

The free energy of dense fishnet graphs

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Project together with
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Prague Spring Amplitudes Workshop
15th May 2023

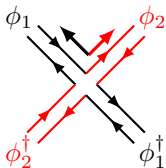


Bi-Scalar Fishnet Theory

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[Gürdoğan, Kazakov:1512.06704]

$$\mathcal{L} = \frac{N}{2} \cdot \text{tr} \left[\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 \right] + N(4\pi)^2 \xi^2 \cdot \text{tr} \left[\phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 \right]$$



Bi-Scalar Fishnet Theory

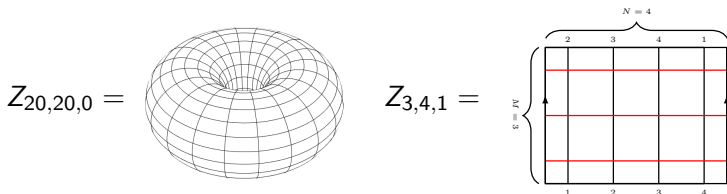
Bi-Scalar Fishnet Theory

[Gürdoğan, Kazakov:1512.06704]

$$\mathcal{L} = \frac{N}{2} \cdot \text{tr} \left[\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 \right] + N(4\pi)^2 \xi^2 \cdot \text{tr} \left[\phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 \right]$$

Free energy:

$$Z = \sum_{M,N=1}^{\infty} (\xi^2)^{MN} \sum_{k=0}^{N-1} Z_{MN,k} + \text{counter terms at } M \text{ or } N \leq 2$$



$$Z_{MN,k} = \int \left[\prod_{m,n=1}^{M,N} d^4 x_{m,n} \frac{1}{(x_{m,n} - x_{m+1,n})^2} \frac{1}{(x_{m,n} - x_{m,n+1})^2} \right]$$

with $x_{M+1,n} = x_{1,n+k}$ and $x_{m,N+1} = x_{m,1}$



Free energy and R-matrix

Free energy in thermodynamic limit (i.e. Radius of convergence)

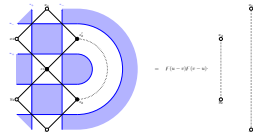
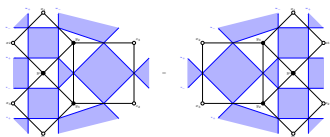
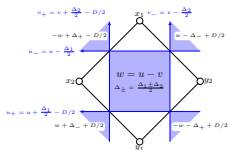
[Zamolodchikov'80]

$$\kappa := \lim_{\substack{M, N \rightarrow \infty \\ M/N=1}} [Z_{MN,0}]^{\frac{1}{MN}}$$

Generalization: R-matrix as lattice face weights

[Bazhanov, Kels, Sergeev'16]

[Chicherin, Derkachov, Isaev'12]

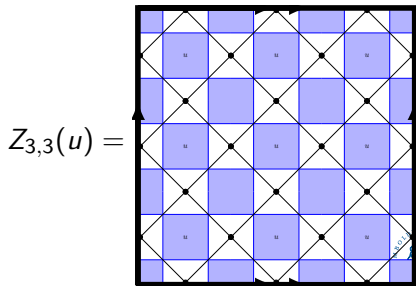
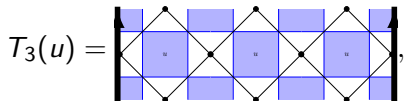


Free energy and R-matrix

Free energy in thermodynamic limit (i.e. Radius of convergence)

[Zamolodchikov'80]

$$\begin{aligned}\kappa(u) &:= \lim_{M,N \rightarrow \infty} [Z_{MN}(u)]^{\frac{1}{MN}} = \lim_{M,N \rightarrow \infty} \text{tr} \left[T_N(u)^M \right]^{\frac{1}{MN}} \\ &= \lim_{M,N \rightarrow \infty} [\lambda_{N,\max}(u)]^{\frac{1}{N}}\end{aligned}$$



Free energy and R-matrix

			$\kappa_e = 2\pi^{3/2} \frac{\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{2}{3}\right)}$ <p>ABJM fishnet</p>
		$\kappa_e = \frac{\pi^{1/2}}{4\sqrt{2}} \Gamma\left(\frac{1}{4}\right)^2$ <p>$\mathcal{N} = 4$ fishnet</p>	
		$\kappa_e = \frac{\pi^{5/2} \Gamma\left(\frac{4}{3}\right)}{2\Gamma\left(\frac{5}{6}\right)}$ <p>?</p>	

Inversion Relation

$$T_3(w) \circ T_3(-w) = \left[\begin{array}{c} \text{Diagram of a 3x3 grid of blue squares with diagonal lines and arrows} \end{array} \right] = [F(w)F(-w)]^3 \left[\begin{array}{c} \text{Diagram of three vertical lines: a solid line on the left, a dotted line in the middle, and a solid line on the right} \end{array} \right]$$

$$\left[\begin{array}{c} \text{Diagram of a diamond-shaped grid with blue shaded regions and points labeled } z_1, z_2, z_1', z_2' \end{array} \right] = F(u-v)F(v-u) \left[\begin{array}{c} \text{Diagram of a vertical dotted line with five points labeled } z_1, z_2, z_1', z_2' \end{array} \right]$$

Inversion Relation

$$T_3(w) \circ T_3(-w) = \left[\begin{array}{c} \text{3x3 grid of blue squares with diagonal lines} \end{array} \right] = [F(w)F(-w)]^3 \left| \begin{array}{c} \text{3 vertical lines: solid, dotted, dotted, solid} \end{array} \right.$$

$$\Rightarrow T_N(w) \circ T_N(-w) = [F(w)F(-w)]^N \mathbb{1}$$

$$\Rightarrow \lambda_{N,\max}(w) \cdot \lambda_{N,\max}(-w) = [F(w)F(-w)]^N$$

$$\Rightarrow \kappa(w)\kappa(-w) = F(w)F(-w)$$

Solving Inversion Relations

Inversion relations:

$$\kappa(w)\kappa(-w) = F(w)F(-w)$$

$$\kappa(w) = \kappa\left(\frac{D}{2} - w\right)$$

Solution:

$$\begin{aligned} \kappa(w) = & \pi^D \pi^{-2w} \frac{\Gamma(w + \Delta_-)}{\Gamma\left(\frac{D}{2} - (w + \Delta_-)\right)} \kappa_e\left(\frac{2\pi}{D}(w + \Delta_-)\right)^2 \\ & \cdot \pi^{-2w} \frac{\Gamma(w - \Delta_-)}{\Gamma\left(\frac{D}{2} - (w - \Delta_-)\right)} \kappa_e\left(\frac{2\pi}{D}(w - \Delta_-)\right)^2 \end{aligned}$$

with

$$\kappa_e(\alpha) = \pi^{D\alpha/2\pi} \frac{\Gamma\left(\frac{D}{2} - \frac{D\alpha}{2\pi}\right)}{\Gamma\left(\frac{D}{2}\right)} \prod_{l=1}^{\infty} \frac{\Gamma\left(Dl + \frac{D}{2} - \frac{D\alpha}{2\pi}\right) \Gamma\left(Dl + \frac{D\alpha}{2\pi}\right) \Gamma\left(Dl - \frac{D}{2}\right)}{\Gamma\left(Dl - \frac{D}{2} + \frac{D\alpha}{2\pi}\right) \Gamma\left(Dl - \frac{D\alpha}{2\pi}\right) \Gamma\left(Dl + \frac{D}{2}\right)}$$



Outlook

- Use fermionic R-matrix to obtain free energy in the thermodynamic limit of the brick-wall model

[Chicherin, Derkachov, Isaev'12][Chicherin, Kazakov, Loebbert, Müller, Zhong'17]

- Check the result via TBA [Basso, Zhong'18]
- Inhomogeneous transfer matrix to study the effect of the twist
- Apply method of inversion relations to other models (e.g. WZW)



Thanks for your attention!

