The (nonprojective) nonperturbative-hedron ?

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Prague Spring Amplitudes
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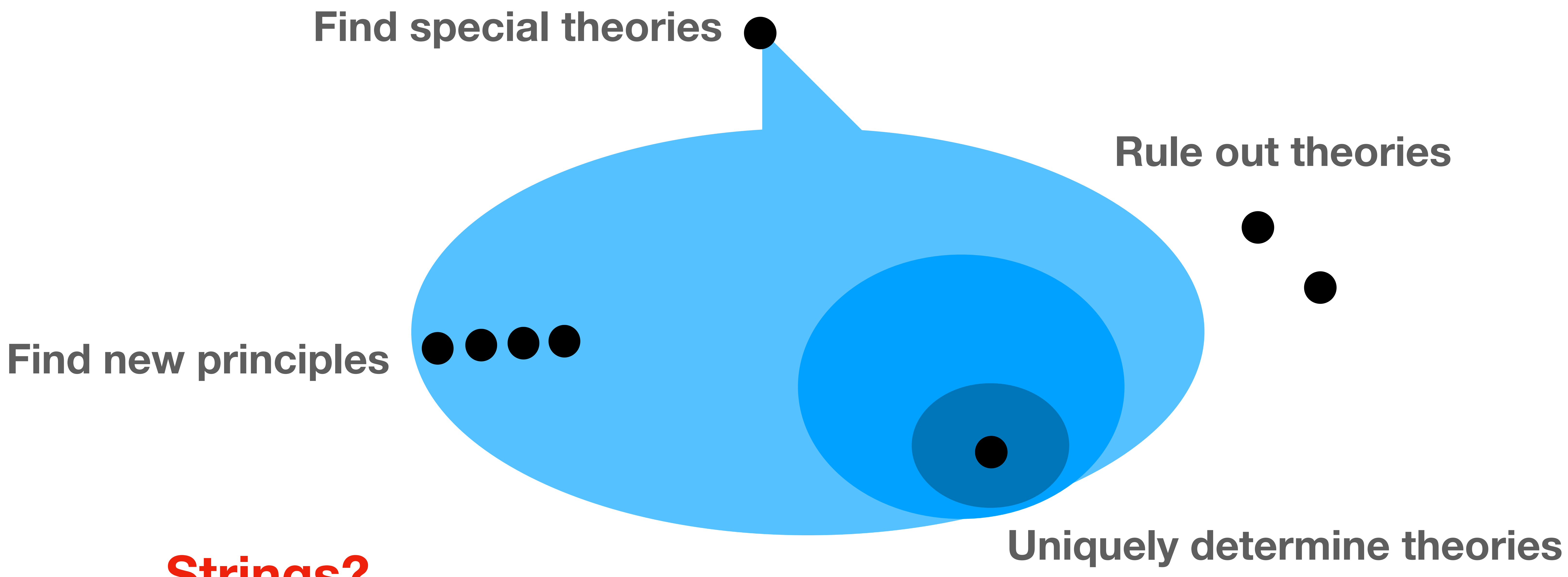
MARIE CURIE ACTIONS

Bootstraping the space of theories

S matrix bootstrap program(s) : given some general physical principles, how far can we go?

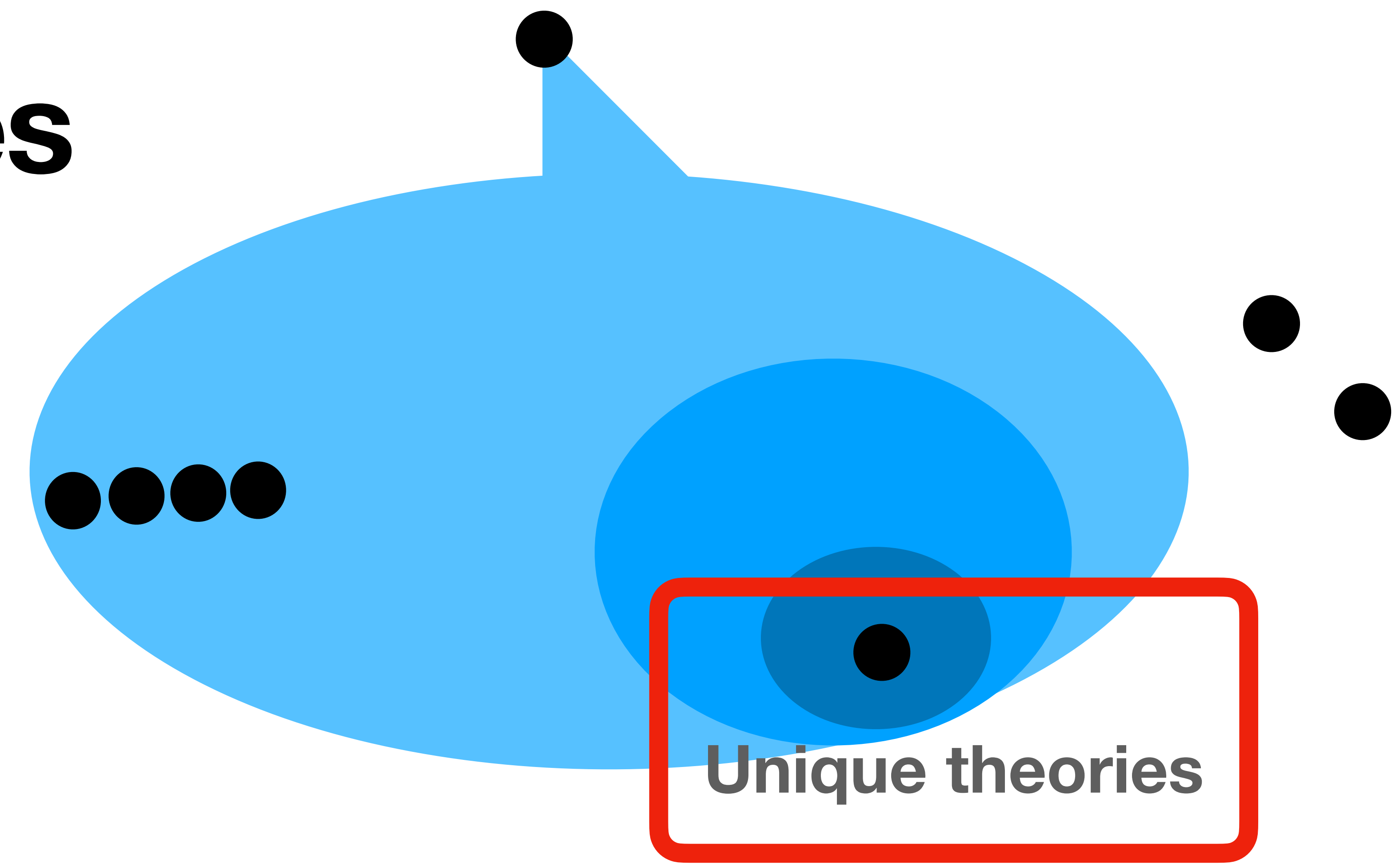
Bootstrap program A: unique special theories (YM, GR, etc)

Bootstrap program B: space of general theories (eg. all EFT with consistent UV completion)



Strings?

The space of massless theories



- Given an expression, how do you check it represents an amplitude?
- Is there a minimal, sufficient set of conditions for this check?
- Surprising answer at tree level : Mass dimension + Analyticity (locality) + one principle => Uniqueness
- Unitarity (factorization) is an automatic consequence in all cases

	gauge inv.	Adler zero	soft theorems	UV scaling	BCJ
YM	x		x	x	
GR	x		x	x	
bi-adjoint			x	x	x
NLSM		x	x	x	x
DBI		x	x	x	
sGal		x	x	x	
dilaton			x	x	

Strings?

[Cheung, Kampf, Novotny, Shen, Trnka]
[Arkani-Hamed, LR, Trnka]
[Carrasco, LR]
[Brown, Kampf, Oktem, Paranjape, Trnka]

More general space of theories

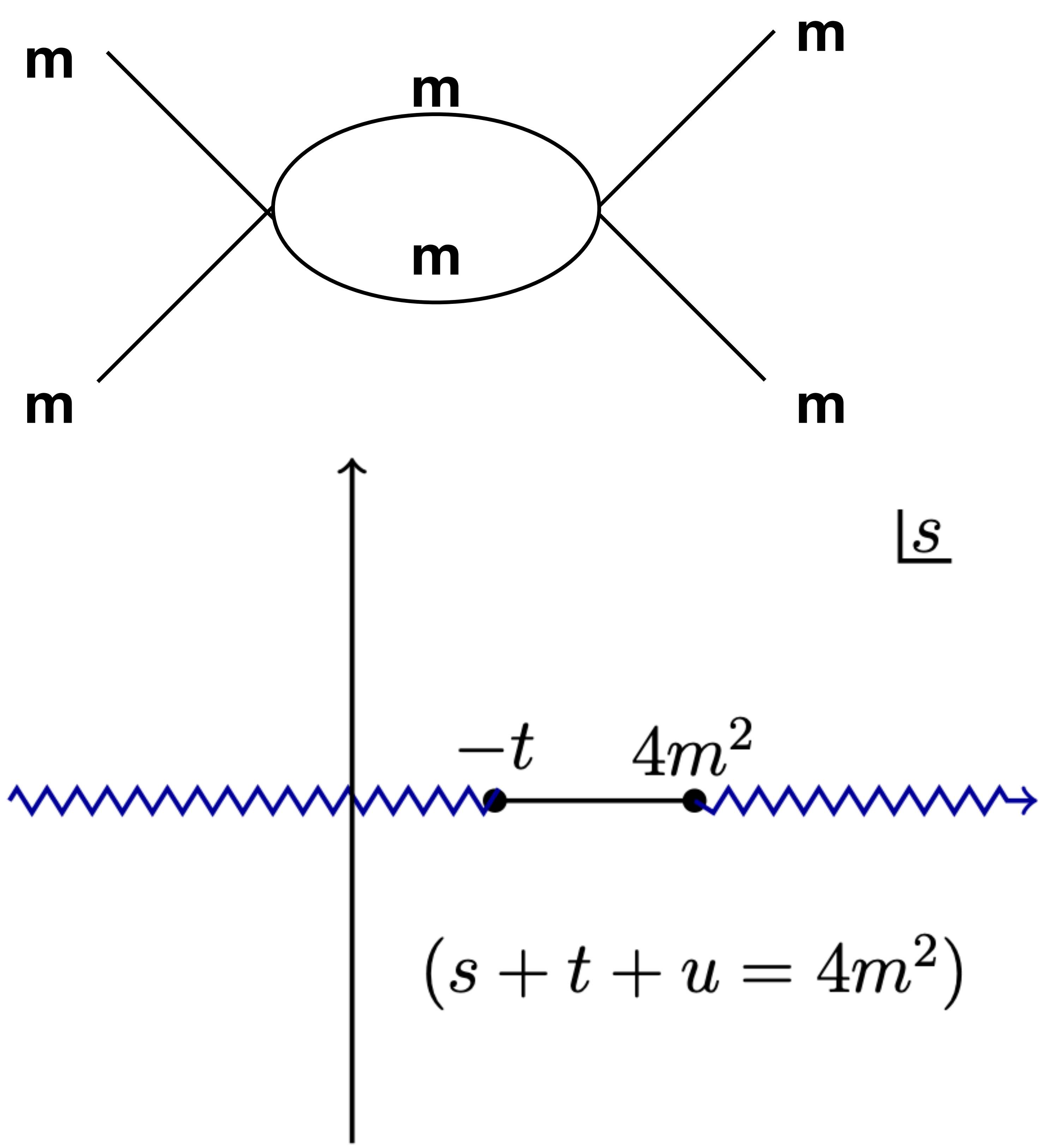
What amplitudes are consistent with

1. **Analyticity**: $M(s,t,u)$ is an analytic function except physical branch cuts or poles

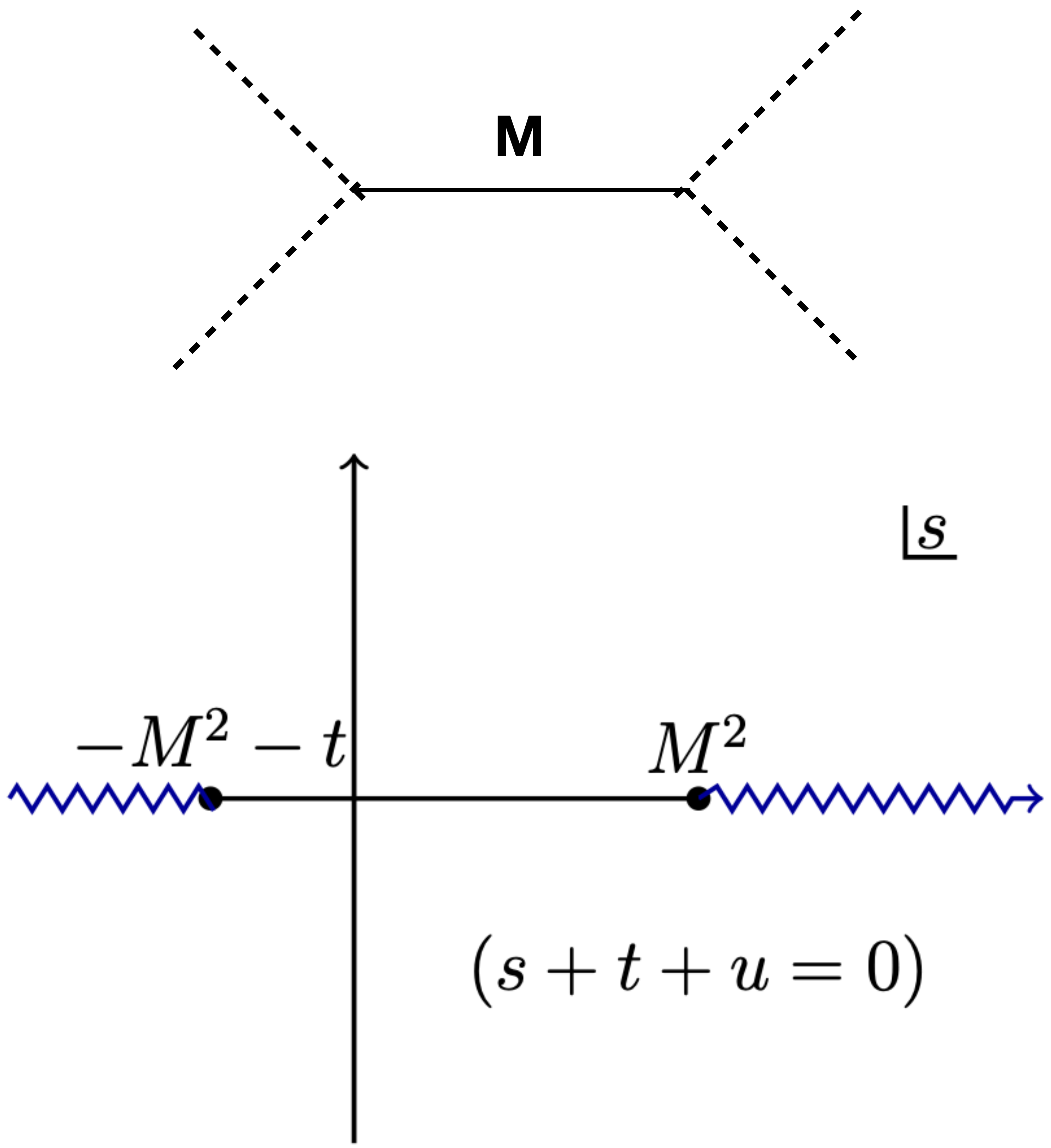
2. **Crossing**: $M(s,t,u)$ is invariant under s,t,u exchange

3. **Unitarity**: partial amplitudes satisfy $|S_J|^2 \leq 1$
   Positivity: $Im[S_j] \geq 0$ (projective)
   Linear unitarity: $Im[S_j] \leq 1$ (non-projective)

Non perturbative



Perturbative EFT



Dispersion relations (can only do linear unitarity)

- Analytic - EFThedron
- Numerical - SDPB (for positivity),
  - Maximize (also for upper bound)

Numerical ansatz for amplitude (full unitarity)

[Paulos, Penedones, Toledo, van Rees, Vieira]

Perturbative EFT, using positivity

Unitarity: partial amplitudes satisfy $|S_J|^2 \leq 1$

Positivity: $Im[S_j] \geq 0$ (projective)

Linear unitarity: $Im[S_j] \leq 1$ (non-projective)

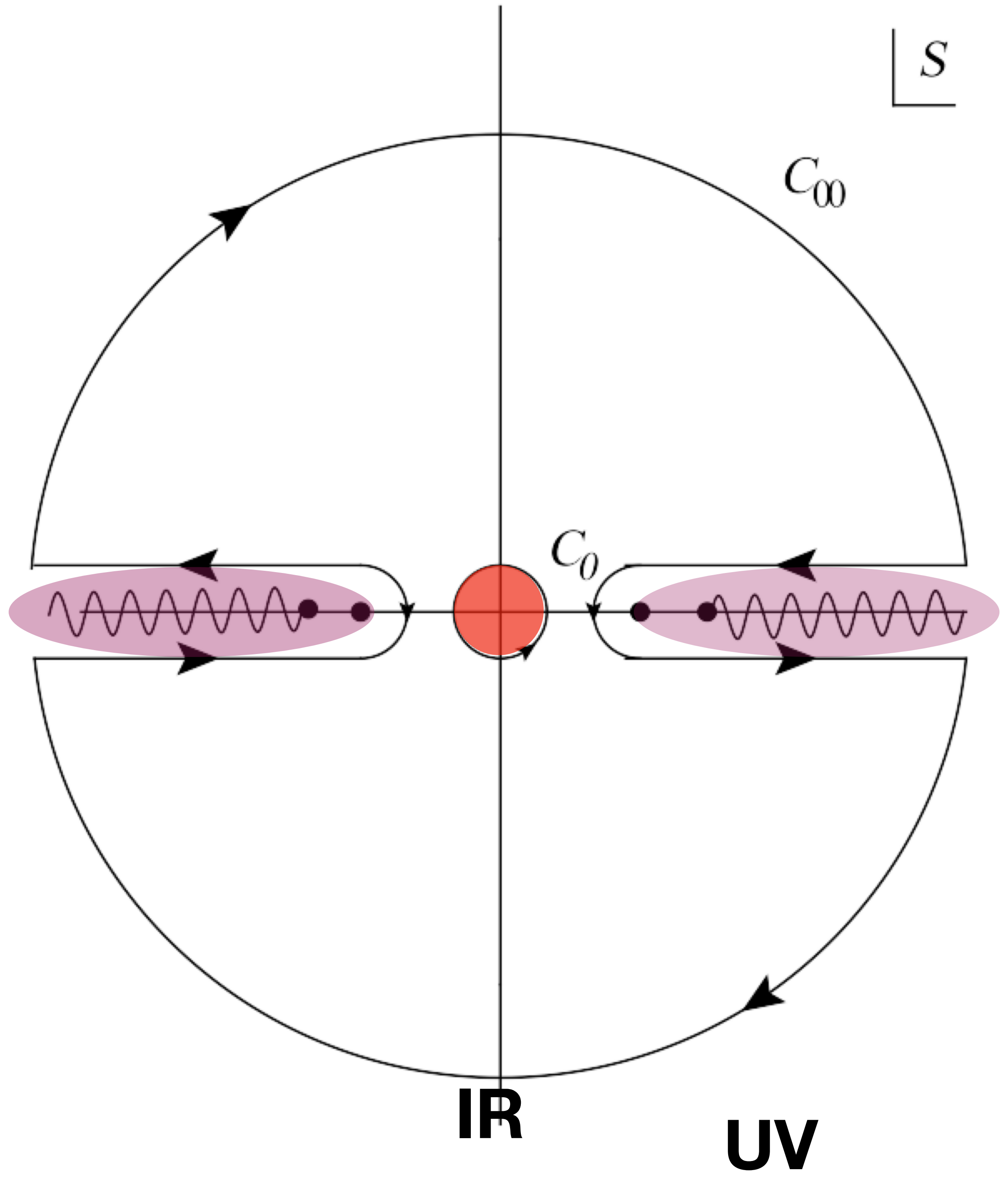
Dispersion relations

Analyticity: $\oint_{IR} M(s,t) + \oint_{UV} M(s,t) + \oint_{\infty} M(s,t) = 0$

$$M(s,t) = g_{00} + g_{10}s + g_{11}t + \ldots .$$

$$\sum_J \int_M^\infty dm\ \rho_J(m)K(m)G_J(m)$$

$$a_{kq} = \sum_J \int_M^\infty dm\, \rho_j(m)\, J^q\, m^k$$



Unitarity: $0 < \rho_J < 1$

Crossing: $g_{10} = g_{11}, \ldots$

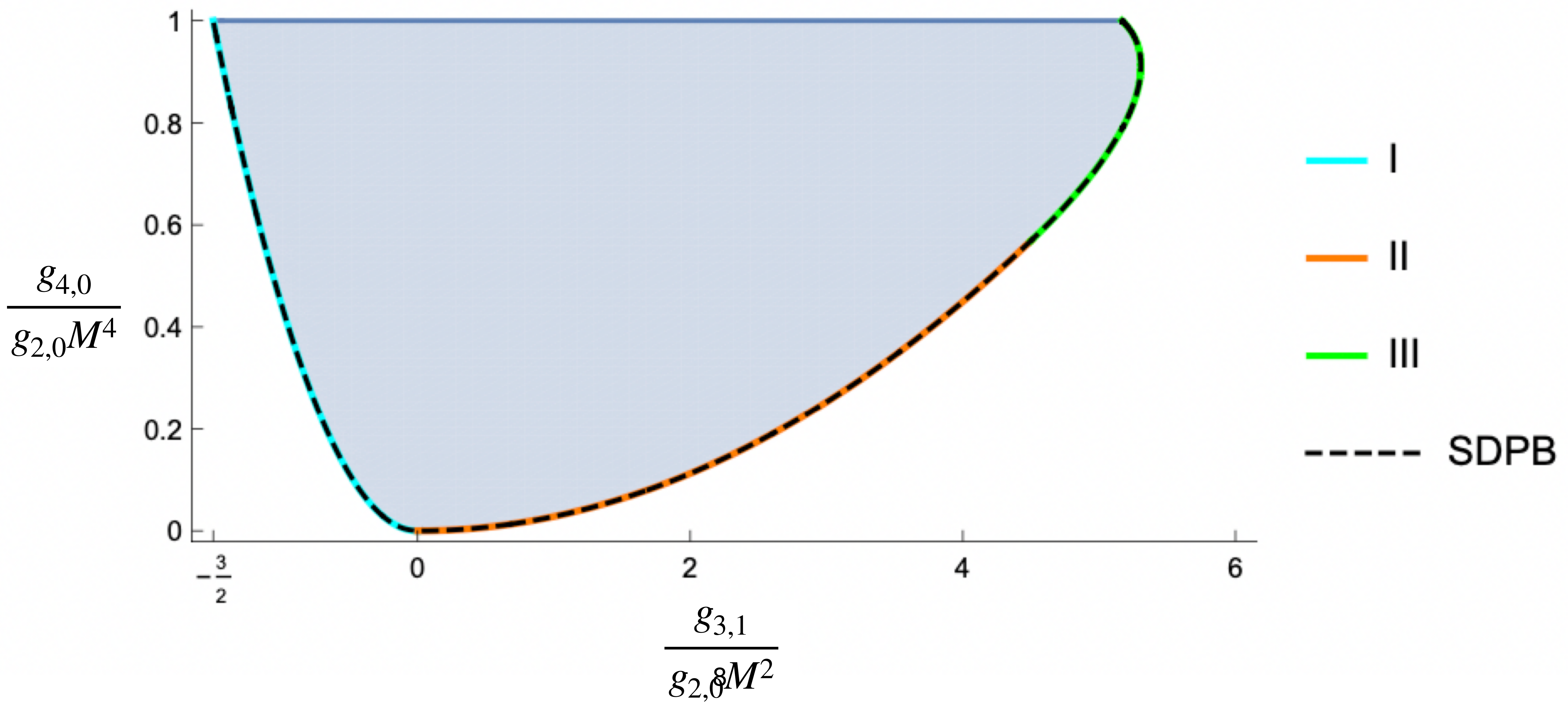
(projective) EFT hedron:

(completes *Arkani-Hamed, Huang, Huang '20*)

$$a_{kq} = \sum_J \int_M^\infty dm\, \rho_j(m)\, J^q\, m^k$$

$$\rho \geq 0 \qquad\qquad J \in \mathbb{N} \qquad\qquad a_{k,q} + \ldots = 0$$

$$\begin{pmatrix} a_{0,0} & a_{1,0} & a_{0,1} & \cdots \\ a_{1,0} & a_{2,0} & a_{1,1} & \\ a_{0,1} & & & \\ \vdots & & & \ddots \end{pmatrix} \geq 0 \qquad \begin{pmatrix} a_{0,0} & 1 & 1 & \cdots \\ a_{0,1} & J_i & J_{i+1} & \\ a_{0,2} & J_i^2 & J_{i+1}^2 & \\ \vdots & & & \ddots \end{pmatrix} \geq 0$$



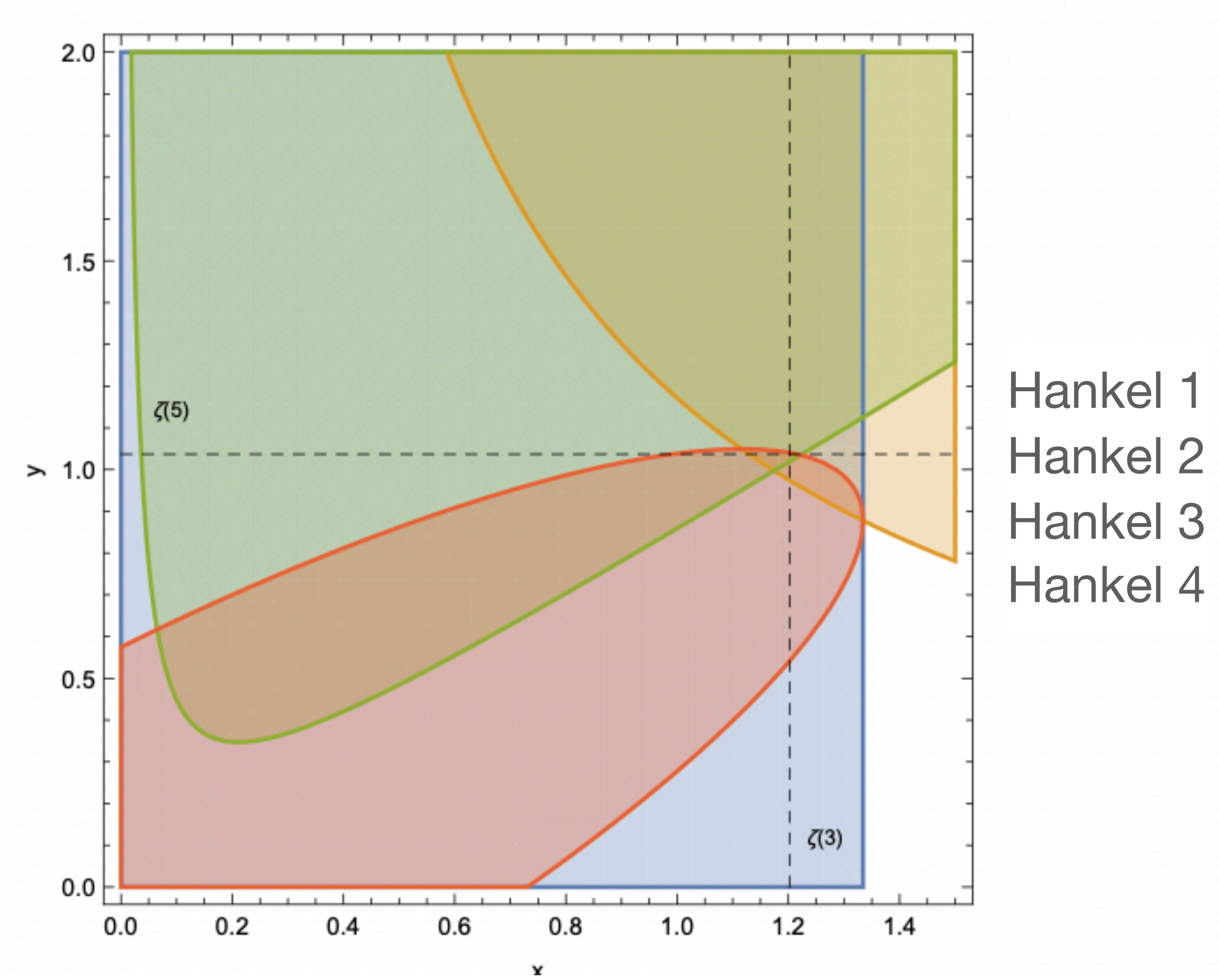
Uniquely fixing string theory

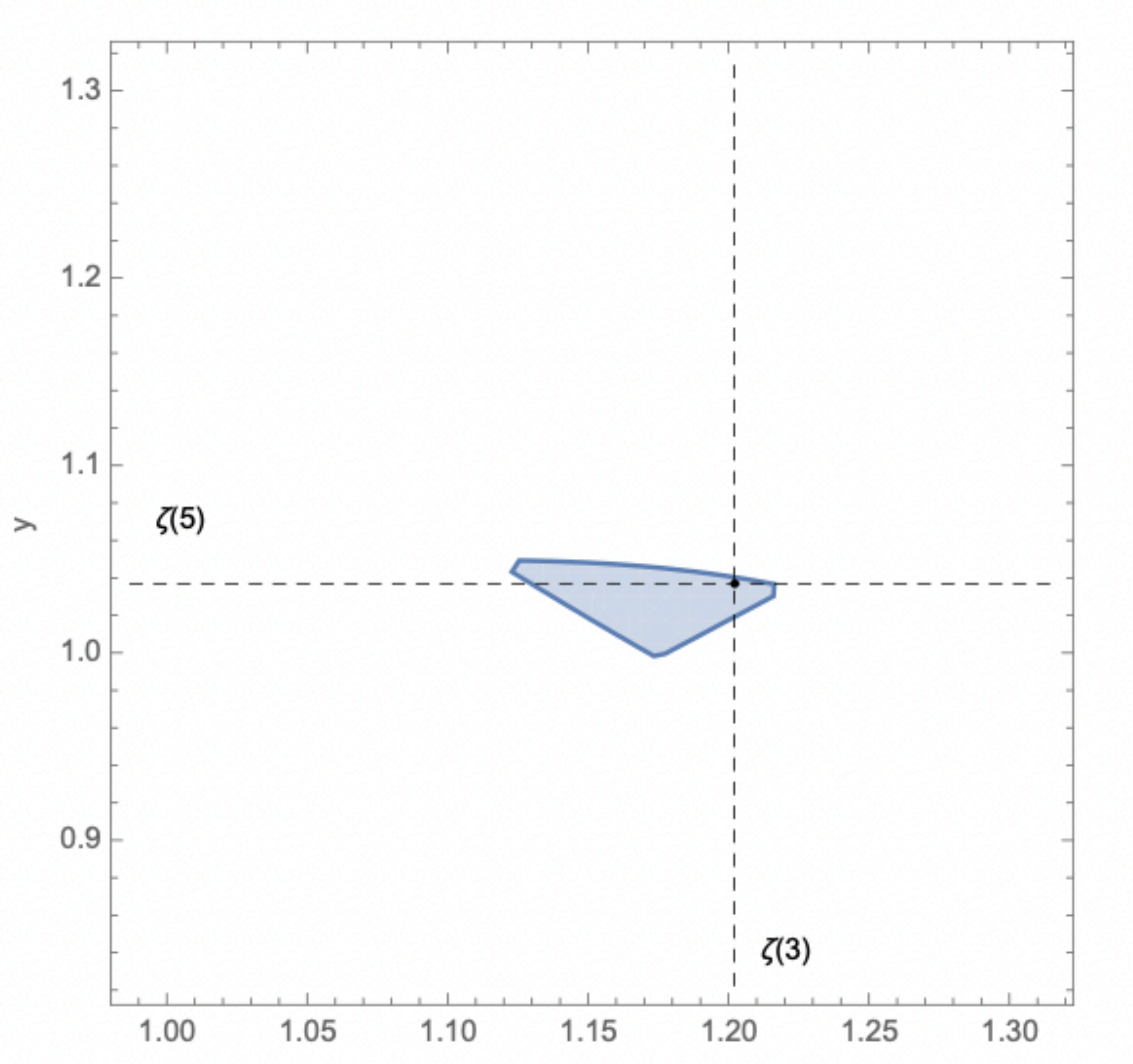
EFThedron + monodromy relations ⇒ Z theory, open string theory

$$A = g_{00} + g_{10}s + g_{11}t + \ldots$$

$$A_{string} = \zeta_2 \ + \ \zeta_3 s + \ldots$$

$$A_u + e^{i\pi\alpha' s}A_s + e^{-i\pi\alpha' t}A_t = 0$$





[Huang, Liu, LR, Wang ’20]

Recent work: KLT/BCJ + factorization at higher multiplicity

[Chen, Elvang, Herderschee], [Brown, Kampf, Oktem, Paranjape, Trnka]

Perturbative EFT, using linear unitarity

Unitarity: partial amplitudes satisfy $|S_J|^2 \leq 1$

Positivity: $Im[S_j] \geq 0$ (projective)

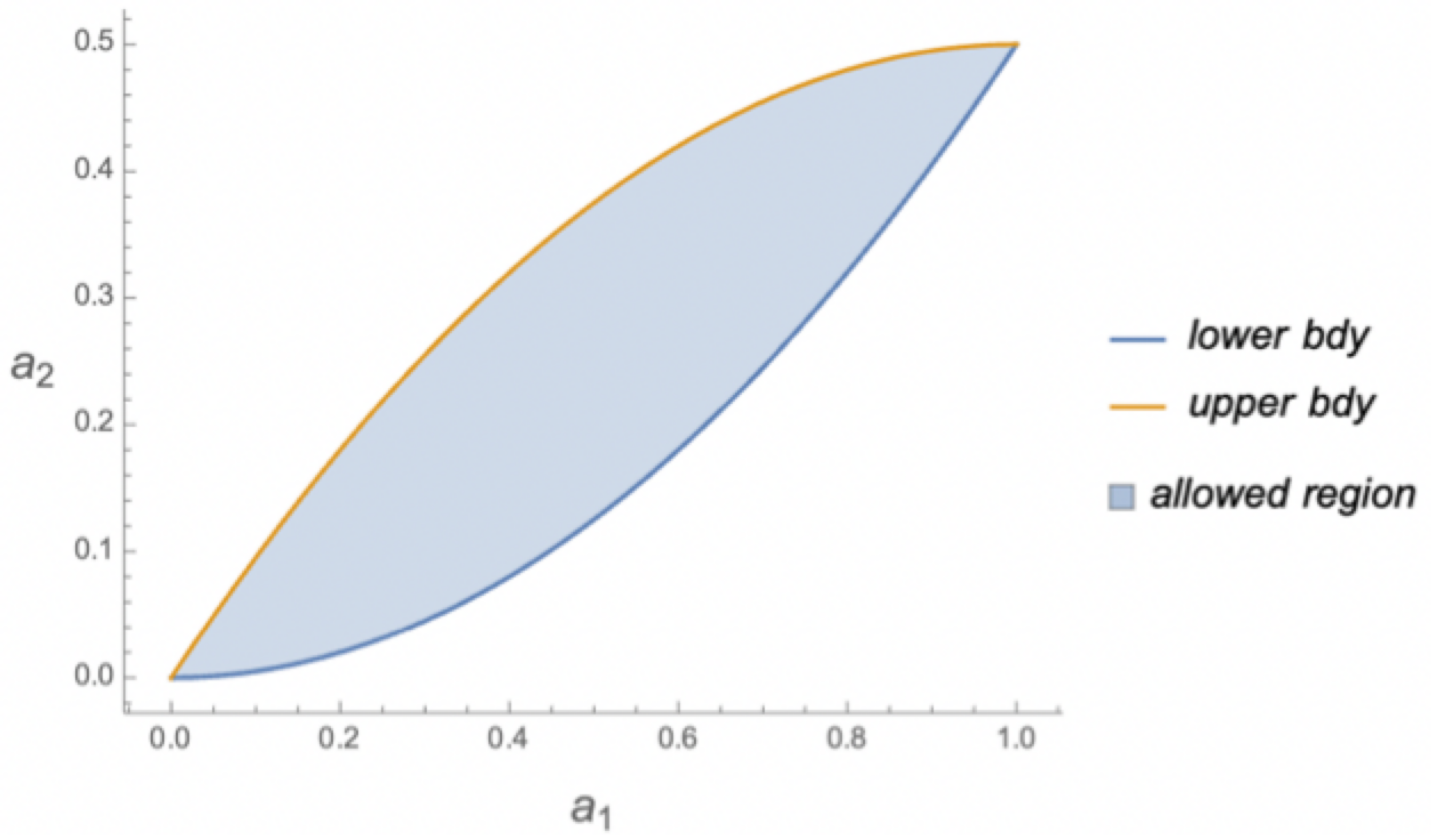
Linear unitarity: $Im[S_j] \leq 1$ (non-projective)

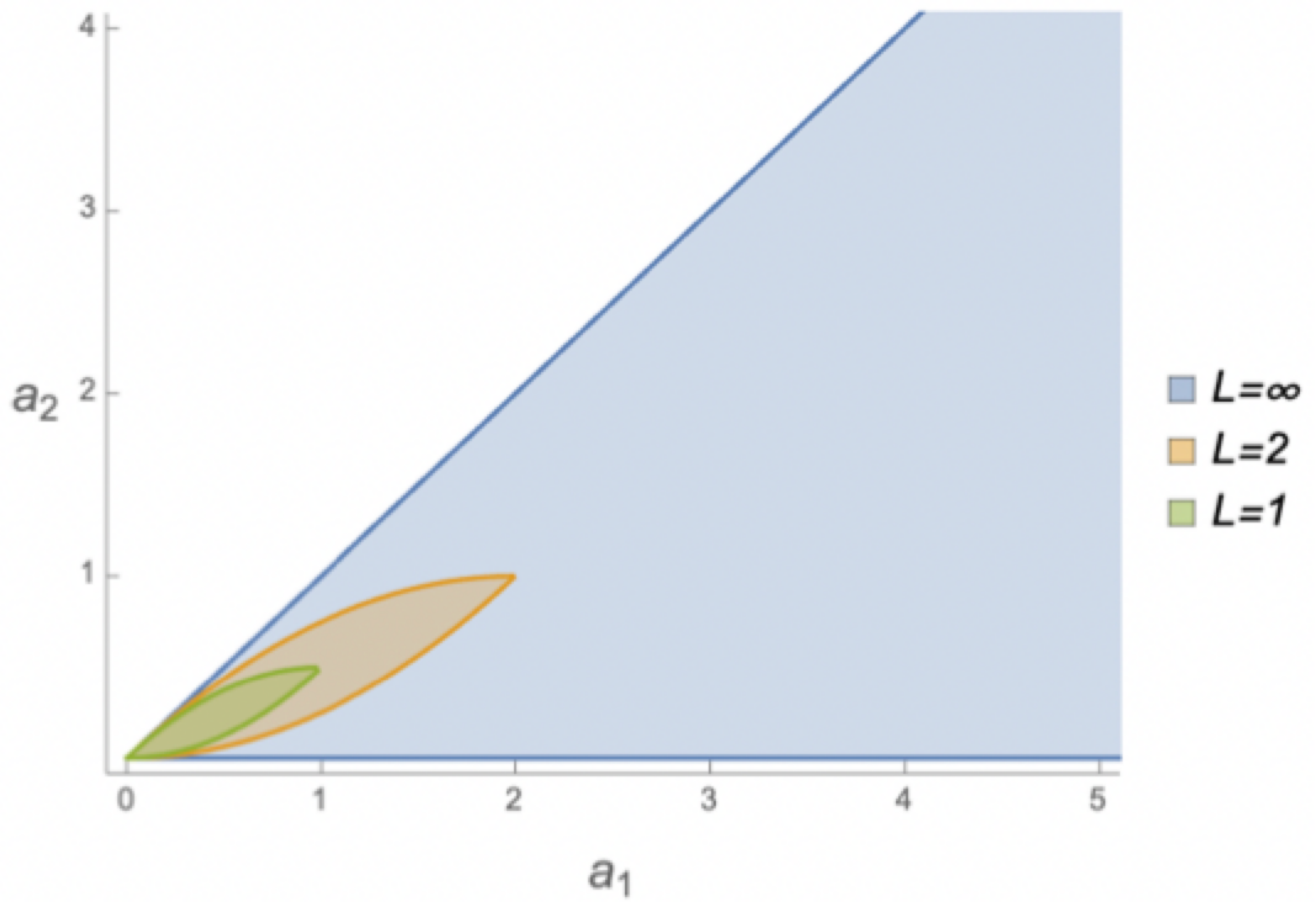
Linear unitarity and the non-projective EFThedron

What is the space defined by $a_k = \int_0^1 \rho(z) z^k dz \qquad 0 \le \rho \le L$

Boundary solution is given by $\rho(z) = L\chi_{[0,m]}$ $\qquad \chi_{[a,b]} \begin{cases} 1 \text{ for } z \in [a,b] \\ 0 \text{ otherwise} \end{cases}$

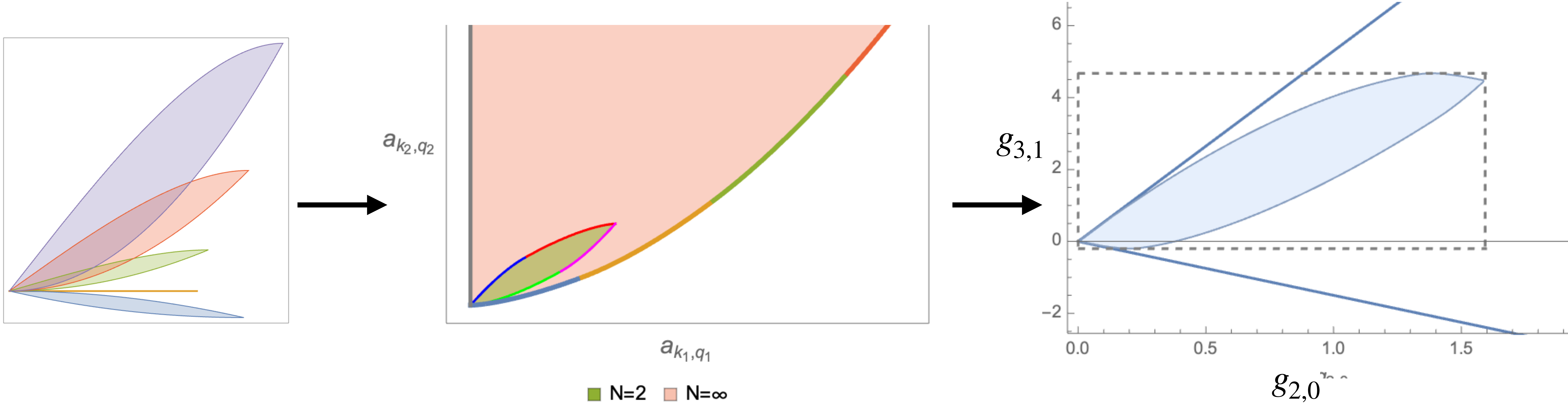
$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = L \begin{pmatrix} m \\ m^2/2 \end{pmatrix}$$





Linear unitarity and the non-projective EFThedron

$$a_{k,q} = \sum_J J^q \int_0^1 \rho(z) z^k dz$$



Non-perturbative, using linear unitarity

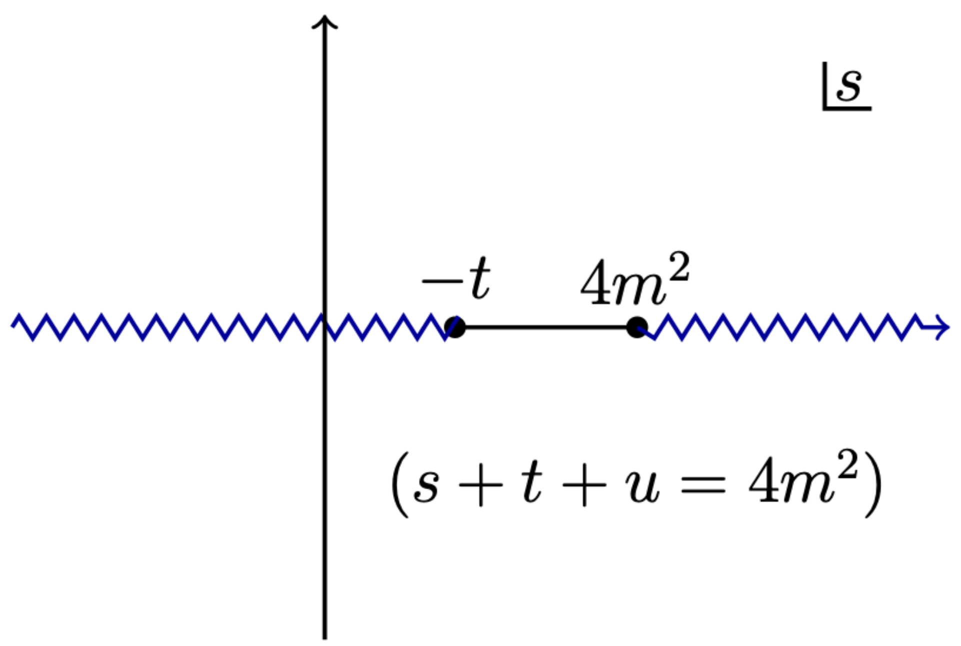
Unitarity: partial amplitudes satisfy $|S_J|^2 \leq 1$

Positivity: $Im[S_j] \geq 0$ (projective)

Linear unitarity: $Im[S_j] \leq 1$ (non-projective)

Beyond the EFThedron

Non-perturbative bounds from positivity and linear unitarity



$$\Lambda_{2,0} = \int_4^\infty ds'\rho(s')\frac{1}{(s'-2)^3} \qquad \Lambda_{4,0} = \int_4^\infty ds'\rho(s')\frac{1}{(s'-2)^5}$$

Positivity: A simple rescaling and change of variables:

$$\Lambda_{2,0} = \int_2^\infty \bar{\rho} w^2 dw \qquad \Lambda_{4,0} = \int_2^\infty \bar{\rho} dw$$

Projective bounds given in terms of Hankels, cyclic poltopes, etc, identical to EFThedron!

Linear unitarity: The same type of extremal solution gives the boundary : $\rho(s) \sim \chi_{[0,m]}$

Bounds obtained by Minkowski sum of L moments, identical to non-projective EFThedron

Full unitarity?

Unitarity: partial amplitudes satisfy $|S_J|^2 \leq 1$

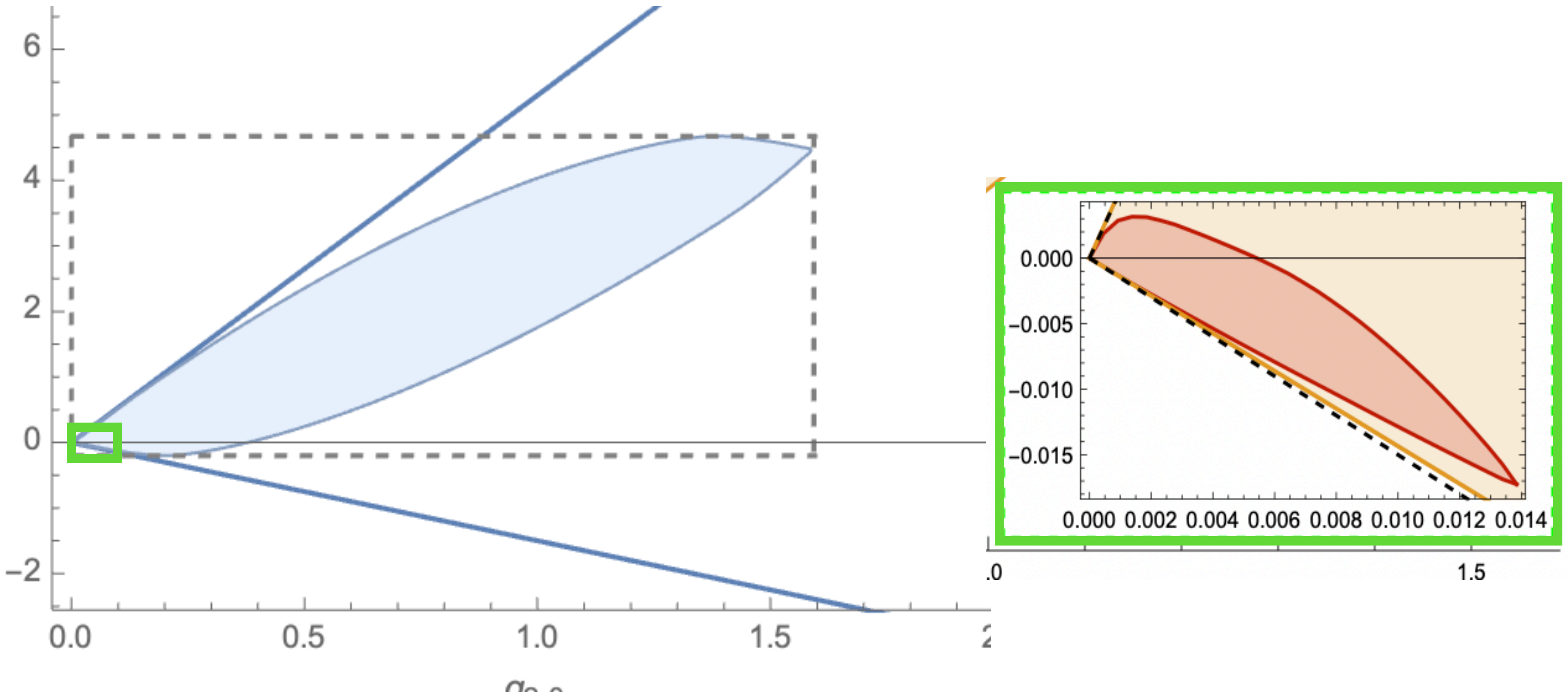
Positivity: $Im[S_j] \geq 0$ (projective)

Linear unitarity: $Im[S_j] \leq 1$ (non-projective)

Full unitarity

Dispersion relations $$a_{k,q} = \sum_J \int_0^1 \rho_J(z) z^k J^q dz$$

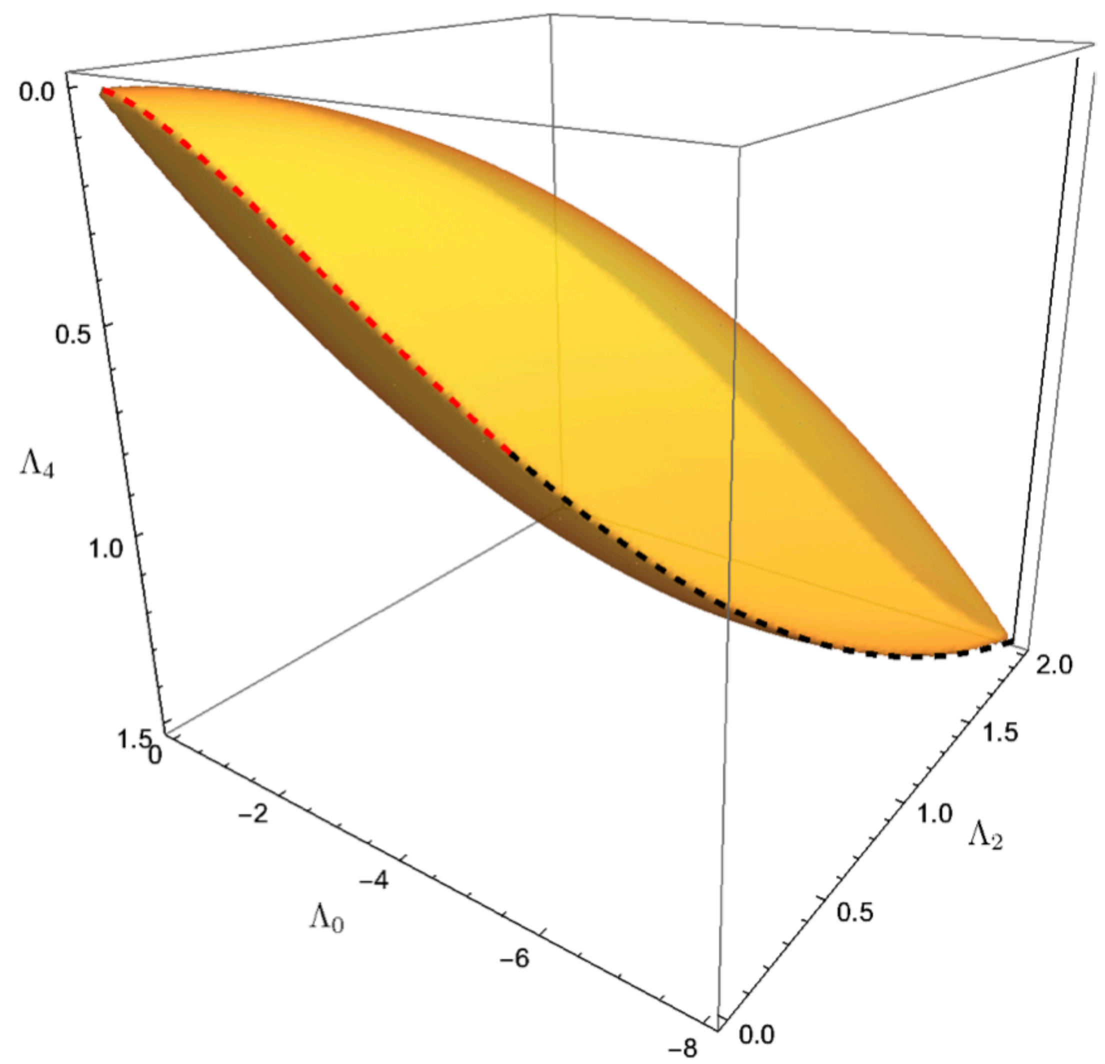
Unitarity $$0 \leq Im[S_J] = \rho_J(z) \leq 1, \quad |S_J|^2 \leq 1$$



[Chen, Fitzpatrick, Karateev ’22]

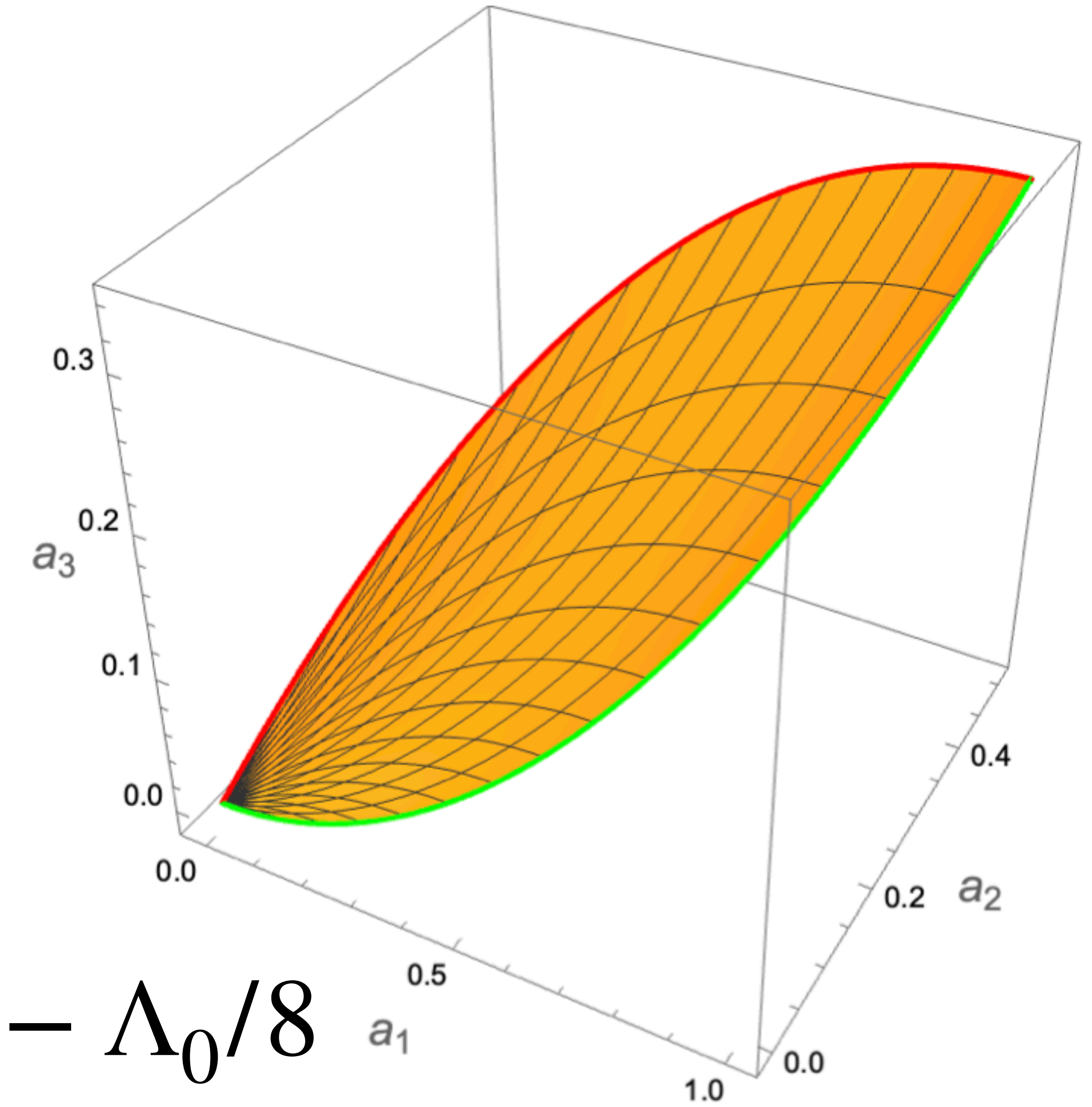
[Chen, Fitzpatrick, Karateev '22]

$$\left|8\left(3\Lambda_0^3+96\left(\Lambda_0+4\right)\Lambda_2^2+48\left(\Lambda_0+4\right)\Lambda_0\Lambda_2-16\left(\Lambda_0+8\right)\Lambda_0\Lambda_4\right)\right|$$
$$\leq 3\left(\Lambda_0^2-32\Lambda_2\right)\left(\Lambda_0\left(\Lambda_0+16\right)+32\Lambda_2\right)$$



L-moments:

$$a_k=\int_0^1 \rho(z)z^k, \quad 0\leq\rho(z)\leq 1$$



$$a_0 \to -\Lambda_0/8$$

$$a_1 \to \Lambda_2/4$$

$$a_2 \to \Lambda_0^3/6144-\Lambda_0^2/512-+\Lambda_2/4-\Lambda_4/12$$

In 2D, full non linear unitarity is a non-linear combination of solutions to linear unitarity

Outlook

- We have an analytic understanding of positivity and linear unitarity for non-perturbative regime, beyond perturbative EFT
- In 2D, the full unitarity condition can be obtained from linear unitarity. This suggest new types of dispersion relations must exist, in terms of |S|, not just Im[S].
- One could now also impose other constraints, such that |S|^2=1 for $s \in [4m^2, 16m^2]$. Avoid all sort of numerical stability, large spin issues, etc.
- Bonus! This machinery has applications in the CFT bootstrap

EFT hedron (space of EFT couplings)

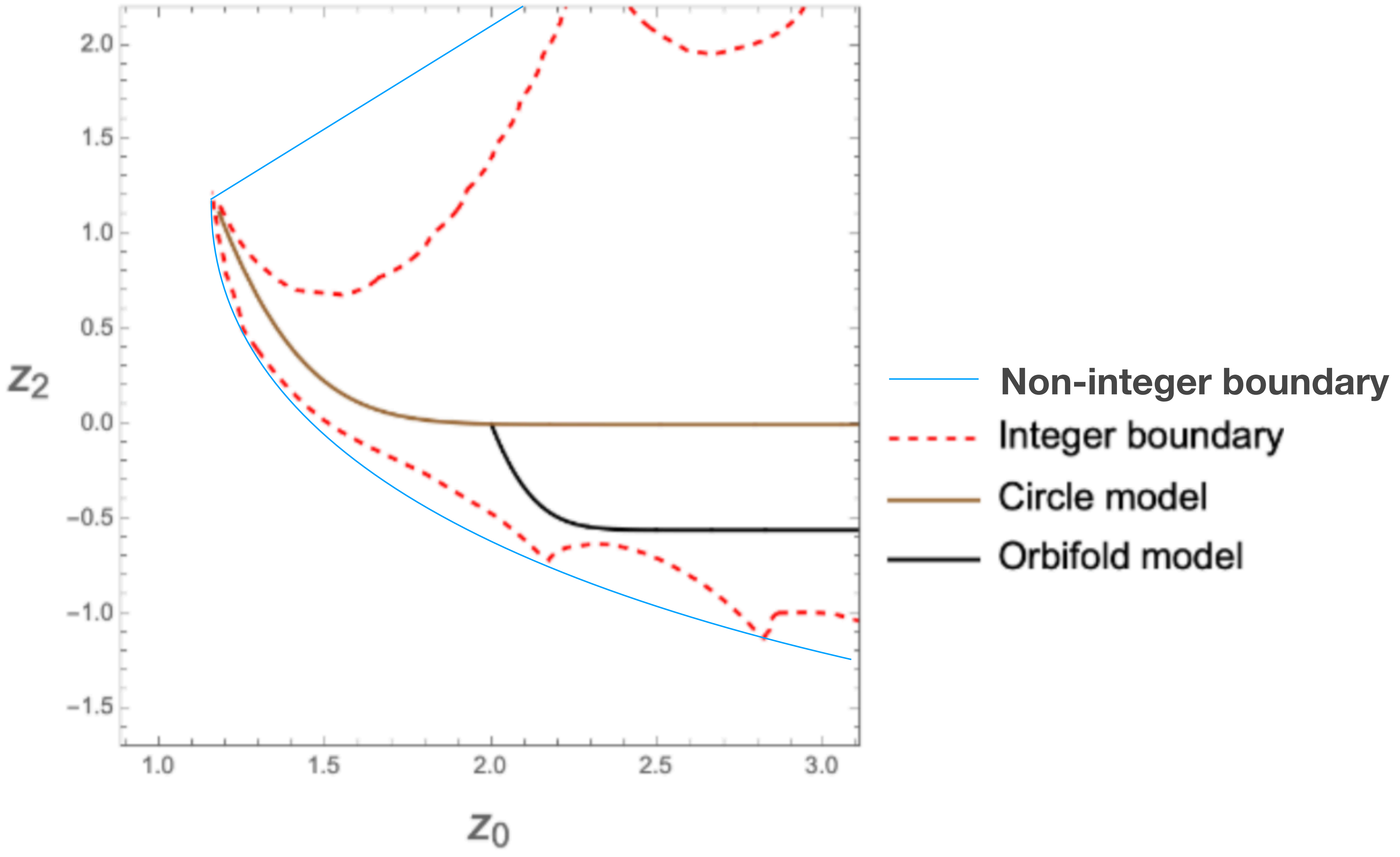
$$g_{k,q} = \sum p_i \, m_i^k \, J_i^q \qquad g_{k,q} + \ldots = 0$$

$p_i \geq 0 \quad m_i \geq M \quad J \in \mathbb{N}$

Modular hedron (space of modular partition functions)

$$z_{k,q} = \sum n_i e^{-\Delta_i} \, \Delta_i^k \, J_i^q \qquad z_{k,q} + \ldots. = 0$$

$n_i \geq 0,\ \in \mathbb{N} \quad \Delta_i \geq \Delta_{gap} \quad J \in \mathbb{N}$



Last chance!!