Mechanization of scalar field theory

a gradual exploration of soliton dynamics in one-dimensional scalar field theories

Prague Spring Amplitudes Workshop, 18.05. 2023

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PART I:
Scattering of kinks in a field theory

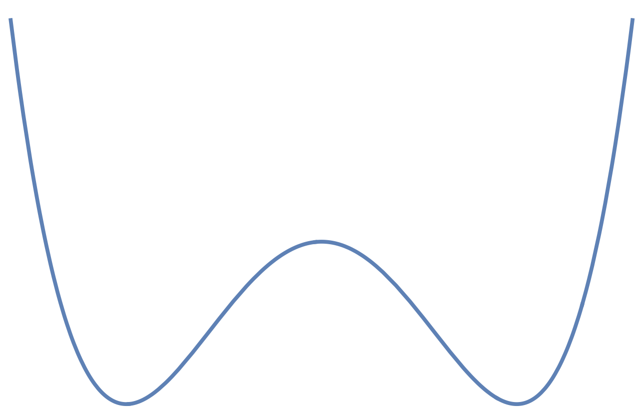
Playground

Classical scalar field theory in (1+1)-dimensions

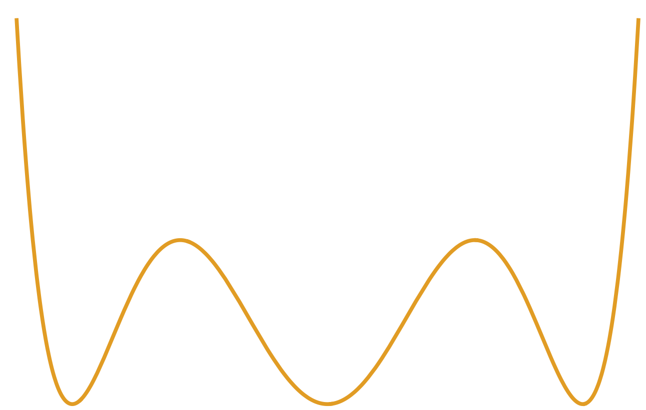
$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

Discrete vacua: Kinks

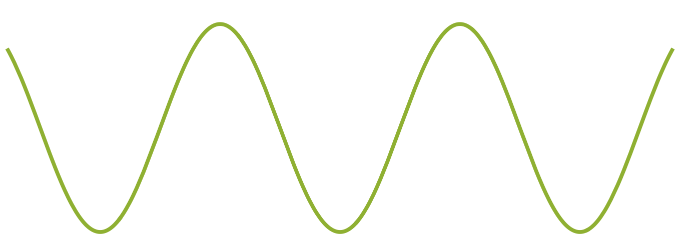
$$V(\phi) = \left\{ \frac{1}{2}(1-\phi^2)^2\,,\ \frac{1}{2}\phi^2(1-\phi^2)^2\,,\ 2\sin^2(\phi/2)\,,\ \dots \right\}$$



$$\phi_K = \tanh(x)$$



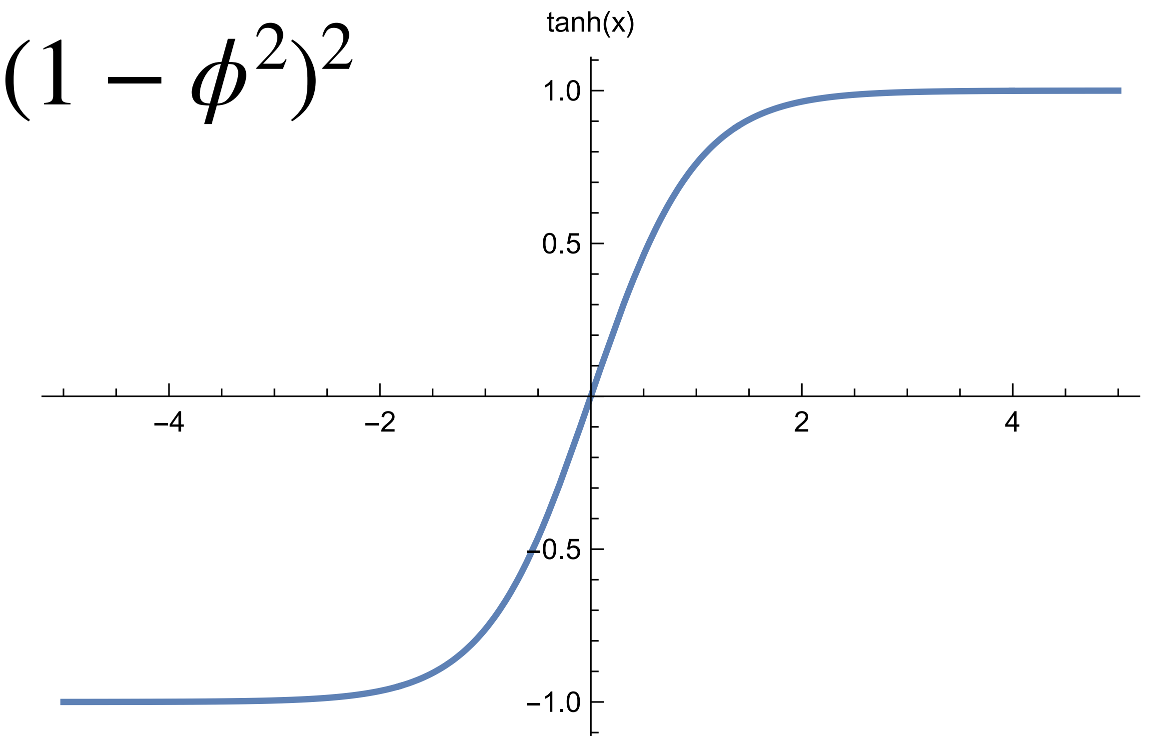
$$\phi_K = \frac{e^x}{\sqrt{1+e^{2x}}}$$



$$\phi_K = 4\arctan\left(e^x\right)$$

Canonical kink

$$V(\phi) = \frac{1}{2}(1 - \phi^2)^2$$



$$\phi_K = \tanh\left(\frac{x - x_0 - vt}{\sqrt{1 - v^2}}\right)$$

$$E = M\gamma \quad P = M\gamma v \quad M = 4/3$$

$$j^\mu = \frac{1}{2}\varepsilon^{\mu\nu}\partial_\nu\phi \qquad \partial_\mu j^\mu = 0$$

$$Q = \int dx\, j^0 = \frac{\phi(\infty) - \phi(-\infty)}{2}$$

Behaves as a relativistic massive particle. Stability guaranteed by conservation of topological charge.

Normal modes

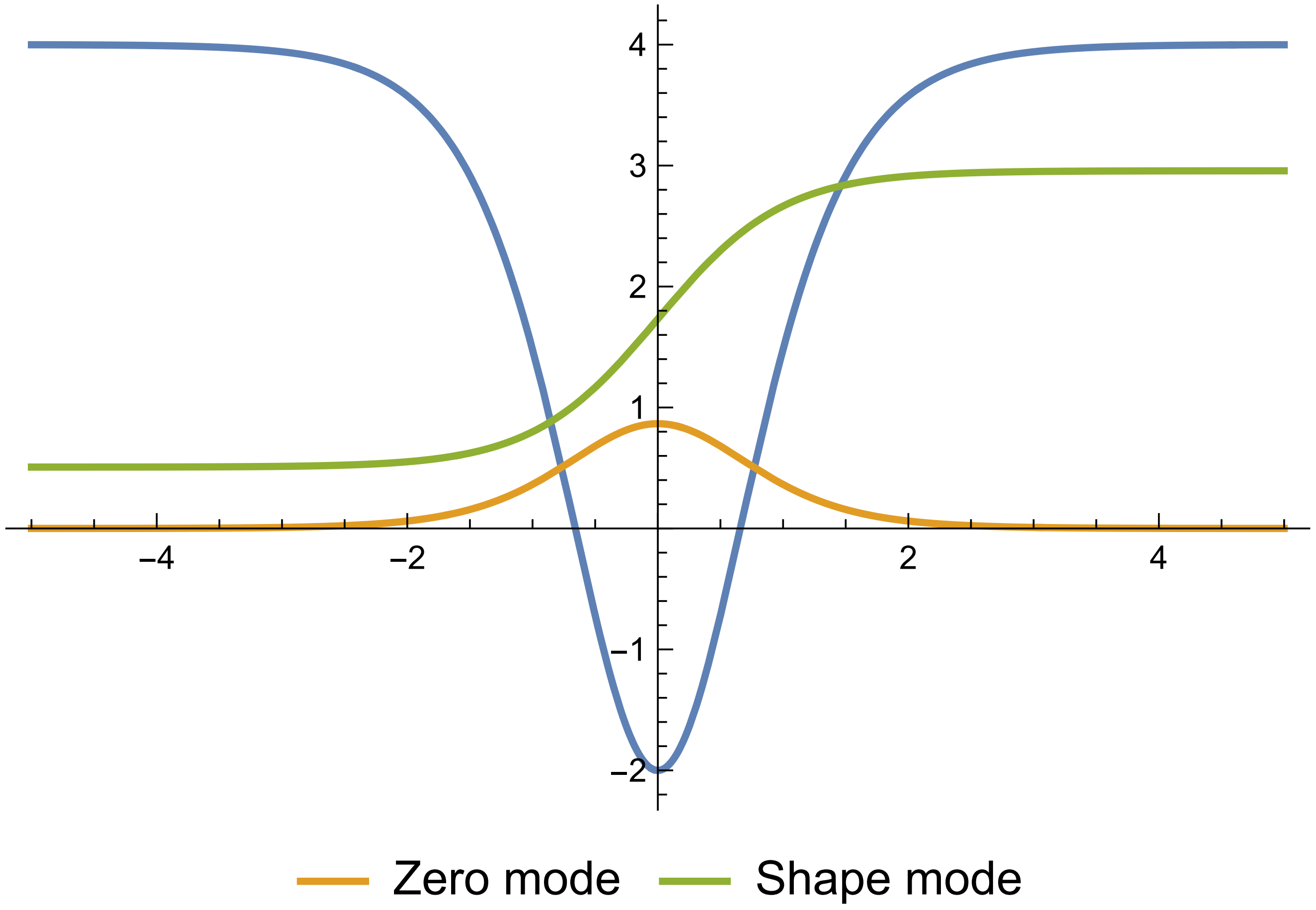
$$\phi = \tanh(x) + e^{-i\omega t} b(x) \qquad -\partial_x^2 b + \left(4 - \frac{6}{\cosh^2(x)}\right) b = \omega^2 b$$

Zero mode:

$$b_0(x) = \sqrt{\frac{3}{4}} \frac{1}{\cosh^2(x)} \qquad \omega_0^2 = 0$$

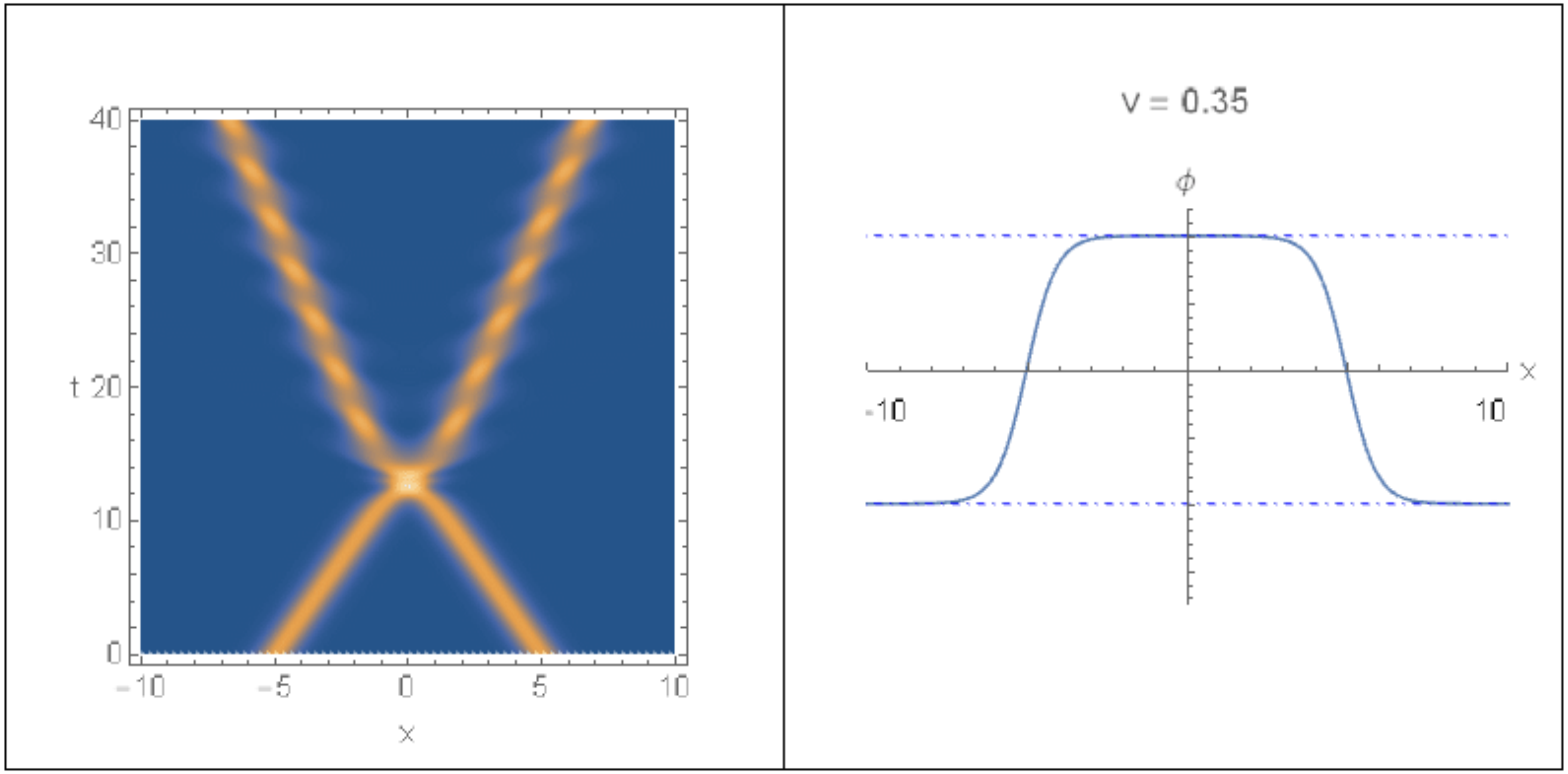
Vibration (shape) mode:

$$b_1(x) = \sqrt{\frac{3}{2}} \frac{\sinh(x)}{\cosh^2(x)} \qquad \omega_1^2 = 3$$



Scattering of kinks

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\left(1-\phi^2\right)^2$$



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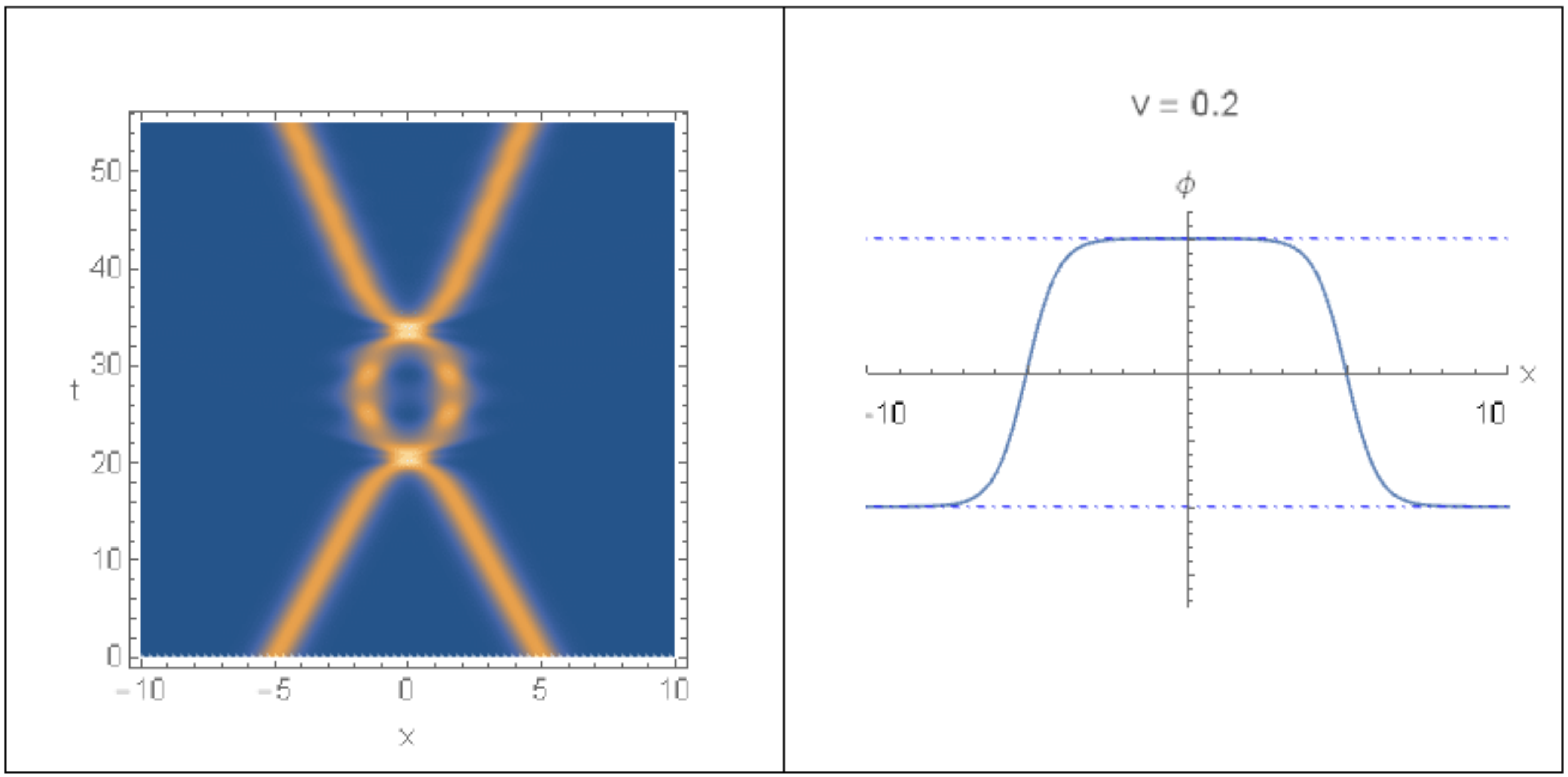

Scattering of kinks

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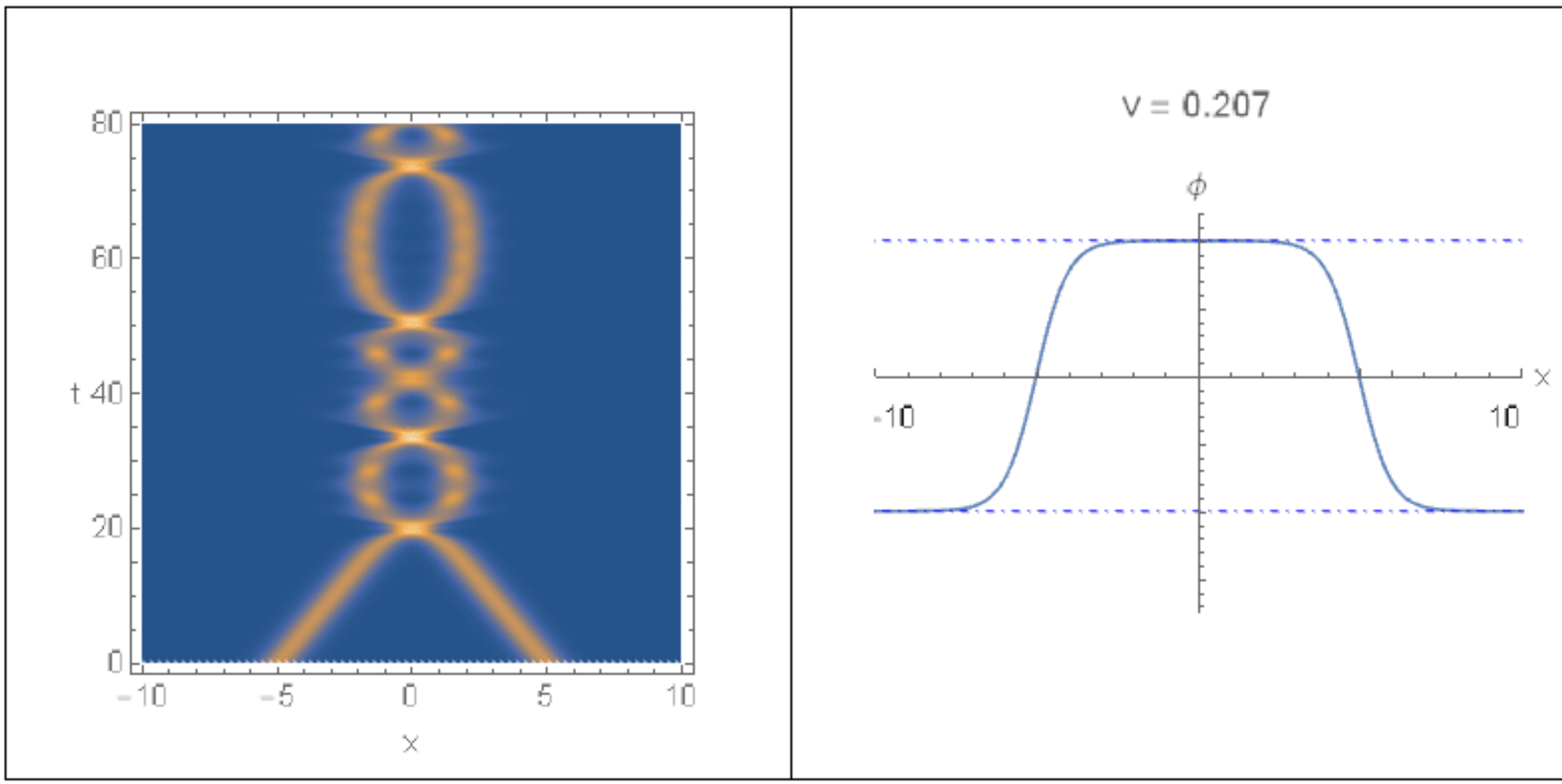
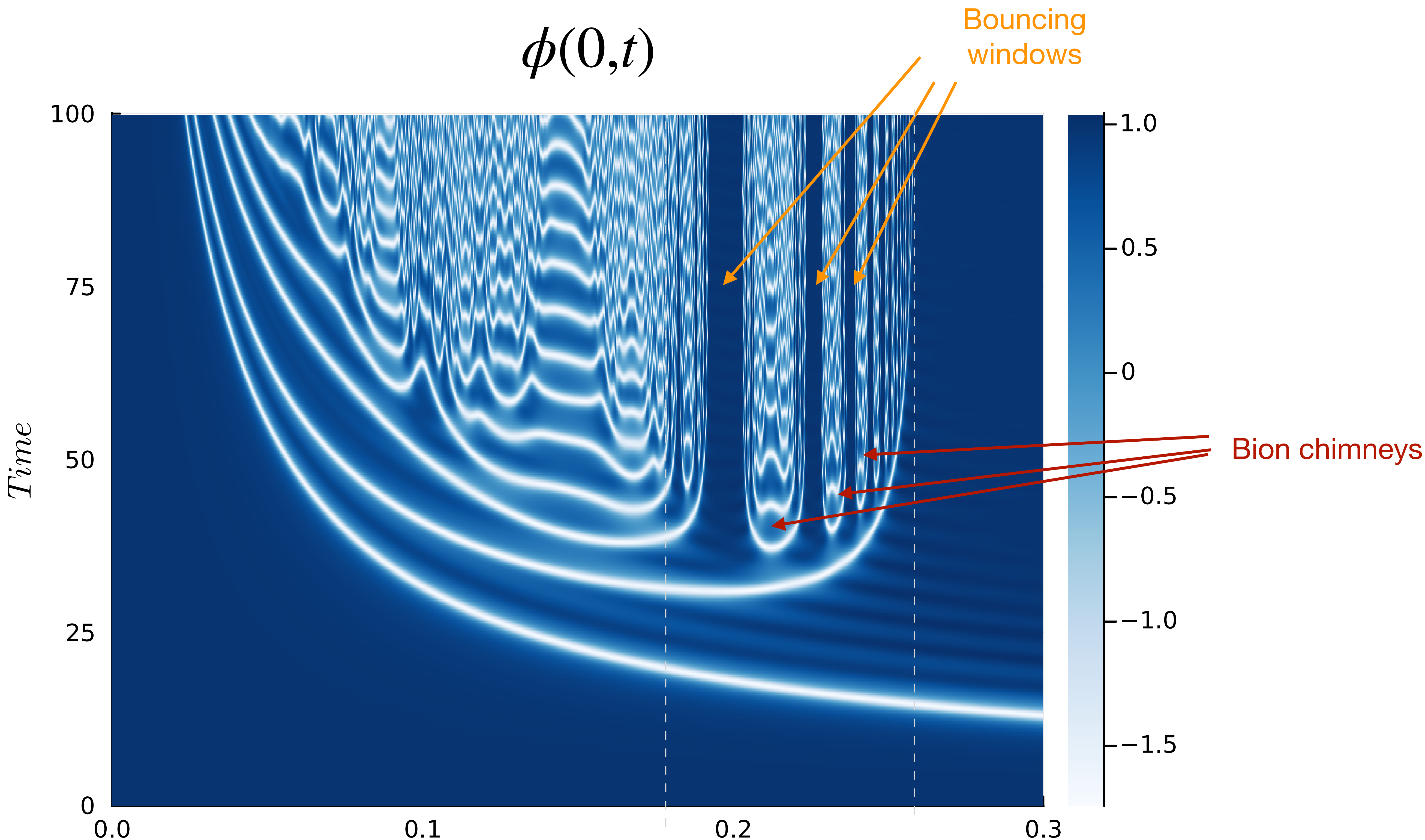


A `map’ of scatterings



Minimal velocity: $v_{\mathrm{min}} \approx 0.18$

Critical velocity: $v_{\mathrm{crit}} \approx 0.26$

Resonance at the heart of the collision

$$\phi_{\text{bkg}} = \tanh(x + X(t)) - \tanh(x - X(t)) - 1 + A(t)\big(b_1(x + X(t)) + b_1(x - X(t))\big)$$

Qualitative understanding of bouncing as resonant energy-transfer

RESONANCE STRUCTURE IN KINK–ANTIKINK INTERACTIONS IN ϕ^4 THEORY

David K. CAMPBELL†, Jonathan F. SCHONFELD††, and Charles A. WINGATE†

Received 29 November 1982

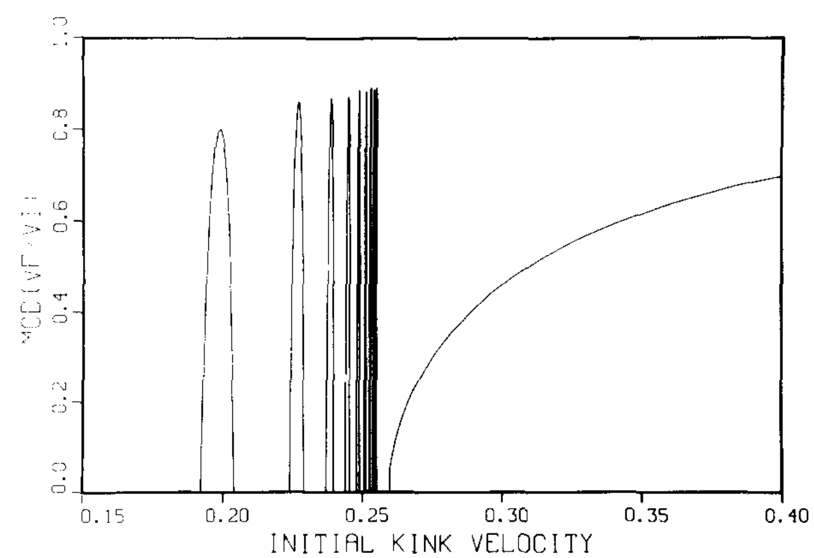


Fig. 8. The ratio of the (time-averaged – see section 4) kink speed after a $K\bar{K}$ collision to the initial speed, as a function of the initial velocity. Note the relatively elastic nature of the reflections below v_c.

Four Decades of Kink Interactions in Nonlinear Klein-Gordon Models: A Crucial Typo, Recent Developments and the Challenges Ahead

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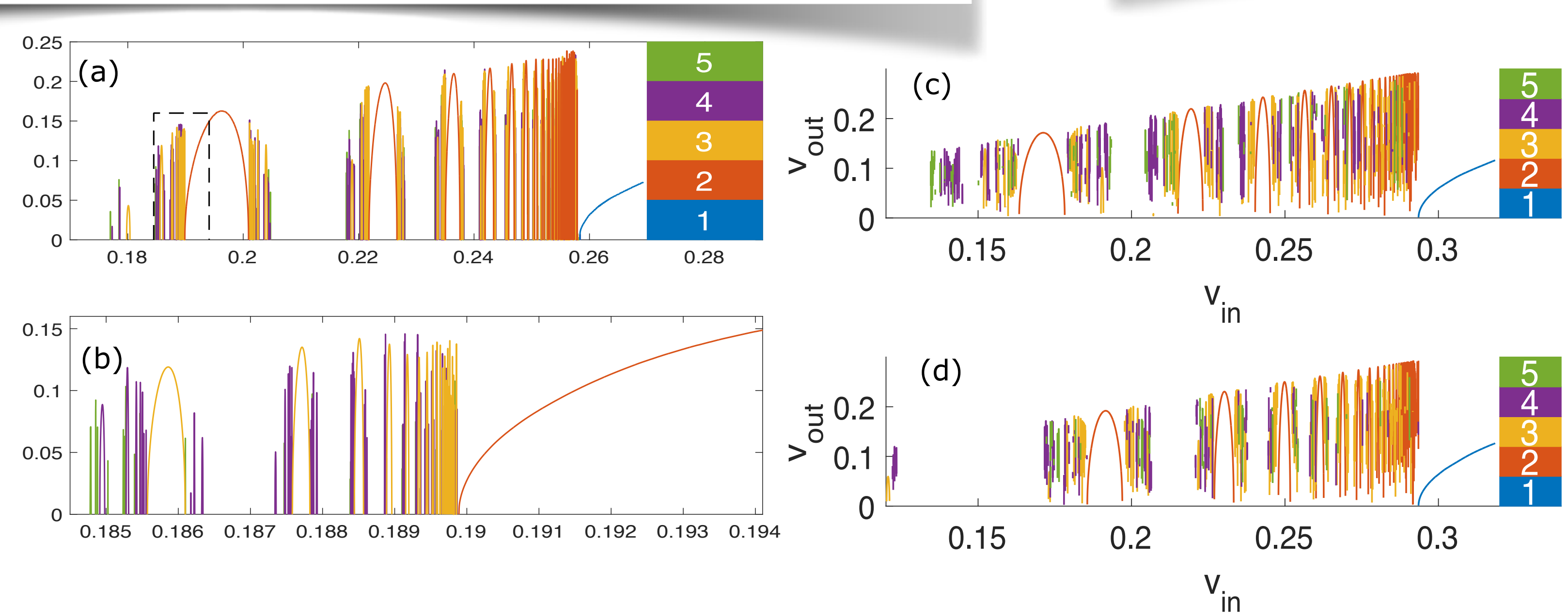


Collective Coordinate Models

Field theory:
Too much degrees
of freedom

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

Idea: curtail the
configuration space

$$\phi \to \phi_{\mathrm{bkg}}\big(x, \{X_a(t)\}\big)$$

Mechanics of
particles on a
curved Moduli
space with a
potential

$$L_{\mathrm{eff}} \equiv \int_{-\infty}^{\infty} \mathrm{d}x\, \mathscr{L} = \frac{1}{2}g_{ab}\dot{X}_a\dot{X}_b - U(X)$$

$$g_{ab} = \int_{-\infty}^{\infty} \mathrm{d}x\, \frac{\partial\phi_{\mathrm{bkg}}}{\partial X_a}\frac{\partial\phi_{\mathrm{bkg}}}{\partial X_b}$$

$$U(X) = \int_{-\infty}^{\infty} \mathrm{d}x\Big(\frac{1}{2}\phi'^2 - V(\phi)\Big)$$

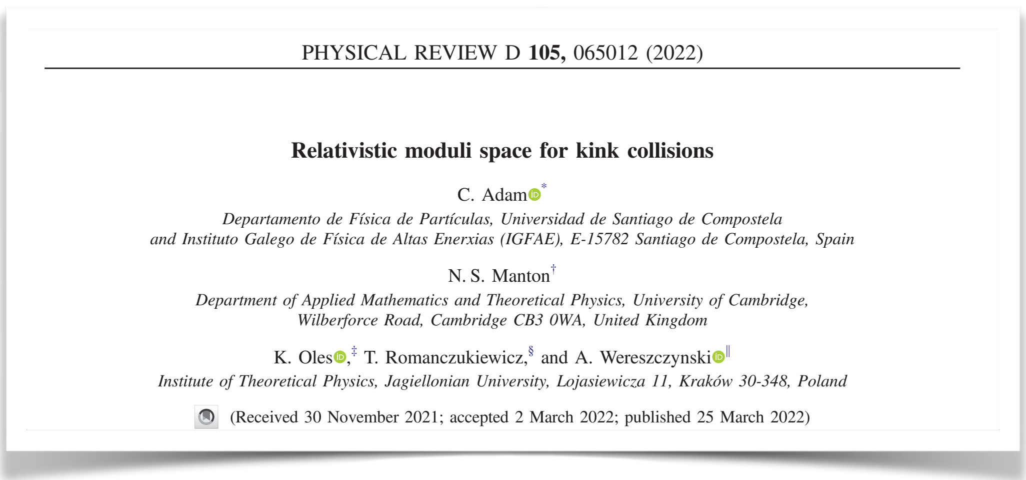
No obvious way how to do this and not make a mess.

Collective Coordinate Models

Idea #1: Engineering CCM:
Sculpting the configuration space so that only the most relevant configurations for a given problem are included.

Applied with great success to KK scattering in various models

$$V(\phi) = \left\{ \frac{1}{2}\left(1-\phi^2\right)^2,\ \frac{1}{2}\phi^2\left(1-\phi^2\right)^2,\ 2\sin^2(\phi/2) \right\}$$


PHYSICAL REVIEW D **105,** 065012 (2022)

Relativistic moduli space for kink collisions

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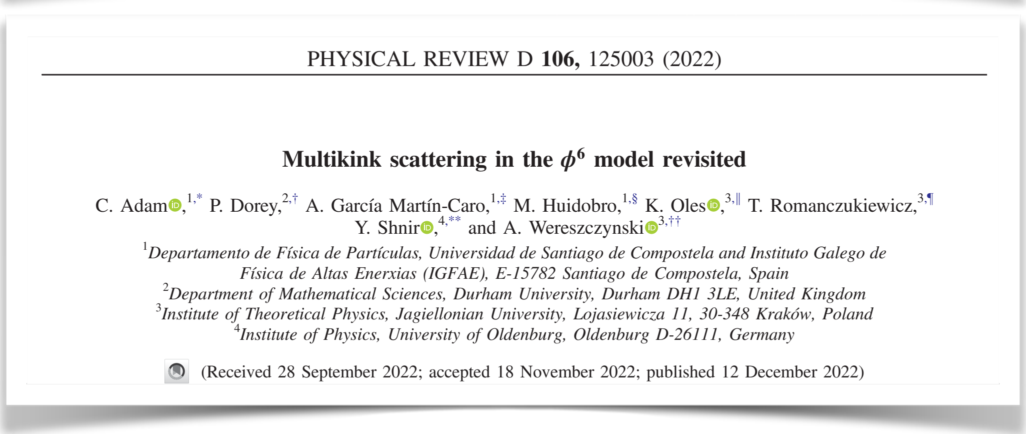
PHYSICAL REVIEW D **106,** 125003 (2022)

Multikink scattering in the ϕ^6 model revisited

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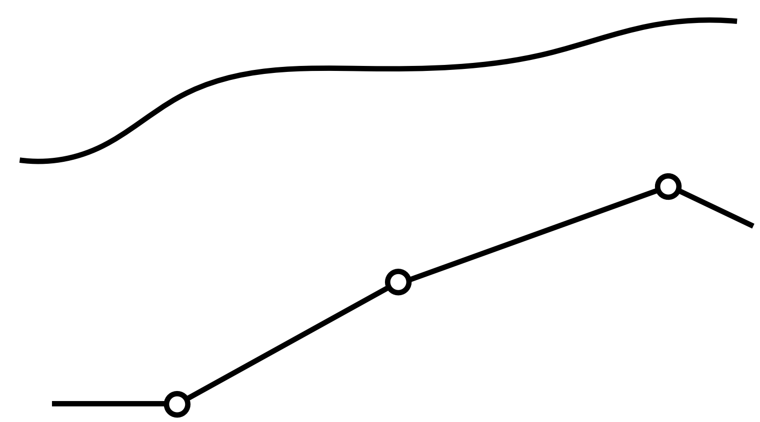
Tailor-made tools for specific problems.

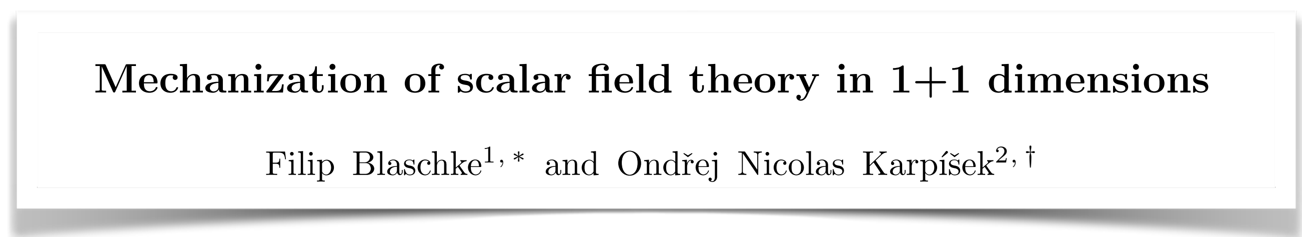
Idea #2: Agnostic CCM:
Triangulating the entire configuration space. No prior insight. General tool.

Mechanization:

$\phi(x,t) \equiv$

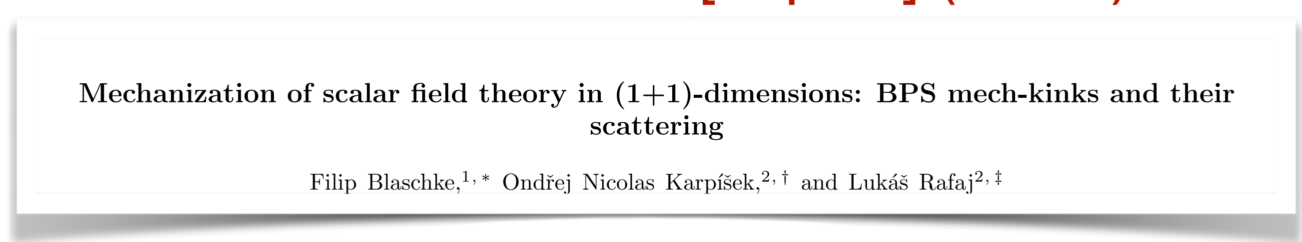
$\phi_M(x, X_a) \equiv$




Mechanization of scalar field theory in 1+1 dimensions

Filip Blaschke[1,*] and Ondřej Nicolas Karpíšek[2,†]

arXiv:2202.05675 [hep-th] (PTEP)

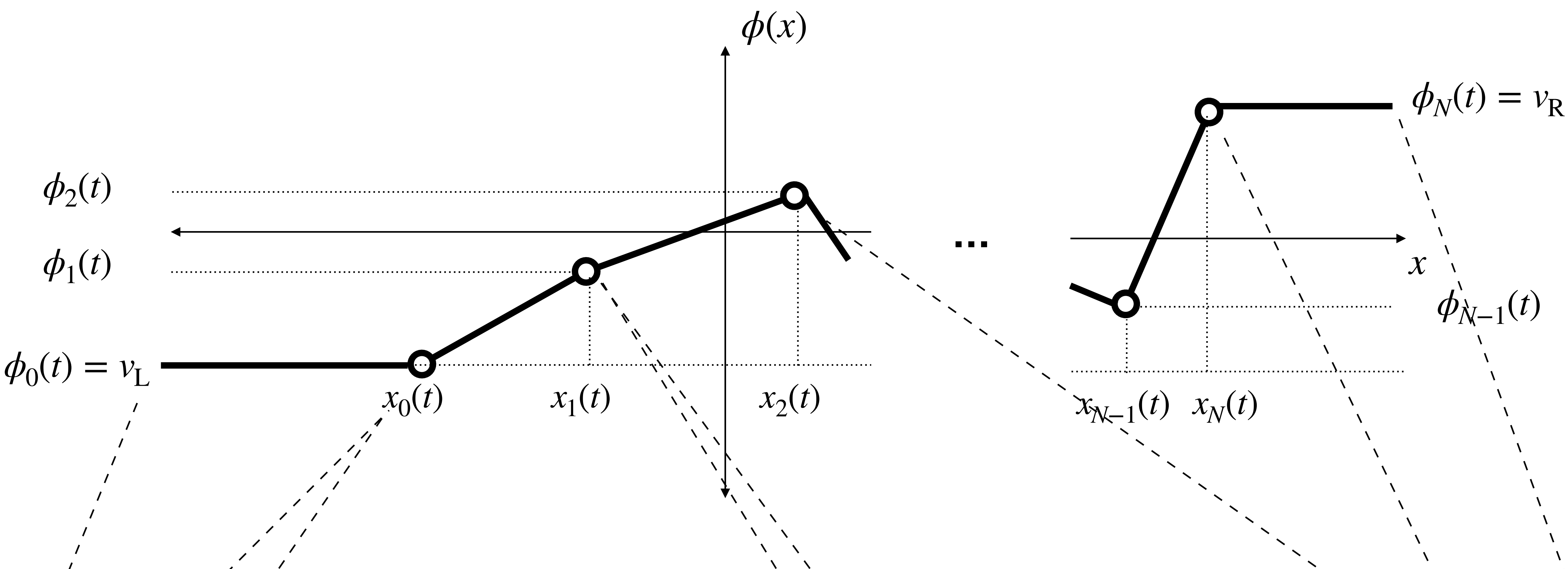

Mechanization of scalar field theory in (1+1)-dimensions: BPS mech-kinks and their scattering

Filip Blaschke,[1,*] Ondřej Nicolas Karpíšek,[2,†] and Lukáš Rafaj[2,‡]

arXiv:2305.09814 [hep-th]

General-purpose CCM ?

Mechanization



$$\phi_M = v_L\theta(-x+x_0) + \big(\theta(x-x_0)-\theta(x-x_1)\big)\left(\frac{\phi_1 - v_L}{x_1 - x_0}(x-x_0)+v_L\right) + \big(\theta(x-x_1)-\theta(x-x_2)\big)\left(\frac{\phi_2-\phi_1}{x_2-x_1}(x-x_1)+\phi_1\right) + \ldots + v_R\theta(x-x_N)$$

$X = \{x_0, x_1, \ldots\ x_N, \phi_1, \phi_2, \ldots\ \phi_{N-1}\}$ Moduli space with *2N* dimensions

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \longrightarrow L_M = \frac{1}{2}g_{\alpha\beta}\dot{X}_\alpha\dot{X}_\beta - U(X) \quad U(X) = \sum_{a=0}^{N-1}\left\{\frac{(\Delta\phi_a)^2}{2\Delta x_a} + \Delta x_a\frac{\mathscr{V}(\phi_{a+1})-\mathscr{V}(\phi_a)}{\phi_{a+1}-\phi_a}\right\}$$

Assuming: $x_0(t) < x_1(t) < \ldots < x_N(t)$

$$\mathscr{V}'(\phi) = V(\phi)$$

Mechanization

$$X = \{x_0, x_1, \ldots\ x_N, \phi_1, \phi_2, \ldots\ \phi_{N-1}\}$$

$$L_M = \frac{1}{2} g_{\alpha\beta} \dot{X}_\alpha \dot{X}_\beta - U(X)$$

Metric is block-tri-diagonal

$$g = \begin{pmatrix}
\frac{(\Delta\phi_0)^2}{3\Delta x_0} & \frac{(\Delta\phi_0)^2}{6\Delta x_0} & 0 & \ldots & (\phi_0-\phi_1)/6 & 0 & 0 & \ldots \\
\frac{(\Delta\phi_0)^2}{6\Delta x_0} & \frac{(\Delta\phi_0)^2}{3\Delta x_0}+\frac{(\Delta\phi_1)^2}{3\Delta x_1} & \frac{(\Delta\phi_1)^2}{6\Delta x_1} & \ldots & (\phi_0-\phi_2)/3 & (\phi_1-\phi_2)/6 & 0 & \ldots \\
0 & \frac{(\Delta\phi_1)^2}{6\Delta x_1} & \frac{(\Delta\phi_1)^2}{3\Delta x_1}+\frac{(\Delta\phi_2)^2}{3\Delta x_2} & \ldots & (\phi_1-\phi_2)/6 & (\phi_1-\phi_3)/3 & (\phi_2-\phi_3)/6 & \ldots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\
(\phi_0-\phi_1)/6 & (\phi_0-\phi_2)/3 & (\phi_1-\phi_2)/6 & \ldots & (x_2-x_0)/3 & (x_2-x_1)/6 & 0 & \ldots \\
0 & (\phi_1-\phi_2)/6 & (\phi_1-\phi_3)/6 & \ldots & (x_2-x_1)/6 & (x_3-x_1)/3 & (x_3-x_2)/6 & \ldots \\
0 & 0 & (\phi_2-\phi_3)/6 & \ldots & 0 & (x_3-x_2)/6 & (x_4-x_2)/6 & \ldots \\
\vdots & \vdots & \ldots & \ddots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}$$

$$\frac{1}{2} g_{\alpha\beta} \dot{X}_\alpha \dot{X}_\beta = \sum_{a=0}^{N-1} \left\{ \frac{1}{6}(\Delta x_a)^3 \dot{k}_a^2 + \frac{1}{2}\Delta x_a \left(\dot{\phi}_{a+1} - k_a \dot{x}_{a+1}\right)\left(\dot{\phi}_a - k_a \dot{x}_a\right) \right\}$$

$$k_a \equiv \frac{\Delta\phi_a}{\Delta x_a} \equiv \frac{\phi_{a+1}-\phi_a}{x_{a+1}-x_a}$$

$$\det(g) = \frac{1}{12^N} \prod_{a=-1}^{N-1} \left(k_{a+1} - k_a\right)^2 \prod_{b=0}^{N-1} (x_{b+1} - x_b)^2$$

Singularity whenever two joint meets
& when neighbouring slopes coincide

Mechanization

Better coordinates:

$$Y = \{k_0, k_1, \dots\ k_{N-1}, \Phi_0, \Phi_1, \dots\ \Phi_{N-1}\} \qquad L_M = \frac{1}{2} g_{\alpha\beta} \dot{Y}_\alpha \dot{Y}_\beta - U(Y)$$

$$k_a = \frac{\phi_{a+1} - \phi_a}{x_{a+1} - x_a} \quad \Phi_a = \frac{x_{a+1}\phi_a - x_a\phi_{a+1}}{x_{a+1} - x_a} \qquad \Leftrightarrow \qquad x_{a+1} = -\frac{\Phi_{a+1} - \Phi_a}{k_{a+1} - k_a} \quad \phi_{a+1} = \frac{k_{a+1}\Phi_a - k_a\Phi_{a+1}}{k_{a+1} - k_a}$$

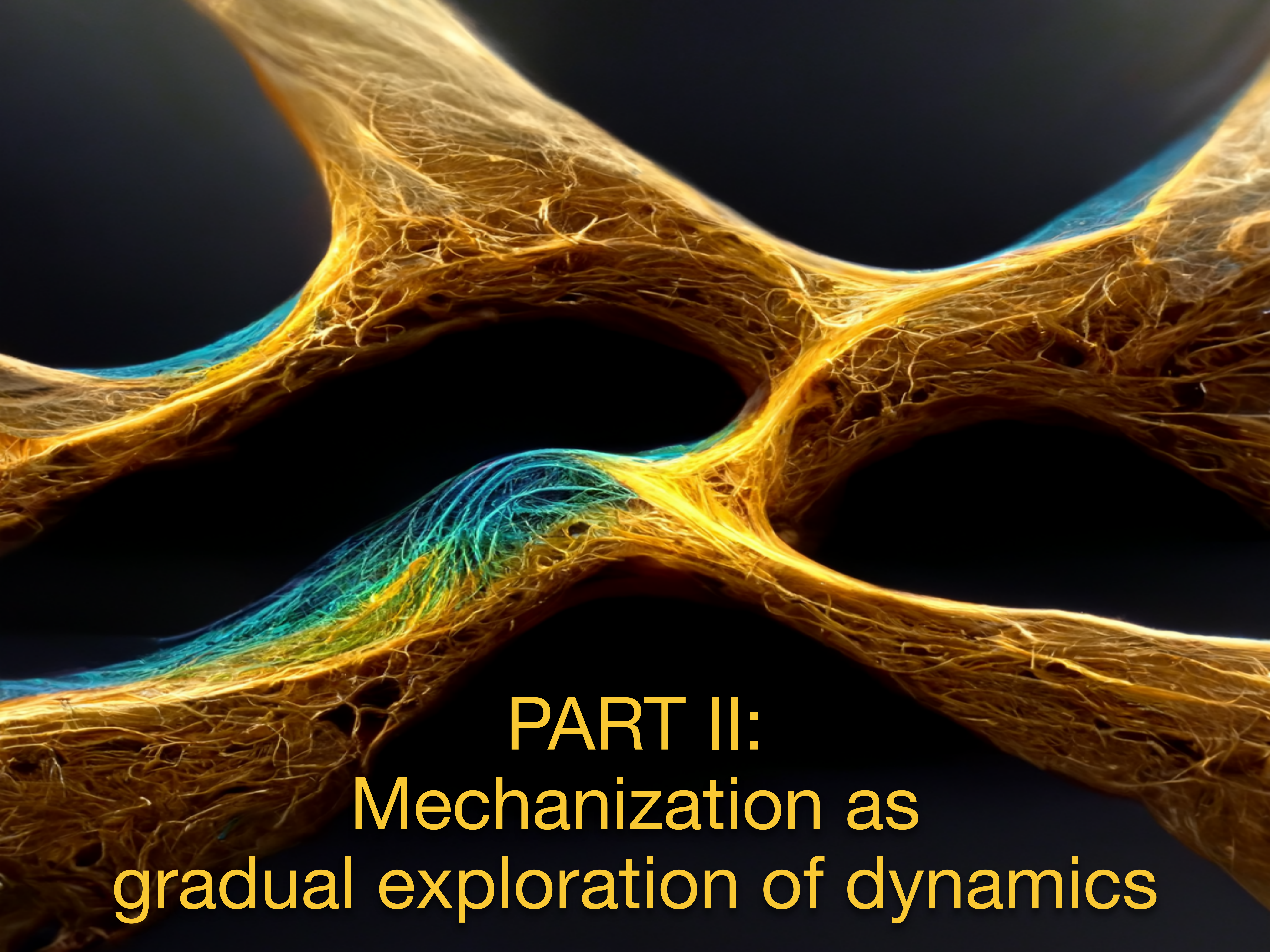
Metric is block-diagonal

$$g = \begin{pmatrix} (x_1^3 - x_0^3)/3 & \dots & 0 & (x_1^2 - x_0^2)/2 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & (x_N^3 - x_{N-1}^3)/3 & 0 & \dots & (x_N^2 - x_{N-1}^2)/2 \\ (x_1^2 - x_0^2)/2 & \dots & 0 & x_1 - x_0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & (x_N^2 - x_{N-1}^2)/2 & 0 & \dots & x_N - x_{N-1} \end{pmatrix}$$

$$\frac{1}{2} g_{\alpha\beta} \dot{Y}_\alpha \dot{Y}_\beta = \sum_{a=0}^{N-1} \left\{ \frac{1}{6}(x_{a+1}^3 - x_a^3)\dot{k}_a^2 + \frac{1}{2}(x_{a+1}^2 - x_a^2)\dot{k}_a \dot{\Phi}_a + \frac{1}{2}(x_{a+1} - x_a)\dot{\Phi}_a^2 \right\}$$

$$\det(g) = \frac{1}{12^N} \prod_{a=0}^{N-1} (x_{a+1} - x_a)^4$$

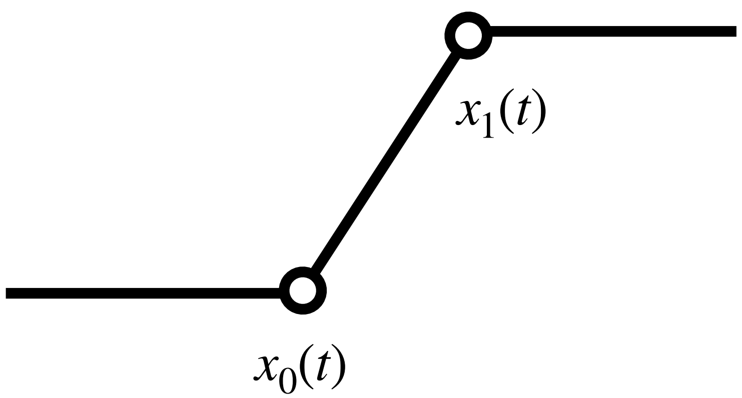
Singularity “only” when two joint meets



PART II:
Mechanization as gradual exploration of dynamics

N=1

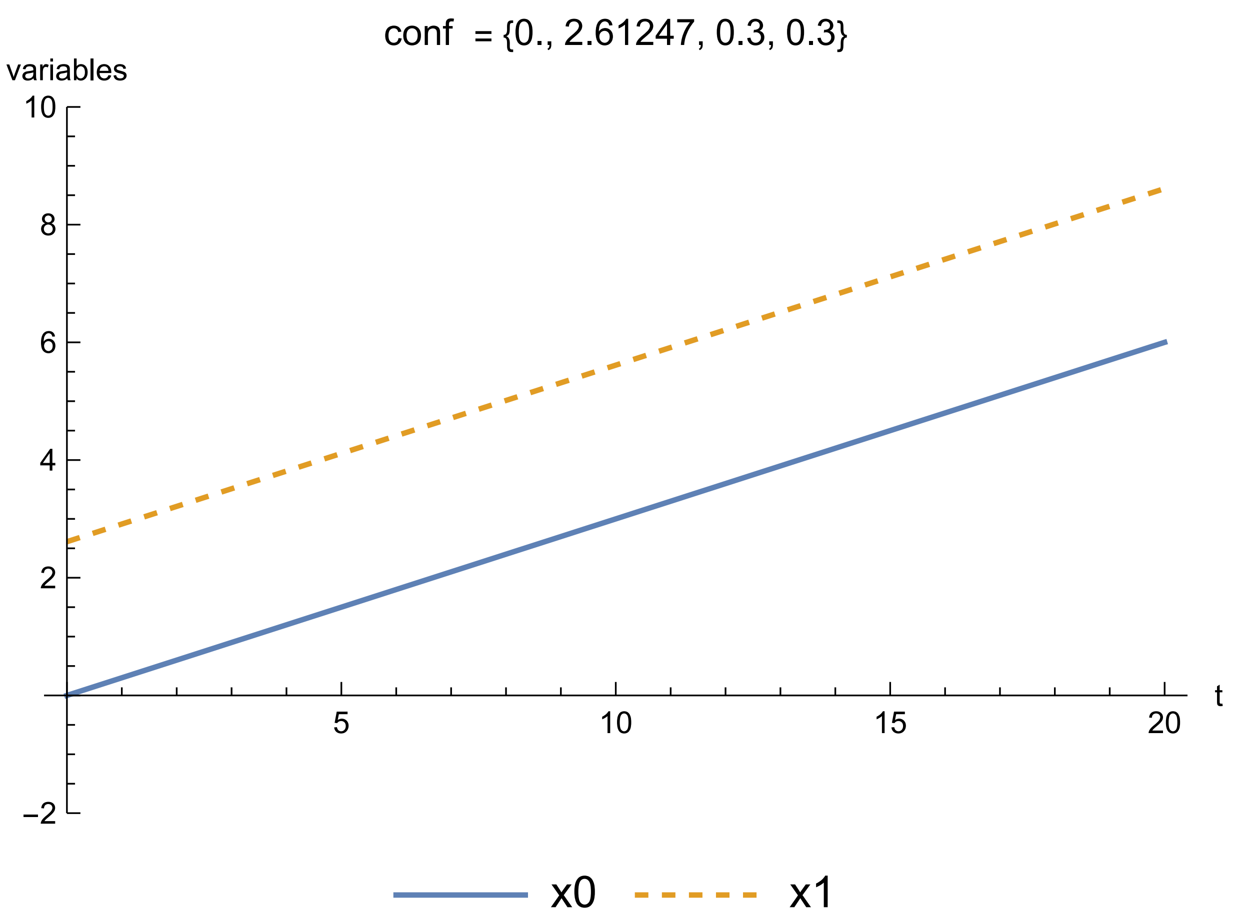
mech-kink



Simplest topologically non-trivial mech-field

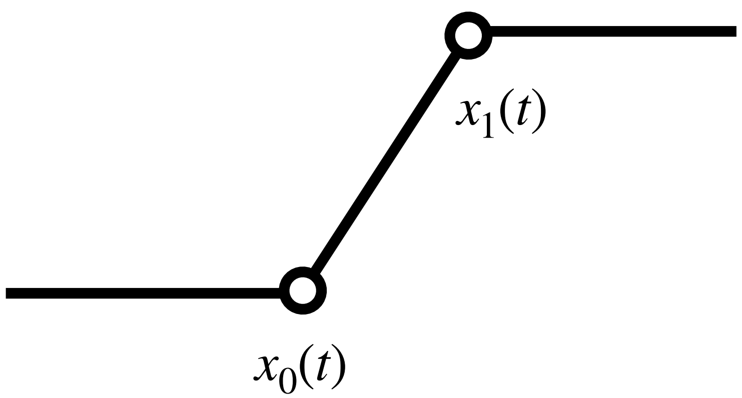
$$m_K = \sqrt{\frac{32}{15}} \qquad R_K = \sqrt{\frac{15}{2}}$$

Can be boosted: $E = m_K/\sqrt{1-v^2} \quad R = R_K\sqrt{1-v^2}$



N=1

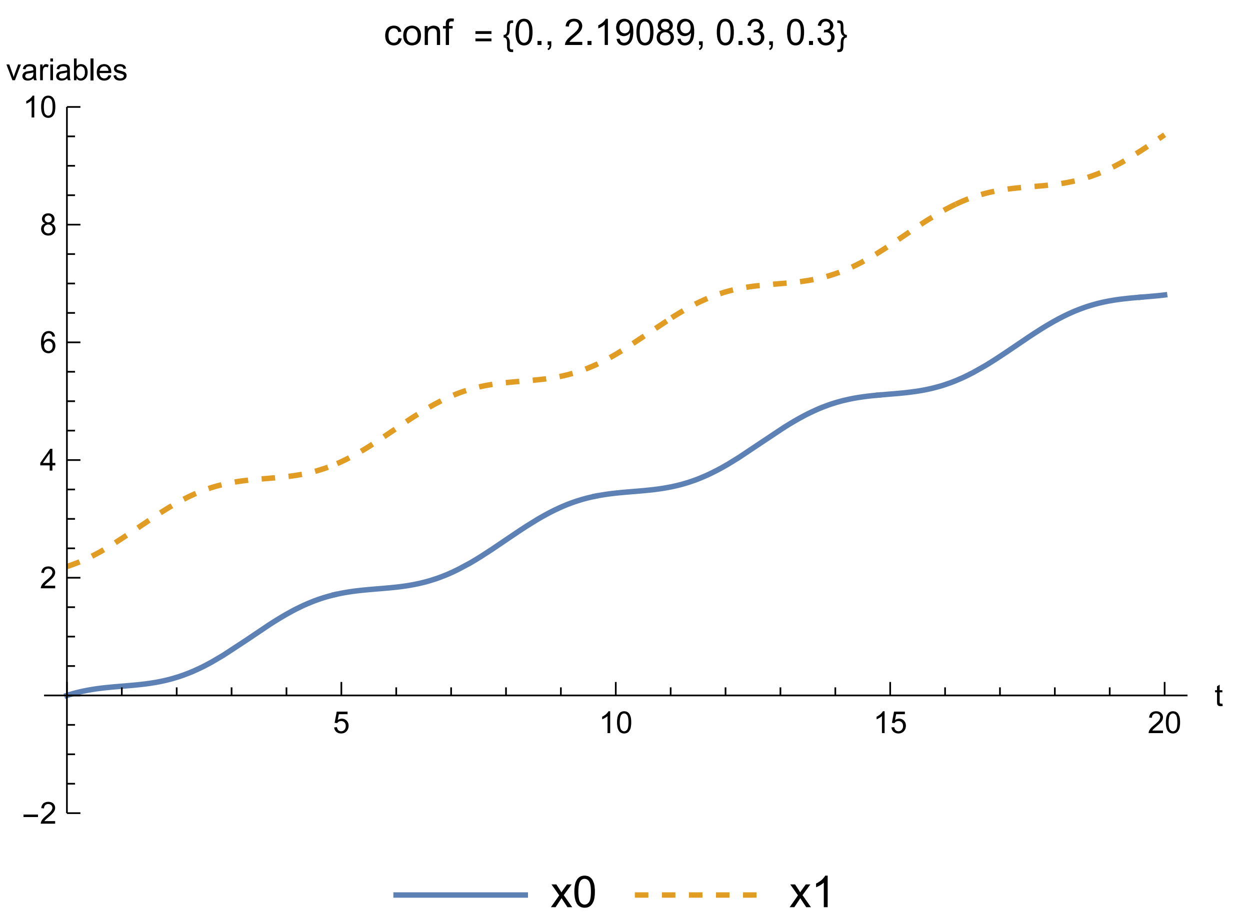
Mech-kink



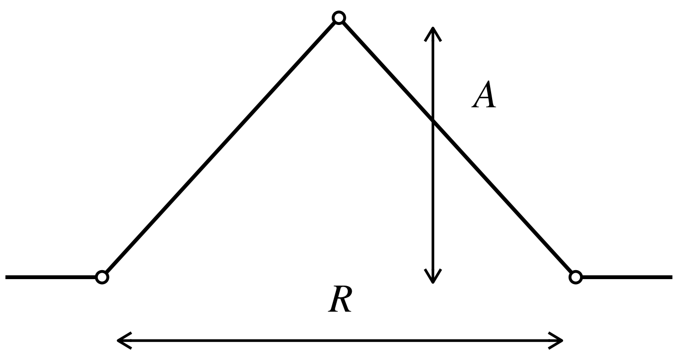
Simplest topologically non-trivial mech-field

$$m_K = \sqrt{\frac{32}{15}} \qquad R_K = \sqrt{\frac{15}{2}}$$

Has a vibration mode: $\omega^2 = m_K/q_K = 8/5$

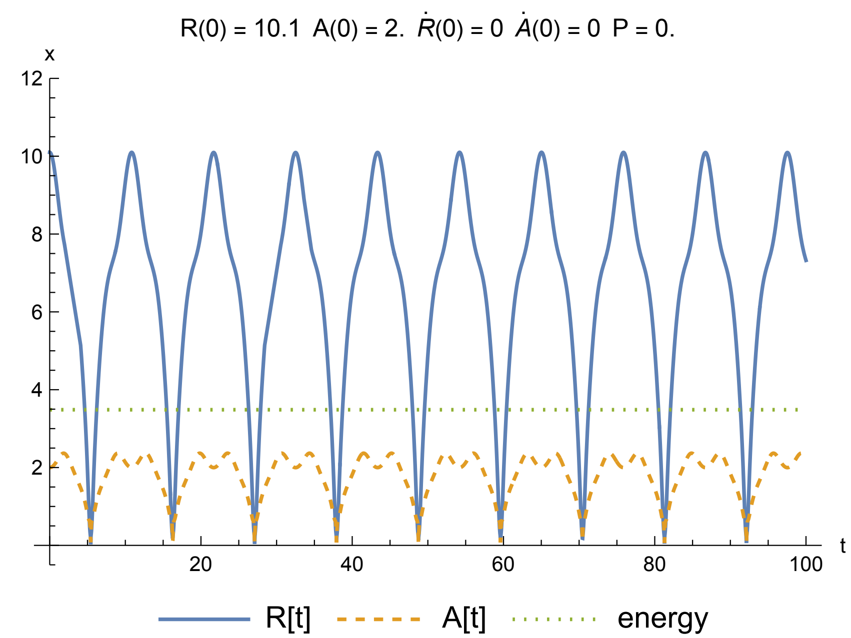


N=2



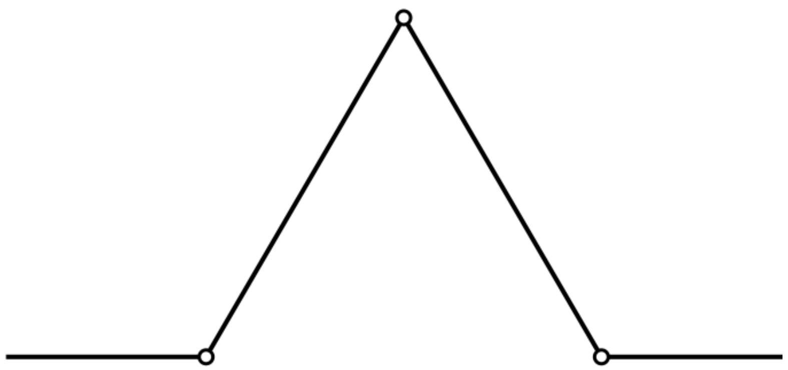
Mech-oscillon: A simplest topologically trivial mech-field

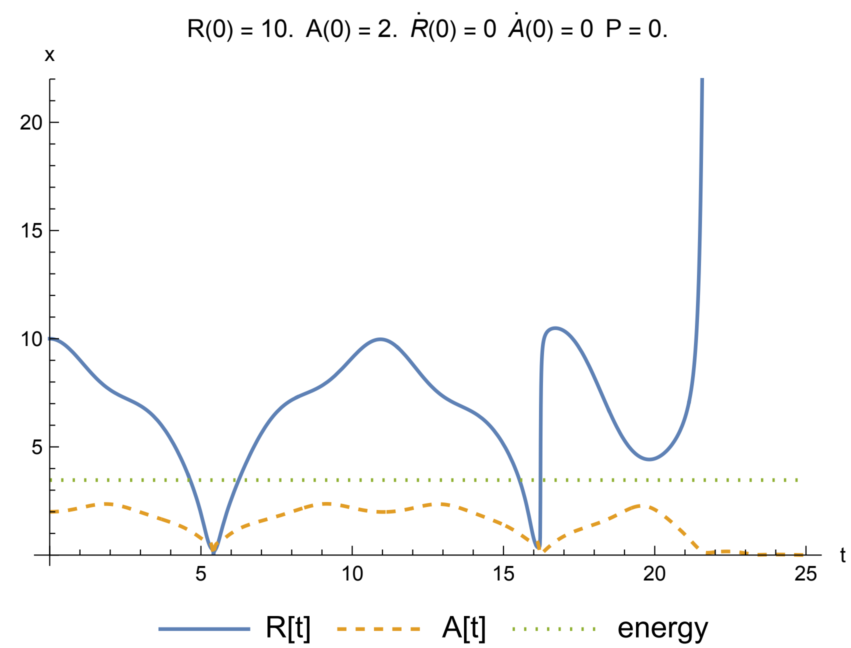
$$L_M = \frac{1}{6} R \dot{A}^2 + \frac{1}{6} A \dot{A} \dot{R} + \frac{1}{6R} A^2 \dot{R}^2 - \frac{2A^2}{R} - R \frac{\mathcal{V}(v+A) - \mathcal{V}(v)}{A}$$

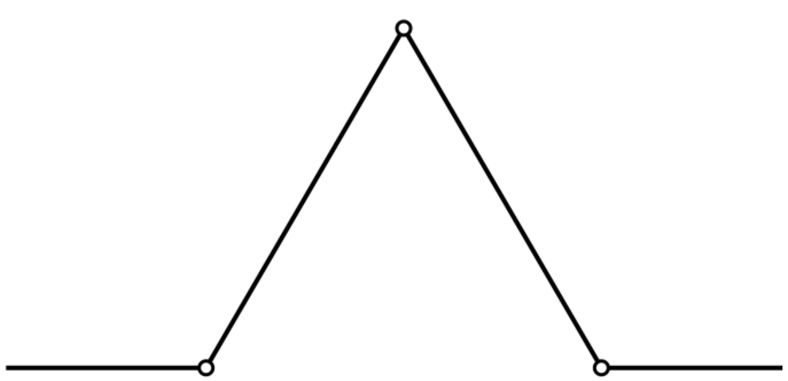


Mech-oscillons with zero momentum decay!

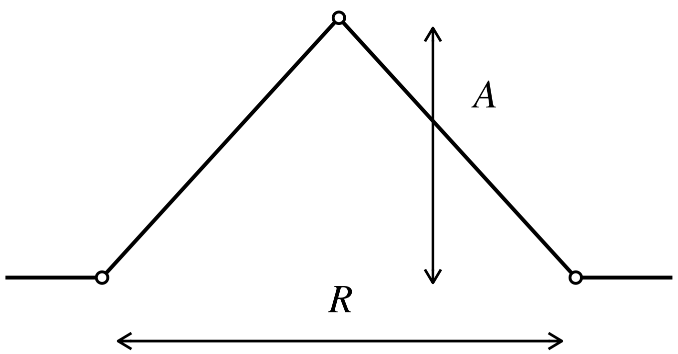
$$R \sim e^{2t\sqrt{V''(v)/3}} \qquad A \sim e^{-t\sqrt{V''(v)/3}}$$







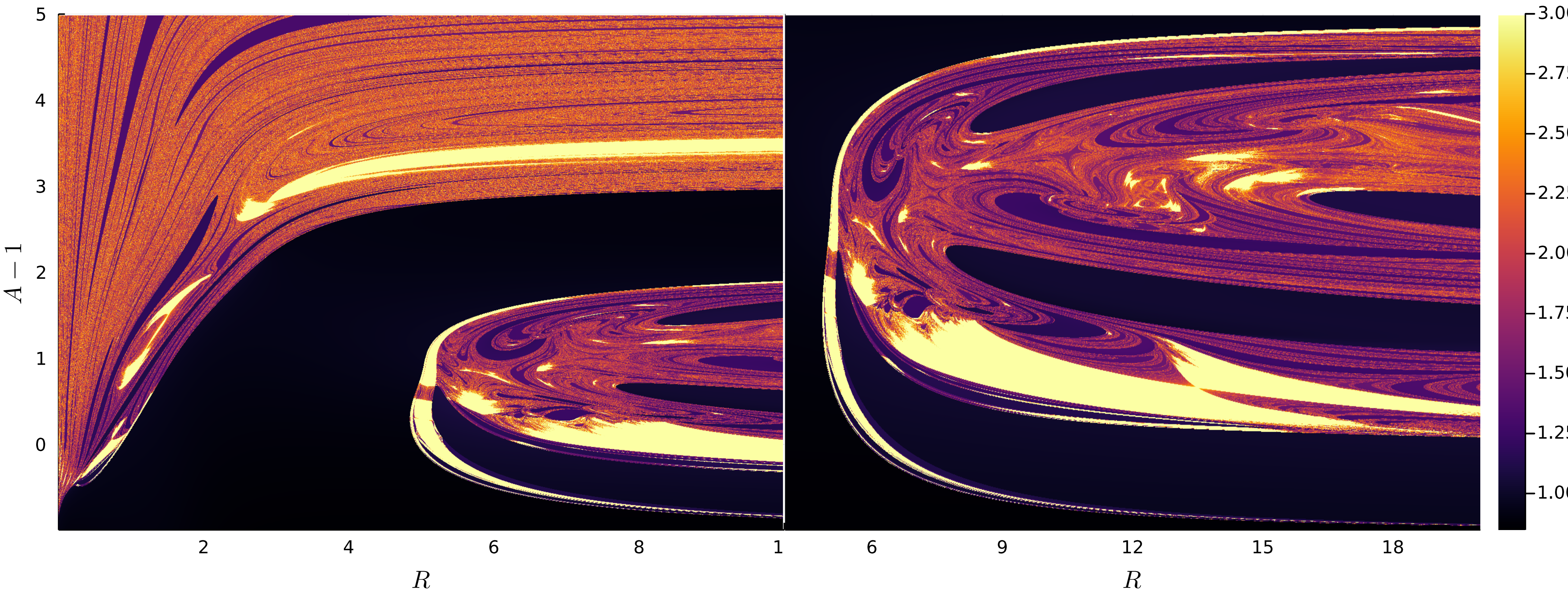
N=2



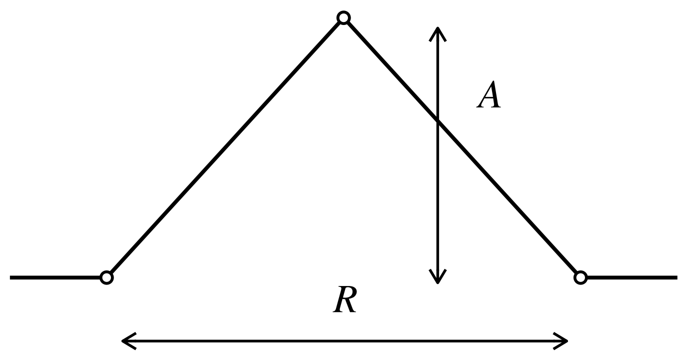
Mech-oscillon: A simplest topologically trivial mech-field

$$L_M = \frac{1}{6} R \dot{A}^2 + \frac{1}{6} A \dot{A} \dot{R} + \frac{1}{6R} A^2 \dot{R}^2 - \frac{2A^2}{R} - R \frac{\mathscr{V}(v+A) - \mathscr{V}(v)}{A}$$

$$V = \frac{1}{2}(1-\phi^2)^2 \quad \mathscr{V} = \frac{\phi}{2} - \frac{\phi^3}{3} + \frac{\phi^5}{10}$$



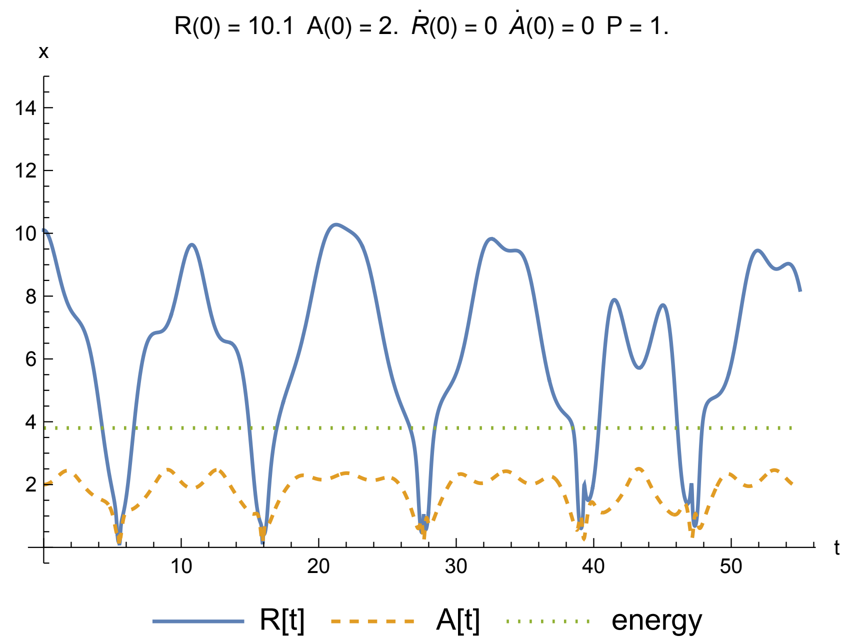
N=2

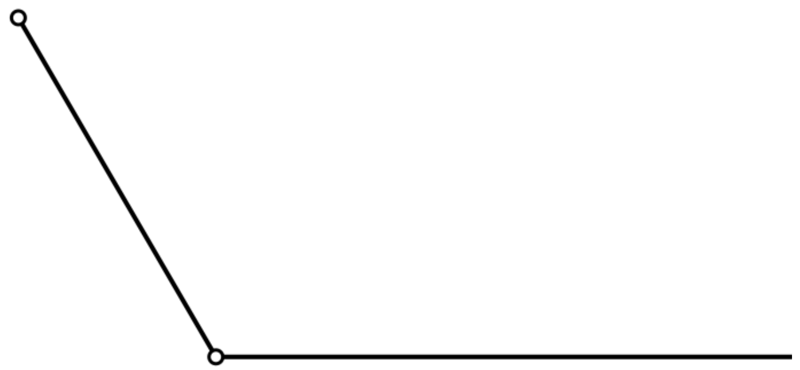


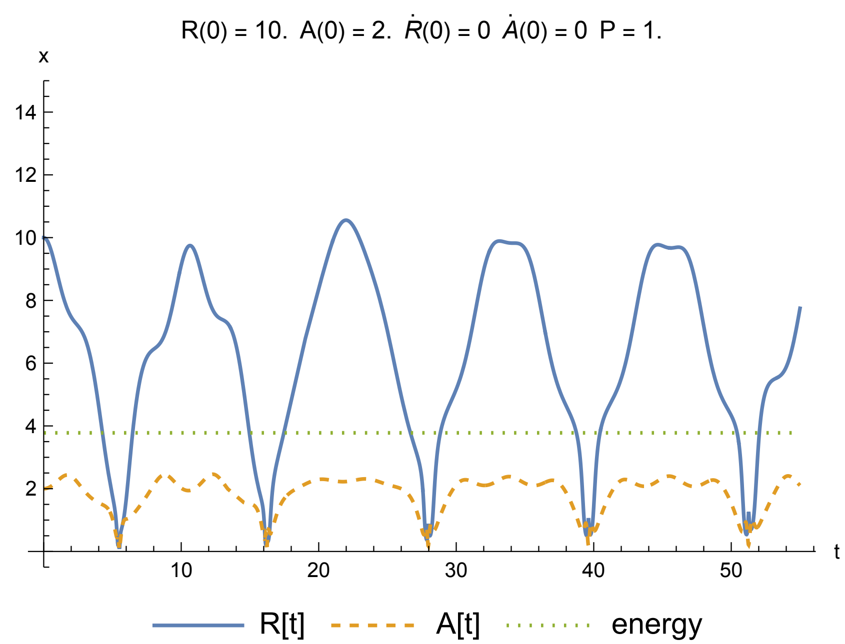
$$L_M = \frac{1}{6}R\dot{A}^2 + \frac{1}{6}A\dot{A}\dot{R} + \frac{1}{6R}A^2\dot{R}^2 - \frac{2A^2(1-\dot{a}^2)}{R} - R\frac{\mathscr{V}(v+A) - \mathscr{V}(v)}{A}$$

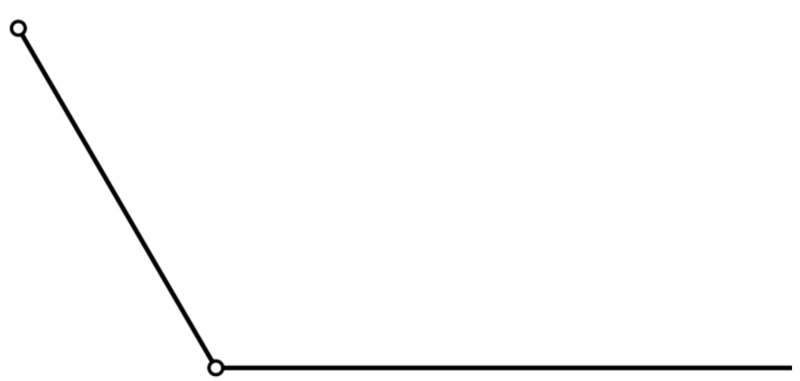
Mech-oscillons with non-zero momentum **do not decay!**

$$R \propto e^{2t\sqrt{V''(v)/3}} \quad A \propto e^{-t\sqrt{V''(v)/3}}$$

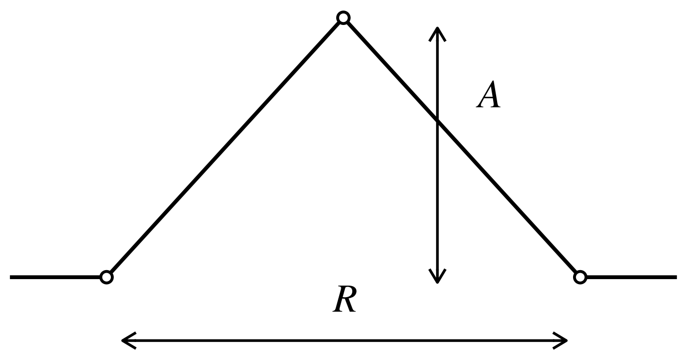








N=2



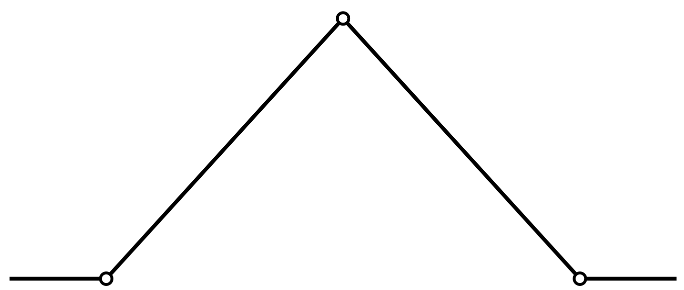
$$L_M = \frac{1}{6}R\dot{A}^2 + \frac{1}{6}A\dot{A}\dot{R} + \frac{1}{6R}A^2\dot{R}^2 - \frac{2A^2(1-\dot{a}^2)}{R} - R\frac{\mathscr{V}(v+A) - \mathscr{V}(v)}{A}$$

Mech-oscillons with non-zero momentum **do not decay!**

$$R \sim e^{2t\sqrt{V''(v)/3}} \quad A \sim e^{-t\sqrt{V''(v)/3}}$$

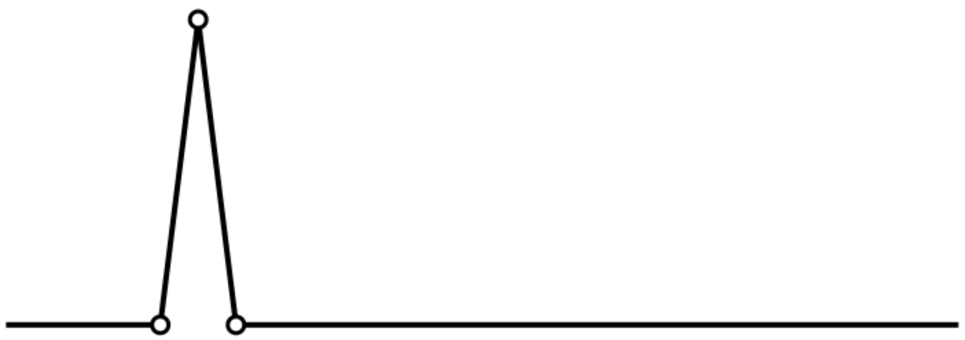
Super-luminal solutions also exists!

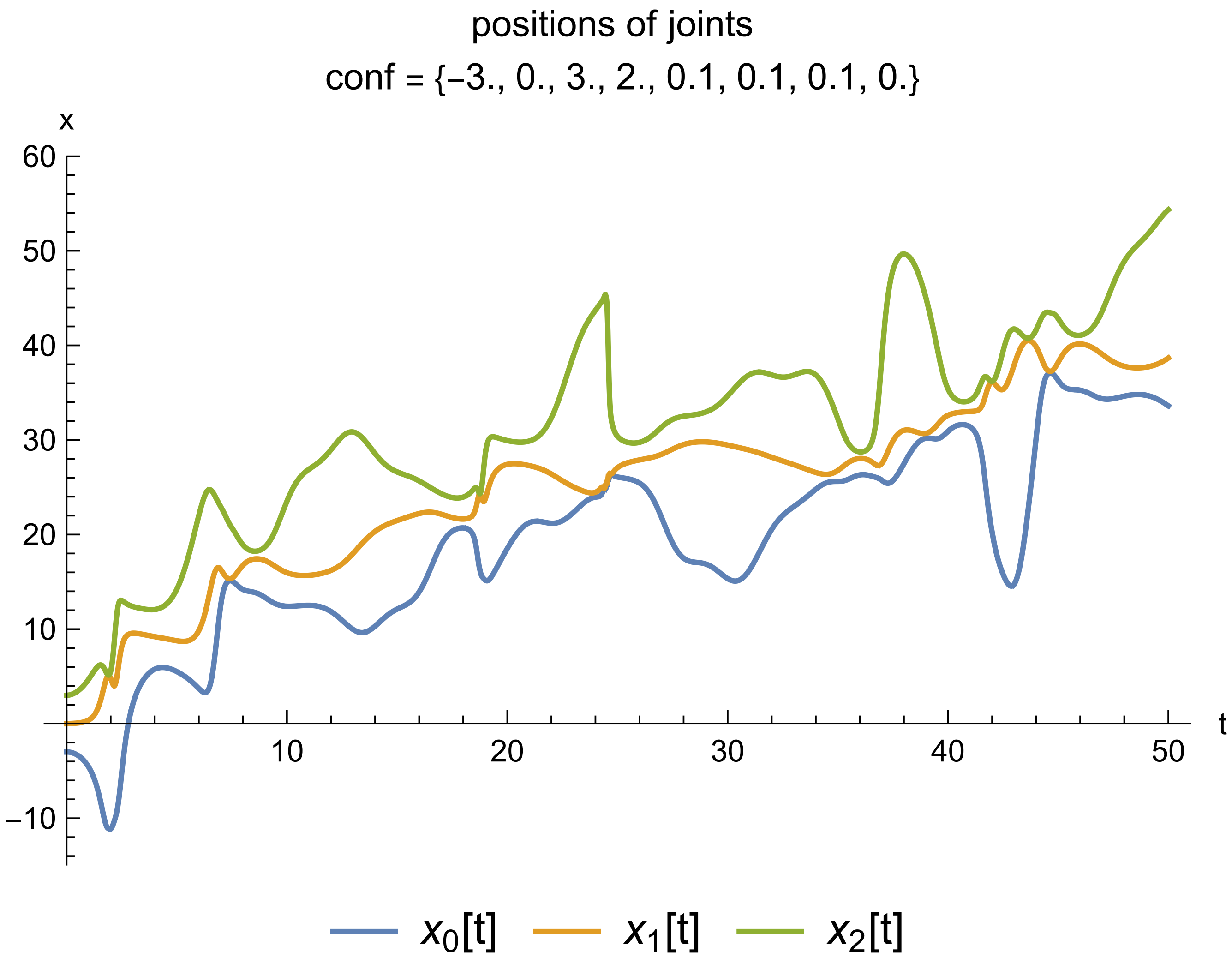
$$\dot{a}^2 = 1 + \frac{R^2}{2A^2}V(v+A) \geq 1 \qquad E = \frac{4A^2}{R}$$



N=2

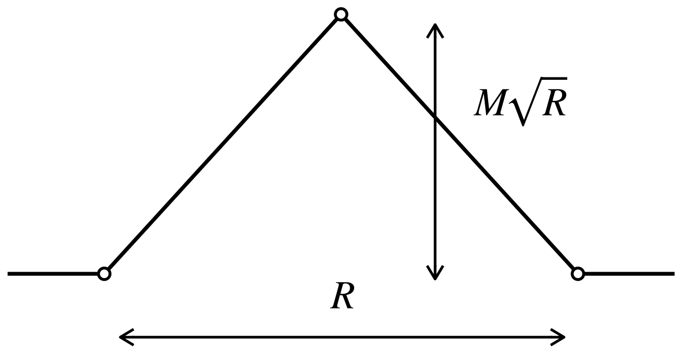
Asymmetric mech-oscillon





N=2

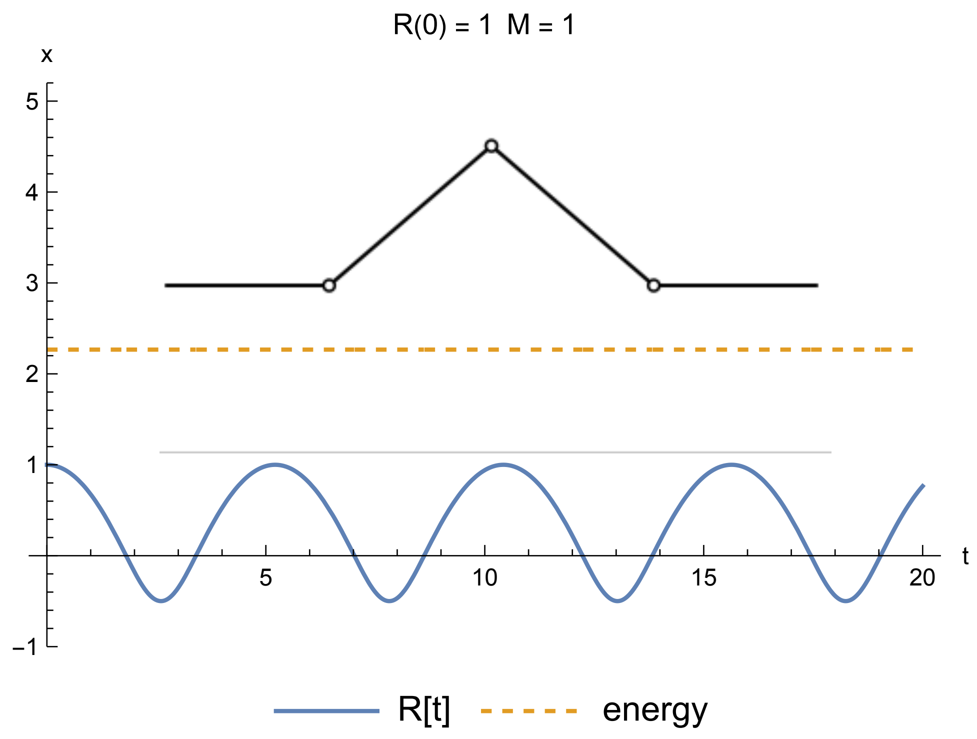
Rigid mech-oscillons

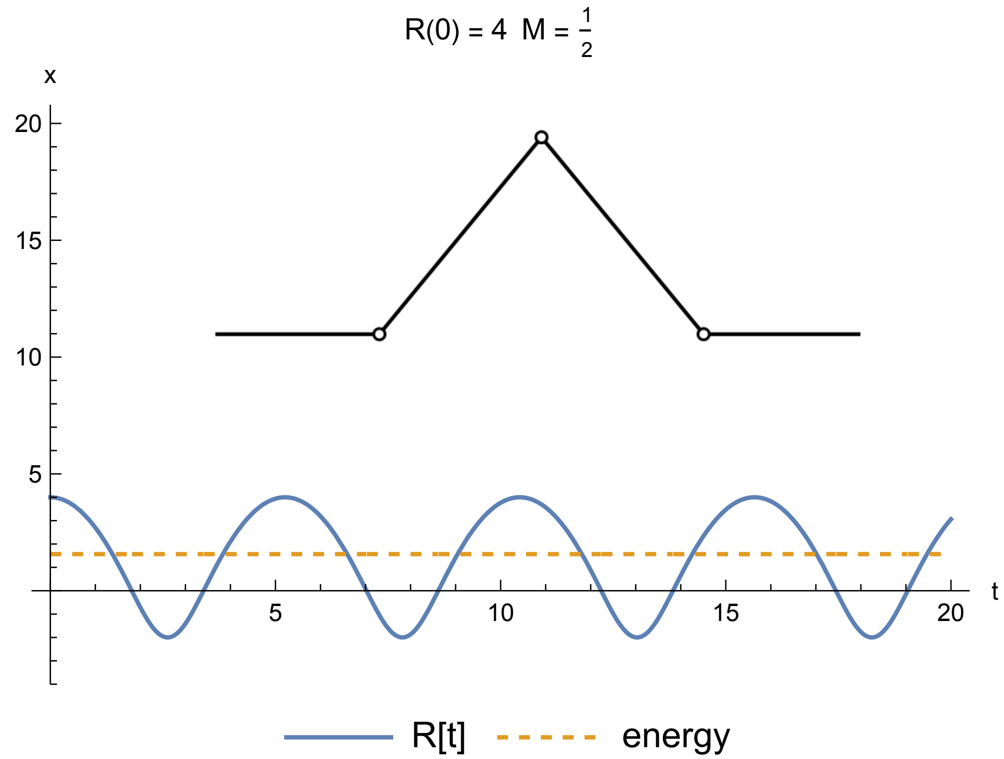


$$L_{\text{oom}} = \frac{7}{24} M^2 \dot{R}^2 - U\left(R/\sqrt{|R|}\right)$$

$$U(x) = 2M^2 + \frac{x}{M}\left(\mathcal{V}\left(v + Mx\right) - \mathcal{V}(v)\right)$$

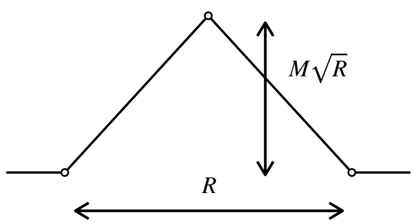
For small mass $\qquad \omega = \sqrt{\frac{4}{7} V''(v)}$





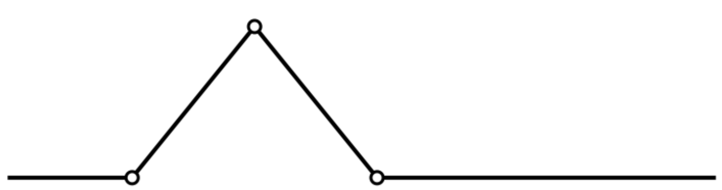
N=2

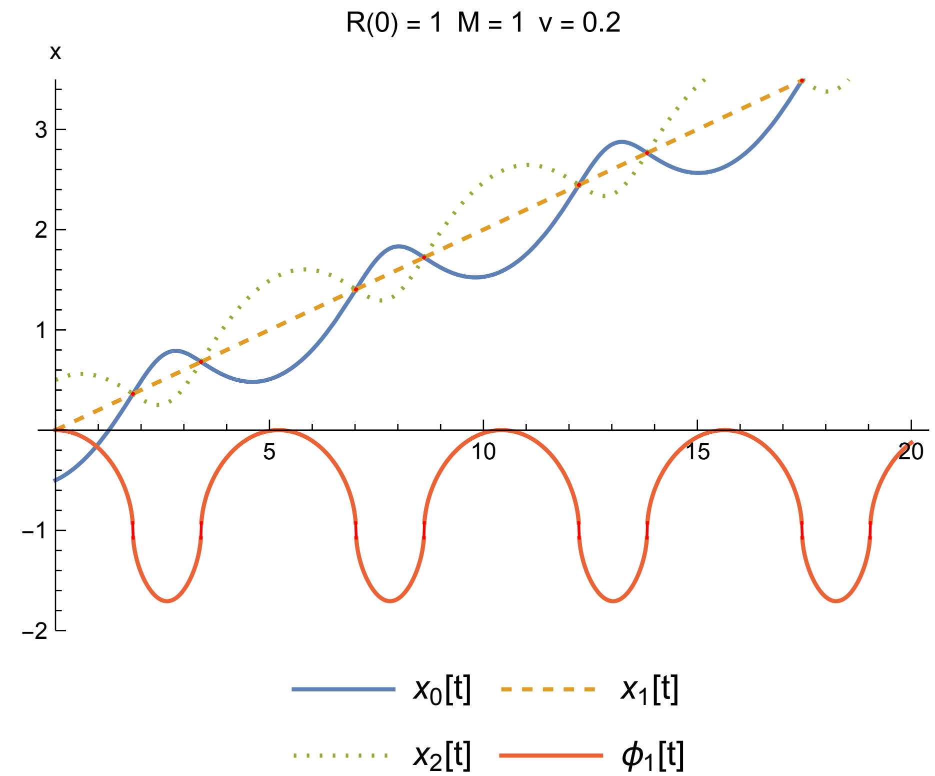
Boosted rigid mech-oscillon

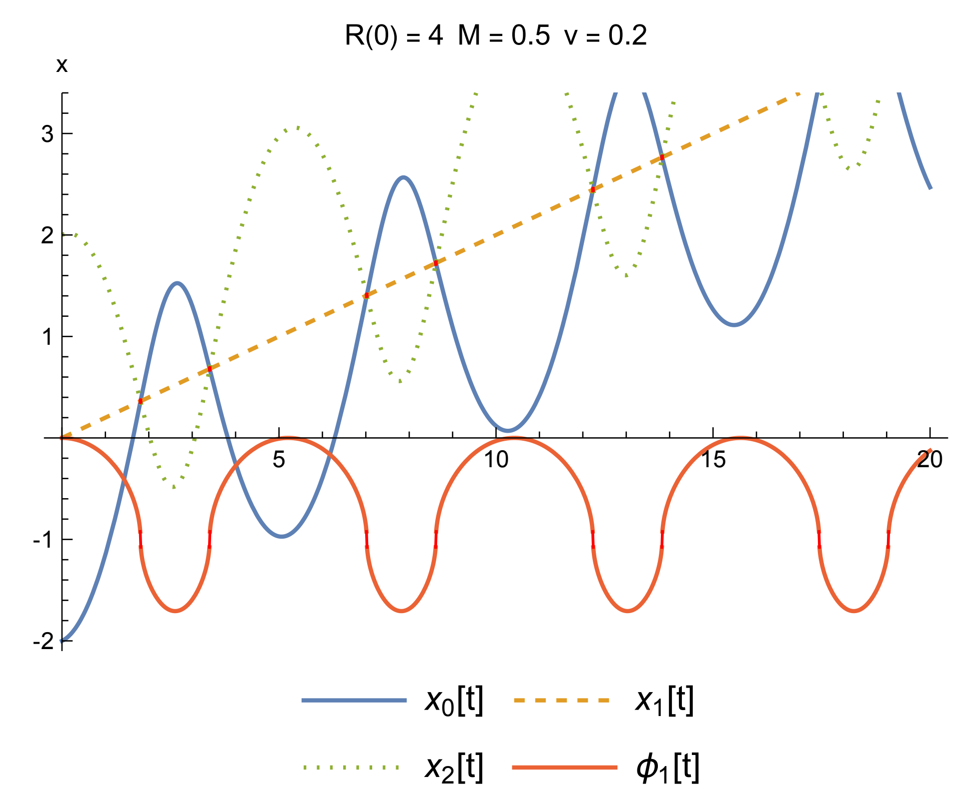


$$L_{\mathrm{oom}} = 2M^2\dot{a}^2 + \frac{7}{24}M^2\dot{R}^2 - U\left(R/\sqrt{|R|}\right)$$

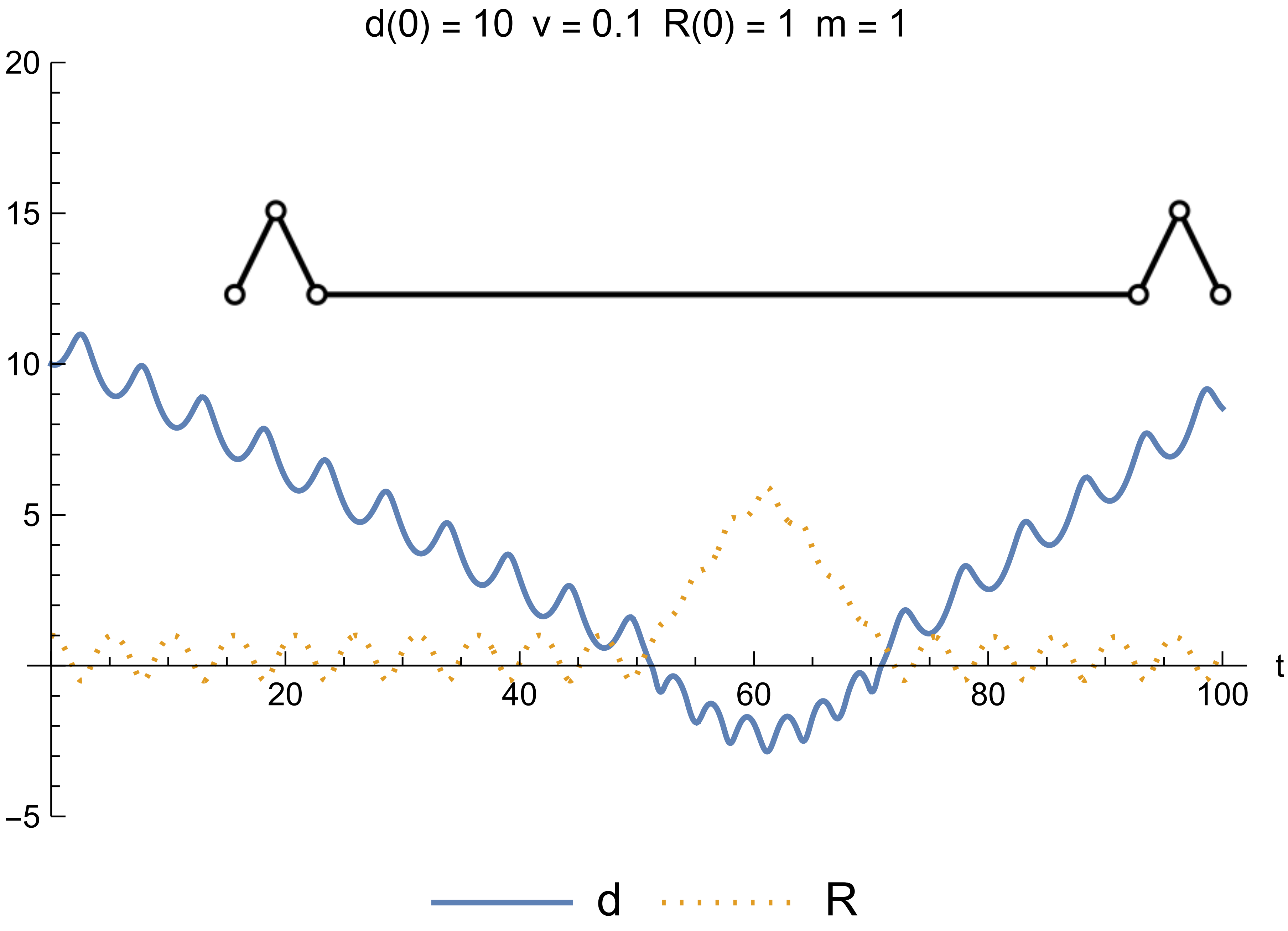
$$U(x) = 2M^2 + \frac{x}{M}\Big(\mathscr{V}\left(v + Mx\right) - \mathscr{V}(v)\Big)$$







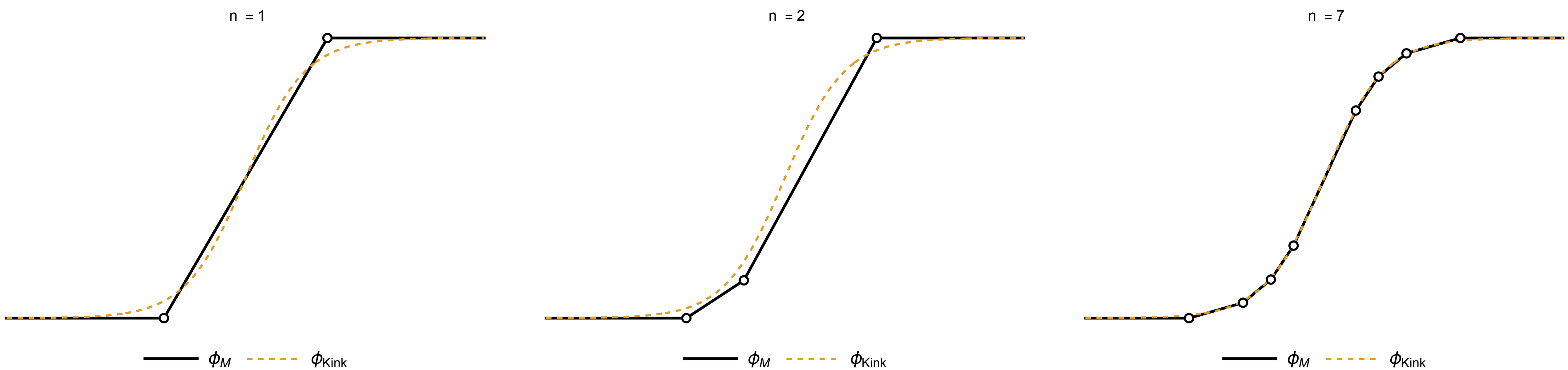
N=5



Mech-kinks

Mech-kinks are found by minimising energy with topologically non-trivial boundary conditions: $\phi_0 = v_\mathrm{L} \neq v_\mathrm{R} = \phi_N$

$$V(\phi) = \frac{1}{2}\left(1 - \phi^2\right)^2 \qquad \phi_K = \tanh(x)$$



In general case the static equations of motion reduce to system of 2N non-linear algebraic equations

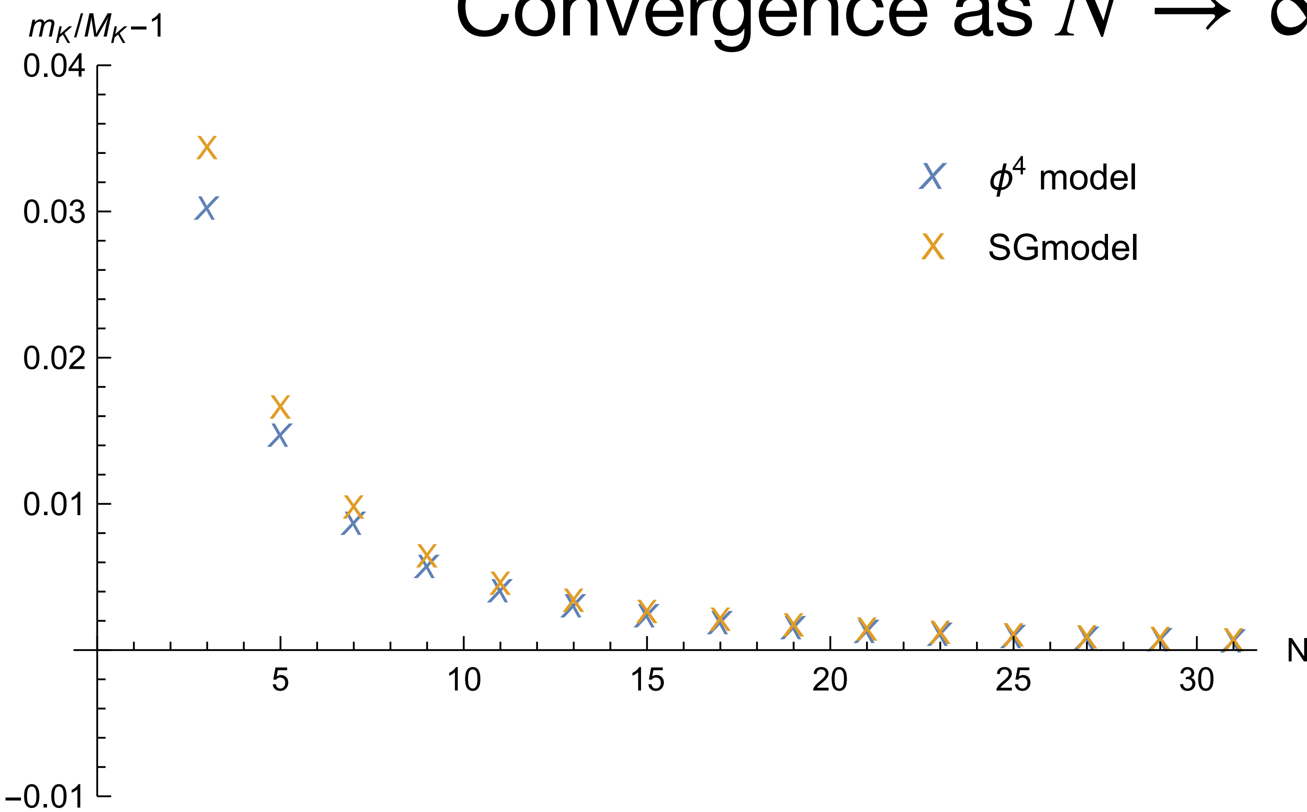
$$\frac{\Delta\phi_a}{\Delta x_a} = \sqrt{2\frac{\mathcal{V}(\phi_{a+1}) - \mathcal{V}(\phi_a)}{\phi_{a+1} - \phi_a}} \qquad V(\phi_a)^2 = \frac{\mathcal{V}(\phi_{a+1}) - \mathcal{V}(\phi_a)}{\phi_{a+1} - \phi_a}\frac{\mathcal{V}(\phi_a) - \mathcal{V}(\phi_{a-1})}{\phi_a - \phi_{a-1}}$$

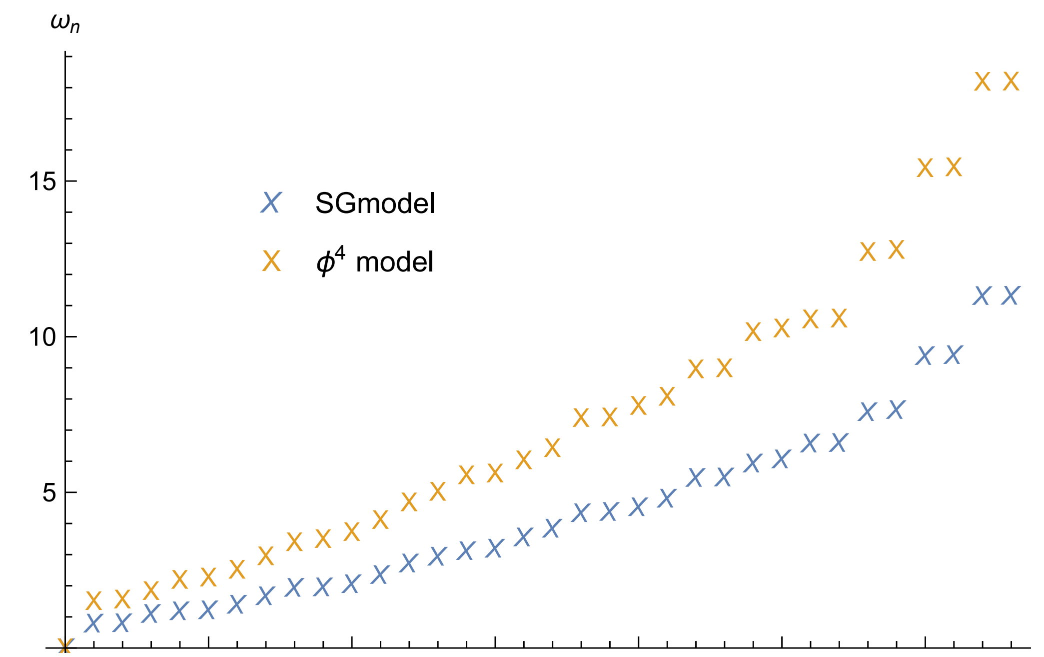
$N \to \infty$

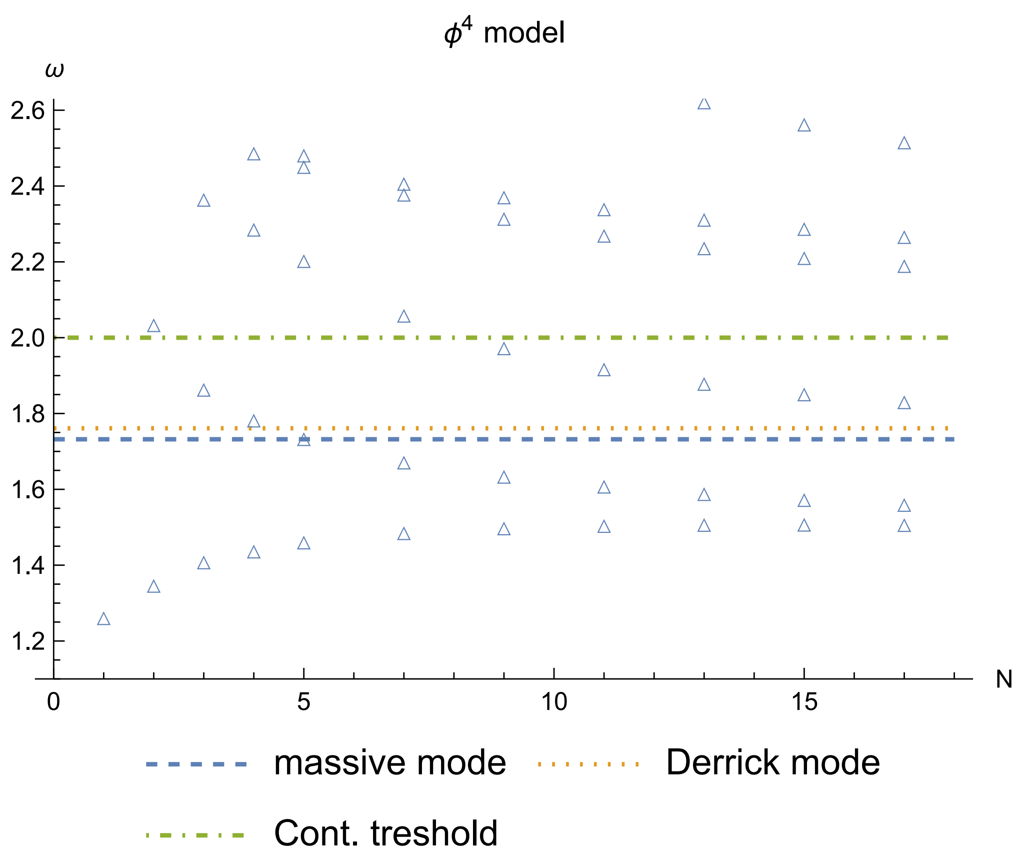
$$\frac{d\phi}{dx} = \sqrt{2V(\phi)} \qquad V(\phi)^2 = V(\phi)^2$$

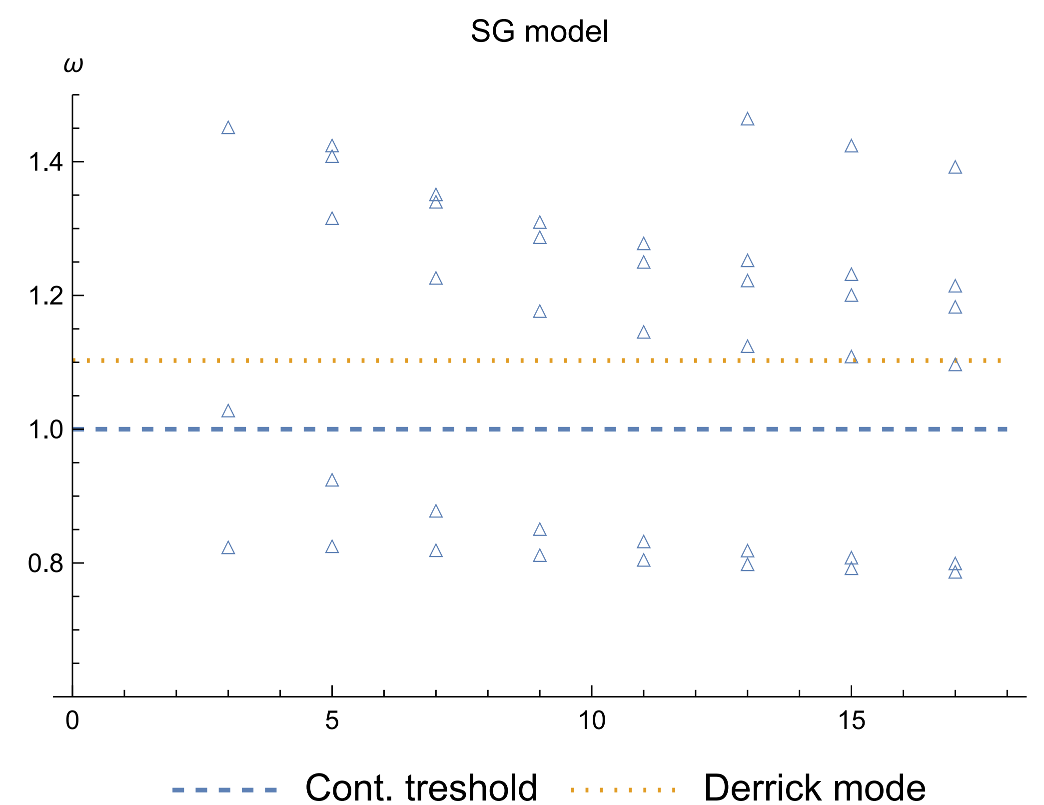
Mech-kinks

Convergence as $N \to \infty$? If $\exists$ $\Rightarrow$ very slow!







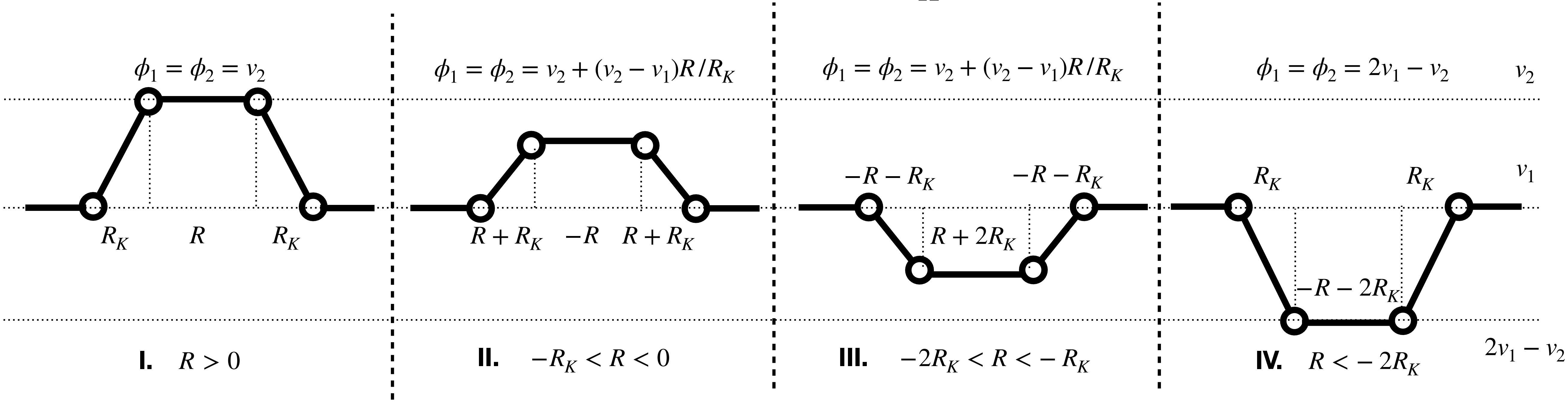


N=3

Scattering of mech-kinks

Scattering of mech-kinks

Both the distance R and the width R_K change with time $\quad X_a = \{R, R_K\}$



$\mathscr{V}(-1) = 0 \quad v_1 = -1 \quad v_2 = 1$

$$L_{\rm I} = \frac{1}{2} g^I_{ab} \dot{X}_a \dot{X}_b - U^I$$

$$g^I = \frac{2}{3R_K} \begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix}$$

$$U^I = \frac{4}{R_K} + R_K \mathscr{V}(1)$$

No singularity at $R = 0$

$$L_{\rm II} = \frac{1}{2} g^{II}_{ab} \dot{X}_a \dot{X}_b - U^{II}$$

$$g^{II} = \frac{1}{3R_K^4} \begin{pmatrix} 6R_K^2\left(R_K - R\right) & 6R_K\left(R_K^2 + R^2\right) \\ 6R_K\left(R_K^2 + R^2\right) & -4\left(R^3 - 2R_K^3\right) \end{pmatrix}$$

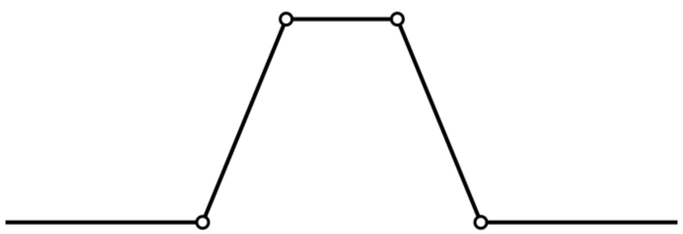
$$U^{II} = \frac{4(R + R_K)}{R_K^2} + \mathscr{V}\left(\frac{2R}{R_K} + 1\right) R_K - V\left(\frac{2R}{R_K} + 1\right) R$$

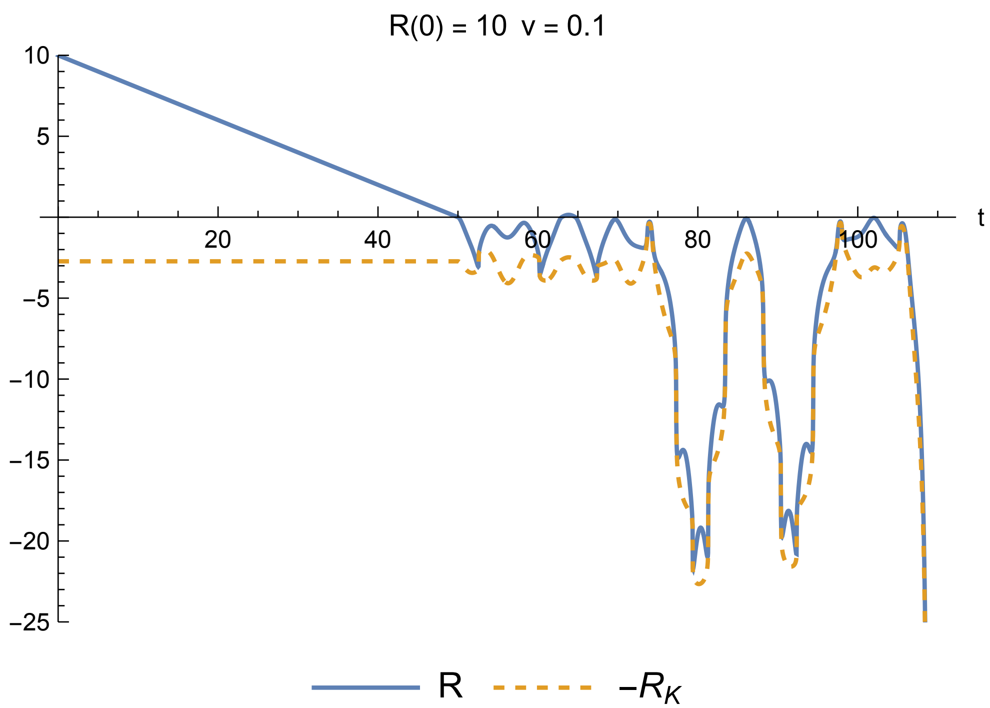
$$\mathscr{R} = \frac{9R_K^4\left(R^2 R_K - 2RR_K^2 - R_K^3 + R^3\right)}{2\left(R_K + R\right)^2 \left(R^2 R_K + 5RR_K^2 - R_K^3 + R^3\right)^2}$$

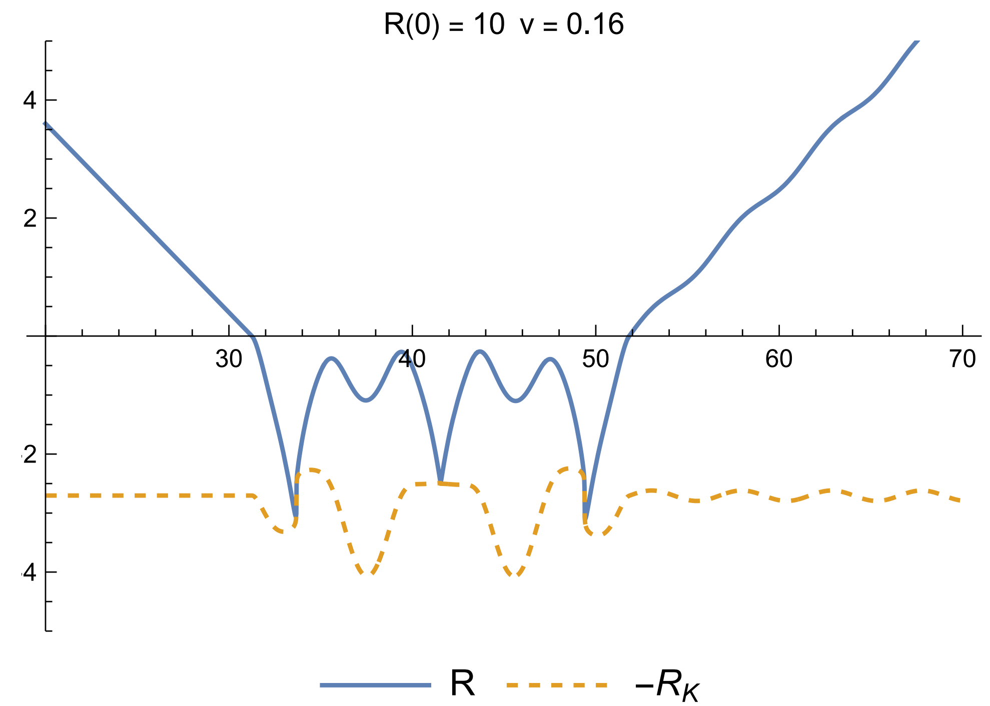
Singularity at $R = -R_K$

No continuation past
$R = -R_K$

Scattering of mech-kinks

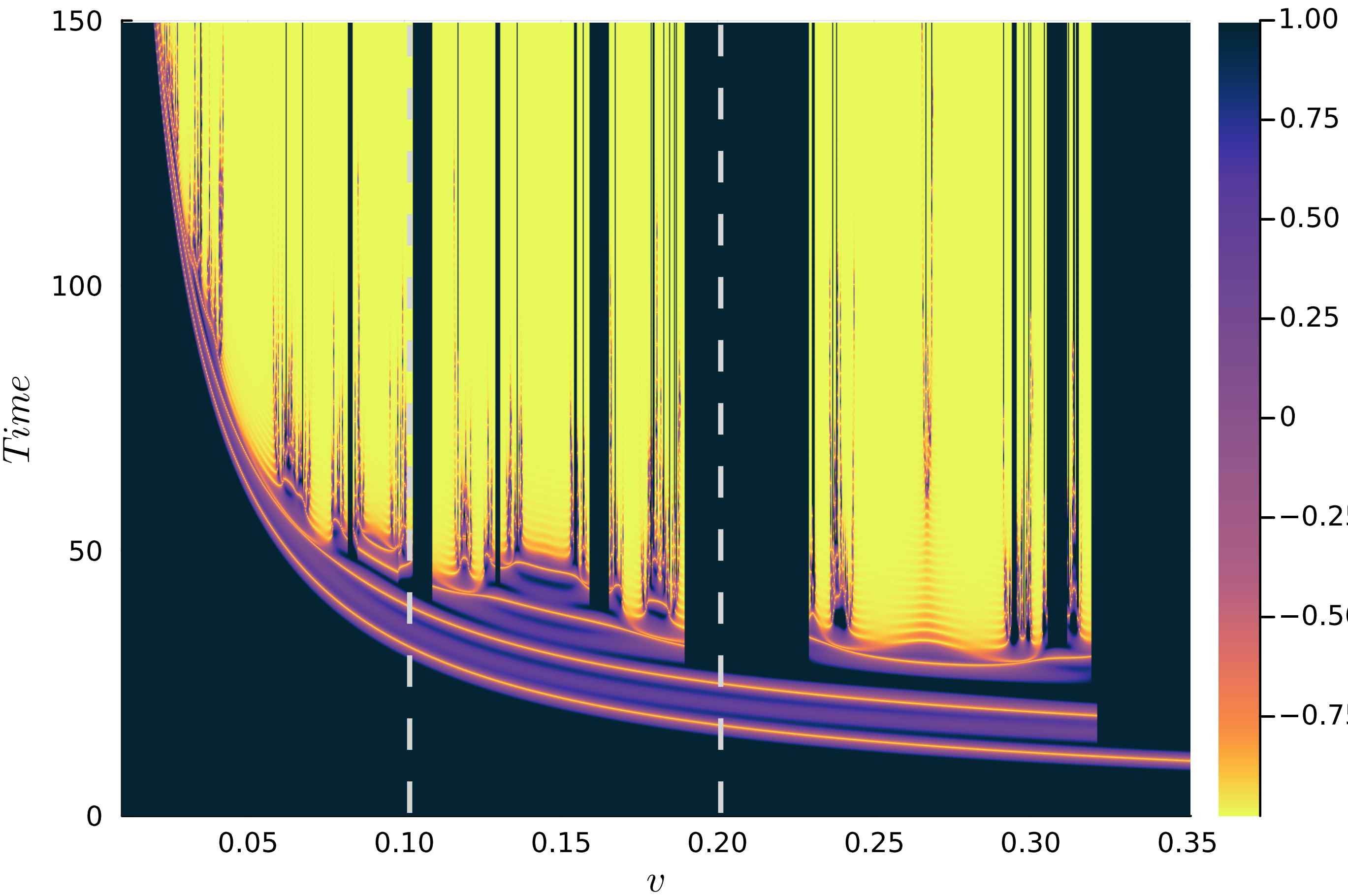




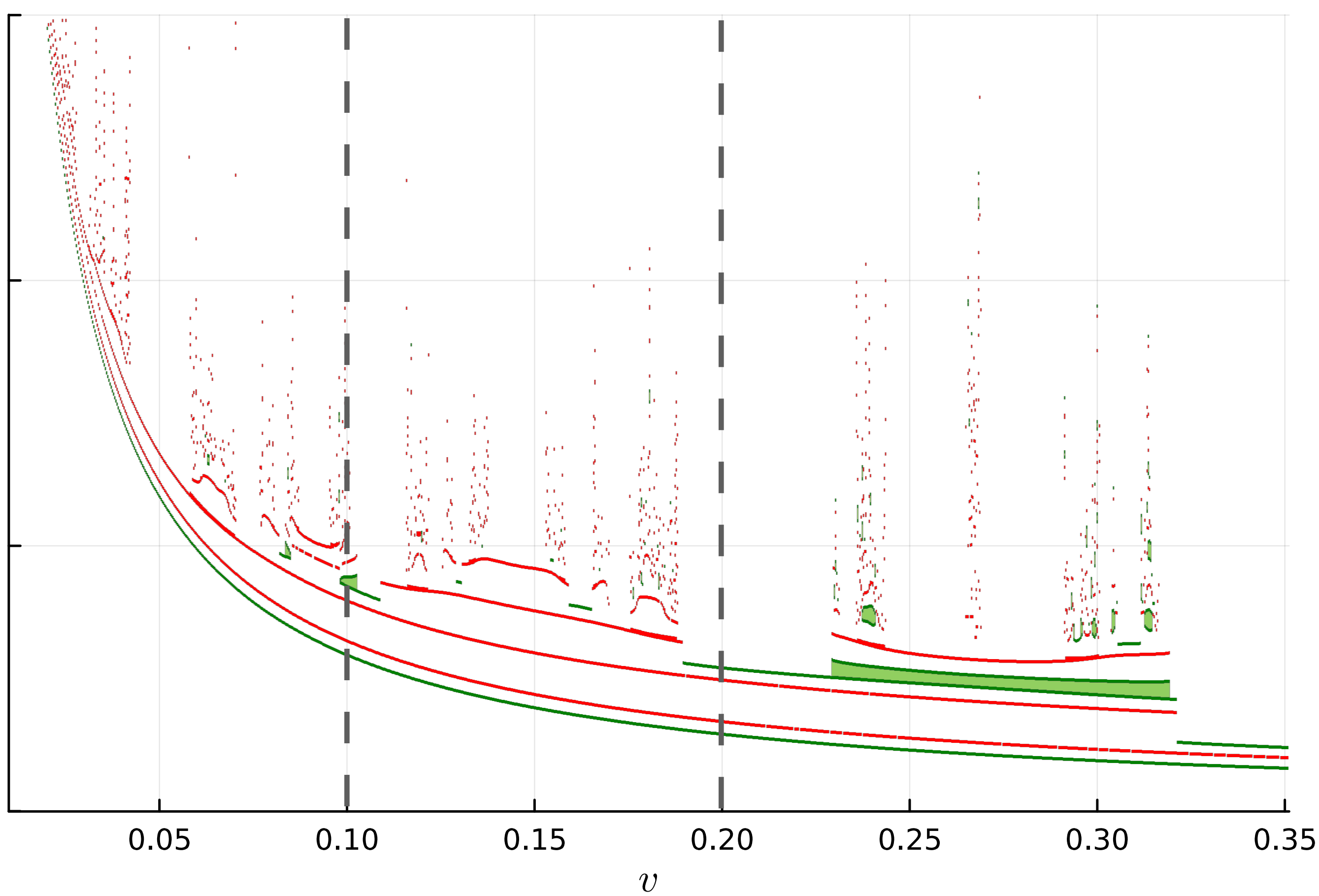


Bouncing and/or mech-oscillon decay

Scattering of mech-kinks

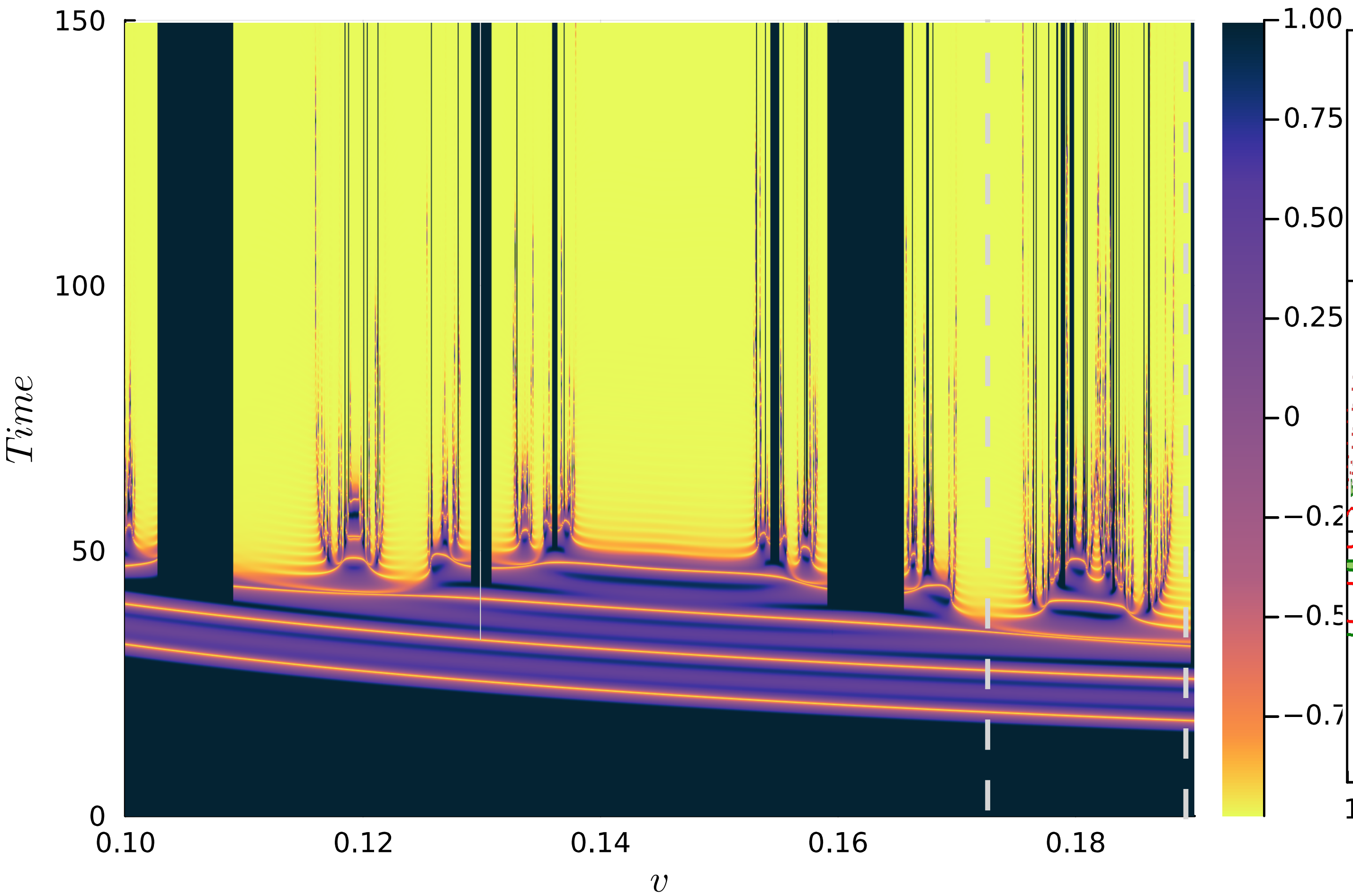


Value of the field at the center

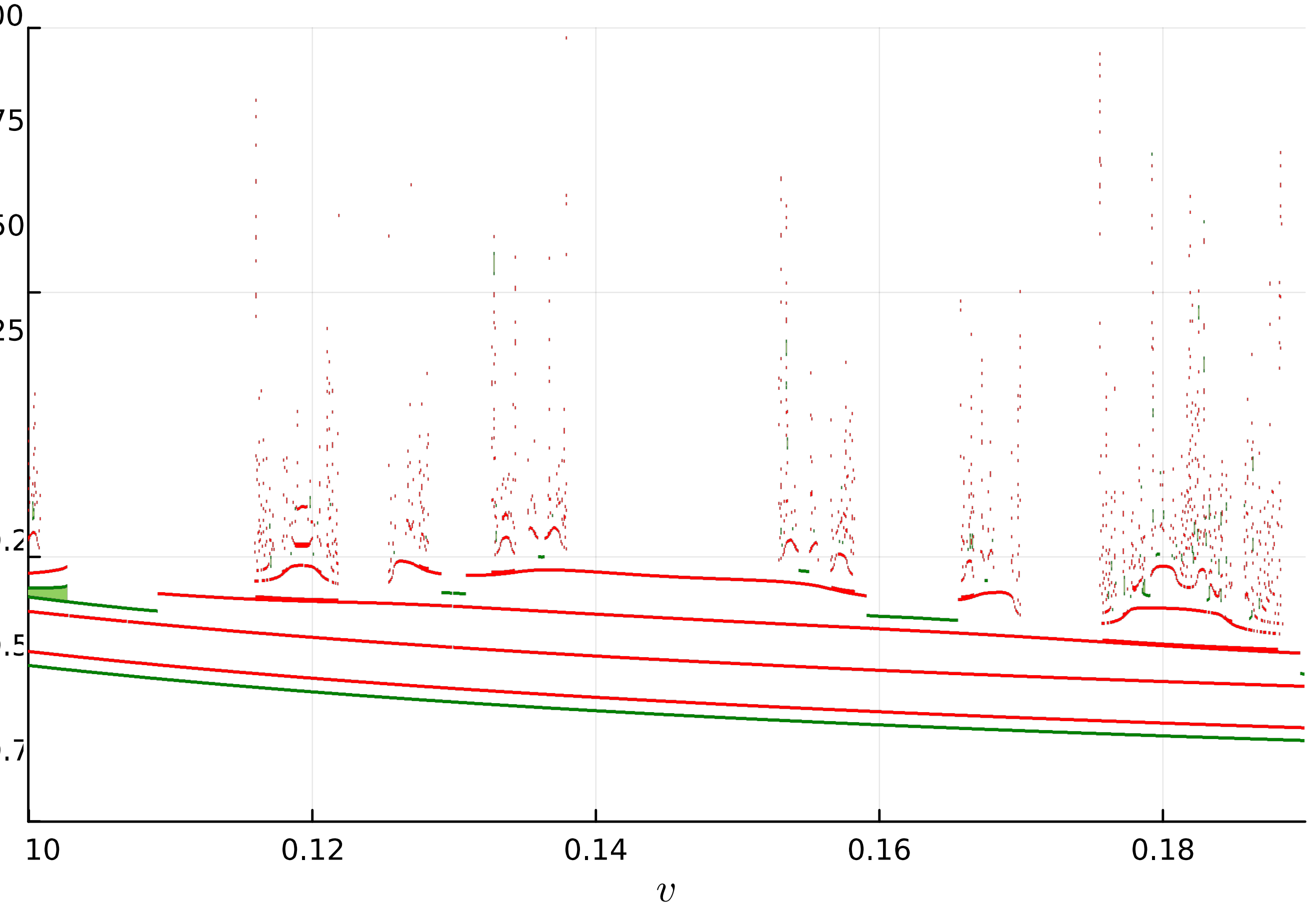


Green: $R = 0$ Red: $R = -R_K$

Scattering of mech-kinks

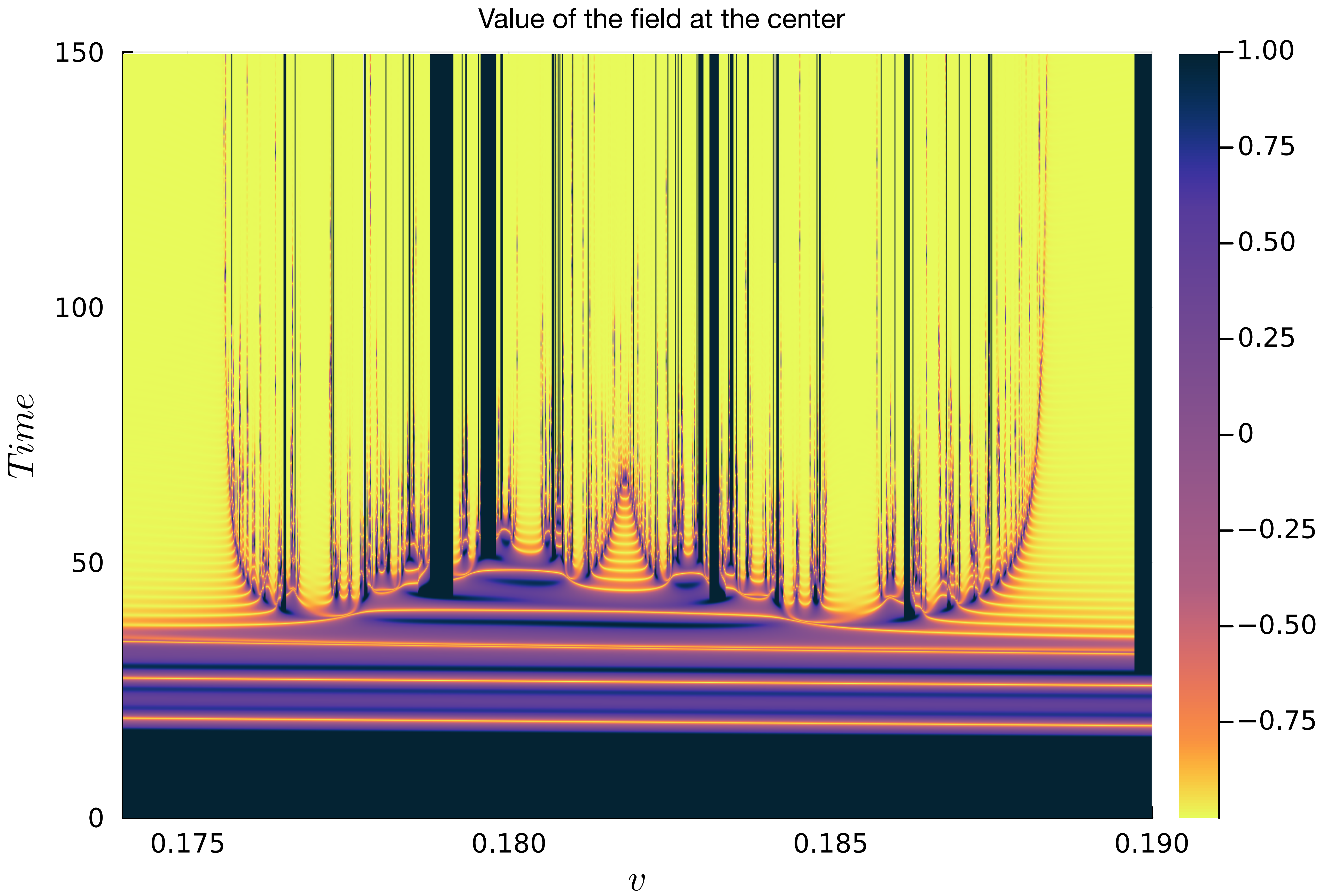


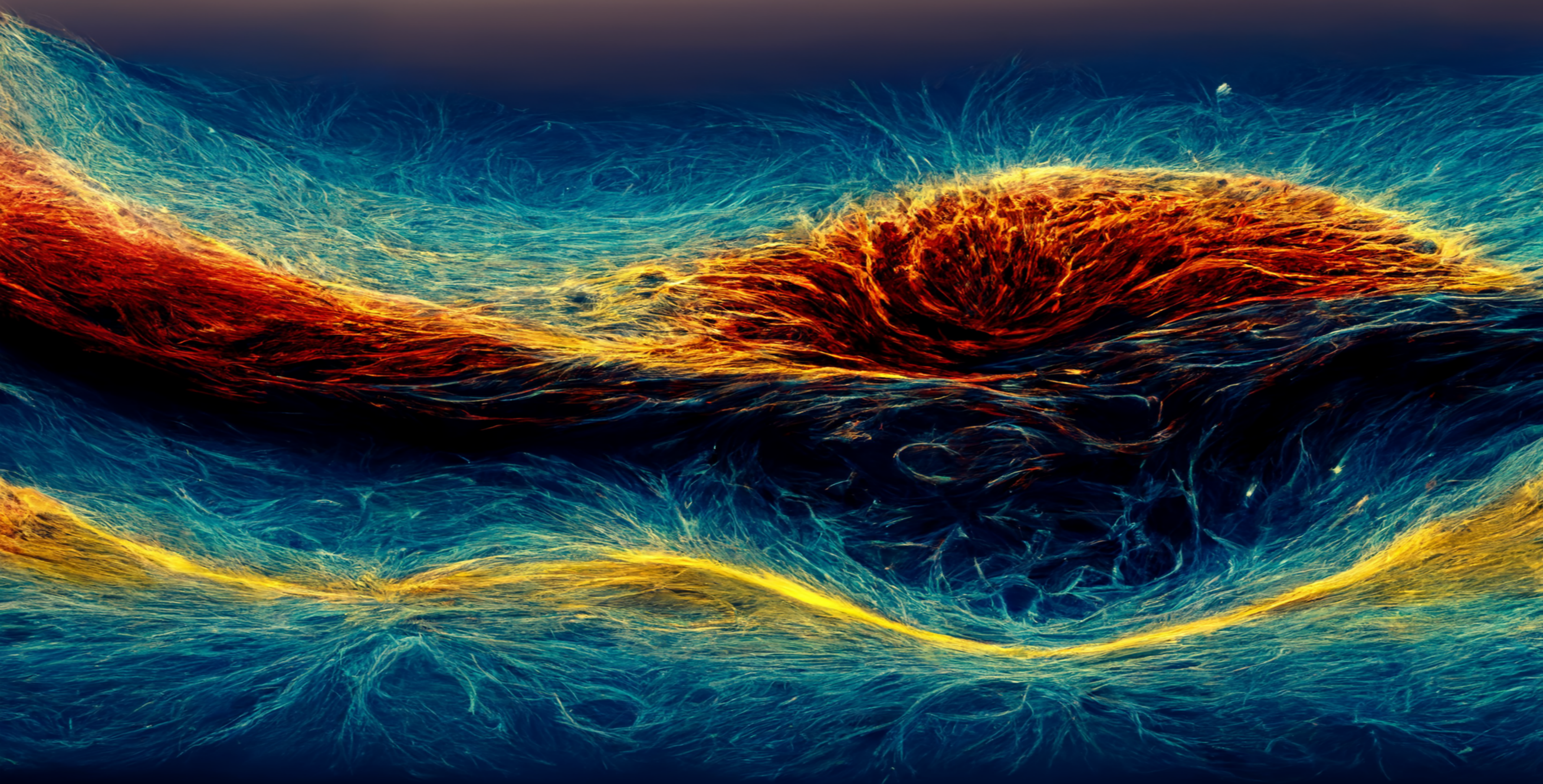
Value of the field at the center



Green: $R = 0$ Red: $R = -R_K$

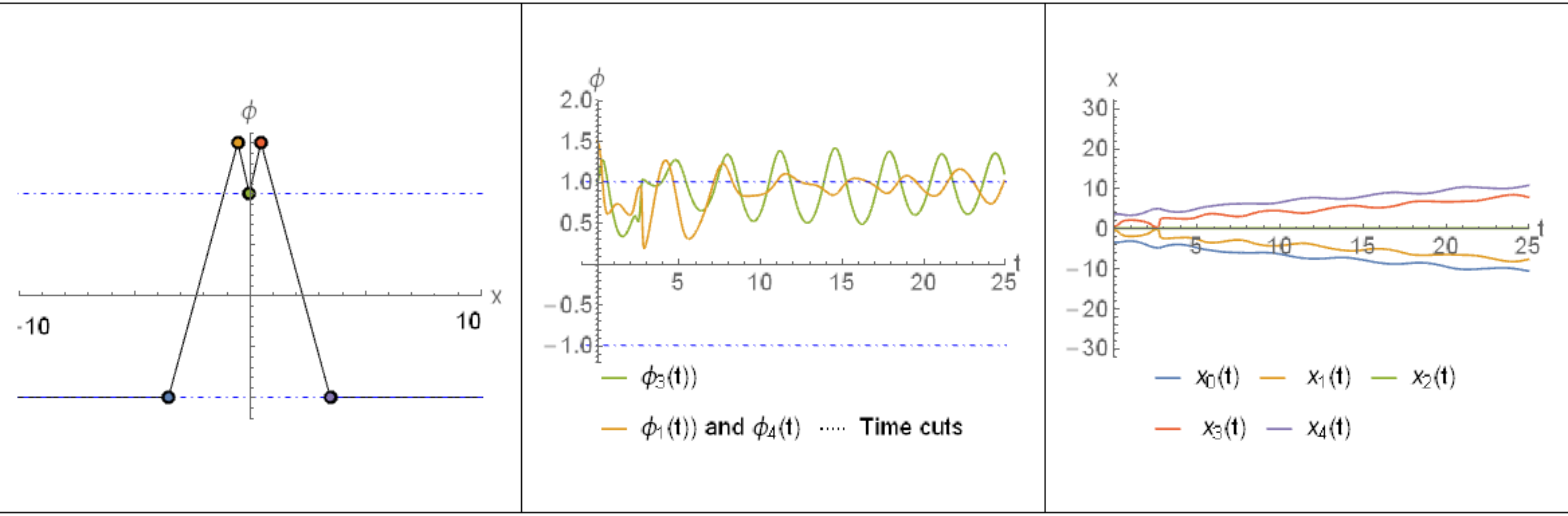
Scattering of mech-kinks



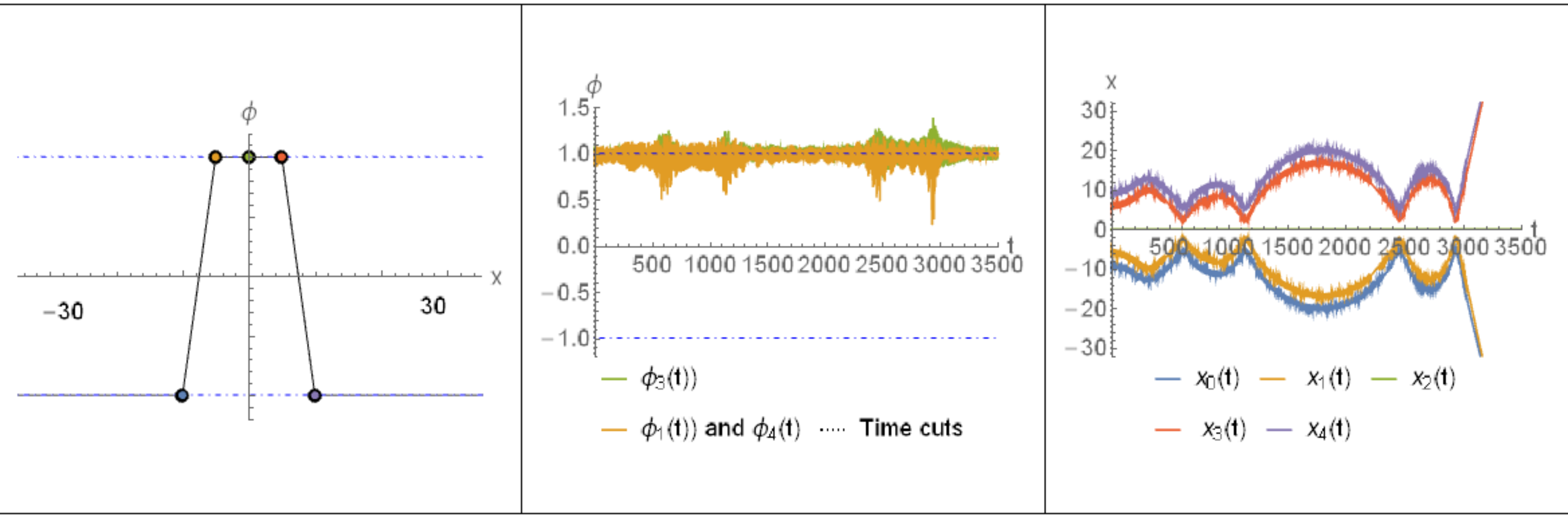


GALLERY

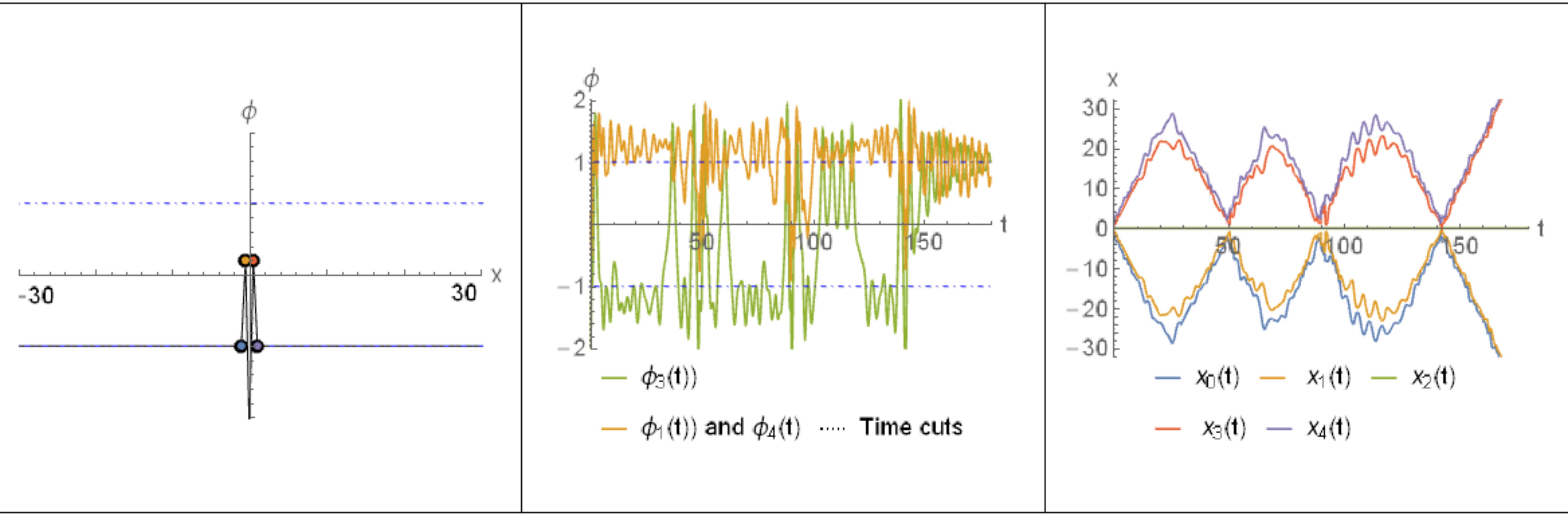
MECH-KK PAIR PRODUCTION



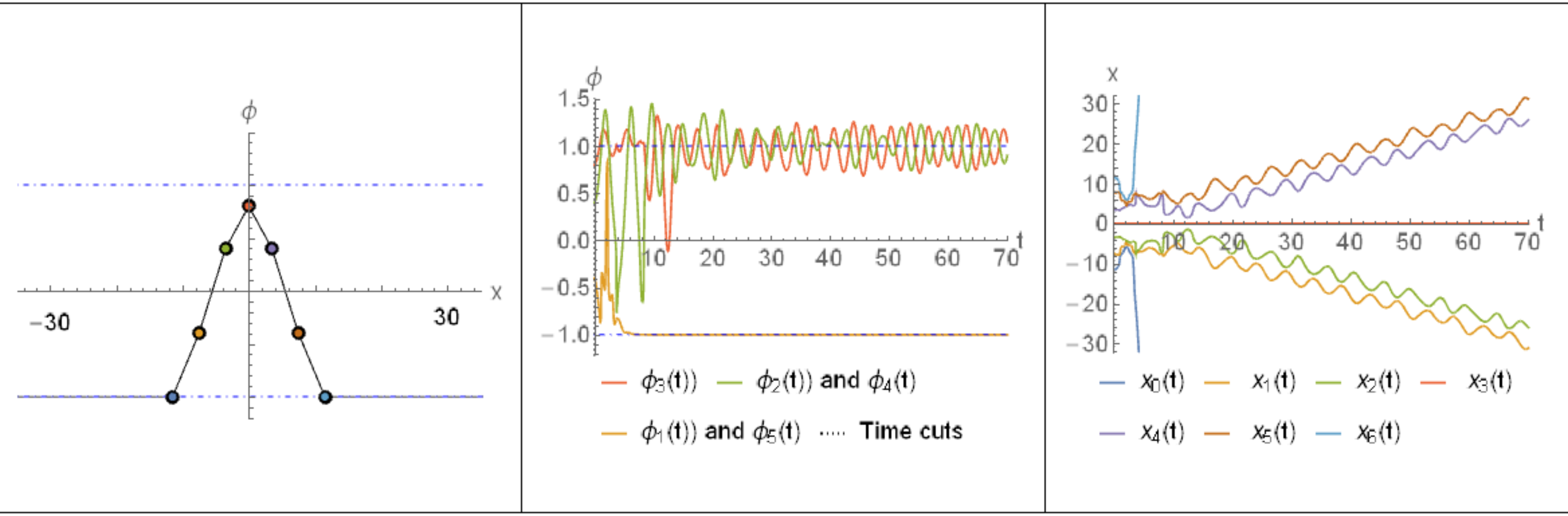
MECH-BOUNCING



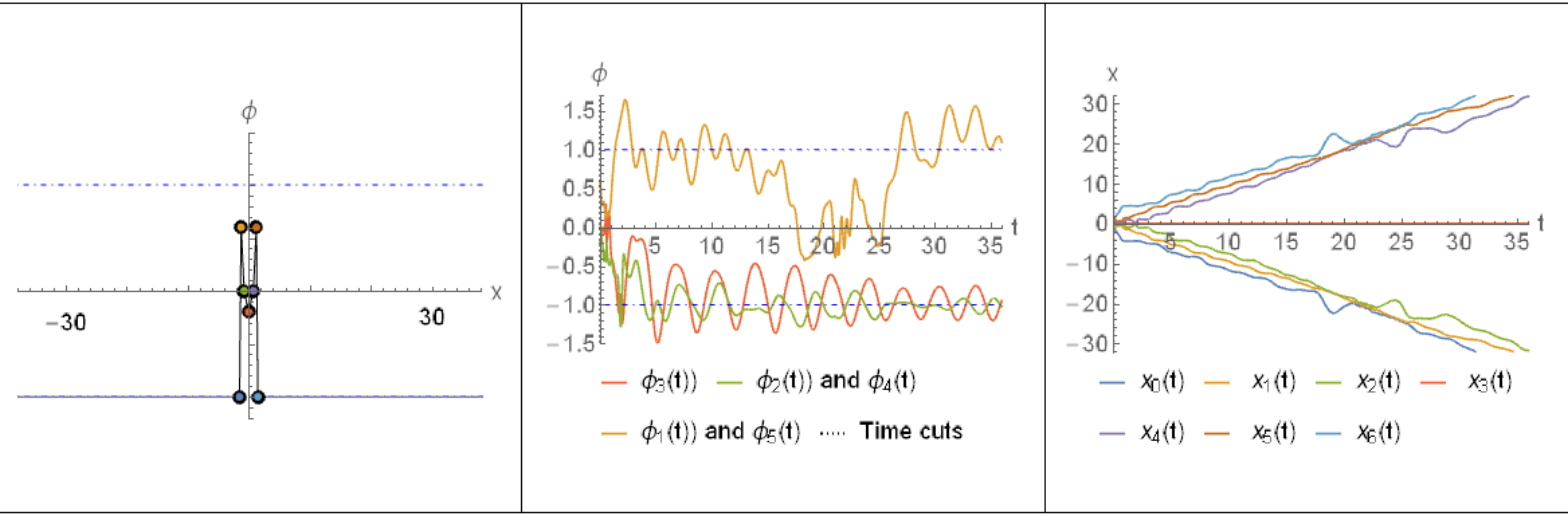
MECH-BOUNCING



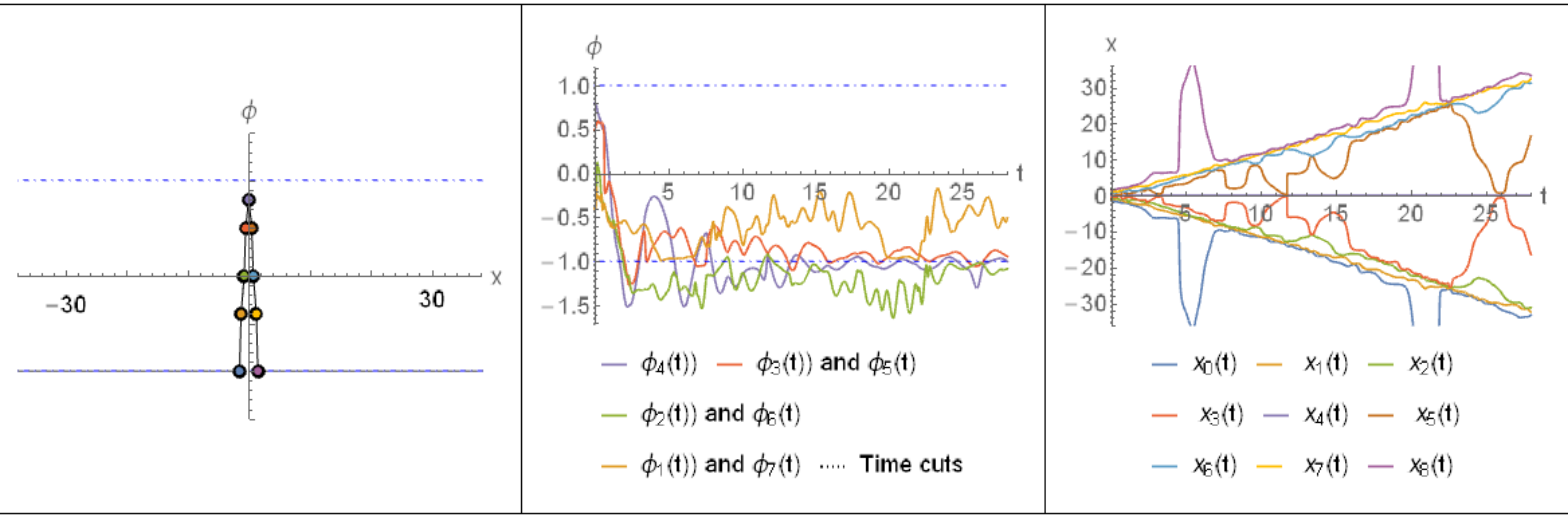
JOINT EJECTION+KK PRODUCTION



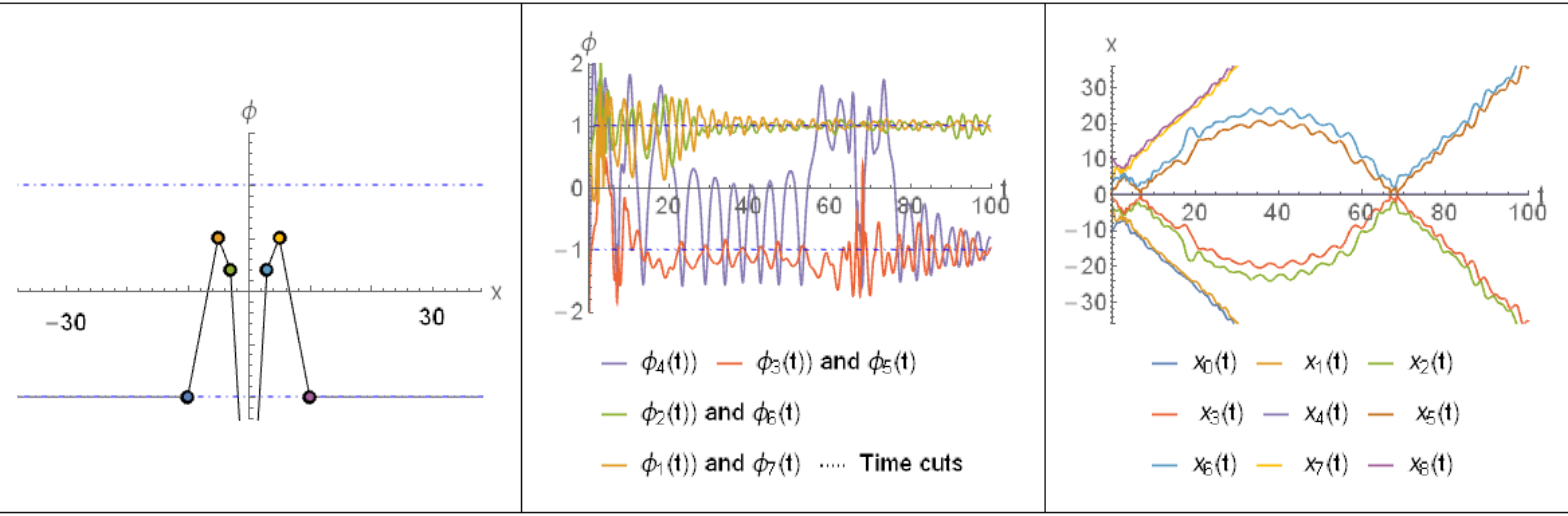
MECH-BION EJECTION



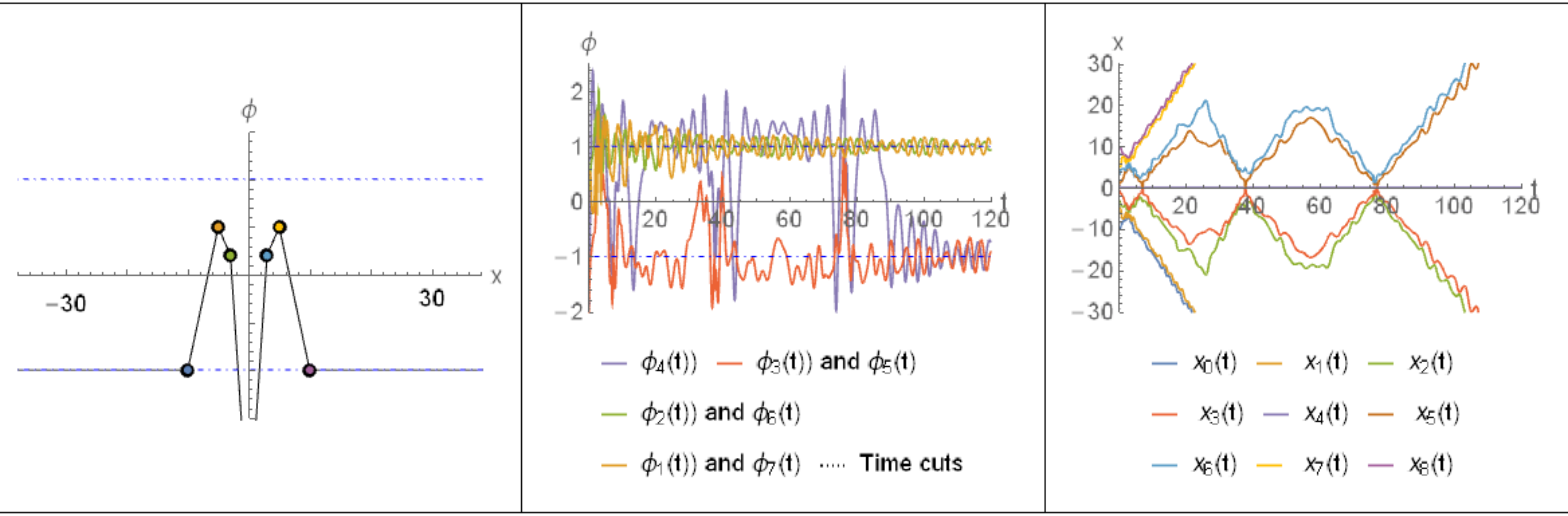
SMALL OSCILLONS/RADIATION



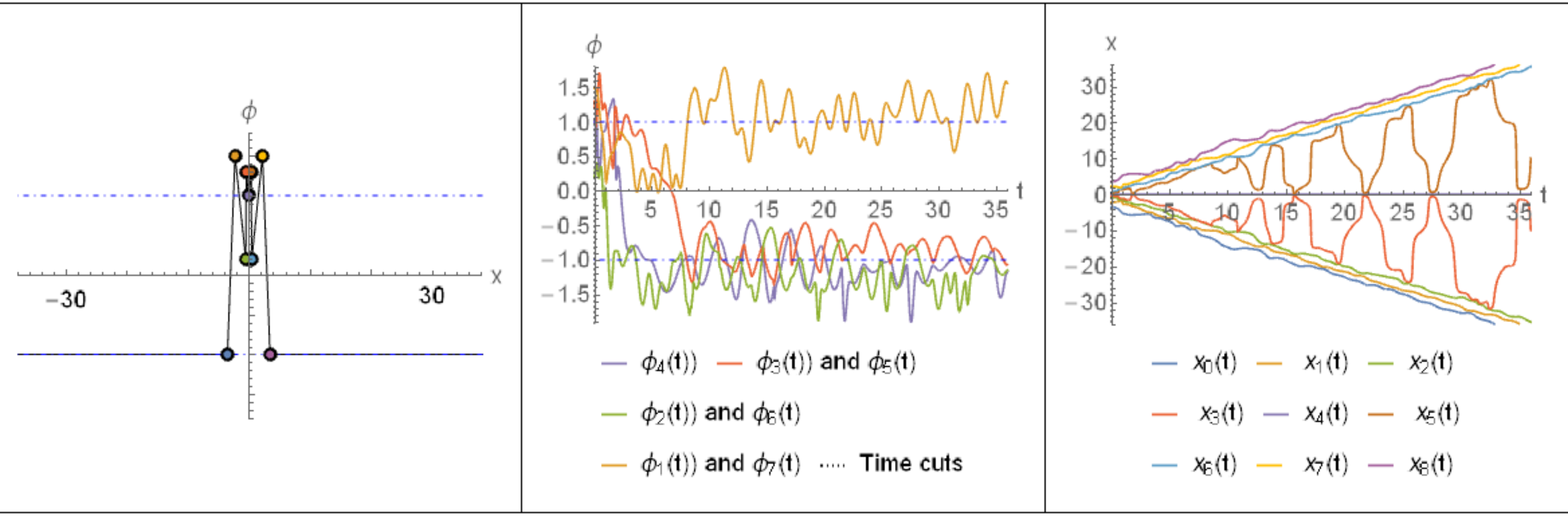
DOUBLE KK PRODUCTION+BOUNCE



DOUBLE KK PRODUCTION+2BOUNCE



BION-BION+OSCILLON



Thanks!