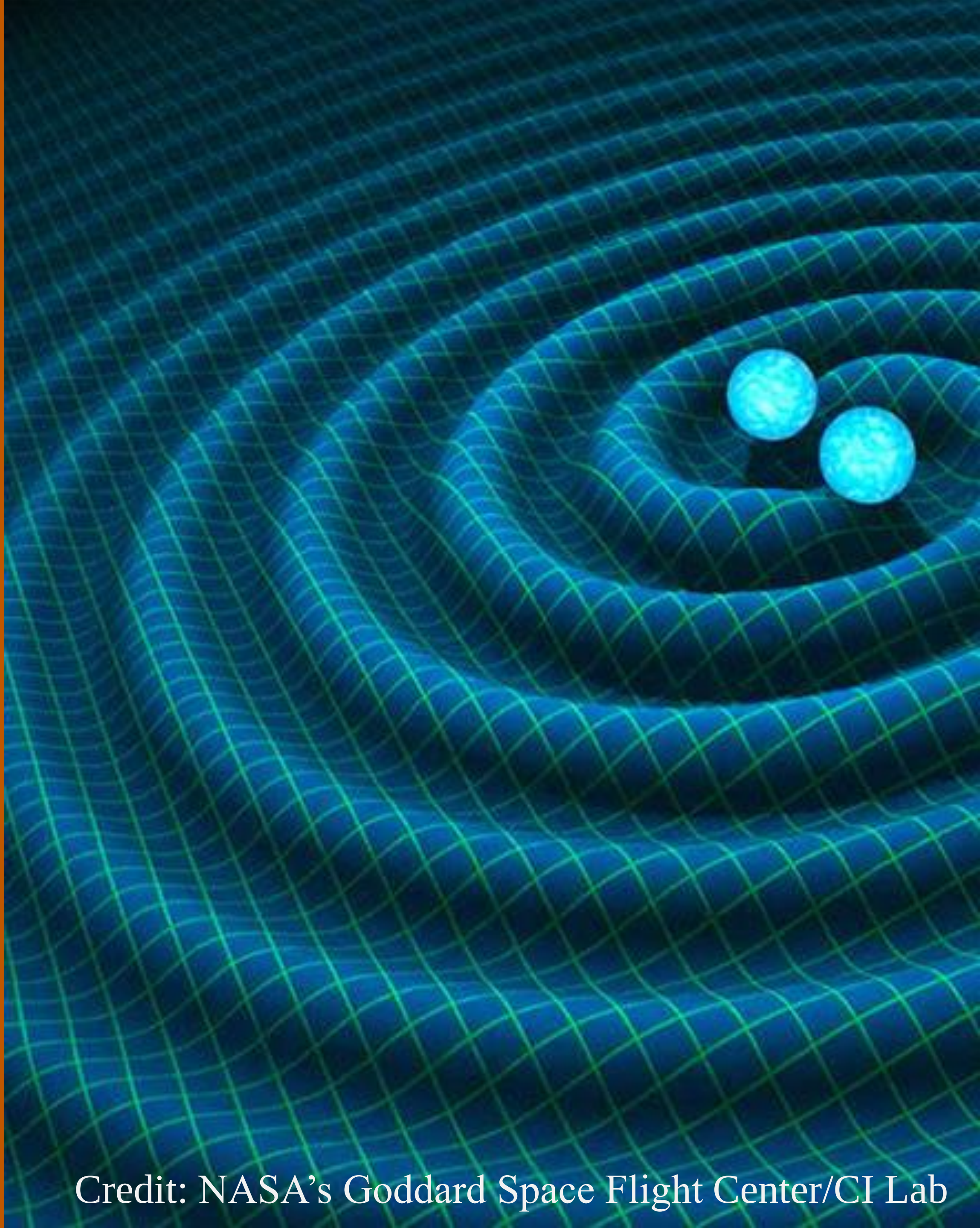
The University of Texas at Austin
College of Natural Sciences

Mitigating the inclination angle bias for standard sirens

Alberto Salvarese, Hsin-Yu Chen



Credit: NASA's Goddard Space Flight Center/CI Lab

Hubble constant with Standard Sirens

Hubble constant: $v = H_0 D$

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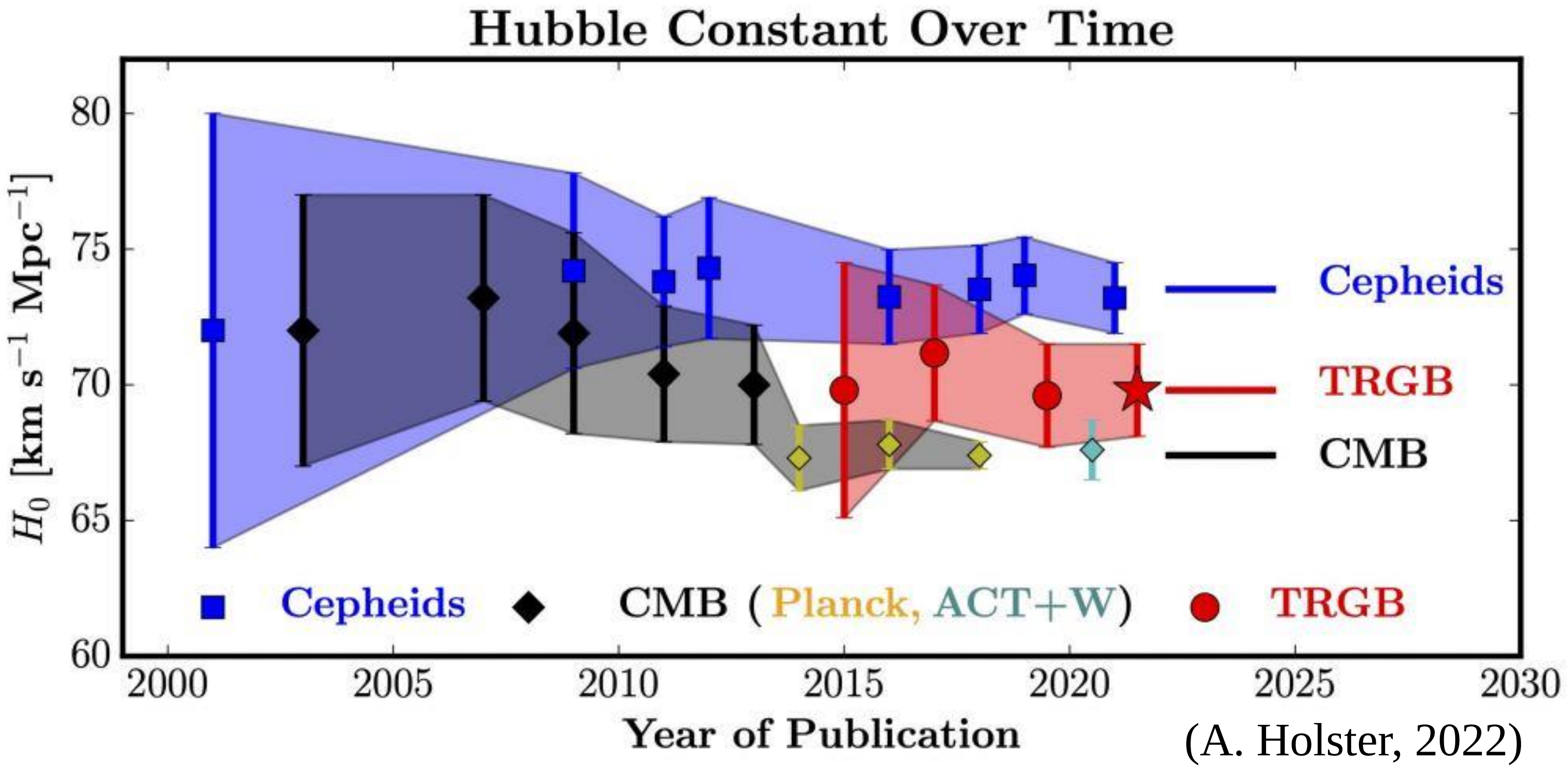

(A. Holster, 2022)

Hubble constant with Standard Sirens

$$H_0 = \frac{c(1+z)}{D_L} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_k (1+z')^2 + \Omega_\Lambda}}$$

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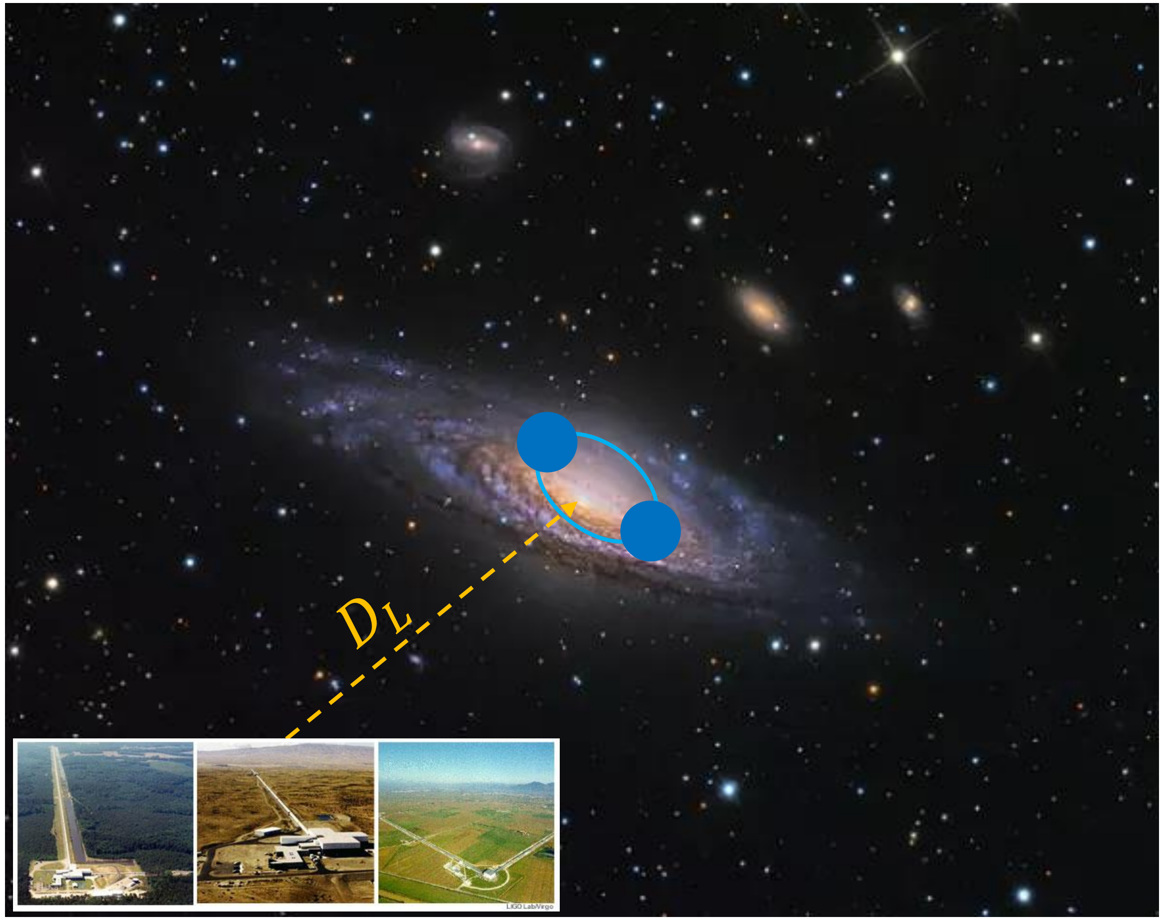
- Luminosity distance D_L from compact binaries gravitational wave signal



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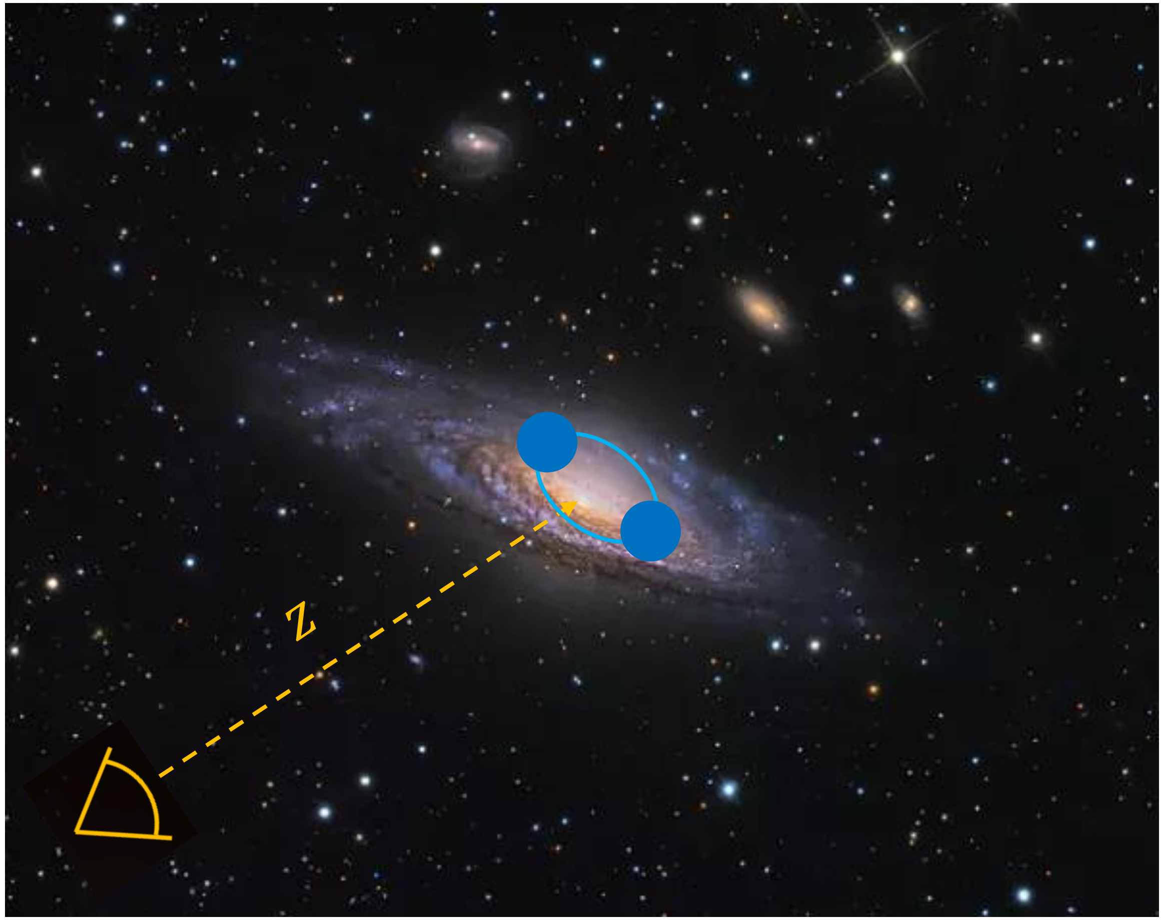
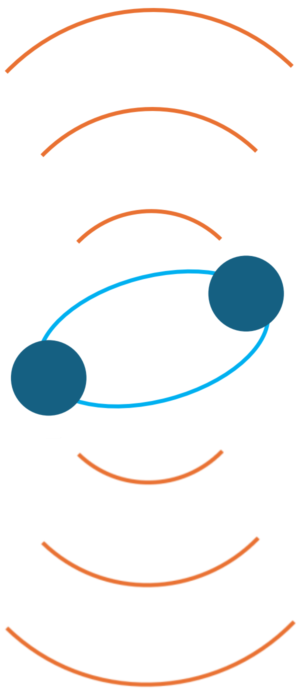
- Redshift z: binaries with electromagnetic counterpart



Inclination angle – distance dependence



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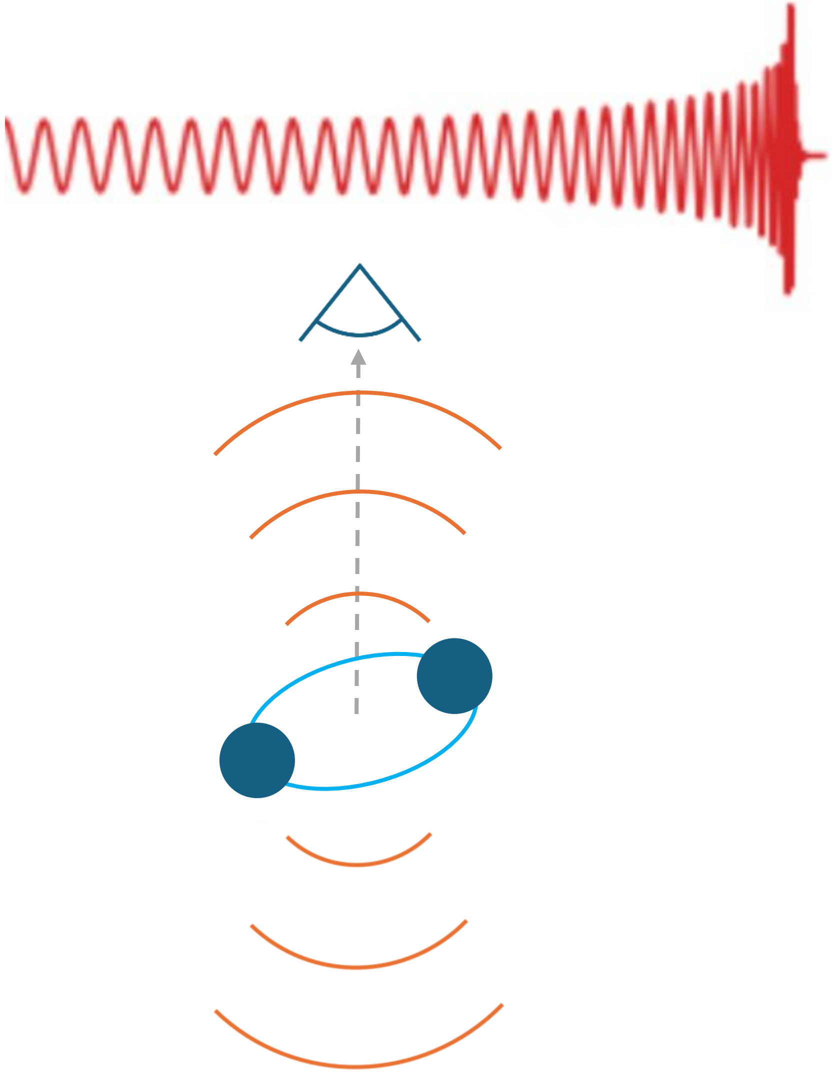

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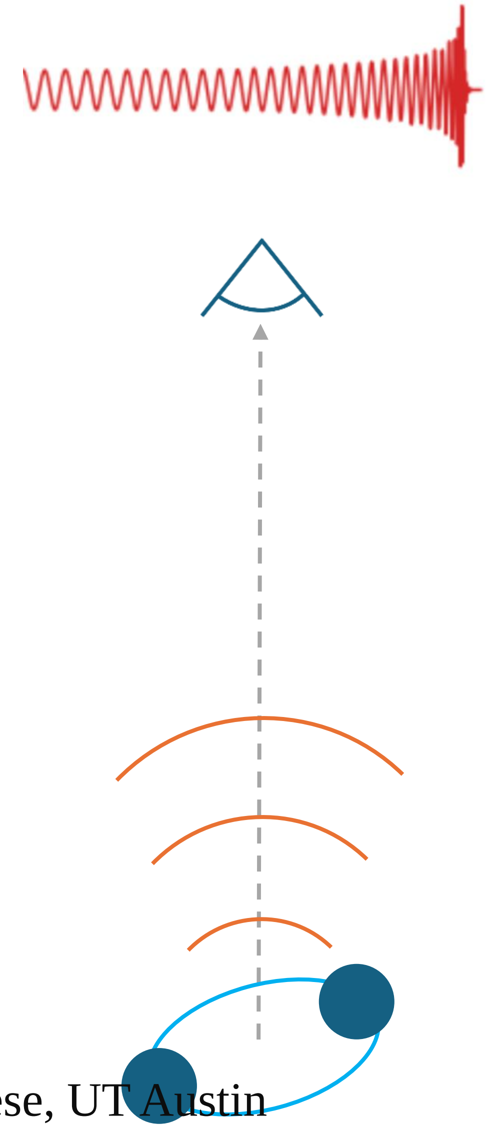

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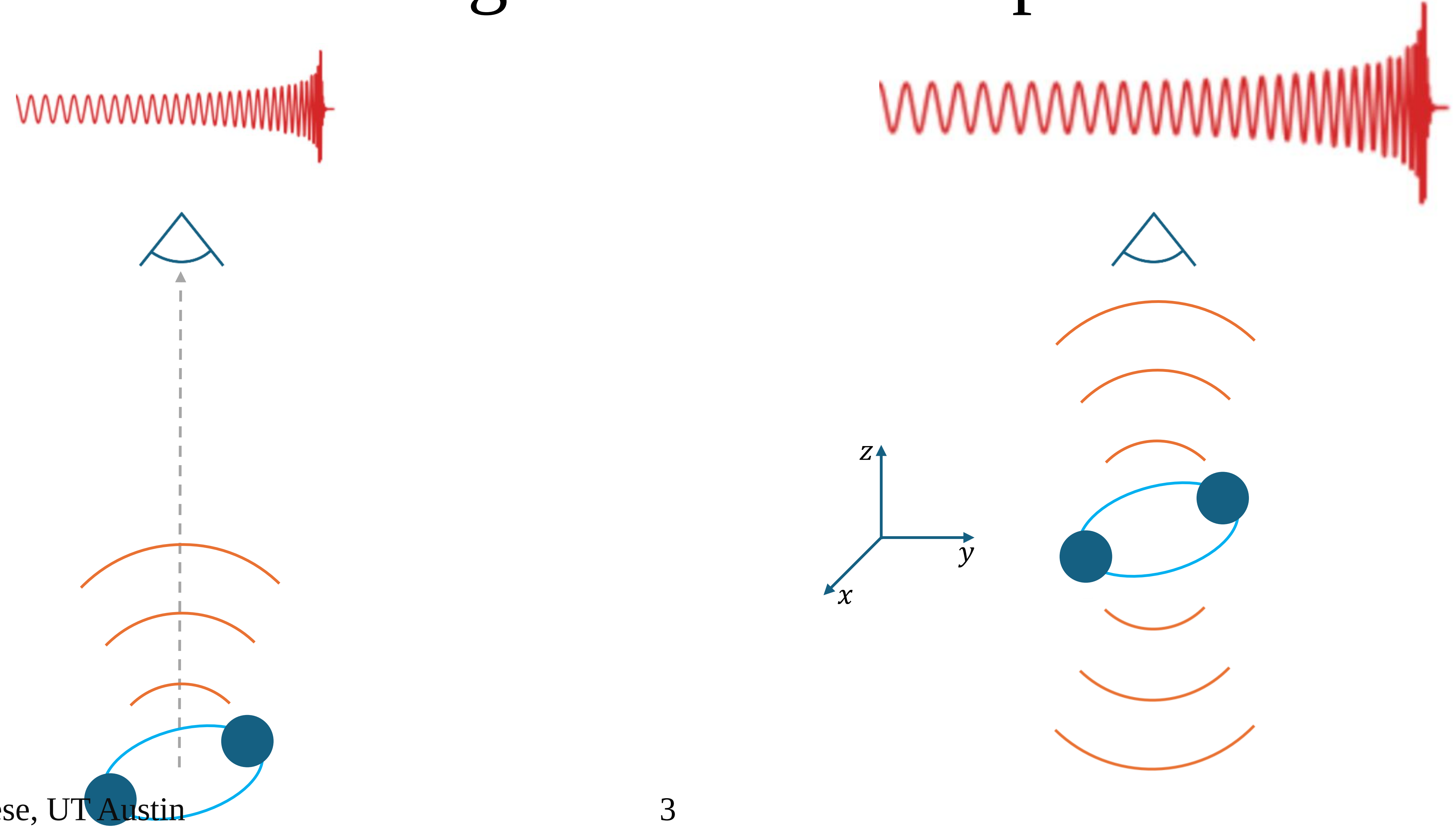

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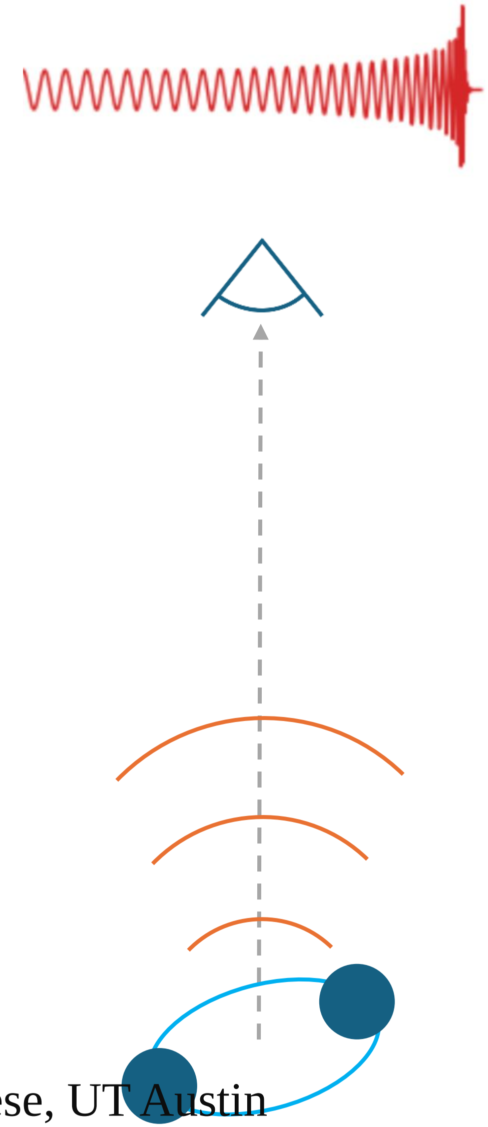

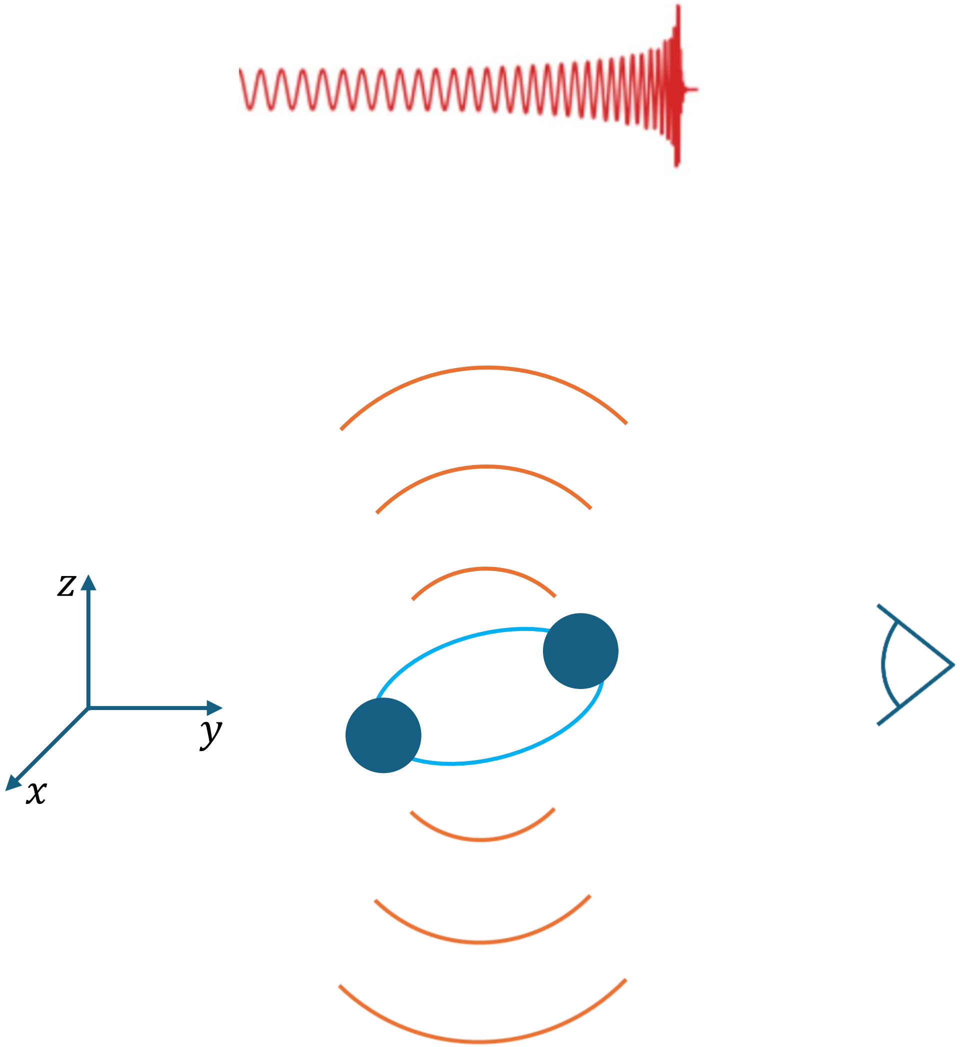

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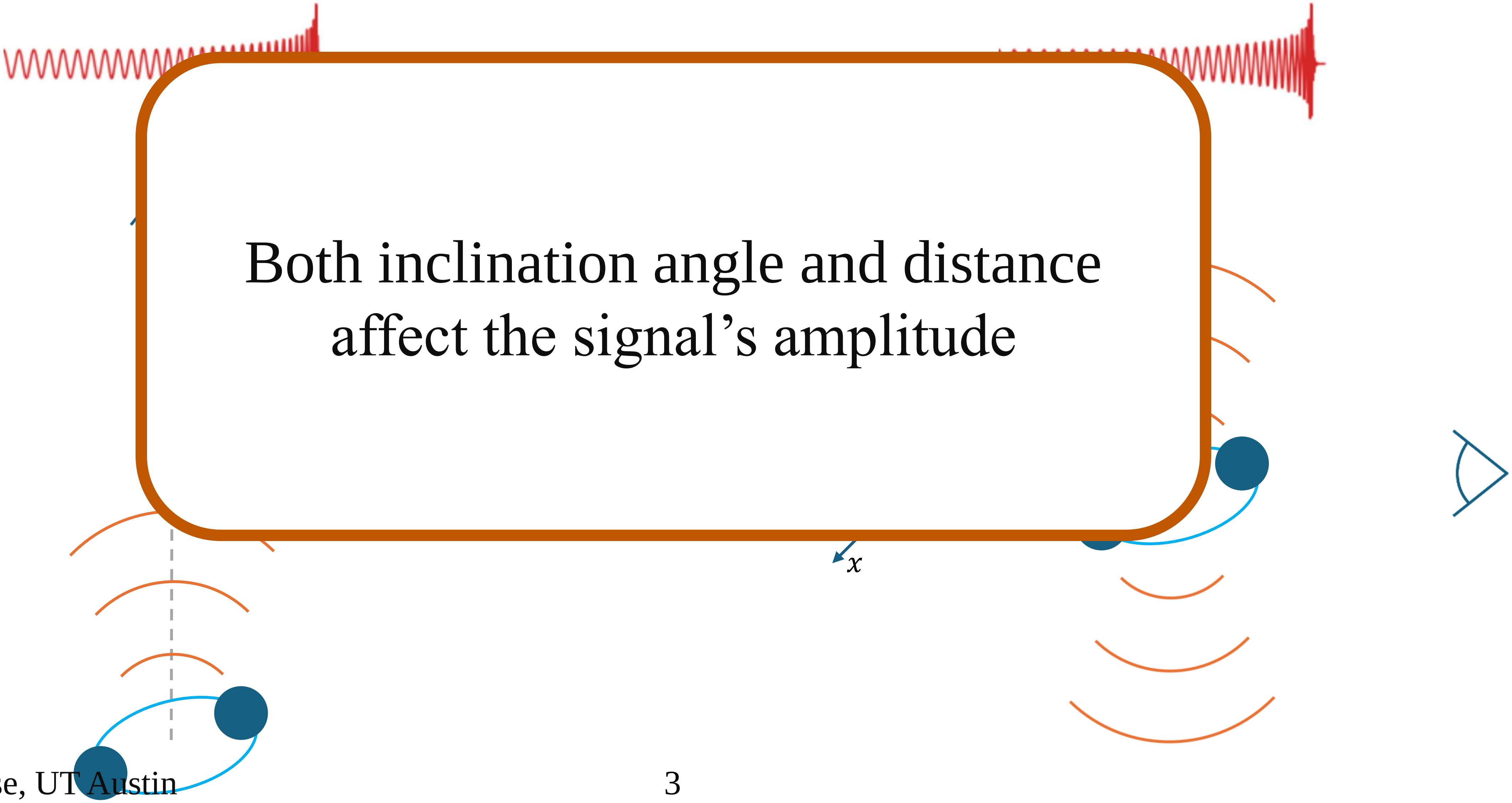
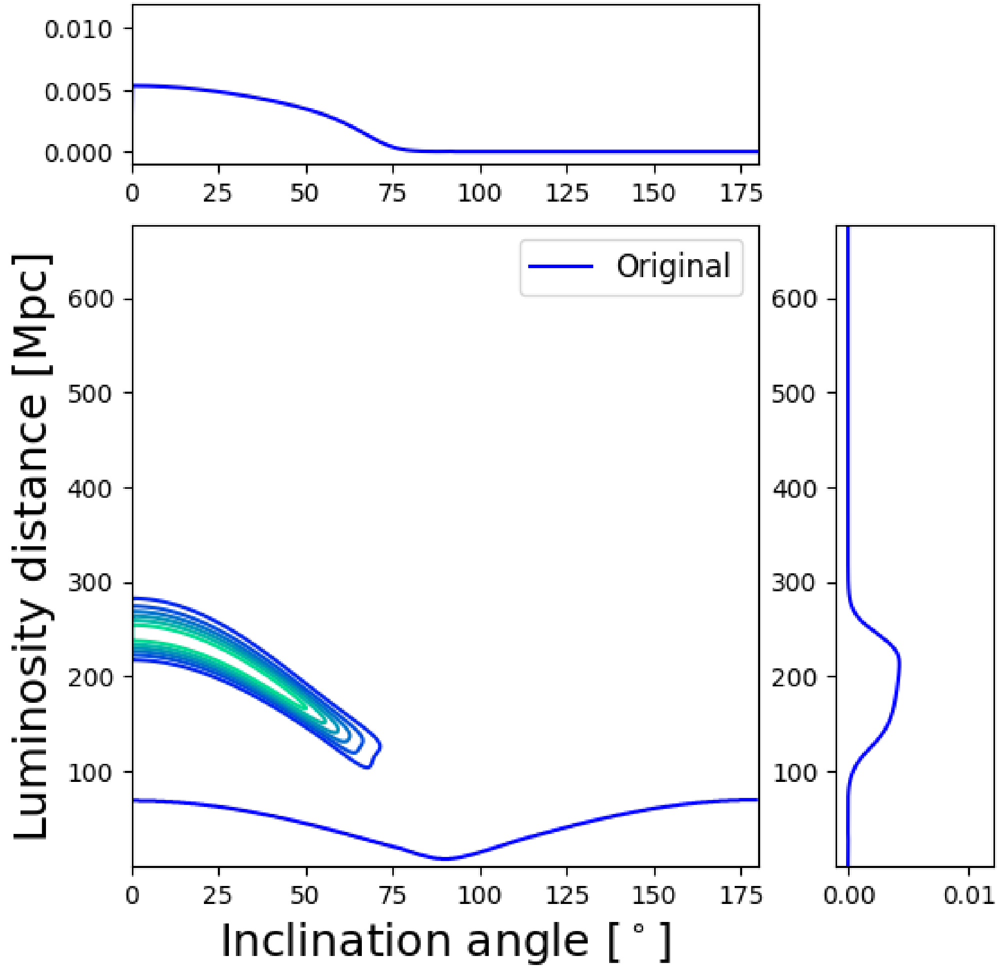


Both inclination angle and distance affect the signal's amplitude

Inclination angle – distance dependence



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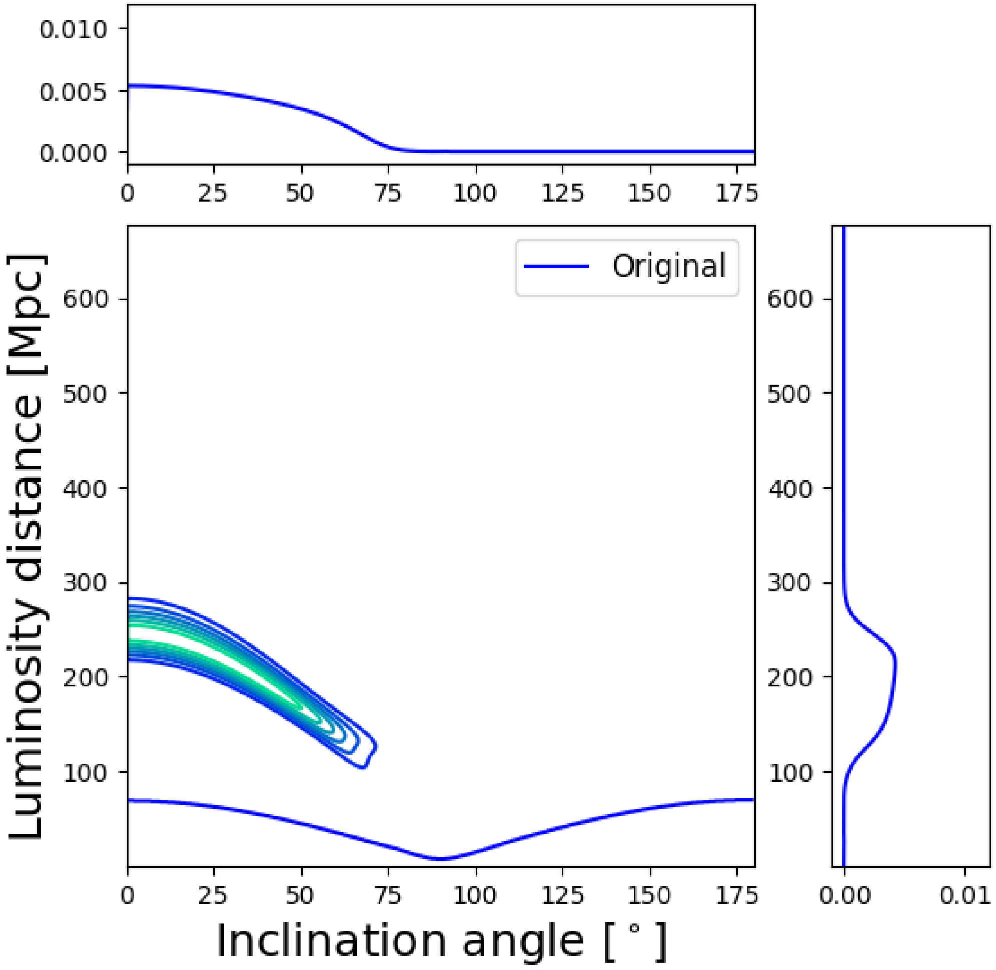

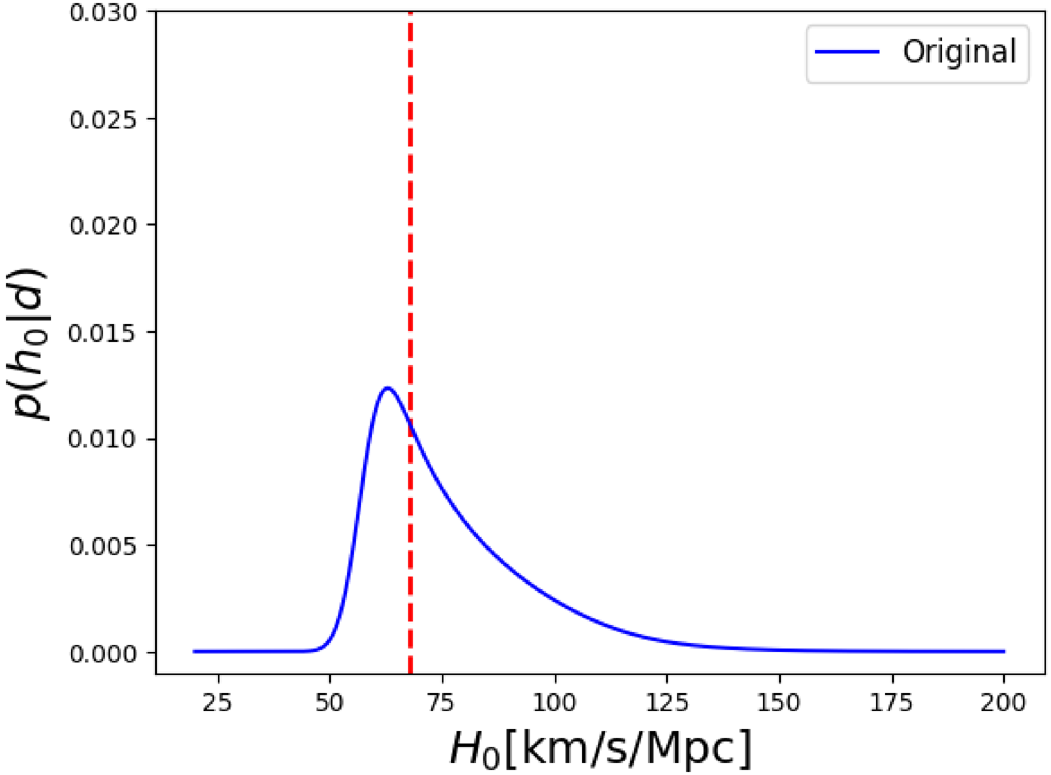

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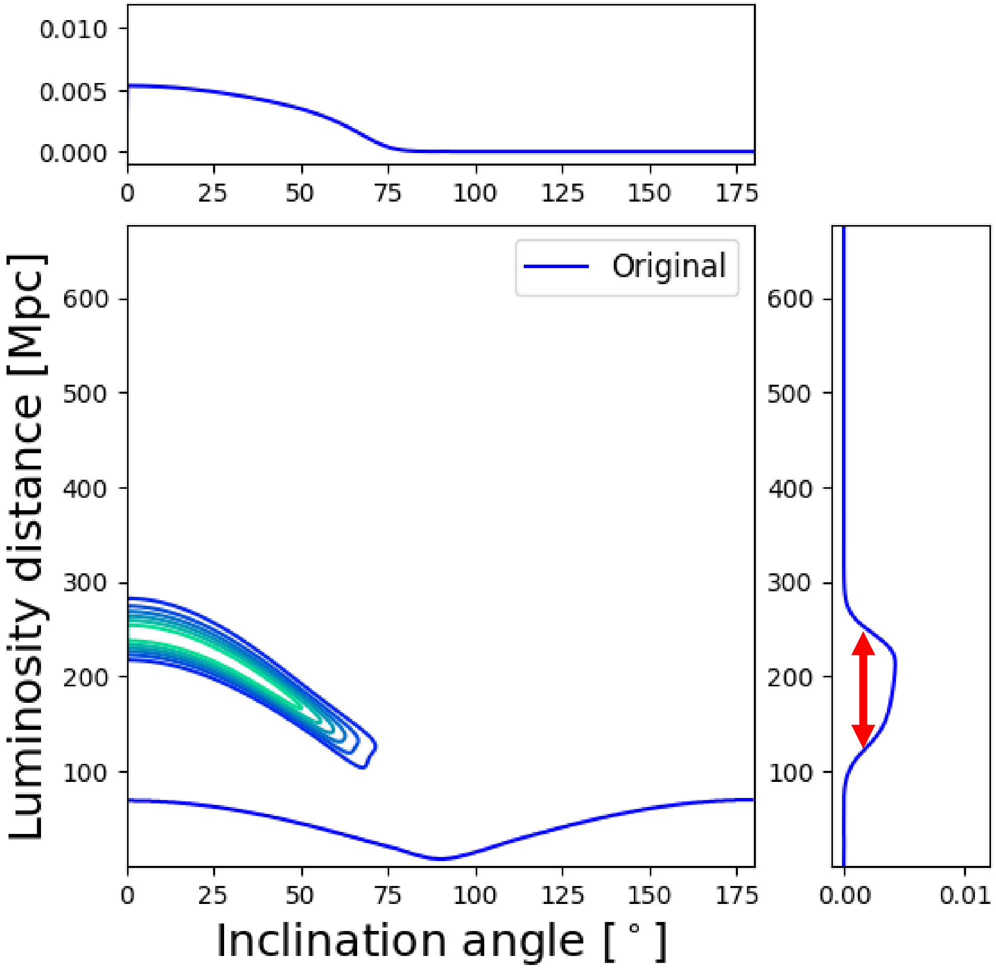
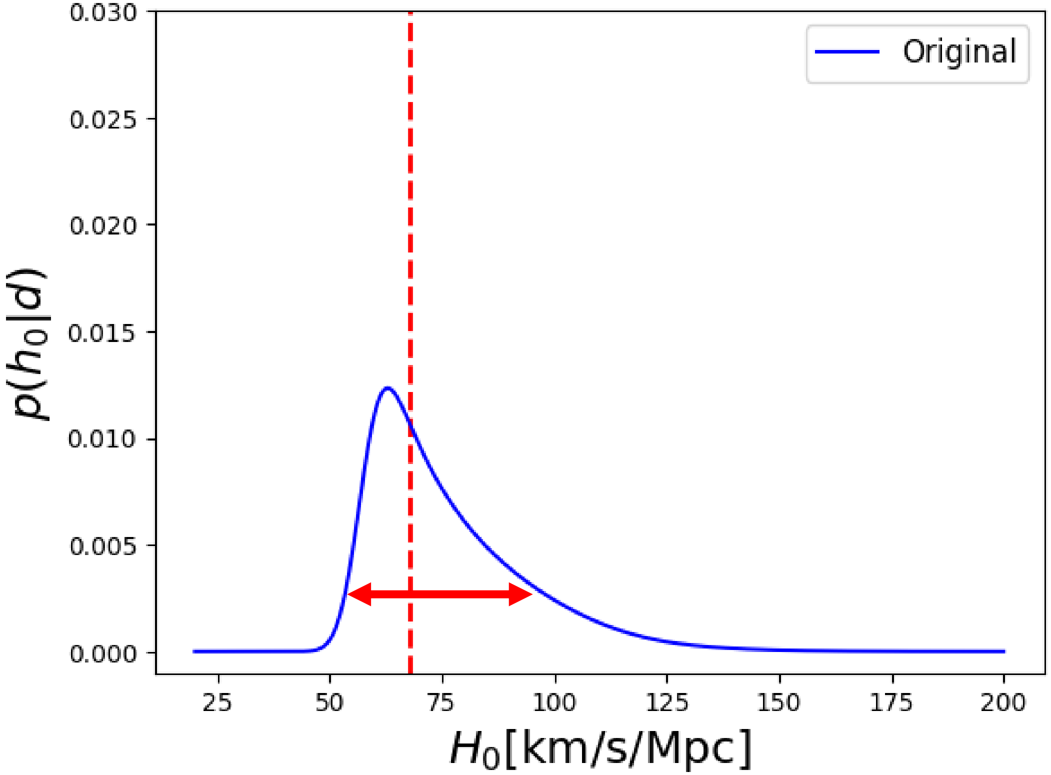
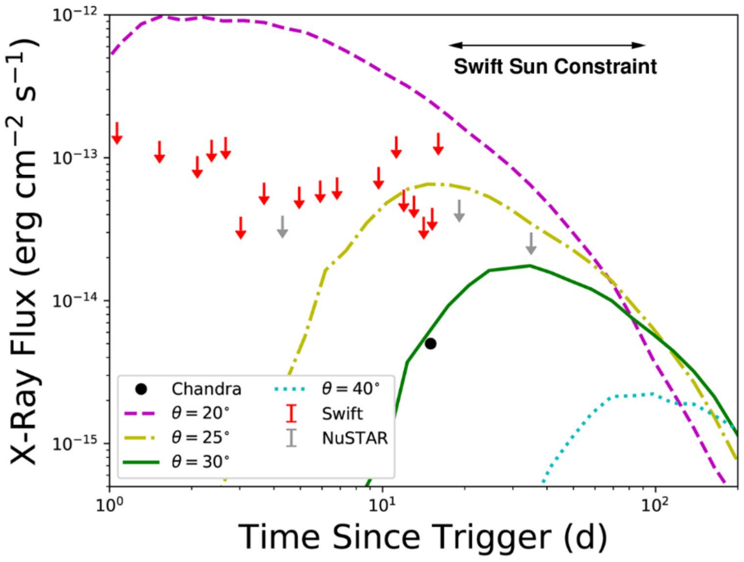




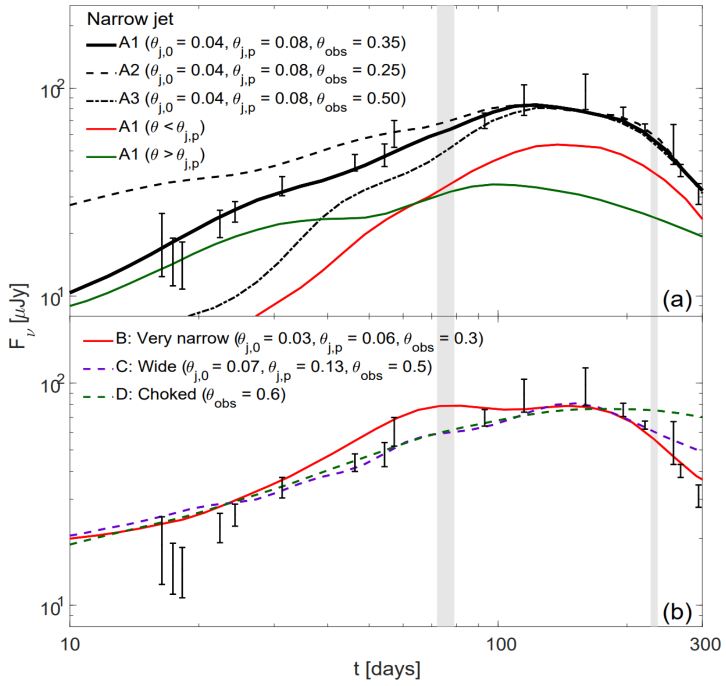
(H.Y. Chen, et al., 2018)

Electromagnetic constraints

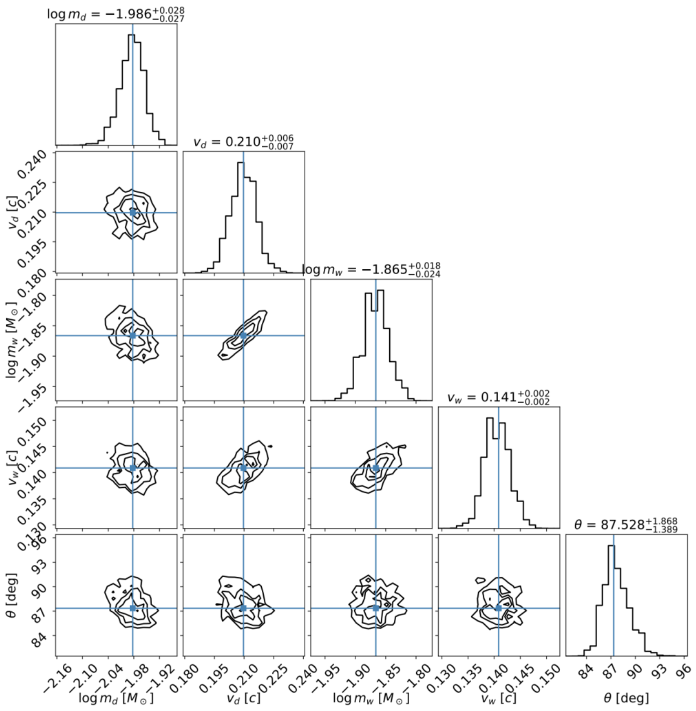
Constraints from: GRB detection (H.Y. Chen, et al., 2019), Kilonova light-curves (Y. Peng, et al., 2024)



(P. A. Evans, et al., 2017)



(K. P. Mooley, et al., 2018)



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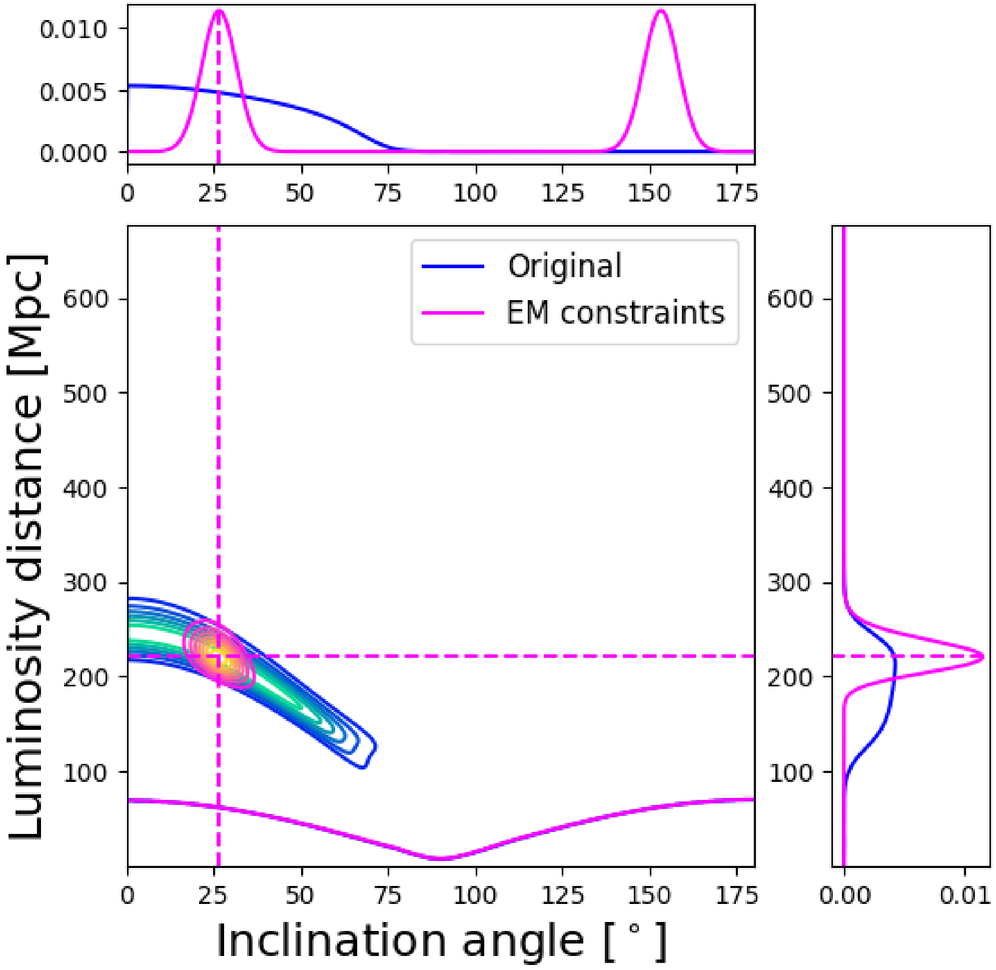

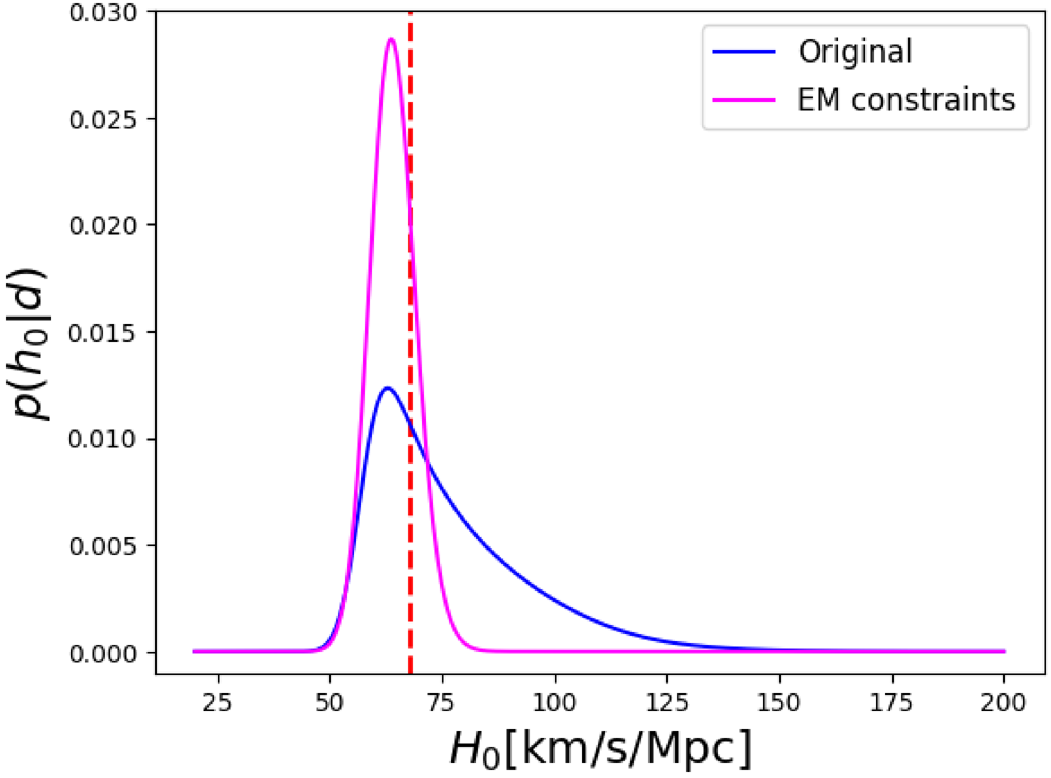

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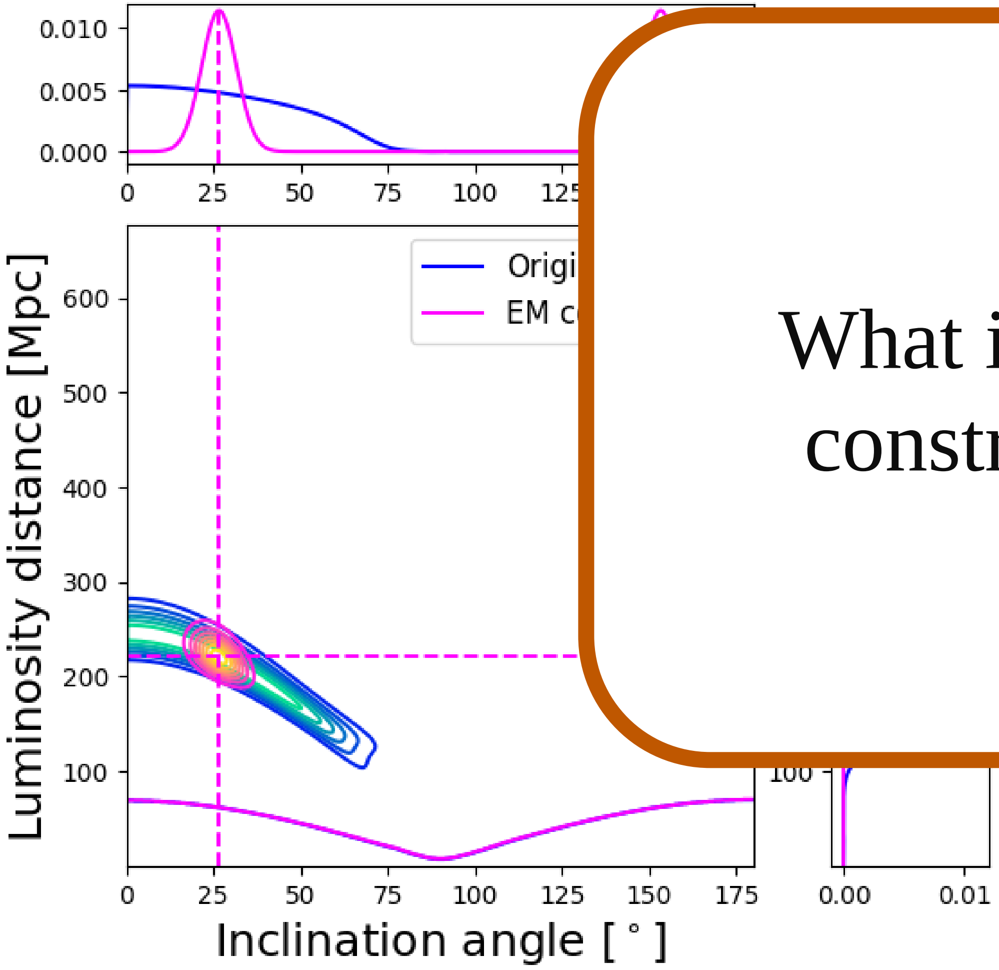
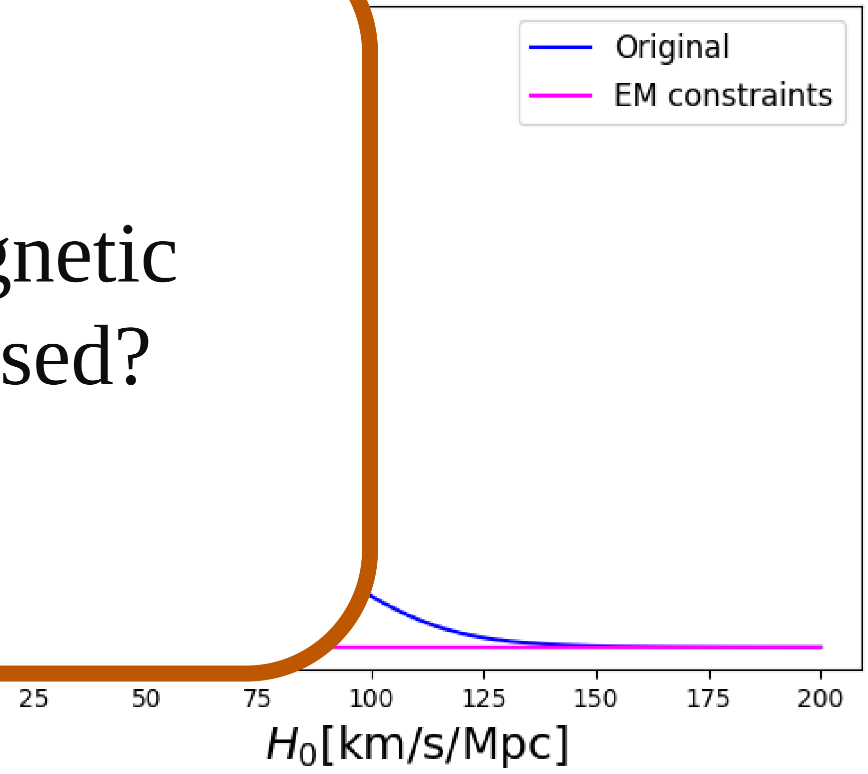
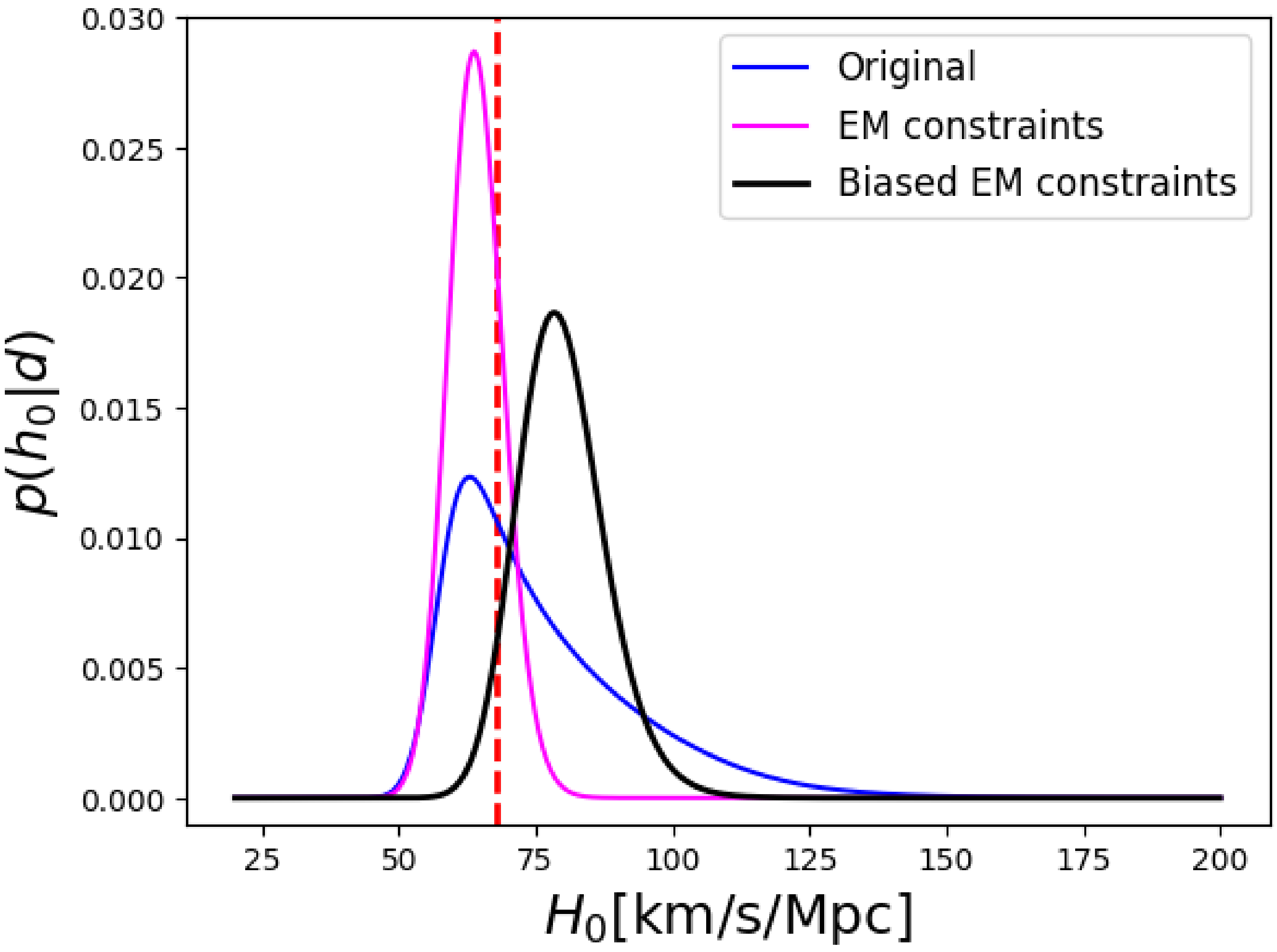




What if electromagnetic constraints are biased?

Electromagnetic constraints



Proposed method

Goal: to develope a Bayesian pipeline that mitigates incliantion angle's systematics effects

Strategy: consider a joint posterior for h_0 and the systematics, and marginalize over the latter

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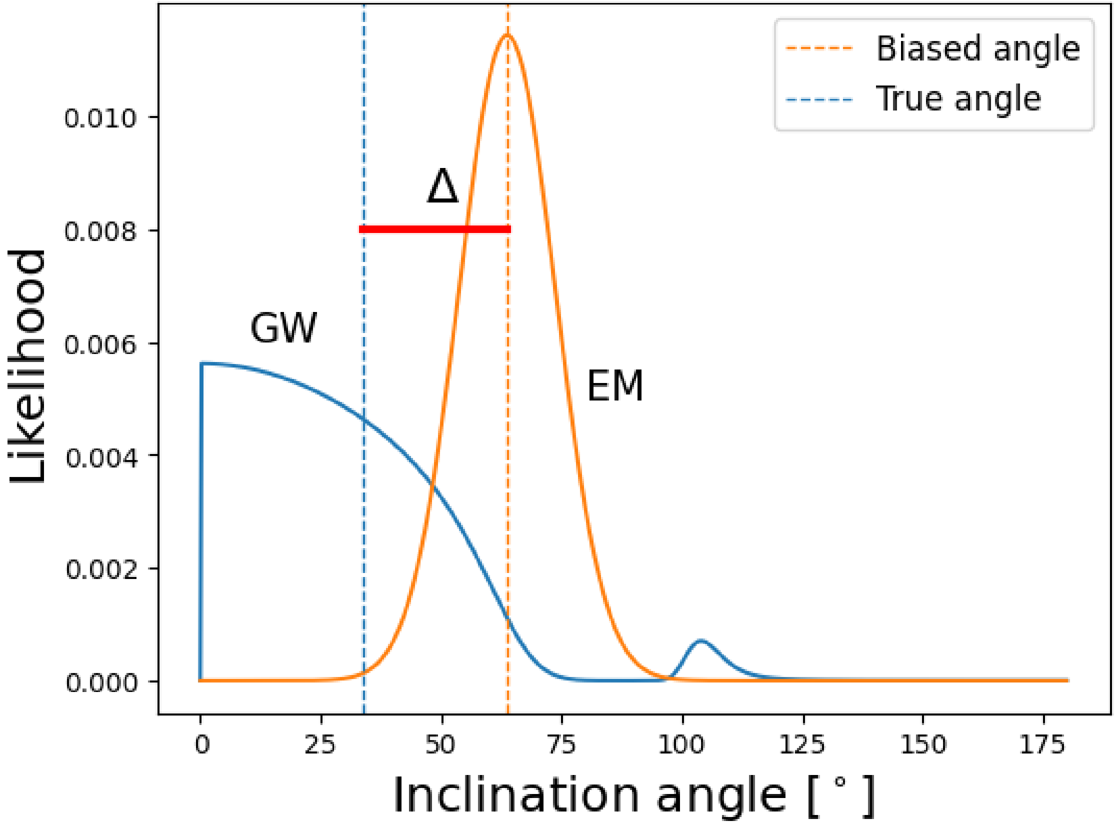

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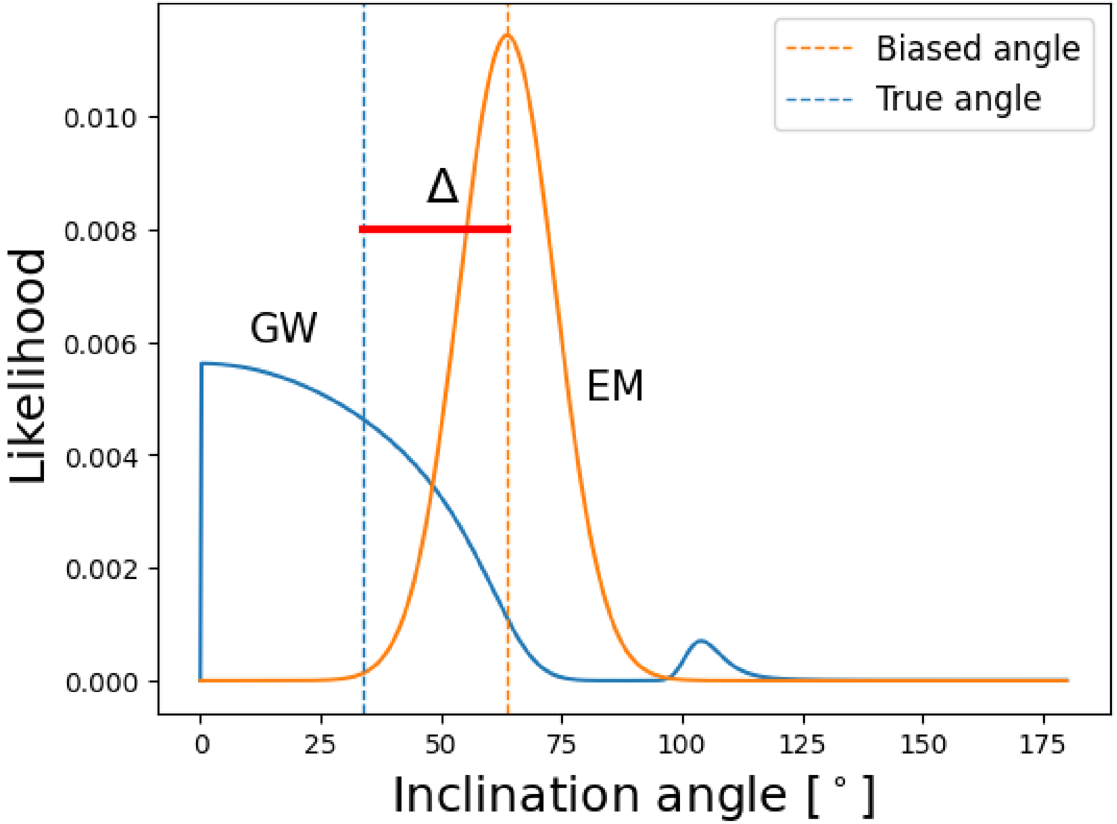
- Consider both gravitational and electromagnetic signals: systematic is captured

- Use multiple events: same systematic is repeated



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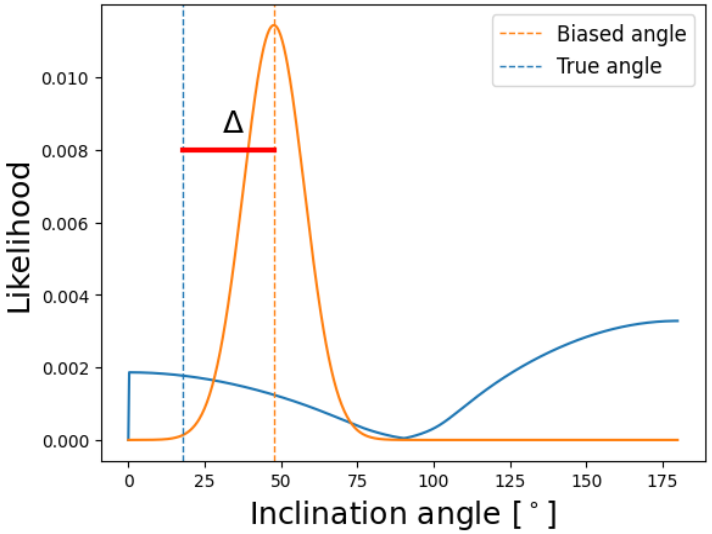

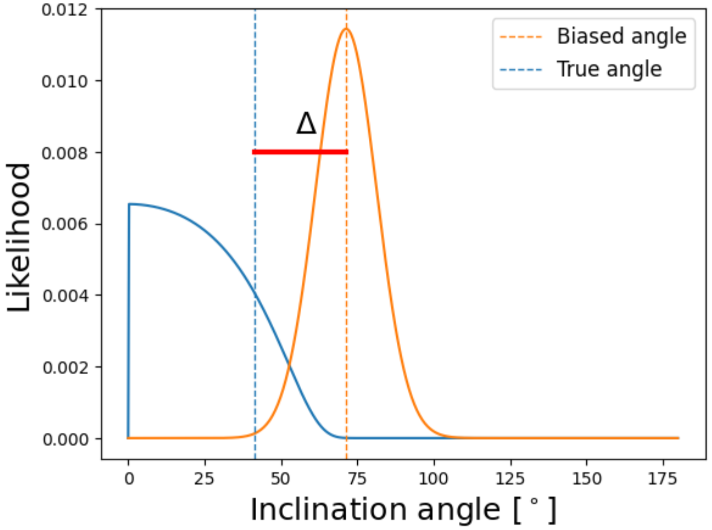

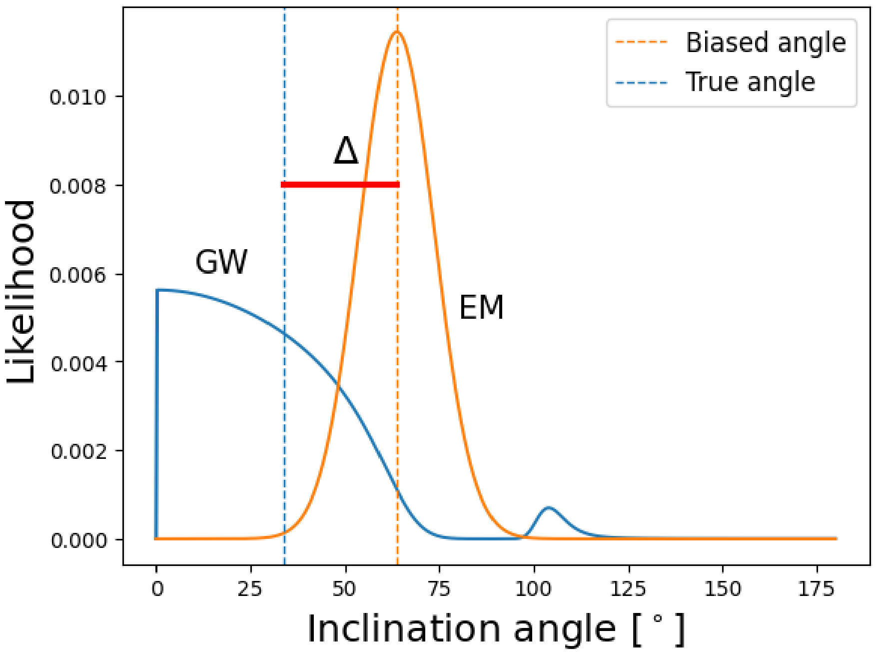

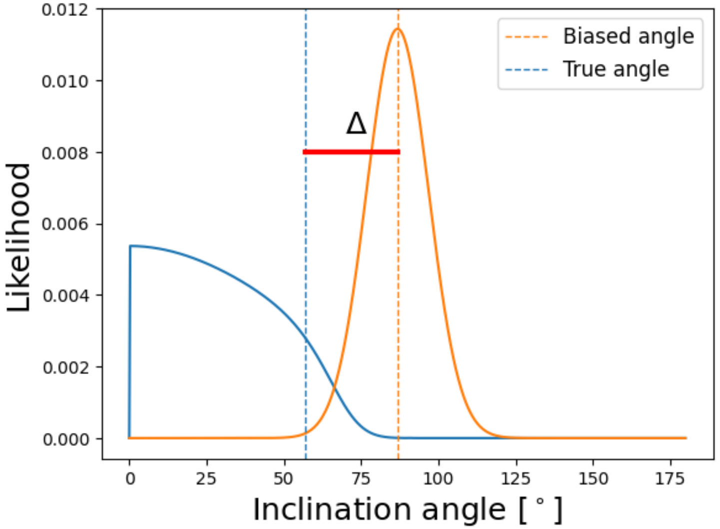

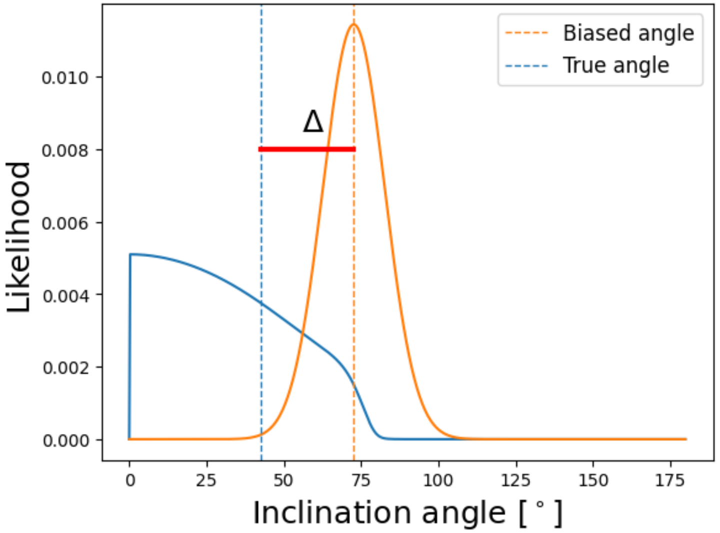

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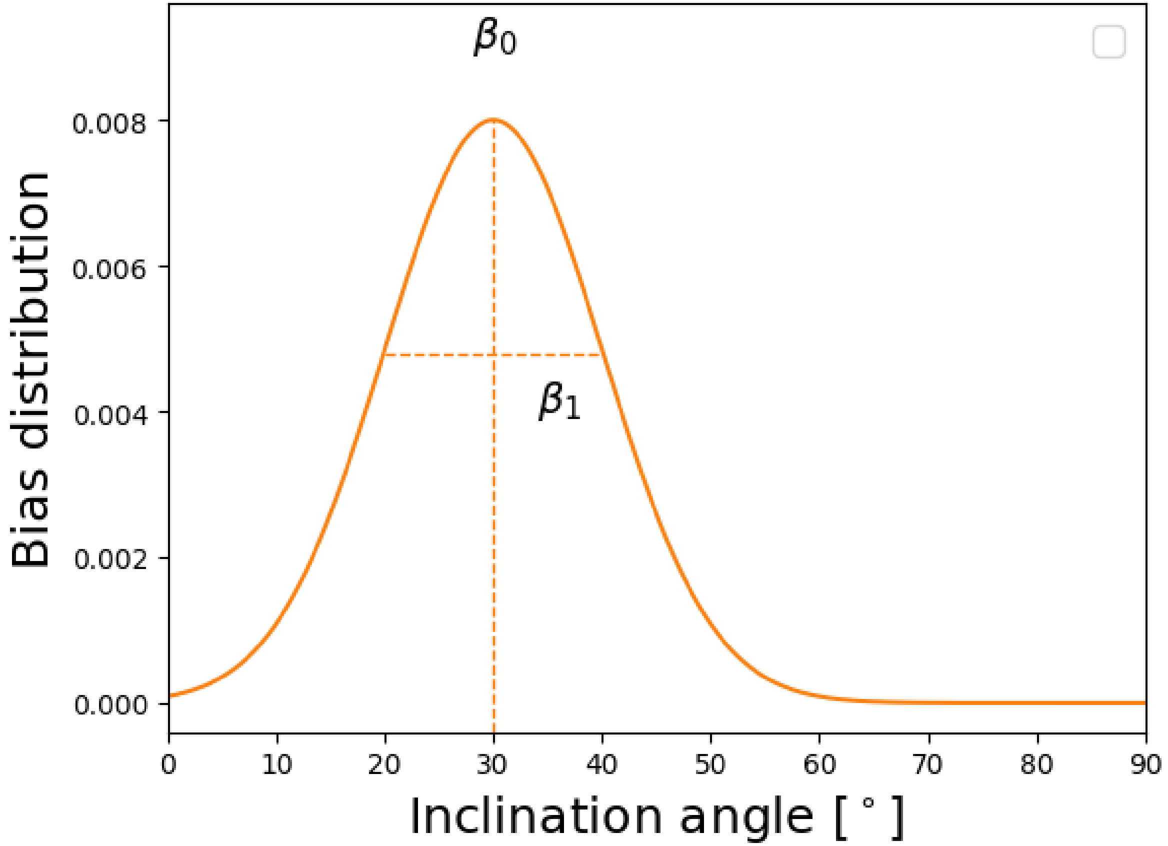
More complex model for the systematic:



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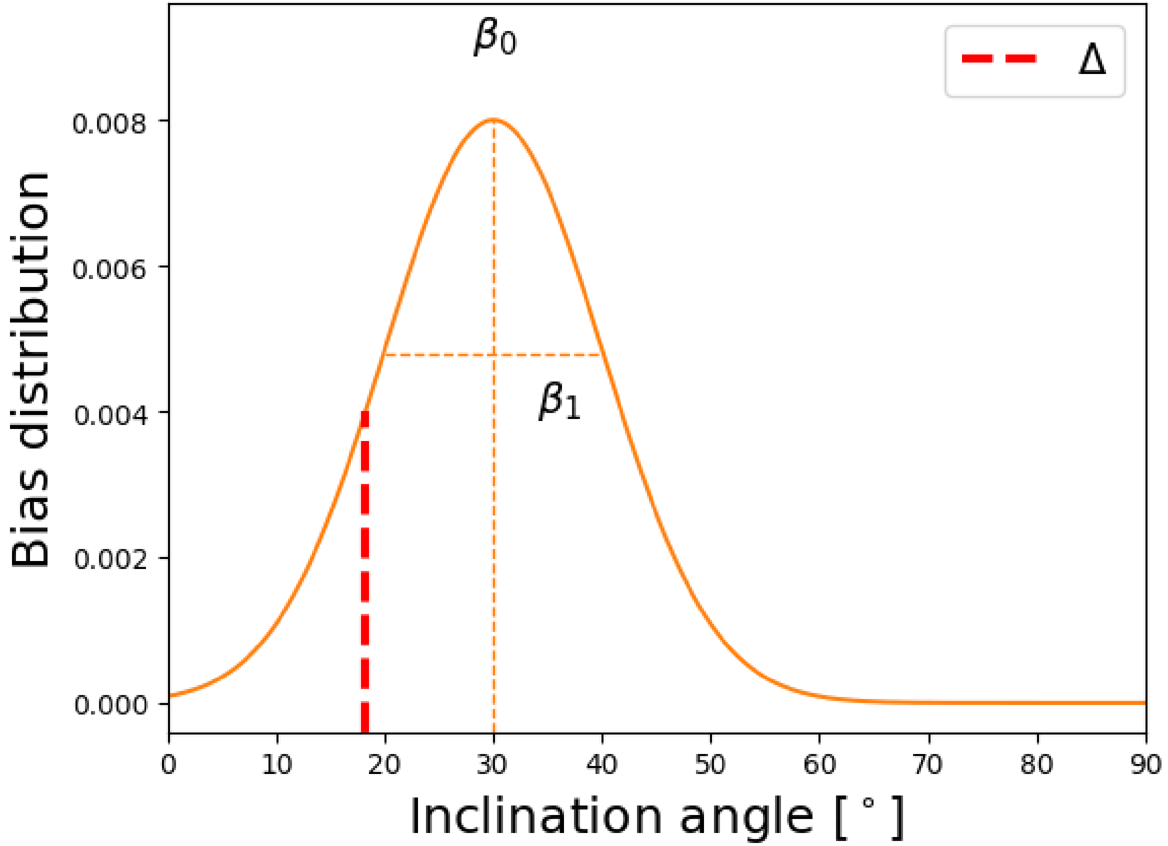
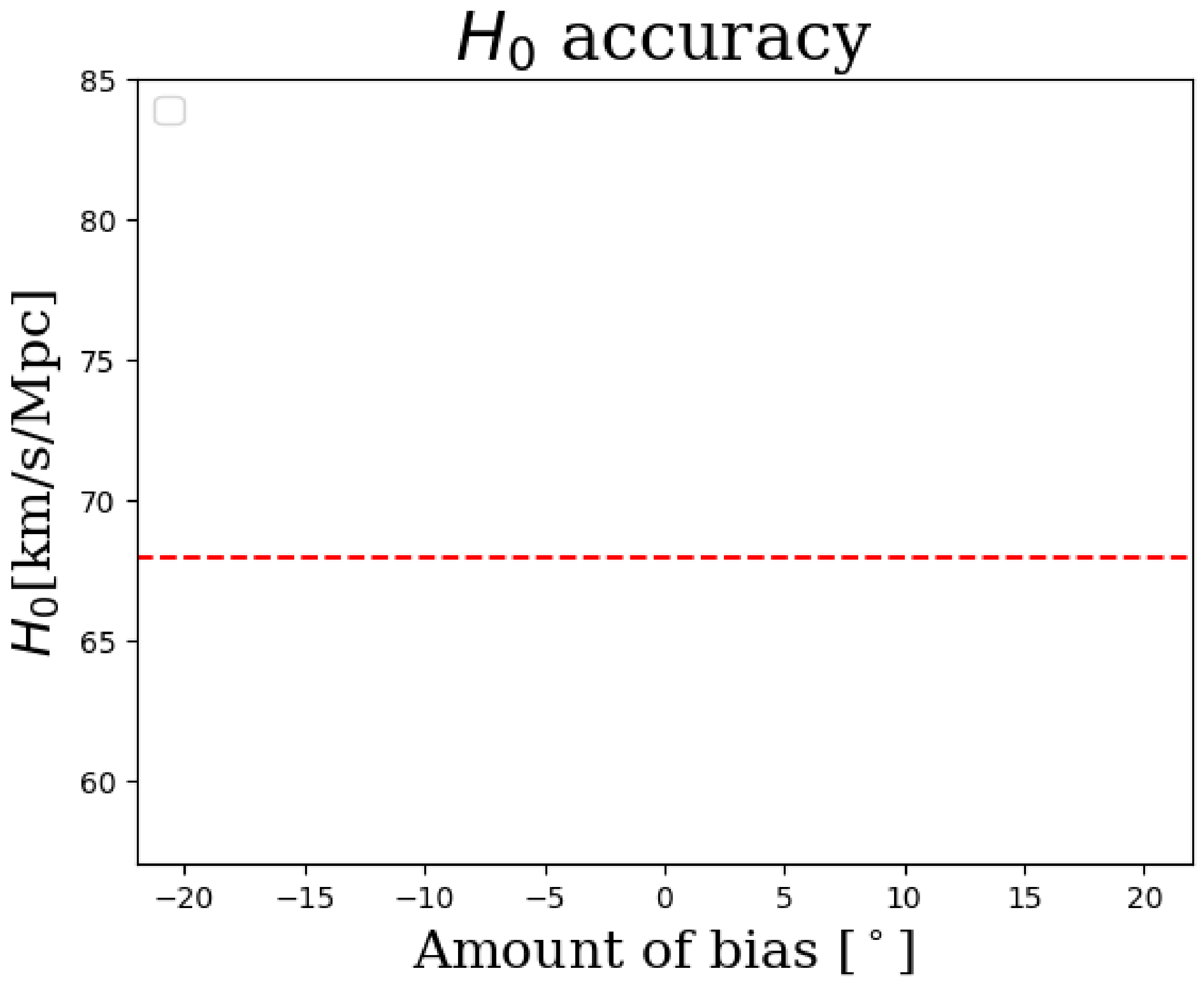


Combination of 20 events



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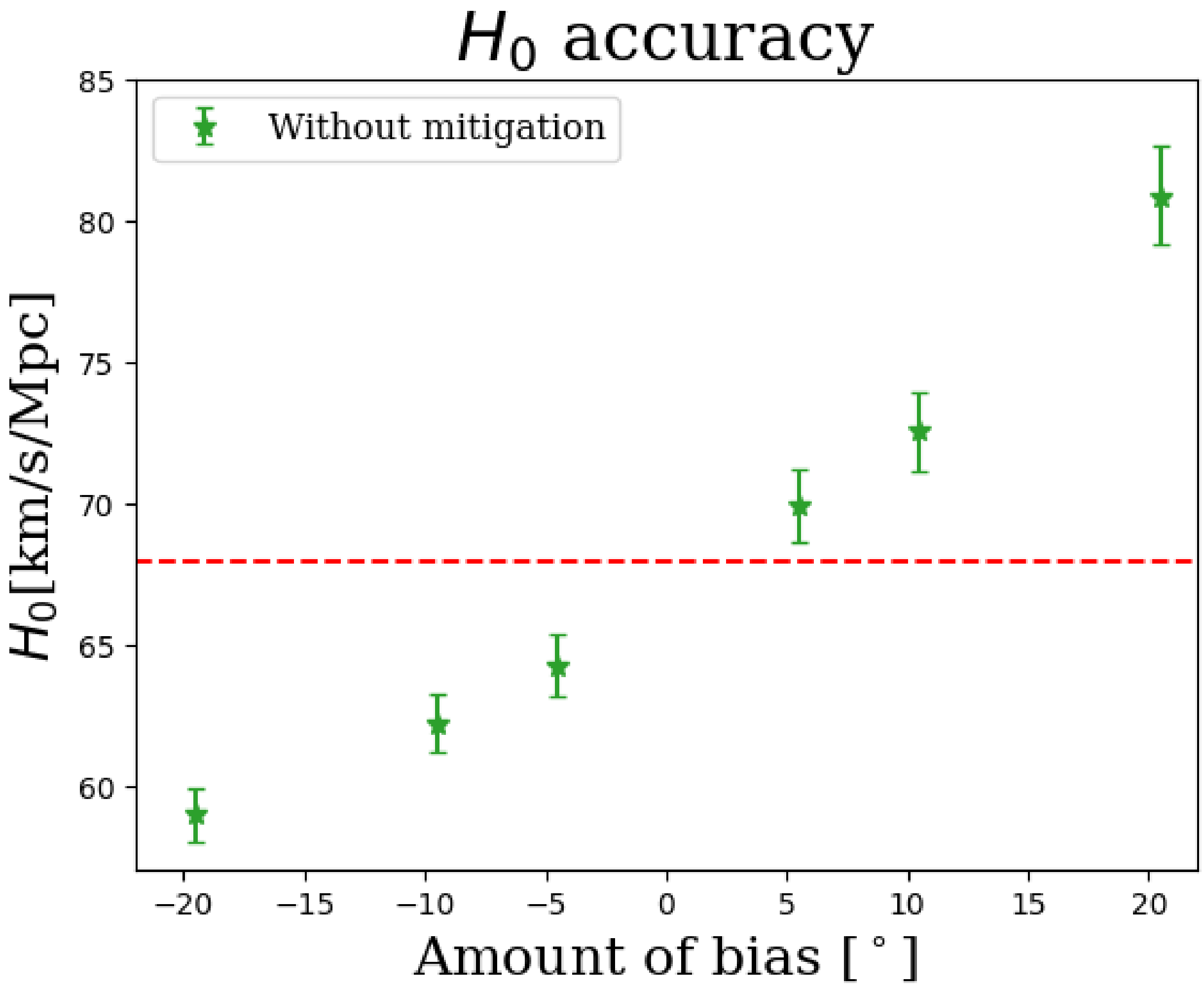
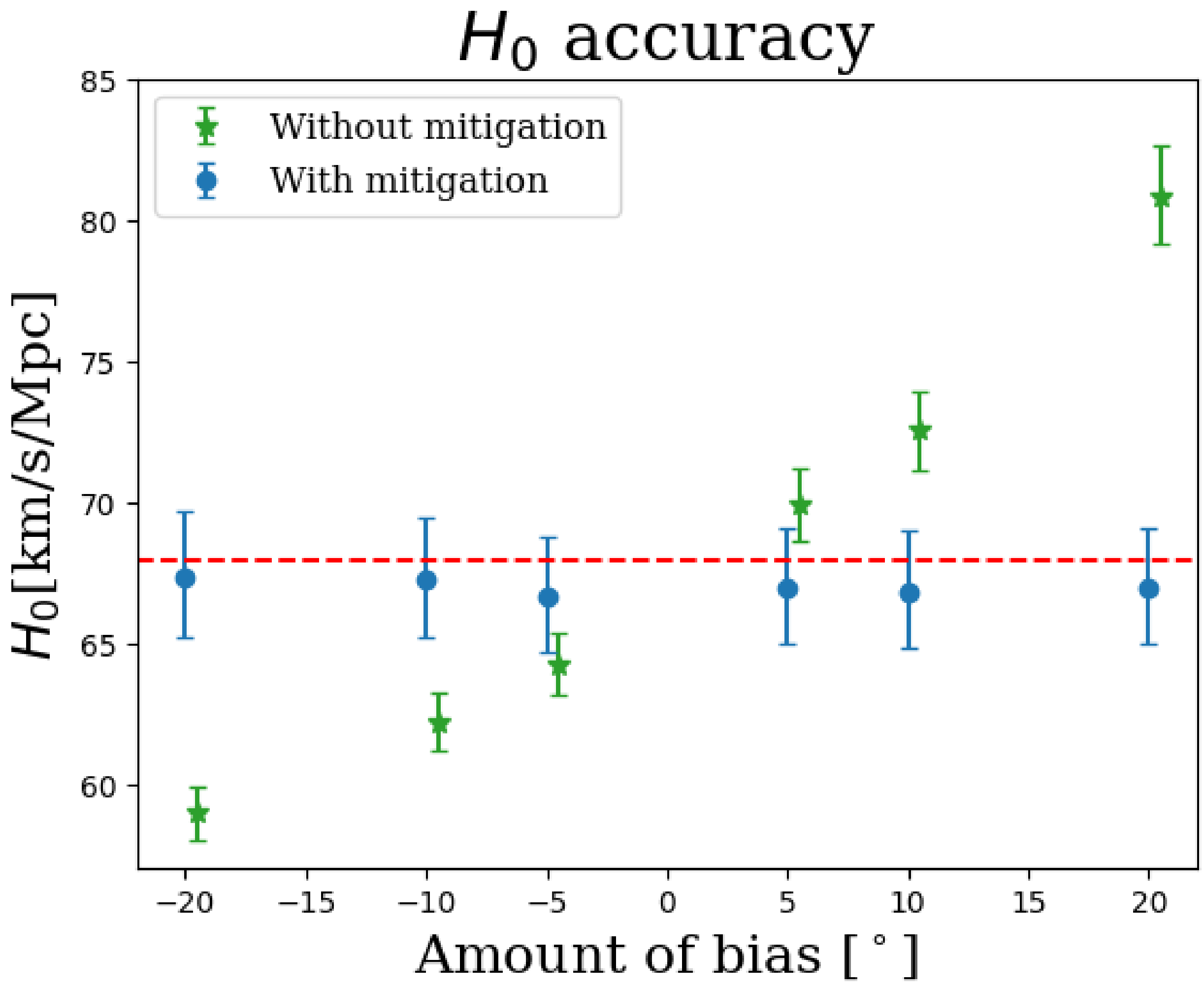


Combination of 20 events



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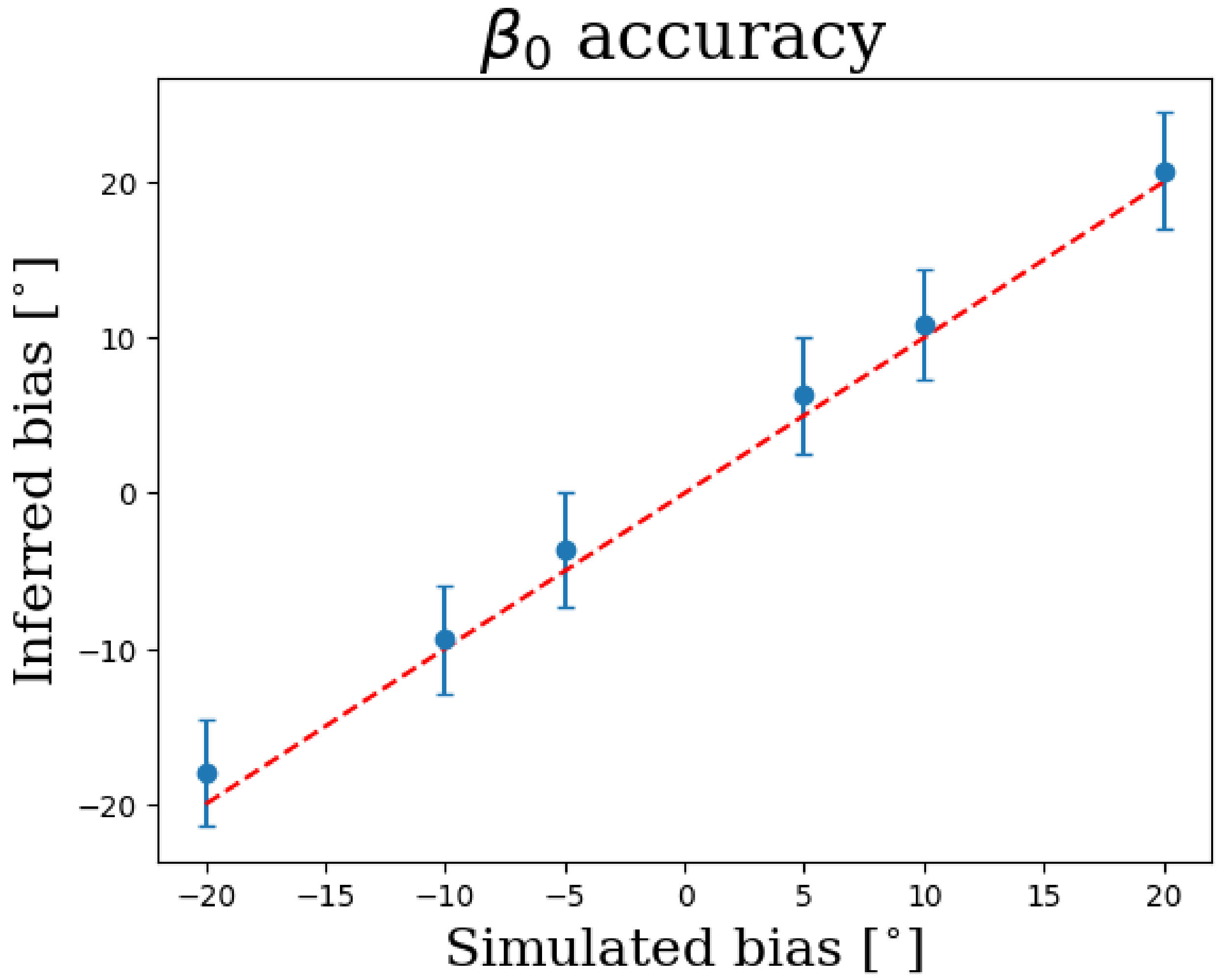

Changing bias distribution

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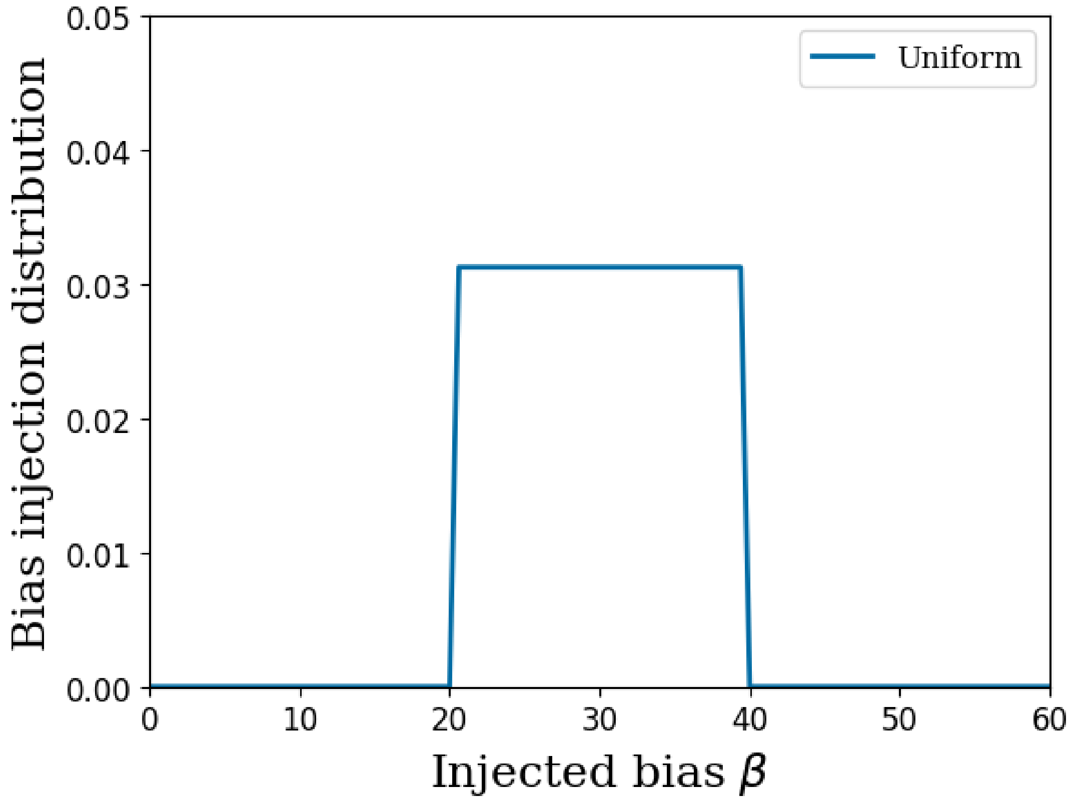

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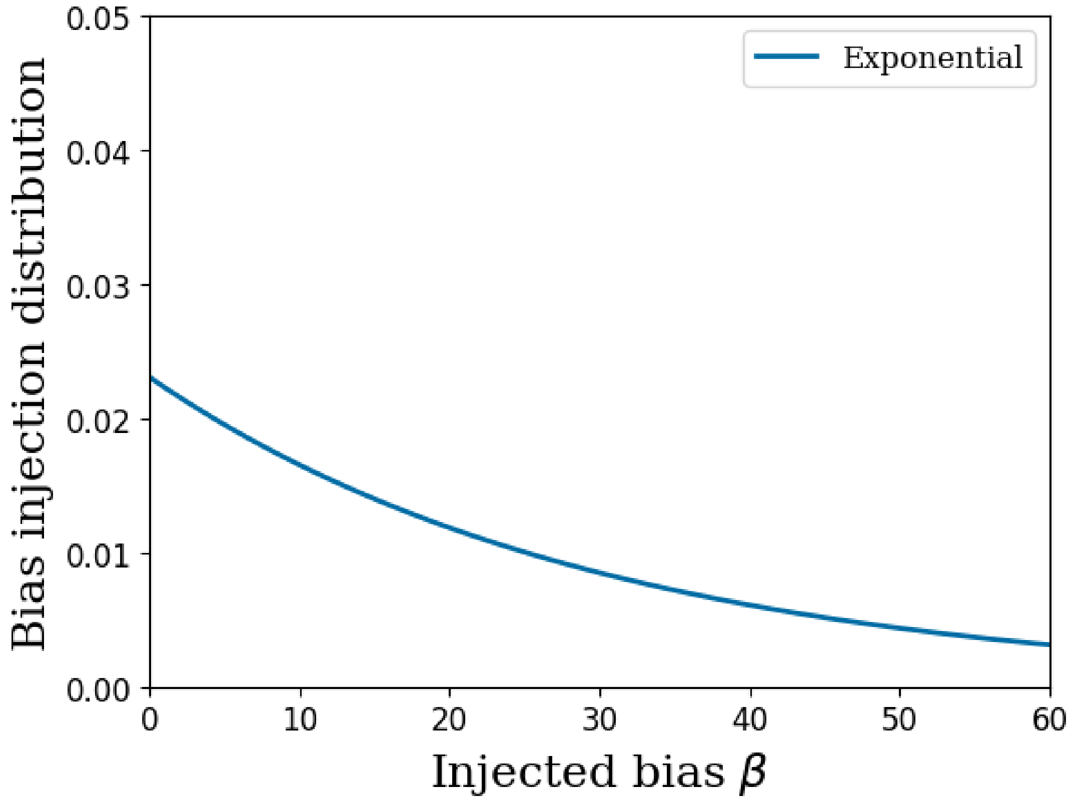

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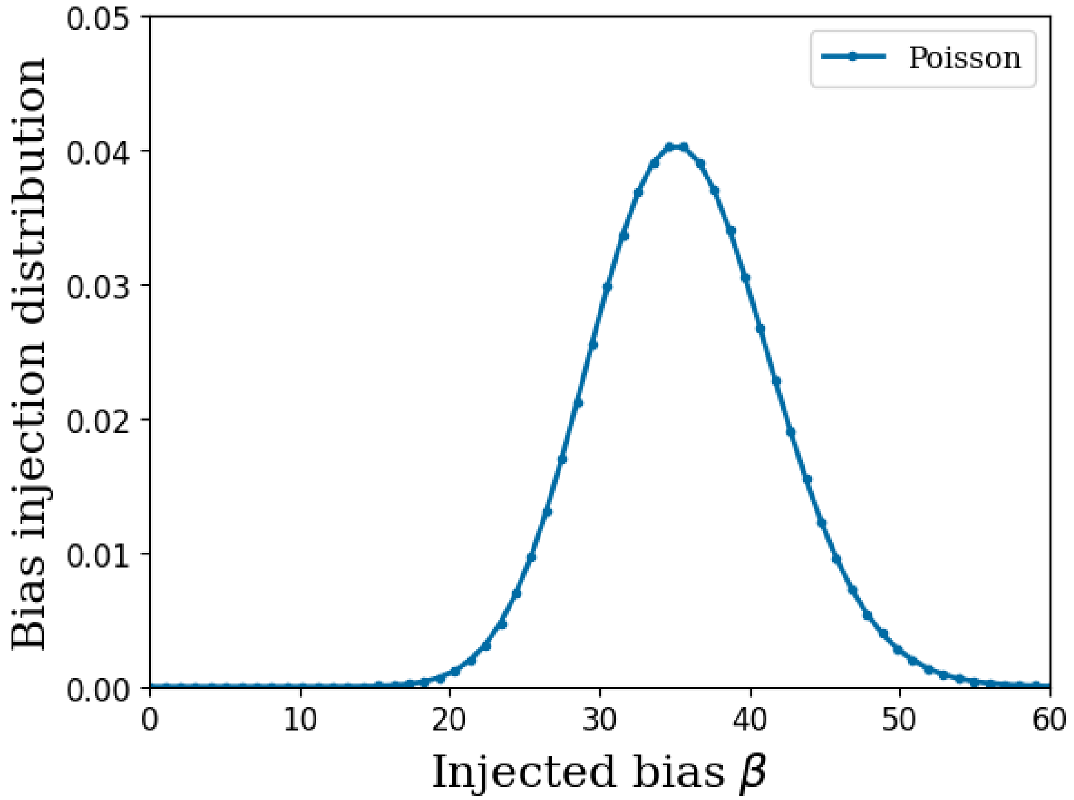
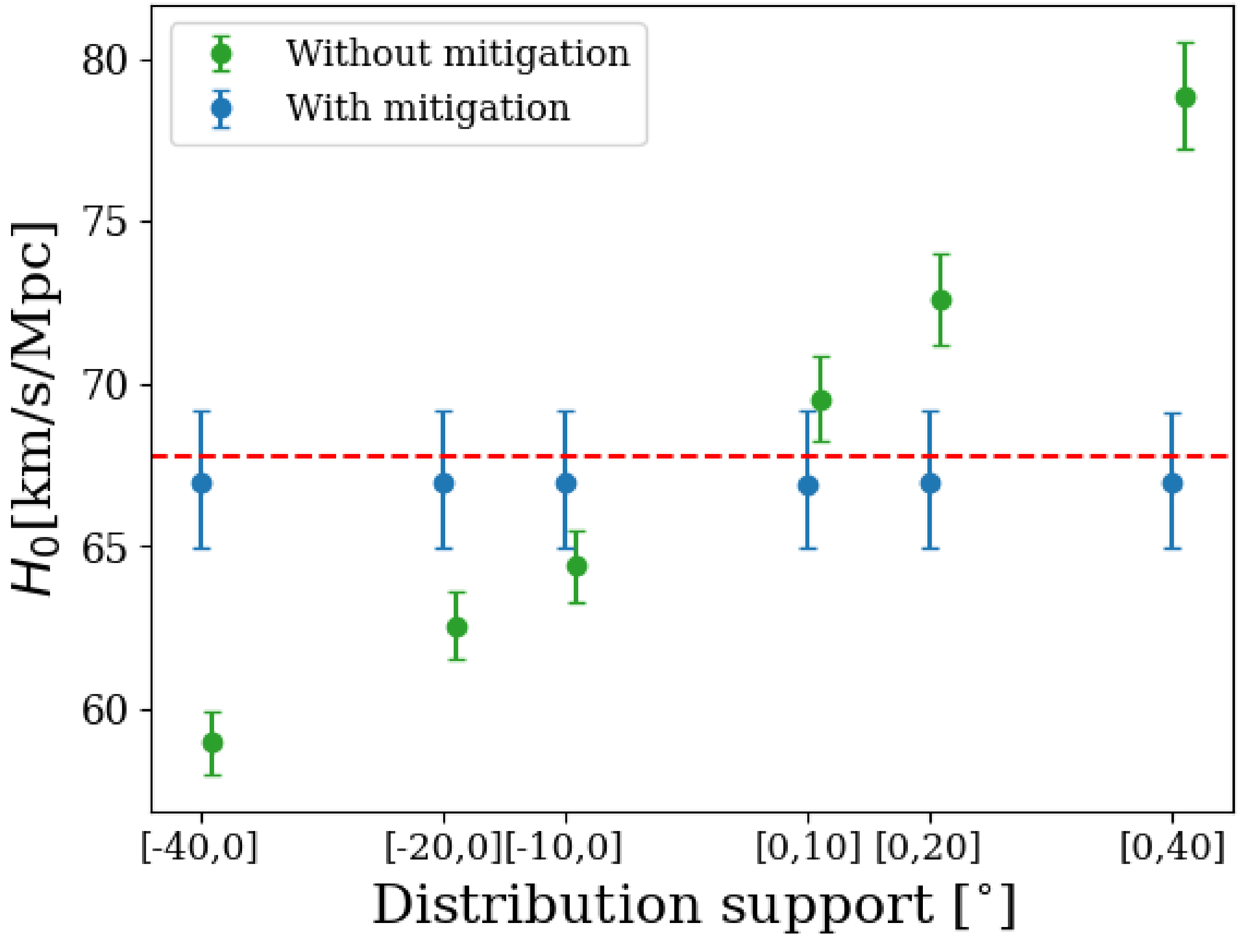


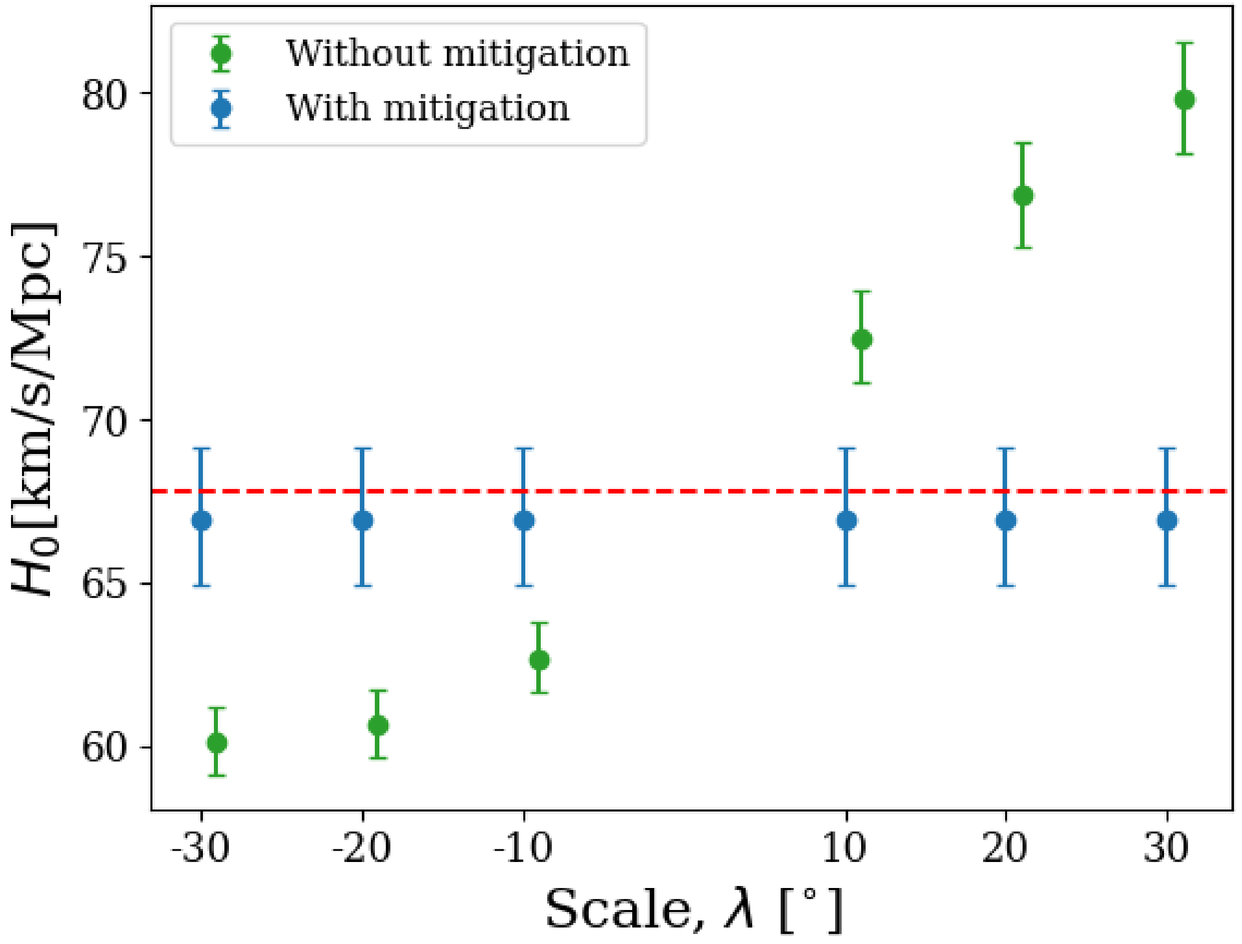
Changing injection bias distribution

Uniform distribution



Changing injection bias distribution

Exponential distribution



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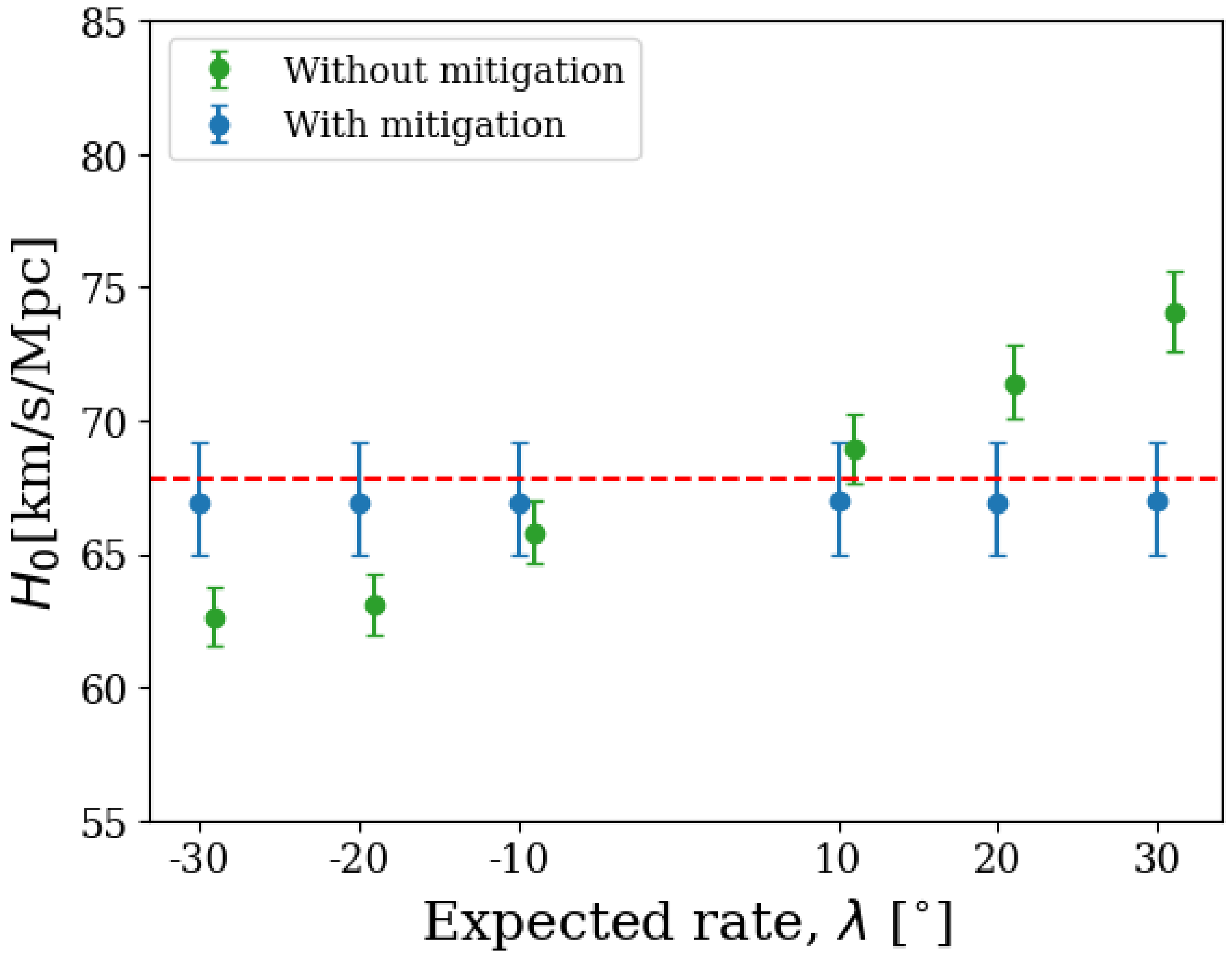


Conclusions

- Estimates of a bright sirens inclination angle are crucial to strongly constrain the Hubble constant

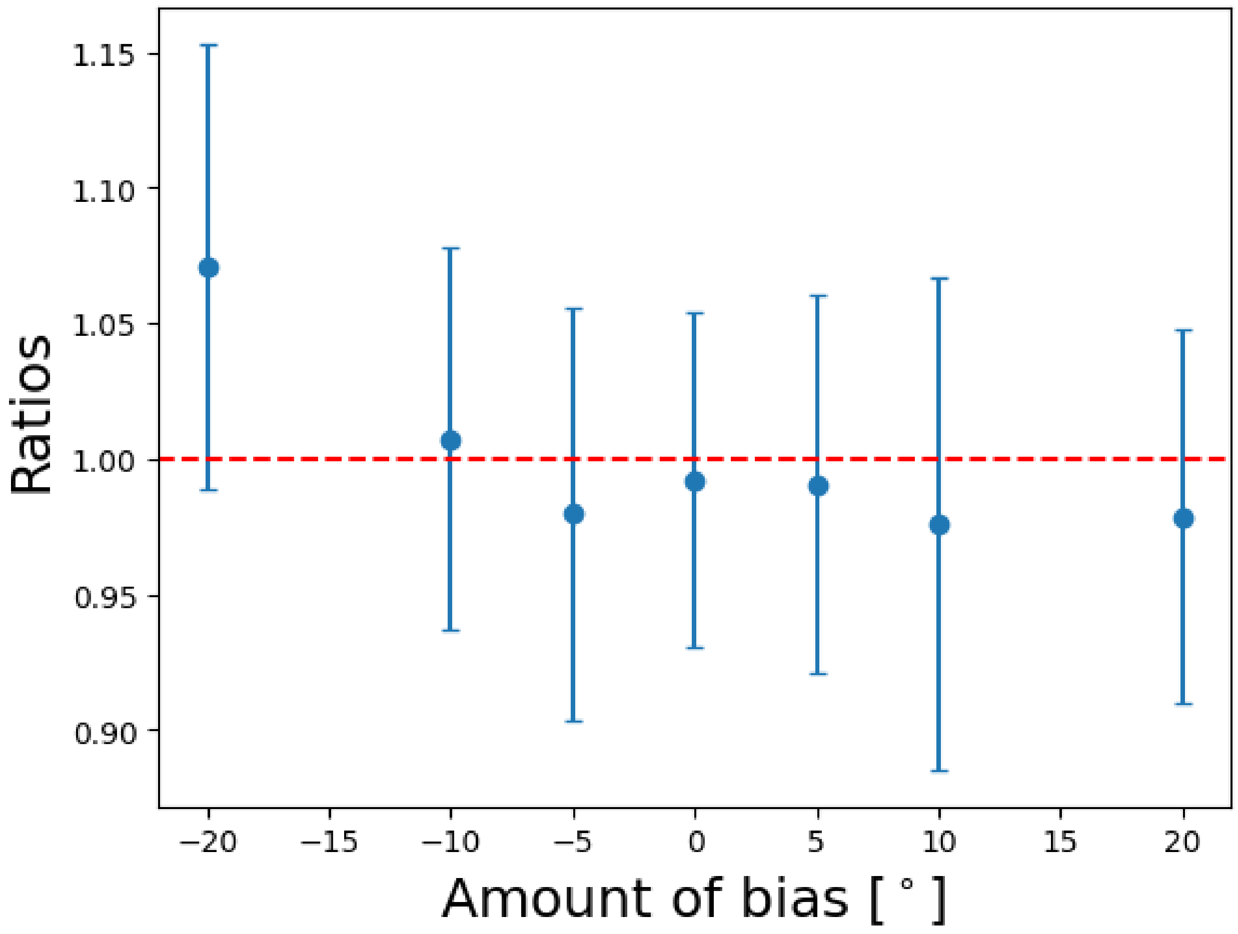
- Electromagnetic information must be used very carefully due to their possible systematics

- We developed a method that mitigates this systematic bias, allowing us to safely consider electromagnetic observations

- The method remains accurate even if the distributions for the injection and recovery bias models differ

Thank you

Precision ratios



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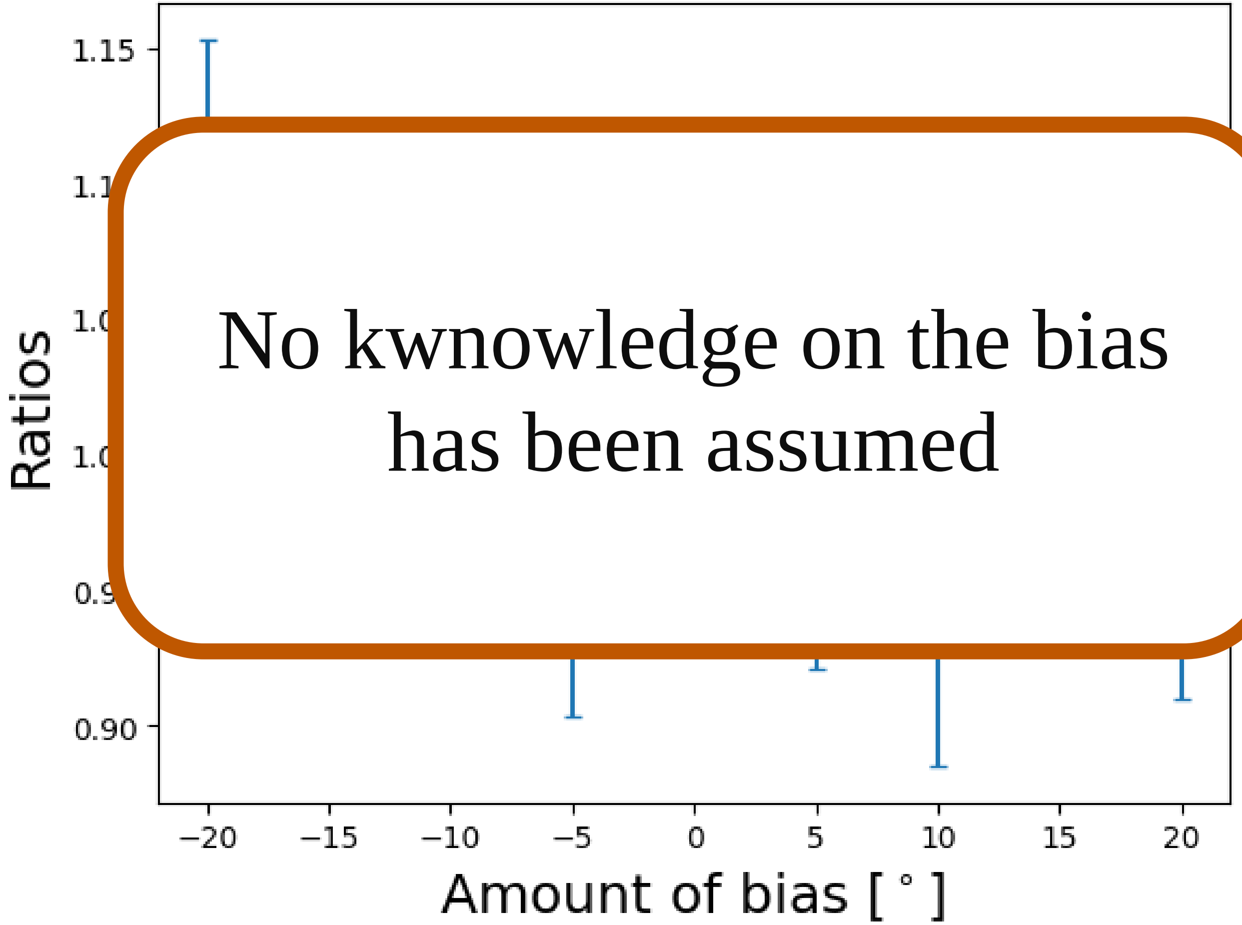
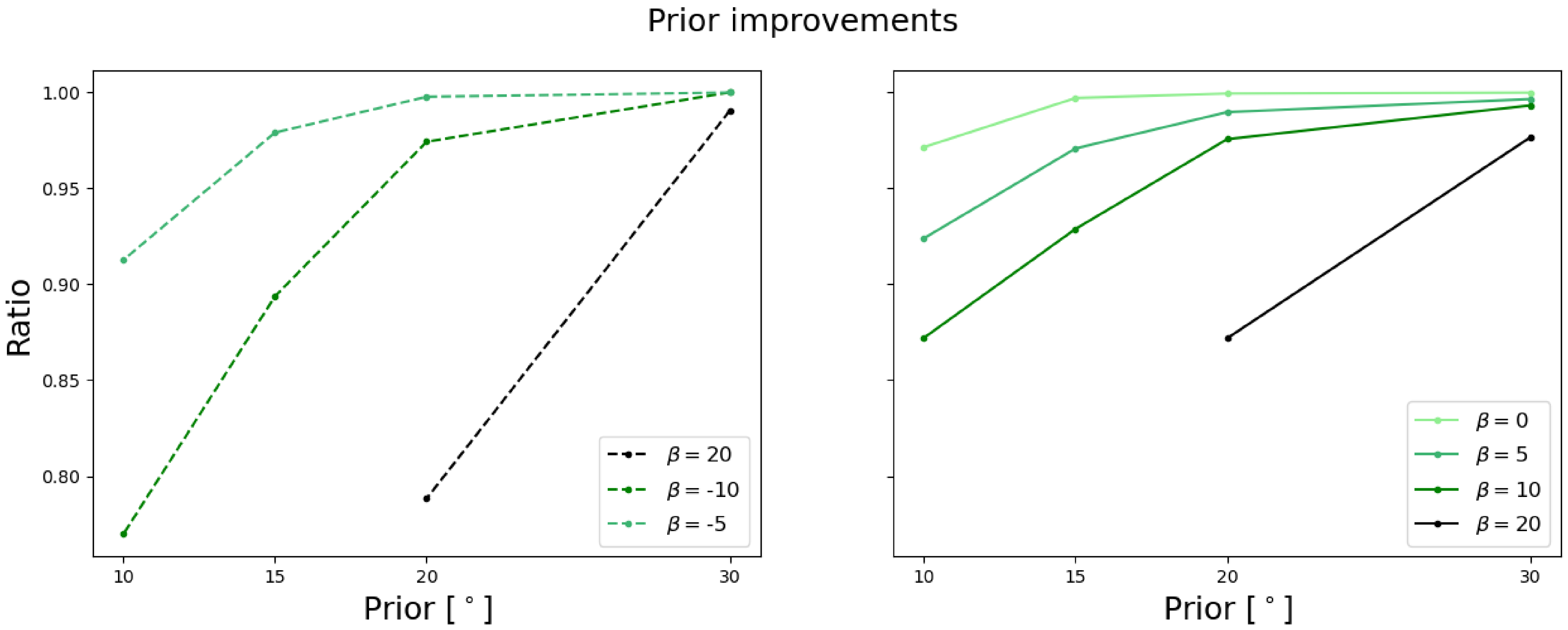
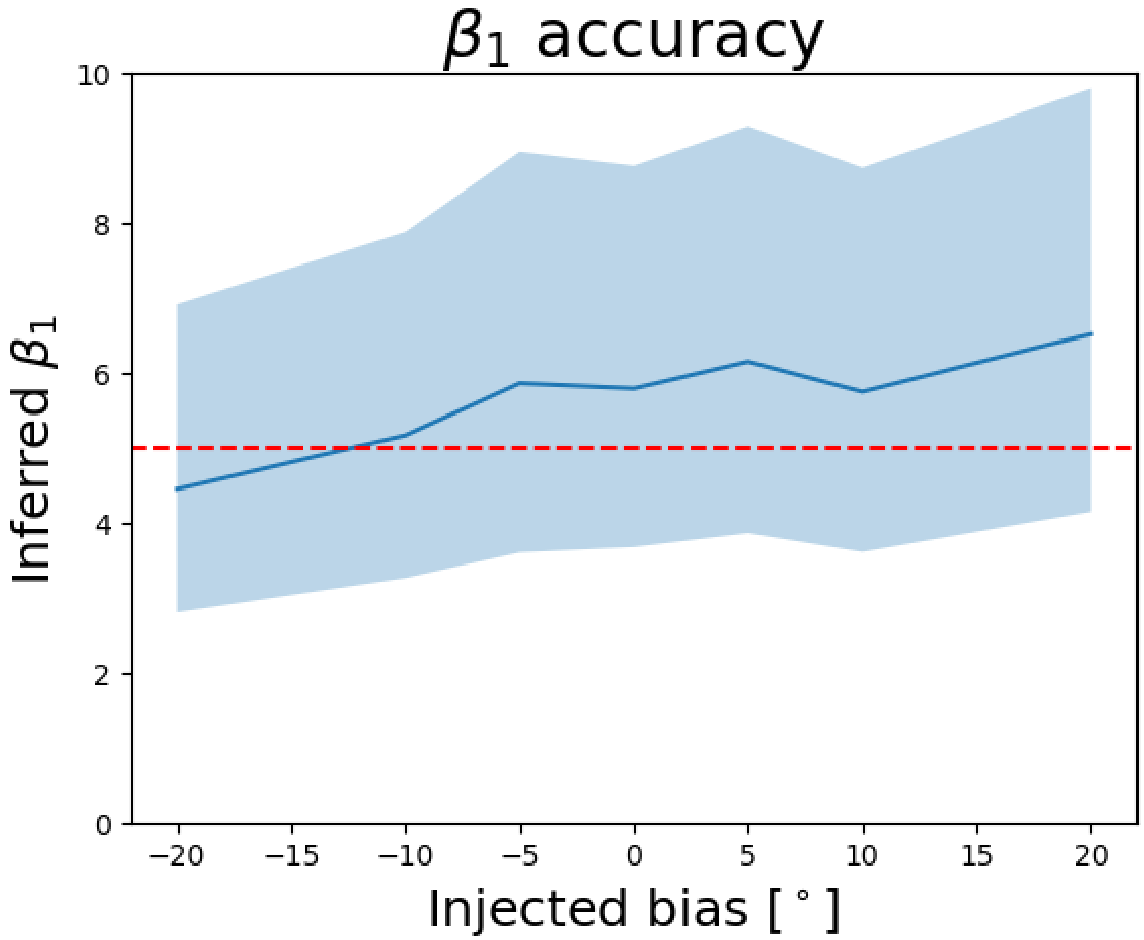


No kwnowledge on the bias has been assumed

Improving the precision

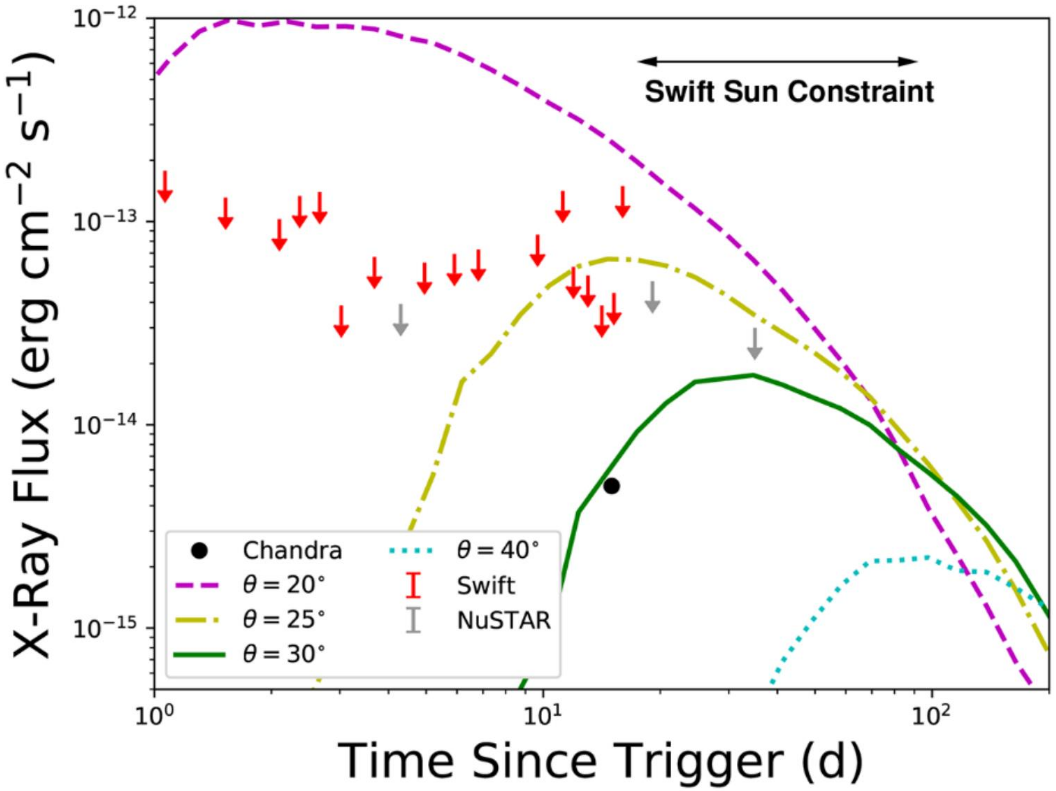



β_1 accuracy
Inferred β_1
Injected bias [°]
10
8
6
4
2
0
−20
−15
−10
−5
0
5
10
15
20

Electromagnetic constraints

- Detections of short GRB: constraints on the binary viewing angle (H.Y. Chen, et al., 2019)

  - GRB EM components (P. A. Evans, et al., 2017)



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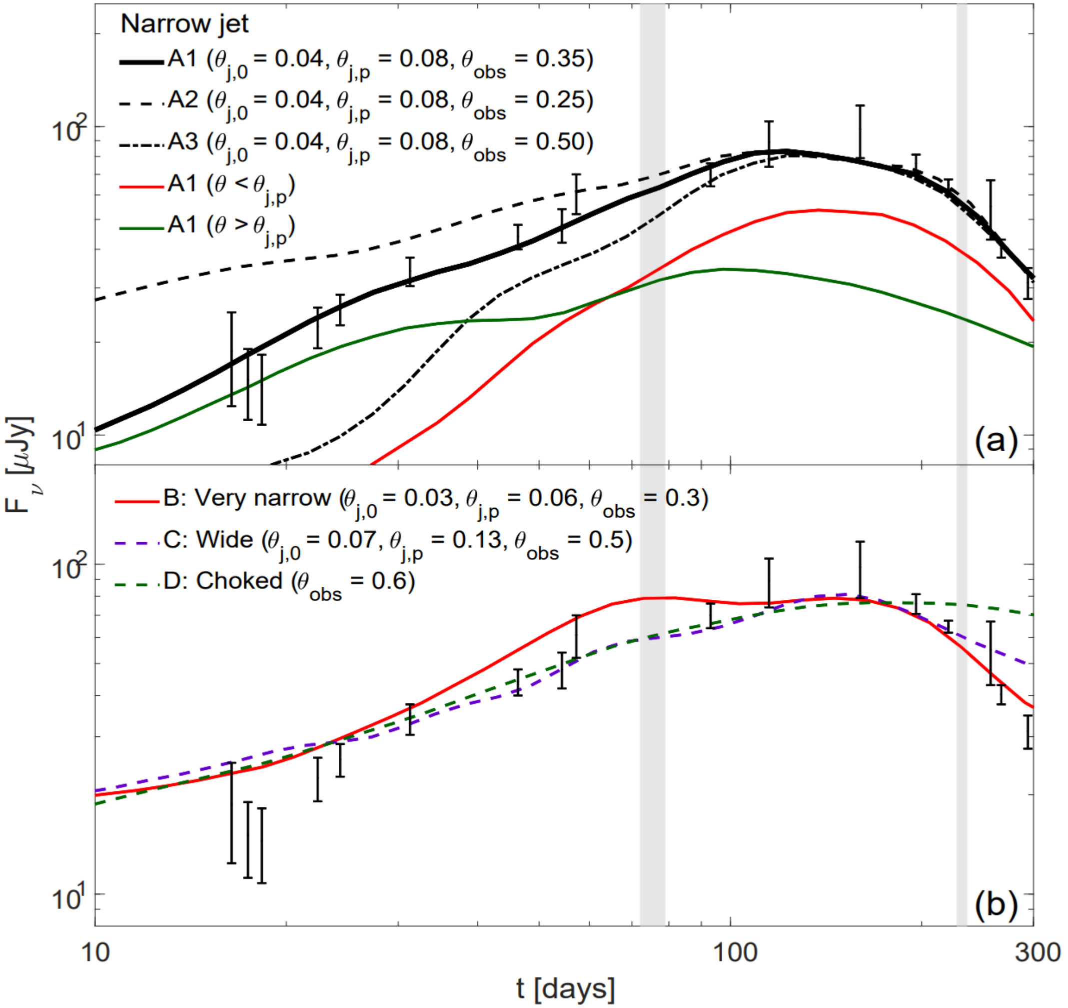

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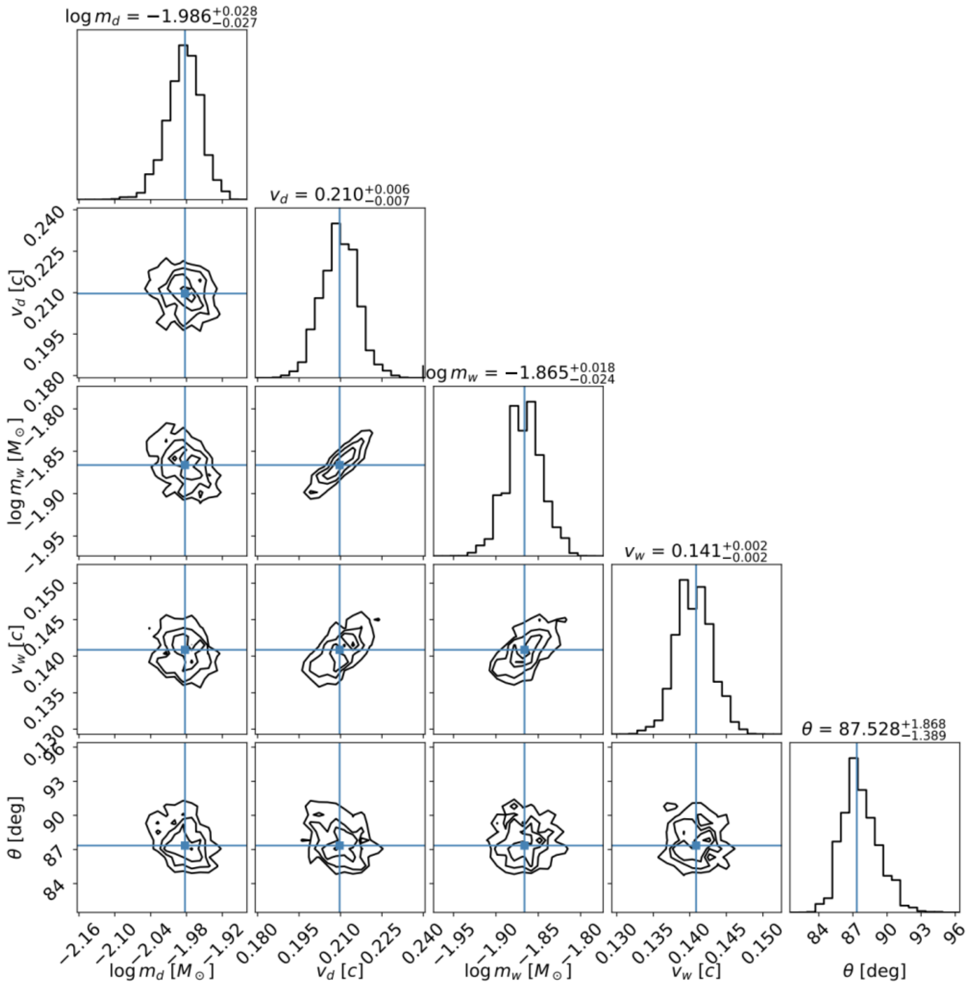
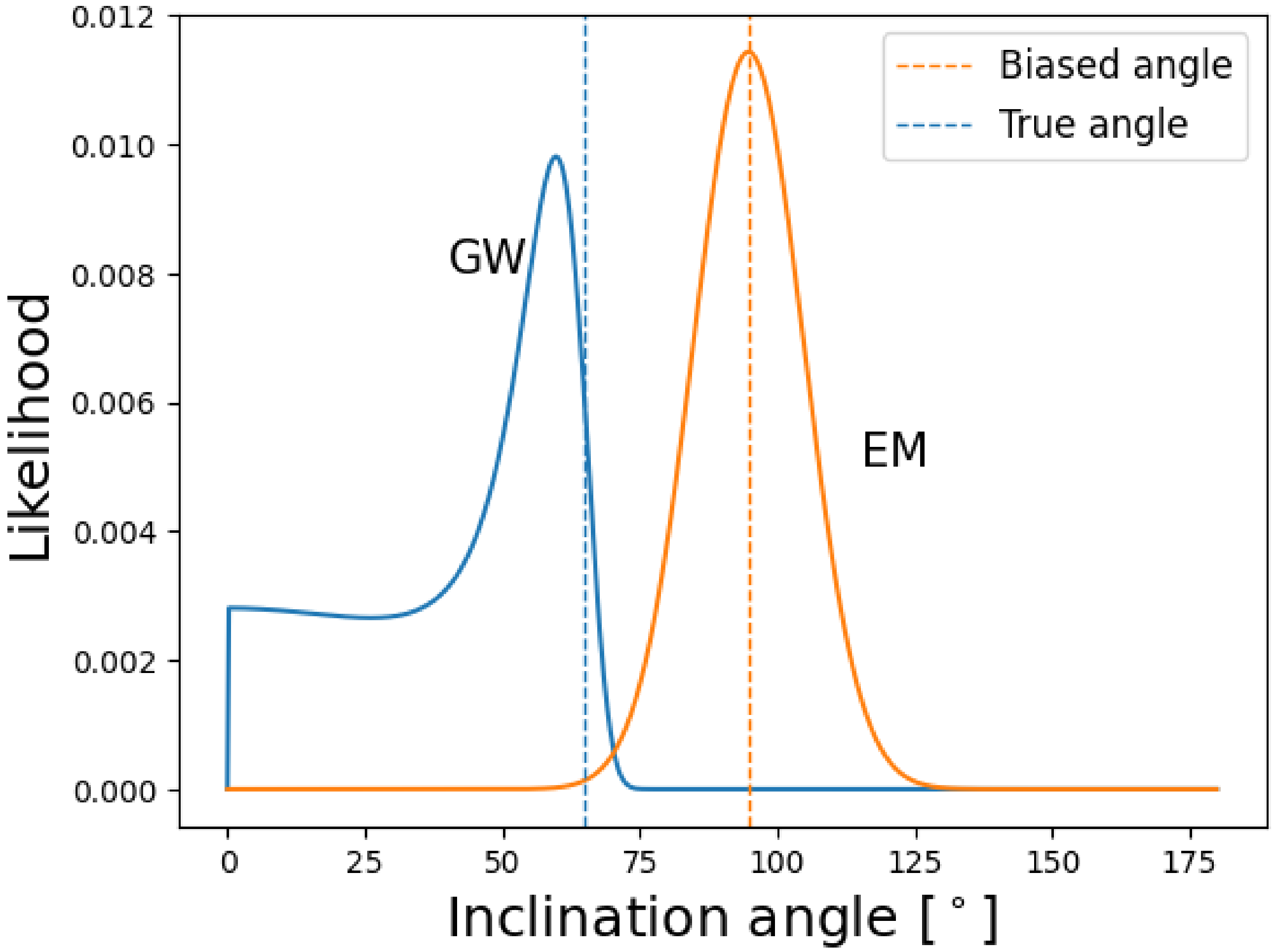
- Possibly: Kilonova light-curves (Y. Peng, et al., 2024)



Inferring the bias

$$p(h_0, \Delta | D) \propto \int p(D_{GW}) \times p(D_{EM})\, p_0(z, \iota) d\iota$$

$$p(D_{EM}) \propto \exp\left\{-\frac{(\iota + \Delta - \tilde{\iota})^2}{2\sigma^2}\right\}$$



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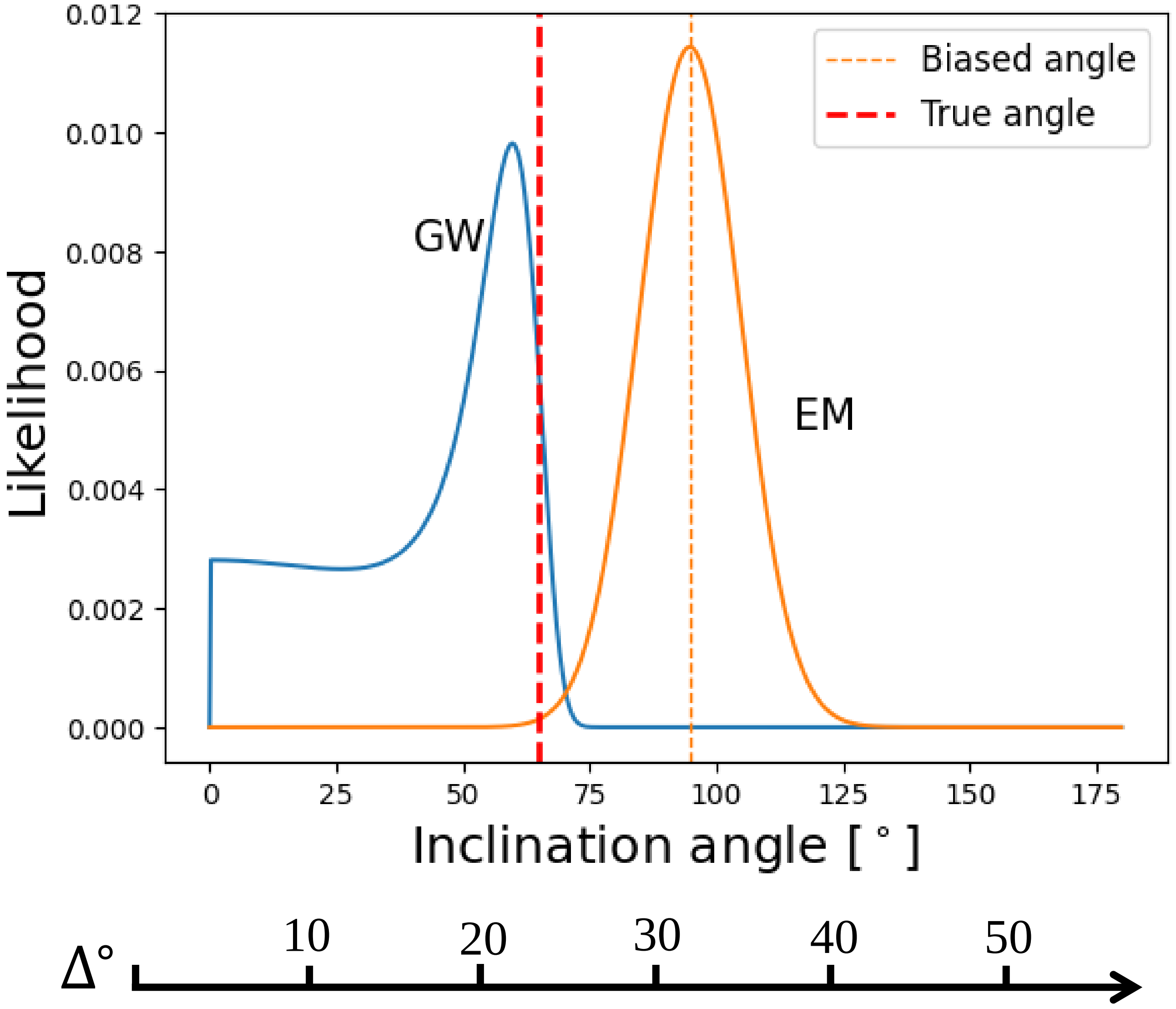

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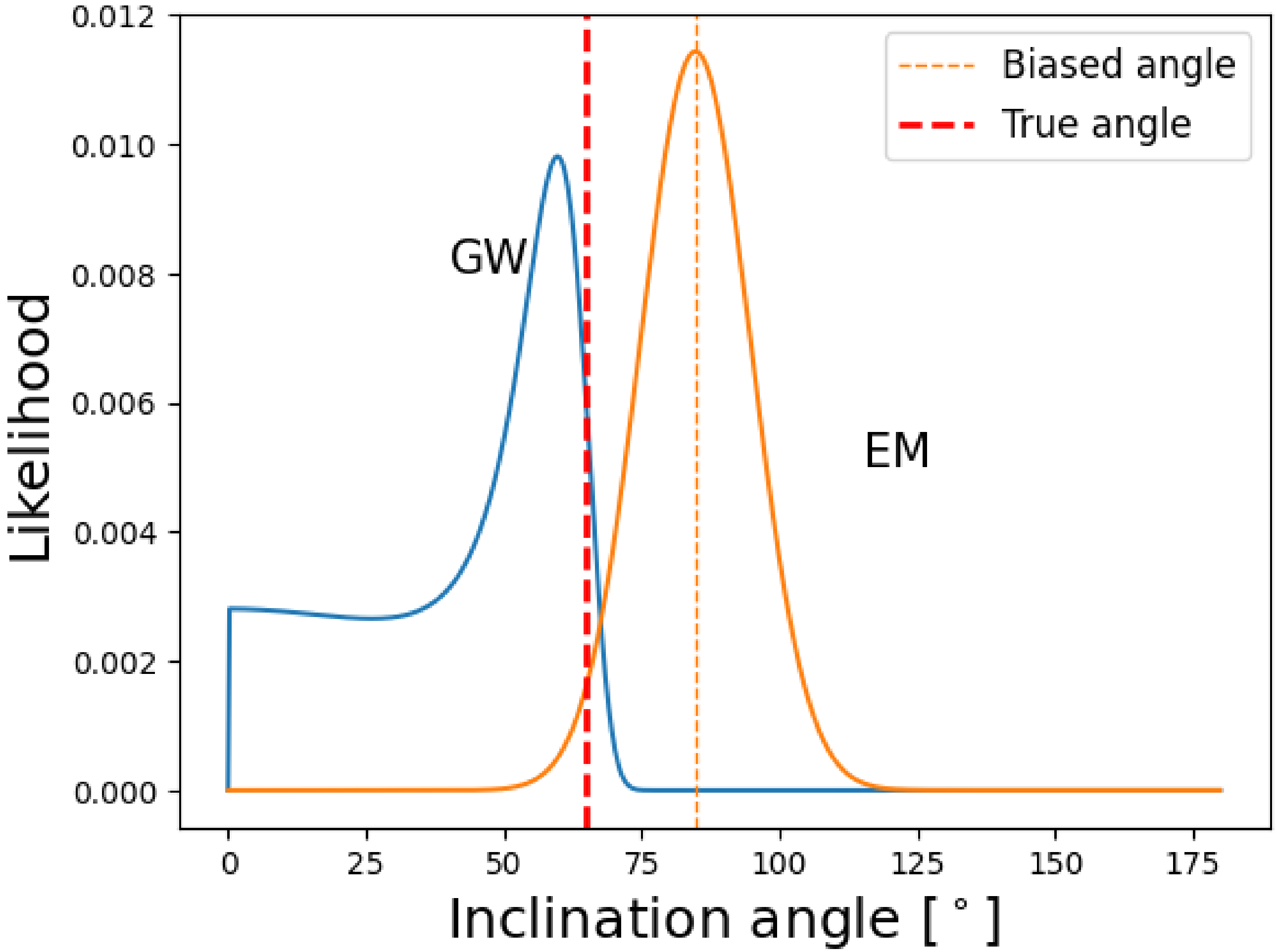

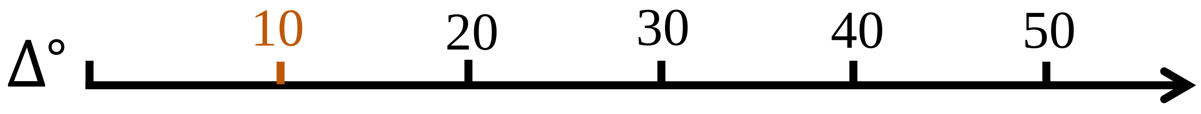

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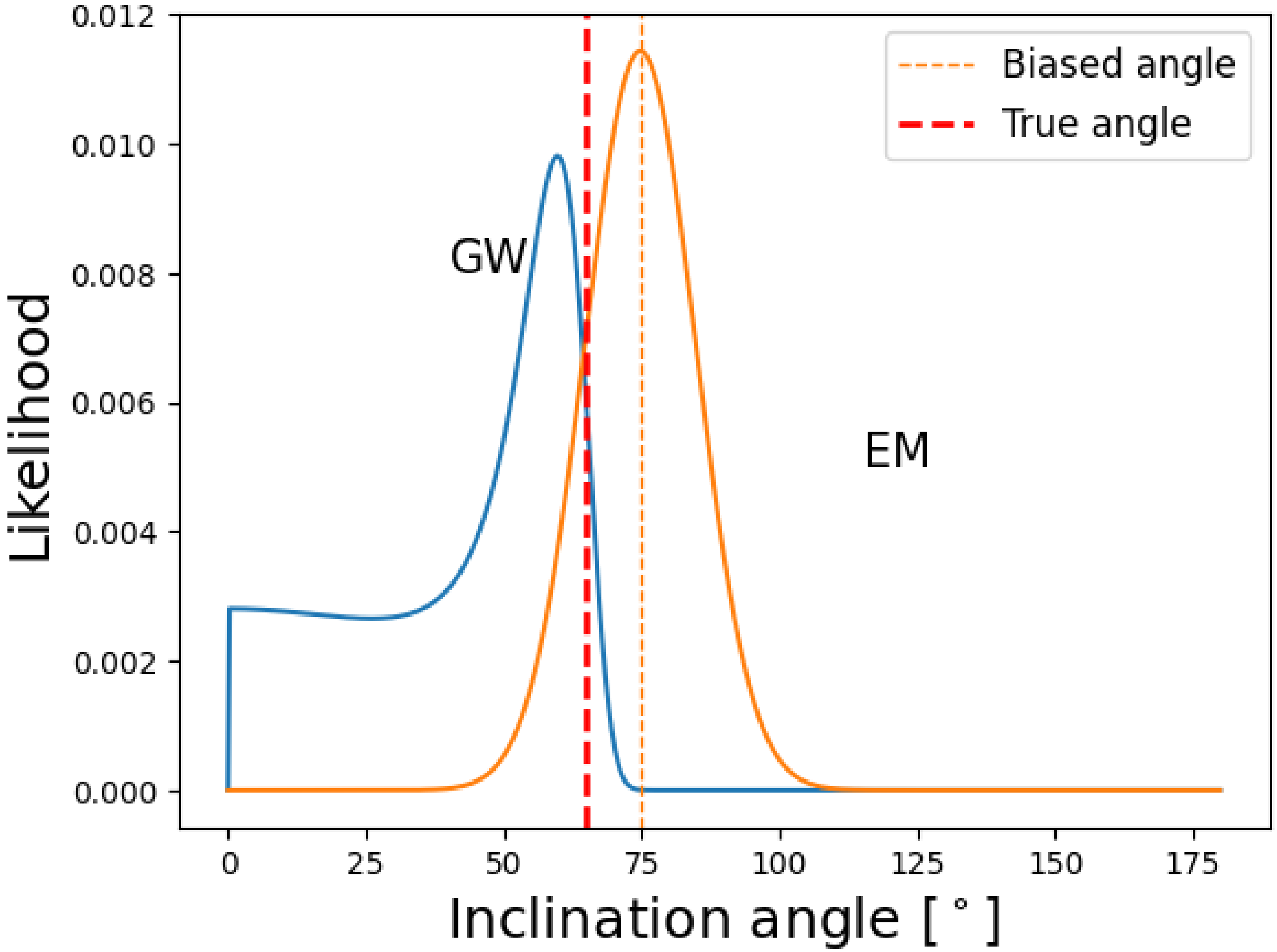

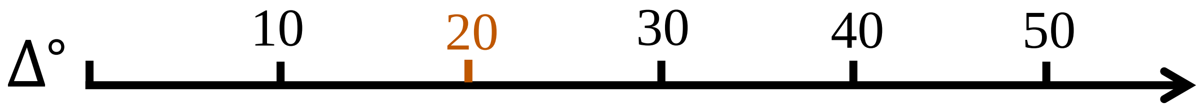

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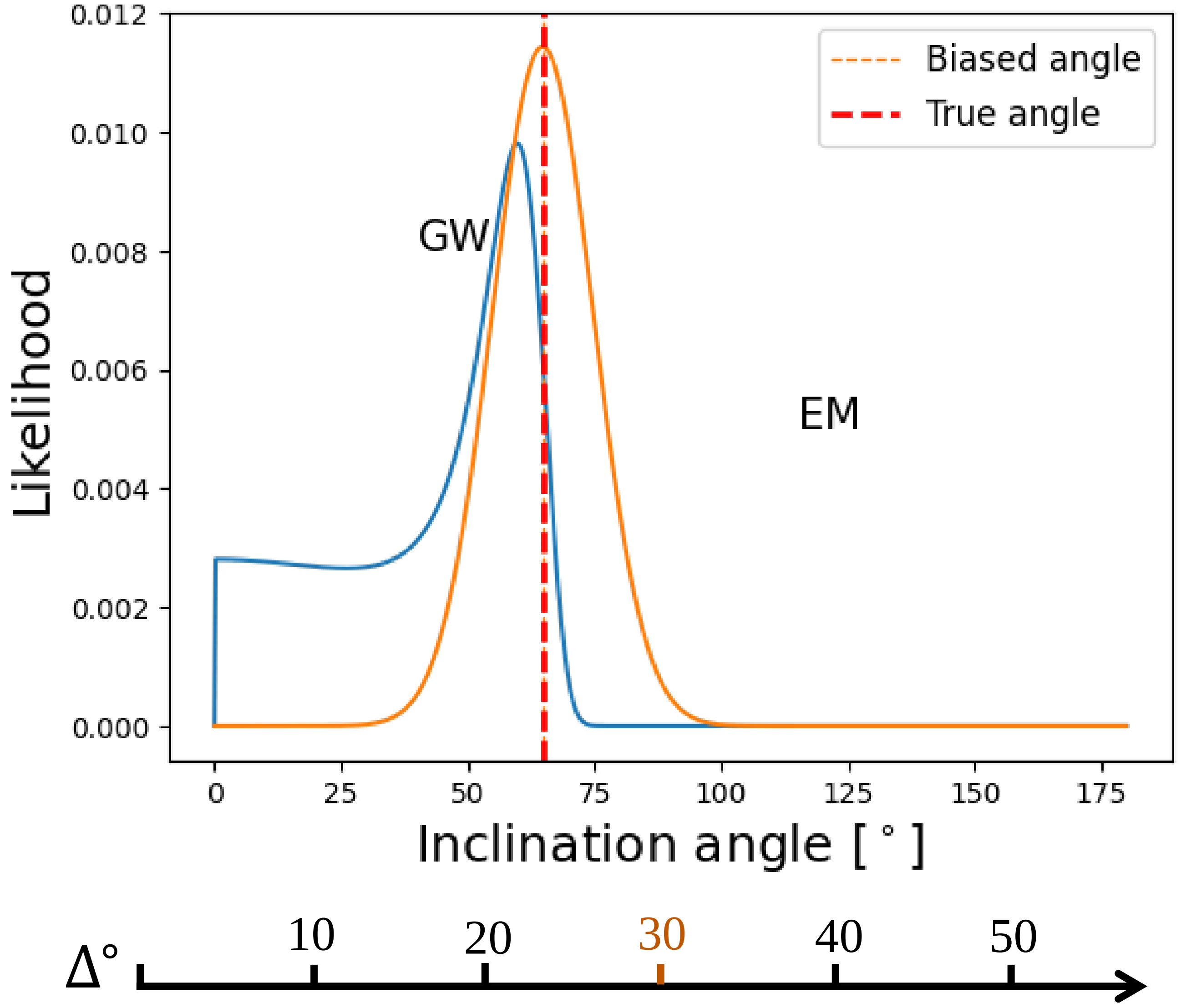
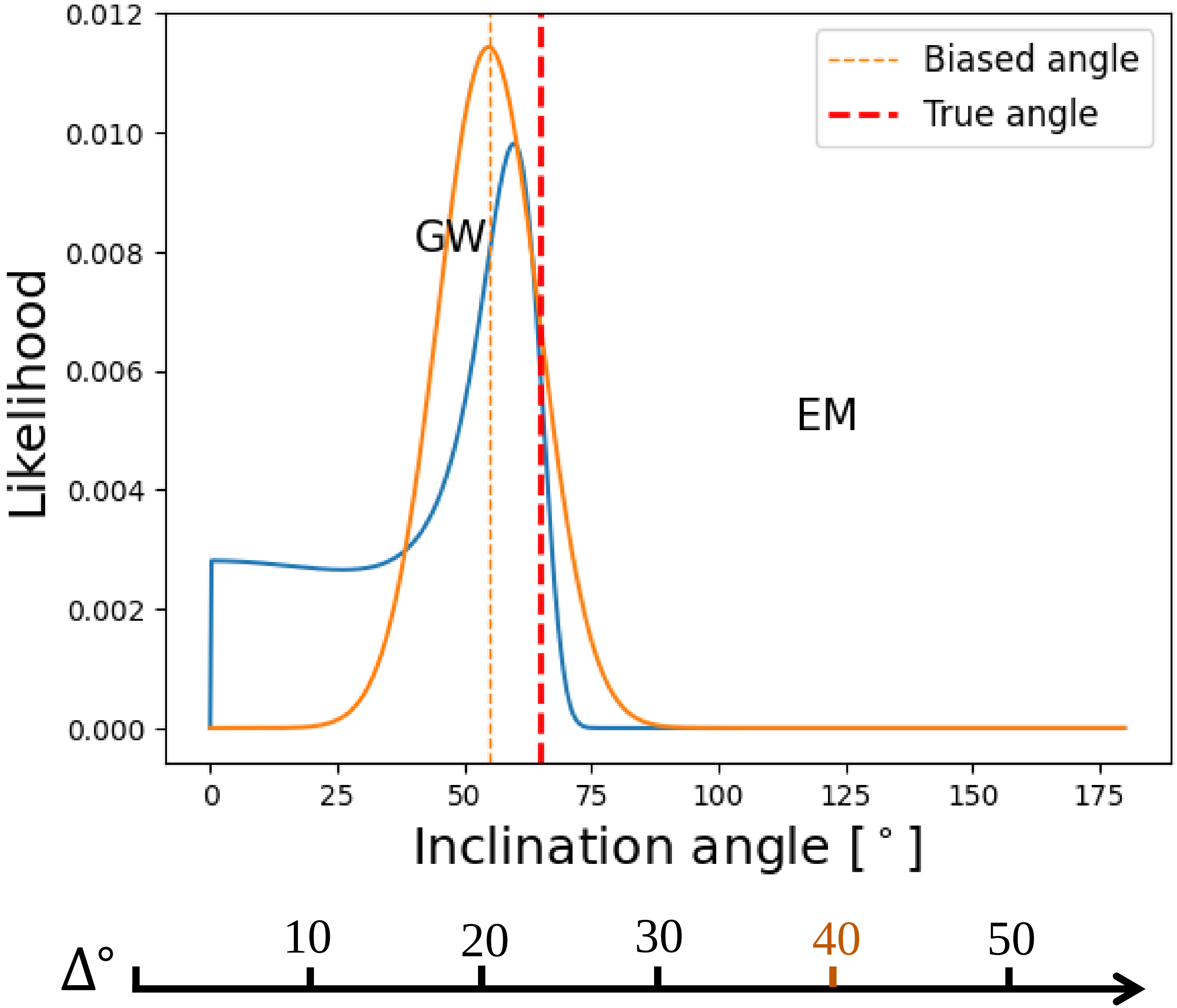


Inferring the bias

$$p(h_0, \Delta | D) \propto \int p(D_{GW}) \times p(D_{EM})\, p_0(z, \iota) d\iota$$

$$p(D_{EM}) \propto \exp\left\{-\frac{(\iota + \Delta - \tilde{\iota})^2}{2\sigma^2}\right\}$$



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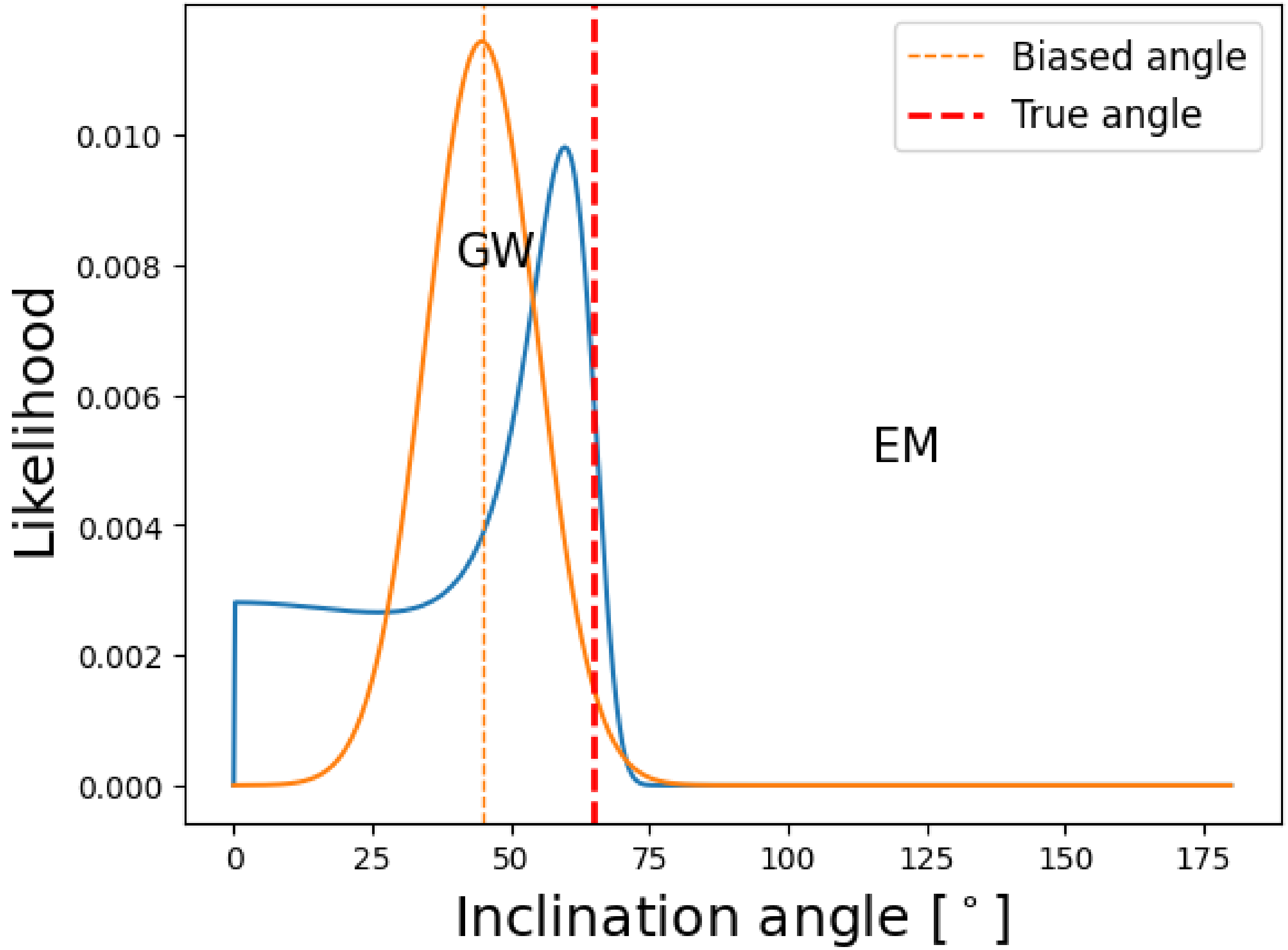
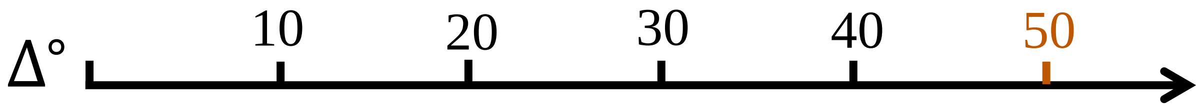


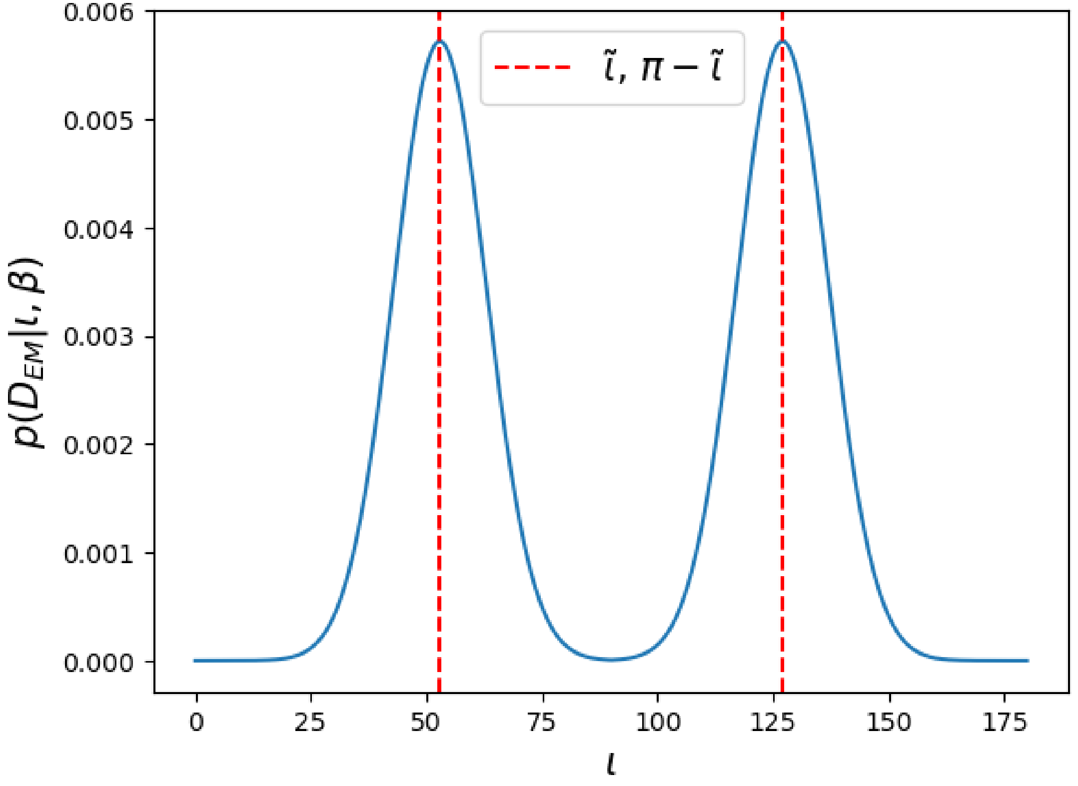


EM likelihood

EM likelihood: double-Normal distribution to account for $> 90°$ angles

- GW detections care about orbital motion orientation: inclination angle $\iota \in [0°, 180°]$
- EM estimates: viewing angle $\theta = \min\{\iota, 180° - \iota\}$ (H. Y. Chen, et al., 2019)

$$D_{EM} \sim N(\tilde{\iota}, \pi - \tilde{\iota}; \sigma)$$



Method: application

- 30 realizations of 20 simulated events: $\tilde{\iota} = \iota + N(0, \sigma) + N(\beta_0, \beta_1)$
- Uniform priors: $h_0 \in [0.2, 2]$, $\beta_0 \in [-90°, 90°]$, $\beta_1 \in [2°, 90° - \beta_0]$
- Three posteriors were estimated through MCMC:
  - $p(h_0|D_{GW})$: only GW information (H.Y. Chen, et al., 2018)
  - $p(h_0|D_{GW})$: only GW modified by biased EM information
  - $p(h_0, \beta_0, \beta_1|D_{GW+EM})$: debiased GW + EM information