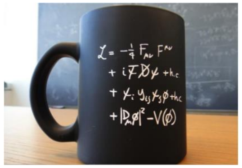
Introduction to Particle Physics

SWEDISH PHYSICS TEACHERS 2023

Handout 2

- Symmetry, invariance and conservation laws
- Toward the SM Lagrangian
- Higgs field
- the SM Lagrangian



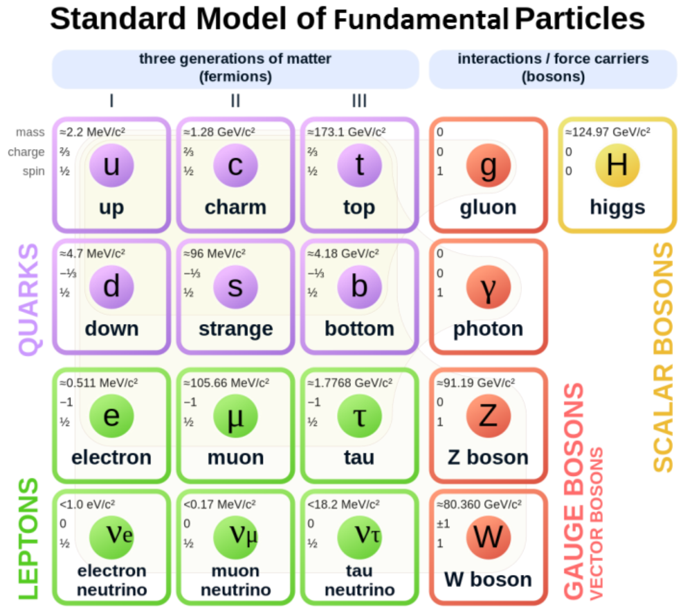
Credits to:

C. Grosjean and D. Tong : https://indico.cern.ch/event/1254879/timetable/

Summary of Fundamental Interactions

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak	Electromagnetic	Strong	
		Unification		Fundamental	Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20



Invariance of the physics laws

The laws of nature should be invariant under certain transformations since "nature does not know" how we observe it!

For example, the physics laws should be invariant w.r.t. the changing of reference systems ☺ (Special Relativity)

Abandon the anthropocentric view and move towards a more abstract mathematical view, universally valid!

Symmetry, invariance and conservation laws

1. System is invariant under a transformation ↔ it posses a symmetry;
2. System is invariant under a transformation ↔ conserved quantities (A.E. Nother Theorem)
3. SIMMETRY ↔ INVARIANCE ↔ CONSERVED QUANTITY (A.E. Nother Theorem)

Lagrangian in the classical Mechanics

The Newton law of classical mechanics

$$\vec{F} = m\vec{a} \quad \text{or} \quad V'(x) = -m\ddot{x}$$

can be obtained by requiring the least action principle

$$\delta S = 0$$

where

the action: $S = \int_{t_1}^{t_2} dt \mathcal{L}(x, \dot{x})$ with the (classical) Lagrangian: $\mathcal{L}(x, \dot{x}) = \frac{1}{2} m\dot{x}^2 - V(x)$

(Hamiltonian/energy: $\mathcal{H} = \dot{x}\frac{\delta \mathcal{L}}{\delta \dot{x}} - \mathcal{L} = \frac{1}{2} m\dot{x}^2 + V(x)$)

Kinetic term

Potential term

Euler-Lagrange equations

$$\delta S = \int_{t_1}^{t_2} dt \left(\frac{\delta \mathcal{L}}{\delta x} - \frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{x}} \right) \delta x + \text{boundary terms} = 0 \quad \Rightarrow \quad \frac{\delta \mathcal{L}}{\delta x} = \frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{x}}$$

For the classical Lagrangian: $-V'(x) = m\ddot{x}$

CG SSLP2023

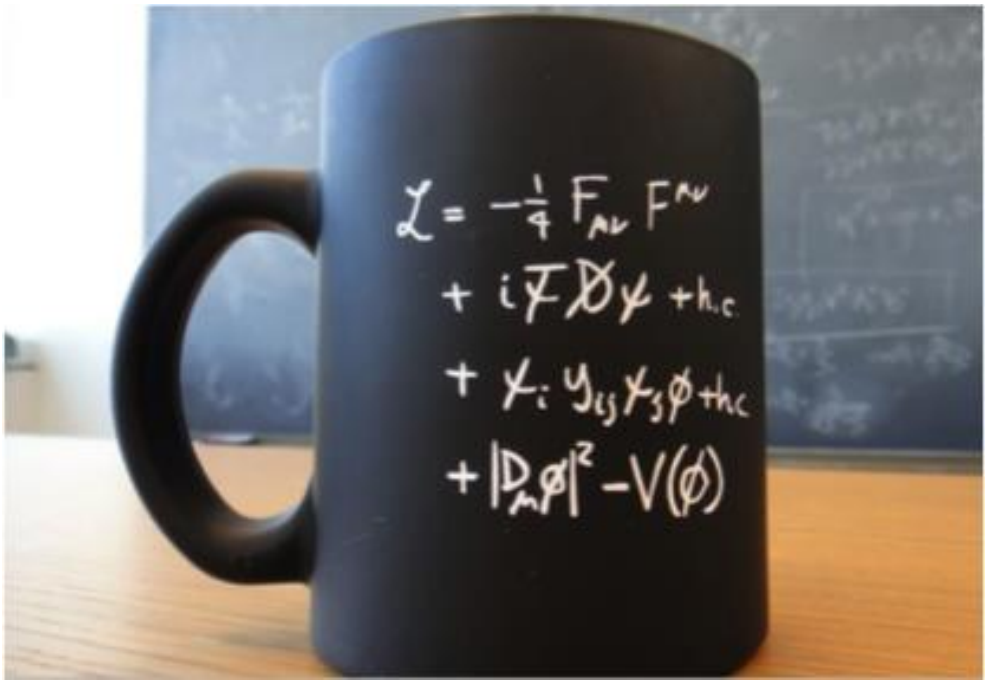
6

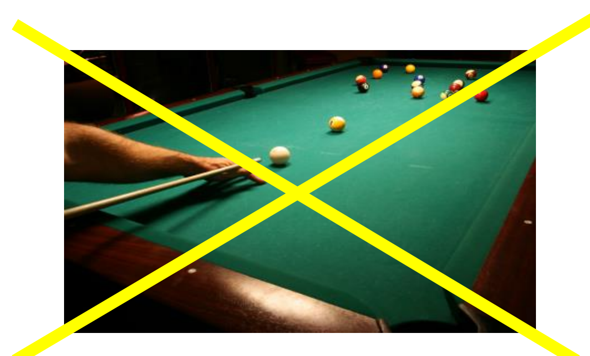
Classical invariance and conservation laws

The evolution of a physical system of interacting particles can be fully described by equations in the Lagrangian (or Hamiltonian) formulation;(Eulero-Lagrange eq. ->x=x(t))

1. If L is invariant under time translation ($t \to t+t_0$) → E_{tot} is conserved;
2. If L is invariant under space translation ($x \to x+x_0$) → the momentum **p** is conserved;
3. If L is invariant under axes rotation(θ, φ) → the angular momentum J is conserved.

Toward the “Lagrangian of the SM





REM:
- particle <-> quantum field
- Complementary principle;
- Heisenberg indetermination principle.

Lagrangians for Particle Physics

Equations of motion, like $\vec{F} = m\vec{a}$, are **covariant** under the action of a symmetry.

(Under the Transformation of Galileo between two reference systems) gdc

Lagrangians are **invariant**.

That makes identifying the symmetries of Nature much easier.

—Particle Physics—

particles ↔ fields with specific transformation properties under some fundamental symmetries

build a Lagrangian (i.e. a function of the these fields and their space-time derivatives) that remains invariant under the action of the symmetry transformations.

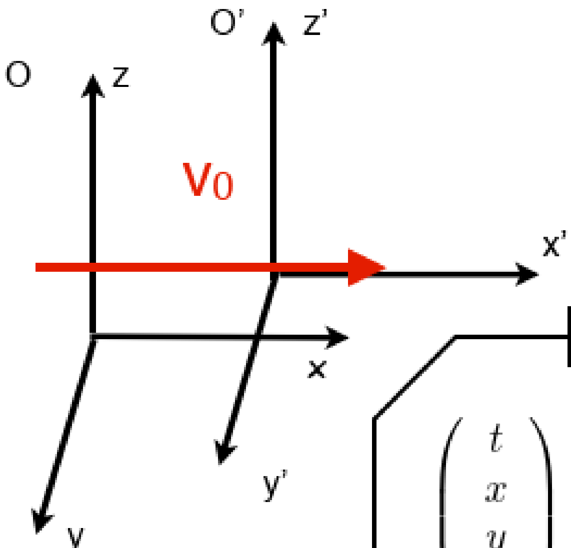
1) L is invariant under the Special Relativity space-time transform (Lorentz invariant)

2) L is invariant under (local)transformations of the particle fields and of the potentials (gauge invariant)

CG SSLP2023

7

Galilean and Lorentz Transformations



Consider two observers

in relative motion with a constant speed v_0 along the x-axis

they use their own systems of coordinates (t,x,y,z) and (t',x',y',z')

Galilean transformations

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} t' = t \\ x' = -\beta_0 ct + x \\ y' = y \\ z' = z \end{pmatrix} \quad \text{with} \quad \beta_0 = \frac{v_0}{c}$$

in particular

$$v' = v - v_0$$

the speed can be arbitrarily large.

Lorentz transformations

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0 (ct - \beta_0 x) \\ x' = \gamma_0 (-\beta_0 ct + x) \\ y' = y \\ z' = z \end{pmatrix} \quad \text{with} \quad \beta_0 = \frac{v_0}{c}, \quad \gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}}$$

in particular

$$v' = \frac{v - v_0}{1 - v \cdot v_0 / c^2}$$

The speed of light is

the same for all observers:

if v=c than v'=c too

Note: $\Delta^2 \equiv (ct)^2 - x^2 - y^2 - z^2 = (ct')^2 - x'^2 - y'^2 - z'^2 \equiv \Delta'^2$

Summary on Simmetries

SYMMETRY name	INVARIANCE	CONSERVED QUANTITY
Classical Space and time translation symmetries; axes rotation symmetry. Space and time symmetry (Galilean transformation)	System invariance under: • space and time translations; • Axes rotation • Change of reference sys.	Momentum **p**, Energy E and angular momentum **L,** respectively
SPACE-TIME symmetries	Lorentz invariance	Square modules of four-vectors: **x**, E-**p**
Internal local symmetry of Lagrangian intercation: GAUGE symmetry (classic ED, QED, QCD, EW)	System Invariant under GAUGE transform	Strength of the interaction

The SM ingredients

Feynman diagram of the muon beta decay

ψ_i

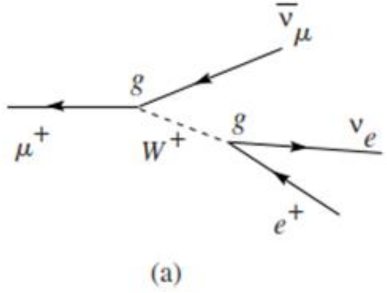


Fig. 7.4. Muon beta decay.

ψ_f

The lifetime τ of the μ depends on the transition amplitude from ψ_i to ψ_f.

It is given by the "matrix element" of the interaction Lagrangian between the final $|f\rangle$ and the initial $|i\rangle$ states. With QM symbols:

$$M_{fi} = \langle \psi_f | L_{int} | \psi_i \rangle$$

$$|M_{fi}|^2 \rightarrow \tau_\mu = 10^{-6} s \ that\ is\ the\ experimental\ value!$$

L properties

Fermion Lagrangian

$$\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \psi$$

ψ 4-component Dirac spinor describes a spin-1/2 particle when quantised

$\gamma^\mu \ (\mu = 0, 1, 2, 3)$ are four 4x4 matrices

Kinetic term

Potential term

- **Equation of motion:**

$$0 = \delta\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \delta\psi$$

Dirac equation $\left(i\gamma^\mu \partial_\mu - m \right) \psi = 0$

Dirac Equation

Equation of motion of a fermion :

Dirac Equation (1928):

$$\left(i\gamma^{\mu}\partial_{\mu} - \frac{mc}{\hbar}\right)\Psi = 0$$

$$E = \begin{cases} +\sqrt{p^2c^2 + m^2c^4} & \text{matter} \\ -\sqrt{p^2c^2 + m^2c^4} & \text{antimatter} \end{cases}$$

$$E = \vec{\alpha}\vec{p}c + \beta mc^2$$

$$\gamma^0 = \beta,\ \gamma^i = \beta\alpha^i,\ \{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}$$

positron (e^+) discovered by C. Anderson in 1932

In this equation, ψ is the Dirac bi-spinor

$$\psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} \qquad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \qquad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \qquad (2.28)$$

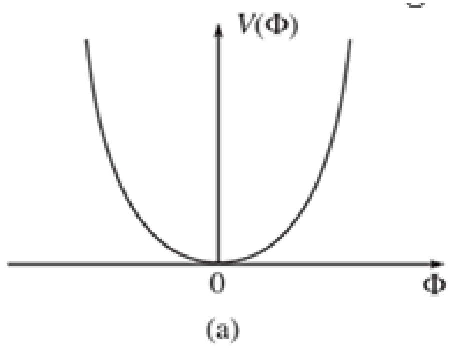
The Higgs mechanism

The Higgs mechanism was proposed (1960) to provide mass to the weak gauge bosons $W^{\pm}$ and Z°, the interaction carriers, without destroying the gauge invariance of the EW Lagrangian !

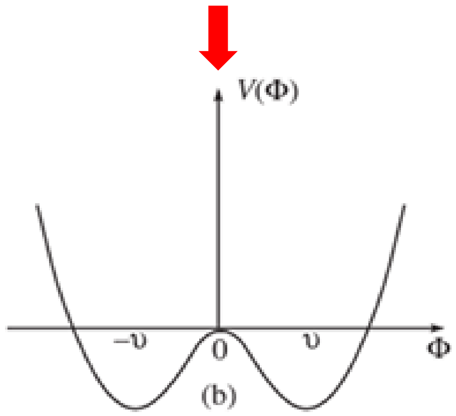
this was to unify the Weak and EM interactions in a gauge QFT (Electroweak interaction);

After about 50 years, The Higgs boson was observed at the LHC and presented on the 4th July 2012 at CERN;

The Higgs field fills the Universe with a non zero value and provides mass to the fundamental particles;

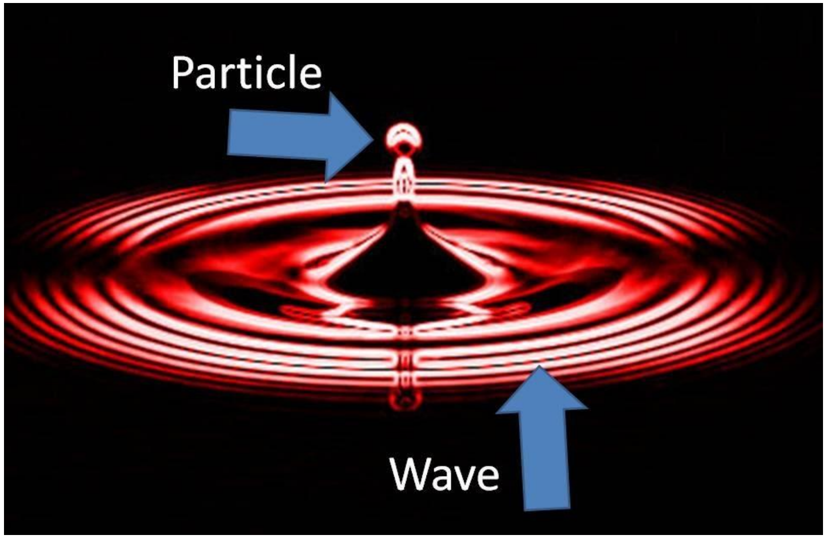


~ 10^{-12} s after the Big-Bang

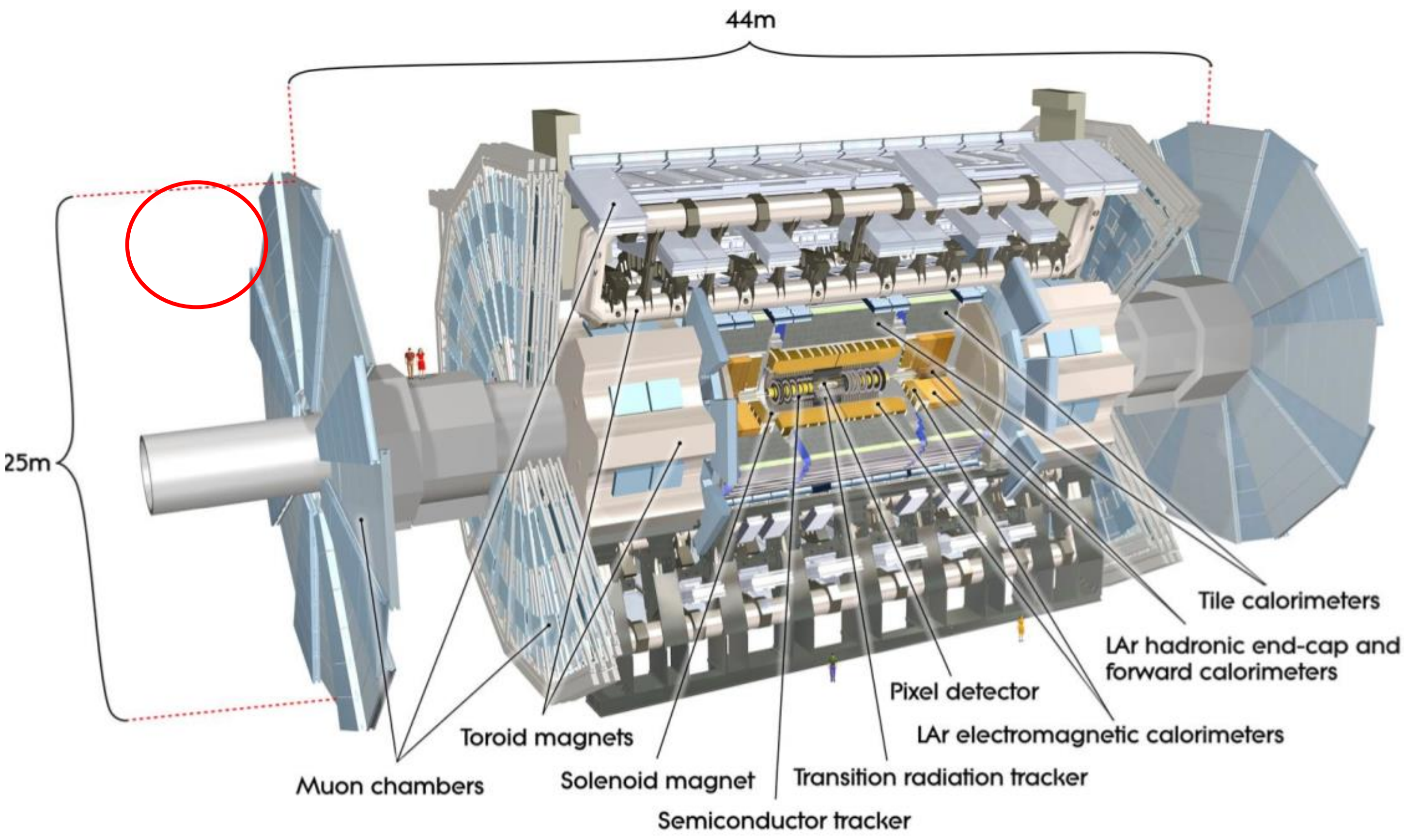


Observation of the Higgs boson

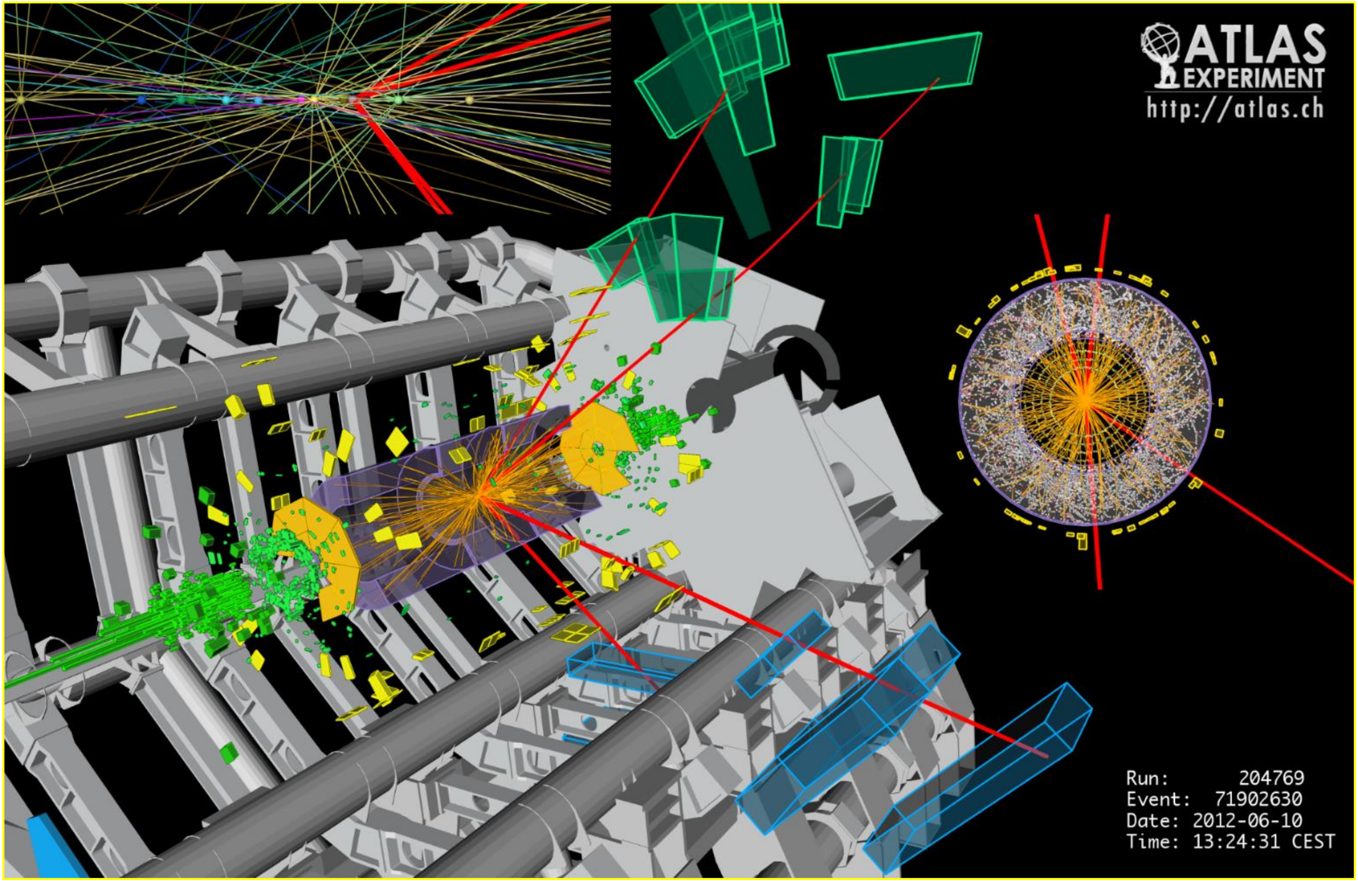
Two protons interacting in the LHC excite the Higgs field that manifests itself with the emission of the boson with a mass of 126 GeV.



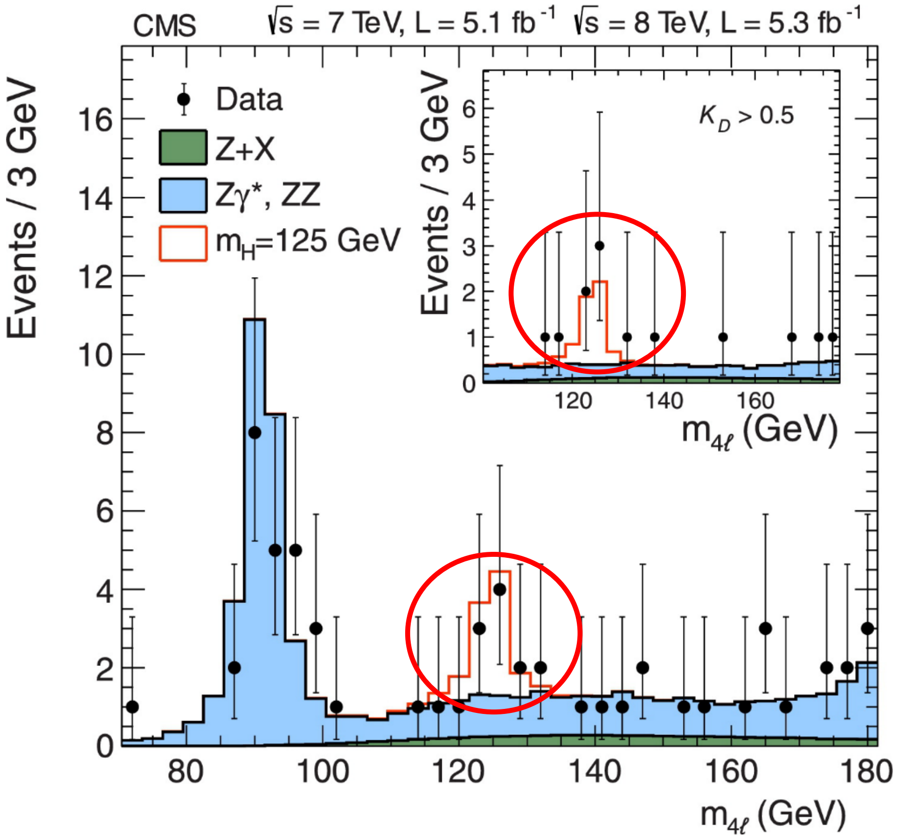
The “giant” of LHC: ATLAS

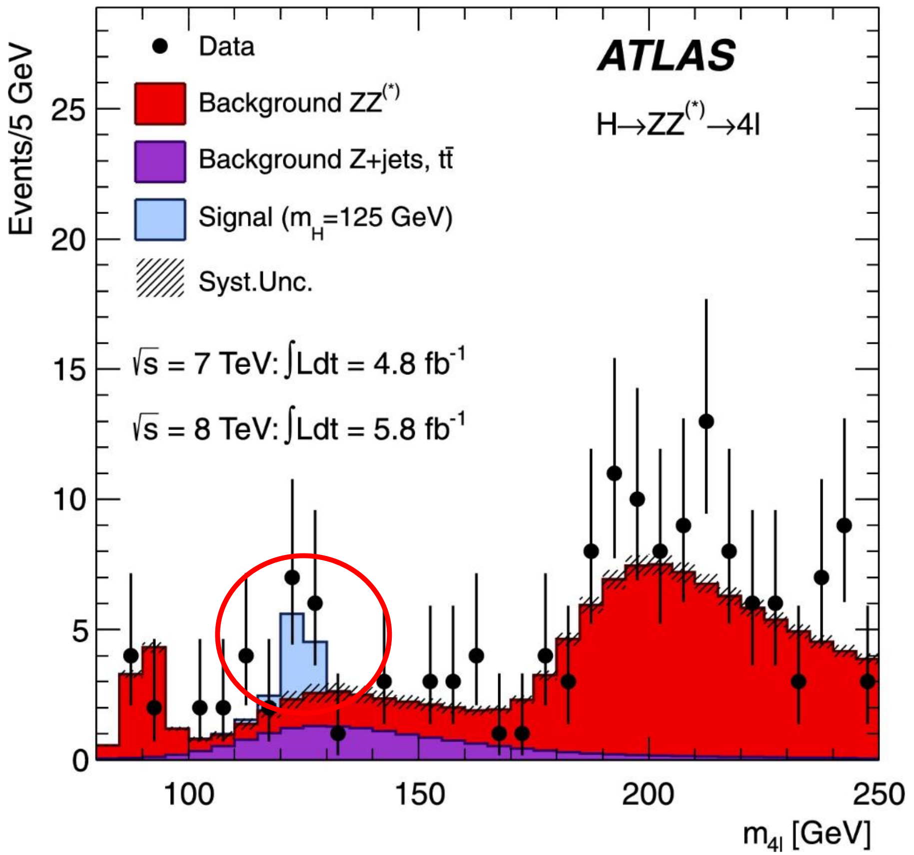


H→Z^0Z^0→4 leptons

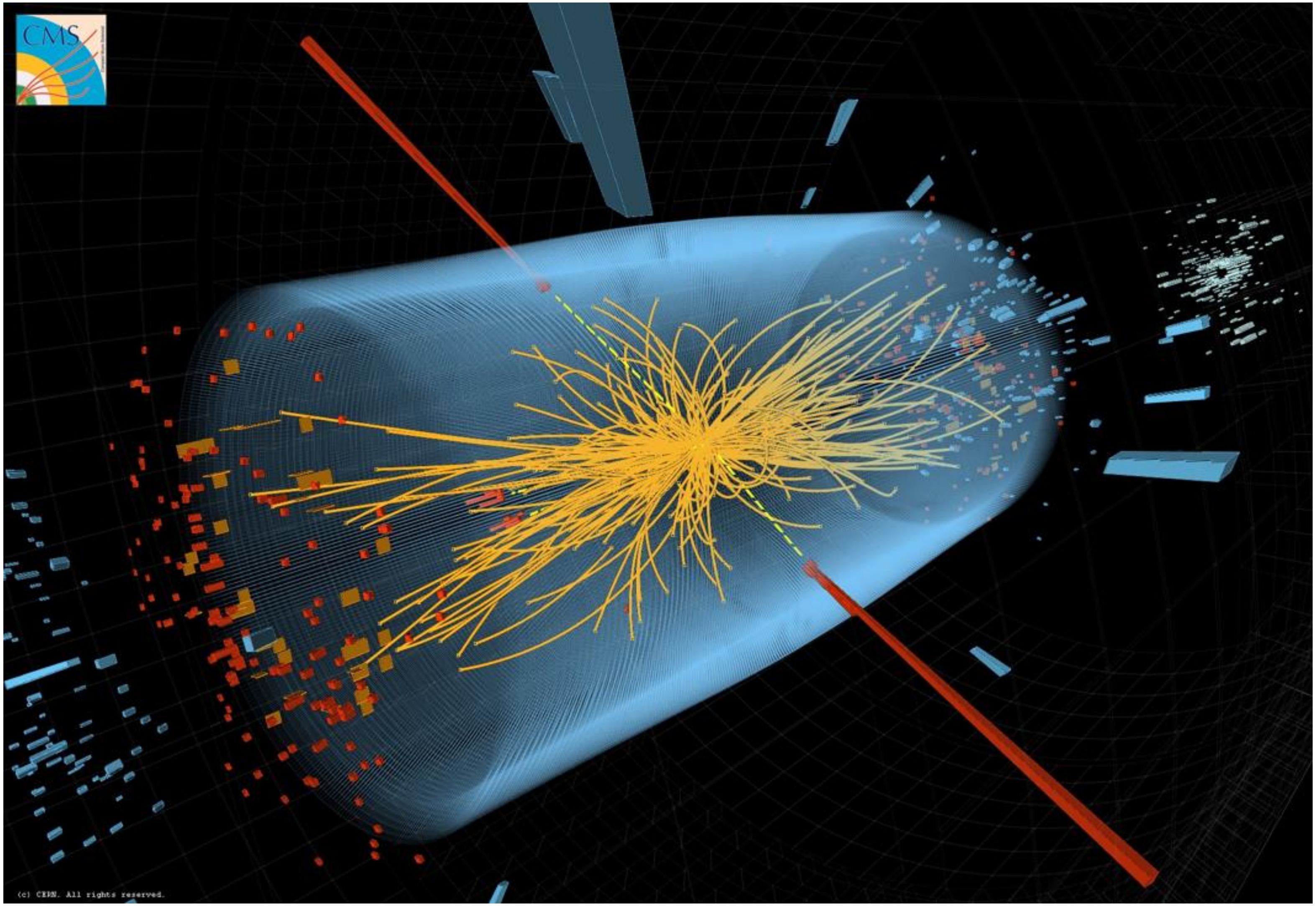


Invariant mass plot H→ZZ→4 leptons

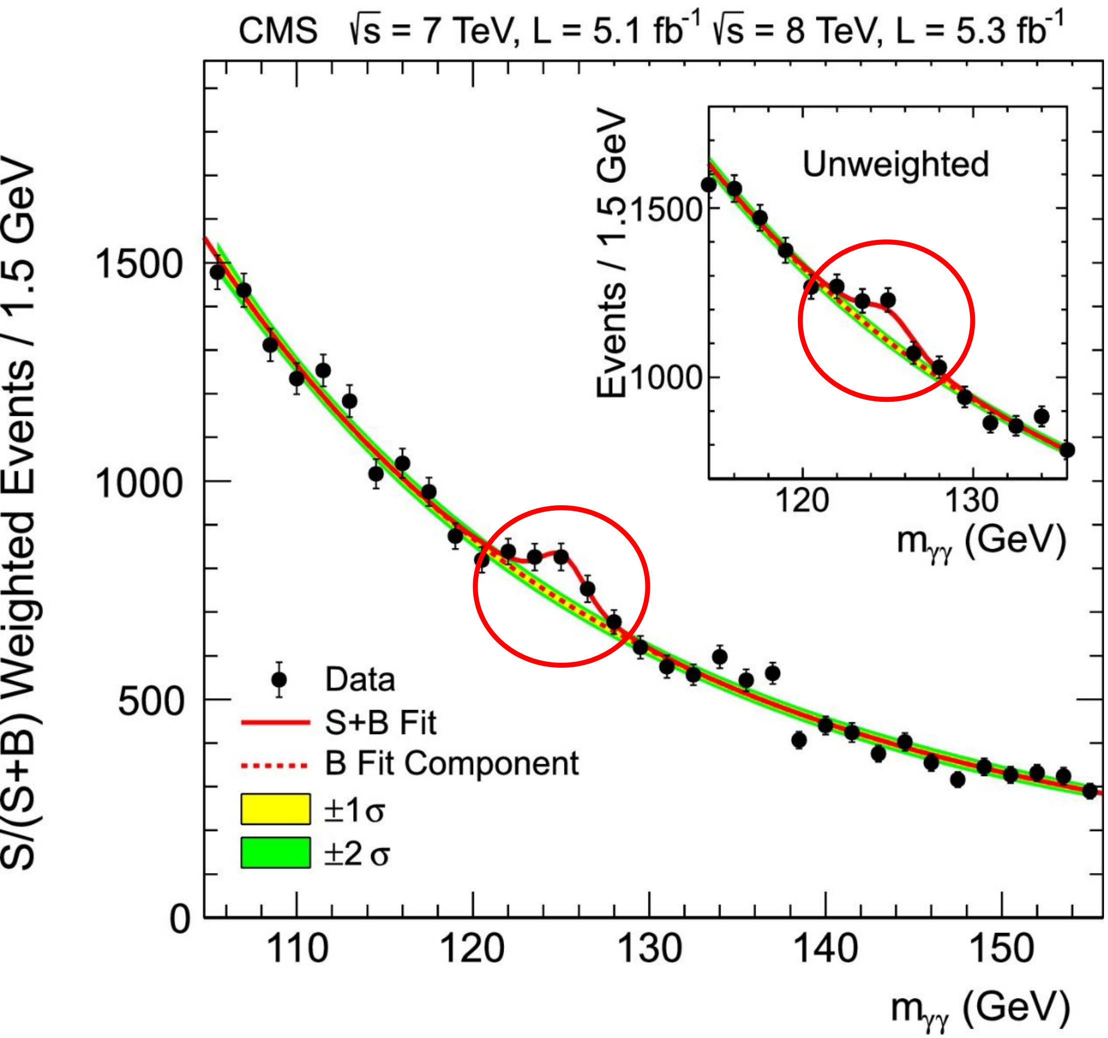


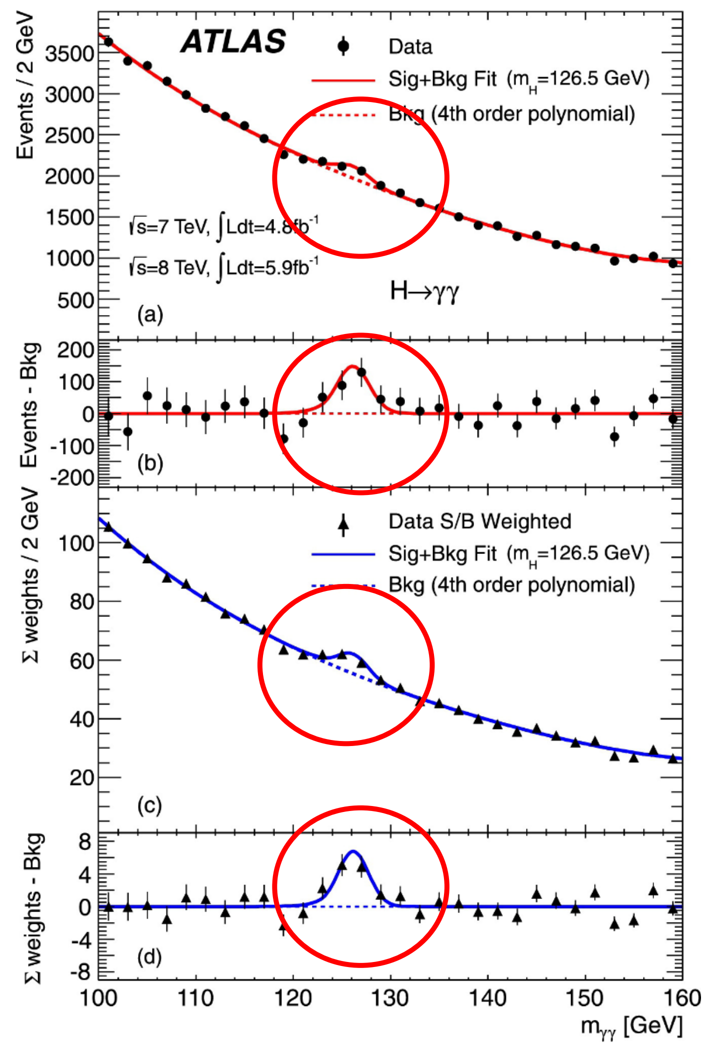


H→γγ

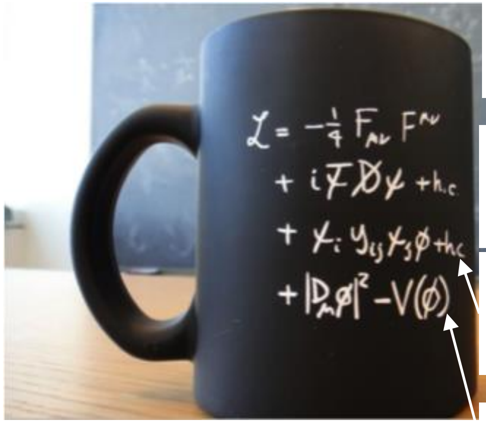


H→γγ


CMS √s = 7 TeV, L = 5.1 fb-1 √s = 8 TeV, L = 5.3 fb-1
S/(S+B) Weighted Events / 1.5 GeV
Events / 1.5 GeV
Unweighted
Data
S+B Fit
B Fit Component
±1σ
±2σ
mγγ (GeV)


ATLAS
Data
Sig+Bkg Fit (mH=126.5 GeV)
Bkg (4th order polynomial)
√s=7 TeV, ∫Ldt=4.8fb-1
√s=8 TeV, ∫Ldt=5.9fb-1
H→γγ
Events / 2 GeV
Events - Bkg
Data S/B Weighted
Σ weights / 2 GeV
Σ weights - Bkg
(a)
(b)
(c)
(d)
mγγ [GeV]

The SM ingredients



Kinetic term of all gauge bosons fields (W's, Zo, photon and gluons)

Kinetic term of fermion fields (quarks and leptons)

Interaction with H field giving mass to the q's and leptons

kinetic term for Higgs boson and H potential

!!

Summary

- From the classical Lagrangian to the particle physics Lagrangian
- Dirac equation and the anti-matter
- Feynman diagram and the computation of the muon the life-time
- Brief description of the SM Lagrangian .