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Color-kinematics duality from Lagrangians

Maor Ben-Shahar

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Prague spring amplitudes

[M.B.S, H. Johansson, 2112.11452]

[M.B.S, M. Guillen, 2108.11708]

[M.B.S, L. Garozzo, H. Johansson, 2301.00233]

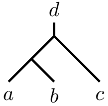
Outline

- Color-Kinemtics duality
- NMHV Lagrangian for CK duality
- Off-shell CK duality in CS
- Summary of results
- Conclusion

Color-Kinematics duality

Relationship between scattering amplitude building blocks: [Bern, Carassco, Johansson]

$$\mathcal{A}_n = \sum_{i \in \Gamma_n} \frac{c_i n_i}{D_i}$$



$$= \frac{f^{abx} f^{xcd} \times n\left(\quad\right)}{(p_a + p_b)^2}$$

Color-Kinematics duality

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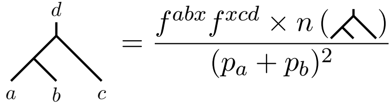
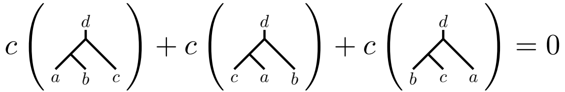
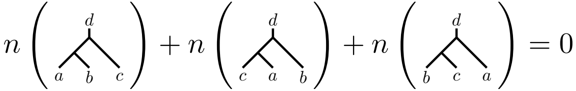


$$c_i + c_j + c_k = 0 \quad \Leftrightarrow \quad n_i + n_j + n_k = 0 \ ,$$





Double copy:

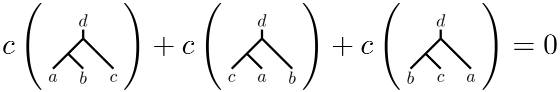
$$\mathcal{M}_n = \sum_{i \in \Gamma_n} \frac{\tilde{n}_i n_i}{D_i} \ .$$

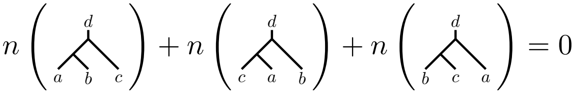
Color-Kinematics duality

Color-Kinematics duality

Can Lagrangians manifest this duality?
Can CK duality hold off-shell?
Is there a kinematic algebra responsible for the kinematic identities?

$$c_i + c_j + c_k = 0 \quad \Leftrightarrow \quad n_i + n_j + n_k = 0 \ ,$$





Lagrangians for CK duality

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If local modifications to YM come with $\Box$ they contribute to NMHV.

The NMHV Lagrangian

$$\mathcal{L}_5 = \mathsf{Tr}\Big(\frac{1}{2}A_\mu\Box A^\mu - \partial_\mu A_\nu[A^\mu, A^\nu] + B_{\mu\nu}\Box\tilde{B}^{\mu\nu} + \frac{1}{2}[A_\mu, A_\nu](B^{\mu\nu} + \Box\tilde{B}^{\mu\nu})$$
$$+ 4\partial_\nu\tilde{B}_{\mu\rho}[A^\mu, B^{\nu\rho}] - 2\partial_\rho\tilde{B}_{\mu\nu}[A^\rho, B^{\mu\nu}]\Big)\,.$$

Same amplitudes as YM, correct NMHV scalar sector numerators, full numerators at 5 points!

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$$\mathcal{L} = \mathcal{L}_5 + \mathsf{Tr}\Big(Z^\mu\Box\tilde{Z}_\mu + X\Box\tilde{X} + [\partial_\nu A^\mu, A_\mu]Z^\nu + [A^\mu, \Box A_\mu]\tilde{X} - 2[A_\mu, Z_\nu]\partial^\mu\tilde{Z}^\nu$$
$$- 2[A_\mu, X]\partial^\mu\tilde{X} + [A_\mu, \tilde{Z}_\nu]\big(B^{\mu\nu} - \Box\tilde{B}^{\mu\nu}\big) - [A^\mu, X]\tilde{Z}_\mu + \frac{1}{2}[A^\mu, \Box\tilde{X}]\tilde{Z}_\mu$$
$$- 2[A_\mu, B^{\mu\nu}]\partial_\nu\tilde{X} + \frac{1}{2}[\tilde{Z}_\mu, \tilde{Z}_\nu]\big(B^{\mu\nu} + \Box\tilde{B}^{\mu\nu}\big) + 4[B^{\mu\nu}, \partial_\mu\tilde{B}_{\nu\rho}]\tilde{Z}^\rho$$
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Tested to 11 points.

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$$- 2[B^{\mu\nu}, \partial_\nu\tilde{X}]\tilde{Z}_\mu\Big)\,.$$

Tested to 11 points. How about off-shell CK duality?

Kinematic algebra in Chern-Simons theory

Consider Axelrod-Singer formulation:

$$S = \frac{k}{2\pi} \int d^3x d^3\theta \, \mathrm{Tr} \left(\frac{1}{2} \Psi Q \Psi + \frac{i}{3} \Psi \Psi \Psi \right) ,$$

Superfield for Faddeev-Popov ghosts and vector:

$$\Psi = c + \theta_\mu A^\mu + \theta_\mu \theta_\nu \epsilon^{\mu\nu\rho} \partial_\rho \bar{c} ,$$

Kinetic (exterior derivative, world line BRST operator)

$$Q = \theta_\mu \partial^\mu .$$

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To define the propagator $b = \frac{\partial}{\partial \theta^\mu} \partial^\mu$ (co-differential, b-ghost):

- $b^2 = 0$
- $bQ + Qb = \partial_\mu \partial^\mu = \Box$

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Feynman rules in gauge $b\Psi = 0 \to \partial_\mu A^\mu = 0$:

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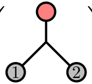
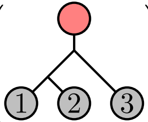
n () $= b(\Psi_1 \Psi_2)$

n () $= b(b(\Psi_1 \Psi_2)\Psi_3)$

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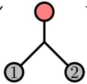
$n\Big($ 

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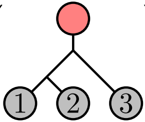
$n\Big($ 

$\Big) = b(b(\Psi_1 \Psi_2)\Psi_3)$

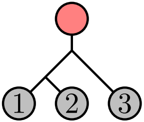
From Leibniz rule: $b = \frac{\partial}{\partial \theta^\mu} \partial^\mu$:

$$b(b(\Psi_1 \Psi_2)\Psi_3) + \text{cyclic}(1,2,3) = 0$$

Using $b^2 = 0$ which implies $b(\Psi_i) = 0$.

Kinematic algebra in Chern-Simons theory

Algebraic interpretation of:



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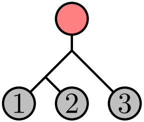

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Jacobi identity for diffeo generators:

$$L_\psi(f) \equiv b(\psi f)\ , \qquad [L_\psi, L_\phi] = L_{b(\psi\phi)}$$

$$f(x,\theta) \to f + b(\psi f) + b(\psi b(\psi f))/2 + ... \approx f(x + \tfrac{\partial}{\partial\theta}\psi, \theta + \partial\psi)$$

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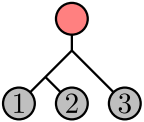

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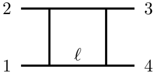
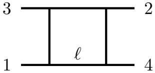
diffeos in (x,θ) space that preserve $d^3x \to d^3x$, $d^3\theta \to d^3\theta$.

Kinematic algebra in Chern-Simons theory

One loop example:



$$= \int d^3\theta d^3\tilde{\theta}\, \Psi_4 \delta^3_{\theta,\tilde{\theta}}\; b\Big(b\big(b(\Psi_1 \tilde{b}(\delta^3_{\theta,\tilde{\theta}}))\Psi_2\big)\Psi_3\Big)\ ,$$



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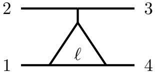

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Define $X \equiv b(\Psi_1 \tilde{b}(\delta^3_{\theta-\tilde{\theta}}))$, which satisfies $bX = 0$,

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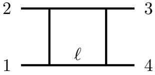

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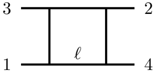

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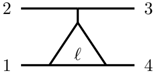

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$$\begin{aligned} \rightarrow\quad & b(b(X\Psi_2)\Psi_3) - b(b(X\Psi_3)\Psi_2) - b(Xb(\Psi_2\Psi_3)) \\ =\quad & b(b(X\Psi_2)\Psi_3) + b(b(\Psi_3 X)\Psi_2) + b(b(\Psi_2\Psi_3)X) = 0 \end{aligned}$$

Summary of results

- Obtained Lagrangian for NMHV CK duality with fields $A,\ (B,\tilde{B}),\ (Z,\tilde{Z}),\ (X,\tilde{X})$.
- CS action: $S \sim \langle \Psi Q \Psi + \Psi^3 \rangle$
- Propagator-numerator b: $b^2 = 0$, second-order wrpt vertex
- Algebra follows from $b\Psi = 0$ and second order b operator
- See related: [Reiterer; MBS, Guillen 2108.11708]
- Additionally: $\mathcal{N} = 4$ CS-matter theories satisfying CK duality
- Double copy to DBI, up to maximal $\mathcal{N} = 8$ SUSY

Future research directions

- Off shell double copy? [See Felipes talk]
- NNMHV Lagrangians
- Loops in YM theory
- What is the double copy of CS?
- CK duality for more $\langle \Psi Q \Psi + \Psi^3 \rangle$ actions