



Modified Gravity ??

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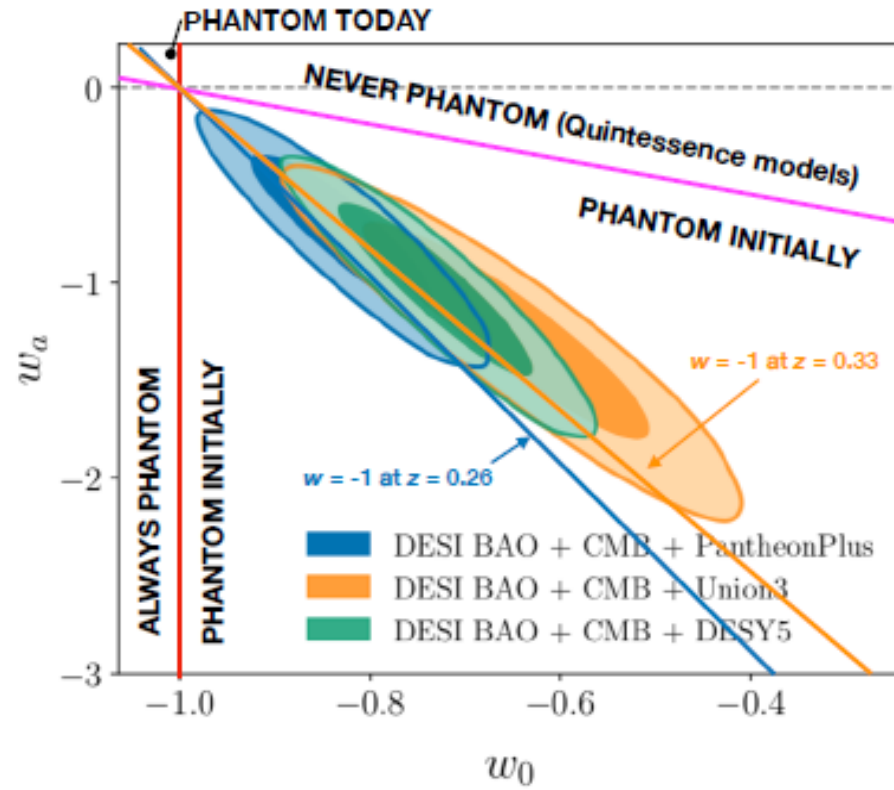
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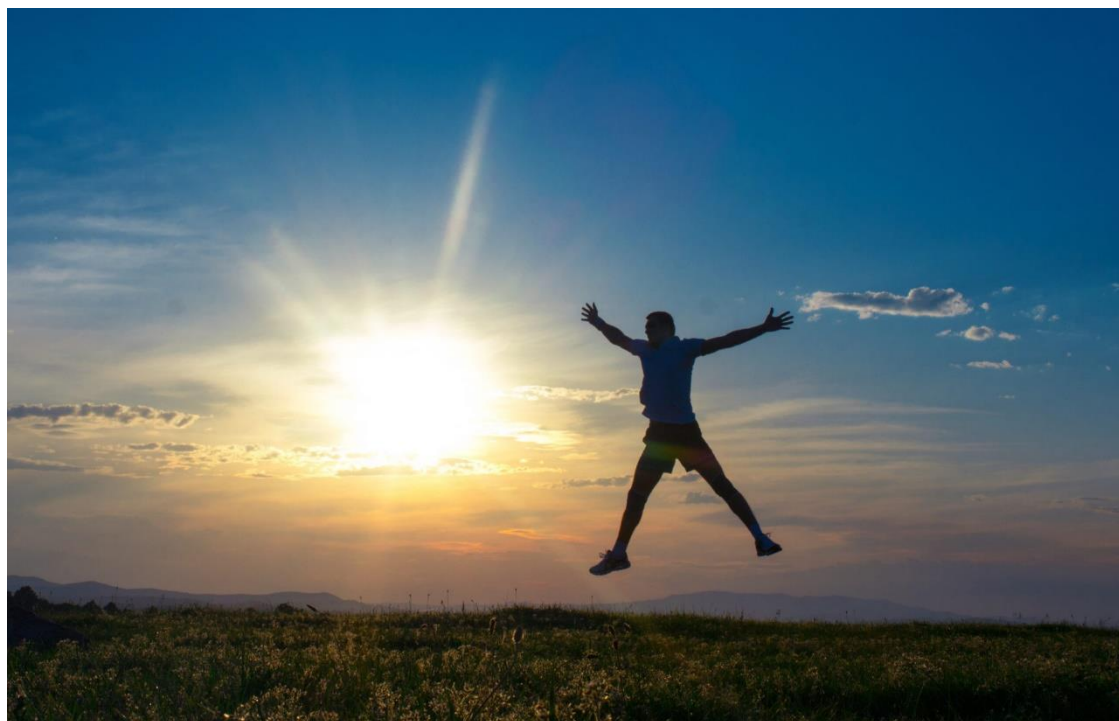
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+ to appear





E par si muove!



$$w_0 = -0.827 \pm 0.063 \quad w_a = -0.75^{+0.29}_{-0.25}$$

DESI + CMB + Pantheon+ $\Rightarrow 2.5\sigma$

$$w_0 = -0.64 \pm 0.11 \quad w_a = -1.27^{+0.40}_{-0.34}$$

DESI + CMB + Union3 $\Rightarrow 3.5\sigma$

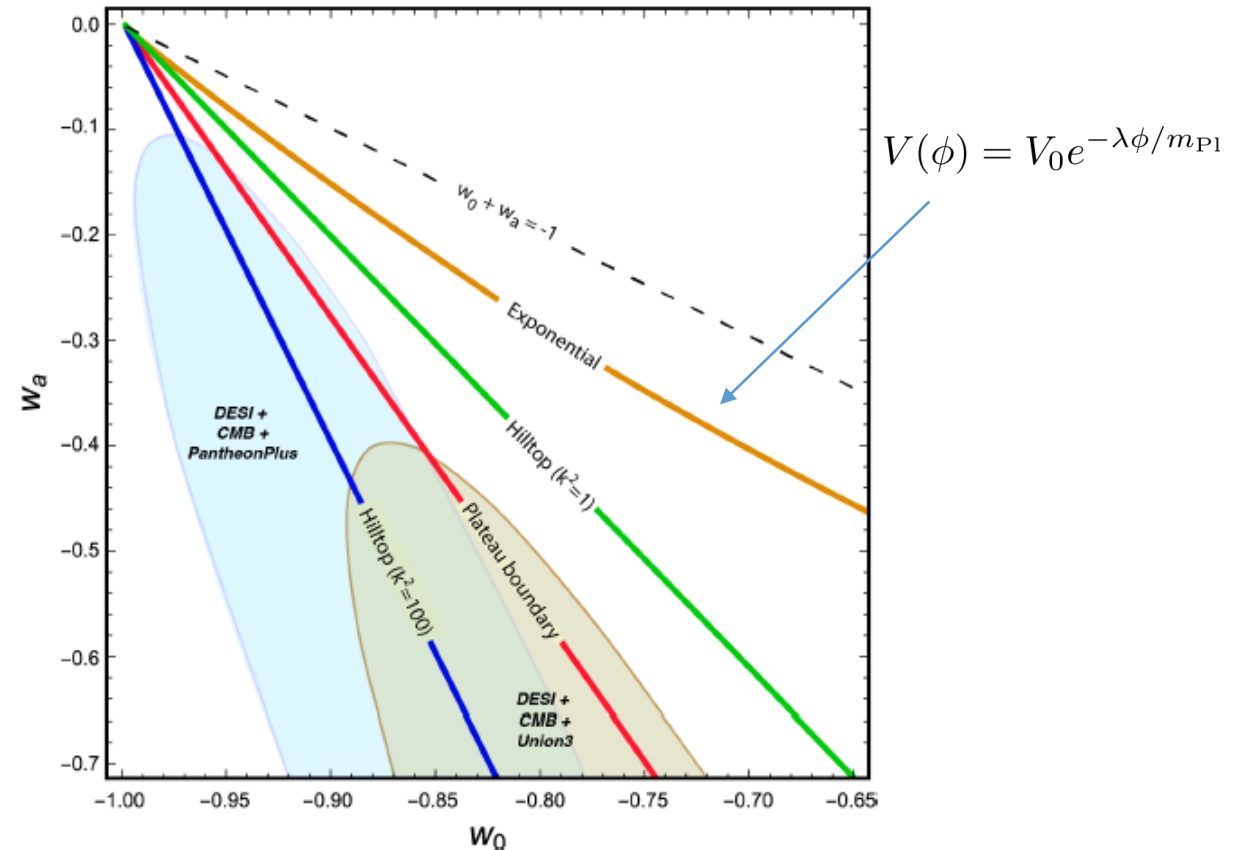
$$w_0 = -0.727 \pm 0.067 \quad w_a = -1.05^{+0.31}_{-0.27}$$

DESI + CMB + DES-SN5YR $\Rightarrow 3.9\sigma$

$$\omega(z) = \omega_0 + \omega_a \frac{z}{1+z}$$

Simple models can reproduce the data!

Is it expected?



The dark energy scale is in the ***pico-eV range***: apparent fine-tuning compared to standard model scales.

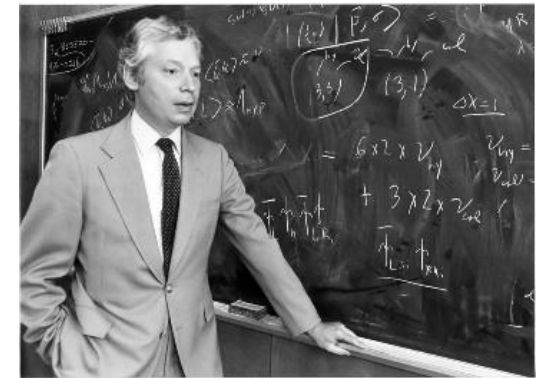
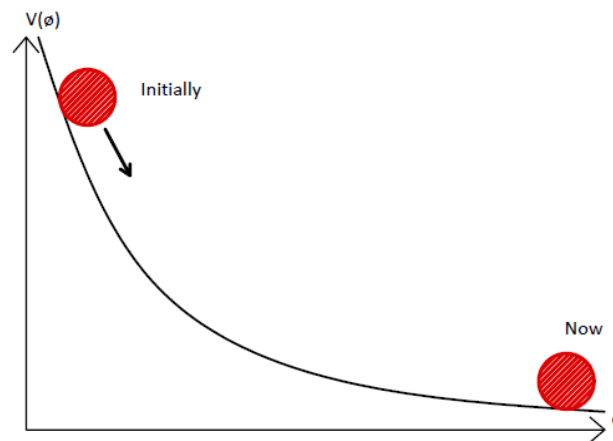
$$\delta\rho_\Lambda = M^4, \quad M \sim 100\text{GeV}$$

Weinberg's theorem states that there is no non-fine-tuned vacuum in a 4d quantum field theory respecting **Poincaré invariance**.

Dynamical configurations



Dark energy

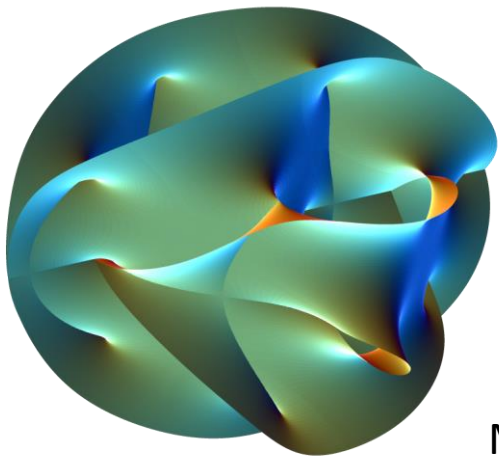


Scalar field rolling down its potential
Lorentz invariance implies the existence of a dark energy field which can be seen as the Goldstone model for the breaking of time translation invariance.

The most important stringy *conjectures* for dark energy are:

In an ideal world, string theory or any other version of quantum gravity would be finite so the vacuum energy could be calculable. Not the case in ordinary Quantum Field Theory.

- ✓ **The de Sitter conjecture:** a pure vacuum energy with no dynamics is not compatible with string theory.
- ✓ **The vacuum conjecture:** Empty space-time is described by the dynamics of at least one scalar field with a potential such that



Moduli could be “sizes” of extra-dimensions

$$\left| \frac{dV}{d\phi} \right| \geq c \frac{V}{m_{\text{Pl}}}$$

$$c = \mathcal{O}(\sqrt{2})$$

This forbids very flat potentials. This favours runaway potentials where the field is a “moduli”.

Some expected features:


- Dark energy is determined by the position of the field now:

$$3\Omega_\Lambda H_0^2 m_{\text{Pl}}^2 = V(\phi_{\text{now}})$$

- The field is ***extremely light***:

$$m_\phi^2 = \left. \frac{d^2 V}{d\phi^2} \right|_{\text{now}} \sim \frac{V_{\text{now}}}{m_{\text{Pl}}^2} = 3\Omega_\Lambda H_0^2$$

Mass of the order
of the Hubble rate

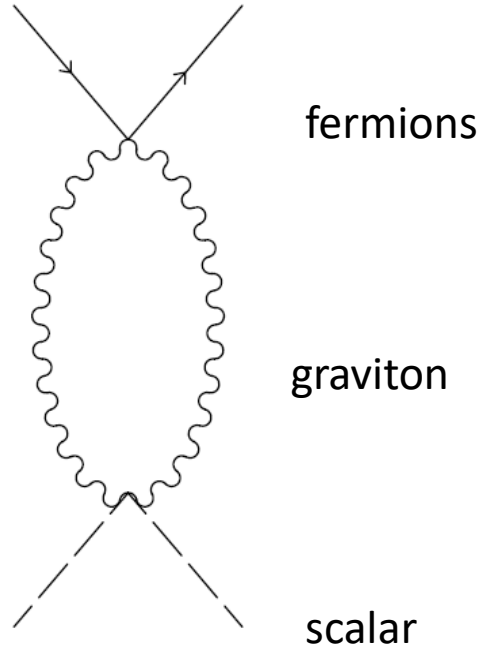

$$H_0 \sim 10^{-42} \text{ GeV}$$

Problem: The coupling to matter

small

$$\beta \sim \frac{H^2}{m_{\text{Pl}}^2} \int \frac{d^4 p}{p^4}$$

Divergent



$$\mathcal{L} \supset -\frac{\beta}{m_{\text{Pl}}} m_\psi \phi \bar{\psi} \psi$$

Yukawa interaction similar to the Higgs interaction to matter.

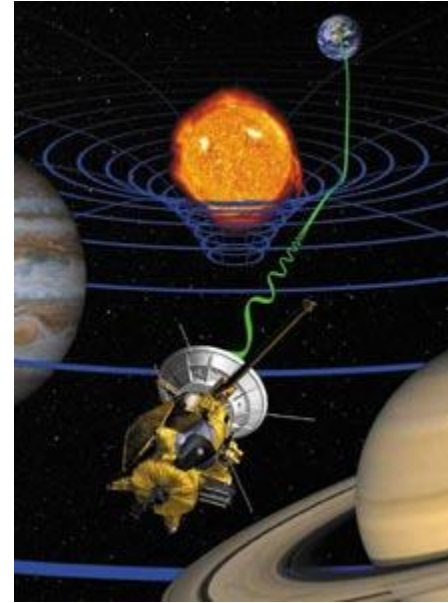
No *reason* to assume $\beta = 0$

Deviations from Newton's law are parametrised by:

$$\Phi = -\frac{G_N M}{r} (1 + 2\beta^2 e^{-r/\lambda})$$

For long range forces with large λ , the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay around a big object: the Sun):

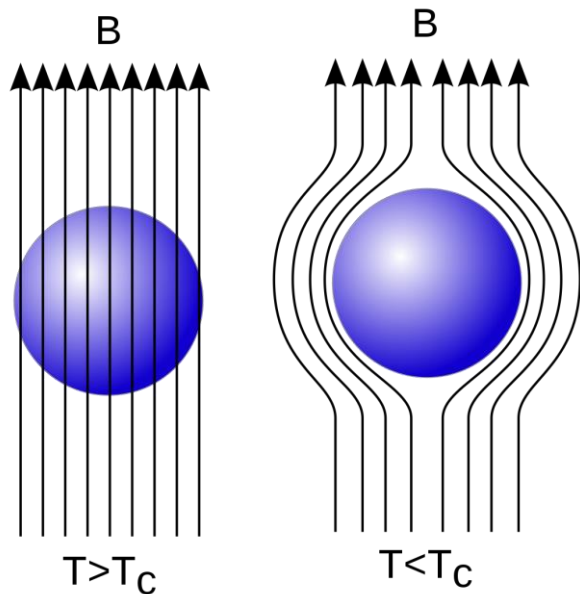
$$\beta^2 \leq 4 \cdot 10^{-5}$$



Bertotti et al. (2004)

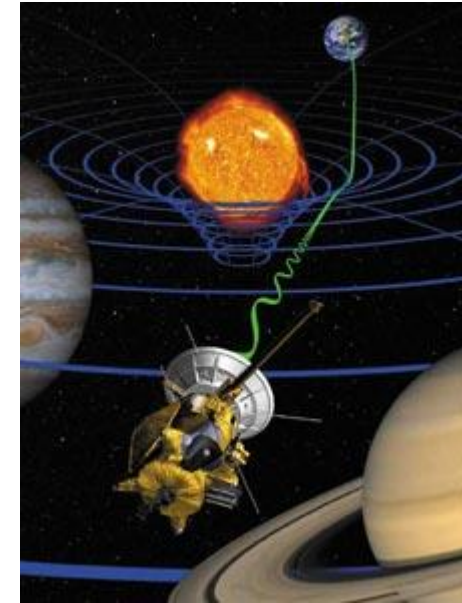
Two options:

Symmetry: the shift symmetry of a Goldstone boson prevents such a coupling BUT the symmetry is broken by the potential so the problem is reintroduced!



Screening

Analogous to the Meissner effect in superconductors: the inside of the solar system is free of scalar field lines.

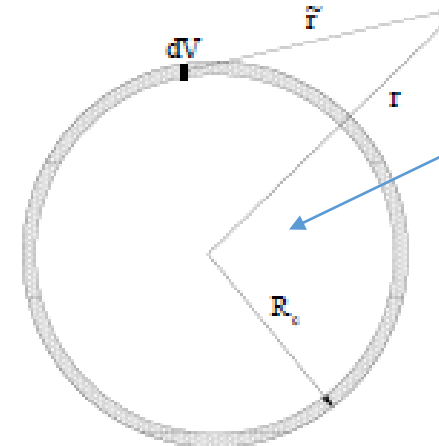
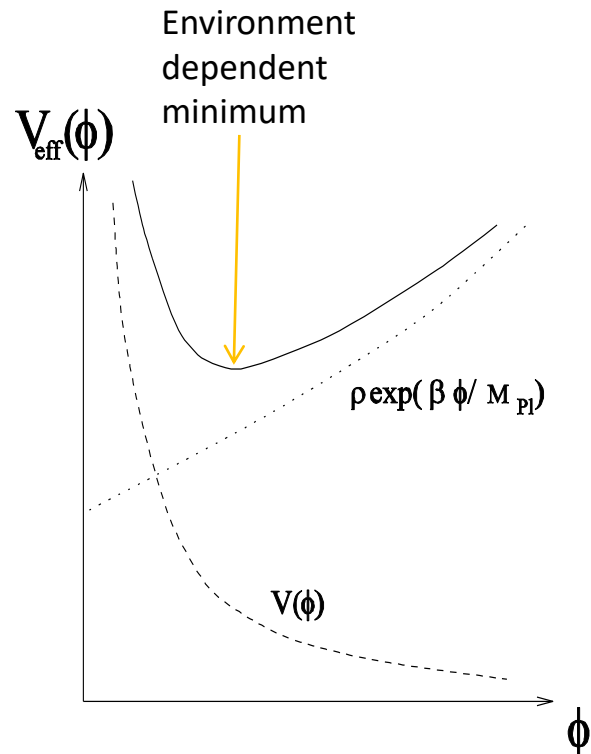


—————> No gradient=no fifth force

Chameleon Screening

When coupled to matter, scalar fields have a *matter dependent effective potential*

$$V_{\text{eff}}(\phi) = V(\phi) + \rho_m (A(\phi) - 1)$$



Large mass
inside object

$$m_{\text{in}} R \gg 1$$

The field generated from deep inside is Yukawa suppressed. Only a *thin shell* radiates outside the body. Hence suppressed scalar contribution to the fifth force.

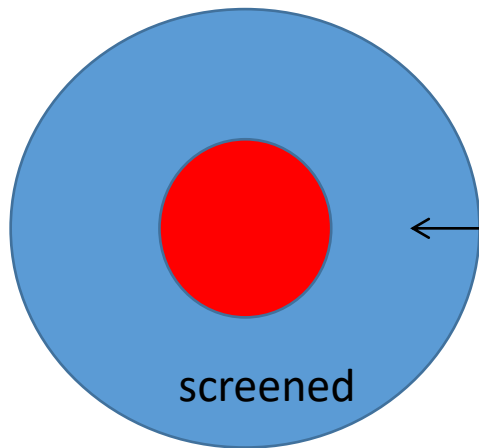
Vainshtein Mechanism in a nutshell

We can use a simple example with higher derivatives:

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2\Lambda^3}(\partial\phi)^2\Box\phi + \frac{\beta\phi}{M_P}T.$$

$$\Lambda^3 = H_0^2 m_{\text{Pl}}$$

Non-linearity



$$\frac{F_\phi}{F_N} = 2\beta^2 \left(\frac{r}{R_V}\right)^{3/2}.$$

Very low cutoff scale!

$$R_V = \left(\frac{\beta M_c}{2\pi M_P}\right)^{1/3} \frac{1}{\Lambda}.$$

Well inside the Vainshtein radius, Newtonian gravity is restored. Well outside, gravity is modified.

The Vainshtein radius is very large for stars, typically 0.1 kpc for the sun.

These theories are **beyond** normal effective theories

$$V(\phi) = \Lambda_0^4 + \frac{\Lambda^{n+4}}{\phi^n}$$

Inverse power laws... and need a cosmological constant

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2\Lambda^3}(\partial\phi)^2 \square\phi + \frac{\beta\phi}{M_P} T .$$

Vainshtein when:

$$\square \geq H_0^2$$

Need to work beyond the validity of the derivative expansion.... (possible way out: non-renormalisation theorems)

Bringing modified gravity back to the fold:

Take heed from successful physics models:

The standard model of particle physics:

$$\mathcal{L} \supset \frac{\mu^2}{2} H^2 - \frac{\lambda}{4} H^4 + y H \bar{\psi} \psi$$

Inflation :

$$\mathcal{L} = \frac{R}{16\pi G_N} + cR^2 + \dots$$

Starobinski: simple curvature expansion around GR...

Simplest lowest order effective theory coupling Higgs and fermions

Multi-field dark energy sector

$$\mathcal{L} = -\frac{1}{2}g_{ij}(\phi^k)\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi^i)$$

Screening can only happen with more than one field and a non-trivial σ -model metric.

Your favourite dark energy potential with no wacky potential in it.

The axio-dilaton system

$$\mathcal{L} \supset -\frac{1}{2}((\partial\phi)^2 + W^2(\phi)(\partial a)^2)$$

Choose W to have a minimum:

$$W^2(\phi) = 1 + \frac{(\phi - \phi_\star)^2}{2\Lambda_\phi^2}$$

The axion is chosen to have a simple potential:

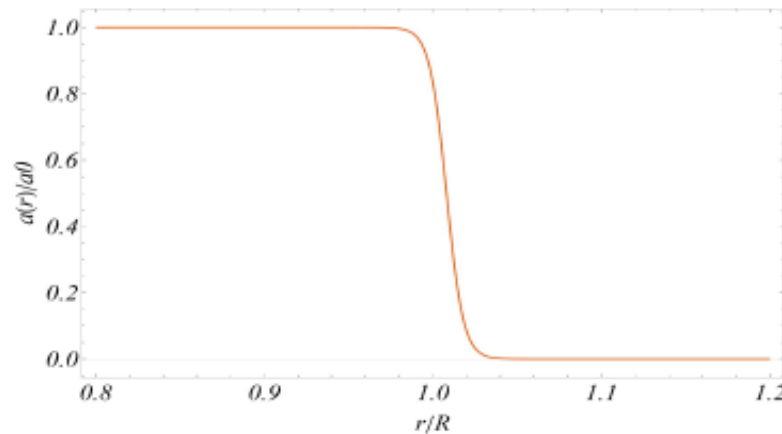
$$V(a) = \frac{1}{2}m_a^2(a - a_+)^2 + \rho_m \frac{(a - a_-)^2}{2\Lambda_a^2}$$

$$V_{\text{QCD}}(a) = -\Lambda_{\text{QCD}}^4 \left(1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2}\right)\right)^{1/2}$$

$$U(a) = \sigma_B \cos \frac{a}{2}$$

The axion interpolates between the two minima close to stars.

QCD-inspired...



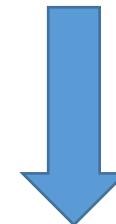
$$\mathcal{L} \supset -\frac{1}{2}((\partial\phi)^2 + W^2(\phi)(\partial a)^2)$$

The large gradients impose that the dilaton does not vary much locally:

$$\phi \approx \phi_\star$$

Screening !

Large gradients



Minimised when W is minimal

Early dark energy for free!

$$V(a) = \frac{1}{2}m_a^2(a - a_+)^2 + \rho_m \frac{(a - a_-)^2}{2\Lambda_a^2}$$

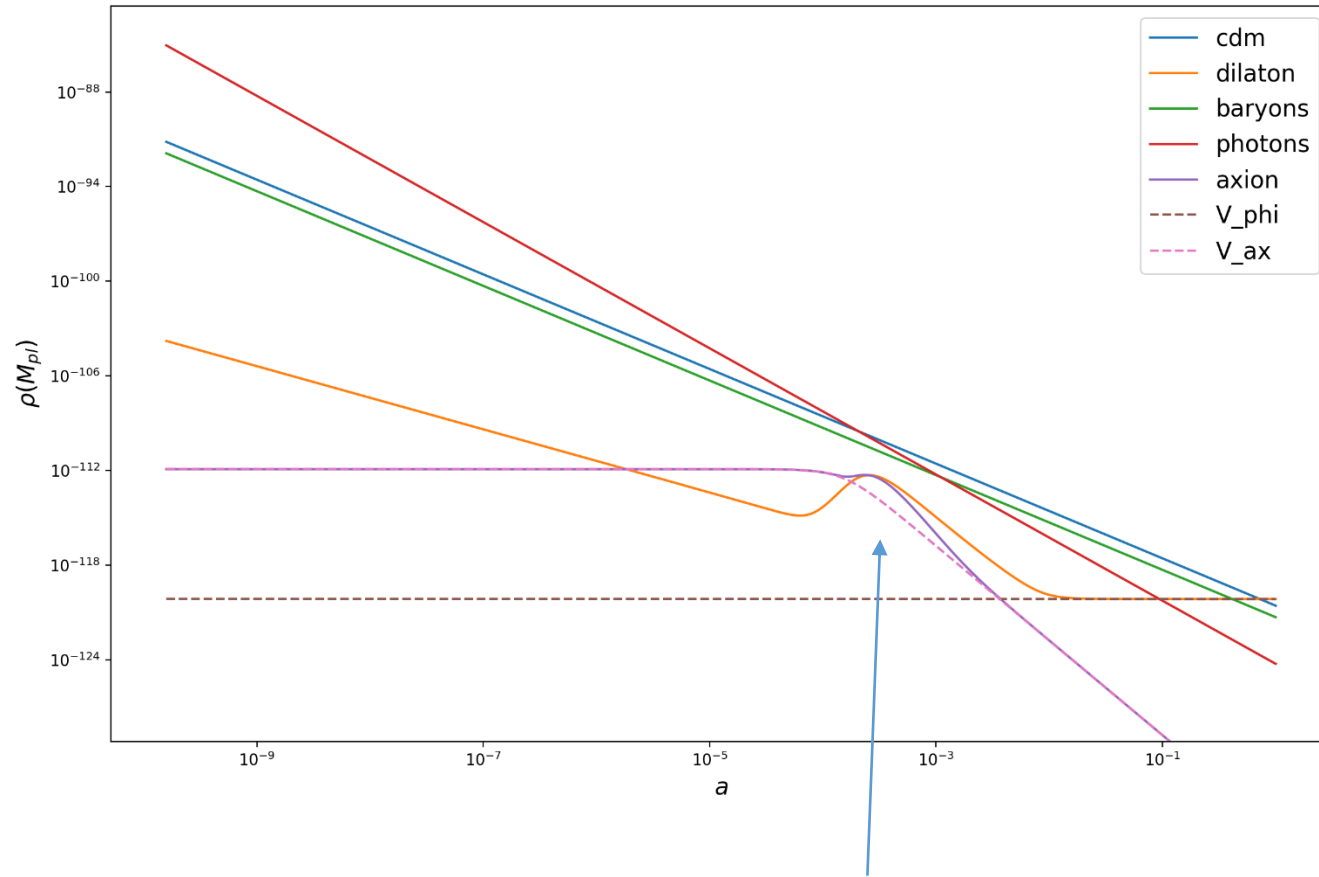
A fraction of added matter

$$\rho_m \gg \Lambda_a^2 m_a^2 \quad V(a_-) = \frac{1}{2}m_a^2(a_+ - a_-)^2$$

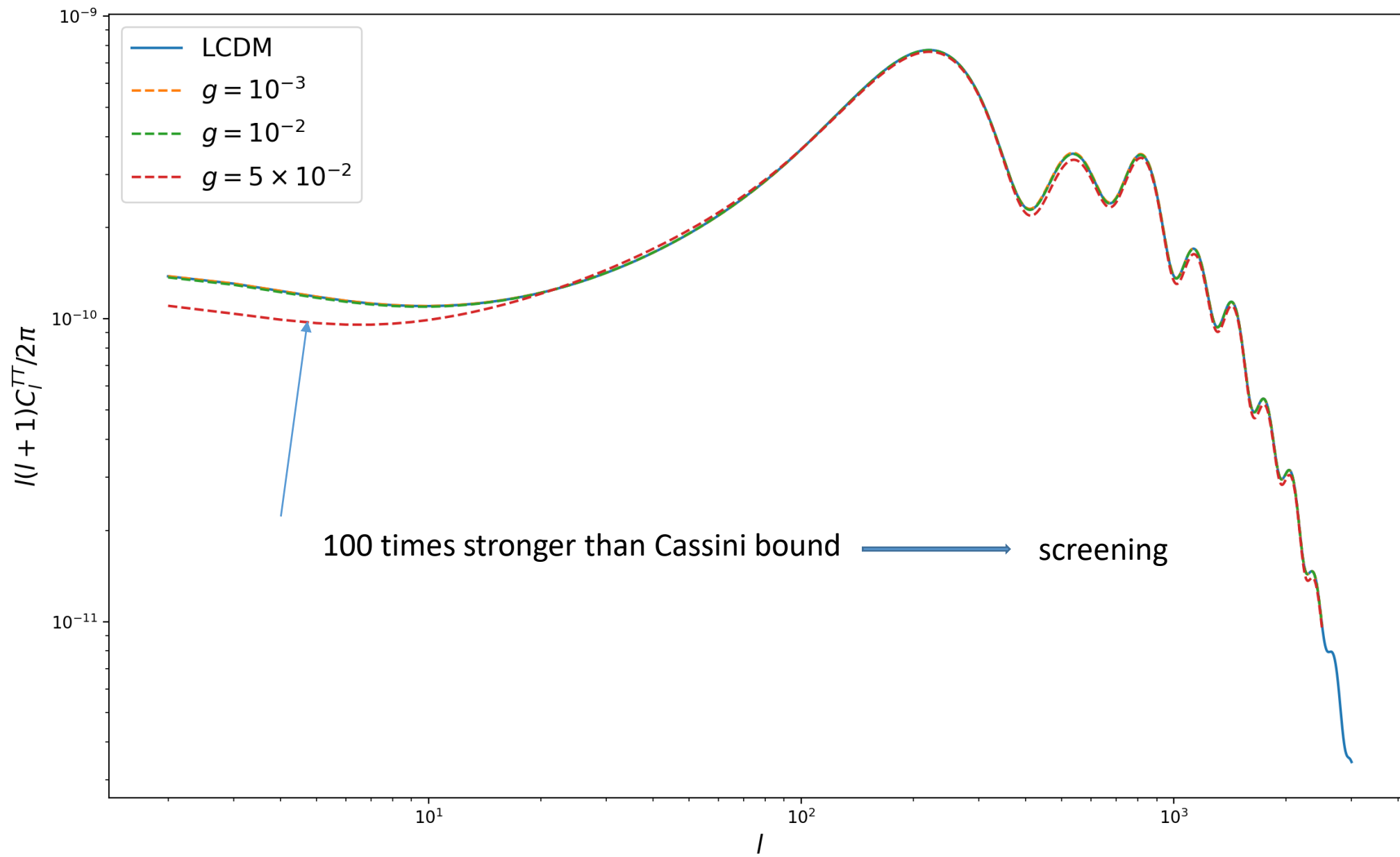
Early dark energy

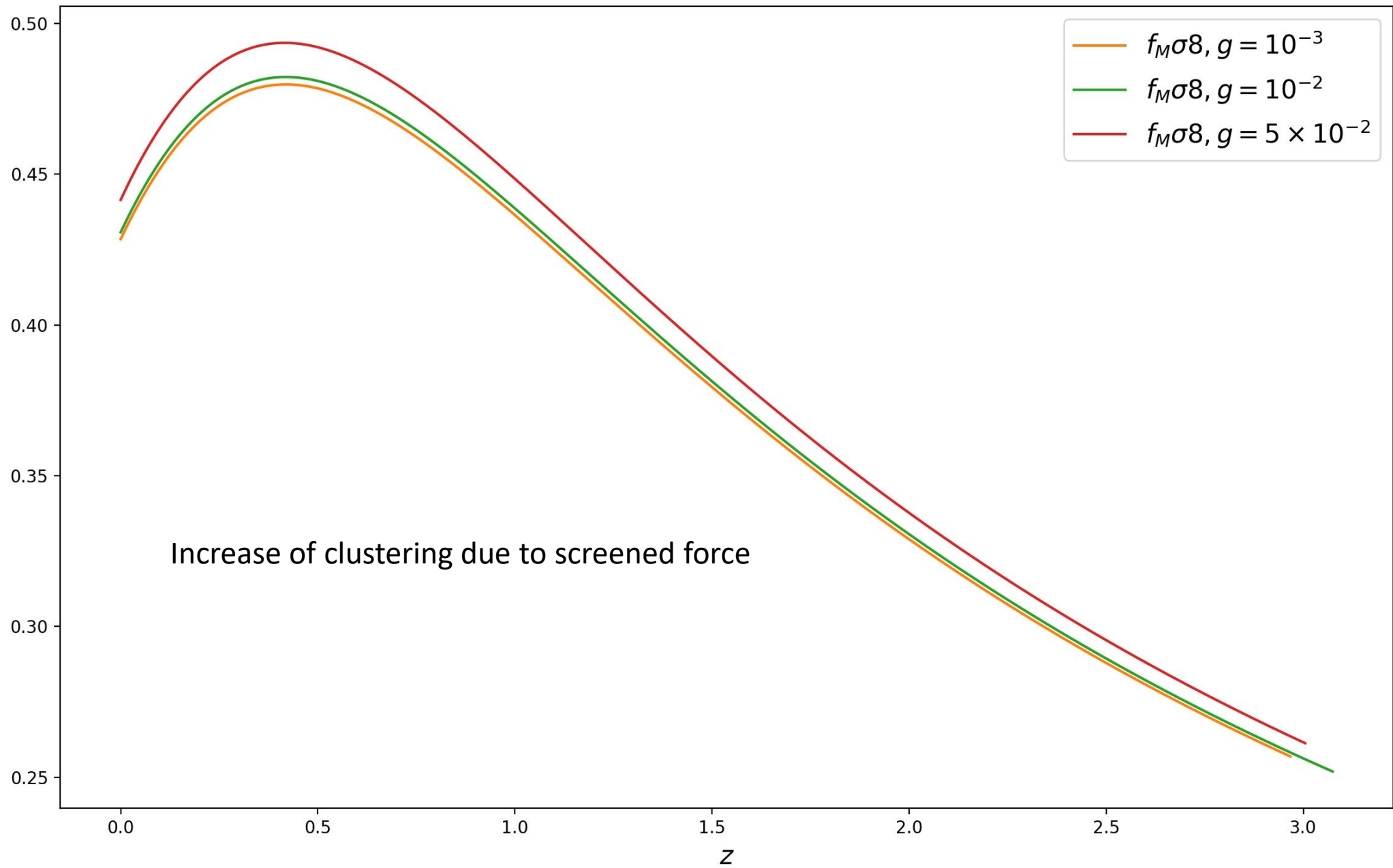
$$\rho_m \ll \Lambda_a^2 m_a^2 \quad V(a_+) = \rho_m \frac{(a_+^2 - a_-)^2}{2\Lambda_a^2}$$

Exponential potential

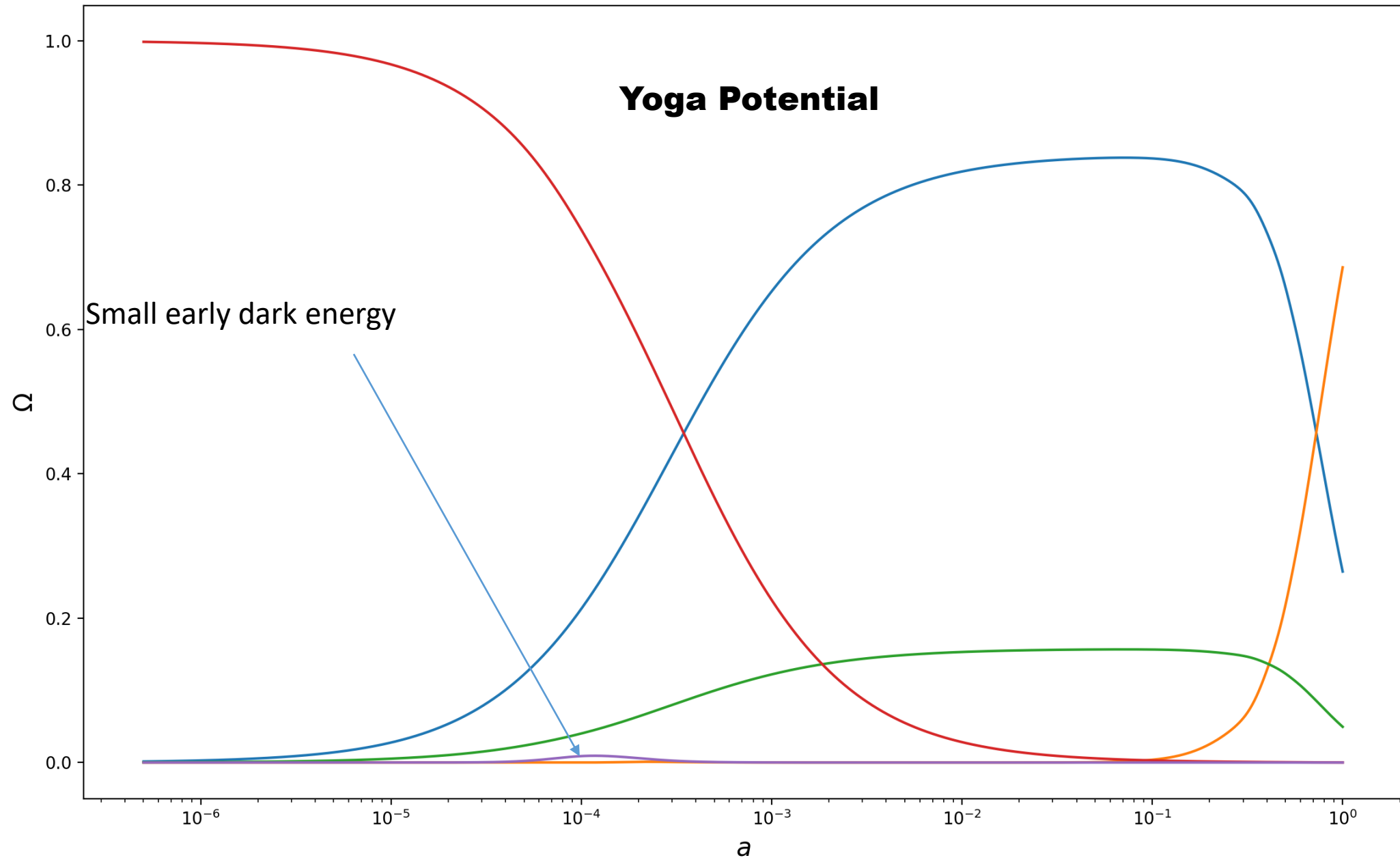


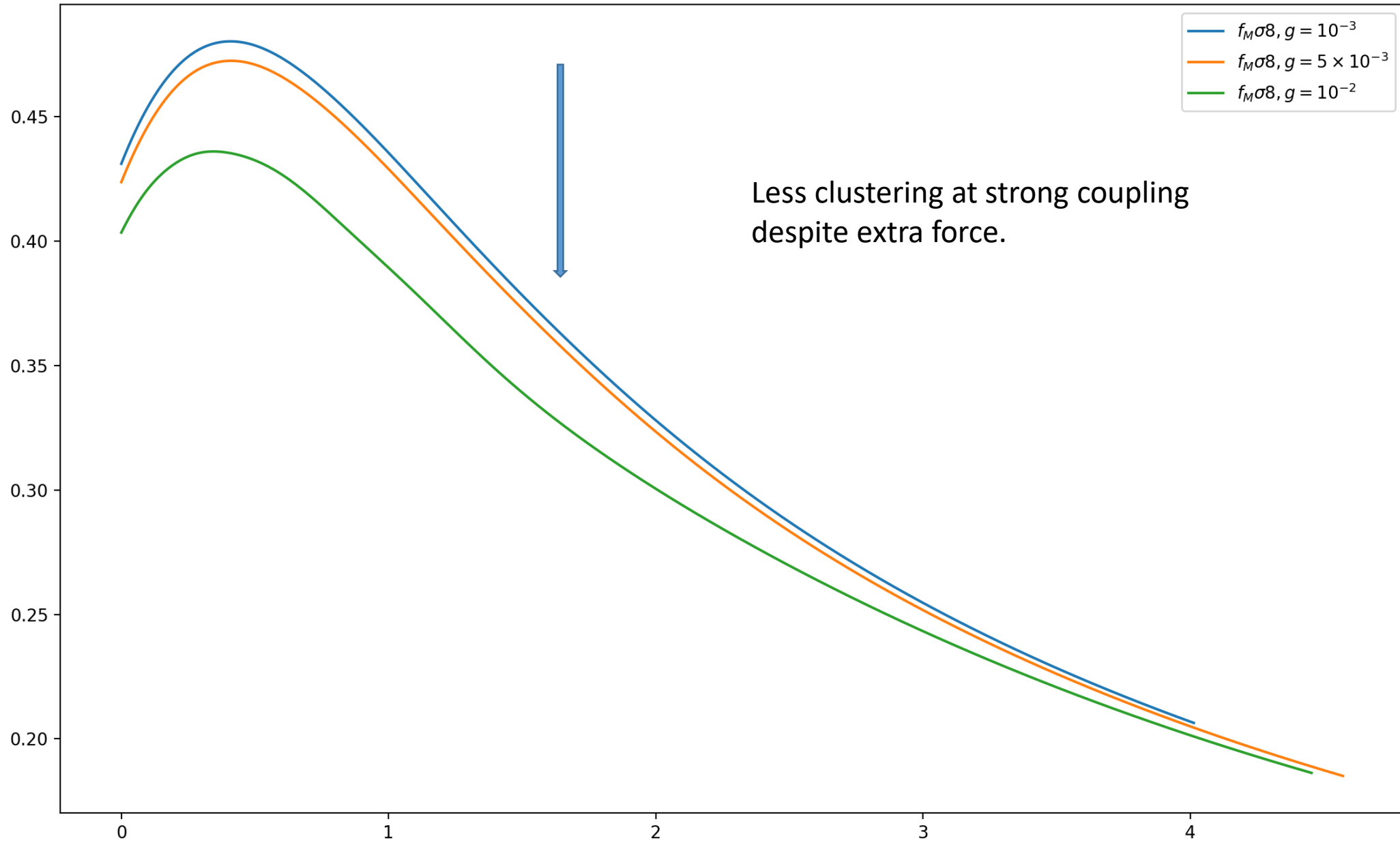
Screening induces an instability limiting the amount of early dark energy





$$V(\phi) = \text{polynomial}(\ln \phi) e^{-\lambda\phi/m_{\text{Pl}}}$$





Summary

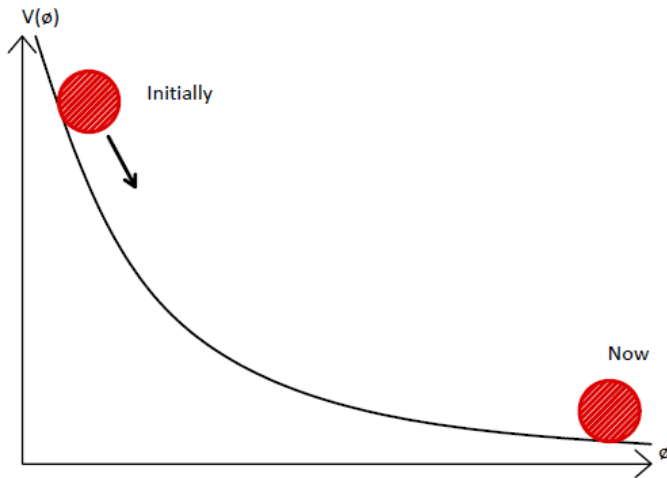
- ❖ Dynamical dark energy would lead to large deviations from General Relativity locally: needs screening.
- ❖ Screening can be achieved in the multi-field setting with nothing beyond standard field theory.
- ❖ Questions:
 - With one field there are 3 possible screening mechanisms: here with multiple fields??
 - Is there a cosmological signature: ISW? Clustering?
 - Are there effects on the equivalence principle: Microscope?
 - Could screening be obtained from the moduli space of a string compactification ?
- ❖ Could we construct a realistic model of dynamical dark energy with screening?

Back up

Runaway dilaton model:

Focus on simplest potential with runaway behaviour:

$$\gamma = \frac{2}{\lambda}, \quad \alpha = \frac{2}{\lambda^2}$$



Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$V(\phi) = V_0 e^{-\lambda\phi/m_{\text{Pl}}}$$

Friedmann:

$$H^2 = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho}{3m_{\text{Pl}}^2}$$

← matter

Initially dark energy is subdominant and then starts dominating when matter becomes very small.

$$\phi = \phi_* + \gamma m_{\text{Pl}} \ln \frac{t}{t_*}$$

$$a = a_* \left(\frac{t}{t_*} \right)^\alpha$$