CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Maximal loop-loop cuts from the Amplituhedron

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AMPLITUHEDRON

Based on
G. D., P. Heslop, A. Stewart 2207.12464

Planar amplitudes integrands are top rational differential forms in the loop variables.

Multiresidues of amplitude integrands

- are at the core of generalized unitarity techniques [Bern, Dixon, Kossover],
- can be used as building block for recursion relations as BCFW and more recently [Bourjaily,Caron-Huot]

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The amplituhedron

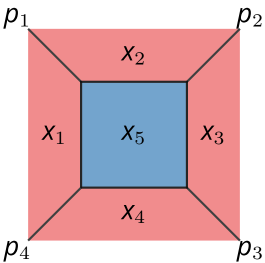
Super amplitudes and their residues can be computed as the canonical form of respectively the Amplituhedron and its boundaries. [Nima, Trnka]

Main points

- **Cut's Geometry:** Derive the geometry of the boundaries of the Amplituhedron corresponding to some loop-loop cuts.
- **Computaing Cuts:** Show a strategy to compute the cuts from the geometry.
- **Universality of Maximal Cuts:** Show how many loop-loop residues we can take and describe their universal geometry.

Region variables

Planar integrands can be written using region variables



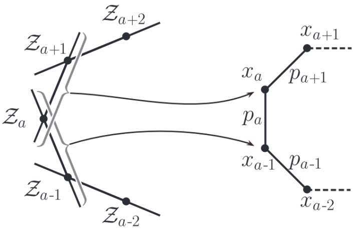
The momenta flowing in the edges are equal to distances between adjacent regions. Example:

$$p_1^2 = (x_2 - x_1)^2$$

Points and Lines

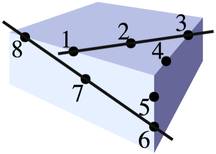
To every region variable, we can associate a line in twistor space that is a line in $\mathbb{P}^3$, aka $Gr(2,4)$.

Null-separated regions correspond to intersecting lines



The internal regions correspond to loop momenta and are usually indicated with a pair of points $A_i B_i$

Tree-level amplitudes poles are related to the boundaries of the configurations space of points.



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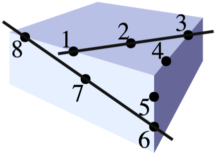

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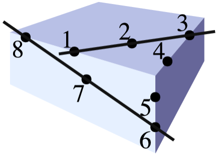

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The Amplituhedron

The 4-point MHV loop amplitude is a function of 4 points in $\mathbb{P}^3$ $Z_1, \ldots, Z_4$ and L lines represented by pairs of points $A_1B_1, \cdots, A_LB_L$. Angle-bracket notation:

$$\langle A_iB_ijk\rangle := \det(A_iB_iZ_jZ_k), \qquad \langle A_iB_iA_jB_j\rangle := \det(A_iB_iA_jB_j)$$

The MHV loop amplituhedron $\mathcal{A}_{4,0,L}$ [Arkani-Hamed, Thomas, Trnka] is the space of oriented lines A_iB_i such that

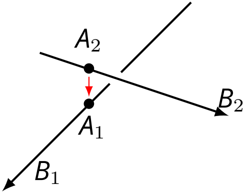
$$\begin{aligned}
&\langle 1234\rangle > 0,\\
&\langle A_iB_i12\rangle > 0, \qquad \langle A_iB_i23\rangle > 0, \qquad \langle A_iB_i34\rangle > 0, \qquad \langle A_iB_i14\rangle > 0,\\
&\langle A_iB_i13\rangle < 0,\\
&\langle A_iB_iA_jB_j\rangle > 0.
\end{aligned}$$

Useful notation for $\mathcal{A}_{4,0,L}$

$$A_iB_i \in \mathcal{A}_{4,0,1} \wedge \langle A_iB_iA_jB_j\rangle > 0$$

2-loop cut

$$\langle A_1 B_1 A_2 B_2 \rangle \to 0^+$$



Natural change of variable $A := A_1 A_2 \cap A_2 B_2$ For L loops we define

$$\mathcal{A}_{\mathrm{dc}}^{(L)} := AB_i \in \mathcal{A}_{4,0,1} \qquad \forall\, i \leq L$$

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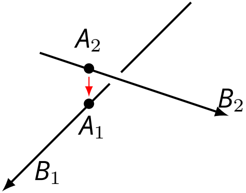

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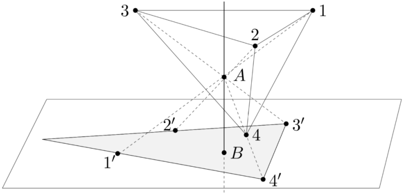
This residue is equal to a sum of factorized of 1-loop terms [Arkani-Hamed, Langer, Yelleshpur Srikant, Trnka]

Geometric factorization

The canonical form of the Cartesian product of geometries is equal to the product of the canonical forms

$$\Omega(\mathcal{X}_1 \otimes \mathcal{X}_1) = \Omega(\mathcal{X}_1) \wedge \Omega(\mathcal{X}_2)$$



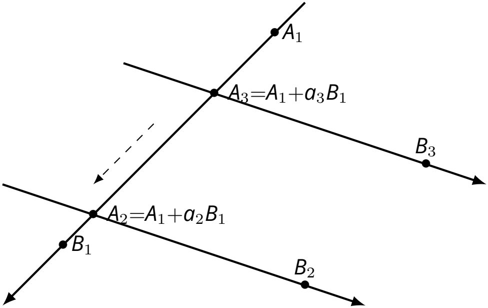
$$\mathcal{A}_{\mathrm{dc}}^{(L)} = \bigcup_i \omega_i(A) \bigotimes_j \lambda_i(B_j)$$

where $\omega_i(A)$ is a tetrahedron and $\lambda_i(B_j)$ is a triangle.

The form will read

$$\mathcal{A}_{\mathrm{dc}}^{(L)} = \sum_i \Omega(\omega_i)(A) \prod_j \Omega(\lambda_i)(B_j)$$

New tree loops feature



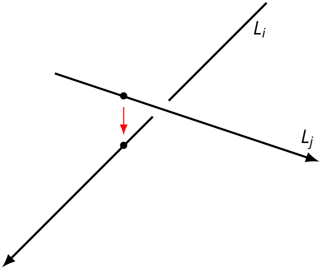
$$\langle A_2B_2A_3B_3\rangle = (a_3 - a_2)\langle A_1B_1B_2B_3\rangle > 0$$

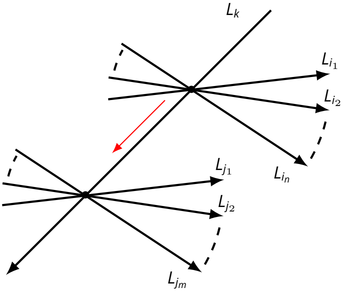
That corresponds to

$$((a_3 - a_2) > 0 \wedge \langle A_1B_1B_2B_3\rangle > 0) \vee ((a_3 - a_2) < \wedge \langle A_1B_1B_2B_3\rangle < 0)$$

Intersecting and sliding

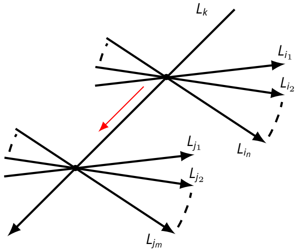
We consider all loop-loop cuts that can eventually land on the configuration where all loops intersect in one point.
Rule: No coplanar lines, that is no $\langle ABBB \rangle = 0$





Rules for determining the geometry

When we intersect two lines A_iB_i, A_jB_j we just remove $\langle A_iB_iA_jB_j\rangle > 0$, and introduce an intersection point $A_iB_i \cap A_jB_j$.



Sliding produces two regions with opposite orientations:

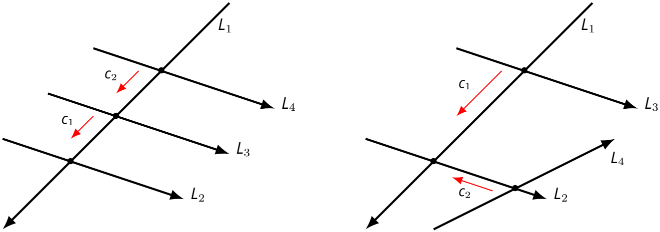
$$\langle A_iB_iA_jB_j\rangle > 0 \ \rightarrow \ \langle AB_iB_kB_j\rangle > 0$$

and

$$\langle A_iB_iA_jB_j\rangle > 0 \ \rightarrow \ \langle AB_iB_kB_j\rangle < 0$$

A family of residues

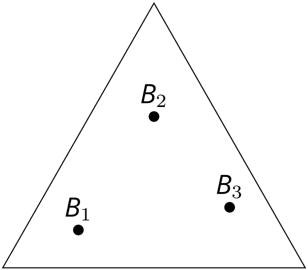
Sliding order matters



Intersections instead commute.

A triangle in a triangle

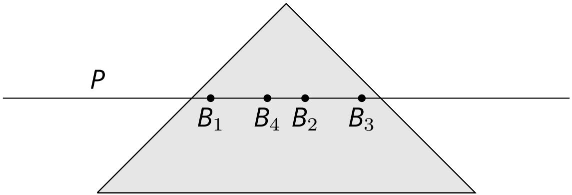
For the MHV amplitude, after we project through A, the B_i geometry corresponds to points in a triangle.



with $\langle B_1 B_2 B_3 \rangle > 0$ or $\langle B_1 B_2 B_3 \rangle < 0$

All-in-on-point-and-plane cuts

Let's study the boundaries $\langle AB_iB_jB_k\rangle = 0$, that is the 3 lines i, j, k lie on the same plane.



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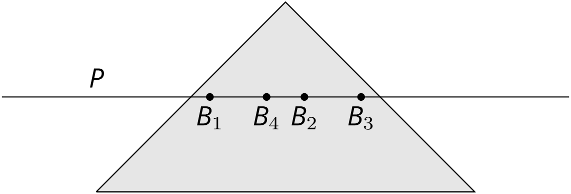

For more than 3 points the $\langle AB_iB_jB_k\rangle > 0$ factorize and gives a partial ordering on the line

$$\langle AP^{\perp}B_iB_j\rangle > 0$$

plus a factor of 2 on the weight for each internal boundary.

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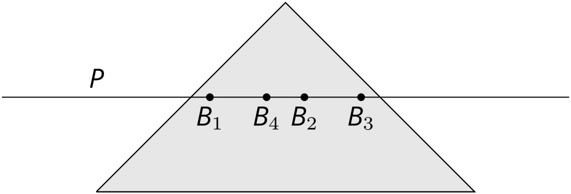

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Solvable at all loops!

Maximal loop-loop cuts

Let's set also all $\langle AP^{\perp}B_iB_j\rangle = 0 \quad \Rightarrow \quad AB_i = AB_j$.

Conjecture

If we exhaust all $\langle AP^{\perp}B_iB_j\rangle = 0$ we always get the $3 -$ *loop* all-in-one-point-all-in-and-plane cut up geometry up to the weight.

The weight is given by $2^{\text{\#internal boundaries}}$.

Conclusions

- We showed how to derive the geometry of a big class of boundaries of the amplituhedron.
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