Kerr amplitudes and higher-spin gauge symmetry

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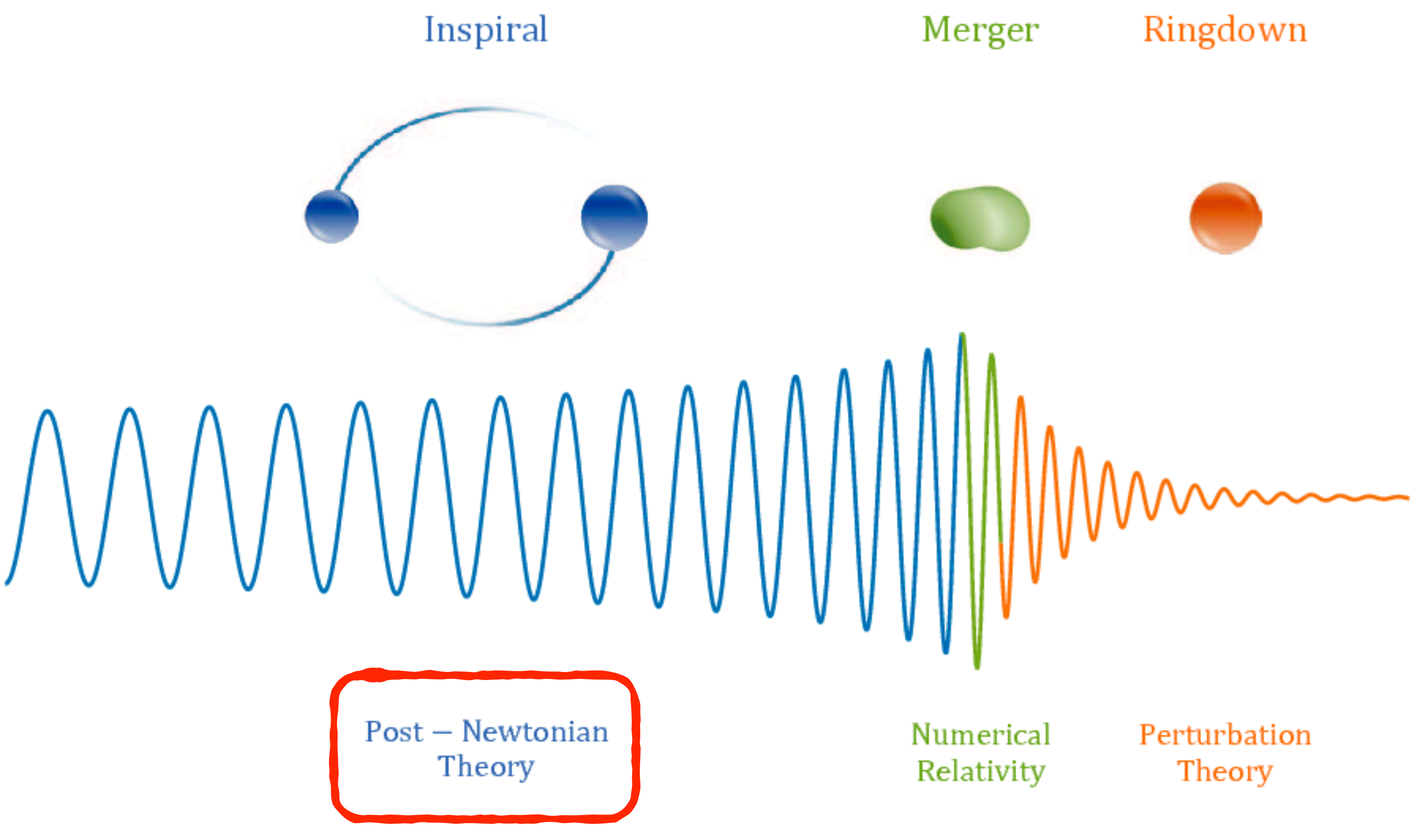
Based on refs:
Chiodaroli, HJ, Pichini [2107.14779];
Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov [2212.06120]

Motivation: gravitational waves



Inspiral phase: high experimental sentitivity ←→ need theoretical precision

→ errors accumulate

→ analytic control possible

→ important for LISA band

PN, PM and spin-multipole expansions

Post-Newtonian (PN) expansion:

Bound systems:

expand in G and v

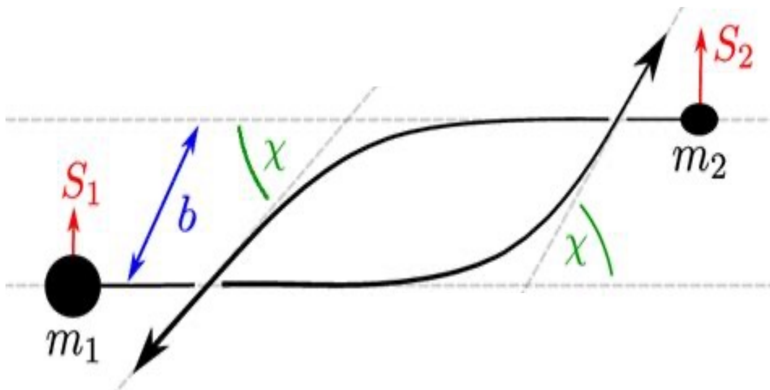
$$v^2 \sim \frac{GM}{r}$$

(virial theorem)

Post-Minkowskian (PM) expansion:

Gravitational scattering:

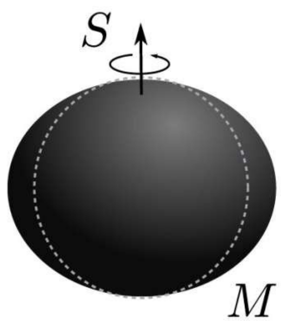
expand in G → loop expansion



Spin-multipole expansion:

Rotating black holes

expand in S_1 and S_2



Methods

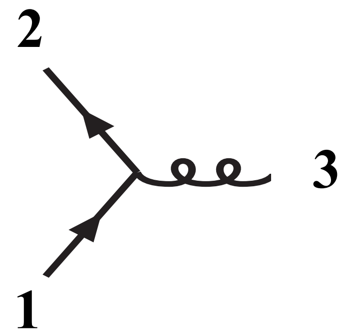
- BH perturbation theory
- Worldline EFTs
- Quantum scattering ampl's
- higher-spin QFTs

Outline

- Motivation
- The AHH higher-spin amplitudes
- The problem of Compton scattering for Kerr BHs
- EFTs describing Kerr and root-Kerr
- Higher-spin gauge symmetry and EFTs
- Conclusion

Higher-spin 3pt amplitudes & Kerr BH

Natural higher-spin gravitational 3pt amplitudes:



$$M(1\phi^s, 2\bar{\phi}^s, 3h^+) = im^2x^2\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}},$$

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-) = i\frac{m^2}{x^2}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Arkani-Hamed, Huang, Huang ('17)

Linearized energy-momentum tensor for Kerr source

Vines ('17)

$$T^{\mu\nu}(-k) = 2\pi\,\delta(p\cdot k)\,p^{(\mu}\exp(m^{-1}S * ik)^{\nu)}{}_{\rho}\,p^{\rho}$$

Non-minimal worldline action for Kerr:

Levi, Steinhoff ('15)

$$L_{\rm SI} = \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}\frac{C_{ES^{2n}}}{m^{2n-1}}D_{\mu_{2n}}\cdots D_{\mu_3}\frac{E_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}$$
$$+\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{C_{BS^{2n+1}}}{m^{2n}}D_{\mu_{2n+1}}\cdots D_{\mu_3}\frac{B_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}S^{\mu_{2n+1}}$$

(spin-multipole expansion)

root-Kerr gauge theory

Classical double copy → Kerr-Schild form

Monteiro, O'Connell ('14)

metric: $$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$$ (Kerr, double copy)

gauge field: $$A_\mu = \phi k_\mu$$ (root-Kerr, single copy)

$$k^\mu k_\mu = 0 \qquad \phi(r) = \frac{2MGr^3}{r^4 + a^2 z^2} \qquad \frac{x^2+y^2}{r^2+a^2} + \frac{z^2}{r^2} = 1$$

Newman-Janis shift:

$$\Psi^{\text{Kerr}}(x) = \Psi^{\text{Schwarzschild}}(x+ia)$$

$$\Phi^{\sqrt{\text{Kerr}}}(x) = \Phi^{\text{Coulomb}}(x+ia)$$

(Newman-Penrose curvature scalars)

→ classical 3pt amplitudes

$$M_{3,\pm}^{\text{Kerr}} = e^{\pm p\cdot a} M_{3,\pm}^{\text{Schwarzschild}}$$

$$A_{3,\pm}^{\sqrt{\text{Kerr}}} = e^{\pm p\cdot a} A_{3,\pm}^{\text{Coulomb}}$$

Guevara, Ochirov, Vines;
Arkani-Hamed, Huang, O'Connell;
Guevara, Maybee, Ochirov, O'Connell, Vines

AHH amplitudes → Kerr BH ?

Arkani-Hamed, Huang, Huang. ('17)

Spin-s gravitational 3pt amplitiudes:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+) = im^2x^2\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}}\,, \qquad M(1\phi^s, 2\bar{\phi}^s, 3h^-) = i\frac{m^2}{x^2}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Spin-s gauge theory 3pt amplitudes

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}}\,, \qquad A(1\phi^s, 2\bar{\phi}^s, 3A^-) = \frac{m}{x}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

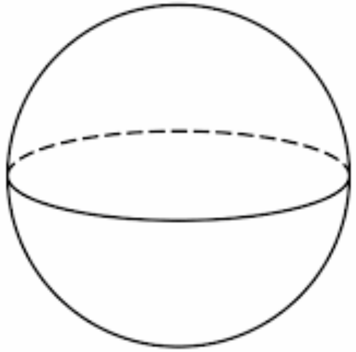
Q1: Where is the spin vector ? $S^\mu = ma^\mu$

Q2: Where is the exponential factor ? $e^{\pm p\cdot a}$

Q3: What are the quantum theories ? (before classical limit)

Quantum spin operator

Introduce projective 3-sphere coordinates



$$z^a = (x_1 + ix_2, x_3 + ix_4) \quad \rightarrow \quad 1 = z^a \bar{z}_a = |x|^2$$

parametrizes SU(2) <-> spin $\qquad z^a \sim (|\uparrow\rangle, |\downarrow\rangle)$

Relation between classical spin vector and quantum spin:

$$S^\mu = \frac{s}{2m} (\bar{z}^a z_a)^{2s-1} \left(\langle \bar{\mathbf{1}} | \sigma^\mu | \mathbf{1}] + \langle \mathbf{1} | \sigma^\mu | \bar{\mathbf{1}}] \right)$$

massive spinor-helicity formalism

Properties:

Transversality of spin vector: $\quad p_1 \cdot S = 0$

Equals an expectation value: $\quad S^\mu = \langle \hat{S}^\mu \rangle \equiv (\bar{z})^{2s} \cdot \hat{S}^\mu \cdot (z)^{2s}$

Gives spin operator: $\quad [\hat{S}^\mu, \hat{S}^\nu] = i\epsilon^{\mu\nu\rho} \hat{S}_\rho \qquad \hat{S}^2 = s(s+1)\mathbb{1}$

Recap of massive spinor helicity

Following AHH bold massive spinors <-> symmetrized little group indices

$$|\,\mathbf{i}\,\rangle \equiv |i^a\rangle z_{i,a}\,, \qquad |\,\mathbf{i}\,] \equiv |i^a] z_{i,a}$$

(spinors define maps: $SL(2,\mathbb{C}) \to SU(2)$)

Analytic functions of spinors now possible:

$$\langle \mathbf{12} \rangle^{2s} = \text{degree-}4s \text{ polynomial in } (z_1^a, z_2^a)$$

Massive polarizations are null vectors Chiodaroli, HJ, Pichini

$$\varepsilon_i^\mu = \frac{\langle \mathbf{i}|\sigma^\mu|\mathbf{i}]}{\sqrt{2}m_i} = \frac{[\mathbf{i}|\bar{\sigma}^\mu|\mathbf{i}\rangle}{\sqrt{2}m_i} = (z_i^1)^2 \varepsilon_{i,-}^\mu - \sqrt{2} z_i^1 z_i^2 \varepsilon_{i,L}^\mu - (z_i^2)^2 \varepsilon_{i,+}^\mu$$

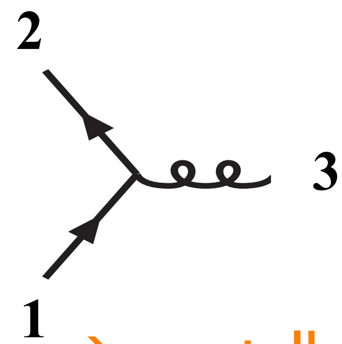
Higher-spin states automatically symmetric, transverse, traceless

$$\varepsilon_i^{\mu_1 \mu_2 \cdots \mu_s} \equiv \varepsilon_i^{\mu_1} \varepsilon_i^{\mu_2} \cdots \varepsilon_i^{\mu_s} = \text{degree-}2s \text{ polynomial in } z_i^a$$

AHH amplitudes = Kerr BHs

Relate in/out states by Lorentz transf.

$$|\mathbf{2}\rangle := |\bar{\mathbf{1}}\rangle + p_3 \cdot \sigma |\bar{\mathbf{1}}]/(2m).$$



AHH factor → exponential of spin operator: → see talk by Cangemi

$$\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}} = \Big\langle \sum_{n=0}^{2s} \frac{1}{n!}\Big(\frac{p_3 \cdot \hat{S}}{m}\Big)^n \Big\rangle = \langle e^{p_3 \cdot \hat{a}} \rangle$$

Quantum Kerr and root Kerr 3pt → Quantum Newman-Janis shift

$$M_{3,\pm}^{\text{Kerr}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle M_{3,\pm}^{\text{Schwarzchild}}$$

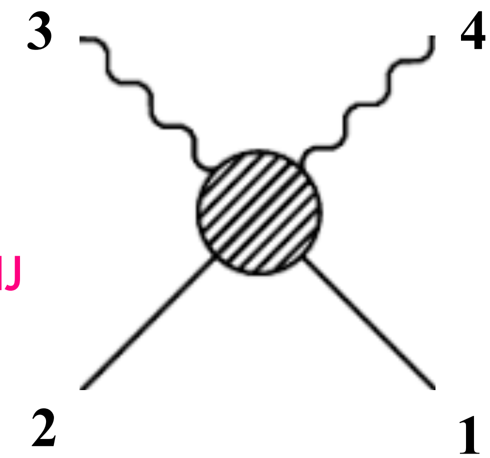
$$A_{3,\pm}^{\sqrt{\text{Kerr}}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle A_{3,\pm}^{\text{Coulomb}}$$

with ring-radius operator: $\hat{a}^\mu = \frac{\hat{S}^\mu}{m}$

(original argument: Guevara, Ochirov, Vines)

Kerr Compton amplitudes

Candidate Compton amplitudes via BCFW:



same helicity case:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+, 4h^+) = i\frac{\langle \mathbf{12}\rangle^{2s}[34]^4}{m^{2s-4}s_{12}t_{13}t_{14}}$$

Ochirov, HJ

oppsite helicity case:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-, 4h^+) = i\frac{[4|p_1|3\rangle^{4-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{s_{12}t_{13}t_{14}}$$

AHH

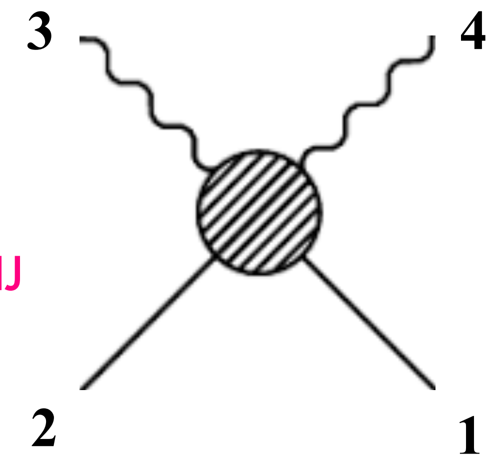
Needed for NLO calculations:

However, for $s > 2$ there is a spurious pole → need corrections

$$\frac{1}{[4|p_1|3\rangle^{2s-4}}$$

root-Kerr Compton amplitudes

Candidate Compton amplitudes via BCFW:



same helicity case:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+, 4A^+) = i\frac{\langle \mathbf{12}\rangle^{2s}[34]^2}{m^{2s-2}t_{13}t_{14}}$$

Ochirov, HJ

oppsite helicity case:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^-, 4A^+) = -i\frac{[4|p_1|3\rangle^{2-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{t_{13}t_{14}}$$

AHH

Not needed for physics purposes, but provide useful toy model!

Again, for $s > 1$ spurious pole $\frac{1}{[4|p_1|3\rangle^{2s-2}}$ → need corrections

→ See talk by Cangemi

Which quantum EFTs give Kerr amplitudes ?

EFTs behind root-Kerr

Identify EFTs from covariant formulas:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}}$$

spin-0: $A(1\phi^0, 2\bar{\phi}^0, 3A) = \varepsilon_3 \cdot (p_1 - p_2) \equiv A_{\phi\phi A}$ (scalar)

spin-1/2: $A(1\phi^{1/2}, 2\bar{\phi}^{1/2}, 3A) = \bar{u}_2 \not{\varepsilon}_3 u_1 \equiv A_{\lambda\lambda A}$ (fermion)

spin-1: $A(1\phi^1, 2\bar{\phi}^1, 3A) = 2(\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2\, \varepsilon_3 \cdot p_2 + \boldsymbol{\varepsilon}_2 \cdot \varepsilon_3\, \boldsymbol{\varepsilon}_1 \cdot p_3 + \varepsilon_3 \cdot \boldsymbol{\varepsilon}_1\, \boldsymbol{\varepsilon}_2 \cdot p_1)$
$\equiv A_{WWA}$ (W-boson)

spin-3/2:

$$A(1\phi^{3/2}, 2\bar{\phi}^{3/2}, 3A) = \bar{u}_2^\mu \not{\varepsilon}_3 u_{1\mu} - \frac{2}{m}\bar{u}_{2\mu} f_3^{\mu\nu} u_{1\nu} - \frac{1}{2m}\bar{u}_2^\mu f_3^{\rho\sigma}\gamma_\rho\gamma_\sigma u_{1\mu} \equiv A_{\psi\psi A}$$

(gravitino)

general spin-s given as a generating function:

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A) = A_{\phi\phi A} + \frac{A_{WWA} - (\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 A_{\phi\phi A}}{(1 + \boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 + \frac{2}{m^2}\boldsymbol{\varepsilon}_1 \cdot p_2\, \boldsymbol{\varepsilon}_2 \cdot p_1}$$

Chiodaroli, HJ, Pichini

For $s > 1$ → higher-derivative HS effective theories (no massless limit)

Kerr/root-Kerr double copy

Are related to the gauge th. ones via KLT

Chiodaroli, HJ, Pichini

$$M(1\phi^s, 2\bar{\phi}^s, 3h^\pm) = iA(1\phi^{s_\mathrm{L}}, 2\bar{\phi}^{s_\mathrm{L}}, 3A^\pm) A(1\phi^{s_\mathrm{R}}, 2\bar{\phi}^{s_\mathrm{R}}, 3A^\pm)$$

Works for any decomposition: $s = s_\mathrm{L} + s_\mathrm{R}$

Preferred decomposition $s = 1 + (s-1)$ give fewest derivatives :

$$\sum_{2s=0}^{\infty} M(1\phi^s, 2\bar{\phi}^s, 3h) = M_{0\oplus 1/2} + A_{WWA}\Big(A_{0\oplus 1/2} + \frac{A_{1\oplus 3/2} - (\varepsilon_1\cdot\varepsilon_2)^2 A_{0\oplus 1/2}}{(1+\varepsilon_1\cdot\varepsilon_2)^2 + \frac{2}{m^2}\varepsilon_1\cdot p_2\, \varepsilon_2\cdot p_1}\Big)$$

From double-copy structure, we can infer:

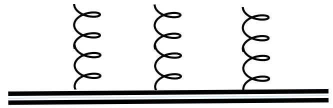
EFTs	$s = {}^1\!/_2$	$s = 1$	$s = {}^3\!/_2$	$s = 2$	$s = {}^5\!/_2$	$s \geq 3$
Kerr	Major.	Proca	Rar.-Sch.	KK grav.	HS	HS
$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	HS	HS	HS

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

For $s > 2$ Kerr → higher-derivative HS EFTs (no massless limit)

Low-spin Compton double copies

Kerr amplitudes for $s \leq 2$ admit Compton double copy (also ***n***-points)



$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \mathrm{scalar}) = (\mathrm{GR} + \mathrm{scalar})$$

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \mathrm{fermion}) = (\mathrm{GR} + \mathrm{fermion})$$

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \mathrm{W\text{-}boson}) = (\mathrm{GR} + \mathrm{Proca})$$

$$(\mathrm{YM} + \mathrm{W\text{-}boson}) \otimes (\mathrm{YM} + \mathrm{fermion}) = (\mathrm{GR} + \mathrm{massive\ gravitino})$$

$$(\mathrm{YM} + \mathrm{W\text{-}boson}) \otimes (\mathrm{YM} + \mathrm{W\text{-}boson}) = (\mathrm{GR} + \mathrm{massive\ KK\ graviton})$$

Lagrangians unique: no new interaction terms beyond cubic order

Can be used for $(S^\mu)^{\leq 4}$ PM/PN calculations

Need new principles to fix interactions of the HS theories!

Higher-spin (HS) theories

What special about the low-spin EFTs ?

Kerr (root-Kerr) EFTs for $s \leq 2 \; (s \leq 1)$

Chiodaroli, HJ, Pichini

→ well-behaved massless limit

→ exhibits gauge symmetry (SSB)

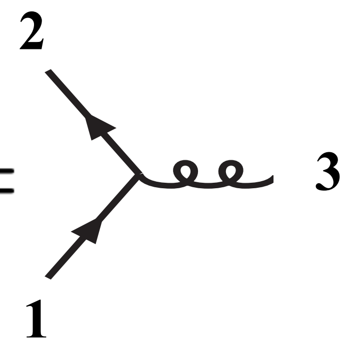
$s = 1$ (YM + W-boson) $\rightarrow$ non-abelian gauge symmetry

$s = 3/2$ (GR + massive gravitino) $\rightarrow$ supersymmetry

$s = 2$ (GR +massive KK graviton) $\rightarrow$ General covariance

Furthermore: satisfy a current constraint

$$p_1 \cdot J = \mathcal{O}(m)$$

$J =$ 

Connected to tree-level unitarity constraint;

Porrati et al.

longitudinal modes suppressed in low-mass (high-energy) limit

Current constraint for s=3/2, 5/2

The current constraint (+ derivative power counting) gives unique amplitudes and EFT Lagrangians up to spin-3/2 root-Kerr

Chiodaroli, HJ, Pichini

$$\mathcal{L} = \bar{\psi}^{\mu}\gamma_{\mu\nu\rho}\left(iD^{\nu} - \frac{1}{2}m\gamma^{\nu}\right)\psi^{\rho} + \frac{ie}{m}\bar{\psi}_{\mu}\mathcal{F}^{\mu\nu}\psi_{\nu}$$

$$\mathcal{F}^{\mu\nu} \equiv F^{\mu\nu} - i/2\,\gamma^{5}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

and also unique spin-5/2 Kerr EFT:

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{min}} = {} & \bar{\psi}_{\mu\nu}(i\nabla\!\!\!/ - m)\psi^{\mu\nu} + 2\bar{\psi}_{\mu\nu}\gamma^{\nu}(i\nabla\!\!\!/ + m)\gamma^{\rho}\psi^{\mu}_{\rho} - \frac{1}{2}\bar{\psi}^{\mu}_{\mu}(i\nabla\!\!\!/ - m)\psi^{\rho}_{\rho} \\ & - (2\bar{\psi}^{\rho\mu}i\nabla_{\rho}\gamma^{\nu}\psi_{\mu\nu} + 2\bar{\psi}_{\mu\nu}\gamma^{\nu}i\nabla_{\rho}\psi^{\rho\mu}) + (\bar{\psi}^{\mu}_{\mu}i\nabla_{\rho}\gamma_{\sigma}\psi^{\rho\sigma} + \bar{\psi}^{\rho\sigma}\gamma_{\sigma}i\nabla_{\rho}\psi^{\mu}_{\mu}) \\ & + m(\bar{\psi}^{\mu}_{\mu}\lambda + \bar{\lambda}\psi^{\mu}_{\mu}) - \frac{12}{5}\bar{\lambda}(i\nabla\!\!\!/ + 3m)\lambda \end{aligned}$$

$$\mathcal{L}_{\text{non-min}} = -\frac{1}{m}\sqrt{-g}\,\bar{\psi}_{\mu\rho}\mathcal{R}^{\mu\nu\rho\sigma}\psi_{\nu\sigma}$$

$$\mathcal{R}^{\mu\nu\rho\sigma} = R^{\mu\nu\rho\sigma} - (i/2)\gamma^{5}\epsilon^{\rho\sigma\alpha\beta}R^{\mu\nu}{}_{\alpha\beta}$$

Using HS gauge invariance

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Consider spin-2 root-Kerr case:

physical field: $\Phi_{\mu\nu}$

Stückelberg fields: $\{B_\mu, \varphi\}$

Imposing a linearized massive higher-spin gauge transformation:

$$\delta\Phi_{\mu\nu} = \frac{1}{2}\partial_\mu\xi_\nu + \frac{1}{2}\partial_\nu\xi_\mu + \frac{m}{\sqrt{2}}\eta_{\mu\nu}\xi,$$

$$\delta B_\mu = \partial_\mu\xi + \frac{m}{\sqrt{2}}\xi_\mu,$$

$$\delta\varphi = \sqrt{3}m\xi,$$

← gauge parameter

Makes sure that:

→ DOFs are correct,

→ small-mass limit better behaved than naively expected

Massive Ward identities

We write down ansatz for off-shell interactions:

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$V_{\Phi\overline{\Phi}A} \sim m\,(\epsilon_1)^2\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p^3}{m^3}+\frac{p}{m}\right),$$

$$V_{B\overline{\Phi}A} \sim m\,(\epsilon_1)\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p^2}{m^2}+1\right),$$

$$V_{\varphi\overline{\Phi}A} \sim m\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p}{m}\right),$$

and constrain them using Ward identities

$$V_{\xi\overline{\Phi}A}\big|_{(2,3)} = V_{\zeta\overline{\Phi}A}\big|_{(2,3)} = 0$$

where the vertices corresponding to gauge parameters are:

$$V_{\xi\overline{\Phi}A} := \frac{m}{\sqrt{2}}V_{B\overline{\Phi}A} - \frac{i}{2}p_1\cdot\frac{\partial}{\partial\epsilon_1}V_{\Phi\overline{\Phi}A},$$

$$V_{\zeta\overline{\Phi}A} := \sqrt{3}mV_{\varphi\overline{\Phi}A} - ip_1\cdot\frac{\partial}{\partial\epsilon_1}V_{B\overline{\Phi}A} + \frac{m}{2\sqrt{2}}\left(\frac{\partial}{\partial\epsilon_1}\right)^2 V_{\Phi\overline{\Phi}A}.$$

→ 3pt amplitude: $A(\Phi_1^2\,\overline{\Phi}_2^2\,A_3^+) = A_0\frac{\langle\mathbf{12}\rangle^3}{m^4}\left(c_1[\mathbf{12}]+(1-c_1)\langle\mathbf{12}\rangle\right)$

unique after current constraint: $c_1 = 0$

General spin-s EFTs

Consider tower $k = 0, 1, 2, \ldots, s$ of HS fields and gauge parameters:

$$\Phi^k := \Phi^{\mu_1\mu_2\cdots\mu_k}, \qquad \xi^k := \xi^{\mu_1\mu_2\cdots\mu_k}$$

(double-traceless) (traceless)

Zinoviev (2001)

Gauge transformation: $\delta\Phi^k = \partial^{(1}\xi^{k-1)} + m\alpha_k\xi^k + m\beta_k\eta^{(2}\xi^{k-2)}$

$$\alpha_k = \frac{1}{k+1}\sqrt{\frac{(s-k)(s+k+1)}{2}}, \qquad \beta_k = \frac{1}{2}\frac{k}{k-1}\alpha_{k-1}$$

Minimal Lagrangian: $\mathcal{L}_0 = \mathcal{L}_{\mathrm{F}} + \frac{1}{2}\sum_{k=0}^{s-1}(-1)^k(k+1)G^kG^k$

Gauge-fixing fn: $G^k = \partial\cdot\Phi^{k+1} - \frac{k}{2}\partial^{(1}\tilde{\Phi}^{k+1)} + m\big(\alpha_k\Phi^k - \gamma_k\tilde{\Phi}^{k+2} - \delta_k\eta^{(2}\tilde{\Phi}^{k)}\big)$

Feynman-gauge Lagr: $\mathcal{L}_{\mathrm{F}} = \sum_{k=0}^{s}\frac{(-1)^k}{2}\Big[\Phi^k(\Box+m^2)\Phi^k - \frac{k(k-1)}{4}\tilde{\Phi}^k(\Box+m^2)\tilde{\Phi}^k\Big]$

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

Non-minimal interactions

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

3pt vertex: $V_{\Phi^k \Phi^s A^\mathfrak{h}} = V^{\text{min.}}_{\Phi^k \Phi^s A^\mathfrak{h}} + V^{\text{non-min.}}_{\Phi^k \Phi^s A^\mathfrak{h}}$

Ward identities:

$$V_{\xi^k \Phi^s A^\mathfrak{h}} := m\alpha_k V_{\Phi^k \Phi^s A^\mathfrak{h}} - \frac{i}{k+1} p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^{k+1} \Phi^s A^\mathfrak{h}} + \frac{m\beta_{k+2}}{(k+2)(k+1)} \frac{\partial}{\partial \epsilon_1} \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^{k+2} \Phi^s A^\mathfrak{h}}$$

Constraints imposed:

(WI) Ward identities $V_{\xi^k \Phi^s A^\mathfrak{h}}\big|_{(2,3),\epsilon_1^2 \to 0} = 0$;

(CC) Current constraint $p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^s \Phi^s A^\mathfrak{h}}\big|_{(2,3),\epsilon_1^2 \to 0} = \mathcal{O}(m)$.

(PC) Power-counting bound on derivatives in non-minimal vertices: $V^{\text{non-min.}}_{\Phi^{s_1} \Phi^{s_2} A^\mathfrak{h}} \sim \partial^{s_1+s_2-2\mathfrak{h}} (F_{\mu\nu})^\mathfrak{h}$;

(ND) Near-diagonal interactions: if $|s_1 - s_2| > \mathfrak{h}$ then $V_{\Phi^{s_1} \Phi^{s_2} A^\mathfrak{h}} = 0$.

Gives unique Kerr and root-Kerr 3pt amplitudes (matching AHH)

HS perturbation theory

Calculations expected to simplify in Feynman gauge:

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

Feynman-gauge propagator for any field obtained as generating fn:

$$\Delta(\epsilon,\bar\epsilon) = \sum_{s=0}^{\infty} (\epsilon)^s \cdot \Delta^{(s)} \cdot (\bar\epsilon)^s = \frac{1}{p^2 - m^2 + i0} \frac{1 - \frac{1}{4}\epsilon^2\bar\epsilon^2}{1 + \epsilon\cdot\bar\epsilon + \frac{1}{4}\epsilon^2\bar\epsilon^2}$$

Focus on root-Kerr Compton amplitude, we obtain

$$A(\Phi_1^s \Phi_2^s A_3^- A_4^+) = \frac{\langle 3|1|4]^2 (U+V)^{2s}}{m^{4s} t_{13} t_{14}} + \frac{\langle 3|1|4] \langle \mathbf{13}\rangle [\mathbf{2}4] P_{2s}}{m^{4s} t_{13}} + \langle \mathbf{13}\rangle \langle 3\mathbf{2}\rangle [\mathbf{1}4][4\mathbf{2}] \frac{P_{2s-1}}{m^{4s}} + C_s,$$

contact term

$C_{s<2} = 0$

with a polynomial: $P_k = \frac{1}{2V}\{(U+V)^k - (U-V)^k\}$

and variables $V = \frac{1}{2}(\langle \mathbf{1}|4|\mathbf{2}] + \langle \mathbf{2}|4|\mathbf{1}]), U = \frac{1}{2}(\langle \mathbf{1}|4|\mathbf{2}] - \langle \mathbf{2}|4|\mathbf{1}]) - m[\mathbf{12}]$

4pt contact terms

root-Kerr Compton amplitude:

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Ochirov, Pichini, Skvortsov

4pt Ward Id leaves 2 parameters unfixed at $s = 2$

$$C_2 = \frac{\langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^6}\left\{c_1(\langle\mathbf{12}\rangle+[\mathbf{12}])^2+c_2(\langle\mathbf{12}\rangle-[\mathbf{12}])^2\right\}$$

Spin-s amplitude in terms of ring-radius operator:

$$A(\Phi_1^s\Phi_2^s A_3^- A_4^+) \xrightarrow{\hbar\sim 0} -e^{-\hat{a}\cdot q_\perp}\left(\frac{(p_1\cdot w)^2}{(p_1\cdot q_\perp)^2}+\frac{(p_1\cdot w)(\hat{a}\cdot w)}{(p_1\cdot q_\perp)}+\frac{1}{2s}(\hat{a}\cdot w)^2\right) + \hat{C}_s + \mathcal{O}(\hat{a}^2)+\mathcal{O}(\hbar),$$

→ See talk by Cangemi

Conclusion: Kerr dynamics proposal

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

We propose that Kerr dynamics is non-trivially constrained by

Massive Higher Spin Gauge Symmetry

Checks: → uniquely predicts previously known Kerr 3,4pt amplitudes

→ gives non-trivial constraints on unknown Compton contact terms

Further checks needed:

→ Analysis of classical limit → Cangemi
→ Comparison with Teukolsky equation (BH-PT)
→ Newman-Janis shift at Compton level ?
→ Uniqueness of EFTs ?

Possible future directions: implications for quantum BHs,
→ including absorption and emission effects