All-Order Celestial OPE from On-Shell Recursion

Anders Schreiber

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Max Planck Institute for Physics

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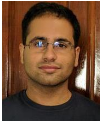

Introduction

- Celestial CFT aims to formulate holography in flat space [Pasterski, Shao, Strominger, 1701.00049] (see also reviews [Pasterski, 2108.04801] and [McLoughlin, Puhm, Raclariu, 2203.13022]).
- A key observation [Fan, Fotopoulos, Taylor, 1903.01676] is that collinear limits $\leftrightarrow$ OPE in celestial CFT.
- We will compute the all-order OPE in the MHV sector of gluon amplitudes at tree-level, using inverse soft recursion [Arkani-Hamed, Cachazo, Cheung, Kaplan, 0907.5418] [Boucher-Veronneau, Larkoski, 1108.5385].
- Expands on previous works [Bannerjee, Ghosh, 2011.00017] [Himwich, Pate, Singh, 2108.07763] [Adamo, Bu, Casali, Sharma, 2211.17124].

Inverse Soft Construction

- The n-point MHV gluon tree-amplitude satisfies inverse soft recursion via BCFW recursion [Arkani-Hamed, Cachazo, Cheung, Kaplan, 0907.5418] [Boucher-Veronneau, Larkoski, 1108.5385]

$$A[1_+\, 2\, 3 \cdots n] = \frac{\langle n\, 2\rangle}{\langle n\, 1\rangle\langle 1\, 2\rangle}\, A[\hat{2}\, 3 \cdots n-1\, \hat{n}]\,,$$

with BCFW shifts

$$\hat{\tilde{\lambda}}_2 = \tilde{\lambda}_2 + \frac{\langle n\, 1\rangle}{\langle n\, 2\rangle}\,\tilde{\lambda}_1\,, \qquad \hat{\tilde{\lambda}}_n = \tilde{\lambda}_n + \frac{\langle 1\, 2\rangle}{\langle n\, 2\rangle}\,\tilde{\lambda}_1\,.$$

- In the DDM color decomposition [Del Duca, Dixon, Maltoni, hep-ph/9910563]

$$A(1_+^{a_1}\, 2^{a_2} \cdots n^{a_n}) = -\sum_{i=3}^{n} \frac{\langle i2\rangle}{\langle i1\rangle\langle 12\rangle}\; T_i^{a_1} A(\hat{2}^{a_2} \cdots \hat{i}^{a_i} \cdots n^{a_n})\,,$$

with

$$T_i^{a_1} A(\hat{2}^{a_2} \cdots \hat{i}^{a_i} \cdots n^{a_n}) := f^{a_1 a_i}{}_b\, A(\hat{2}^{a_2} \cdots \hat{i}^{b} \cdots n^{a_n})\,.$$

MHV Gluon OPE: Collinear expansion

- Little group choice:

$$\lambda_i^{\alpha} = \begin{pmatrix} 1 \\ z_i \end{pmatrix}, \qquad \tilde{\lambda}_i^{\dot{\alpha}} = \omega_i \begin{pmatrix} 1 \\ \bar{z}_i \end{pmatrix}, \qquad z_i, \bar{z}_i \in \mathbb{C}\,,\ \omega_i \in \mathbb{R}_+\,.$$

- Inverse soft expression

$$A(1_+^{a_1}\, 2^{a_2} \cdots n^{a_n}) = -\sum_{i=3}^{n} \frac{1}{z_{12}} \left(1 - \frac{z_{12}}{z_{i2}}\right)^{-1} \exp\left\{ \frac{z_{12}}{z_{i2}} \left([1\partial_i] - [1\partial_2]\right) \right\} T_i^{a_1} A(\mathbf{2}^{a_2} \cdots n^{a_n})$$

- Particle 2 now has effective momentum

$$\mathbf{p}_2^{\alpha\dot{\alpha}} = \lambda_2^{\alpha} (\tilde{\lambda}_1 + \tilde{\lambda}_2)^{\dot{\alpha}}\,.$$

MHV Gluon OPE: Collinear expansion

- Performing all order expansion in z_{12}

$$A(1_+^{a_1}\, 2^{a_2} \cdots n^{a_n}) = -\sum_{p=0}^{\infty} z_{12}^{p-1} \sum_{q=0}^{p} \sum_{r=0}^{q} \frac{(-[1\partial_2])^{q-r}}{r!(q-r)!} \sum_{i=3}^{n} \frac{[1\partial_i]^r}{z_{i2}^p}\, T_i^{a_1} A(\mathbf{2}^{a_2} \cdots n^{a_n})\,.$$

- We can always connect back with the usual $\omega_i, z_i, \bar{z}_i$ representation

$$[i\partial_j] \equiv \tilde{\lambda}_i^{\dot\alpha} \frac{\partial}{\partial \tilde{\lambda}_j^{\dot\alpha}} = \frac{\omega_i}{\omega_j} \left(\bar{z}_{ij} \frac{\partial}{\partial \bar{z}_j} + \omega_j \frac{\partial}{\partial \omega_j} \right)\,.$$

- To get the celestial OPE, we will now identify the following operator:

$$\cdots \sum_{r=0}^{q} \frac{(-[1\partial_2])^{q-r}}{(q-r)!} \frac{1}{r!} \sum_{i=3}^{n} \frac{[1\partial_i]^r}{z_{i2}^p}\, T_i^{a_1} A(\mathbf{2}^{a_2} \cdots n^{a_n})\,.$$

Soft gluon currents

- Firstly, to get the OPE at the level of operators in celestial CFT, we introduce the duality statement

$$A(1^{a_1}\, 2^{a_2} \cdots n^{a_n}) = \left\langle O^{a_1}_{s_1}(z_1, \tilde{\lambda}_1)\, O^{a_2}_{s_2}(z_2, \tilde{\lambda}_2) \cdots O^{a_n}_{s_n}(z_n, \tilde{\lambda}_n) \right\rangle \,.$$

- Soft gluon currents

$$O^a_+(z, \tilde{\lambda}) = \sum_{k,\ell=0}^{\infty} \frac{(\tilde{\lambda}^{\dot{1}})^k (\tilde{\lambda}^{\dot{2}})^\ell}{k!\, \ell!}\, J^a[k, \ell](z)\,.$$

- Useful to consider generating functions for the soft gluon currents

$$J^a[r](z, \tilde{\lambda}) = \sum_{k=0}^{r} \frac{(\tilde{\lambda}^{\dot{1}})^k (\tilde{\lambda}^{\dot{2}})^{r-k}}{k!(r-k)!}\, J^a[k, r-k](z)\,, \qquad r \in \mathbb{Z}_{\geq 0}\,,$$

  with hard gluons given by

$$O^a_+(z, \tilde{\lambda}) = \sum_{r=0}^{\infty} J^a[r](z, \tilde{\lambda})\,.$$

Soft gluon currents

– If we start with the leading OPE between two hard gluons

$$O_{+}^{a_1}(z_1,\tilde{\lambda}_1)\, O_{s_i}^{a_i}(z_i,\tilde{\lambda}_i) \sim \frac{f^{a_1 a_i}{}_b}{z_{1i}}\, O_{s_i}^{b}(z_i,\tilde{\lambda}_1+\tilde{\lambda}_i)\,,$$

we can compute the leading OPE between a soft and hard gluon:

$$J^{a_1}[r](z_1,\tilde{\lambda}_1)\, O_{s_i}^{a_i}(z_i,\tilde{\lambda}_i) \sim \frac{f^{a_1 a_i}{}_b}{z_{1i}}\, \frac{[1\partial_i]^r}{r!}\, O_{s_i}^{b}(z_i,\tilde{\lambda}_i)\,.$$

– Descendants of $J^{a_1}[r](z_1,\tilde{\lambda}_1)$

$$J_{-p}^{a_1}[r](\tilde{\lambda}_1) O_{s_2}^{a_2}(z_2,\tilde{\lambda}_2) := \oint_{|z_{12}|=\varepsilon} \frac{\mathrm{d}z_1}{2\pi\mathrm{i}}\, \frac{1}{z_{12}^{p}}\, J^{a_1}[r](z_1,\tilde{\lambda}_1)\, O_{s_2}^{a_2}(z_2,\tilde{\lambda}_2)\,.$$

– You will then find that inserting a soft descendant will result in the following expression

$$\left\langle J_{-p}^{a_1}[r](\tilde{\lambda}_1) O_{s_2}^{a_2}(z_2,\tilde{\lambda}_2) \prod_{i=3}^{n} O_{s_i}^{a_i}(z_i,\tilde{\lambda}_i) \right\rangle = -\frac{1}{r!} \sum_{i=3}^{n} \frac{[1\partial_i]^r}{z_{i2}^{p}}\, T_i^{a_1} A(2^{a_2}\, 3^{a_3} \cdots n^{a_n})\,.$$

All order gluon OPE

– The all-order collinear expansion can be written as

$$A(1_+^{a_1}\, 2^{a_2} \cdots n^{a_n}) = -\sum_{p=0}^{\infty} z_{12}^{p-1} \sum_{q=0}^{p} \sum_{r=0}^{q} \frac{(-[1\partial_2])^{q-r}}{r!(q-r)!} \sum_{i=3}^{n} \frac{[1\partial_i]^r}{z_{i2}^p}\; T_i^{a_1} A(\mathbf{2}^{a_2} \cdots n^{a_n})$$

$$= \sum_{p=0}^{\infty} z_{12}^{p-1} \sum_{q=0}^{p} \sum_{r=0}^{q} \frac{(-[1\partial_2])^{q-r}}{(q-r)!} \left\langle J_{-p}^{a_1}[r](\tilde{\lambda}_1) O_{s_2}^{a_2}(z_2, \tilde{\lambda}_1 + \tilde{\lambda}_2) \prod_{i=3}^{n} O_{s_i}^{a_i}(z_i, \tilde{\lambda}_i) \right\rangle .$$

– The all-order OPE in momentum eigenstates is given by

$$\boxed{O_+^{a_1}(z_1, \tilde{\lambda}_1)\, O_{s_2}^{a_2}(z_2, \tilde{\lambda}_2) = \sum_{p=0}^{\infty} z_{12}^{p-1} \sum_{r=0}^{p} \sum_{q=0}^{p-r} \frac{(-[1\partial_2])^q}{q!}\; J_{-p}^{a_1}[r](\tilde{\lambda}_1) O_{s_2}^{a_2}(z_2, \tilde{\lambda}_1 + \tilde{\lambda}_2)} .$$

Conclusion

– Inverse soft recursion for MHV amplitudes is dual to OPE recursion for celestial CFT.

– The OPE is written in terms of soft gluon currents and their descendants.

– Advantage of our approach: you can also apply it to gravity!

$$M(1_+ \, 2 \cdots n) = \sum_{i=3}^{n} \frac{[1i]}{z_{1i}} \frac{z_{i2}^2}{z_{12}^2} \, M(\hat{2} \cdots \hat{i} \cdots n) \, .$$

– Graviton OPE can similarly be written in terms of soft gravitons and their descendants.

– We would like to apply this to the NMHV sector, but here multiparticle poles appear in the BCFW inverse soft recursion.

Celestial OPE in boost Eigenstates

– We can perform the Mellin transform to go to boost eigenstates. For example when $\varepsilon_1 = \varepsilon_2$

$$O^{\varepsilon_1,a_1}_{\Delta_1,+}(z_1,\bar{z}_1)\, O^{\varepsilon_2,a_2}_{\Delta_2,s_2}(z_2,\bar{z}_2) = \sum_{p,\bar{m}=0}^{\infty} \sum_{r=0}^{p} \frac{\varepsilon_1^r}{\bar{m}!} B(2\bar{h}_1 + r + \bar{m}, 2\bar{h}_2) \frac{\Gamma(2\bar{h}_1 + p + 1)}{(p-r)!\,\Gamma(2\bar{h}_1 + r + 1)} \times z_{12}^{p-1}\, \bar{z}_{12}^{\bar{m}} \bar{\partial}_2^{\bar{m}}\, J^{a_1}_{-p}[r](\bar{z}_1) O^{\varepsilon_1,a_2}_{\Delta_1+\Delta_2+r-1,s_2}(z_2,\bar{z}_2)$$