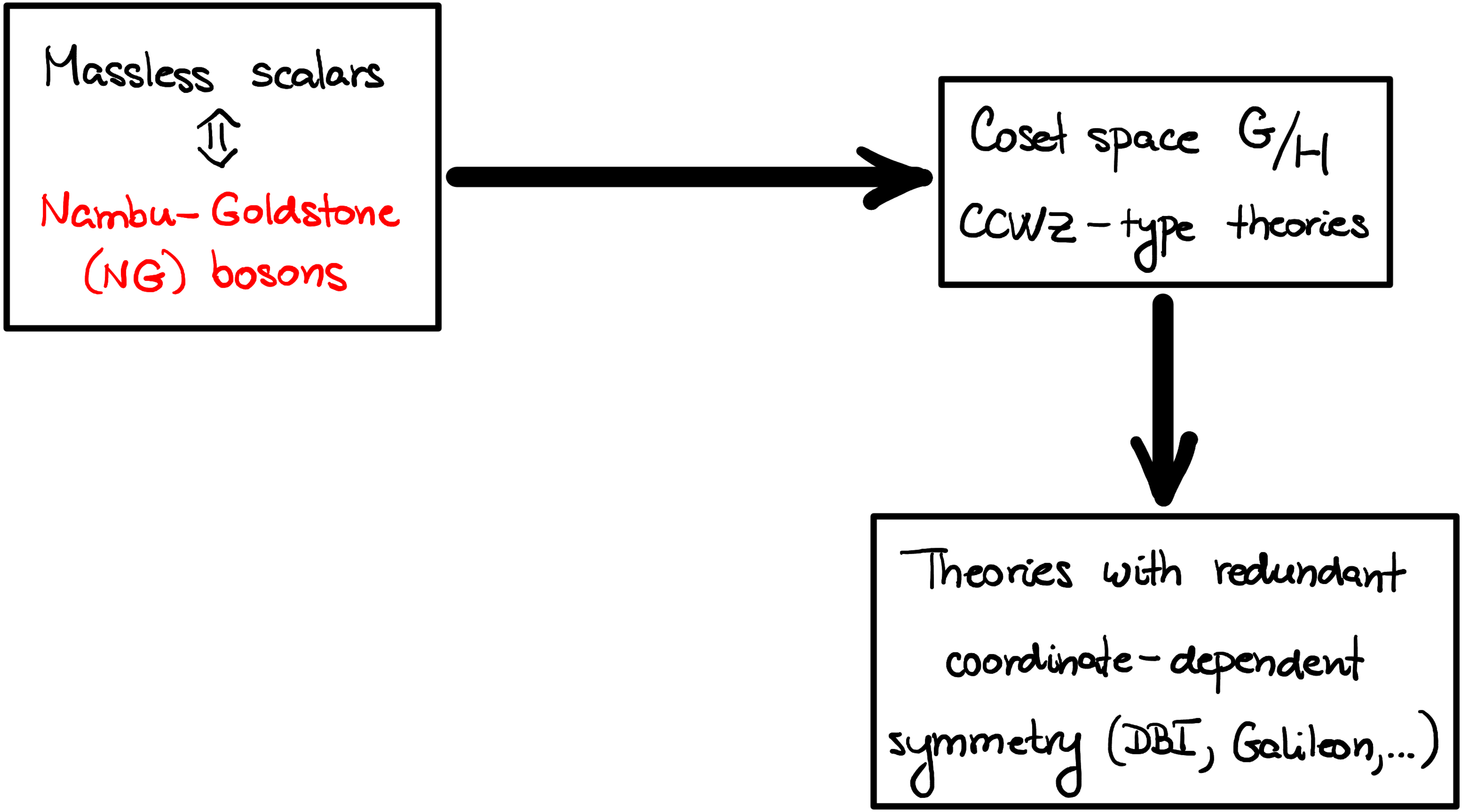
SCATTERING AMPLITUDES IN NONRELATIVISTIC EFT

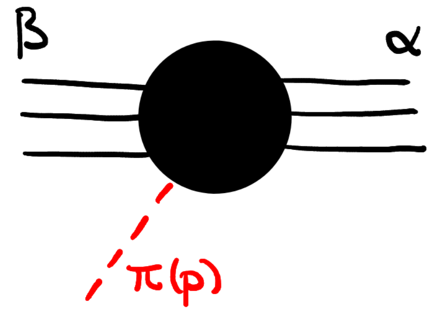
PROLOGUE

RELATIVISTIC EFTS



SOFT LIMIT OF SCATTERING AMPLITUDES

$$\mathcal{A}_{\alpha\to\beta+\pi(p)} =$$



Asymptotic behavior of the amplitude :

$$\mathcal{A}_{\alpha\to\beta+\pi(p)} = \# p^{\sigma} + \dots$$

soft scaling parameter

- $\sigma = 0$: unexpected exception?
- $\sigma = 1$: Adler zero (default)
- $\sigma \geqslant 2$: enhanced soft limit

UNEXPECTED EXCEPTIONS?

CCWZ-type EFTs : $\mathcal{L}_{\text{eff}} = \frac{1}{2} g_{ab}(\pi)\, \partial_\mu \pi^a \partial^\mu \pi^b$

NG fields = coordinates on G/H

G-invariant metric on G/H

$$\lim_{p\to 0} \sum_b F_a^b \mathcal{A}_{\alpha\to\beta+\pi^b(p)} = \frac{1}{2} \sum_{b\in\alpha\cup\beta} \sum_c \left(f_{cab} - f_{bac} - f_{abc}\right) \mathcal{A}^{b\to c}_{\alpha\to\beta}$$

$\langle 0 | J^\mu_a(0) | \pi^b(\vec{p})\rangle = -ip^\mu F_a^b$

structure constants of G

Main conclusion : $\boxed{\text{Adler zero} \iff G/H \text{ is symmetric}}$

Kampf, Novotný, Shifman, Trnka (2020)

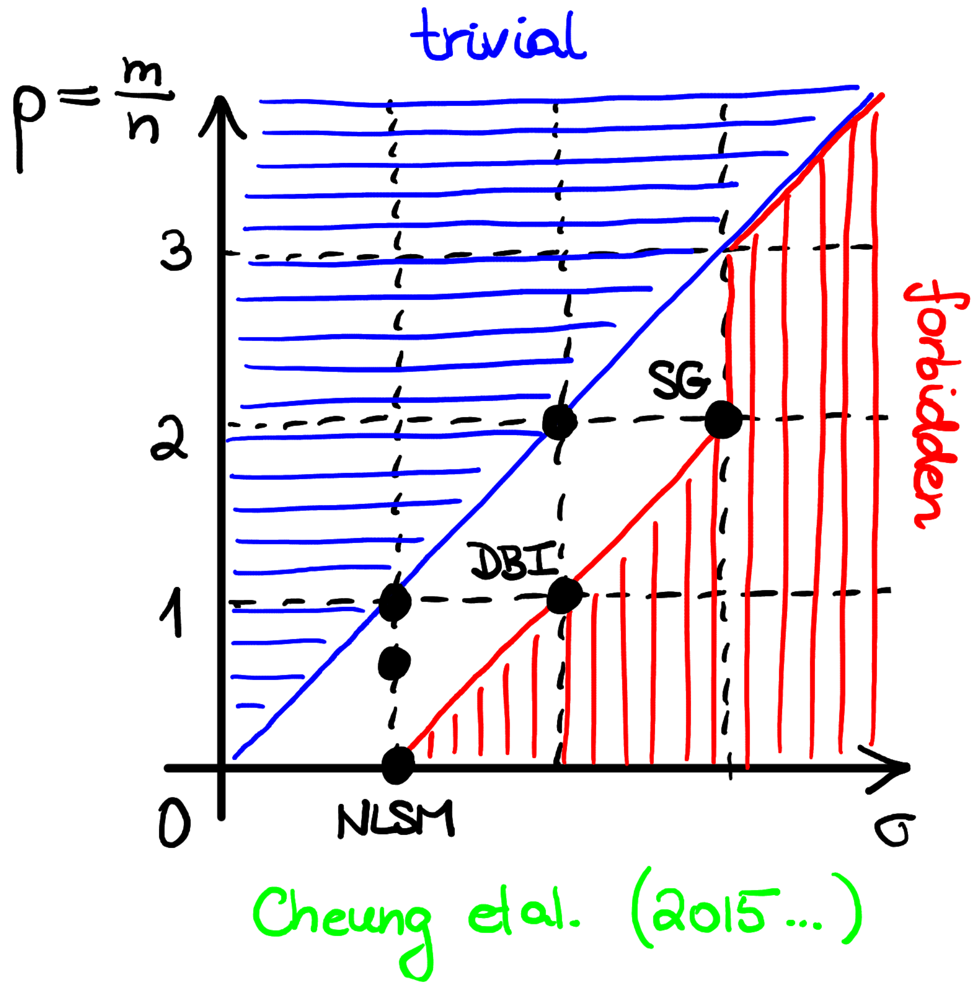
Cheung, Helset, Parra-Martinez (2022)

BEYOND ADLER ZERO

EFTs of a single NG boson: $\mathcal{L}_{\text{eff}} \sim (\partial\pi)^2 \sum_{m,n=0}^{\infty} c_{m,n}\, \partial^m \pi^n$

Not all values of $\underline{\sigma}$ are allowed.

Exceptional theories : maximize $\underline{\sigma}$ for given ρ.



- NLSM : $(\rho,\sigma) = (0,1)$
- DBI : $(\rho,\sigma) = (1,2)$
- SG : $(\rho,\sigma) = (2,3)$

FINDING NEW THEORIES

Bottom-up

soft bootstrap

Top-down

symmetry classification

Cheung et al. (2015...)

Elvang et al. (2018...)

Bogers, TB (2018)

Roest, Stefanyszyn, Werkman (2019)

Importance of coordinate-dependent symmetries: invariance under

$$\delta_\epsilon \pi(x) = \epsilon_{\mu_1 \dots \mu_n} \left[x^{\mu_1} \dots x^{\mu_n} + f_A^{\mu_1 \dots \mu_n}(x)\, \mathcal{O}^A[\pi](x) \right]$$ implies $\sigma = n+1$

Cheung et al. (2017)

NONRELATIVISTIC NG BOSONS

MOTIVATION

- Some kind of NG boson exists in all phases of matter.
- Scattering (amplitudes) also important in a medium.
- Nonrelativistic gravity / string theory ?

CLASSIFICATION OF NG BOSONS

Type	Dispersion relation	Propagator
A_m	$\omega \sim \lvert\vec{p}\rvert^m$	$\dfrac{1}{\omega^2 - \vec{p}^{\,2m}}$
B_{2m}	$\omega \sim \lvert\vec{p}\rvert^{2m}$	$\dfrac{1}{\omega - \vec{p}^{\,2m}}$

TB, Watanabe, Murayama, Hidaka (2011–2012)

Griffin, Grosvenor, Hořava, Yan (2015)

CONSTRAINTS ON THE SPECTRUM

Type	Dispersion relation	Allowed dimension ($T=0$)	Allowed dimension ($T>0$)
A_m	$\omega \sim \lvert\vec{p}\rvert^{m}$	$D \geqslant m+2$	$D \geqslant 2m+2$
B_{2m}	$\omega \sim \lvert\vec{p}\rvert^{2m}$	any $D \geqslant 2$	$D \geqslant 2m+2$

Mermin, Wagner (1966)
Hohenberg (1967)
Coleman (1973)

Griffin, Grosvenor, Hořava, Yan (2013–2015)
Watanabe, Murayama (2014)
Argurio, Naegels, Pasternak (2019)

MASSIVE NG BOSONS

System with chemical potential μ : excitation energy = eigenvalue of $\tilde{H} = H - \mu Q$.

$$[Q, Q_i^{\pm}] = \pm q_i Q_i^{\pm}$$

& $Q_i^{\pm}$ spontaneously broken

state in the spectrum with $\lim_{\vec{p}\to\vec{0}} \omega(\vec{p}) = \mu q_i$

- Exact gap protected by symmetry.
- Interactions dictated by symmetry.
- Scattering amplitudes with Adler zero.

Nicolis, Piazza (2012)

Watanabe, TB, Murayama (2013)

TB, Jakobsen (2018)

ADLER ZERO FOR NG BOSONS?

Generic shift-invariant EFT for a single type-A_1 NG boson:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu\pi)^2 + c_1(\partial_0\pi)^3 + c_2\,\partial_0\pi\,(\vec{\nabla}\pi)^2 + \dots$$

- Such cubic vertices cannot be removed by field redefinition!
- Violates Adler zero unless $c_2 = -c_1$:

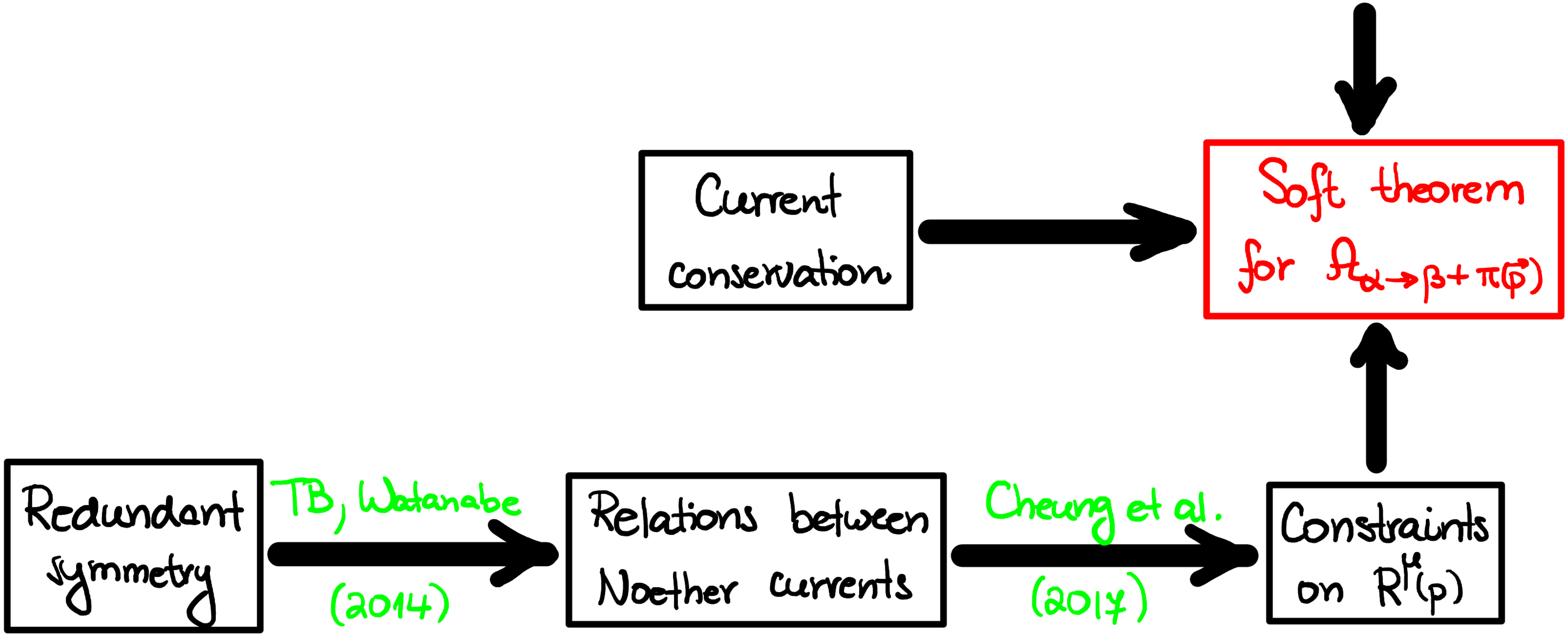
$$\boxed{\mathcal{L}_{\text{eff}} \longrightarrow \frac{1}{2}(\partial_\mu\pi)^2 + c_1\,\partial_0\pi\,(\partial_\mu\pi)^2 + \dots}$$

General criterion for Adler zero in terms of symmetry not available yet.

ENHANCED SOFT LIMIT

General approach to Adler zero and its enhancement :

$$\boxed{\langle\beta|J^\mu(0)|\alpha\rangle = \langle 0|J^\mu(0)|\pi(\vec{p})\rangle \frac{i}{p^0-\omega(|\vec{p}|)} \mathcal{A}_{\alpha\to\beta+\pi(\vec{p})} + R^\mu(p)}$$



ENHANCED SOFT LIMIT

Invariance under spatial polynomial shift symmetry :

$$\delta_\epsilon \pi(x) = \epsilon_{i_1 \dots i_n} \left[x^{i_1} \dots x^{i_n} + f_A^{i_1 \dots i_n}(x)\, \mathcal{O}^A[\pi](x) \right]$$

type-A_m NG bosons

$\sigma = \min(m, n+1)$

type-B_{2m} NG bosons

$\sigma = \min(2m, n+1)$

Mojahed, TB (2022)

SOFT BOOTSTRAP

Mojahed, TB (2021-2022)

NAIVE LAGRANGIAN SCAN – TYPE A_1

Theories with $p=1$ (one derivative per field):

two-parametric family of theories with $\sigma=2$

$$\mathcal{L}_{\text{eff}} \sim \sqrt{(1-c_1\partial_0\pi)^2 - c_2\left[(\partial_0\pi)^2-(\vec{\nabla}\pi)^2\right]}$$

Special cases

- $c_1=0$: relativistic DBI
- $c_1=c_2=1$: $\mathcal{L}_{\text{eff}} \sim \sqrt{1-2\partial_0\pi+(\vec{\nabla}\pi)^2}$

  Galilei-invariant superfluid

Tree-level amplitudes independent of $\underline{c_1}$ and equal to those of the relativistic DBI theory.

Emergent Lorentz invariance!

Soft Recursion – Type A_1

Seed 4-point amplitudes: polynomials in $\underbrace{s, t, u}_{\text{relativistic Mandelstams}}$, $\underbrace{\omega_a}_{\text{energies}}$ $(a = 1, \dots, 4)$

Only three amplitudes consistent with recursion:

- $s^2 + t^2 + u^2$ … relativistic DBI $(\sigma = 2)$
- $s^3 + t^3 + u^3$ … relativistic special Galileon $(\sigma = 3)$
- $s^2(\omega_1\omega_2 + \omega_3\omega_4) + t^2(\omega_1\omega_3 + \omega_2\omega_4) + u^2(\omega_1\omega_4 + \omega_2\omega_3)$

  … "spatial Galileon" $(\sigma = 2)$

Exceptional theories are automatically Lorentz-invariant: DBI & special Galileon.

There are no exceptional type-A_1 theories with $\underline{\sigma > 3}$.

NAIVE LAGRANGIAN SCAN – TYPE B_2

Included $p=1$ theories of the type $\mathcal{L}_{\text{eff}} = \psi^\dagger(i\partial_0 + \vec{\nabla}^2)\psi + \mathcal{L}_{\text{int}}(\partial\psi, \partial\psi^\dagger)$.

The only theory in this class with $\sigma=2$ is the Schrödinger–DBI theory:

$$\mathcal{L}_{\text{eff}} = \psi^\dagger i\partial_0\psi + s\sqrt{1-2s\vec{\nabla}\psi^\dagger\cdot\vec{\nabla}\psi + (\vec{\nabla}\psi^\dagger\cdot\vec{\nabla}\psi)^2 - |\vec{\nabla}\psi\cdot\vec{\nabla}\psi|^2}$$

- $s=+1$
  ISO$(d+2)$ spatial symmetry
- $s=-1$
  ISO$(d,2)$ spatial symmetry

$\psi, \psi^\dagger$ are fluctuations of a d-dimensional brane embedded in a $(d+2)$-dimensional pseudo-Euclidean space

SOFT RECURSION – TYPE B_2

Seed 4-point amplitudes : polynomials in nonrelativistic Mandelstams $s_{ab} = \vec{p}_a \cdot \vec{p}_b$

Only three amplitudes consistent with recursion :

$$\left.\begin{array}{ll} \bullet\ s_{13} + s_{24} \ \ldots & \mathbb{CP}^1 \text{ NLSM } (\sigma=1), \text{ ferromagnetic spin waves} \\ \bullet\ (s_{13} + s_{24})^2 \ \ldots & \text{Schrödinger-DBI } (\sigma=2) \end{array}\right\} \text{exceptional}$$

- lengthy $O(s_{ab}^3)$ expression … "Schrödinger – Galileon" $(\sigma=2)$

There are no exceptional type-B_2 theories with $\underline{\sigma > 2}$.

EPILOGUE

SOME MORALS

- Adler zero and its enhancements, soft recursion & bootstrap are straightforward to generalize to nonrelativistic EFTs.
- Top-down symmetry approach & bottom-up bootstrap complement each other.

easy generation of concrete theories, see Mojahed, TB (2022) for a technically natural type-B_4 theory

easy elimination of large parts of EFT parameter space

- Some surprises in nonrelativistic EFTs, and even more in theories of nonscalar particles: fractional scaling of amplitudes

TB, Esposito, Penco (2022)