Compatibility and combination of W boson mass measurements

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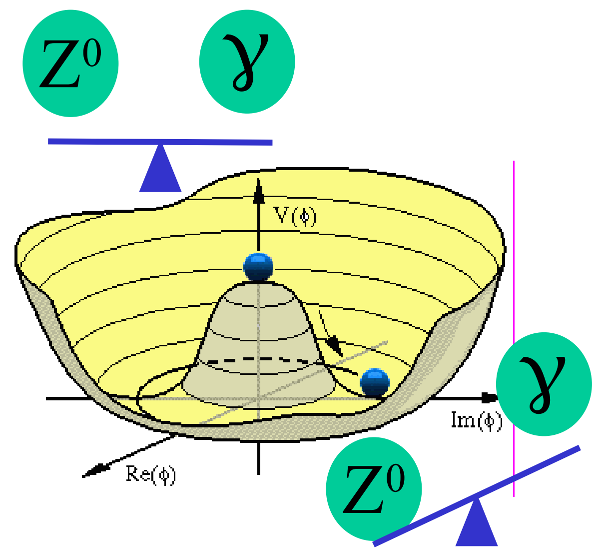
on behalf of the LHC/Tevatron MW Working group

W boson mass and Standard Model

- Standard Model
  - Electroweak symmetry breaking by Higgs mechanism thanks to a $SU(2)_L x U(1)_Y$ doublet
  - Few parameters to describe the EW gauge sector
  - W boson mass determined by 3 other parameters at leading order

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2} G_F}$$

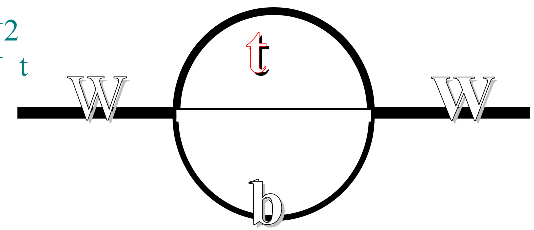


W boson mass and Standard Model

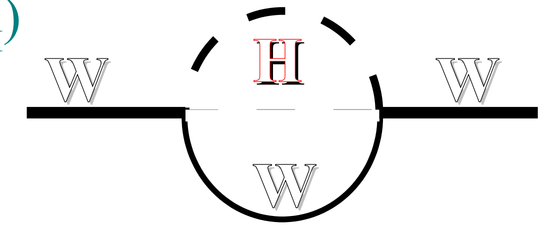
- Standard Model
  - Quantum corrections modify the relation between masses and couplings

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\times\frac{1}{1-\Delta r}$$

$\Delta M_W \alpha\ M_t^2$



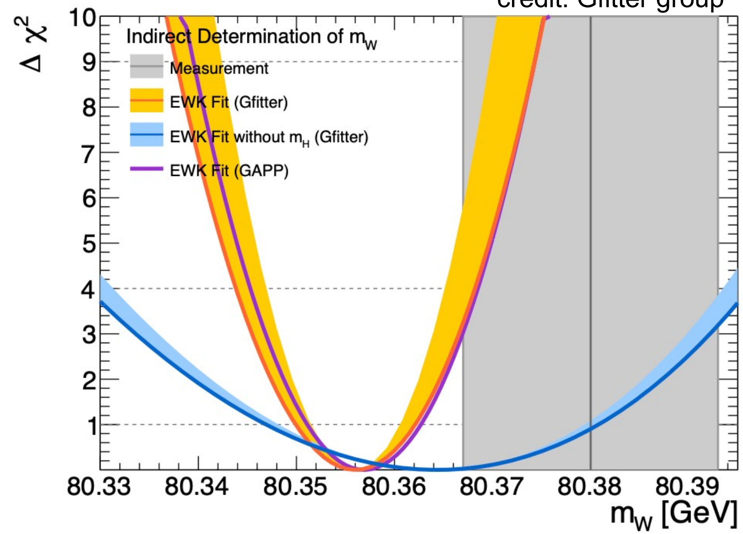
$\Delta M_W \alpha\ \text{-log}(M_H)$



  - Sensitivity to top mass, Higgs mass, and unknown particles

- Precision measurements of W boson mass
  - test Standard Model consistency
  - probe new physics and new particles at multi-TeV scales

credit: Gfitter group



40 years of studies

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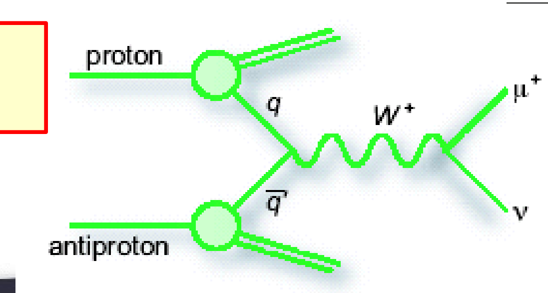
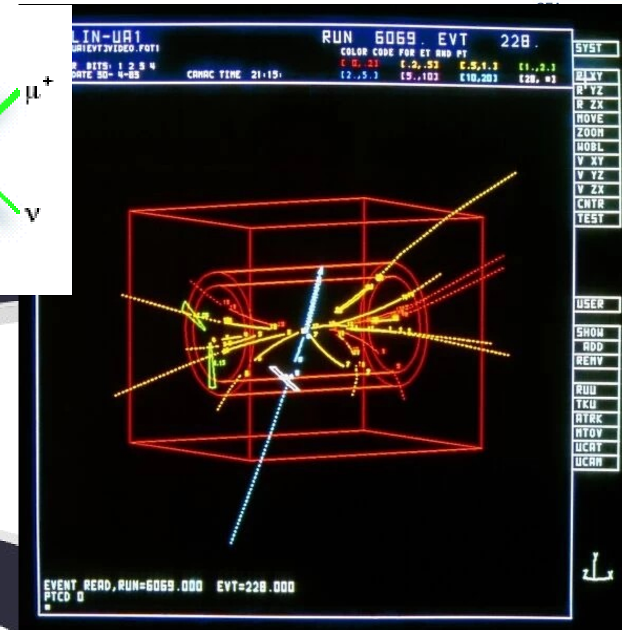
1983: Discovery at CERN UA1 and UA2
1985: M_W=83.1 ± 1.5 GeV





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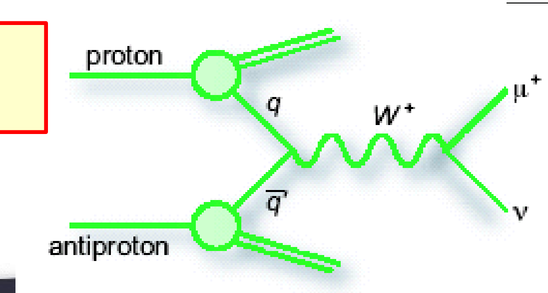

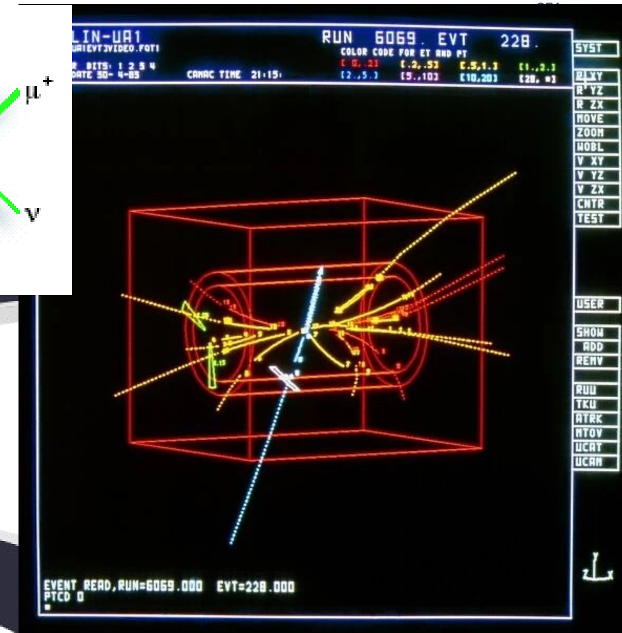

D0 and CDF at Tevatron Run 0/1 1988-1996
1990 CDF: M_W=79.910 ± 0.390 GeV
1993 D0: M_W=80.350 ± 0.270 GeV
2004 D0+CDF: M_W=80.456 ± 0.059 GeV

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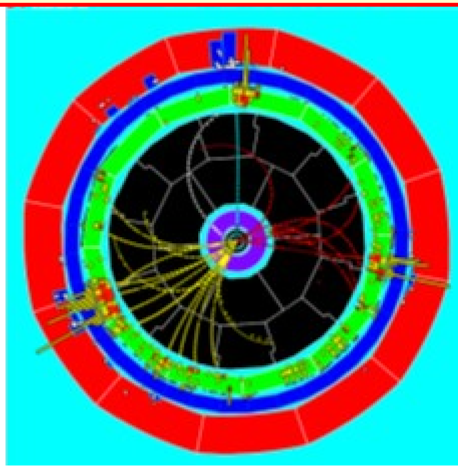

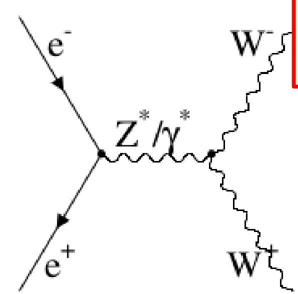

CERN LEP2 e^+e^- collider 1996-2000
LEP2 1998: M_W=80.340 ± 0.098 GeV
LEP2 2006: M_W=80.376 ± 0.033 GeV

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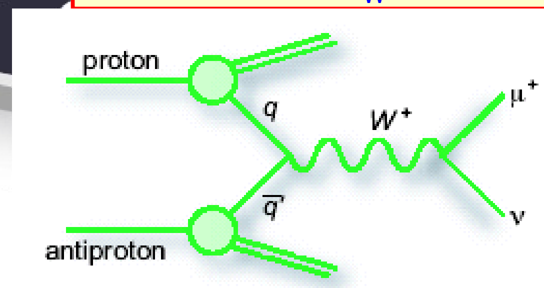
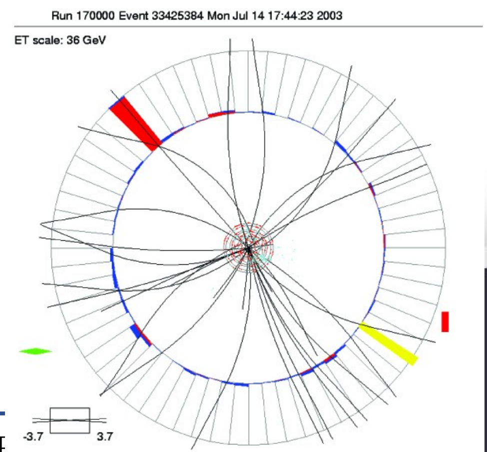


D0 and CDF at Tevatron Run 2 2001-2011
2012 D0: M_W=80.375 ± 0.023 GeV
2012 CDF: M_W=80.387 ± 0.019 GeV
2022 CDF: M_W=80.434 ± 0.009 GeV



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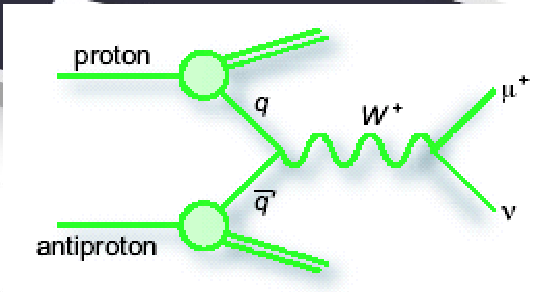
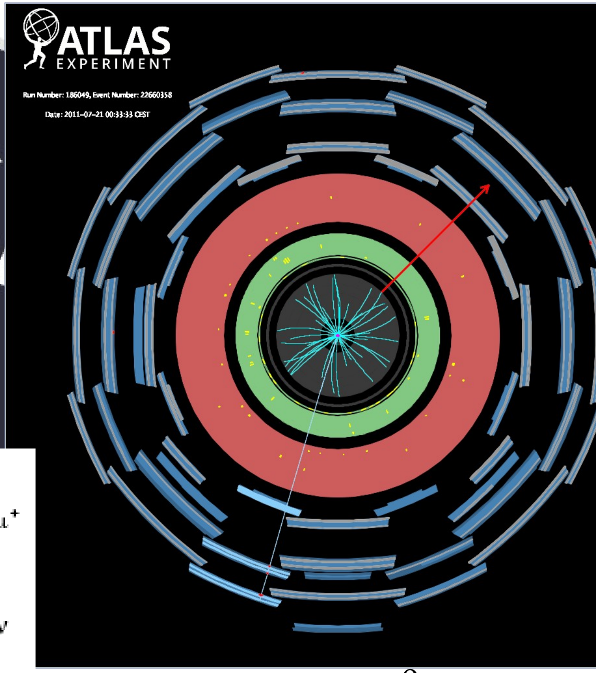
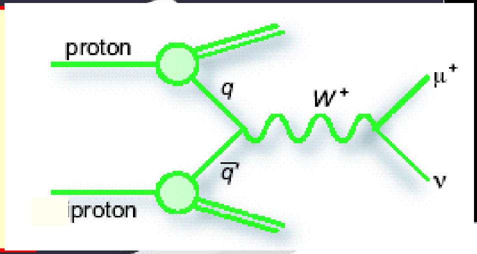




LHC data since 2009
Atlas 2017: M_W=80.370 ± 0.019 GeV
LHCb 2022: M_W=80.354 ± 0.032 GeV
Atlas 2024: M_W=80.367 ± 0.016 GeV



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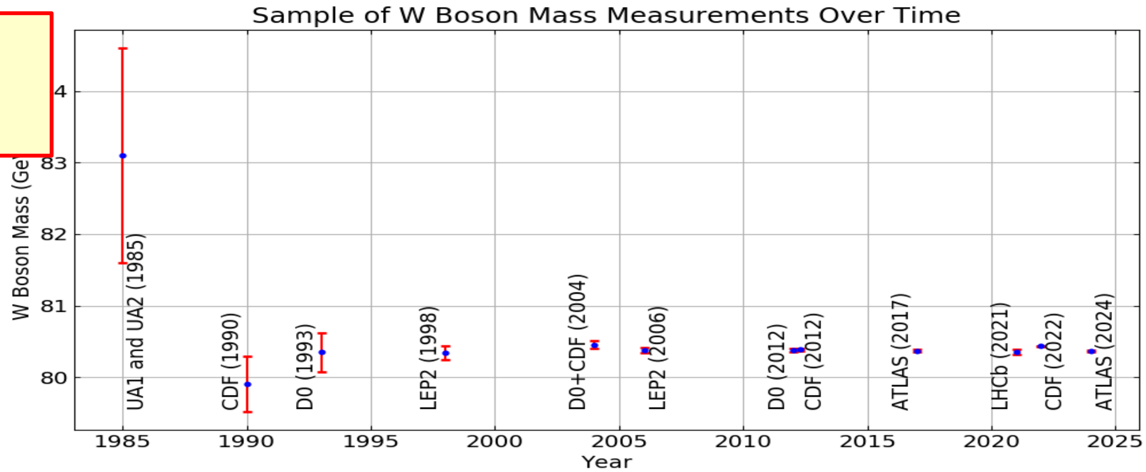
Few percent level precision using first recored events



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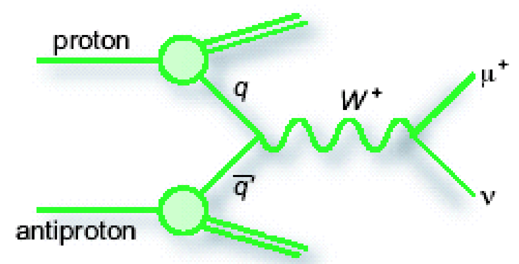

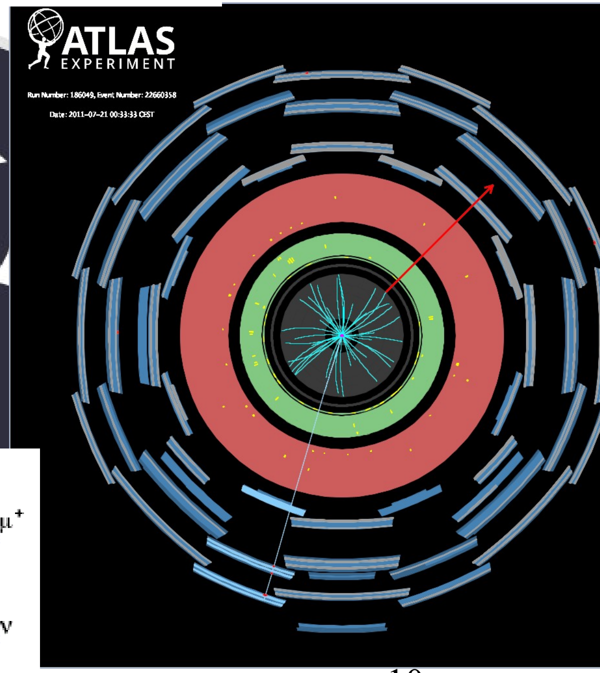

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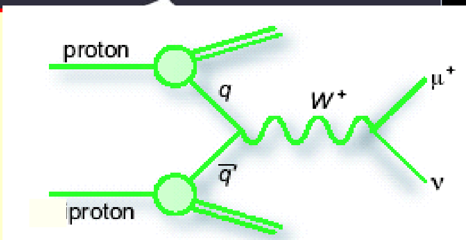

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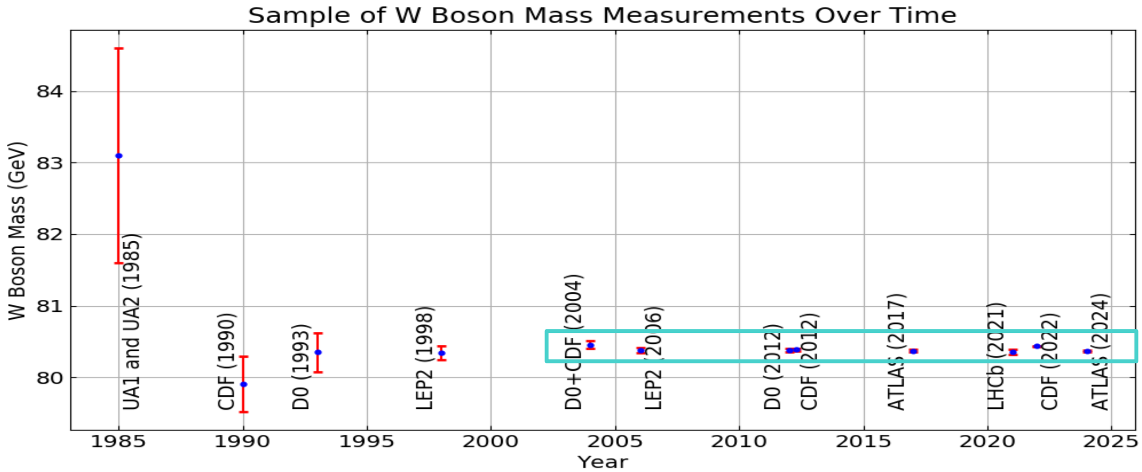

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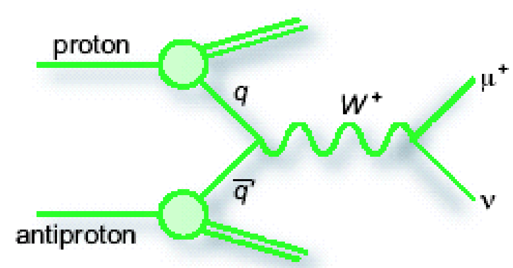

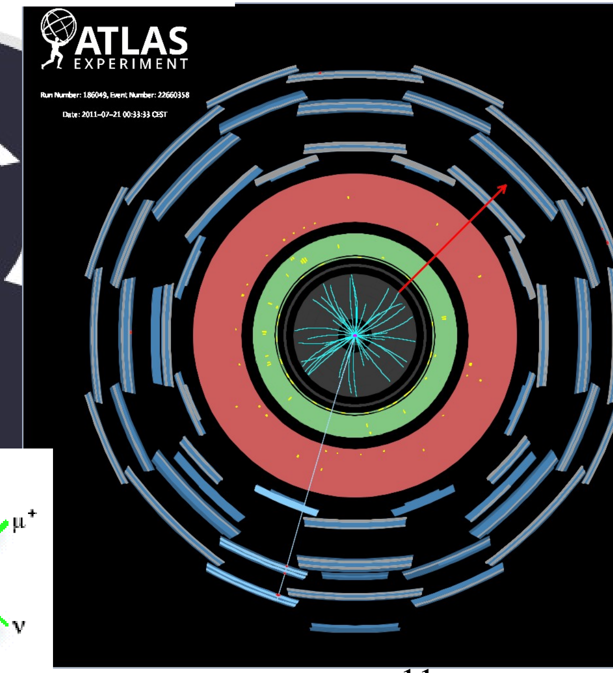

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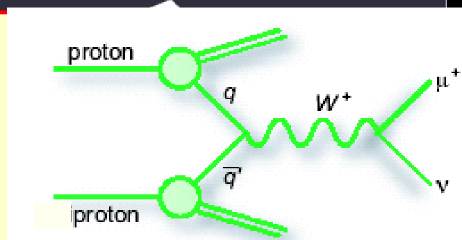

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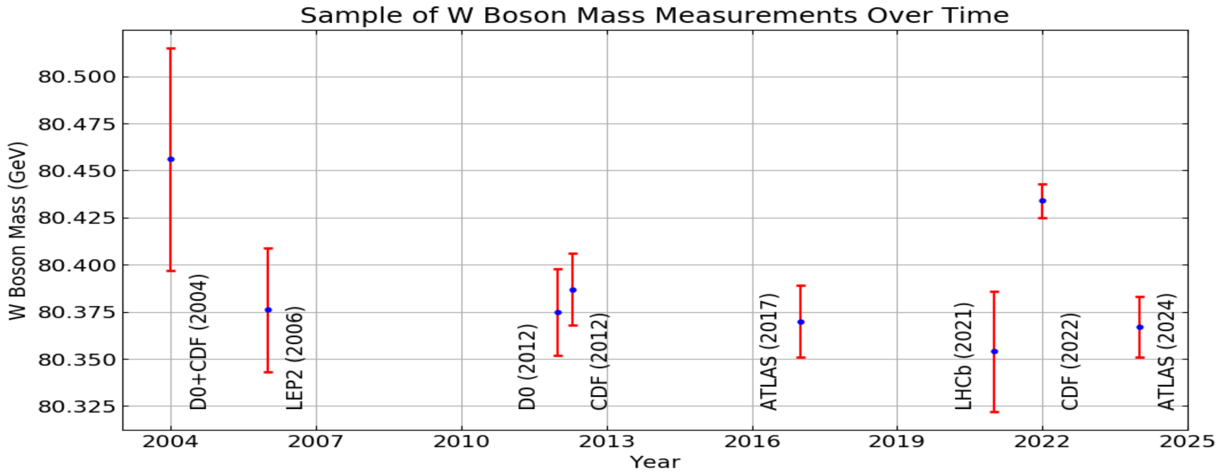

Precision now at ~10^{-4} level

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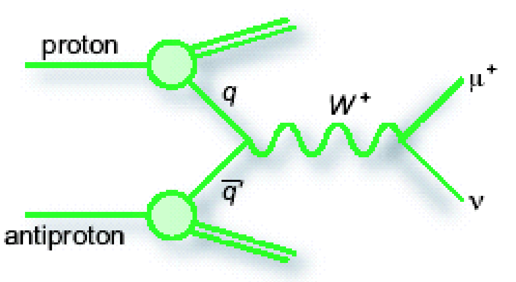

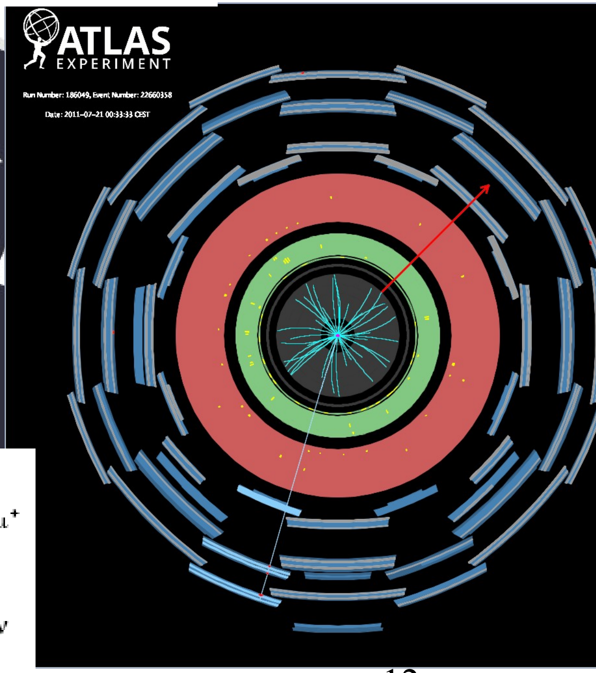

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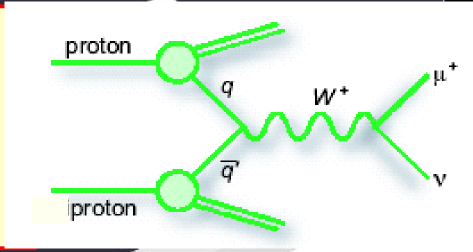
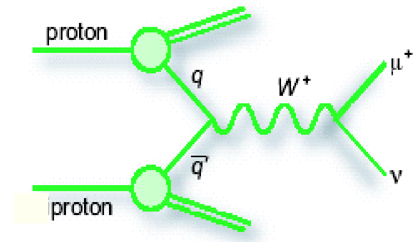
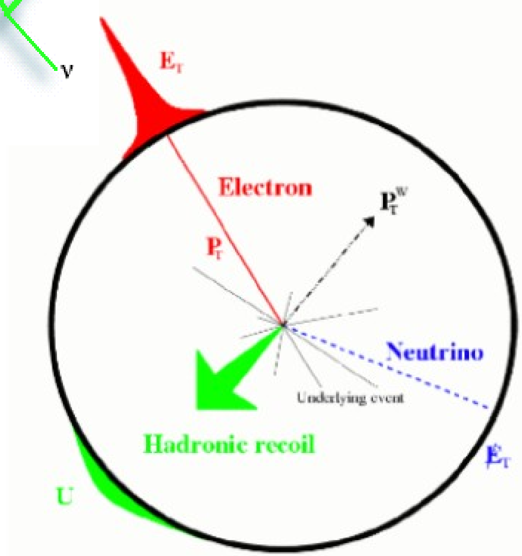
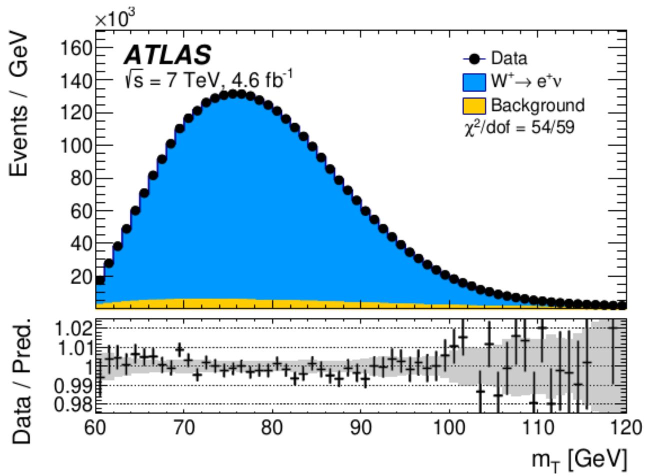


W mass at hadron colliders in a nutshell

- Collisions between quarks from (anti)protons
  - Unknown effective center of mass energy
  - Measure energy/momenta transverse to the collision axis : p_T (lepton), u_T = hadronic recoil
- Hadronic environements:
  - Use W leptonic decay $W \rightarrow \mu\nu$, $W \rightarrow e\nu$
  - Detector calibration from $J/\psi \rightarrow \mu^+\mu^-$, $Z \rightarrow \mu^+\mu^-$, $Z \rightarrow e^+e^-$
  - Measure hadronic recoils to infer missing transverse momentum from neutrino and reconstruct "transverse mass"
  - Proxy: m_T, p_T(lepton), p_T(miss)







- Spectra compared to templates to fit the mass
  - Model dependence of templates → uncertainties
    - Detector response model
    - Final state radiation
    - Model of W transverse recoil (involves QCD at low energy)
    - Model of W production and decay (polarization)
    - Parton distribution functions (PDFs) describing boost along beam axis. Measurements sensitive to boost due to finite rapidity(=angular) acceptance of detectors.

Combination of results

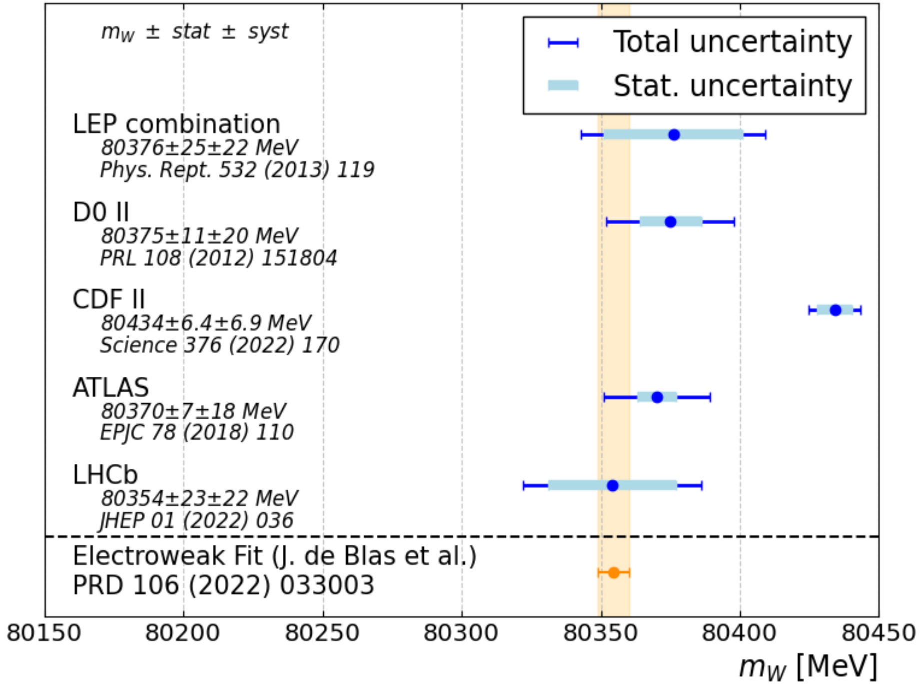
- LHC-Tevatron working group set up to combine latest most precise measurements

- Detailed inputs
  - LEP legacy combination of e^+e^- collider at $\sqrt{s}$=130 – 209 GeV
  - CDF – ppbar $\sqrt{s}$=1.96 TeV , 6 channels
    - fit variables p_T^ℓ, p_T^ν, $m_T^{\ell\nu}$ for ℓ=e,μ
  - D0 – ppbar $\sqrt{s}$=1.96 TeV, 5 channels
    - fit variables p_T^e, p_T^ν, $m_T^{e\nu}$ for two time ranges
  - ATLAS – pp $\sqrt{s}$=7 TeV , 28 channels:
    - fit variables p_T^ℓ, $m_T^{\ell\nu}$ for ℓ=e^+,e^-,μ^+,μ^- and different rapidity bins
  - LHCb – pp $\sqrt{s}$=13 TeV , 1 channel
    - fit variable q/p_T^μ in forward region

Experiment	Event requirements	Fit ranges
CDF	$30 < p_T^\ell < 55$ GeV	$32 < p_T^\ell < 48$ GeV
	$\lvert\eta_\ell\rvert < 1$	$32 < E_T^{miss} < 48$ GeV
	$30 < E_T^{miss} < 55$ GeV	$60 < m_T < 100$ GeV
	$65 < m_T < 90$ GeV	
	$u_T < 15$ GeV	
D0	$p_T^e > 25$ GeV	$32 < p_T^e < 48$ GeV
	$\lvert\eta_\ell\rvert < 1.05$	$65 < m_T < 90$ GeV
	$E_T^{miss} > 25$ GeV	
	$m_T > 50$ GeV	
	$u_T < 15$ GeV	
ATLAS	$p_T^\ell > 30$ GeV	$32 < p_T^\ell < 45$ GeV
	$\lvert\eta_\ell\rvert < 2.4$	$66 < m_T < 99$ GeV
	$E_T^{miss} > 30$ GeV	
	$m_T > 60$ GeV	
	$u_T < 30$ GeV	
LHCb	$p_T^\mu > 24$ GeV	$28 < p_T^\mu < 52$ GeV
	$2.2 < \eta_\mu < 4.4$	

Combination of results

- LHC-Tevatron working group set up to combine latest most precise measurements
- Challenges
  - Measurements made over a number of years. QCD understanding and modeling has improved over time
  - Measurements need to be combined using a common ground
  - Consistent treatment of different theory assumption
  - Proper estimate of correlations



Combining results

- Starting point: different models as inputs to the different measurements
  - D0: Resbos CP (NNLO+NNLL) generated with CTEQ66 (NLO)
  - ATLAS: Powheg+Pythia8 (NLO+PS); y_W + A_i at NNLO with CT10 (NNLO)
  - LHCb: Powheg+Pythia8 (NLO+PS); A_i at NNLO,as PDF the average of NNPDF3.1,CT18,MSHT20 (NLO)
  - CDF: Resbos C (NLO+NNLL) generated with CTEQ6M (NLO). Final results corrected for using NNPDF 3.1

- Strategy for combination:
  - Each individual measurement has to be updated to a common baseline

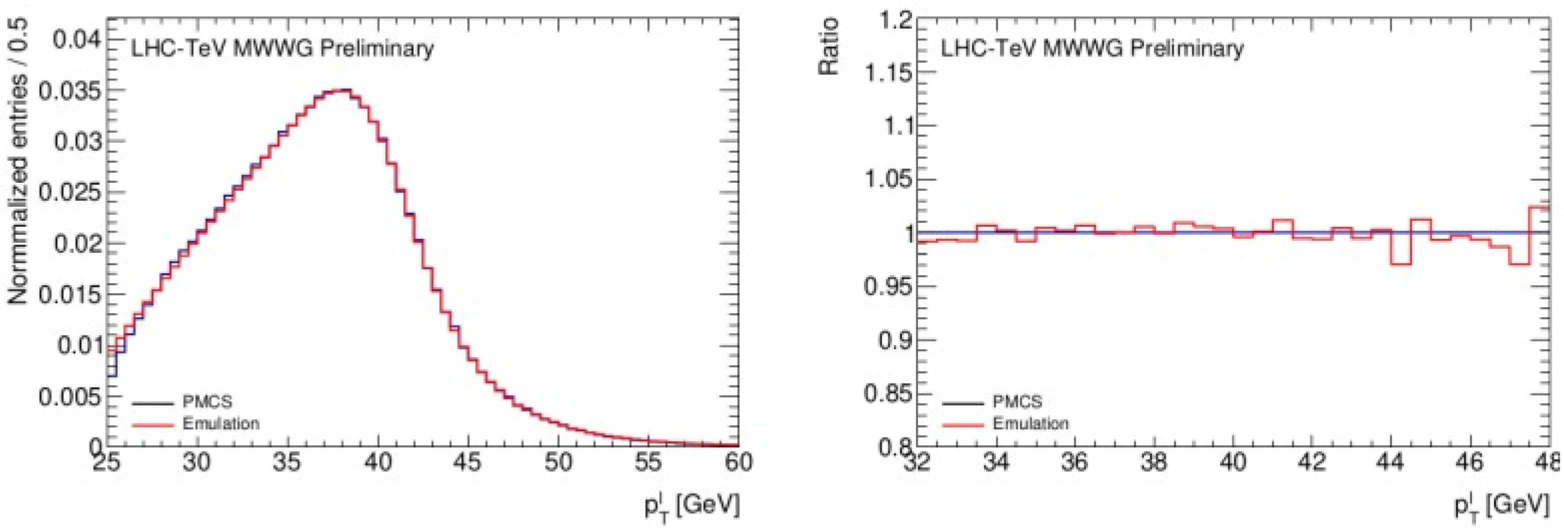
$$m_W^{\mathrm{update}} = m_W^{\mathrm{ref}} + \delta m_W^{\mathrm{PDF}} + \delta m_W^{\mathrm{QCD}} + \delta m_W^{\mathrm{other}}$$

  - Correlation have to be determined by consistently propagating change in model to each different measurement
  - Combination using Best Linear Unbiased Estimate (BLUE)
  - Approach validated by reproducing internal experimental combinations.

- Tool needed:
  - Fast emulation of D0/Atlas/CDF detector simulation to determine the impact of changes in W production model.
    - LHCb was able to re-run its own framework

Detector emulations

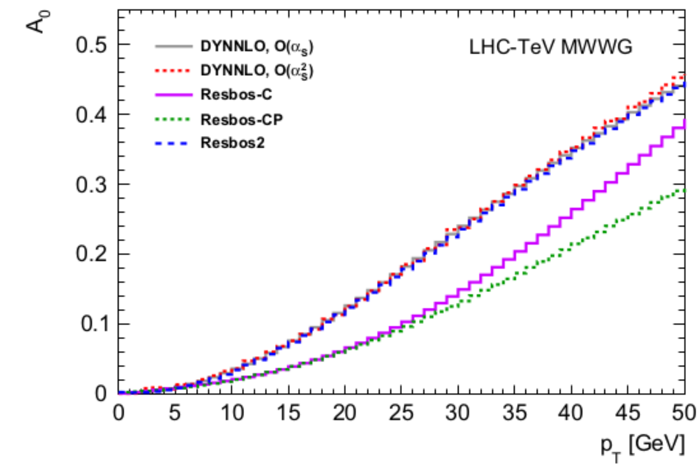
- ATLAS, CDF and D0 detectors emulated.
  - lepton energy smearing, reconstruction efficiency, p_T,η dependent
  - smearing of measured recoil
- The emulations allow for processing quickly billions of events with different generators
  - use MiNNLOPS generator for our main results
- Agreements obtained at the percent level between individual simulations and LHC-TeV MWWG emulations.
- This is translated to MeV-level uncertainties for the estimate of δm's .



QCD model : W polarisation

- Improvement over years for the computation of angular terms

$$\frac{d\sigma}{dp_T^W\,dy\,d\Omega} = \frac{d\sigma}{dp_T^W\,dy}\Big[(1+\cos^2\theta)+\frac{1}{2}A_0(1-3\cos^2\theta)+A_1\sin 2\theta\cos\phi + \frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\Big]$$



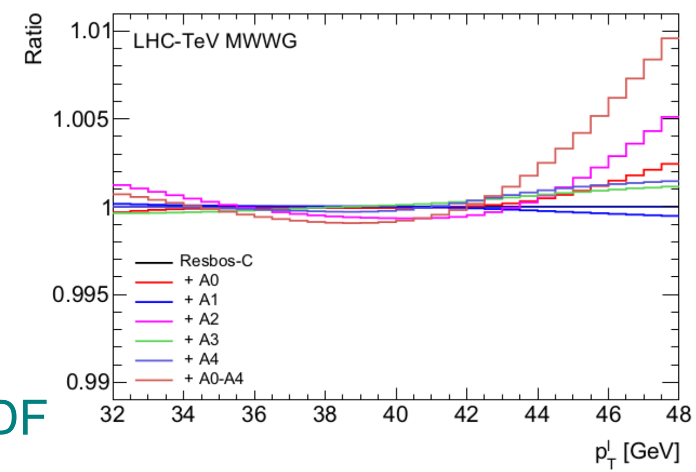
- Older models at D0 and CDF need to be updated
- Consistent results obtained when
  - reweighting angular distributions
  - using more modern generator

D0 δm_W^{QCD} (MeV)

Coefficient	m_T	p_T^ℓ	p_T^ν
A_0	−9.8	−7.3	−15.6
A_1	1.9	2.4	1.8
A_2	3.0	3.3	−2.7
A_3	−1.6	−2.9	0.4
A_4	0.2	−2.3	0.5
$A_0 - A_4$	−6.4	−6.9	−15.8
ResBos2	−7.8 ± 1.0	−6.6 ± 1.1	−16.5 ± 1.2
Difference	−1.4 ± 1.0	0.3 ± 1.1	−0.7 ± 1.2

effects of Ai reweighitng (A_0 to $A_0 - A_4$)

modern generator (ResBos2)



- Overall effects: δm_W ~ -6.5 MeV@D0, -9.5 MeV@CDF

Choice of PDF

- Several PDFs on the phenomenology "market"
- Compatibility of PDF sets tested using pp, ppbar → W,Z measurements (differential distributions, asymmetries) using xFitter package

Measurement	NNPDF3.1	NNPDF4.0	MMHT14	MSHT20	CT14	CT18	ABMP16
CDF y_Z	24 / 28	28 / 28	30 / 28	32 / 28	29 / 28	27 / 28	31 / 28
CDF A_W	11 / 13	14 / 13	12 / 13	28 / 13	12 / 13	11 / 13	21 / 13
D0 y_Z	22 / 28	23 / 28	23 / 28	24 / 28	22 / 28	22 / 28	22 / 28
D0 $W \to e\nu$ A_ℓ	22 / 13	23 / 13	52 / 13	42 / 13	21 / 13	19 / 13	26 / 13
D0 $W \to \mu\nu$ A_ℓ	12 / 10	12 / 10	11 / 10	11 / 10	11 / 10	12 / 10	11 / 10
ATLAS peak CC y_Z	13 / 12	13 / 12	58 / 12	17 / 12	12 / 12	11 / 12	18 / 12
ATLAS W^- y_ℓ	12 / 11	12 / 11	33 / 11	16 / 11	13 / 11	10 / 11	14 / 11
ATLAS W^+ y_ℓ	9 / 11	9 / 11	15 / 11	12 / 11	9 / 11	9 / 11	10 / 11
Correlated χ^2	75	62	210	88	81	41	83
Total χ^2 / d.o.f.	200 / 126	196 / 126	444 / 126	270 / 126	210 / 126	162 / 126	236 / 126
p(χ^2, n)	0.003%	0.007%	$< 10^{-10}$	$< 10^{-10}$	0.0004%	1.5%	10^{-8}

- No PDF set provides a good description of the full Tevatron+LHC dataset.
- Best description given by CT18 (which has slightly larger uncertainties).

- CT18 taken as the main PDF set for the final combination

Impact of PDF choice

- Impact of the different PDF choice is computed
- Typically δm_W = O (10 MeV)

δm_W^{PDF} (MeV), for p_T(lepton), p_T(miss) channels

starting points of published results (red boxes)

For final combination (blue box)

PDF set	D0 p_T^ℓ	D0 p_T^ν	CDF p_T^ℓ	CDF p_T^ν	ATLAS W^+	ATLAS W^-	LHCb
CTEQ6	− 17.0	− 17.7	0.0	0.0	–	–	–
CTEQ6.6	0.0	0.0	15.0	17.0	–	–	–
CT10	0.4	− 1.3	16.0	16.3	0.0	0.0	–
CT14	− 9.7	− 10.6	5.8	6.8	−1.2	−5.8	1.1
CT18	− 8.2	− 9.3	7.2	7.7	12.1	−2.3	−6.0
ABMP16	− 19.6	− 21.5	− 1.4	− 2.4	−22.5	−3.1	7.7
MMHT2014	− 10.4	− 12.7	6.1	5.5	−2.6	9.9	− 10.8
MSHT20	− 13.7	− 15.4	3.6	4.1	−20.9	4.5	−2.0
NNPDF3.1	− 1.0	− 1.2	14.0	15.1	−14.1	−1.8	6.0
NNPDF4.0	6.7	8.1	20.8	24.1	−22.4	6.9	8.3

- Final result using CT18 (see previous slides)

Impact of PDF choice

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- Typically $\delta m_W = O$ (10 MeV)

δm_W^{PDF} (MeV), for m_T channels

starting points of published results (red boxes)

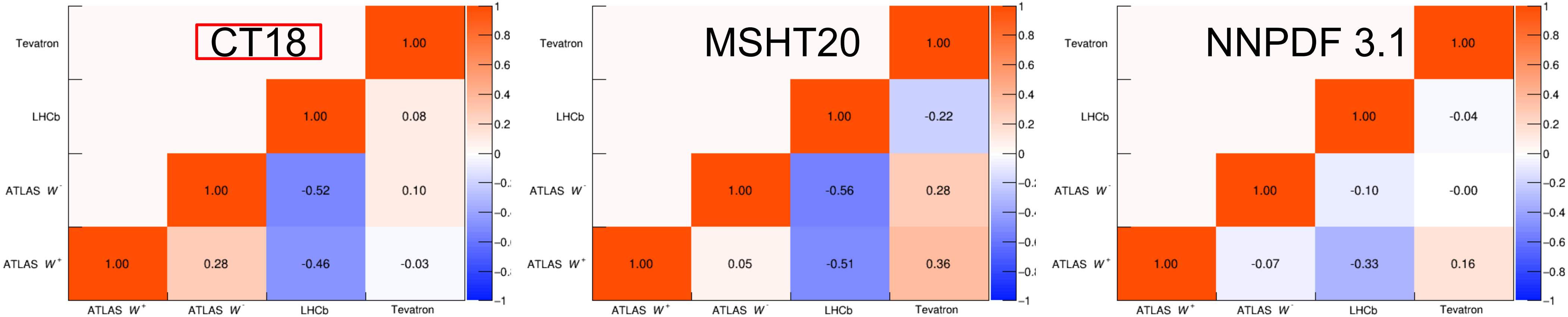
For final combination (blue box)

PDF set	D0	CDF	ATLAS W^+	ATLAS W^-
CTEQ6	−14.6	0.0	–	–
CTEQ6.6	0.0	14.2	–	–
CT10	−0.5	14.3	0.0	0.0
CT14	−8.7	5.2	−0.5	−7.6
CT18	−7.5	6.5	13.4	−5.5
ABMP16	−17.9	−2.4	−25.7	−7.9
MMHT2014	−10.1	4.5	−3.6	9.1
MSHT20	−12.9	2.5	−22.3	4.2
NNPDF3.1	−1.0	13.1	−14.6	−6.3
NNPDF4.0	6.2	20.1	−23.3	4.3

- Final result using CT18 (see previous slides)

PDF correlations

- PDFs are the main source of correlations for final combination
  - Different correlations and uncertainties for each PDF set.
  - Anti-correlations LHCb/Atlas as they are probing different rapidity region
  - CT18 uncertainties 10 –15 MeV



PDF uncertainties (MeV)

PDF set	D0	CDF	ATLAS	LHCb
CTEQ6	–	14.1	–	–
CTEQ6.6	15.1	–	–	–
CT10	–	–	9.2	–
CT14	13.8	12.4	11.4	10.8
CT18	14.9	13.4	10.0	12.2
ABMP16	4.5	3.9	4.0	3.0
MMHT2014	8.8	7.7	8.8	8.0
MSHT20	9.4	8.5	7.8	6.8
NNPDF3.1	7.7	6.6	7.4	7.0
NNPDF4.0	8.6	7.7	5.3	4.1

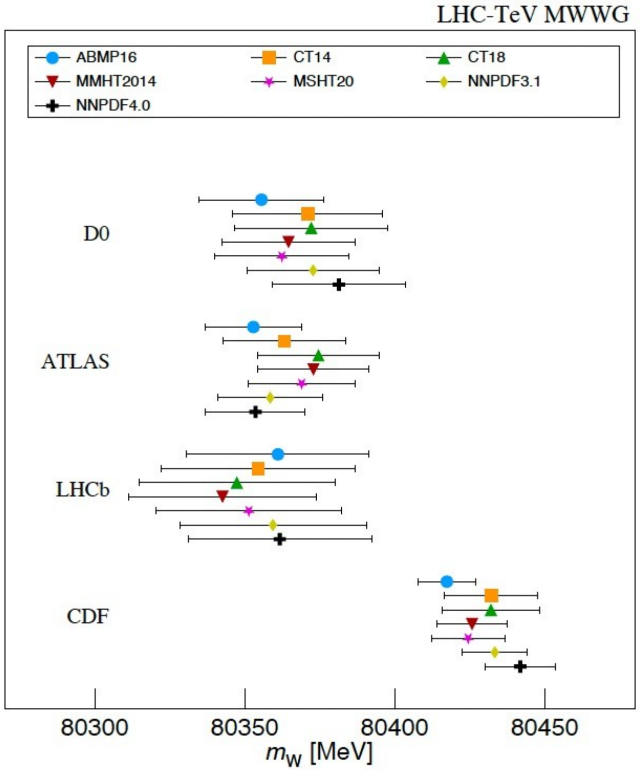
Inputs to combination

- Updates are computed for different input PDFs

$$m_W^{\text{update}} = m_W^{\text{ref}} + \delta m_W^{\text{PDF}} + \delta m_W^{\text{QCD}} + \delta m_W^{\text{other}}$$

- A few other adjustments needed for a consistent treatment
  - O(1) MeV level δm_W for a consistent Γ_W of (SM) 2.089 GeV between experiments
  - Generator level cuts in CDF mass involving a 2 MeV shift
  - When combining sub-channels, another O(3) MeV shift may arise from differences in weights due to modified PDF uncertainties
- Example of update (For our choice of main PDF (CT18)

	Publications (MeV)		CT18 input to combination	
Atlas	80370	±19	80374.5	± 20.3
CDF	80433.5	± 9.4	80432.0	± 16.1
D0	80375	± 23	80372	± 25.5
LHCB	80354	± 32	80347.3	± 32.7

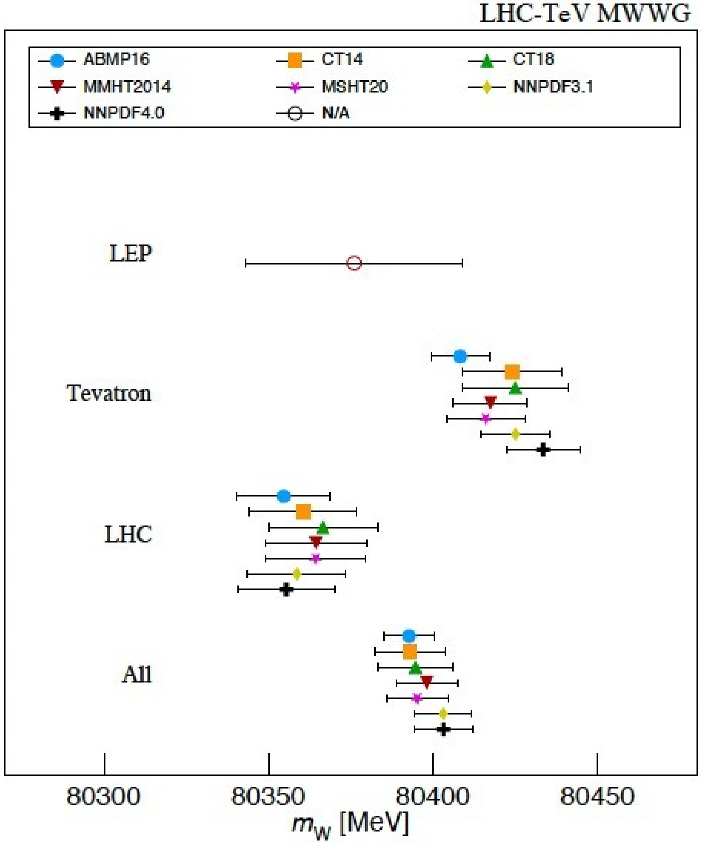


  - At the end the updated values are close ($|\delta m_W|$<7 MeV) to original measurements

Combination

- Account for individual uncertainties
- Account for correlations
  - Statistical correlations between distributions p_T, m_T, p_T^{ν}
  - $P_T(W)$ model is constrained using (Z) data in the different experiments, uncertainties of 2 –11 MeV treated as uncorrelated
  - Small correlated uncertainty ~ 2.5 – 5 MeV between CDF and D0 due to photon radiation model
  - Main source of correlations across experiments is PDF
- Two steps
  - Individual channels combination within each experiment
  - combination across all experiments

Overall combinations

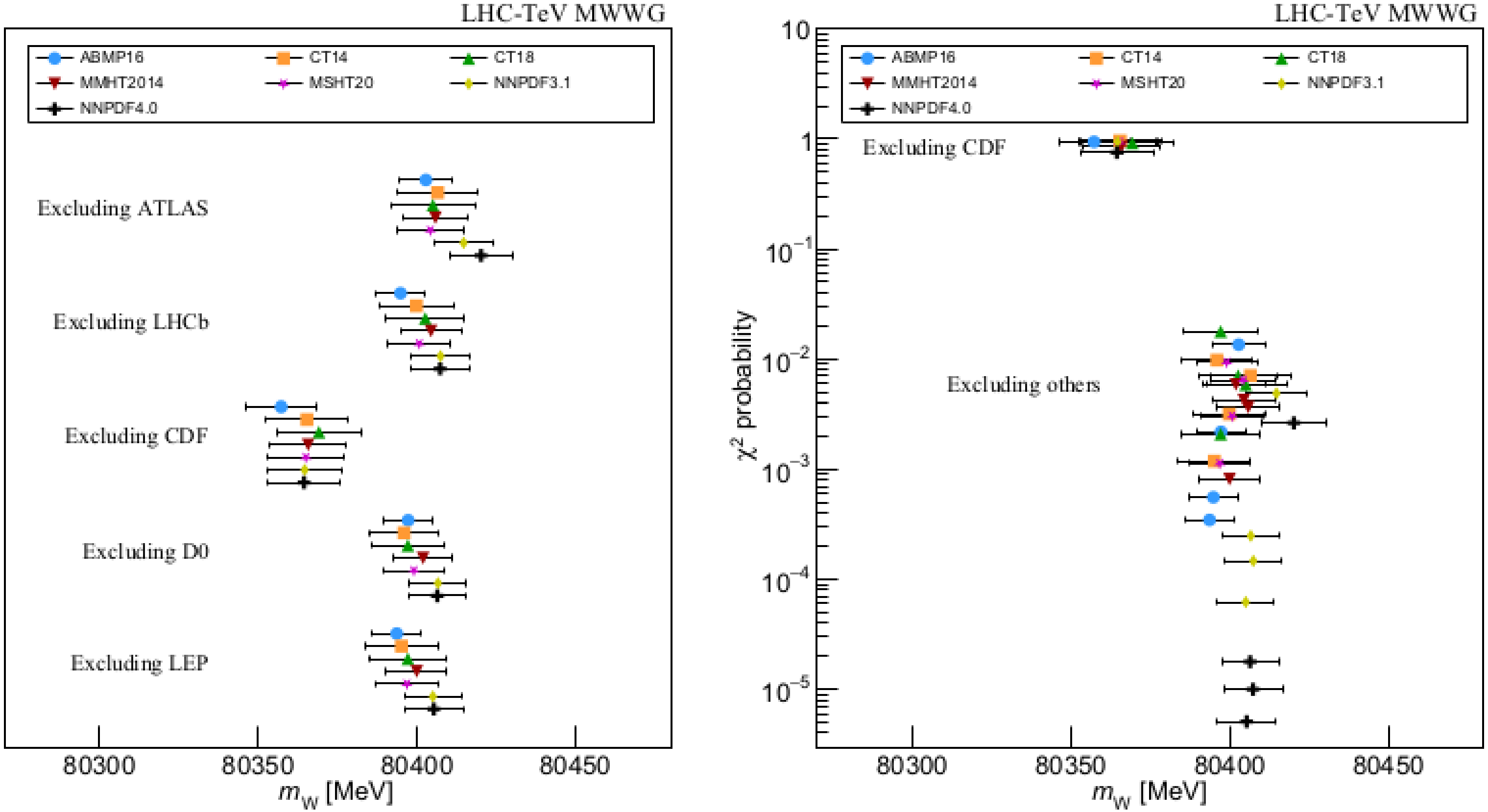


All experiments (4 d.o.f.)				
PDF set	m_W	σ_{PDF}	χ^2	$p(\chi^2, n)$
ABMP16	$80{,}392.7 \pm 7.5$	3.2	29	0.0008%
CT14	$80{,}393.0 \pm 10.9$	7.1	16	0.3%
CT18	$80{,}394.6 \pm 11.5$	7.7	15	0.5%
MMHT2014	$80{,}398.0 \pm 9.2$	5.8	17	0.2%
MSHT20	$80{,}395.1 \pm 9.3$	5.8	16	0.3%
NNPDF3.1	$80{,}403.0 \pm 8.7$	5.3	23	0.1%
NNPDF4.0	$80{,}403.1 \pm 8.9$	5.3	28	0.001%

- For each PDF overall combinations has a poor χ^2
- Combination using CT18
  - m_W=80394.6 ± 11.5 MeV
  - but is disfavored as the probability is quite low Prob(χ^2)=0.5%

Subcombination N-1

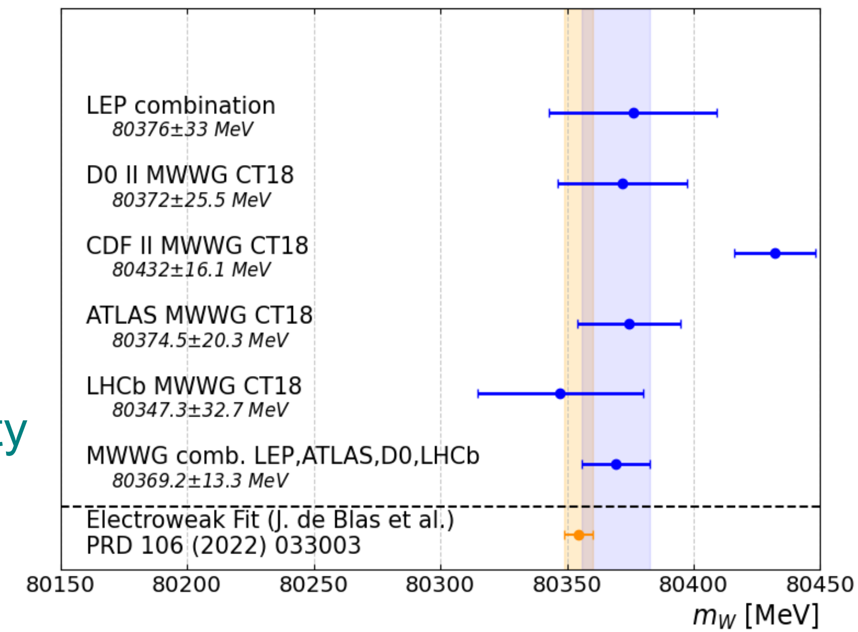
- Combination obtained when subtracting one of the inputs



- Combinations without CDF have good compatibility
  - m_W=80369.2 ± 13.3 MeV with CT18
  - Relative weights: 42% (ATLAS); 23% (D0); 18% (LHCb); 16% (LEP).
- Difference between CDF and the All-CDF combination is 3.6σ using CT18

Summary

- W boson mass is a key parameter of the Standard Model
- Combination of the experimental results within the LHC-TeV W mass group
  - Procedure and tools setup to obtain
    - consistent treatment of inputs for a proper combination
      - update W polarization model
      - common PDF set
    - compute correlations
  - This effort is not able to solve the tension between existing measurements
    - Full combination using CT18 has low probability Prob(χ^2)=0.5%
    - CDF measurement is 3.6σ from the other ones



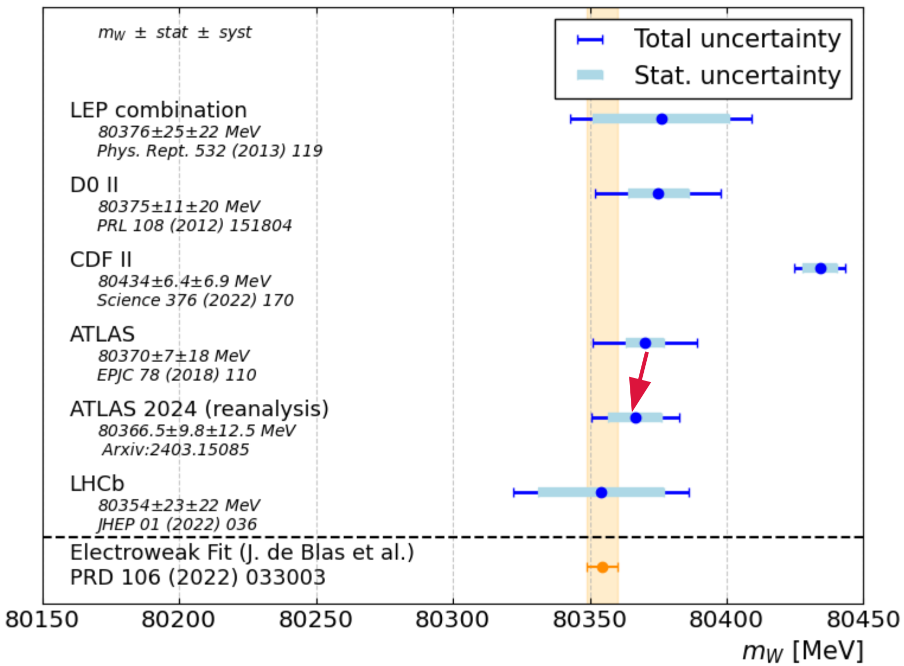
- Combinations without CDF have good compatibility
- Using CT18 we obtain

  m_W=80369.2±13.3 MeV

Prospects

- LHC-TeV MWWG planning to update results
  - Atlas submitted recently an update using profiling techniques
  - This will likely worsen the discrepancy between CDF and the others

- Also aim to include CMS measurement whenever it is out



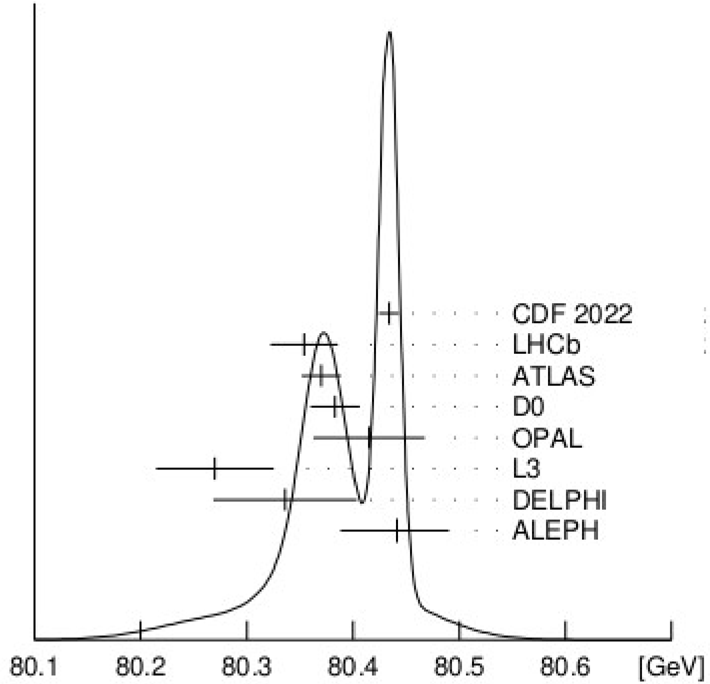
Tev/LHC MWWG papers
- CERN-LPCC-2022-06/FERMILAB-TM-2779-V
- EPJC 84 (2024) 451

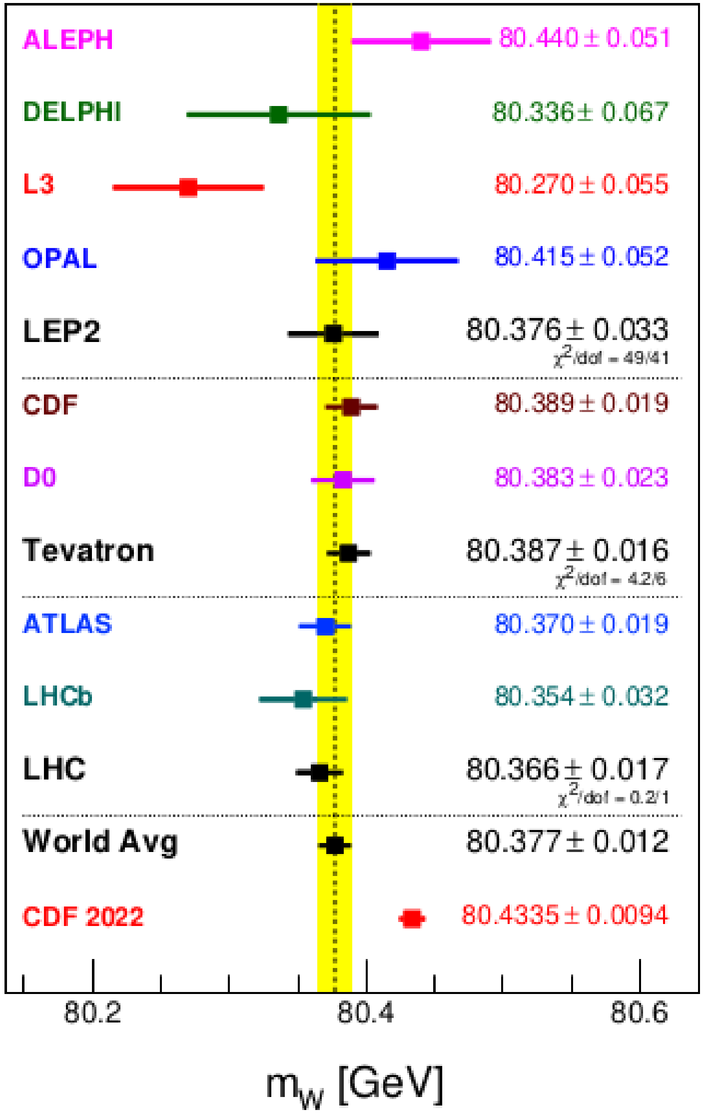
More info: https://twiki.cern.ch/twiki/bin/view/LHCPhysics/LHC-TEV-MWWG

BACKUP

PDG calculation

- No correction for common PDF set
- Does not include CDF 2022 but keep CDF 2012 measurement
- Guess estimate of correlated uncertainties
  - 9 MeV correlated between Atlas&LHCb
  - 7 MeV correlated between Tevatron&LHC





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- CTEQ6.6 PRD 78 (2008) 013004
- CT10 PRD 82 (2010) 074024
- CT14 PRD 93 (2016) 033006
- CT18 PRD 103 (2021) 014013
- ABMP16 PRD 96 (2017) 014011
- MMHT2014 EPJC 75 (1015) 204
- MSHT20 EPJC 81 (2021 341
- NNPDF31 EPJC 77 (2017) 663
- NNPDF40 EPJC 82 (2022) 428

- xFitter, arXiv:1709.01151