Cosmic Inflation at the Crossroads

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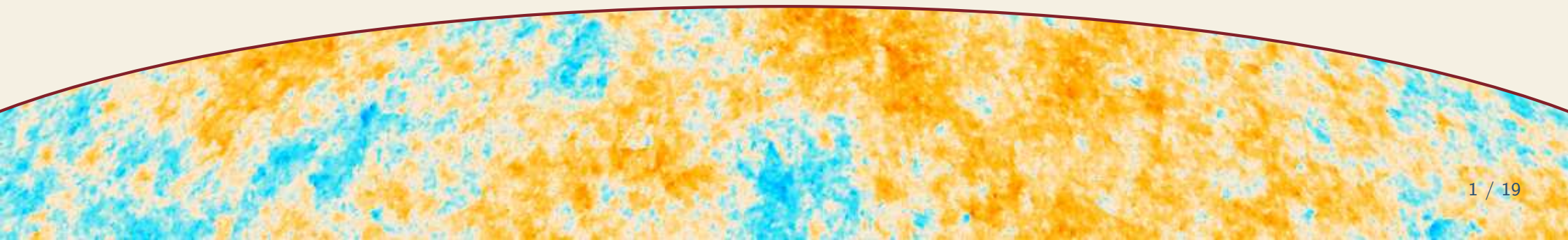

Outline

Qualitative Inflation
Quantitative inflation
Bayesian inference

Qualitative Inflation
- What is inflation?
- Triggering and stopping acceleration
- Cosmological perturbation

Quantitative inflation
- Encyclopædia Inflationaris: Opiparous Edition
- Primordial power spectra
- Reheating consistent model predictions

Bayesian inference
- Machine-learning an effective likelihood
- Marginalized posteriors
- Computing bayesian evidences
- Bayes factors for all models
- Information gain on the reheating
- Vanilla inflation predicts...
- Conclusion

J. Martin, CR and V. Vennin: arXiv:2404.10647, arXiv:2404.15089, arXiv:1303.3787v4

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What is inflation?

- Predictive, testable and tested early universe paradigm
  - ✦ Hypothetical accelerated expansion of the universe at $E_{\rm inf} > {\rm MeV}$ (BBN)
  - ✦ Addresses some unexplainable features of the Friedmann-Lemaître model
- For the simplest incarnation of inflation...
  - ✦ Historically introduced to dilute monopoles formed at GUT
  - ✦ Flatness of the spatial sections ($\Omega_{\rm K} = 0.0009 \pm 0.0018$)
  - ✦ Statistical isotropy of the observable universe (horizon problem)
  - ✦ Origin of CMB and LSS (quantum fluctuations)
  - ✦ Gaussianities of the cosmological perturbations ($f_{\rm NL} < -0.9 \pm 5$)
  - ✦ Adiabaticity of the cosmological perturbations (isocurv. $< 1\%$)
  - ✦ Almost scale invariance ($n_{\rm S} = 0.9649 \pm 0.004$)

Triggering and stopping acceleration

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- The simplest way: single-field inflation

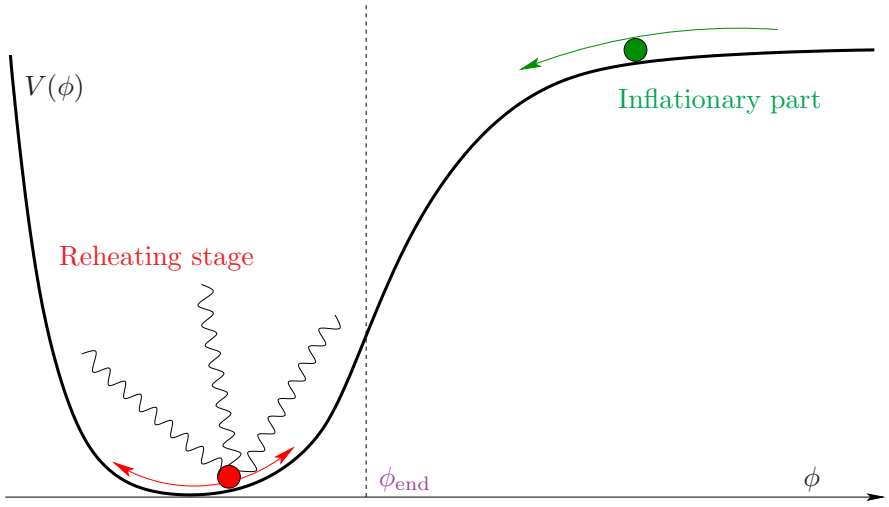
$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \mathcal{L}(\phi) \right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

- Inflation occurs in the plateau and is followed by a reheating era

  - Friedmann-Lemaître

$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V \right)$$
$$\frac{\ddot{a}}{a} = -\frac{1}{3} \left(\dot{\phi}^2 - V \right)$$

  - $H \simeq \text{Constant} \rightarrow a \propto e^{Ht}$



- The reheating stage: everything after ϕ_{end} till radiation domination

Cosmological perturbations of quantum origin

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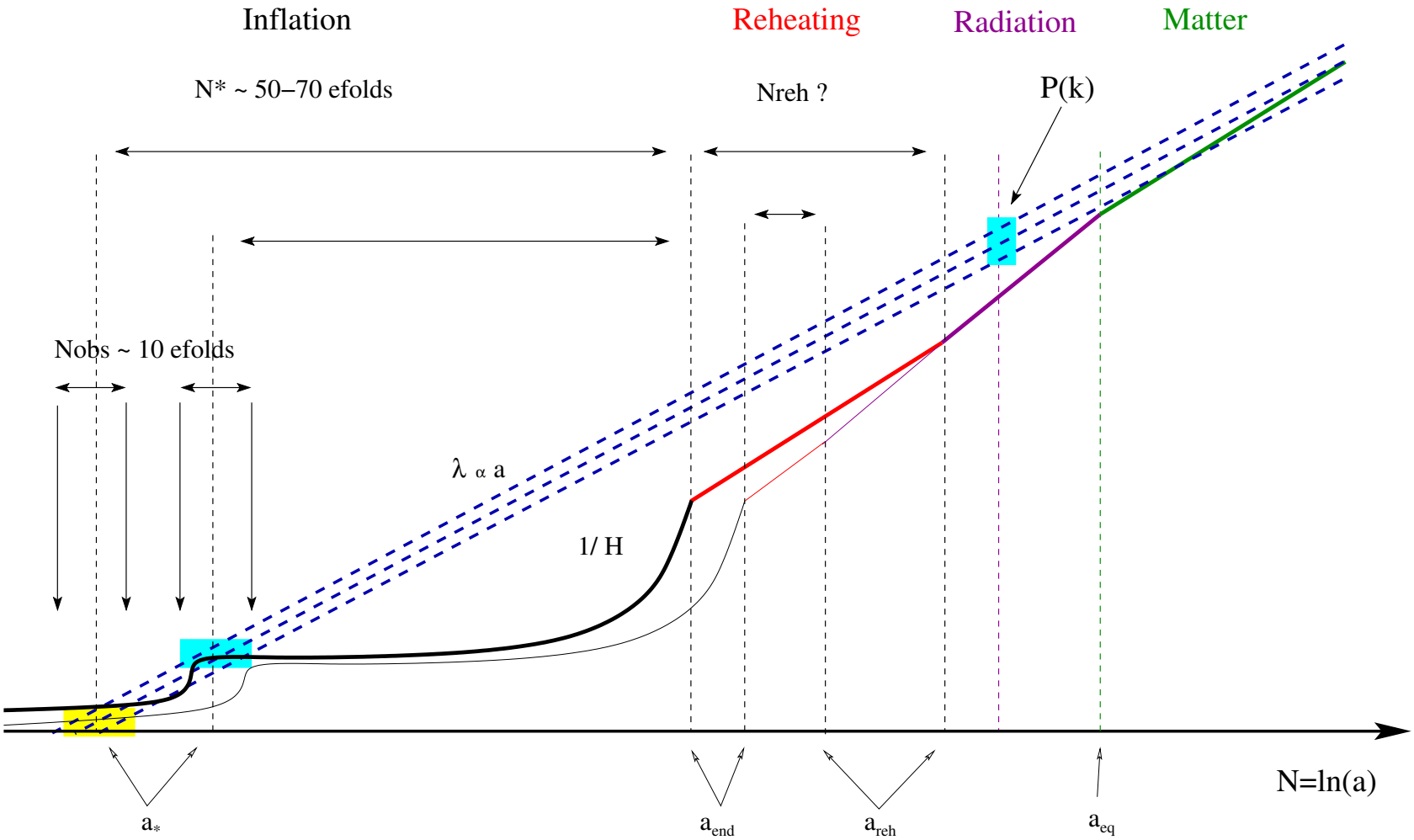

- Model testing: reheating effects must be included!

Redshift at which inflation ends

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- Depends on the redshift of reheating

$$1+z_{\rm end}=\frac{a_0}{a_{\rm end}}=\frac{a_{\rm reh}}{a_{\rm end}}(1+z_{\rm reh})=\frac{a_{\rm reh}}{a_{\rm end}}\left(\frac{\rho_{\rm reh}}{\tilde{\rho}_\gamma}\right)^{1/4}=\frac{1}{R_{\rm rad}}\left(\frac{\rho_{\rm end}}{\tilde{\rho}_\gamma}\right)^{1/4}$$

  - The reheating parameter $R_{\rm rad}\equiv\dfrac{a_{\rm end}}{a_{\rm reh}}\left(\dfrac{\rho_{\rm end}}{\rho_{\rm reh}}\right)^{1/4}$
  - Encodes any observable deviations from a radiation-like or instantaneous reheating $R_{\rm rad}=1$

- $R_{\rm rad}$ can be expressed in terms of $(\rho_{\rm reh},\overline{w}_{\rm reh})$ or $(\Delta N_{\rm reh},\overline{w}_{\rm reh})$

$$\ln R_{\rm rad}=\frac{\Delta N_{\rm reh}}{4}(3\overline{w}_{\rm reh}-1)=\frac{1-3\overline{w}_{\rm reh}}{12(1+\overline{w}_{\rm reh})}\ln\left(\frac{\rho_{\rm reh}}{\rho_{\rm end}}\right)$$

- Scary Astrophysics (early universe)

$$10^{10}<z_{\rm end}<10^{28},\qquad -46<\ln R_{\rm rad}<15$$

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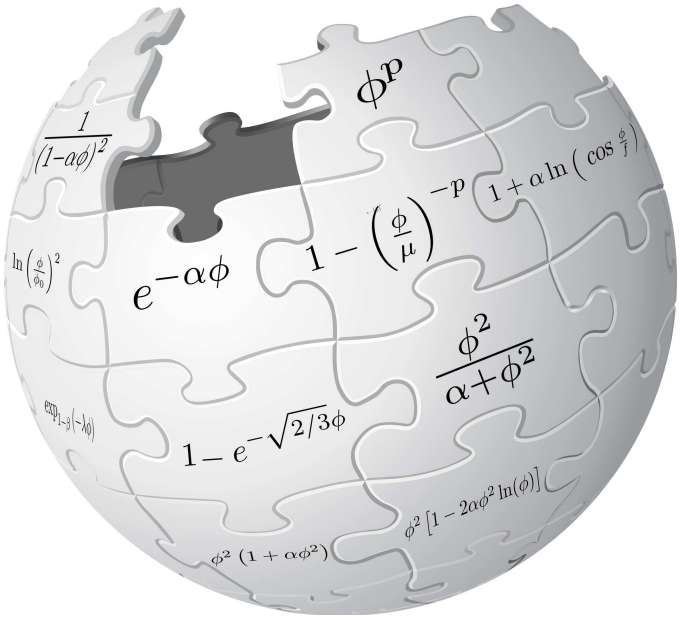
- New version published in May 2023 (arXiv:1303.3787v4)
  - ✦ Deals with accurate slow-roll predictions for 287 models
  - ✦ Comes with a public runtime library `ASPIC`



- Computes the Hubble-flow functions from the model parameters $\boldsymbol{\theta}_{\rm inf}$

$$(\boldsymbol{\theta}_{\rm inf}, R_{\rm rad}) \longrightarrow \texttt{ASPIC} \longrightarrow \epsilon_i(\boldsymbol{\theta}_{\rm inf}, R_{\rm rad}) \equiv \frac{\mathrm{d}\ln|\epsilon_{i-1}|}{\mathrm{d}\ln a}, \quad \epsilon_0 \propto \frac{1}{H}$$

Primordial power spectra

- Uniquely determined from the Hubble-flow functions

$$\mathcal{P}_\zeta = \frac{H_*^2}{8\pi^2 M_{\rm Pl}^2 \epsilon_{1*}} \left\{ 1 - 2(1+C)\epsilon_{1*} - C\epsilon_{2*} + \left(\frac{\pi^2}{2} - 3 + 2C + 2C^2\right)\epsilon_{1*}^2 \right.$$
$$+ \left(\frac{7\pi^2}{12} - 6 - C + C^2\right)\epsilon_{1*}\epsilon_{2*} + \left(\frac{\pi^2}{8} - 1 + \frac{C^2}{2}\right)\epsilon_{2*}^2 + \left(\frac{\pi^2}{24} - \frac{C^2}{2}\right)\epsilon_{2*}\epsilon_{3*}$$
$$+ \left[-2\epsilon_{1*} - \epsilon_{2*} + (2+4C)\epsilon_{1*}^2 + (-1+2C)\epsilon_{1*}\epsilon_{2*} + C\epsilon_{2*}^2 - C\epsilon_{2*}\epsilon_{3*} \right] \ln\left(\frac{k}{k_*}\right)$$
$$\left. + \left[2\epsilon_{1*}^2 + \epsilon_{1*}\epsilon_{2*} + \frac{1}{2}\epsilon_{2*}^2 - \frac{1}{2}\epsilon_{2*}\epsilon_{3*} \right] \ln^2\left(\frac{k}{k_*}\right) + \cdots \right\},$$

$$\mathcal{P}_h = \frac{2H_*^2}{\pi^2 M_{\rm Pl}^2} \left\{ 1 - 2(1+C)\epsilon_{1*} + \left[-3 + \frac{\pi^2}{2} + 2C + 2C^2 \right]\epsilon_{1*}^2 + \left[-2 + \frac{\pi^2}{12} - 2C - C^2 \right]\epsilon_{1*}\epsilon_{2*} \right.$$
$$\left. + \left[-2\epsilon_{1*} + (2+4C)\epsilon_{1*}^2 + (-2-2C)\epsilon_{1*}\epsilon_{2*} \right] \ln\left(\frac{k}{k_*}\right) + \left(2\epsilon_{1*}^2 - \epsilon_{1*}\epsilon_{1*}\right) \ln^2\left(\frac{k}{k_*}\right) + \cdots \right\}$$

- Currently known at third order: arXiv:2205.12608 (involves ϵ_{4*})

Reheating consistent model predictions

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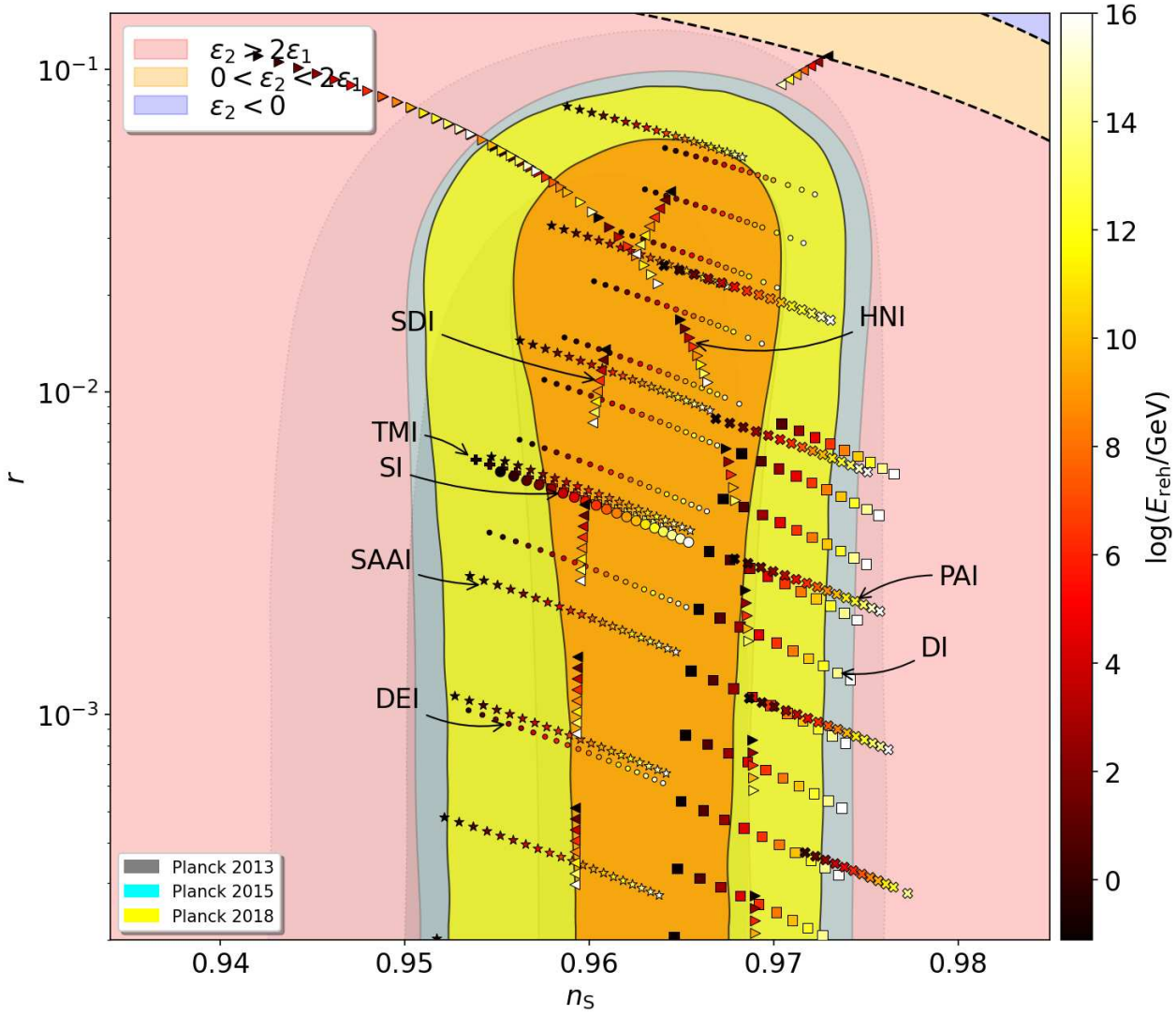
- Quick check for which reheating history a model is compatible with the data



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Machine-learning an effective likelihood

- To speed-up data analysis for 287 models

$$\mathcal{L}_{\rm eff}(\boldsymbol{D}|P_*,\varepsilon_1,\varepsilon_2,\varepsilon_3) \propto \int P(\boldsymbol{D}|\boldsymbol{\theta}_{\rm s},P_*,\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)\,\pi(\varepsilon_4)\,\pi(\boldsymbol{\theta}_{\rm s})\,{\rm d}\varepsilon_4{\rm d}\boldsymbol{\theta}_{\rm s}.$$

- The full likelihood has 55 parameters and is built upon
  - Planck 2020 post-legacy TT, TE, EE data (`PR4/NPIPE` maps)
  - Large scale EE polarization (`lowE`)
  - BICEP/Keck B-mode 2018 (arXiv:2110.00483)
  - Small scale TE and EE from SPT-3G (arXiv:2103.13618)
  - Baryon Acoustic Oscillations (SDSS collaboration)
- MCMC exploration of the 55 dimensions
  - `COSMOMC` up to 25 million samples ($R-1<10^{-3}$)
  - `GetDist` marginalization in 4D: $P_*,\varepsilon_1,\varepsilon_2,\varepsilon_3$
- Basic machine learning: 1 hidden layer with 300 nodes

Marginalized posteriors

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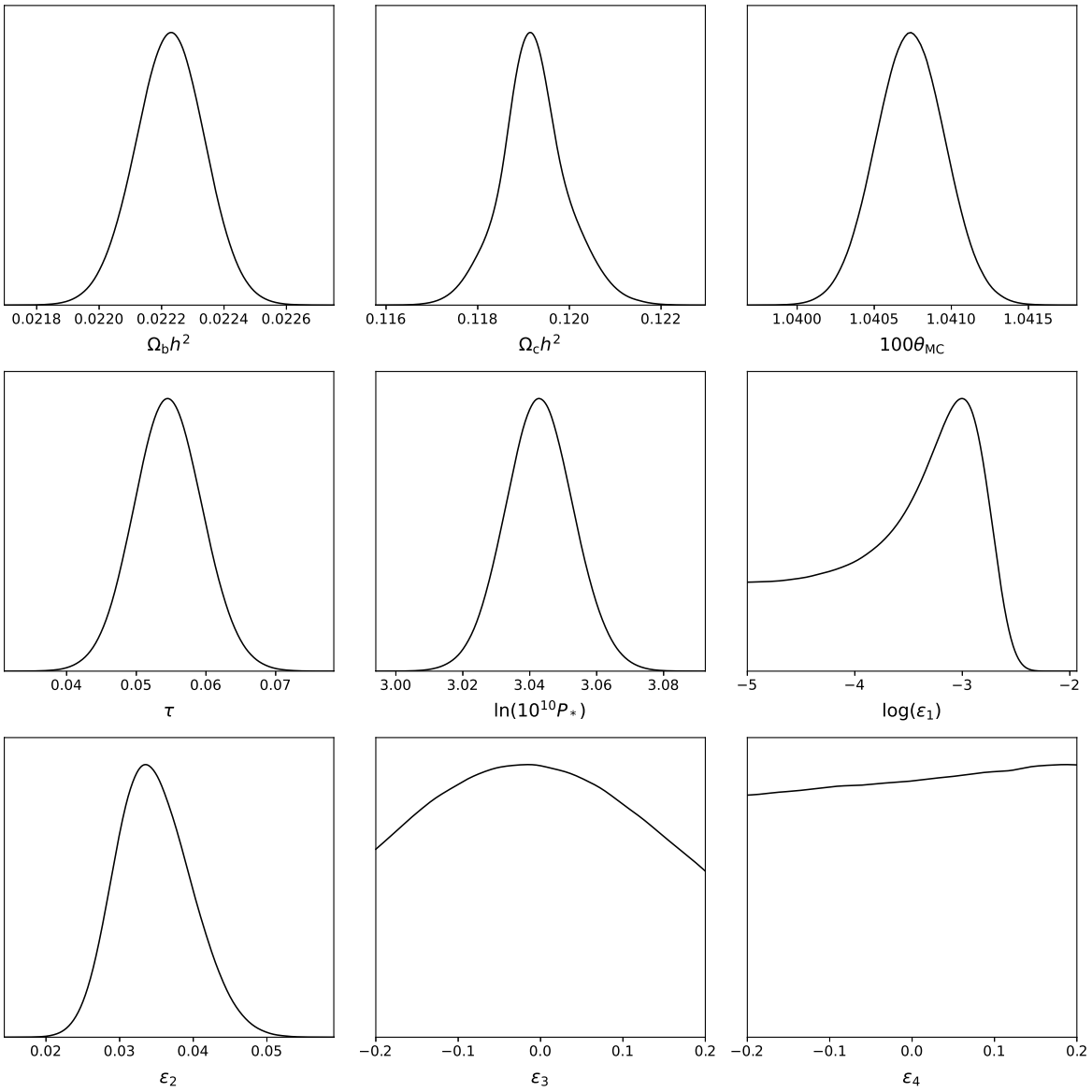
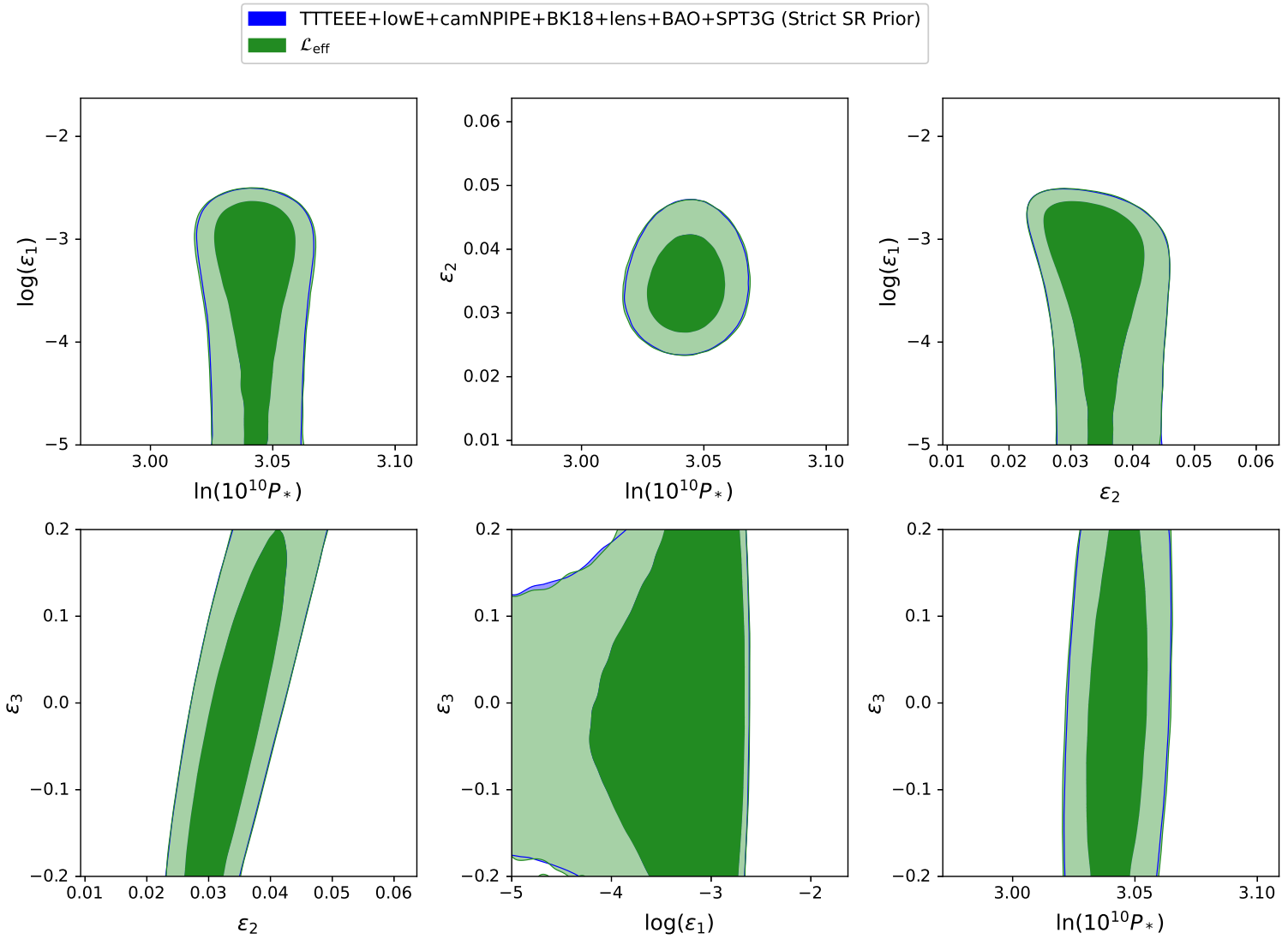
- Cosmo and Hubble-flow parameters



Marginalized posteriors

- Effective likelihood vs exact: 2D



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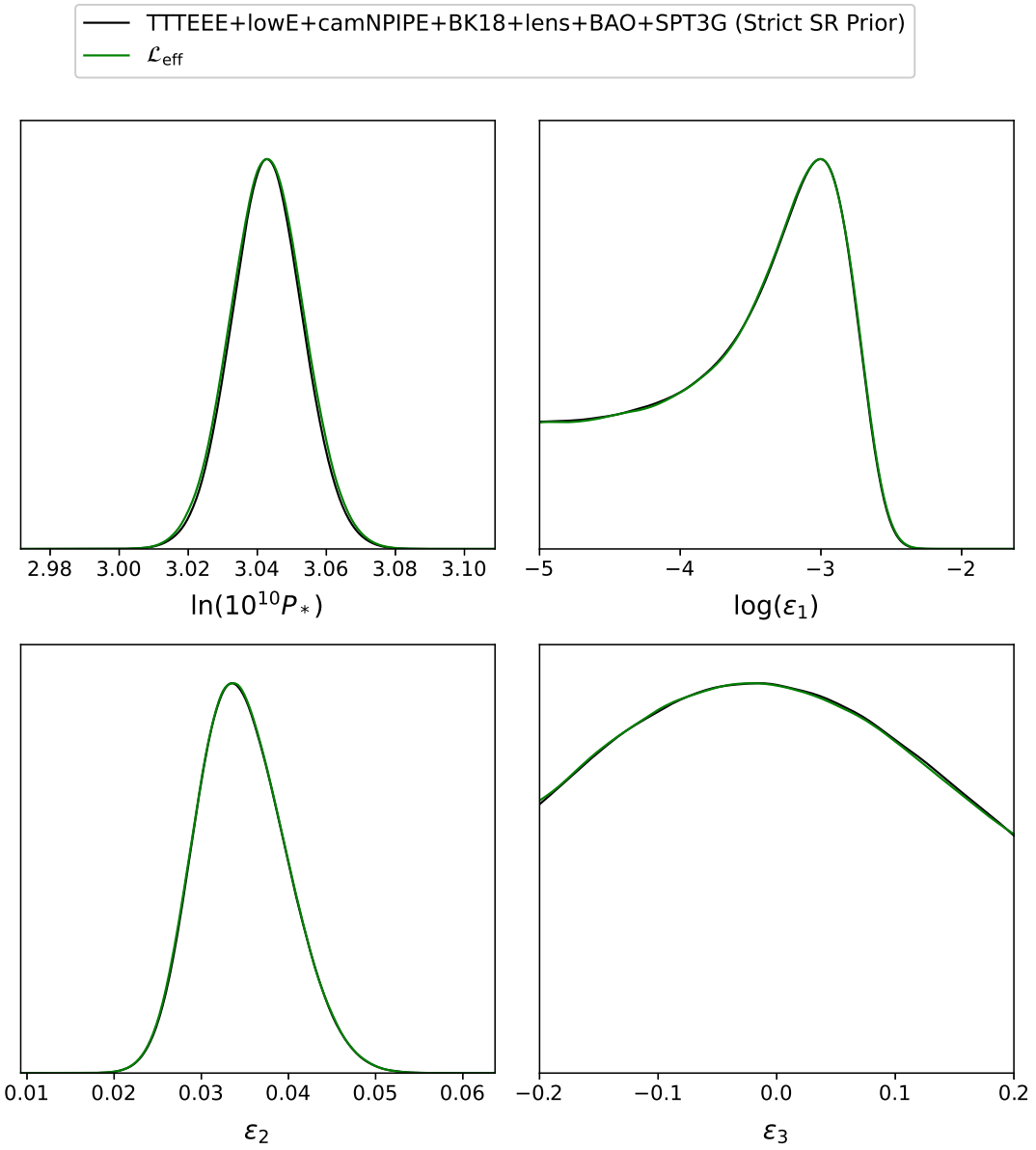
- Effective likelihood vs exact: 1D



Computing bayesian evidences

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- Probability of a model $\mathcal{M}$ to explain the data $\boldsymbol{D}$

$$P(\mathcal{M}|\boldsymbol{D}) = \frac{\mathcal{E}(\boldsymbol{D}|\mathcal{M})\, P(\mathcal{M})}{P(\boldsymbol{D})}$$

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- Bayesian evidence

$$\mathcal{E}(\boldsymbol{D}|\mathcal{M}) \propto \int \mathcal{L}_{\rm eff}(\boldsymbol{D}|P_*, \varepsilon_1, \varepsilon_2, \varepsilon_3)\, \pi(\boldsymbol{\theta}_{\rm inf}, R_{\rm rad})\, {\rm d}\boldsymbol{\theta}_{\rm inf}\, {\rm d}R_{\rm rad}$$

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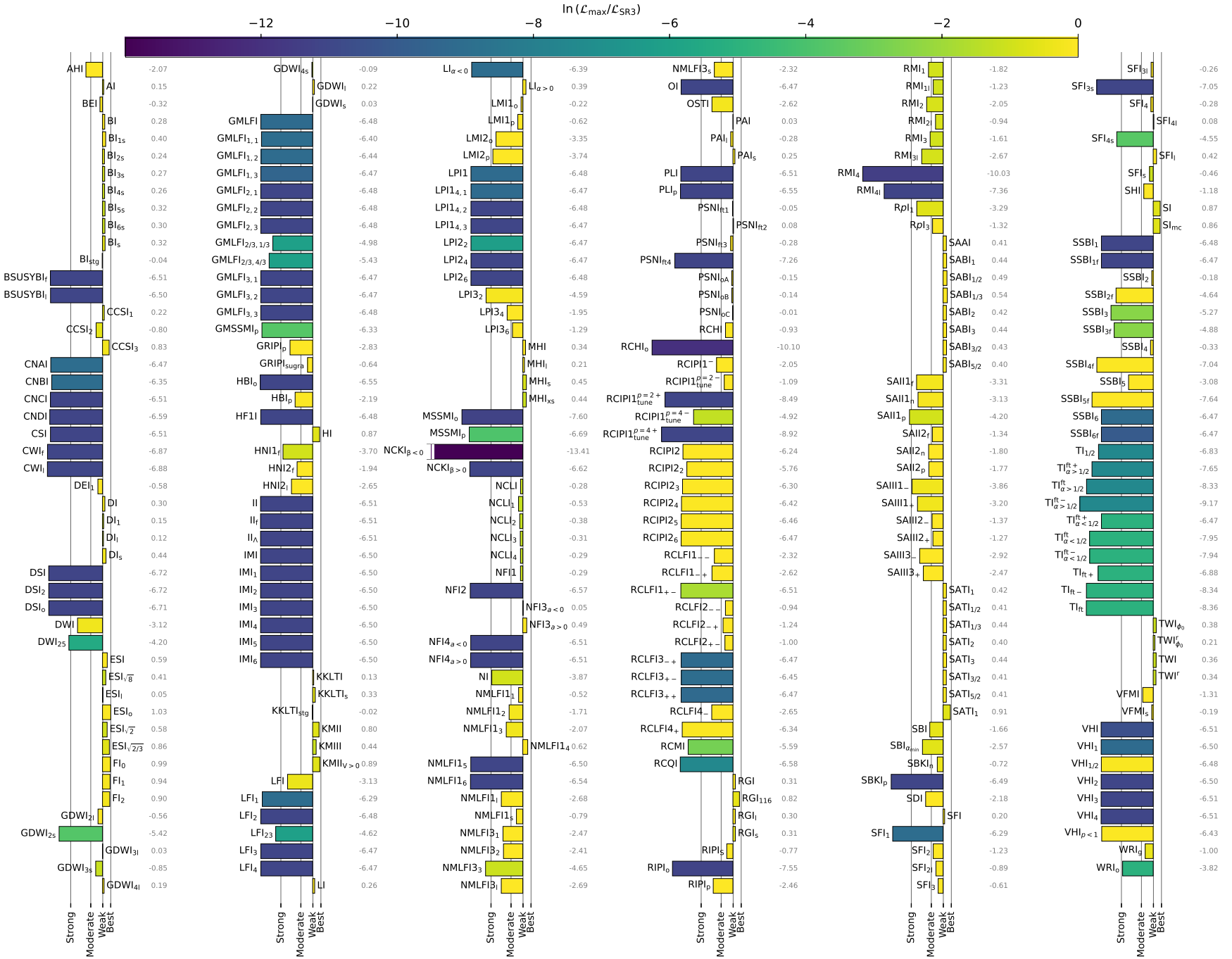
- Computed with `BAYASPIC` running over 287 models
  - ✦ $\texttt{BAYASPIC} \equiv \texttt{ASPIC} + \texttt{PolyChord} + \mathcal{L}_{\mathrm{eff}}$
  - ✦ A few cpu-hours per model $\mathcal{M}$ (but up to 2 days for some)
- $1\,\mathrm{TB}$ of data output (nested chains, posteriors, plots,...)

Bayes factors for all models



Information gain on the reheating

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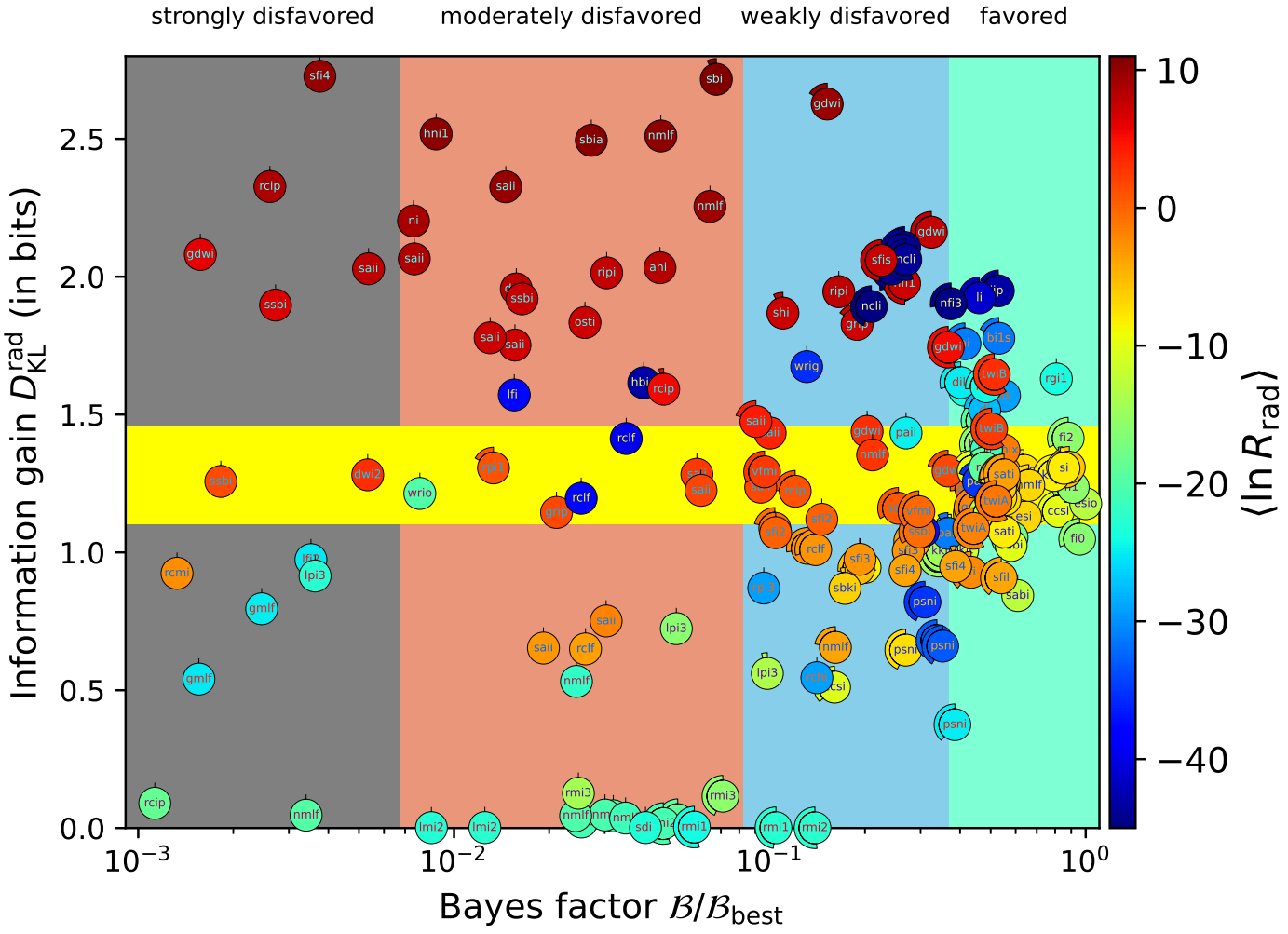
- Kullback-Leibler divergence between the prior and posterior

$$D_{\mathrm{KL}}^{\mathrm{rad}} = \int P(\ln R_{\mathrm{rad}}|\boldsymbol{D}) \ln \left[\frac{P(\ln R_{\mathrm{rad}}|\boldsymbol{D})}{\pi(\ln R_{\mathrm{rad}})} \right] \mathrm{d} \ln R_{\mathrm{rad}},$$



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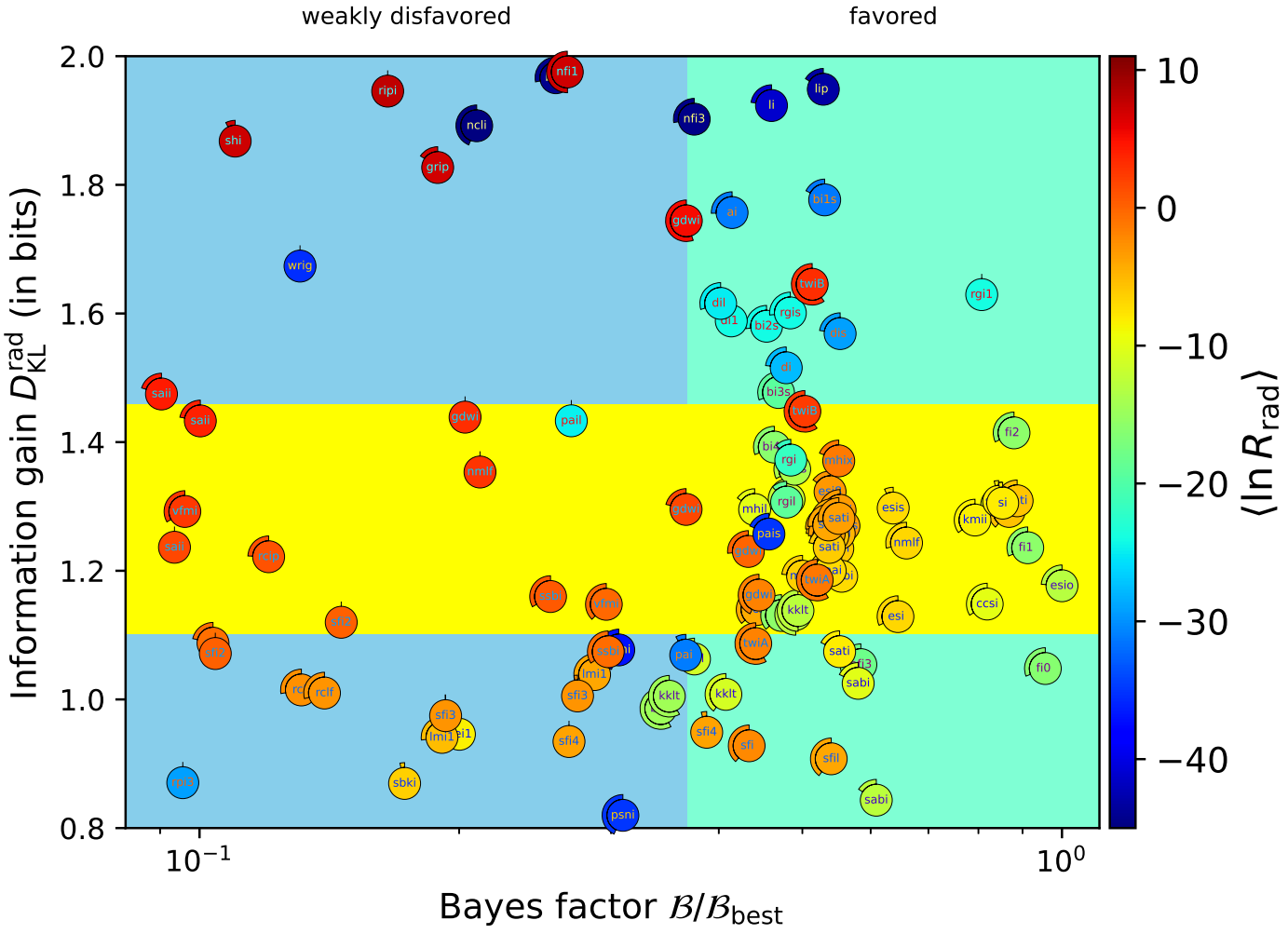

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Vanilla inflation predicts. . .

- Within the space of all slow-roll single-field models $\mathcal{I}_{\rm mod} \equiv \{\mathcal{M}_i\}$
- Posterior probability of the running $\alpha_{\rm S} \simeq -\epsilon_{2*}\left(2\epsilon_{1*} + \epsilon_{3*}\right)$

$$P(\alpha_{\rm S}|\boldsymbol{D}, \mathcal{I}_{\rm mod}) = \sum_i P(\alpha_{\rm S}|\boldsymbol{D}, \mathcal{M}_i)\, P(\mathcal{M}_i|\boldsymbol{D})$$

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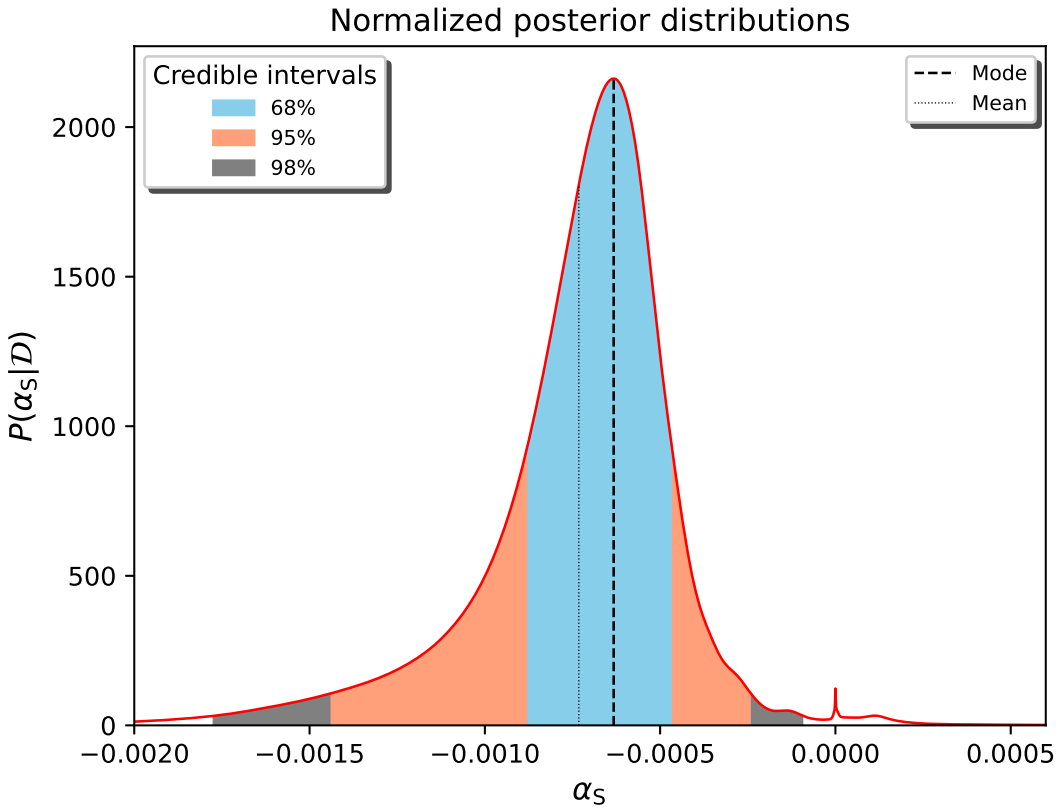

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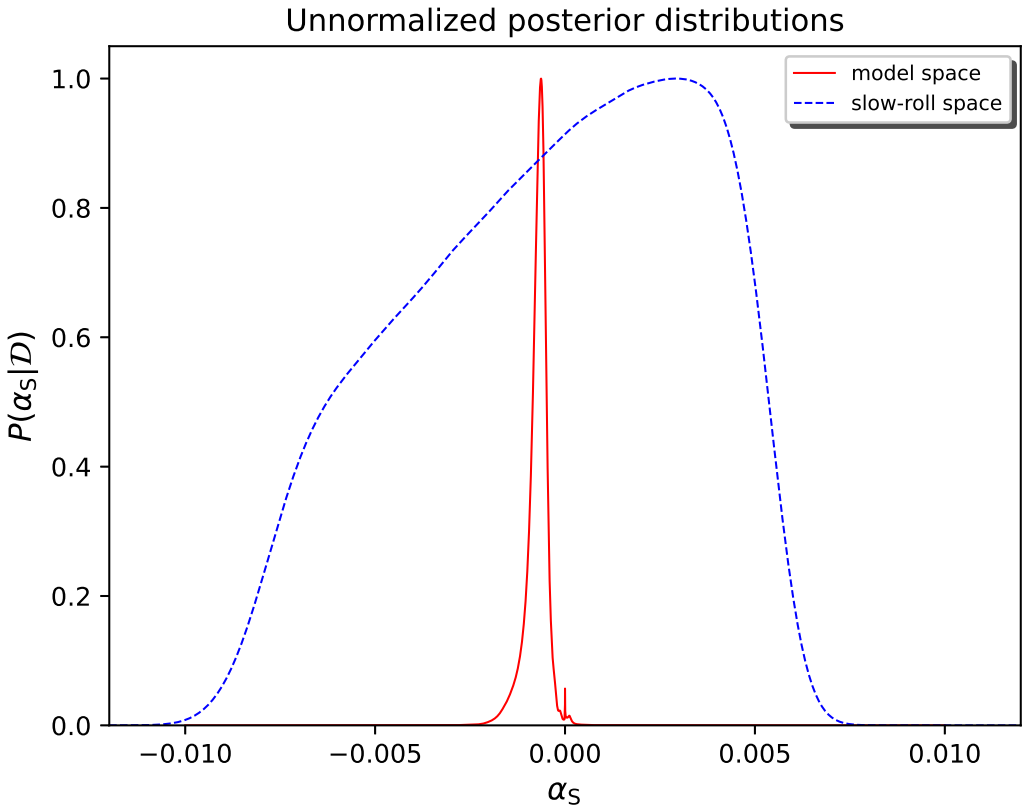

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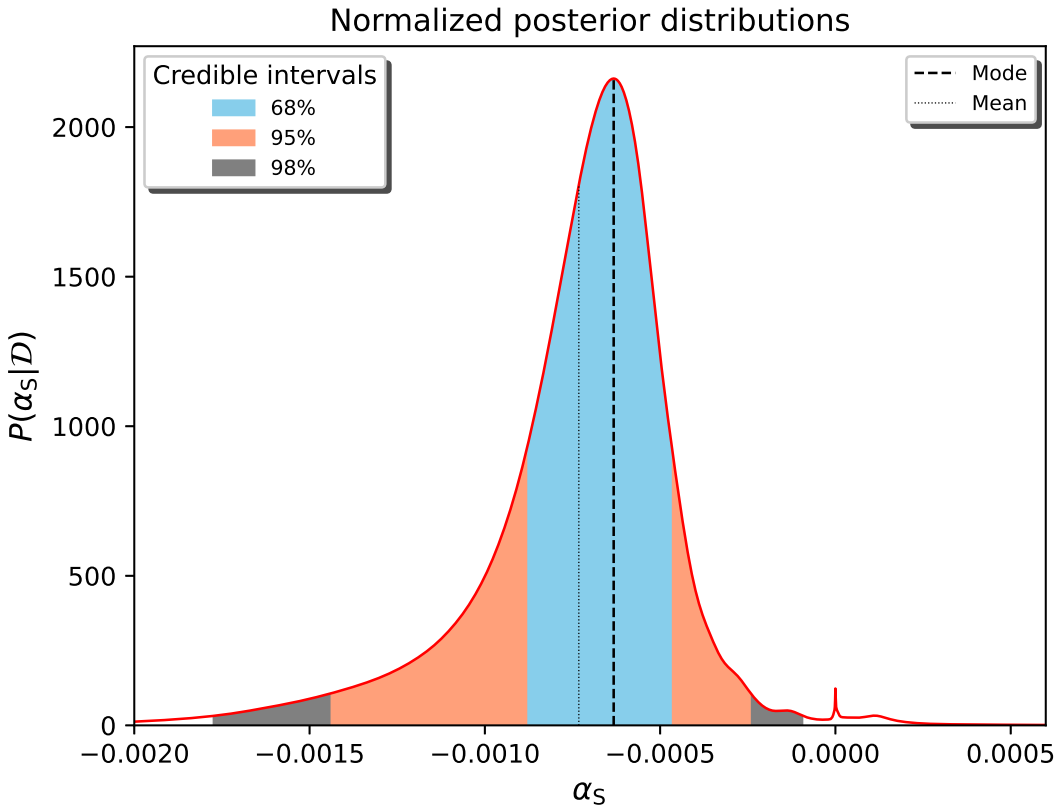

- Credible interval: $-1.8 \times 10^{-3} < \alpha_{\rm S} < -9.1 \times 10^{-5} \quad (98\%)$

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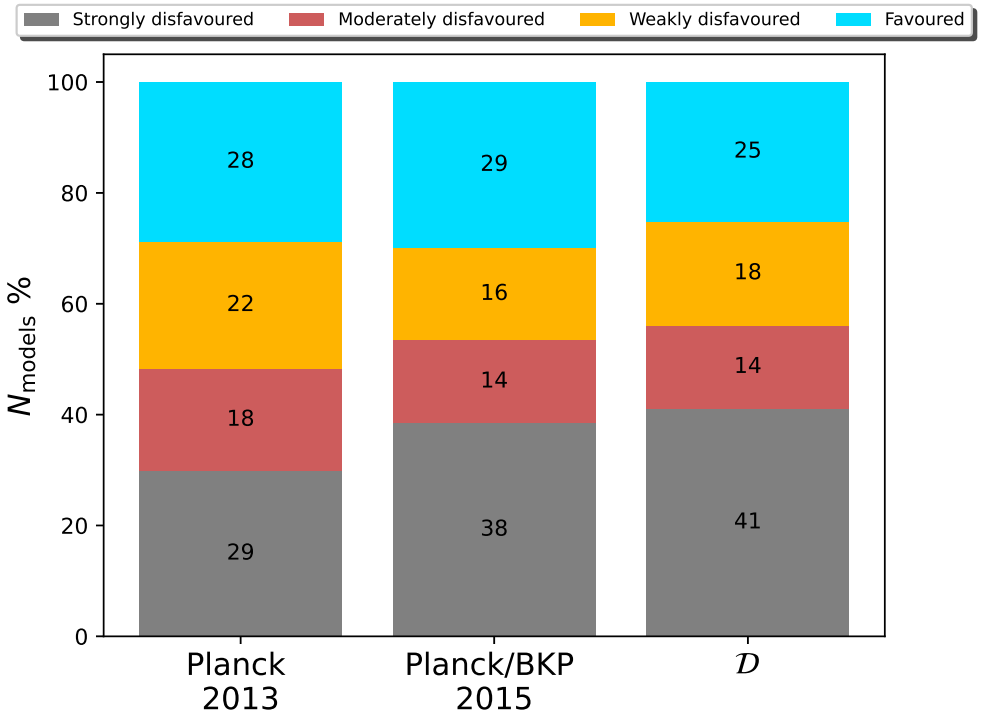
- Bayesian data analysis in model space $\mathcal{I}_{\rm mod}$
  - ✦ Enforces model consistency + new insights on the reheating era
  - ✦ Predicts: $\langle \alpha_{\rm S} \rangle = -7.3 \times 10^{-4}$
- Data constraining power is winning against theoretical proposals



- Looking forward to the Euclid, LSS & CMB-S4 data!