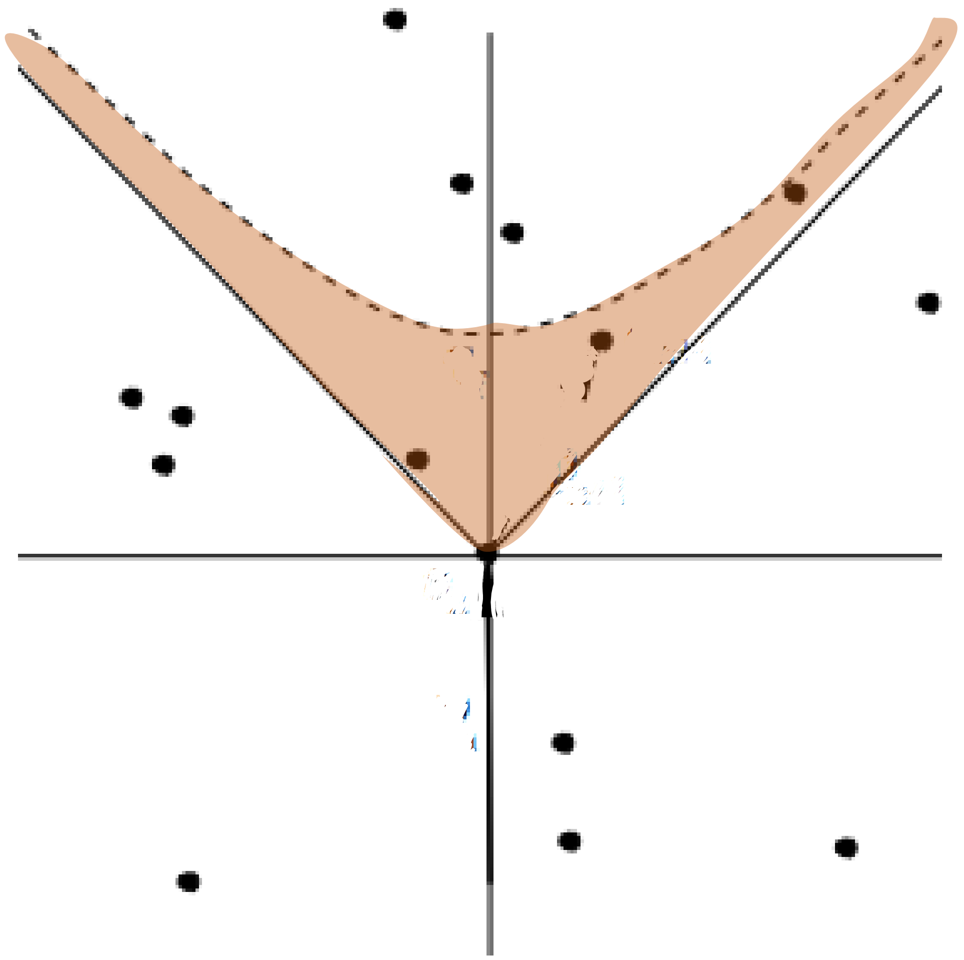

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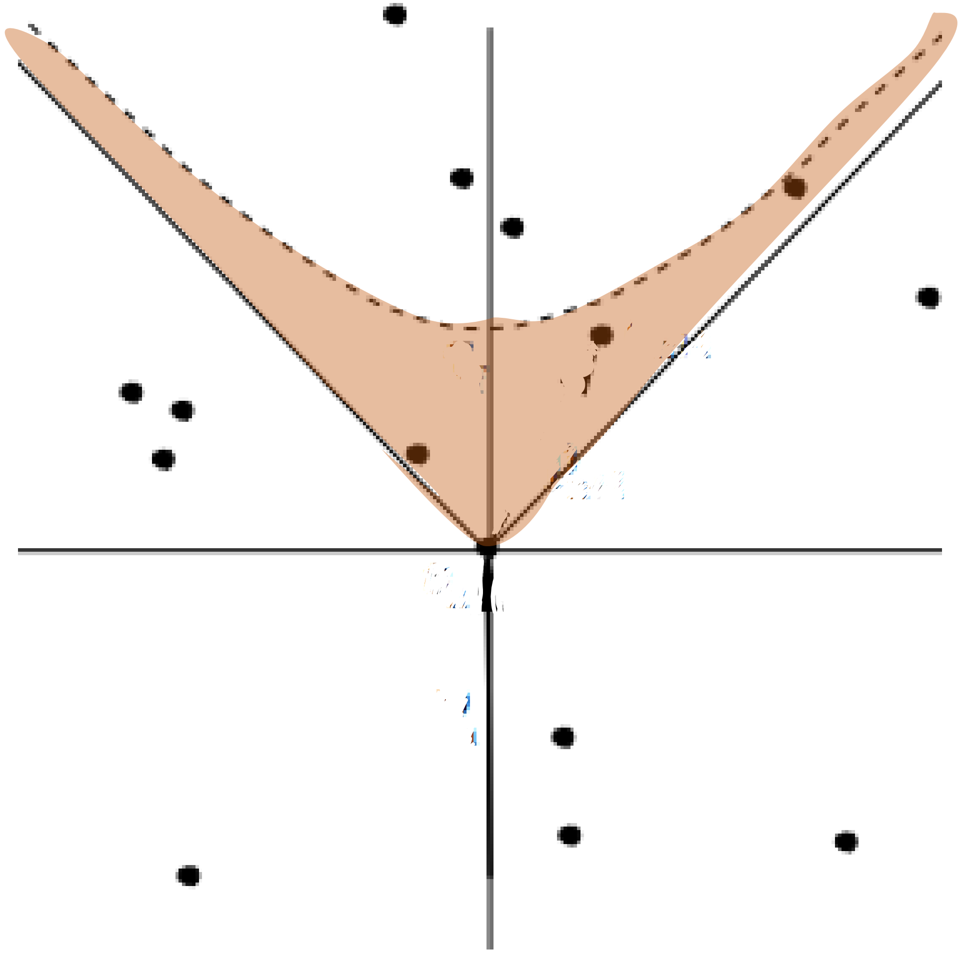

- **Non-locality:** infinitely many nearest neighbours arbitrarily far away.

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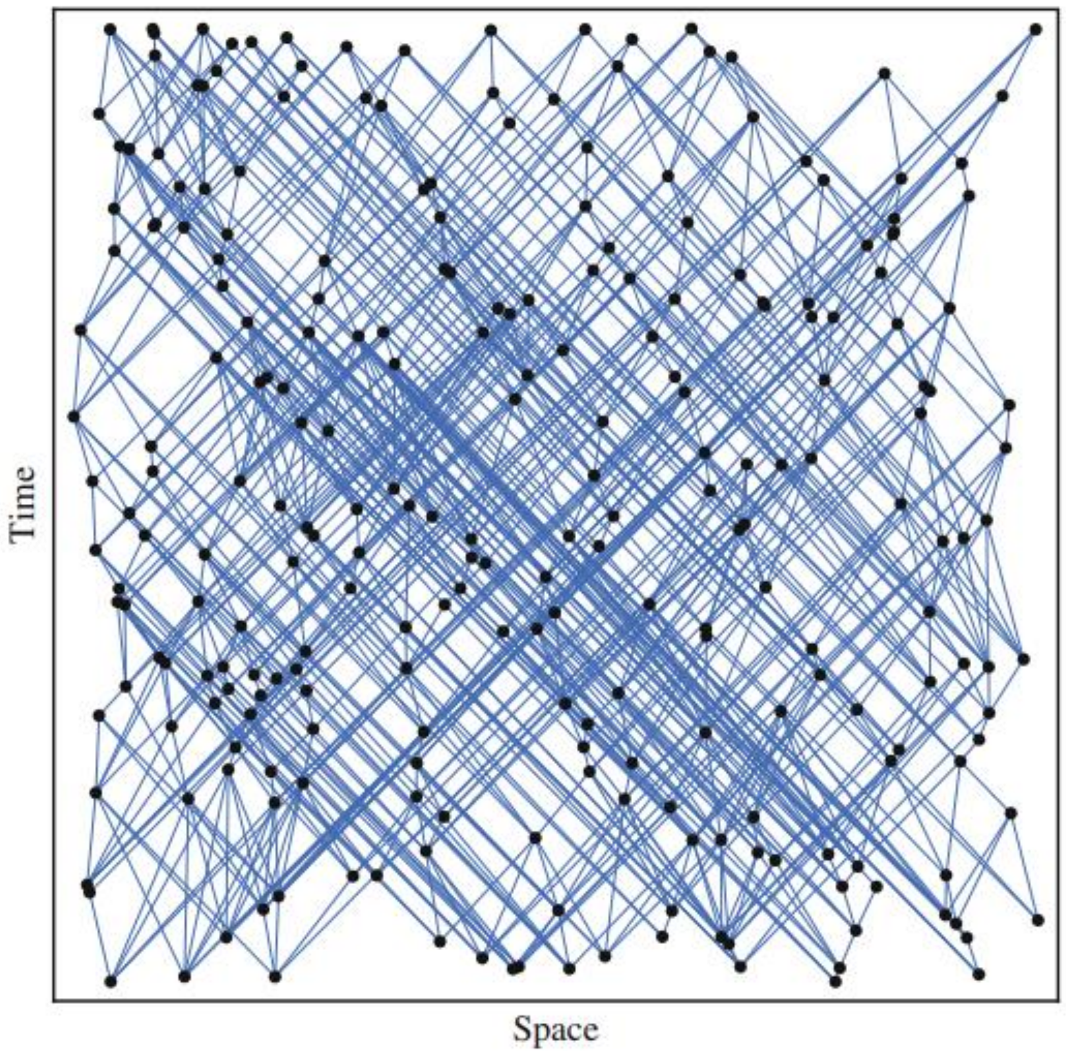

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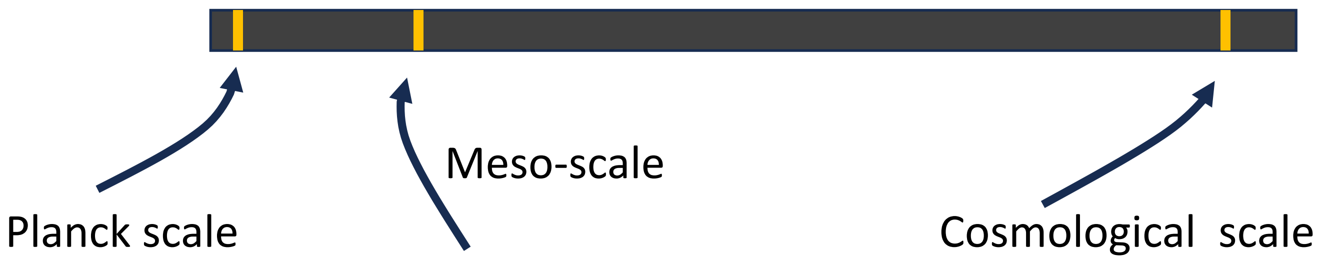
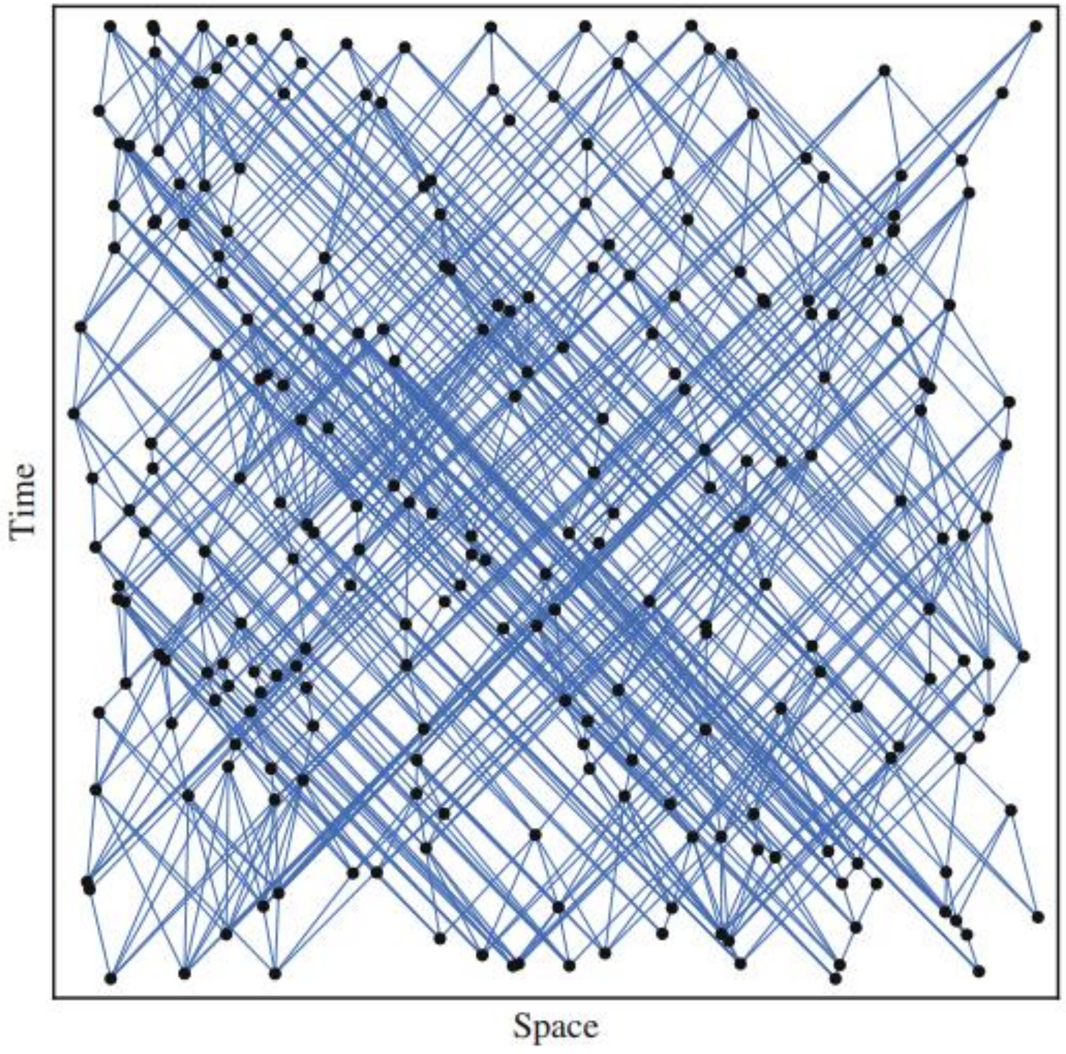
Non-locality: a challenge and an opportunity

- No notion of Cauchy hypersurface / initial value problem
- **Quasi-local** action, d'Alembertian…
- **Meso-scale** where non-locality becomes relevant but the continuum approximation is still valid





Causal Sets as a tool for new discoveries

Cosmological constant

- An order-of-magnitude prediction for the value of the cosmological constant

Sorkin, SILARG VII, 1990

- Detailed studies of the *Everpresent* Λ cosmological model

Das, Nasiri and Yazdi, 2304.03819
2307.13743

Quantum cosmology and the origins of our Universe

- A tool for asking meaningful questions about what happened *before* the Big Bang singularity

Dowker and SZ, 2212.01149
Bento, Dowker and SZ, 2109.10749

- Realising branching universe scenarios, e.g. Smolin's CNS

Dowker and SZ, 1703.07556

Swerves

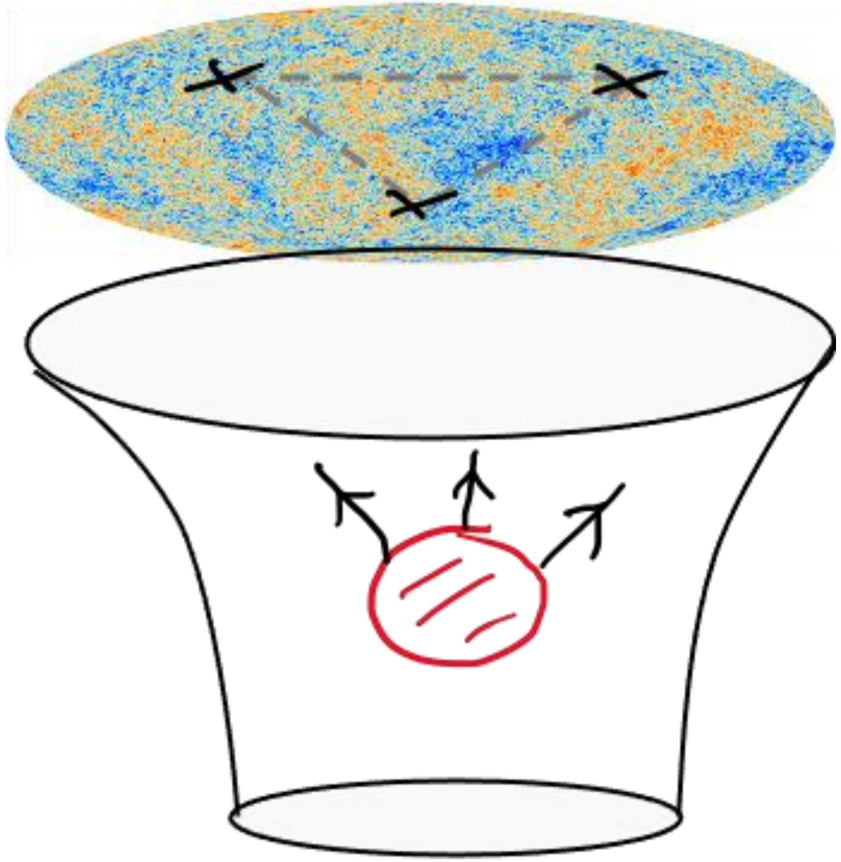
- Deviations from geodesic motion

Dowker, Philpott and Sorkin, 0810.5591

Cosmological collider physics

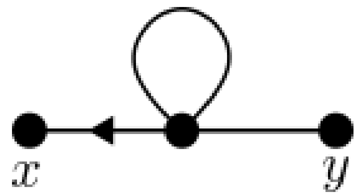
The cosmological collider

- Key idea: the high-energy, high-curvature environment in the Early Universe is an ideal laboratory in which to search for new fundamental physics – including QG!
- Goal: to translate the signals that could be measured by upcoming sky surveys (e.g. primordial non-Gaussianity) into concrete properties of the fundamental physics which produced them in the Early Universe.
- Challenge: to import QFT techniques from flat to cosmological (continuum) spacetimes.



The discrete cosmological collider

- Can we compute cosmological correlators on a causal set background? Yes! We can also define an S-matrix. Albertini, Dowker, Nasiri and SZ, 2402.08555



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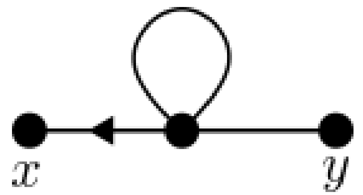
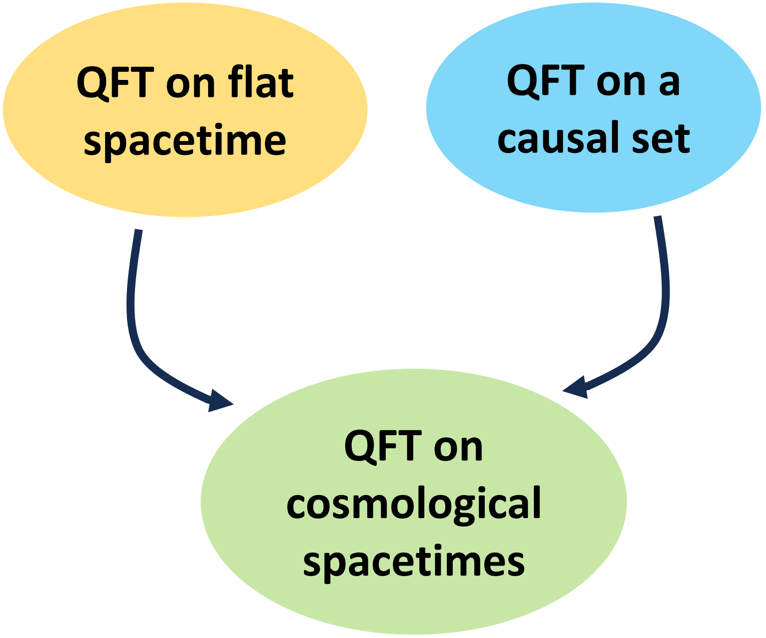


- A new tool for cosmological collider physics, can produce predictions to compare against cosmological data to test for spacetime discreteness.
- Can also help with developing techniques for continuum cosmological spacetimes, for instance defining a unique vacuum state.

Afshordi et al, arXiv:1205.1296

- May offer a novel regularization of the continuum, since there are no UV divergences on a causal set.



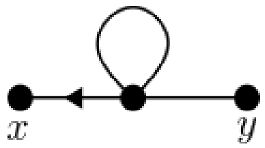
The diagrammatic expansion

Albertini, Dowker, Nasiri and SZ, 2402.08555

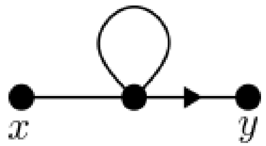
Example: 2pt function $x \succ y$ in ϕ^4,

$$\langle\Omega|\phi^H(x)\phi^H(y)|\Omega\rangle =$$

$$\Delta^F{}_{xy}$$

$+$ 

$$\frac{\lambda}{2}\sum_z \Delta^R{}_{xz}\Delta^F{}_{zz}\Delta^F{}_{yz}$$

$+$ 

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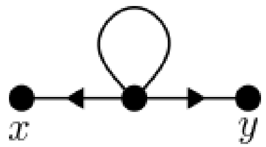
$+$ 

$$\frac{\lambda}{2}(-i)\sum_z \Delta^R{}_{xz}\Delta^F{}_{zz}\Delta^R{}_{yz}$$

$+ \ldots$

- each internal vertex is connected to at least one external vertex by a directed path,
- no edges directed outwards from an external vertex,
- no closed directed cycles

cf. continuum rules by Dickinson et al., arXiv:1312.3871

Summary

- Causal Set Theory is an approach to quantum gravity in which spacetime is fundamentally discrete.

- It's a tool for new discoveries of non-local and Lorentz-invariant physics.

- New developments are enabling us to make concrete predictions, including for cosmological collider physics.