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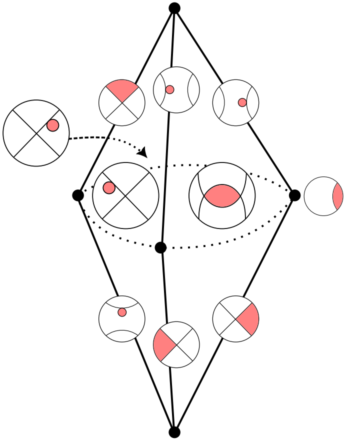

The Loop Momentum Amplituhedron for ABJM

Work in progress with T. Łukowski

Jonah Stalknecht

May 16, 2023

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Loop Momentum Amplituhedron for $\mathcal{N} = 4$ SYM

- The Momentum Amplituhedron (see Tomek's talk)
  [Damgaard, Ferro, Parisi, Łukowski], [Ferro, Łukowski]
  - Geometry in $\underbrace{n\text{-particle spinor helicity space}}_{\text{tree level}} + \underbrace{L\ 2 \times 2 \text{ matrices}}_{L \text{ loops}}$.
  - Describes L-loop integrands for scattering amplitudes in planar $\mathcal{N} = 4$ SYM.
- Canonical differential form $\leftrightarrow$ scattering amplitude/integrand.
- Boundaries of the momentum amplituhedron $\leftrightarrow$ singularities of the amplitude.

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ABJM and 3D Spinor Helicity

ABJM: 3D matter Chern-Simons theory with $\mathcal{N} = \not{6}$ 4 SUSY.

- Only nontrivial for $n = 2k$ particle amplitudes.

3D spinor helicity variables:

- Momentum $\to$ symmetric 2×2 matrix $p^{\alpha\beta} = \begin{pmatrix} -p^0 + p^3 & p^1 \\ p^1 & -p^0 - p^3 \end{pmatrix}$.
- $\det p = m^2, \quad m^2 = 0 \implies p^{\alpha\beta} = \lambda^\alpha \lambda^\beta$.
- $s_{ij} = \langle ij \rangle^2, \quad \langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta$.
- Momentum conservation: $p_1 - p_2 + p_3 - \ldots - p_{2k} = 0$.

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Loop level: work in progress.

Tree level: [Huang, Kojima, Wen, Zhang], [He, Kuo, Zhang]

- $\langle a\, a+1 \rangle > 0$,
- $\{\langle 12 \rangle, \langle 13 \rangle, \ldots, \langle 1n \rangle\}$ has $k = n/2$ sign-flips,
- $\sum_{j=1}^{n} (-1)^j \langle ja \rangle \langle jb \rangle = 0 \quad a, b = 1, \ldots, n.$

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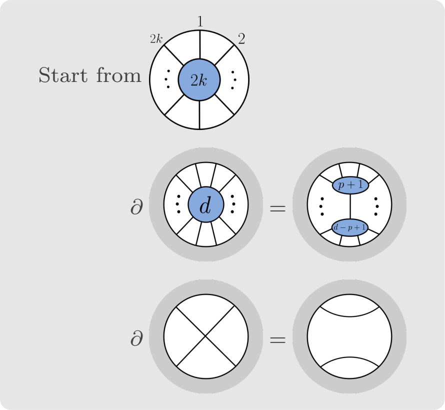
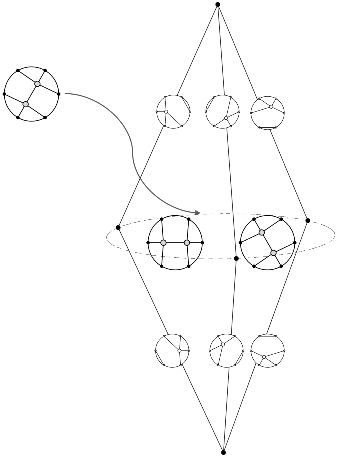
Tree ABJM Momentum Amplituhedron

Full classification of boundaries:





Euler characteristic $\chi = 1$ for all $n = 2k$ [Moerman, Łukowski, JS, 2112.03294]

Loop Level $\mathcal{N}=4$ SYM $(n=4)$

Recall: for $\mathcal{N}=4$ SYM:

- Supplement tree-level with L 2×2 matrices.
- $D_i \in G_+(2,4)$, $i=1,\dots,L$

$$\ell_i = \frac{\lambda\cdot F\cdot\tilde{\lambda}^T}{\det(D_i\cdot\lambda^T)},\quad F=\begin{pmatrix}(12)_i\langle 12\rangle+(23)_i\langle 23\rangle & 0 & 0 & (13)_i\langle 34\rangle\\ 0 & (23)_i\langle 23\rangle & 0 & 0\\ 0 & 0 & 0 & 0\\ -(24)_i\langle 12\rangle & 0 & 0 & -(34)_i\langle 34\rangle\end{pmatrix},$$

$$\text{where } \det\begin{pmatrix}D_i\\ \hdashline D_j\end{pmatrix}>0,\quad i,j=1,\dots,L.$$

Loop Level ABJM ($n = 4$)

For ABJM:

- Supplement tree-level with L symmetric 2×2 matrices.
- Reduce to 3D: $\tilde{\lambda}_a \to (-1)^a \lambda_a$, $\langle 12 \rangle = \langle 34 \rangle$

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- ℓ_i symmetric for $(13)_i = (24)_i$.

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$n = 4$, $L = 1$ ABJM Momentum Amplituhedron

For one loop, $n = 4$:

- Canonical differential form:

$$\Omega_4^{(1)} = \frac{1}{2}\,\mathrm{d}\log\frac{\langle 12\rangle}{\langle 23\rangle} \wedge \underbrace{\mathrm{d}\log\frac{\ell^2}{(\ell+p_1)^2} \wedge \mathrm{d}\log\frac{\ell^2}{(\ell+p_1+p_2)^2} \wedge \mathrm{d}\log\frac{\ell^2}{(\ell-p_4)^2}}_{\mathrm{d}^3\ell\,\frac{\ell^2\epsilon_{\mu\nu\rho}p_1^\mu p_2^\nu p_4^\rho + \langle 12\rangle^2\epsilon_{\mu\nu\rho}\ell^\mu p_1^\nu p_4^\rho}{\ell^2(\ell+p_1)^2(\ell+p_1+p_2)^2(\ell-p_4)^2}}$$

- Agrees with know result for the 4-particle 1-loop integrand [Chen, Huang].

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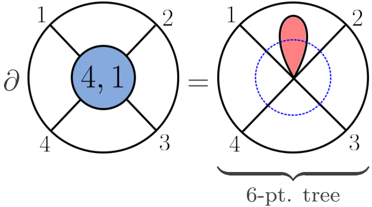
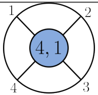
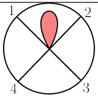
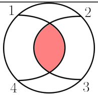
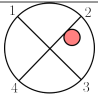
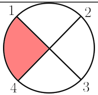
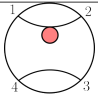
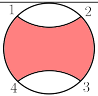
$n = 4,\ L = 1$ ABJM Momentum Amplituhedron

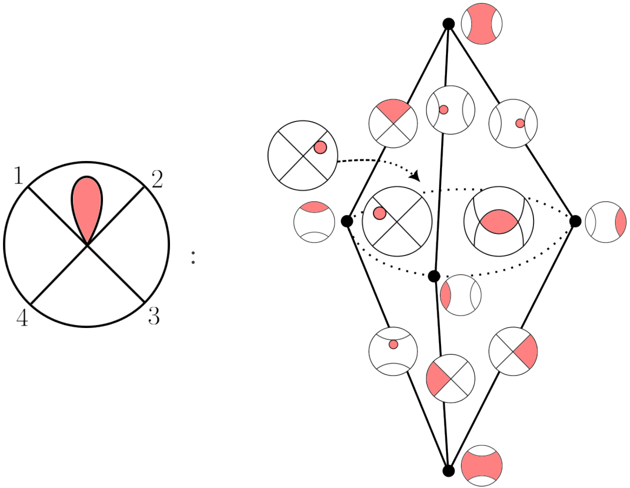
Boundaries captured by a very similar diagrammatics as the ones used for tree-level.



$$\chi = 6 - 8 + 6 - 4 + 1 = 1$$

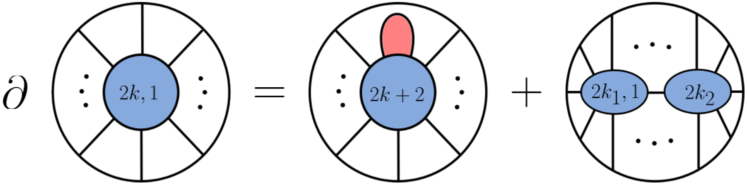
$d = 4$	 $\times 1$
$d = 3$	 $\times 4$
$d = 2$	 $\times 2$  $\times 4$
$d = 1$	 $\times 4$  $\times 4$
$d = 0$	 $\times 6$

$n = 4,\ L = 1$ ABJM Momentum Amplituhedron



$n > 4,\ L = 1$

- For $n \geq 6$, there is no definition yet (work in progress).
  - There is a natural candidate for the geometry: reduce the $\mathcal{N} = 4$ SYM loop momentum amplituhedron to 3D by dimensional reduction.
  - More tests are needed.
- But, we can define the combinatorial problem:



- We checked up to $n = 14$ that $\chi = 1$.

$n = 4,\ L = 2$

For two loops, $n = 4$:

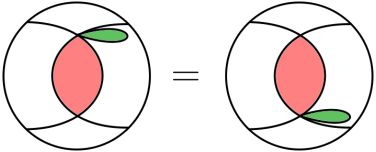
- Canonical form:

$$\Omega_4^{(2)} = \left(\mathrm{d}\log \frac{\langle 12 \rangle}{\langle 23 \rangle} \wedge \frac{2\langle 12\rangle^2 \langle 23 \rangle^2 \mathrm{d}^3\ell_1 \wedge \mathrm{d}^3\ell_2}{\ell_1^2 (\ell_1 + p_1 + p_2)^2 (\ell_1 - \ell_2)^2 (\ell_2 + p_1)^2 (\ell_2 - p_4)^2} + (\ell_1 \leftrightarrow \ell_2) \right)$$
$$- \mathrm{d}\log \frac{\langle 12 \rangle}{\langle 23 \rangle} \wedge \left(\frac{1}{2}\, \mathrm{d}\log \frac{\ell^2}{(\ell + p_1)^2} \wedge \mathrm{d}\log \frac{\ell^2}{(\ell + p_1 + p_2)^2} \wedge \mathrm{d}\log \frac{\ell^2}{(\ell - p_4)^2} \right)^2$$

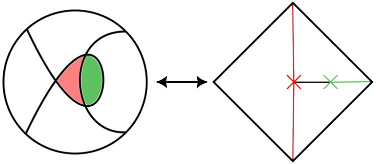
- This agrees with [Chen, Huang].

$n = 4,\ L = 2$

- Diagrammatics similar to tree and one-loop diagrams.
- Some new rules:



- Dual picture seems more natural:



- $f = (1, 9, 36, 72, 82, 58, 40, 18)$, $\chi = -2$.

Conclusion

- ABJM loop momentum amplituhedron: reduce $\mathcal{N}=4$ loop mom. amp. by
  - $\tilde{\lambda}_a = (-1)^a \lambda_a$,
  - $\ell_i = \ell_i^T$.
- Canonical form agrees with the known 4-point integrands.
- Boundary structure captured by natural diagrams
  - For $L = 0, 1$: $\chi = 1$ for all n.
  - For $L \geq 2$: $\chi \neq 1$.
  - Combinatorics also seems to work for $n > 4$.

Q&A

Thank you for listening!