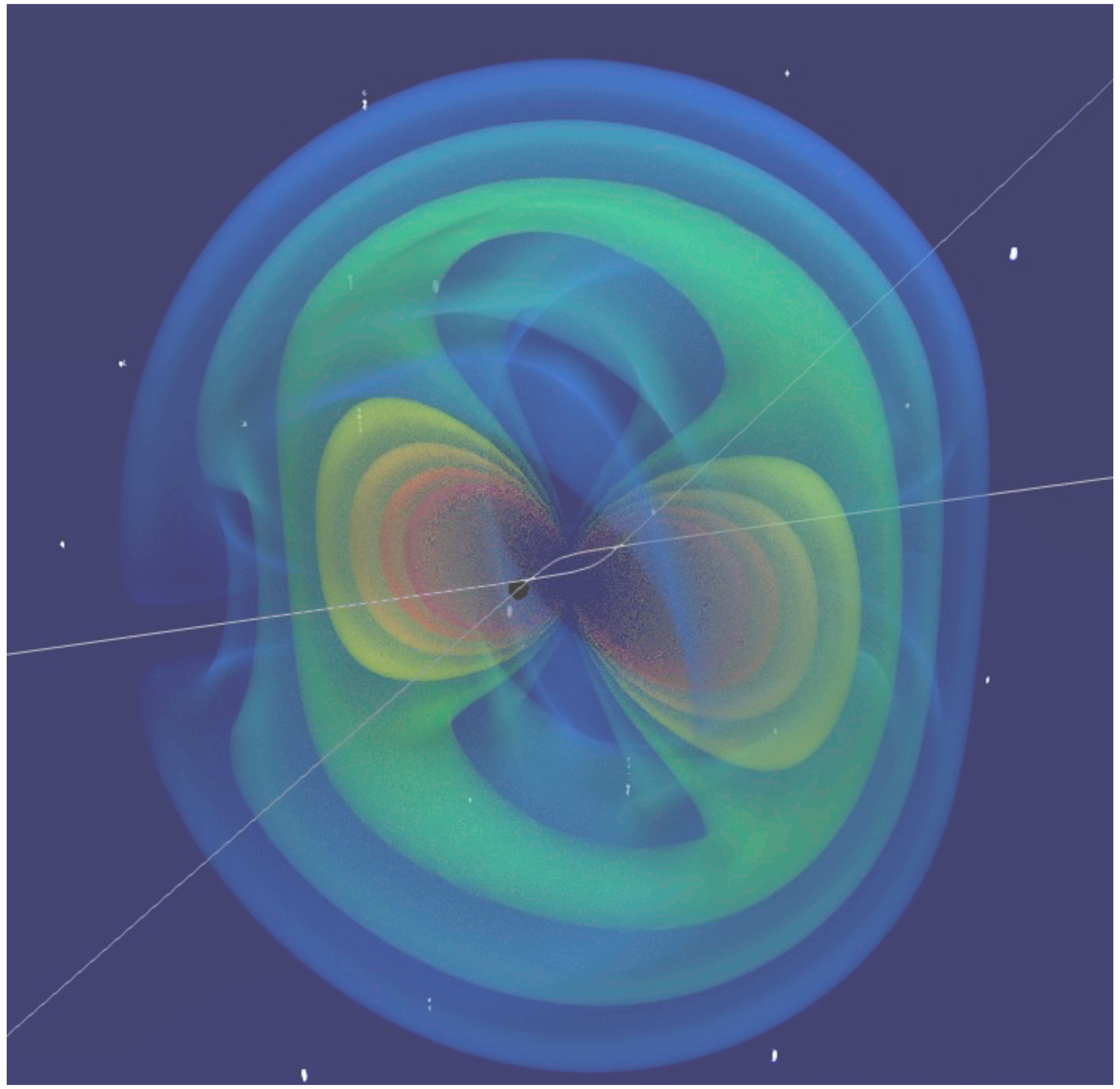
HIGH PRECISION GRAVITATIONAL WAVE PHYSICS FROM A WORLDLINE QUANTUM FIELD THEORY



Jan Plefka

Humboldt Universität zu Berlin

Based on joint work with

Gustav Uhre Jakobsen, Gustav Mogull, Benjamin Sauer, Jan Steinhoff (AEI)

2010:02865, *JHEP* 02 (2021) 048

2101.12688, ***PRL*** 126 (2021) 20

2106.10256, ***PRL*** 128 (2022) 1

2109.04465, *JHEP 01 (2022) 027*

2201.07778, ***PRL*** 128 *(2022) 14*

2207.00569, *JHEP* 10 (2022) 128

RTG 2575: Rethinking Quantum Field Theory

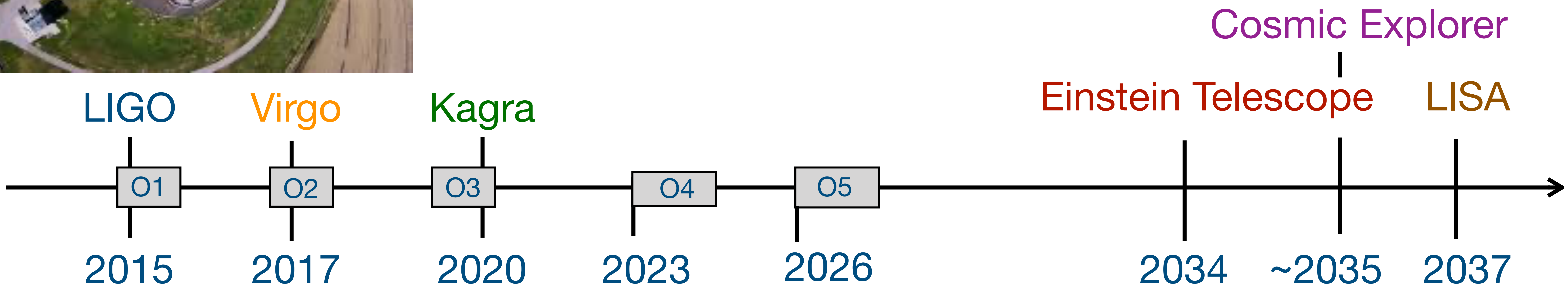

$\gamma = 1.1, \ \frac{m_1}{m_2} = 1, \ \frac{b}{m_1} = 50$
Time: -5.840000

ERA OF GRAVITATIONAL WAVE PHYSICS: NEED FOR HIGH-PRECISION PREDICTIONS

- Upcoming 3rd generation of gravitational wave observatories with 10^2 sensitivity increase
- Need for accurate waveform predictions well beyond state-of-the art

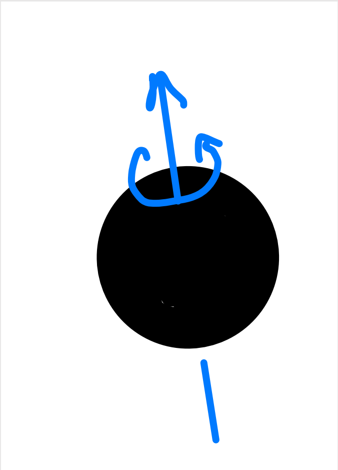
High-precision predictions necessary basis to study fundamental questions in physics:

- Is Einstein's theory correct?
- Black hole formation & population?
- Neutron star properties?
- Physics beyond the standard model?



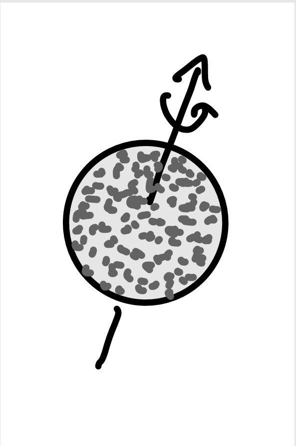
GRAVITATIONAL TWO-BODY PROBLEM

Black Hole



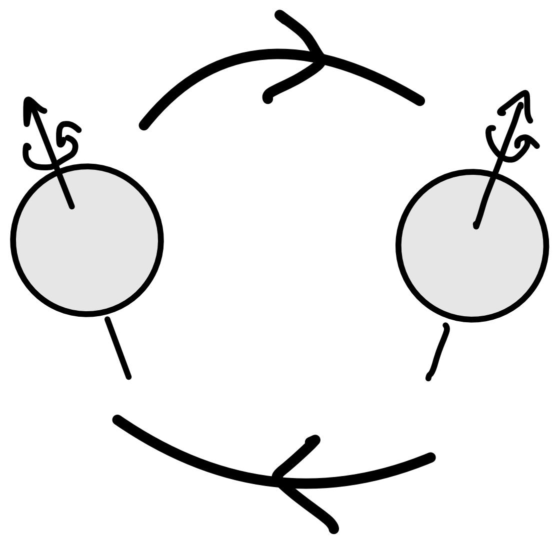
mass, spin

Neutron Star

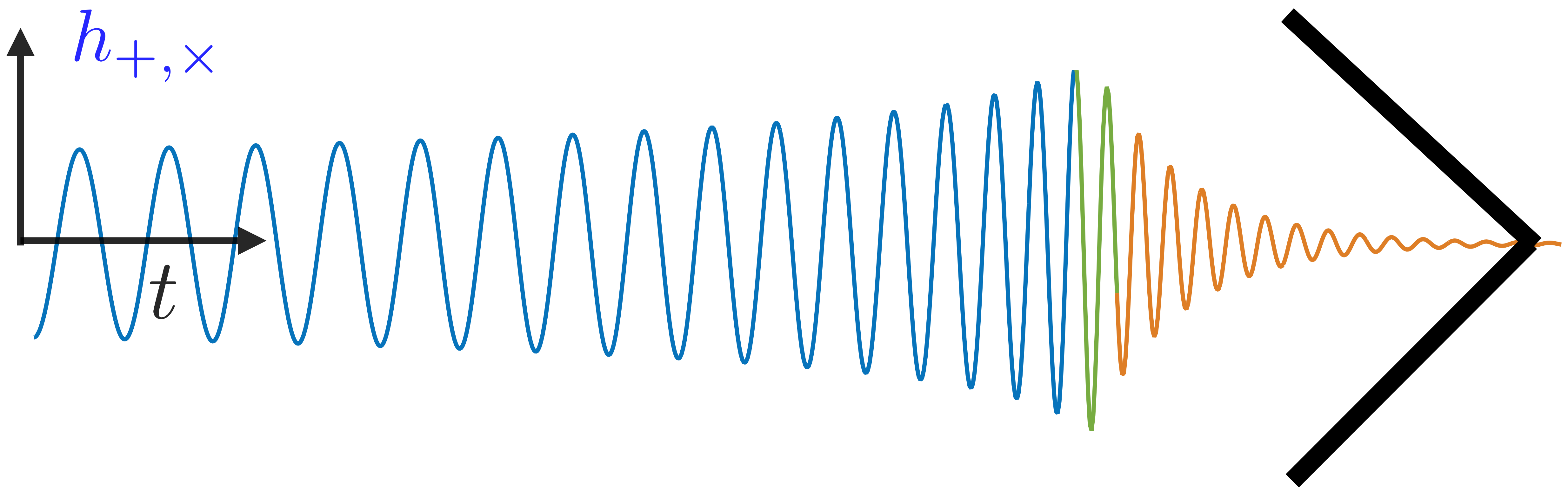


mass, spin, radius, tidal deformability

Black Hole/Neutron Star Binaries:



Bound state



inspiral

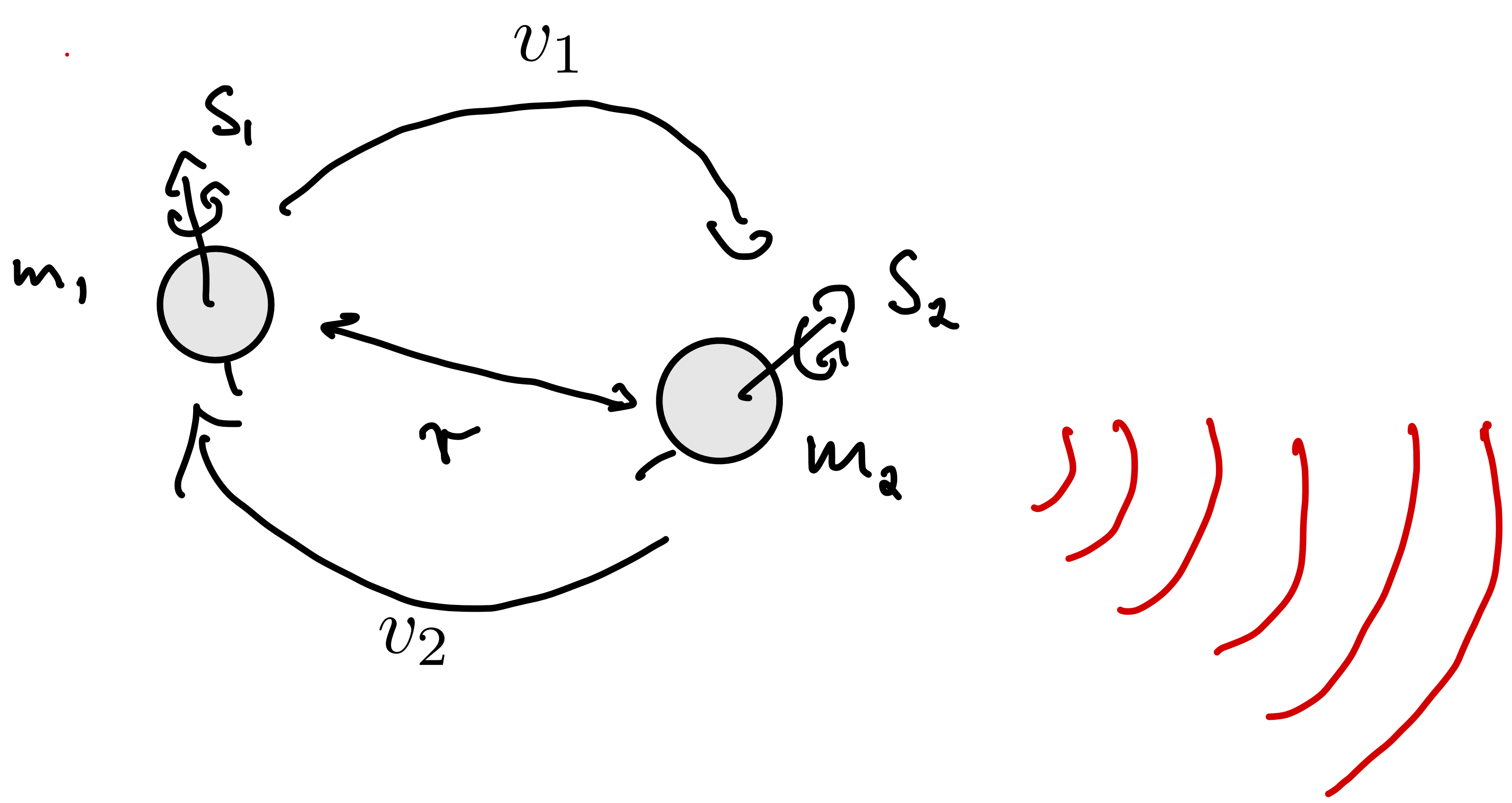
merger

- During **inspiral**: weak gravitational fields $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$
- **Quantum** field theory formalism for **classical** two-body problem:

WORLDLINE QUANTUM FIELD THEORY

THE GENERAL RELATIVISTIC 2-BODY PROBLEM

As in Newtonian case has either **bound** or **unbound** orbits.



Inspiral of 2 black holes or neutron stars:

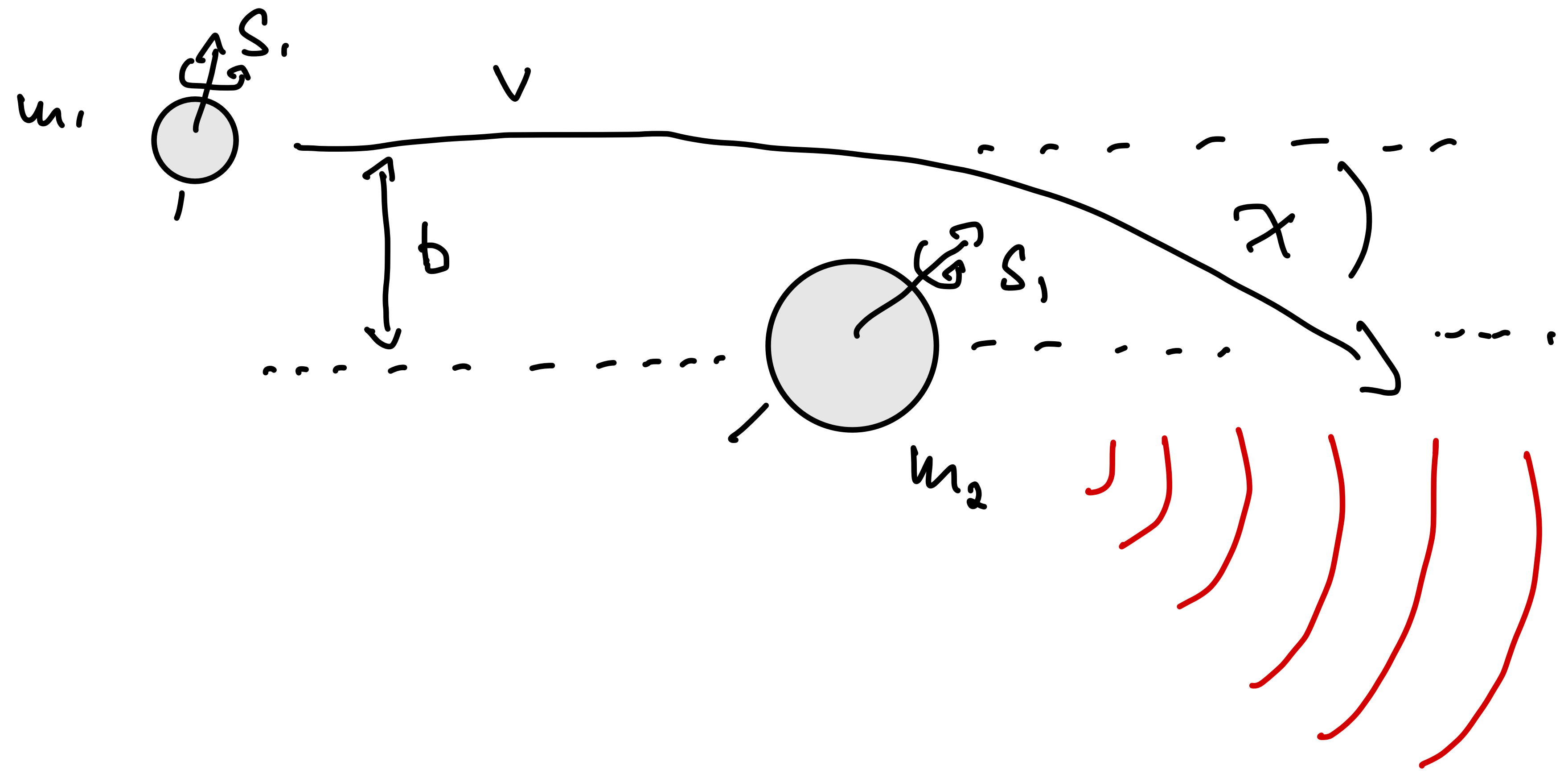
Virial-theorem: $\frac{GM}{r} \sim v^2 \qquad (c=1)$

post-Newtonian (PN) expansion in G & v^2

Weak field expansion: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$

$\kappa = \sqrt{32\pi G}$

Newton's constant



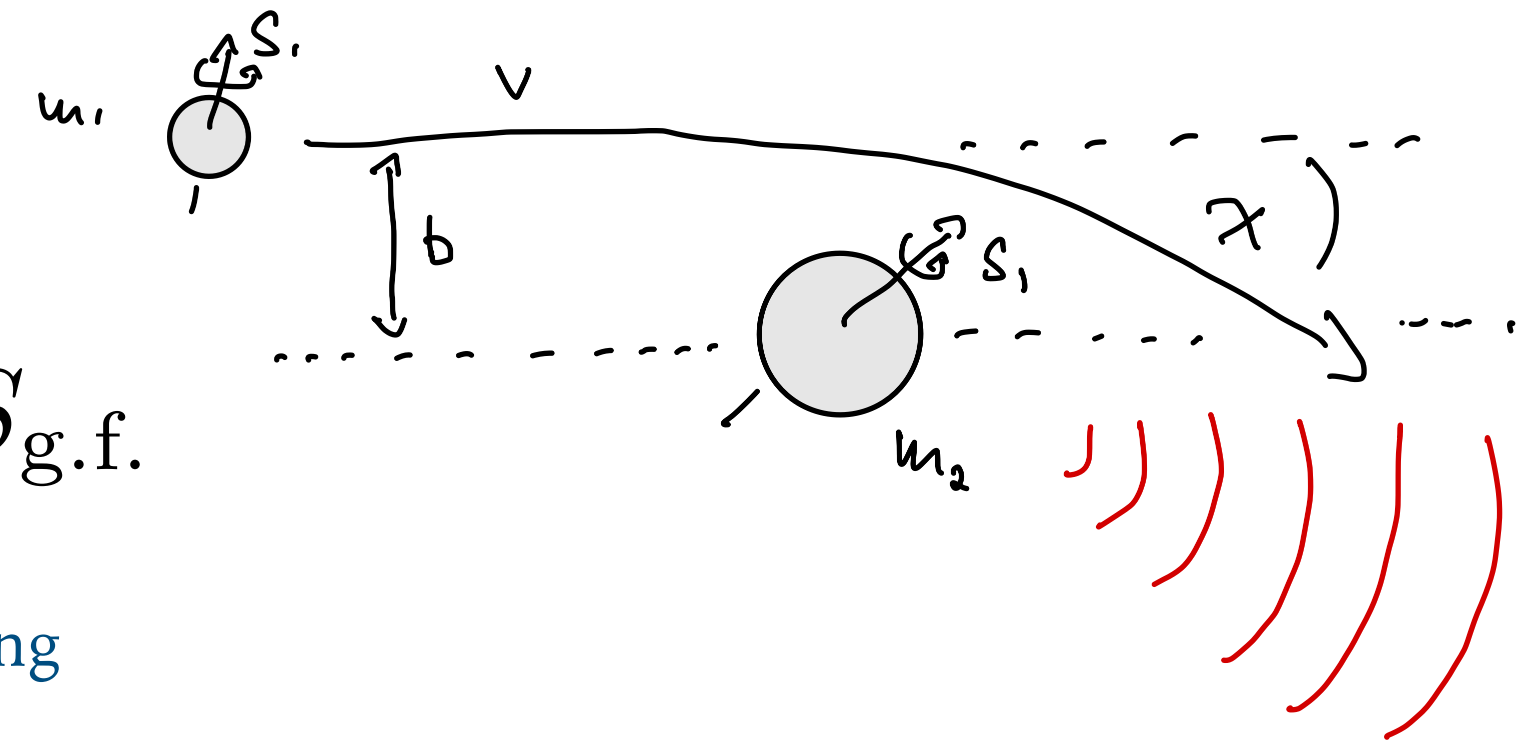
Scattering of 2 black holes or neutron stars:

Weak field (G), but exact in v^2

post-Minkowskian (PM) expansion

RELATIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH

Point-particle approximation for BHs (or NSs)



$$S = -\sum_{i=1}^{2} \int d\tau_i \sqrt{g_{\mu\nu}\dot{x}_i^{\mu}(\tau_i)\dot{x}_i^{\nu}(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\mathrm{g.f.}}$$

Point particle approximation — Bulk gravity & gauge fixing

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \frac{\kappa^2}{8}T_{\mu\nu}$$

Einstein's eqs.

$$\ddot{x}_i^{\mu} + \Gamma^{\mu}{}_{\nu\rho}\dot{x}_i^{\nu}\dot{x}_i^{\rho} = 0$$

Geodesic eqs.

2) Solve iteratively in G

$$g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} G^n\, h_{\mu\nu}^{(n)}(x)$$

emitted radiation

$$x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\,\tau + \sum_{n=1}^{\infty} G^n\, z_i^{(n)\,\mu}(\tau)$$

straight line: „in“ state — deflections

3) Construct observables

Far field waveform:

$$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t-r,\theta,\varphi)}{r} + \mathcal{O}(\frac{1}{r^2})$$

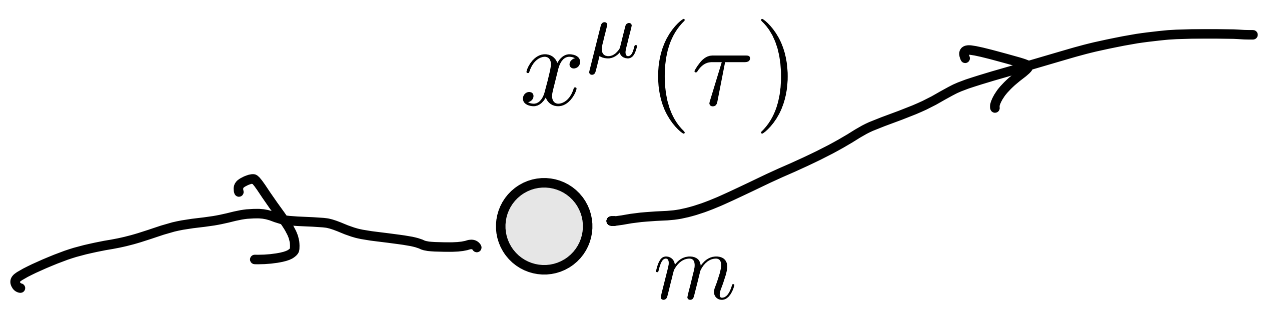
„Impulse“ (change in momentum):

$$\Delta p_i^{\mu} = m_i\dot{x}_i^{\mu}\Big|_{\tau=-\infty}^{\tau=+\infty} = m_i\int d\tau\,\ddot{x}_i^{\mu}(\tau)$$

WORLDLINE QUANTUM FIELD THEORY

Mogull, **JP**, Steinhoff *JHEP* 02 (2021) 048

- Model Black Holes/Neutron Stars as a point particles



$$S_{\mathrm{BH/NS}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\,\dot{x}^\mu(\tau)\,\dot{x}^\nu(\tau) \quad + \text{[spin \& tidal effects]}$$

They interact through Einstein's gravity:

$$S = S_{\mathrm{BH/NS}} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^\mu(\tau) = b_i^\mu + v_i^\mu\tau + z^\mu(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$

- Path integral quantisation perturbative in Newton's constant G but exact in velocity

$$\langle\mathcal{O}\rangle_{\mathrm{WQFT}} = \int D[h,z]\,\mathcal{O}\,e^{-\frac{i}{\hbar}S[z,h]} \quad \xrightarrow{\hbar\to 0}$$

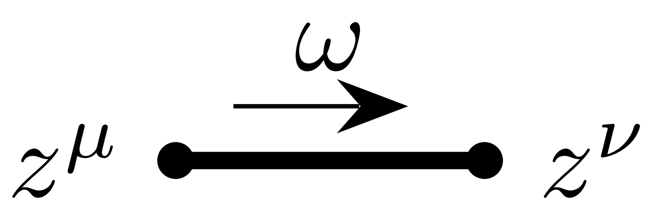
Tree-level one-point functions $\langle h_{\mu\nu}\rangle$ and $\langle z^\mu\rangle$ solve classical equations of motion

$\Rightarrow$ Advanced quantum field theory technology for classical gravitational wave physics

WORLDLINE QUANTUM FIELD THEORY: PERTURBATIVE SETUP

$$S_{\mathrm{WQFT}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\, \dot{x}^{\mu}(\tau)\, \dot{x}^{\nu}(\tau) + \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R(g)$$

- Scattering scenario: $x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z^{\mu}(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$
- Worldline propagators:



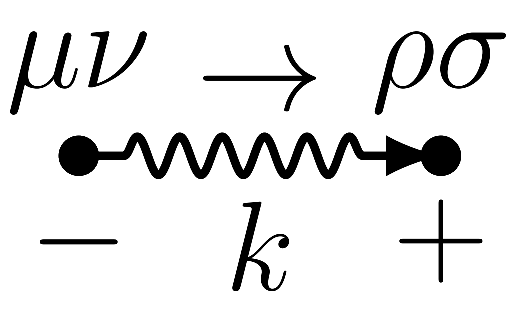
$$\langle z^{\mu}(\omega) z^{\nu}(-\omega)\rangle = -\frac{i}{m}\frac{\eta_{\mu\nu}}{(\omega + i0)^2}$$

- Perturbative (quantum) gravity:

$$\sqrt{-g}\, R(g) \;\xrightarrow{\;g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}\;}\; -\frac{1}{2}h_{\mu\nu}(P^{-1})^{\mu\nu;\rho\sigma}\Box h_{\rho\sigma} + \sqrt{G}[\partial^2 h^3] + \sqrt{G}^2[\partial^2 h^4] + \sqrt{G}^3[\partial^2 h^5] + \ldots$$

$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

⇒ graviton propagator:

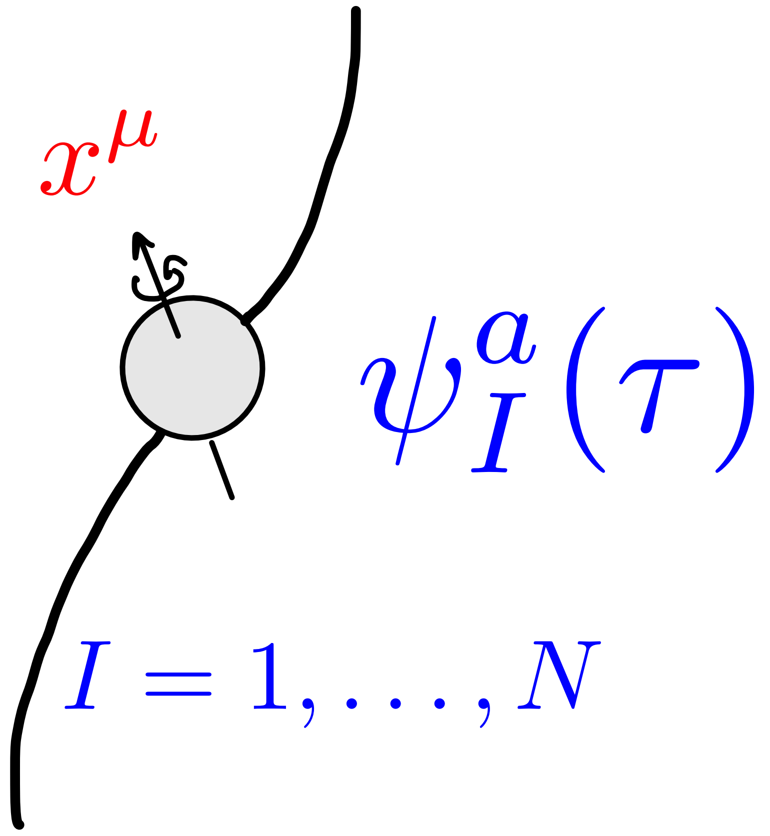


$$= i\frac{P_{\mu\nu;\rho\sigma}}{(k^0 + i0)^2 - \mathbf{k}^2}$$

N.B. need to take retarded propagator (in-in formalism)

PUTTING SPIN ON THE WORLD-LINE

[Jakobsen, Mogull, JP, Steinhoff]



- Hidden **supersymmetry** of spinning-black holes!

  Captures (Spin)^N interactions

- Presently only control spin-orbit & spin-spin interactions ($N = 2$)

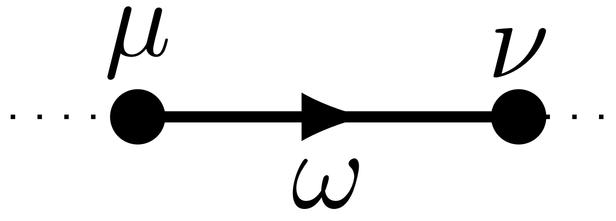
$$S_{\mathrm{BH/NS}} = -m \int d\tau \Big[\tfrac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + i\bar{\psi} D_\tau \psi + \tfrac{1}{2} R_{abcd} \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d + C_E R_{a\mu b\nu} \dot{x}^\mu \dot{x}^\nu \bar{\psi}^a \psi^b \, \bar{\psi} \cdot \psi \Big]$$

spin degrees of freedom — neutron star term

Scattering scenario:

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + z_i^\mu(\tau)$$
$$\psi_i^a(\tau) = \Psi_i^a + \psi_i'^a(\tau)$$

Integrate out $z_i^\mu, \psi_i'^a, \bar{\psi}_i'^a$ perturbatively!

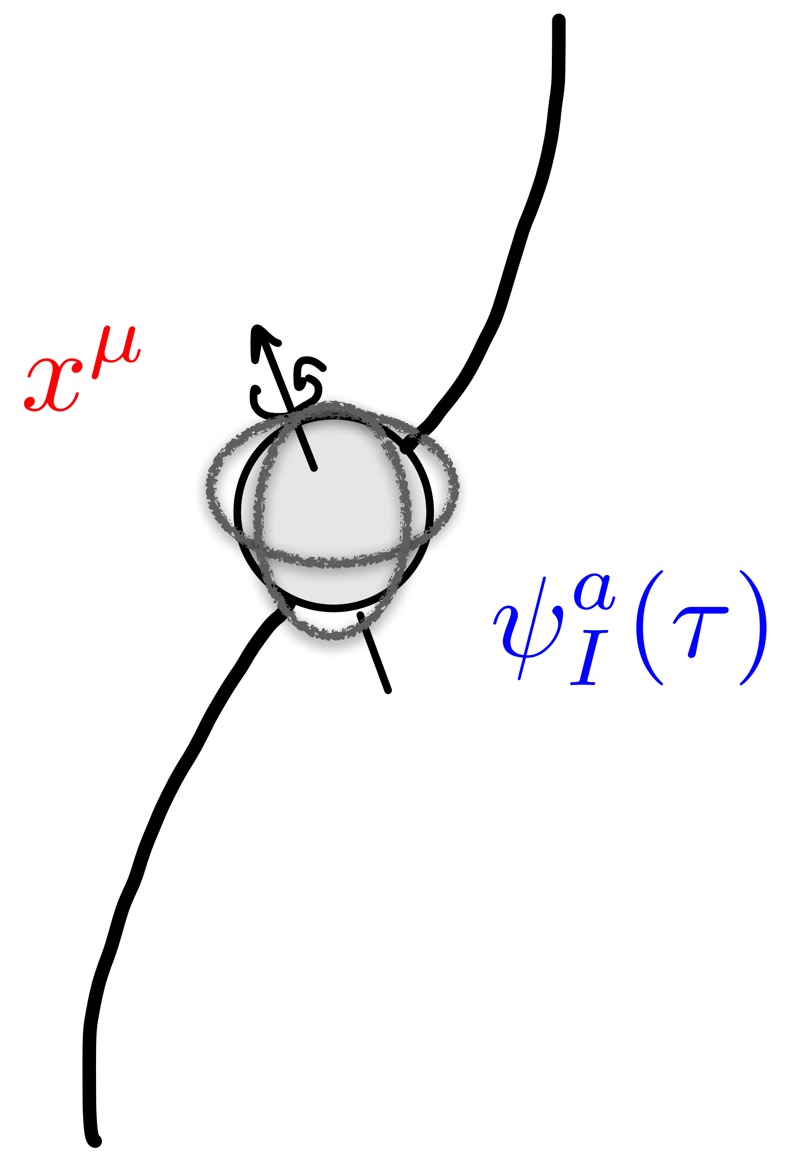


$$\langle \psi^a(\omega) \bar{\psi}^b(-\omega) \rangle = \frac{-i\eta^{ab}}{m(\omega + i0)}$$

$$S_i^{ab} = -2im \bar{\psi}_i^{[a} \psi_i^{b]}$$

Initial spins of BHs/NSs

TIDAL INTERACTIONS



- First layer of tidal & finite size effects:

$$S_{\text{tidal}} = m \int d\tau \left[c_{E^2} E_{\mu\nu} E^{\mu\nu} + c_{B^2} B_{\mu\nu} B^{\mu\nu} \right]$$

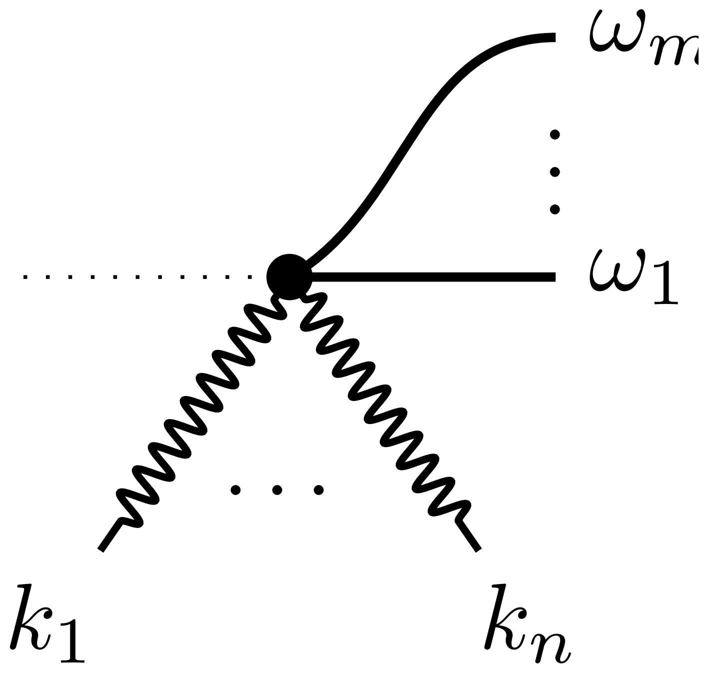
Electric and magnetic curvature:

$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta \qquad B_{\mu\nu} := R^*_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta$$

Wilson coefficients (or „Love numbers“): c_{E^2} & c_{B^2} (vanish for black holes)

WORLDLINE QUANTUM FIELD THEORY: VERTICES

- Worldline vertices: n-gravitons & m world-line fluctuations

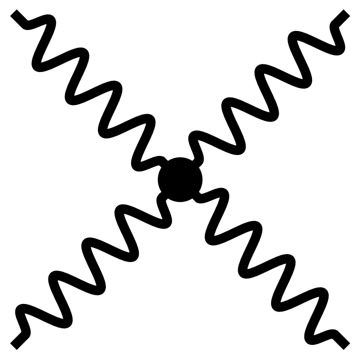
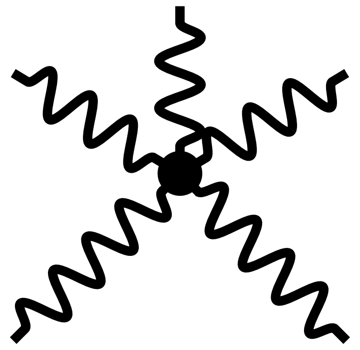
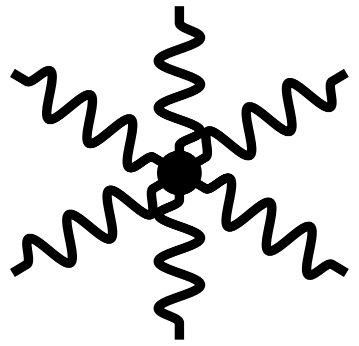
$V_{n|m} =$ 

$$= m\sqrt{G}^{\,n}\, e^{ib\cdot\sum_j k_j}\,\delta\Big(v\cdot\sum_{j=1}^{n}k_j+\sum_{i=1}^{m}\omega_i\Big)\times\begin{pmatrix}\text{polynomial in } \omega_i, k_j\\ \text{of degree } 2n+m\\ \text{depending on } v^\mu,\ S^{\mu\nu}\end{pmatrix}$$

+ C_E, c_{E^2}, c_{B^2} for neutron stars

Energy conservation on worldline

- „Bulk“ graviton vertices:

 $\sim \sqrt{G}^{2} k^2$,  $\sim \sqrt{G}^{3} k^2$  $\sim \sqrt{G}^{4} k^2, \ldots$

Four-momentum conservation in bulk

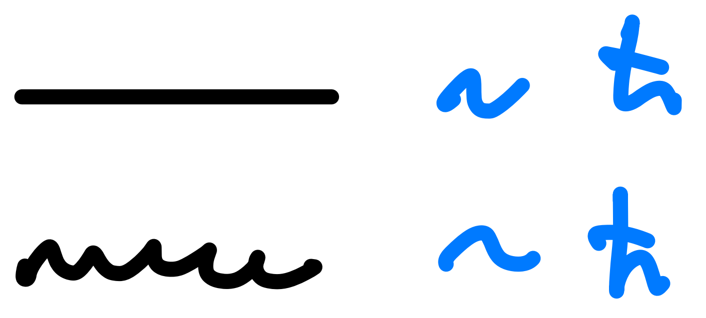
CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

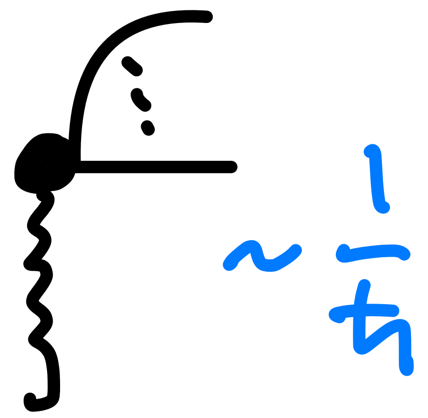
[Jakobsen]

- Action: $S[\Phi_A]$ with fields $\Phi_A(x_A) = \{h_{\mu\nu}(x), z^\mu(\tau)\}$ and coordinates $x_A = \{x^\mu, \tau\}$
- Partition function in the presence of sources

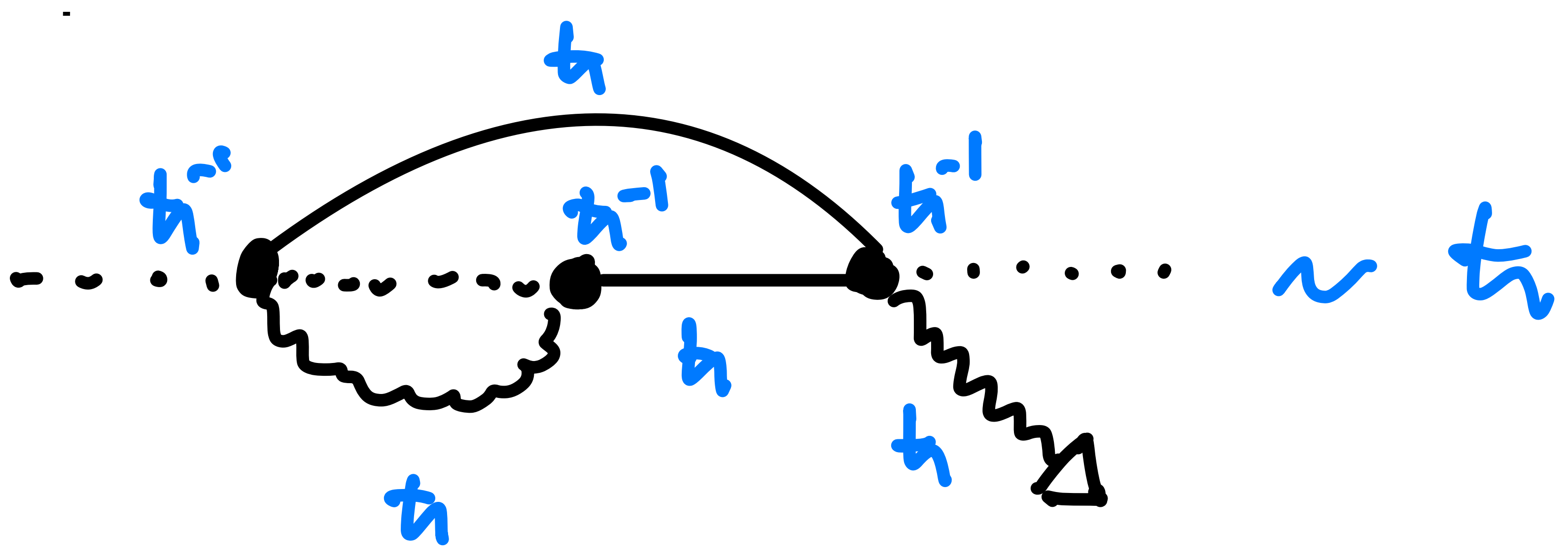
$$Z[J_A] = \int D[\Phi_A] \exp\left[\frac{i}{\hbar}\left(S[\Phi_A] + \sum_A \int dx_A J_A(x_A)\,\Phi_A(x_A)\right)\right]$$

- $\hbar$ counting:





E.g:



$\Rightarrow$ Loops are quantum effects!

- Scalings of connected n-point functions:

$$\langle \Phi_{A_1} \ldots \Phi_{A_n} \rangle_{\text{conn}} \sim \sum_L \hbar^{-1+n+L} \quad \text{(L-loop connected n-point diagrams)}$$

Well defined classical limit only for n=1 and L=0: **Tree-level one-point functions**

CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

[Jakobsen]

- Factorization $$\lim_{\hbar\to 0} \langle \Phi_{A_1}\Phi_{A_2}\dots\Phi_{A_n}\rangle_{\text{discon}} = \langle \Phi_{A_1}\rangle_{\text{con}}^{\text{tree}} \langle \Phi_{A_2}\rangle_{\text{con}}^{\text{tree}} \dots \langle \Phi_{A_n}\rangle_{\text{con}}^{\text{tree}}$$

- Consequence for Schwinger-Dyson equations:

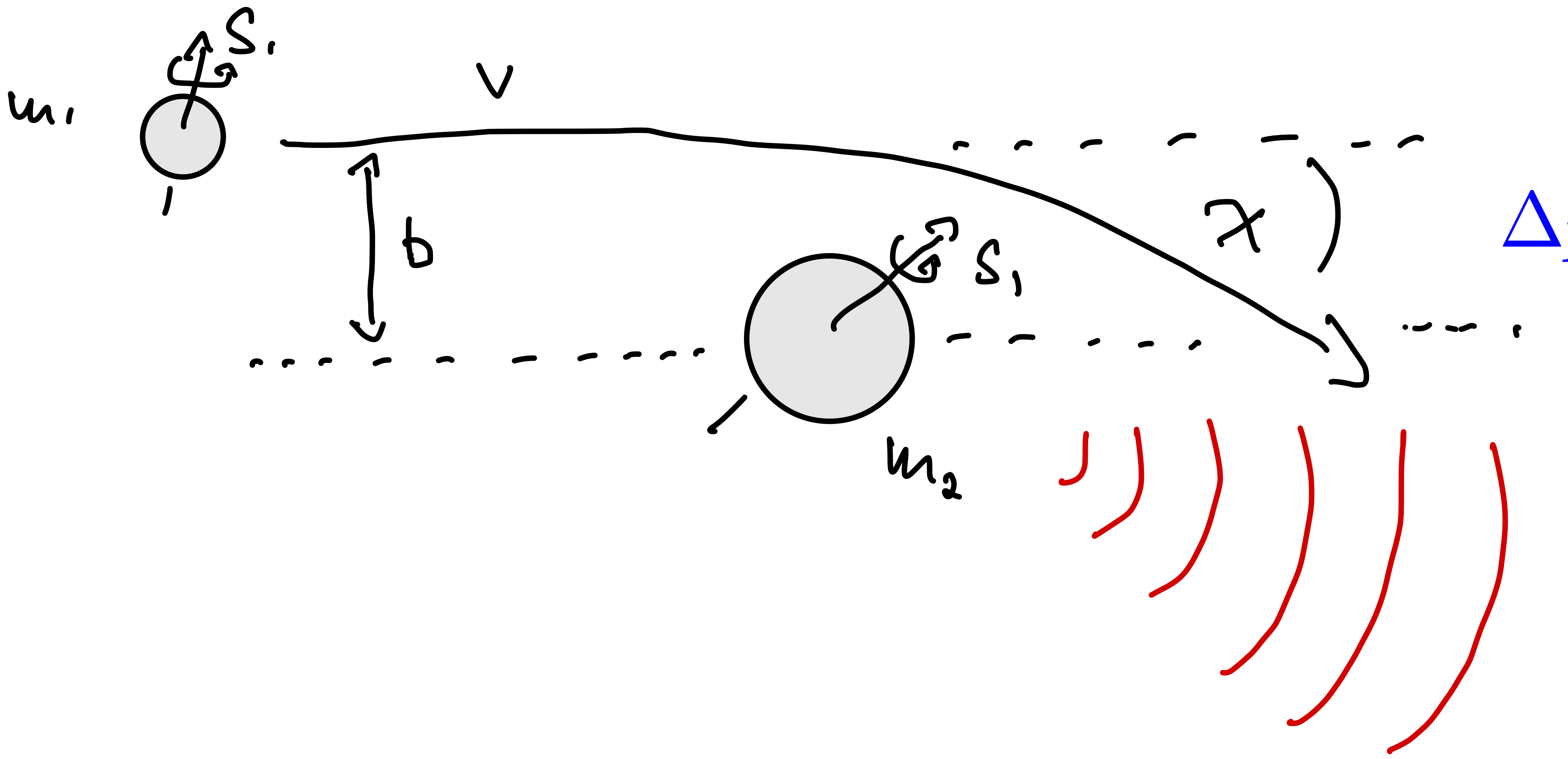
$$\left\langle \frac{\delta S[\Phi_A]}{\delta \Phi_A} \right\rangle = 0 \quad \xrightarrow{\hbar \to 0} \quad \frac{\delta S[\langle \Phi_A\rangle_{\text{tree}}]}{\delta \Phi_A} = 0$$

Tree-level one-point functions solve classical equations of motion

- Importantly $S[\Phi_A]$ must be independent of $\hbar$ (not the case in amplitudes approach - massive field!) -> Key advantage of WQFT approach (no „super classical“ terms)

- Need non-trivial background field configurations for non-vanishing one-point functions

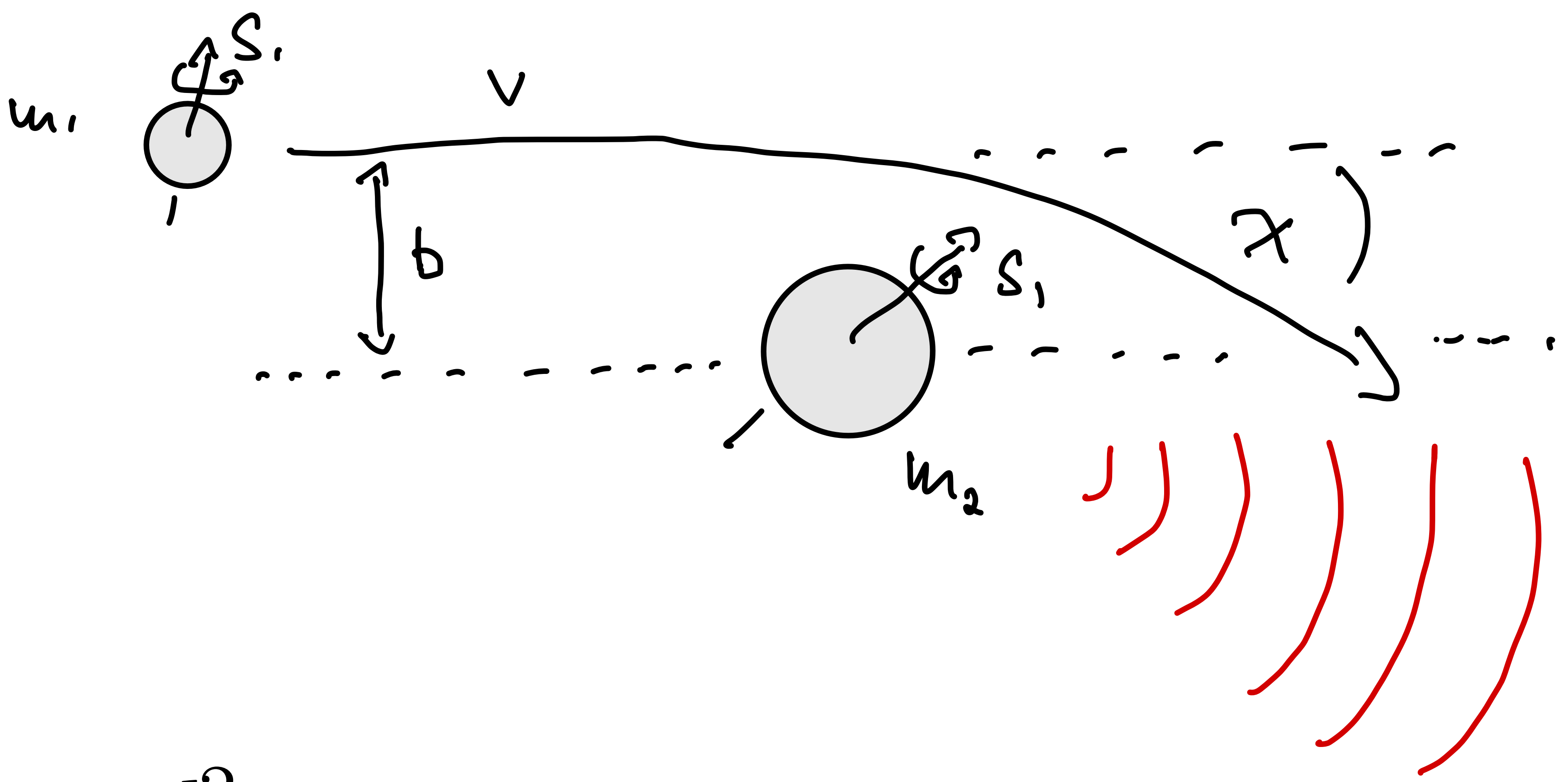
WQFT OBSERVABLES



$$\Delta p_1^\mu = \sum_{n=1}^{\infty} G^n \, \Delta p_1^{(n)\,\mu}$$

$$f_{\mu\nu} = \sum_{n=1}^{\infty} G^n \, f_{\mu\nu}^{(n)}$$

OBSERVABLES OF WQFT: ONE POINT FUNCTIONS



Spin-less BH/NS scattering:

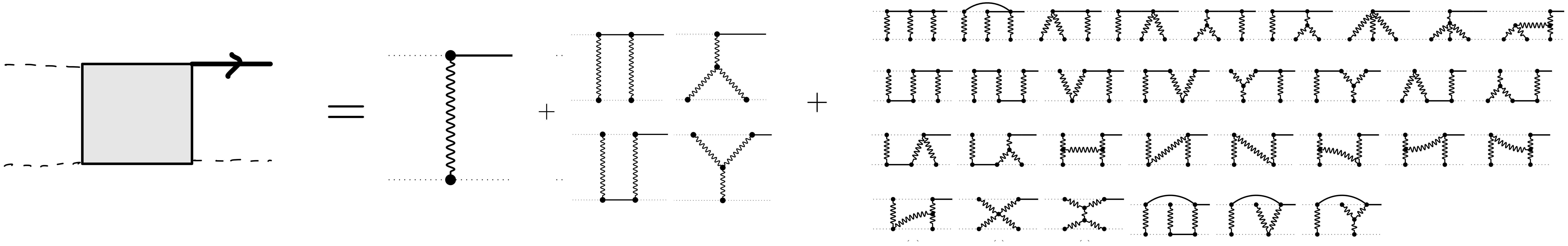
1) Impulse (change of momentum)

$$\Delta p_i^\mu = m_i \langle \dot{x}_i^\mu \rangle \Big|_{\tau=-\infty}^{\tau=+\infty} = m_i \int d\tau \langle \ddot{x}_i^\mu(\tau) \rangle = m_i \int d\tau \frac{d^2}{d\tau^2} \langle z_i^\mu(\tau) \rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega \to 0}$$

Fourier trans. ↑

Needs sum of all graphs with outgoing z-line: Jakobsen, Mogull, *PRL* 128 (2022) 14; Jakobsen, Mogull, **JP**, Sauer, *JHEP* 10 (2022) 128

$$\langle \Delta p^\mu \rangle =$$



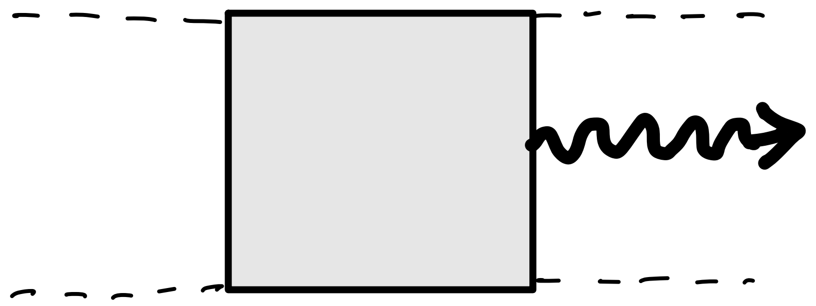
G   G²   G³

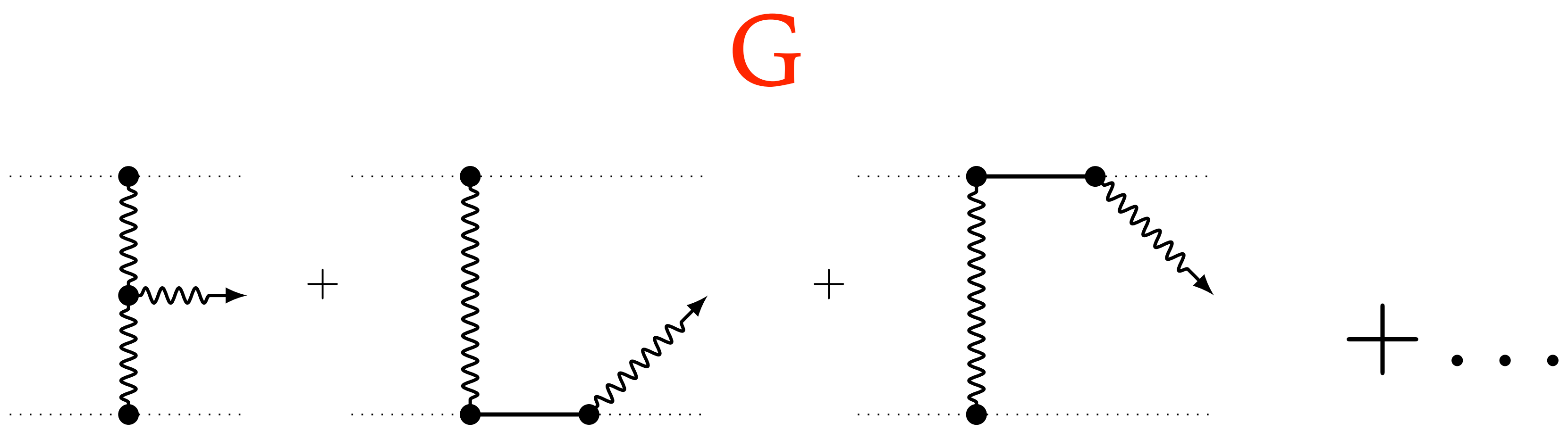
OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

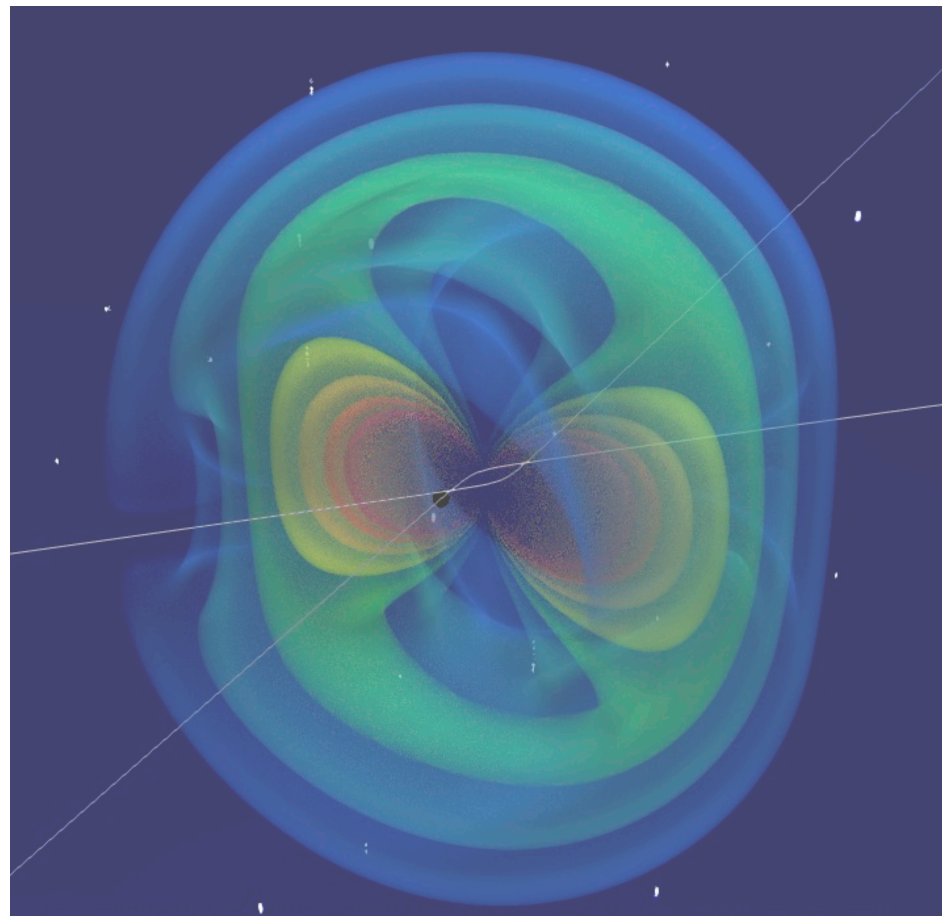
2) **Emitted Waveform** (Gravitational Bremsstrahlung)

Jakobsen, Mogull, **JP**, Steinhoff, ***PRL*** 126 (2021) 20, ***PRL*** 128 (2022) 1

$$\langle h_{\mu\nu} \rangle$$

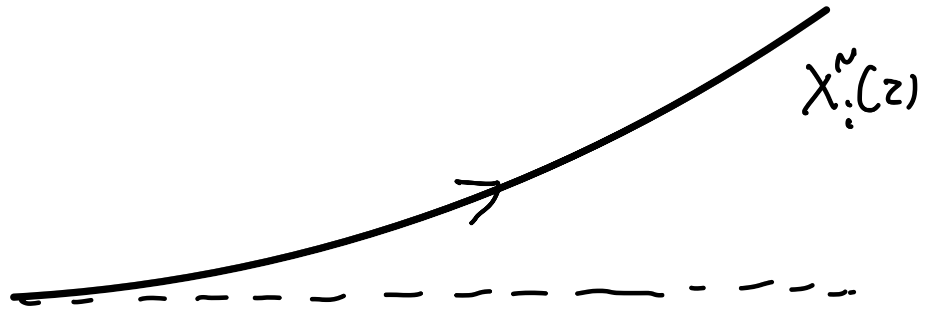






3) Trajectory!

$$X_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + \int d\omega \; e^{i\omega\cdot\tau} \langle z_i^\mu(\omega) \rangle_{WQFT}$$

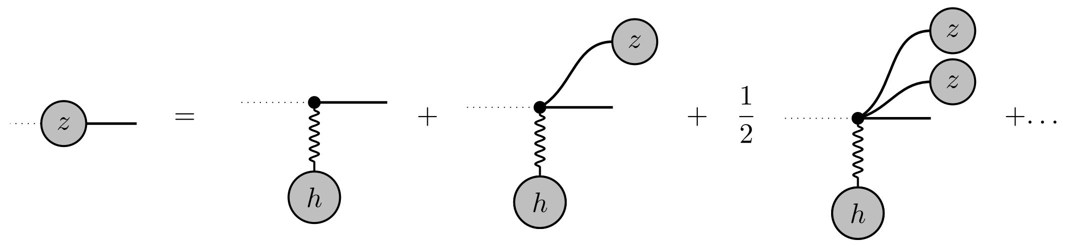


WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion
(tree-level!)



Insert Feynman rules

Form script
(4PM: 5g vertex)

Tensor reduction to scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i\epsilon$

IBPs: Basis of Master Integral

Fire/Litered/Kira
Reduced symmetries
due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

Solve masters by Diff. Eq. technique

Single scale
integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions
(potential & radiation
gravitons)

$i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

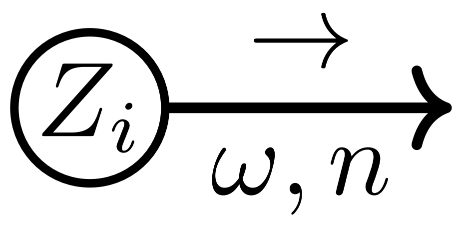
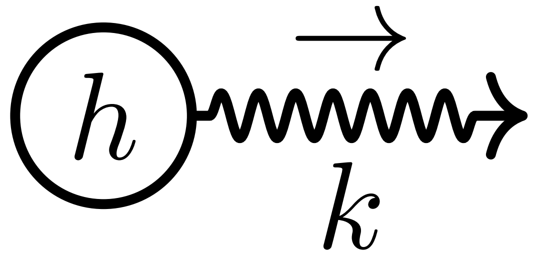
ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

Generate graphs

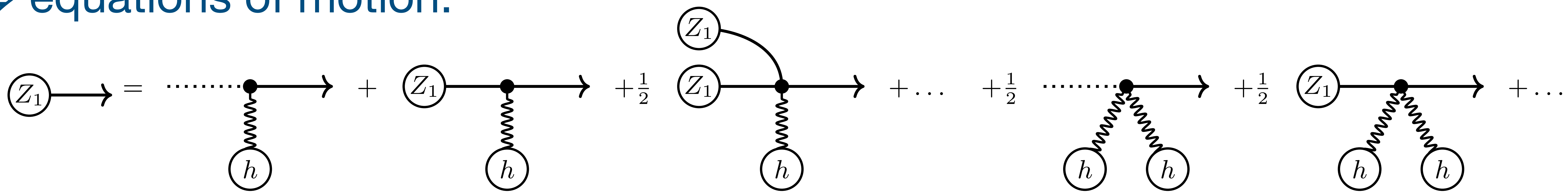
BERENDS-GIELE TYPE RECURSION

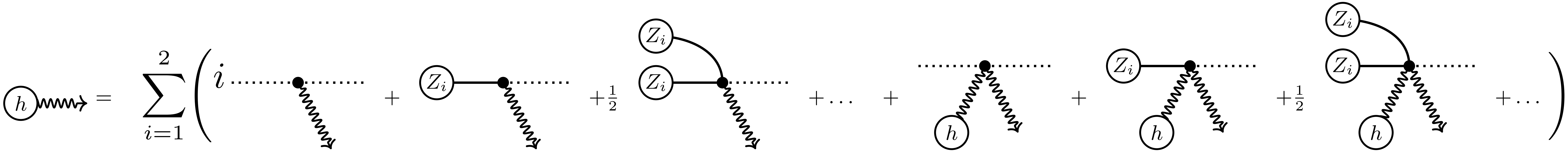
[Jakobsen,Mogull,JP,Sauer]

- One-point functions: $\langle Z_i(\omega)\rangle =$  $\langle h_{\mu\nu}(k)\rangle =$ 

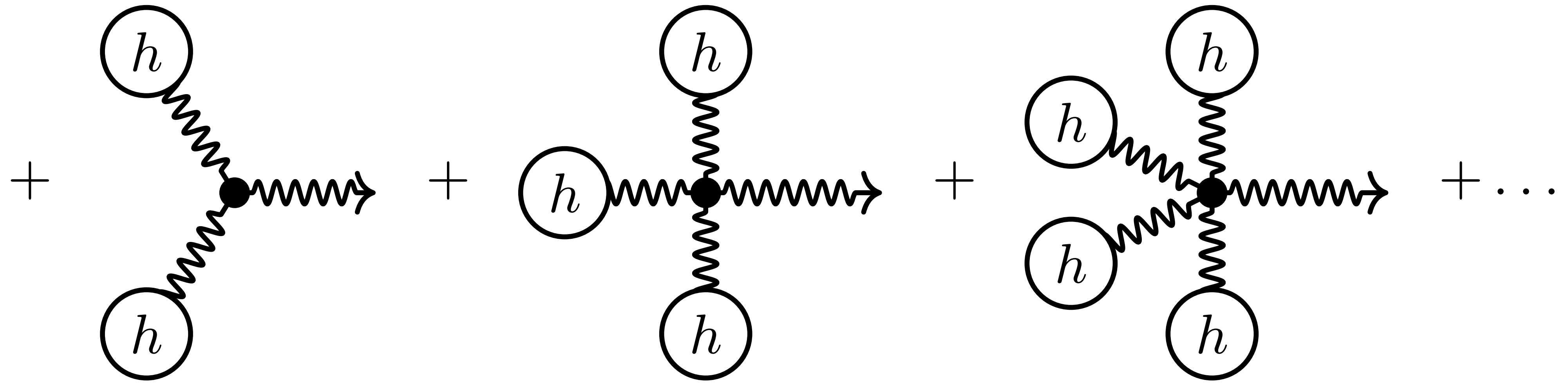
- Recursion ⇔ equations of motion:

Geodesic:





Einstein:



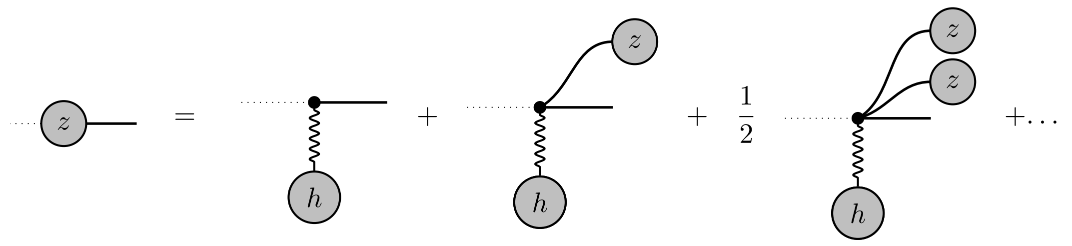
Causality flow implented

WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion
(tree-level!)



Insert Feynman rules

Form script
(4PM: 5g vertex)

Tensor reduction to
scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i\epsilon$

IBPs: Basis of Master
Integral

Fire/Litered/Kira
Reduced symmetries
due to retarded prop.
$l \to -l \quad \epsilon \to -\epsilon$

Solve masters by Diff.
Eq. technique

Single scale
integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions
in static limit (PN) $\gamma \to 1$

Method of regions
(potential & radiation
gravitons)

$i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$
ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

Tensor reduction to scalar integrals

STRUCTURE OF WQFT INTEGRALS: IMPULSE & SPIN KICK

[Jakobsen,Mogull,JP,Sauer]

- Order n-PM : Single scale (n-1)-loop integral

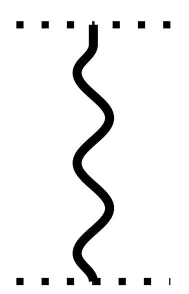
$$I_{\mathrm{nPM}} = \int_q e^{-q\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)\int_{l_1,l_2\ldots l_{n-1}} \frac{\mathrm{num}[l_i]}{D_1\ldots D_j}\delta(l_1\cdot v_*)\delta(l_1\cdot v_*)\ldots\delta(l_{n-1}\cdot v_*)$$

$$v_* \in \{v_1, v_2\}$$

Retarded propagators $D_i(l_i, q, v_*)$ are linear $(l_i \cdot v_*) \pm i0$ or quadratic $(l_i + q)^2$

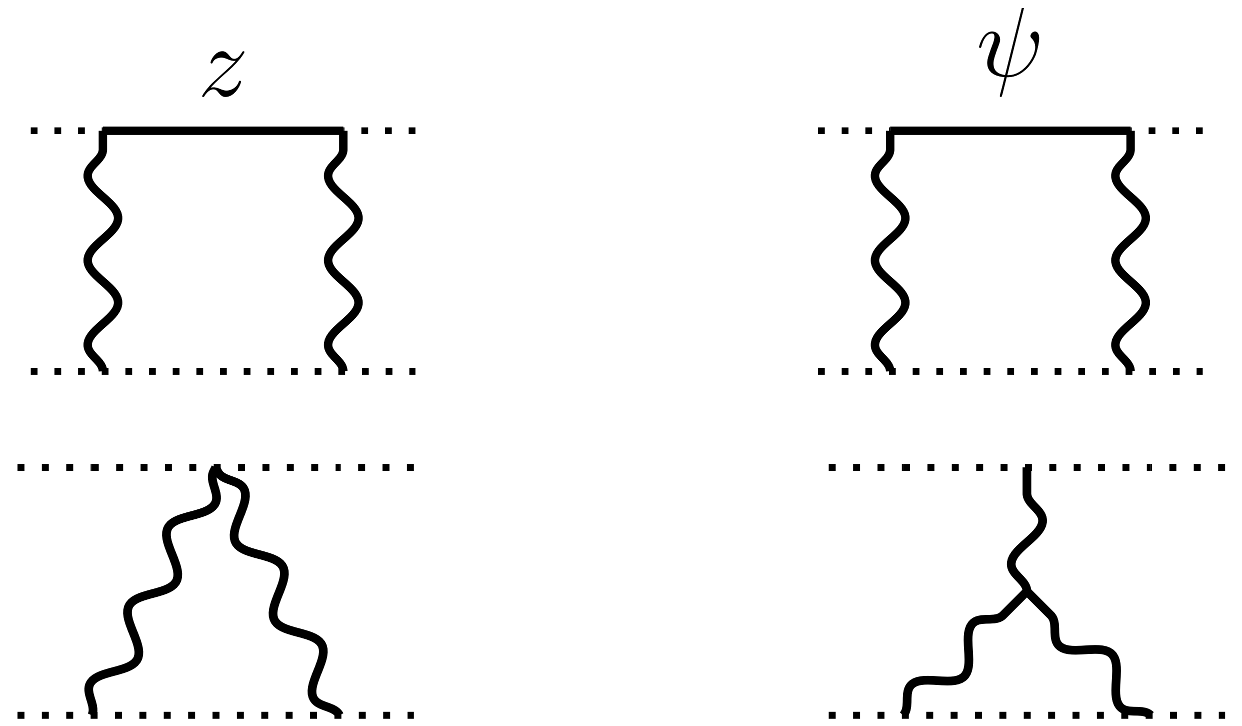
Scale q factor out, left with single parameter integral $\gamma = v_1 \cdot v_2$

- **1PM: Trivial - pure Fourier transform**



$\sim G m_1 m_2$

- **2PM: 1-loop**



$\sim G^2 m_1 m_2^2$

$$D_1 = l\cdot v_1 \pm i\epsilon\ ,$$
$$D_2 = l^2\ ,$$
$$D_3 = (l+q)^2\ .$$

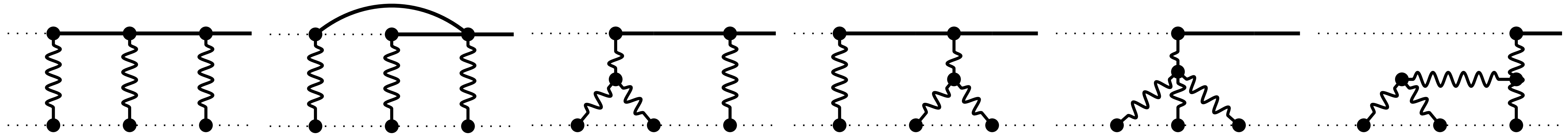
$$\int_l \frac{\mathrm{num}[l]}{D_1 D_2 D_3}\delta(l\cdot v_*)$$

Tensor reduction to scalar integrals

3PM

[Jakobsen,Mogull,JP,Sauer]

1) Test body diagrams (geodesic motion in Schwarzschild background):

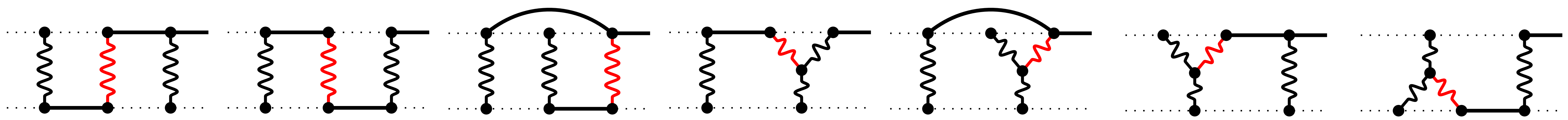


(a) (b) (c) (d) (e) (f)

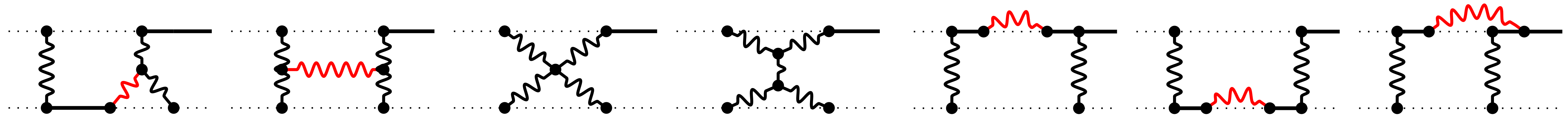
$\sim G^3 m_1 m_2^3$

A 2-loop computation!

2) Comparable mass diagrams (i0 prescription relevant for red propagators):



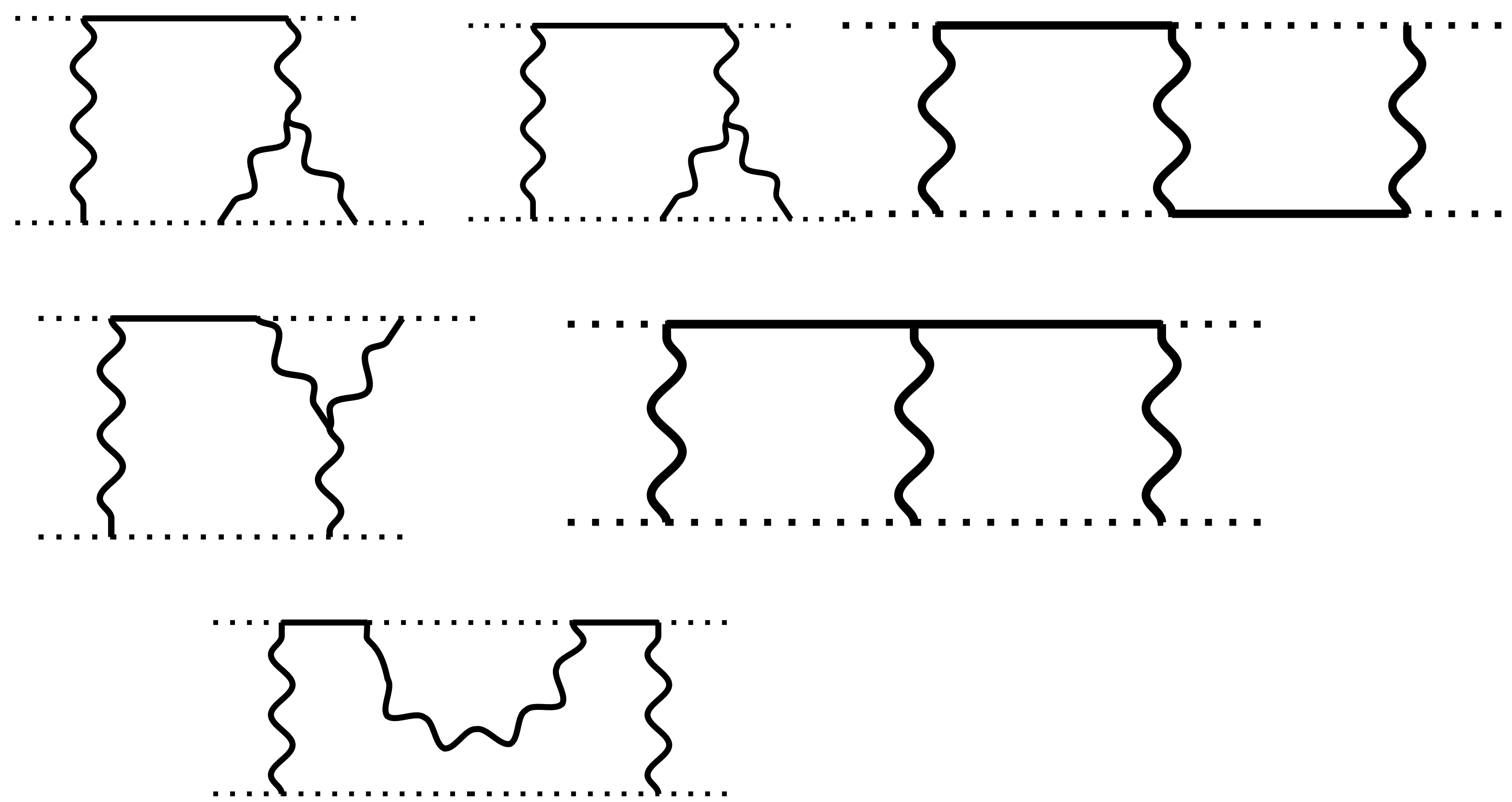
(a) (b) (c) (d) (e) (f) (g)



(h) (i) (j) (k) (l) (m) (n)

$\sim G^3 m_1^2 m_2^2$

6 top sectors integrals:



Integral family

$$\int_{l_1 l_2} \frac{\delta(l_1 \cdot v_{\bar{i}_1})\delta(l_2 \cdot v_{\bar{i}_2})}{\prod_{j=1}^{7} D_j^{n_j}}$$

$$D_1 = l_1 \cdot v_{i_1} + \sigma_1 i0 \,, \qquad D_2 = l_2 \cdot v_{i_2} + \sigma_2 i0 \,,$$

$$D_3 = (k^0 + \sigma_3 i0)^2 - \boldsymbol{k}^2$$

$$D_4 = l_1^2 \,, \qquad D_5 = l_2^2 \,,$$

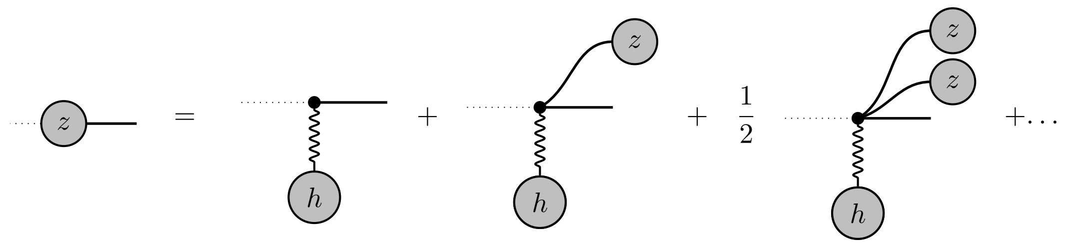
$$D_6 = (l_1 + q)^2 \,, \qquad D_7 = (l_2 + q)^2 \,.$$

WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion (tree-level!)



Insert Feynman rules

Form script (4PM: 5g vertex)

Tensor reduction to scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i0$

IBPs: Basis of Master Integral

Fire/Litered/Kira

Reduced symmetries due to retarded prop.

$l \to -l \qquad i0 \to -i0$

Solve masters by Diff. Eq. technique

Single scale integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

$i0$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

RESULT IMPULSE @ 3PM PRECISION:

[Jakobsen,Mogull,JP,Sauer]

$$\Delta p_1^\mu = p_\infty \sin\theta \frac{b^\mu}{|b|} + (\cos\theta - 1)\frac{m_1 m_2}{E^2}\left[(\gamma m_1 + m_2)v_1^\mu - (\gamma m_2 + m_1)v_2^\mu\right] - v_2 \cdot P_{\rm rad}\, w_2^\mu$$

$$w_1^\mu = \frac{\gamma v_2^\mu - v_1^\mu}{\gamma^2 - 1}$$

$$\gamma = v_1 \cdot v_2$$

- Scattering angle:

$$\frac{\theta}{\Gamma} = \frac{GM}{|b|}\frac{2(2\gamma^2-1)}{\gamma^2-1} + \left(\frac{GM}{|b|}\right)^2 \frac{3\pi(5\gamma^2-1)}{4(\gamma^2-1)} + \left(\frac{GM}{|b|}\right)^3 \left(2\frac{64\gamma^6 - 120\gamma^4 + 60\gamma^2 - 5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{(4\gamma^4 - 12\gamma^2 - 3)}{(\gamma^2-1)}\frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right)$$

1PM

2PM

3PM conservative

$$+ \left(\frac{GM}{|b|}\right)^3 \frac{4\nu(2\gamma^2-1)^2}{(\gamma^2-1)^{3/2}}\left(-\frac{8}{3} + \frac{1}{v^2} + \frac{(3v^2-1)}{v^3}\text{arccosh}\gamma\right)$$

3PM radiation-reaction

$$\Gamma = E/M = \sqrt{1 + 2\nu(\gamma - 1)}$$

$$\nu = \frac{m_1 m_2}{M^2}$$

- Radiated 4-momentum: $P_{\rm rad}^\mu = -\Delta p_1^\mu - \Delta p_2^\mu$

$$P_{\rm rad}^\mu = \frac{G^3 m_1^2 m_2^2 \pi}{|b|^3}\frac{v_1^\mu + v_2^\mu}{\gamma + 1}\left[e_1 + e_2 \log\left(\frac{\gamma+1}{2}\right) + e_3 \frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right]$$

$$e_1 = \frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{48(\gamma^2-1)^{3/2}}$$

$$e_2 = -\frac{35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5}{8\sqrt{\gamma^2-1}},$$

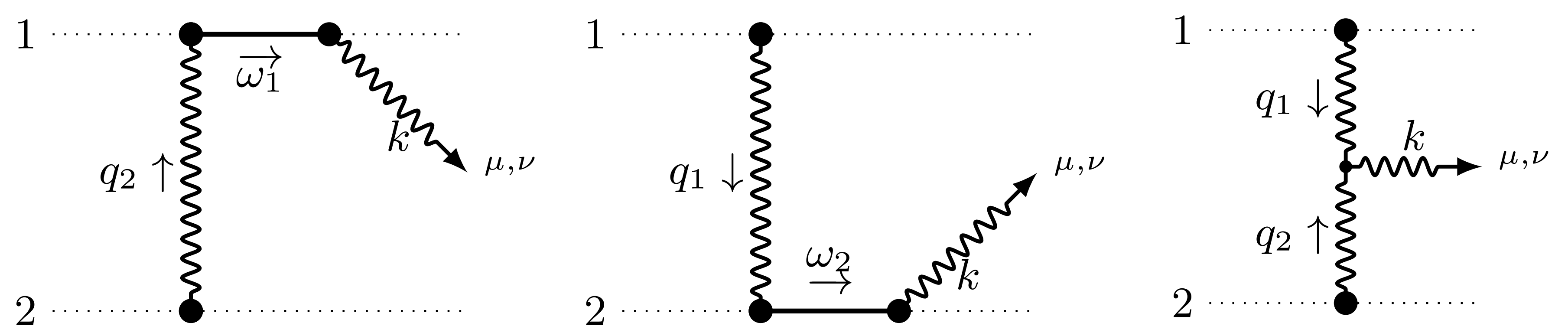
$$e_3 = \frac{\gamma(2\gamma^2-3)(35\gamma^4 - 30\gamma^2 + 11)}{16(\gamma^2-1)^{3/2}}.$$

FAR FIELD WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

Sum of diagrams with outgoing graviton:

$$\langle h_{\mu\nu}(k)\rangle =$$



For time-domain waveform needs to integrate over outgoing energy : Ω

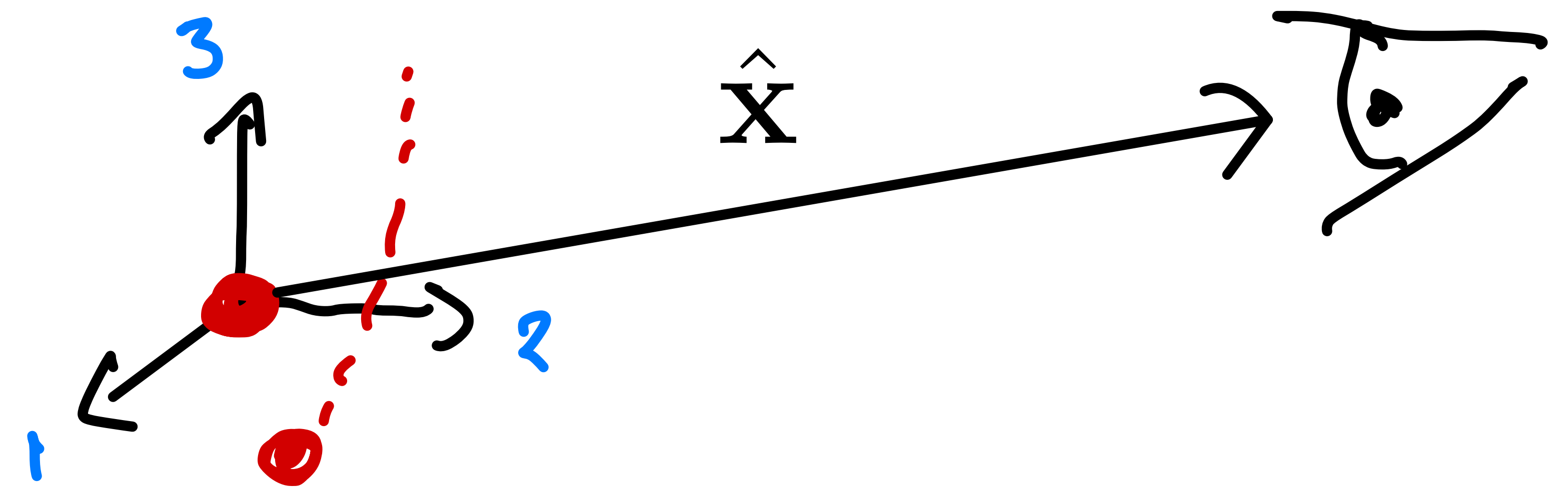
$$\frac{f_{+,\times}(t-r,\hat{\mathbf{x}})}{r} = \frac{4G}{r}\int d\Omega\, e^{-i\Omega(t-r)}\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega(1,\hat{\mathbf{x}}))\rangle$$

where unit vector $\hat{\mathbf{x}}$ points towards the observer

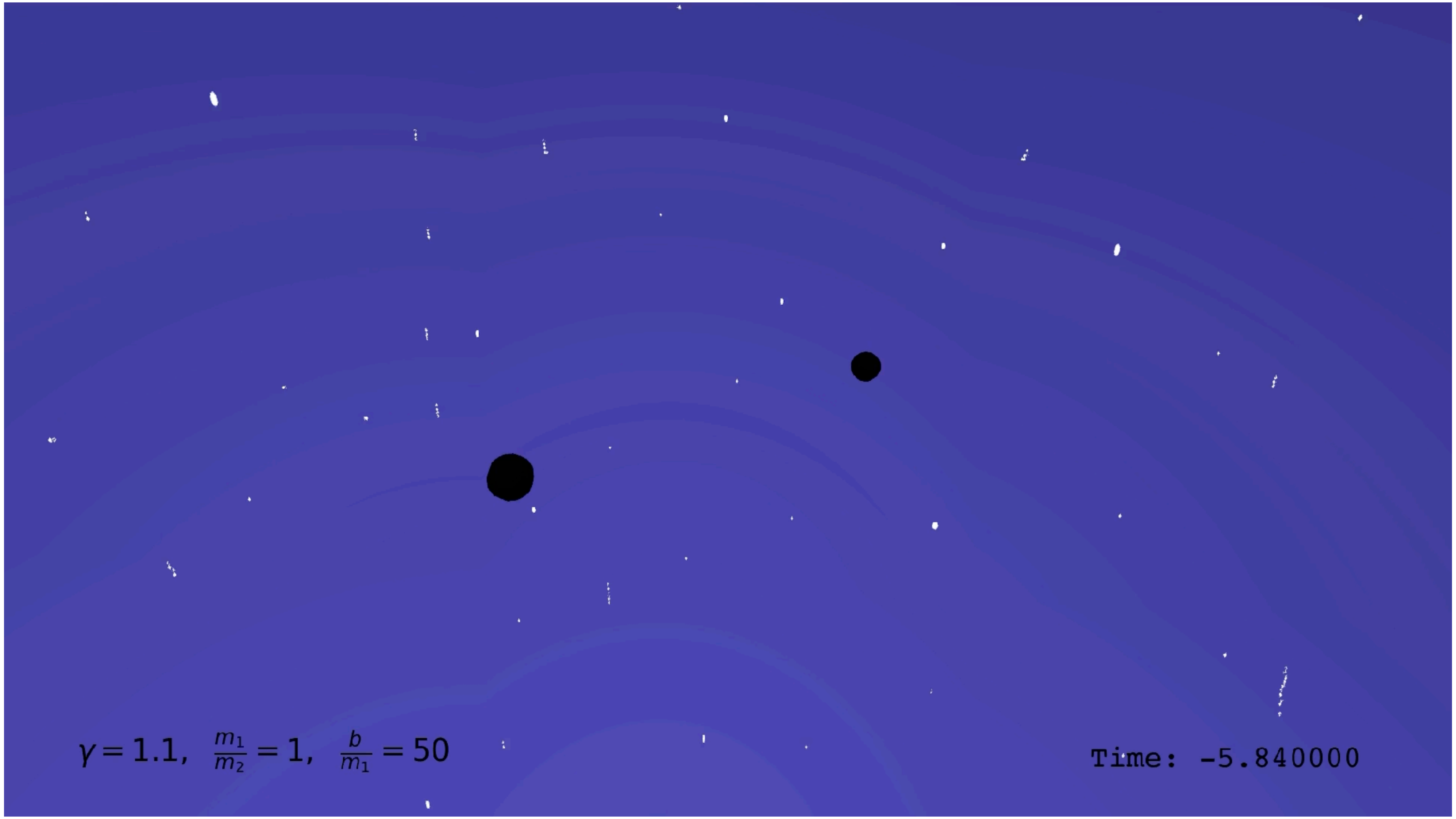
The waveform has two polarizations

$$f_{+,\times}(\underbrace{t-r}_{u},\underbrace{\theta,\phi}_{\hat{\mathbf{x}}};v,|b|,m_1,m_2)$$

retarded time



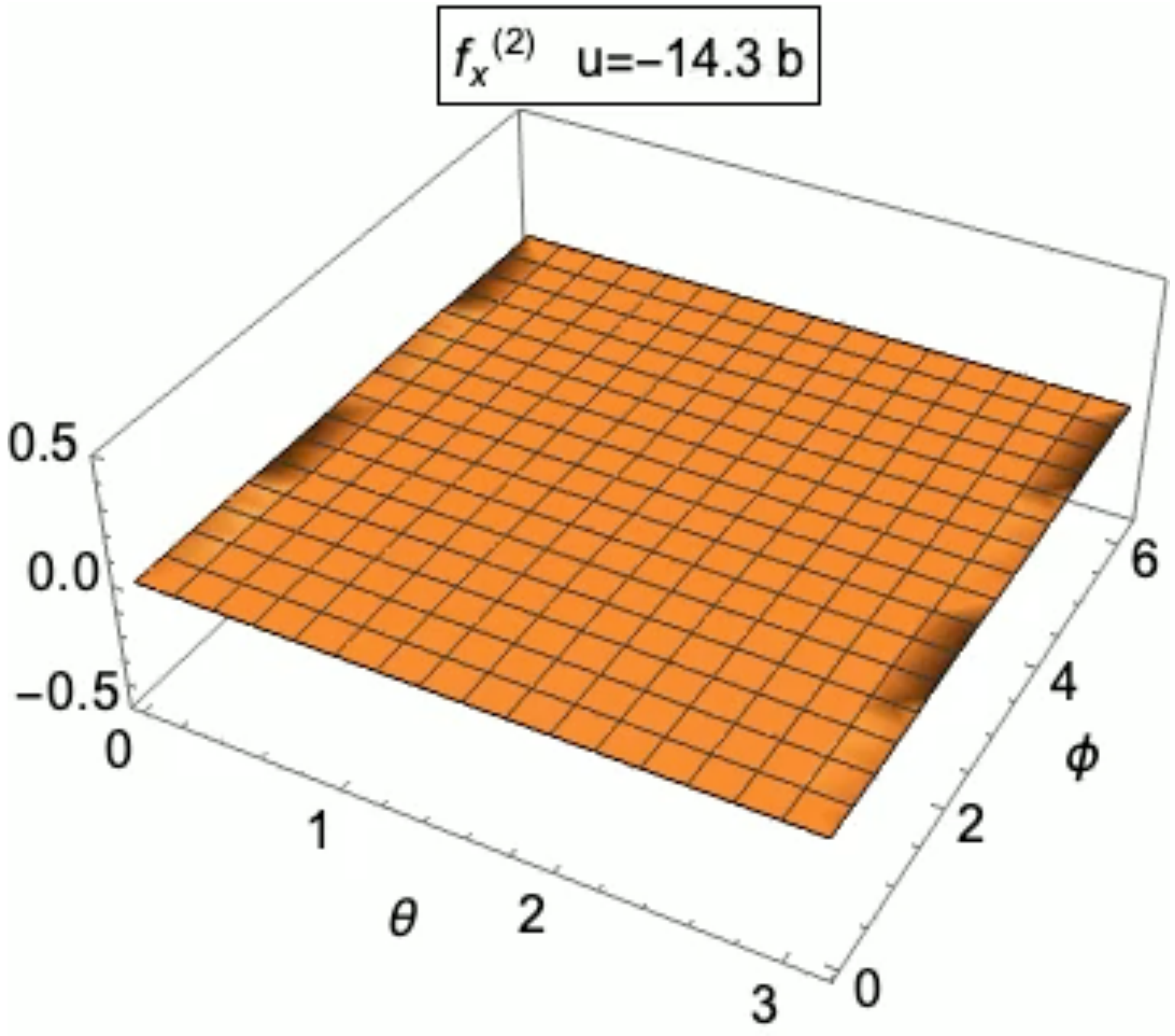
Visualization: Plus-Polarization $f_+^{(2)}$

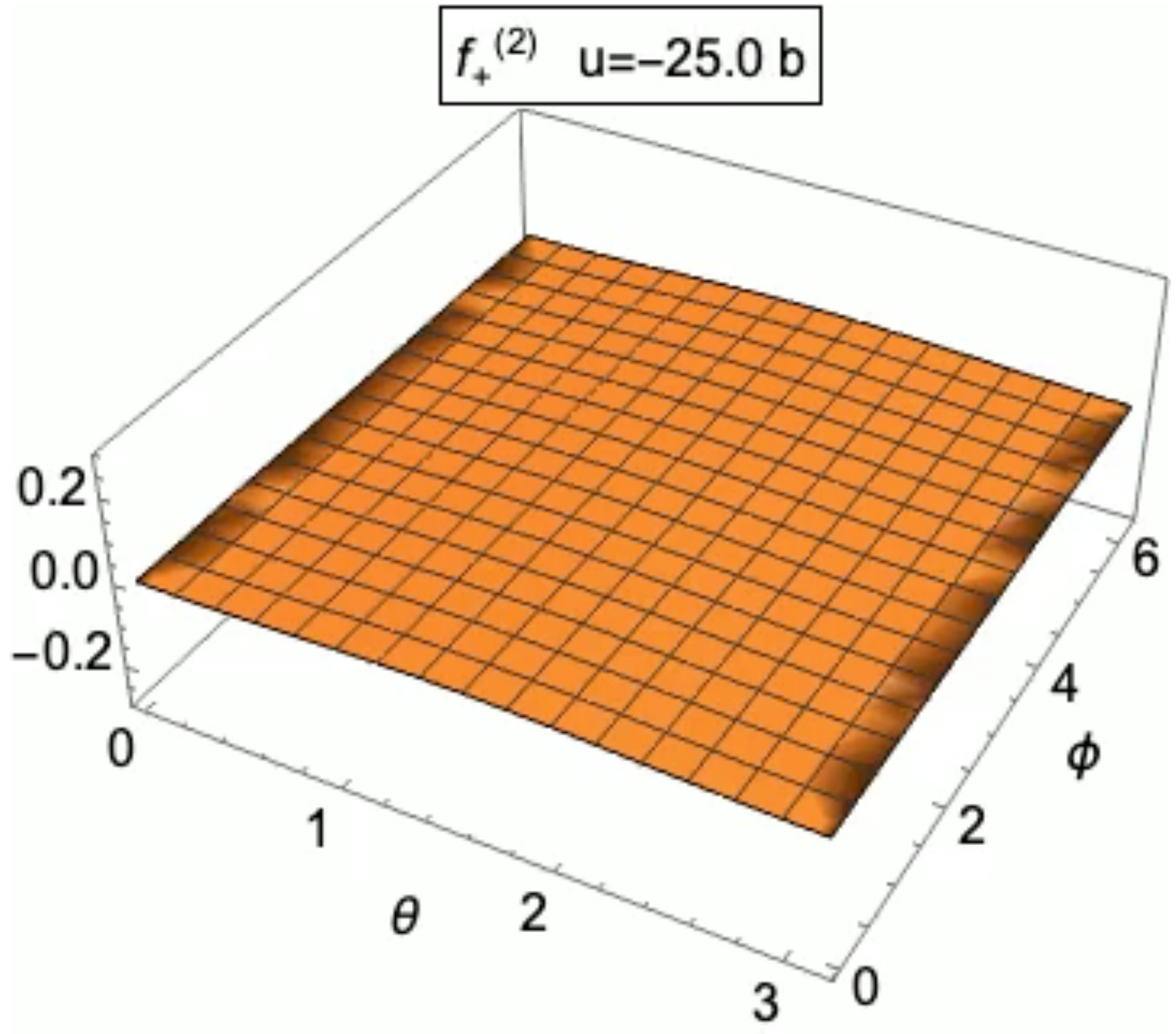


$v = 0.4$

[HU/AEI BSc thesis Babayemi]

Memory effect





$$v = 0.2$$

EXTRACT GRAVITATIONAL POTENTIAL IN PM EXPANSION

[Jakobsen,Mogull]

- Make ansatz for 2-body Hamiltonian

$$H = \sqrt{\vec{p}^2 + m_1^2} + \sqrt{\vec{p}^2 + m_2^2} + V(\vec{r}, \vec{p}, \vec{S}_i) \qquad \vec{L} = \vec{r} \times \vec{p}$$

$$V = \sum_A \mathcal{O}^A V^A \qquad \mathcal{O}^A = \left\{ 1, \frac{\vec{L}\cdot\vec{S}_i}{r^2}, \frac{\vec{r}\cdot\vec{S}_i\,\vec{r}\cdot\vec{S}_j}{r^4}, \frac{\vec{S}_i\cdot\vec{S}_j}{r^2}, \frac{\vec{p}\cdot\vec{S}_i\,\vec{p}\cdot\vec{S}_j}{r^2} \right\}$$

$$V^A = \sum_n \left(\frac{G}{r}\right)^n c_n^A(|\vec{p}|)$$

Fix free coefficients c_n^A by matching to scattering data Δp_1^μ and $\Delta s_1^{\mu\nu}$ by solving Hamilton's eqs.

$$\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}}, \quad \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}}, \quad \dot{\vec{S}}_i = -\vec{S}_i \times \frac{\partial H}{\partial \vec{S}_i}$$

Can make contact to bound problem

BINDING ENERGY: COMPARISON TO NUMERICAL RELATIVITY

Use 2-body Hamiltonian to compute binding energy

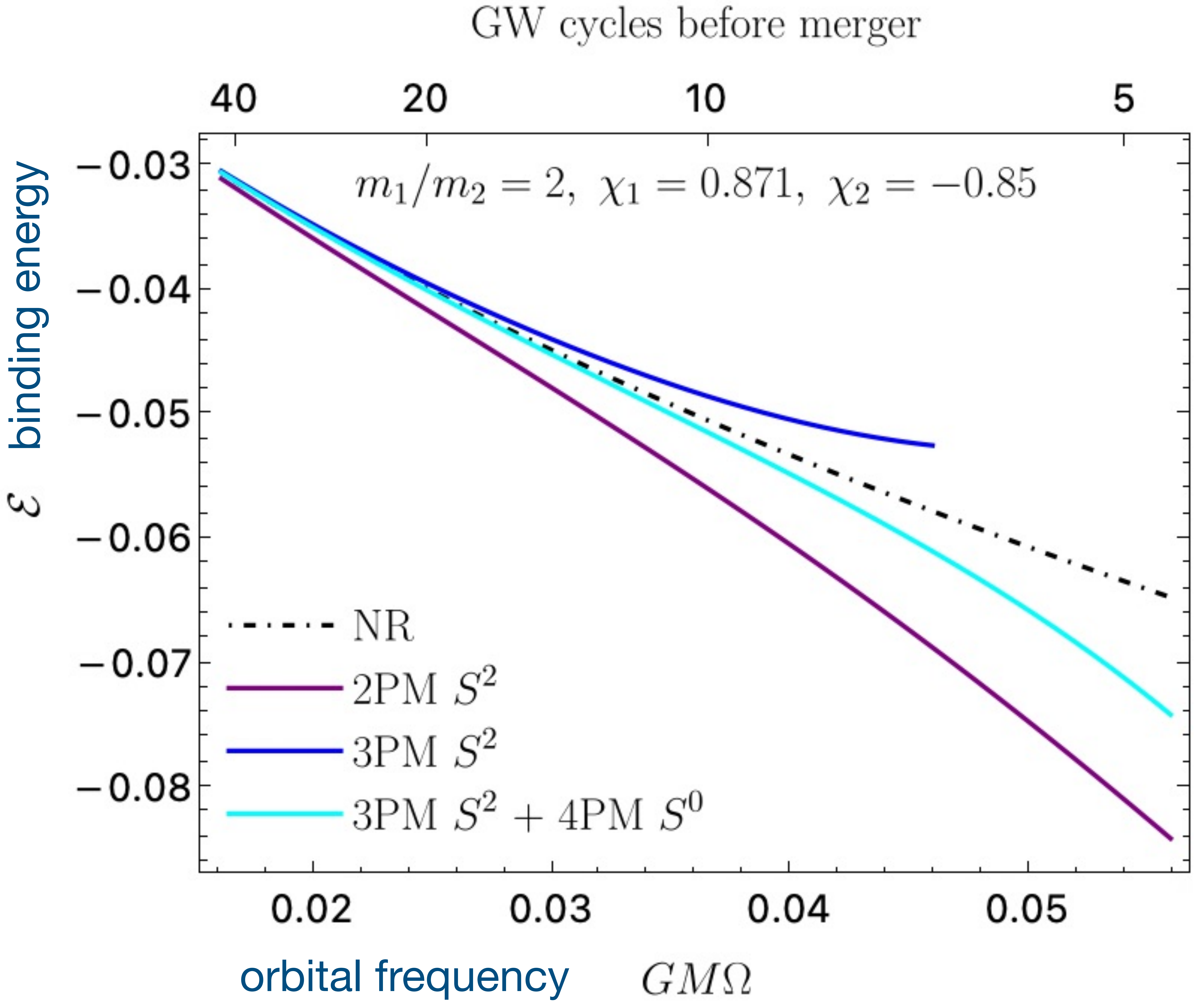
$\mathcal{E} = \dfrac{E - M}{\mu}$ for circular orbits

and and aligned spins to compare to numerical relativity

$$E = H(p_r, r, J; \chi_1, \chi_2)$$

$$\chi_1 = \frac{a_1}{m_1}, \quad \chi_2 = \frac{a_2}{m_2}$$

$$\Omega(J, r) = \partial H / \partial J$$



PM SCATTERING STATE-OF-THE-ART

WQFT [us]

WEFT **Worldline effective theory** [Källin,Porto,Dlapa,Cho,Liu,..] [Riva,Vernizzi,Mougiakakos..]

HEFT **Heavy BH effective theory** [Aoude,Haddad,Helset,Damgaard] [Brandhuber,Travaglini,Chen]

Amps **Scattering amplitudes** [Bern,Roiban,Shen,Parra-Martinez,Ruf,..] [Bjerrum-Bohr,Damgaard,Vanhove,..] [Di Vecchia,Veneziano,Heissenberg,Russo] [Solon,Cheung,..] [Huang,..] [Guevera,Ochirov,Vines,…] [Johansson,Pichini [Kosower,O'Connell,Maybee,Cristofoli, Gonzo…]

	deflection & spin kick				waveform			
	plain	spin²	spin>2	tidal	plain	spin²	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps	WQFT WEFT Amps	WQFT WEFT (Amps)	WQFT WEFT	WQFT WEFT	~ 1-loop
3PM w/o r-r	WQFT WEFT Amps HEFT	WQFT (Amps)		WQFT WEFT	HEFT Amps			~ 2-loop
3PM r-r	WQFT WEFT Amps HEFT	WQFT		WQFT WEFT				
4PM w/o r-r	WEFT Amps							~ 3-loop
4PM w r-r	WEFT							

r-r: Radiation-reaction (…) : partial results

SUMMARY

WQFT: Highly efficient technology for classical scattering in GR

- „Quantize“ world-line degrees of freedom & focus on observables (=one-point functions)
- Only compute tree-level diagrams (=classical theory). No „super-classical“ contributions
- IN-IN Formalism: Take all propagators retarded.
- Include spin degrees of freedom through Graßmann odd vectors on the world-line (spinning particle)
- Hidden Supersymmetry = Spin Supplementary Condition

OUTLOOK

WQFT still needs to be extended:

- Higher precision (4PM)
- Higher spin (beyond Spin squared)
- Bound orbits? Relation to EOB
- Relation to self force expansion
-

WE ARE HIRING!

- Fall 23: Long Term Postdoc (5y), 1 PhD
- Fall 24: Postdoc (4y), 1 PhD

erc
European Research Council
Established by the European Commission

Thank you for your attention!