[Cotler, Jensen *to appear*]

[Cotler, Jensen 2302.06603]

[Cotler, Strominger 2201.11658]

Non-perturbative de Sitter Jackiw-Teitelboim Gravity

JORDAN COTLER

HARVARD SOCIETY OF FELLOWS

Pretext

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So little is known about non-perturbative quantum gravity in dS

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Goal: Address foundational questions about dS quantum gravity in the simplest possible setting

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We solve dS JT gravity non-perturbatively, providing the first exactly solvable model of dS quantum gravity

A brief history of this work

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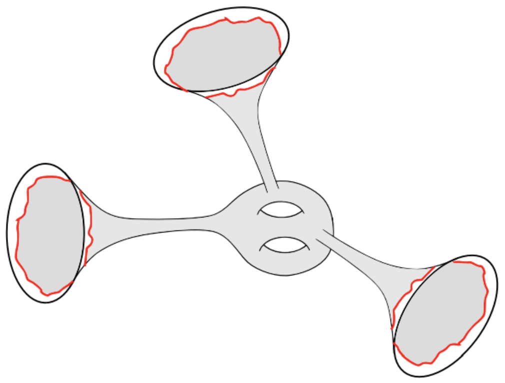
[Saad, Shenker, Stanford '19]

Solved AdS JT gravity, $R = -2$



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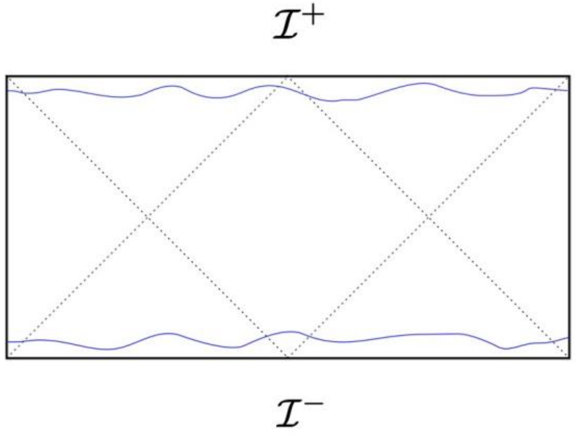

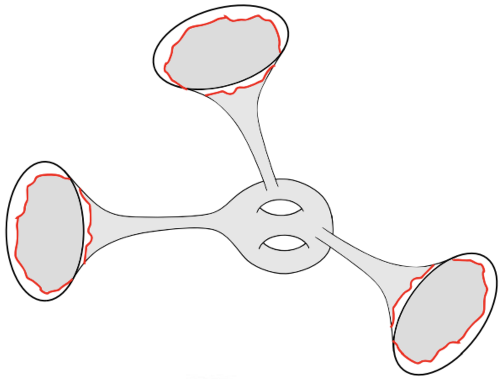

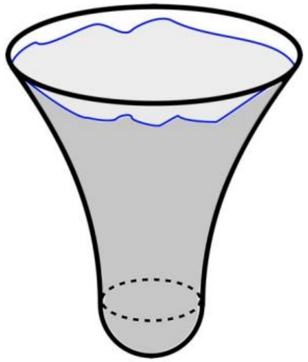

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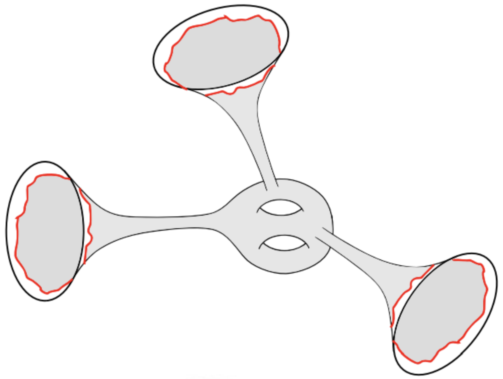

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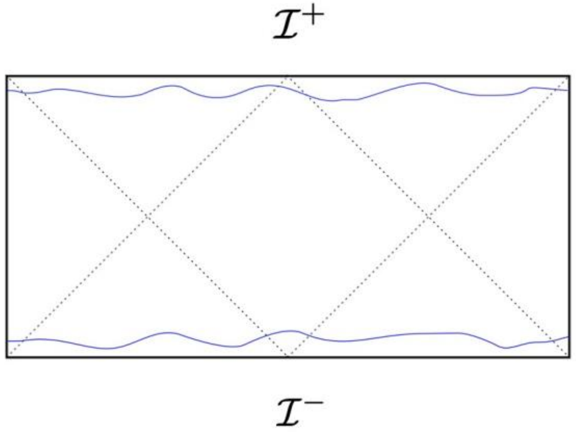

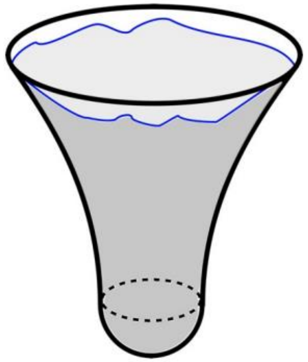

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Inner product on asymptotic states, S-matrix
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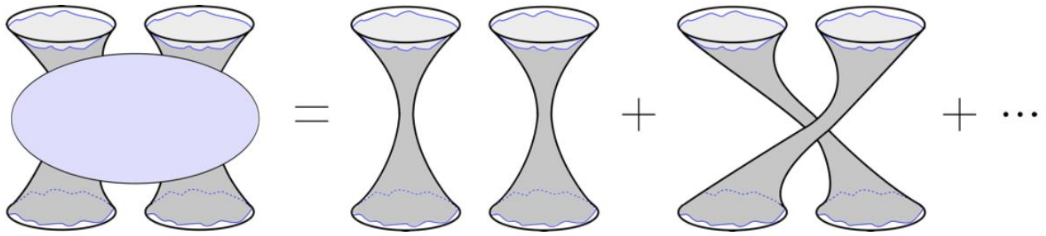

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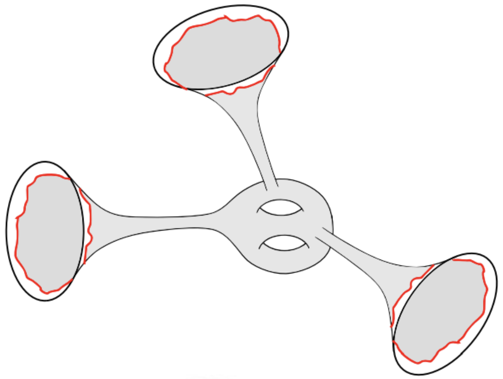

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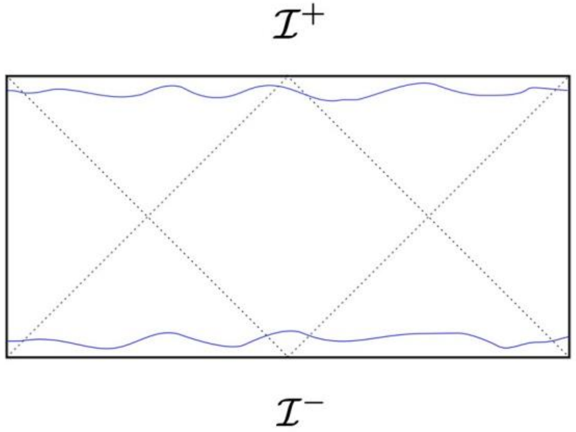

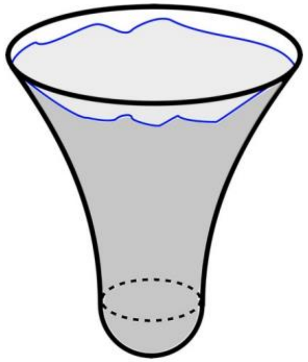

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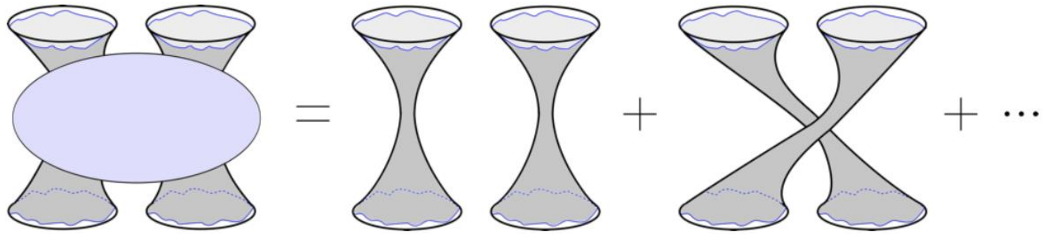

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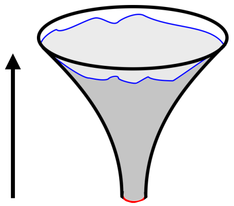

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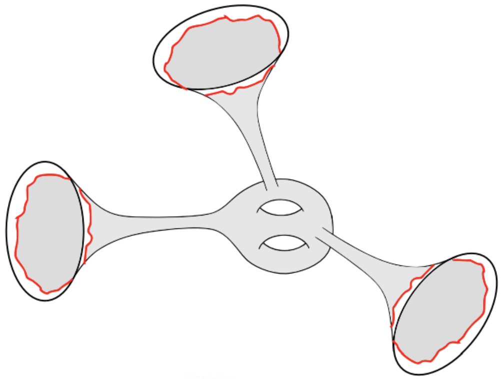

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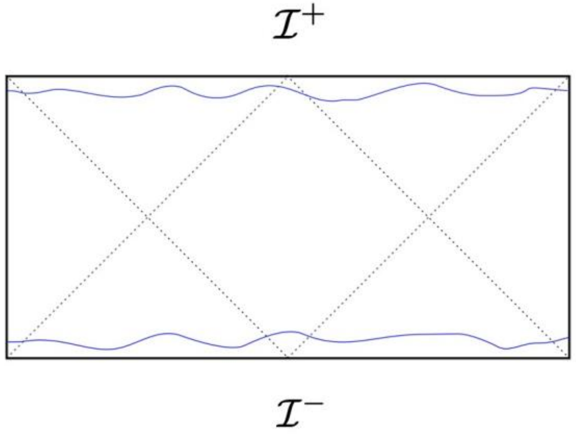

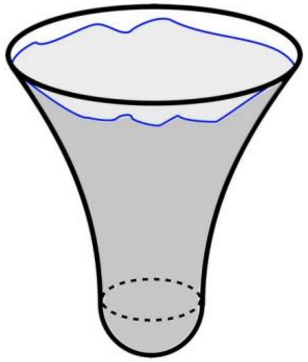

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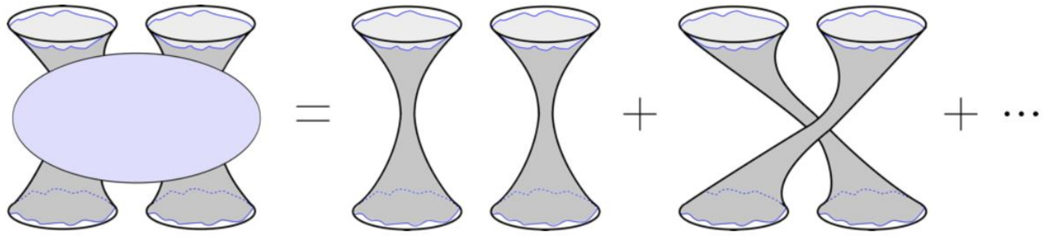

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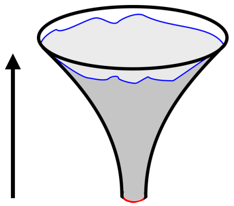

(See also **[Cotler, Strominger '22]**)

Summary of results

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Carefully treat the dS JT path integral measure

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Continuation of β's

Holographically dual matrix integral with $N_{\mathrm{eff}} < 0$

Three Parts

Part I

Review of dS JT gravity

Part II

Genus expansion

Part III

Discussion

Part I

Review of dS JT gravity

dS JT gravity

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$$S_{\rm JT} = \int_{\mathcal{M}} d^2x \sqrt{-g}\, \phi(R-2) - 2\int_{\partial\mathcal{M}} dx \sqrt{h}\, \phi(K-1) - iS_0\chi$$

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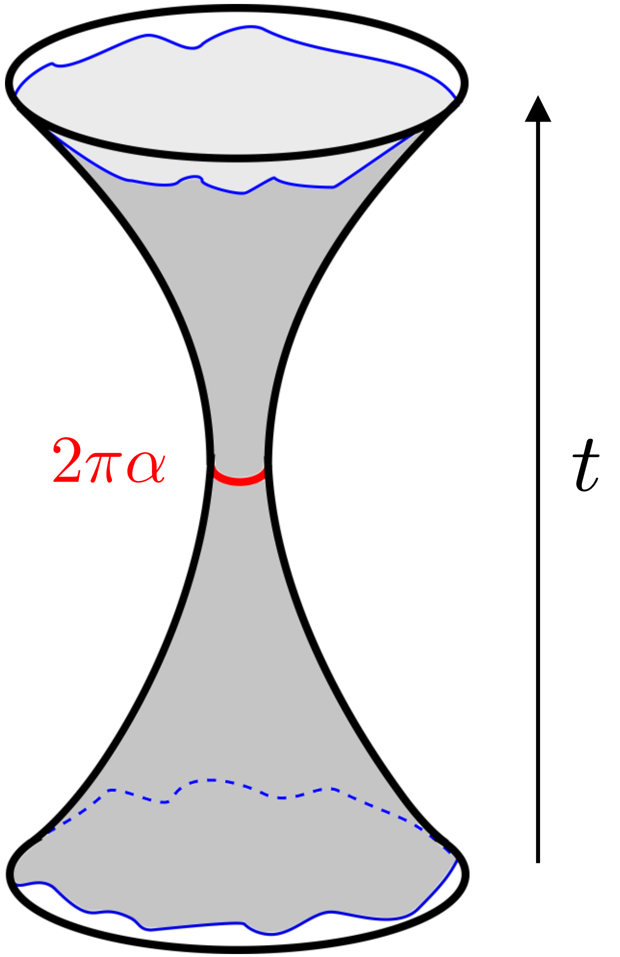

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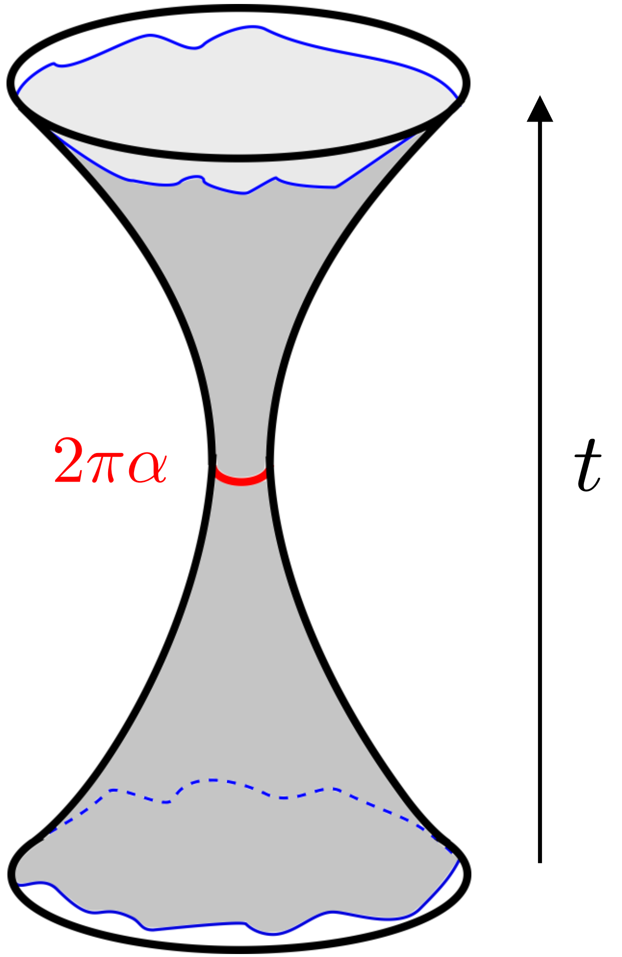
No propagating degrees of freedom

Boundary gravitons (Schwarzian modes)

Moduli space



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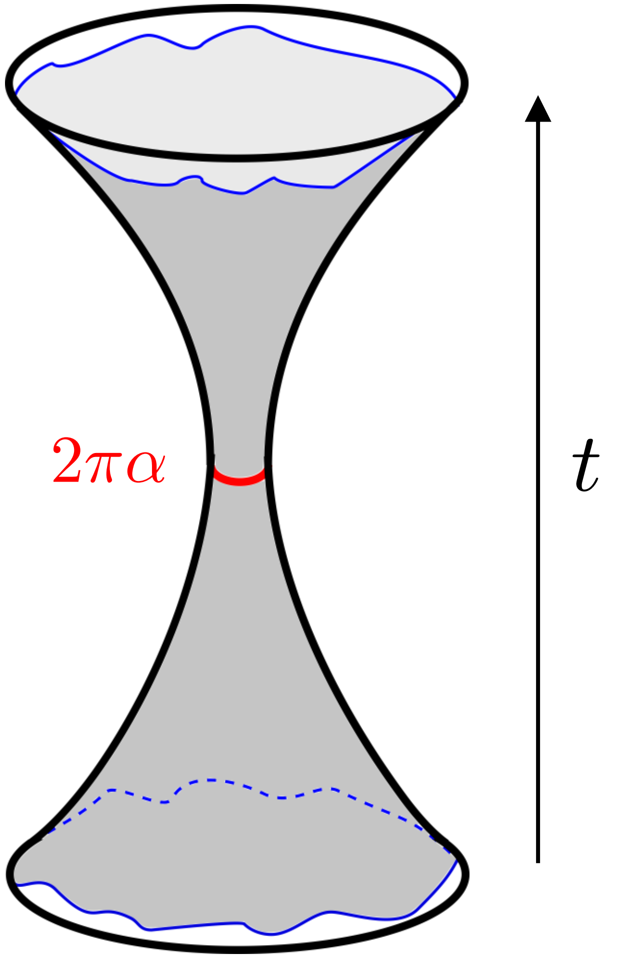

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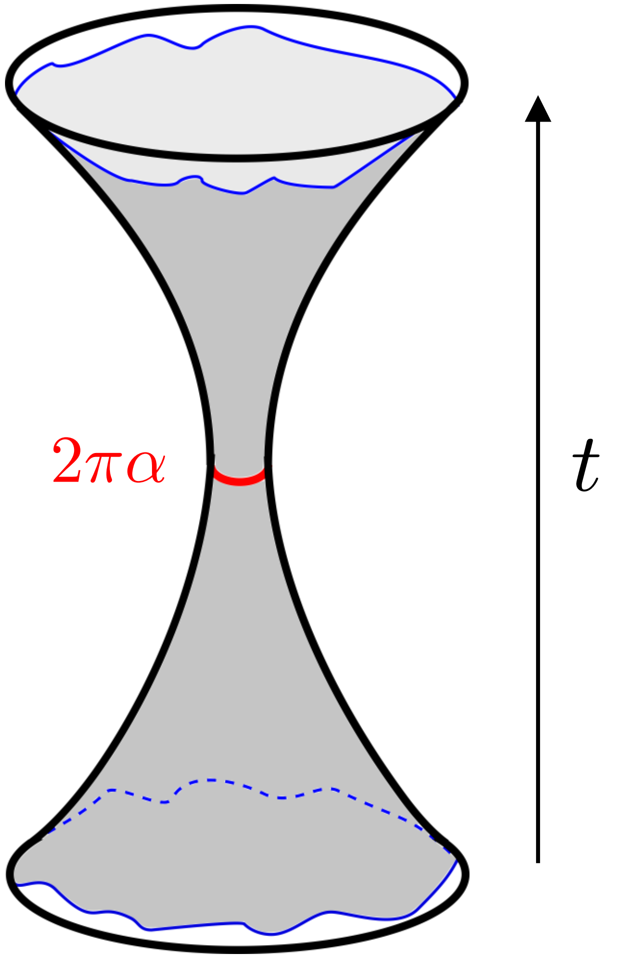

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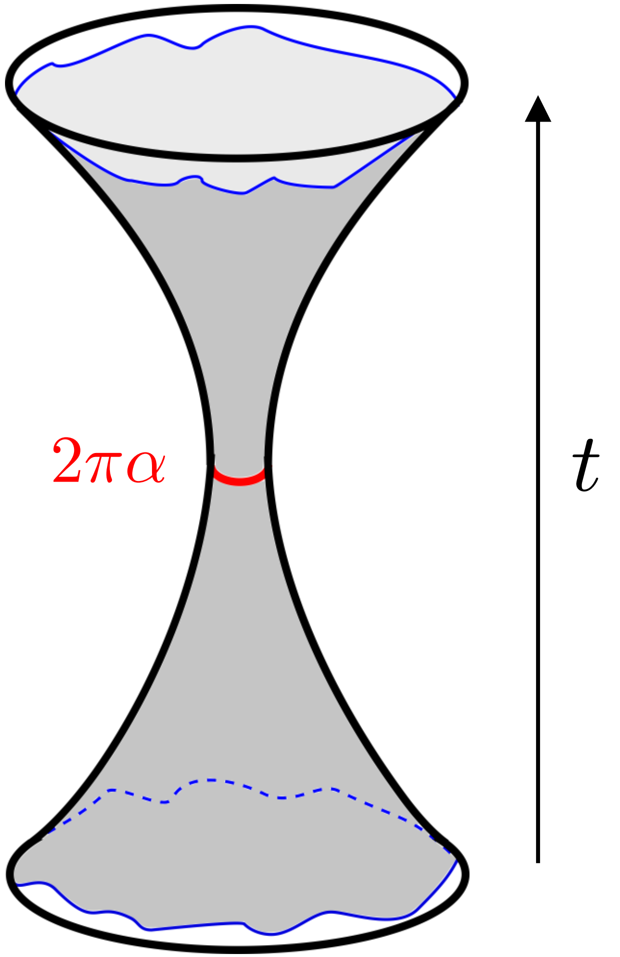

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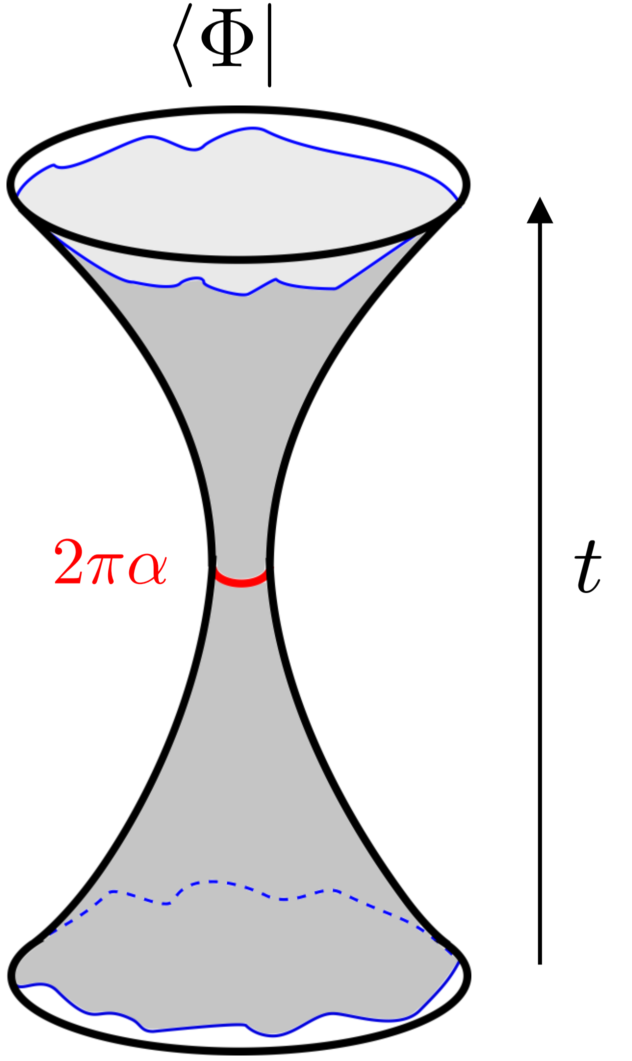

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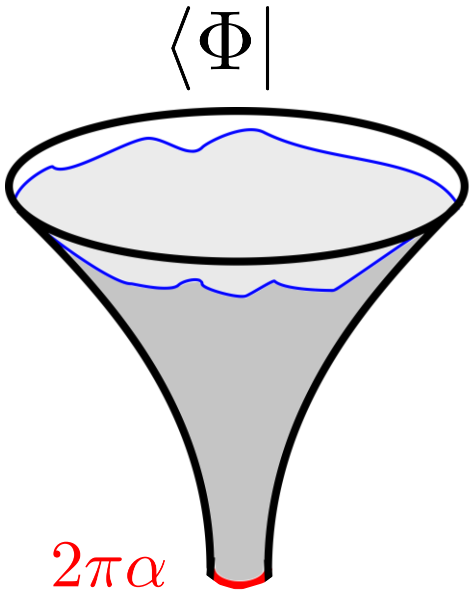

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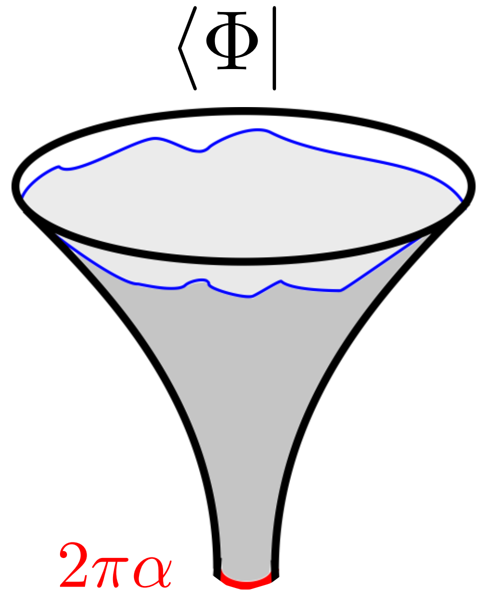

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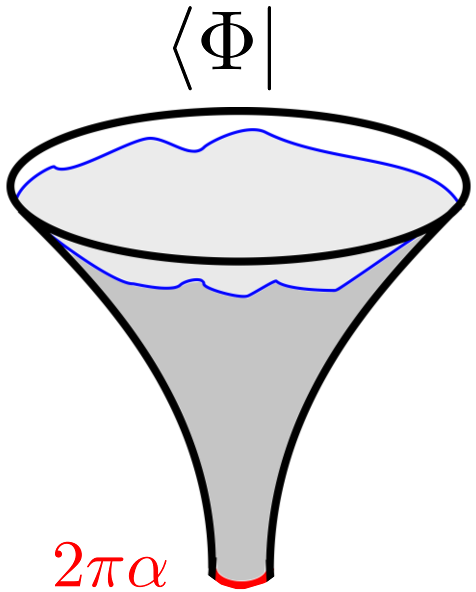

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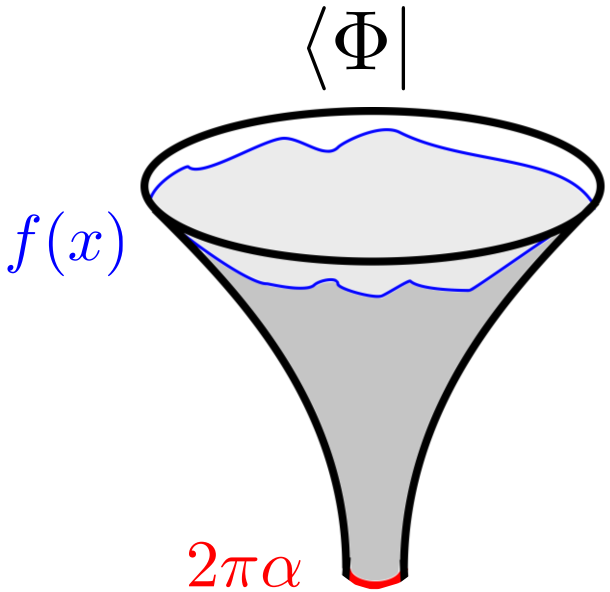

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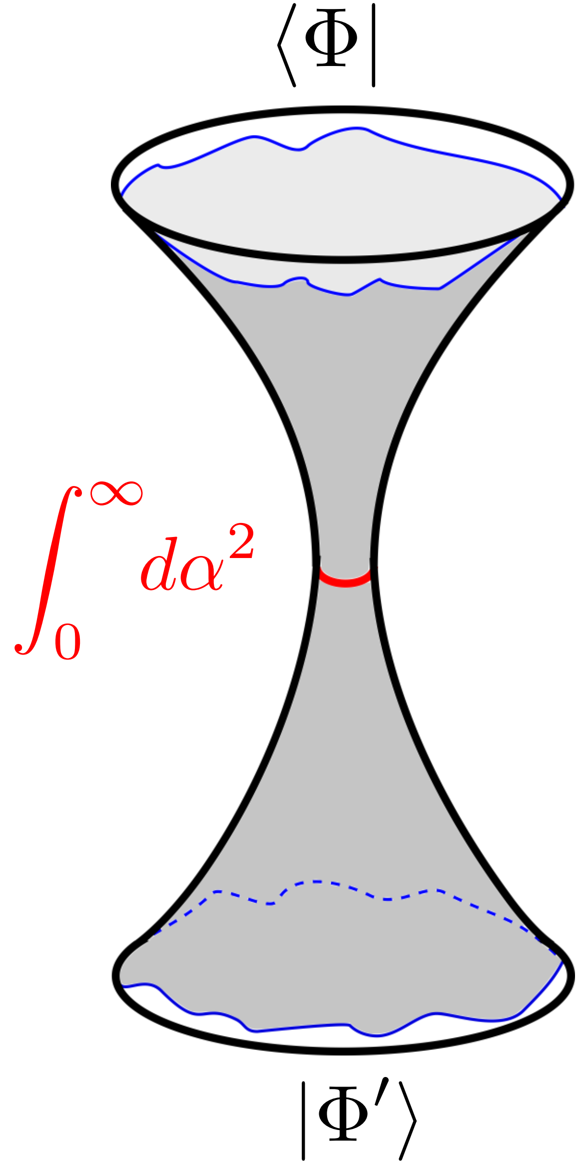

dS JT amplitudes

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Meta-observables are S-matrix elements:

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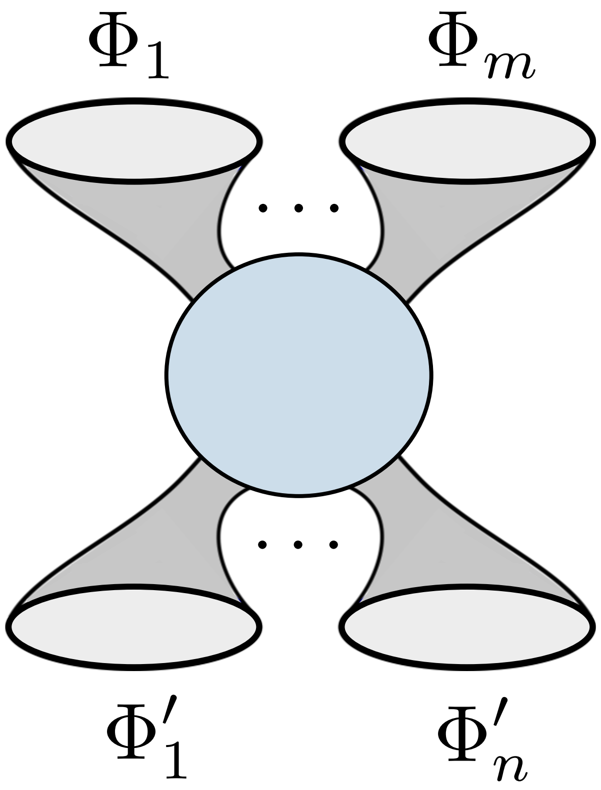

$$= \langle \Phi_1, \ldots, \Phi_m | \, \widehat{\mathcal{S}} \, | \Phi'_1, \ldots, \Phi'_n \rangle$$

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Sphere:

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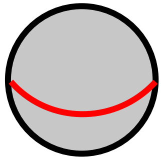
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

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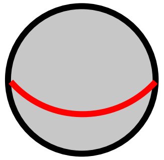
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

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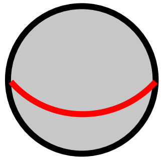
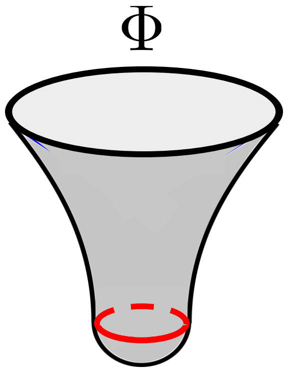
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

Disk: 

$$= \frac{(i\Phi)^{\frac{3}{2}}}{\sqrt{2\pi}} e^{S_0 + i\Phi}$$

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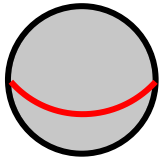
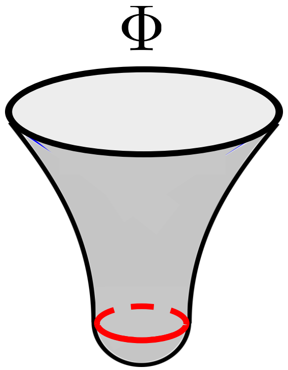
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

Disk: 

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Cylinder:

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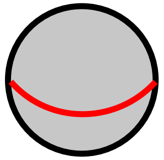
Sphere: 

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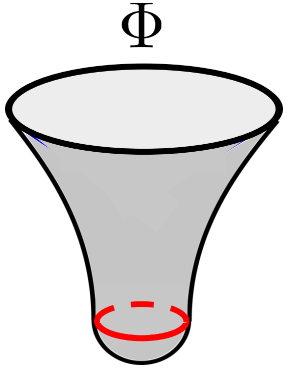
Disk: 

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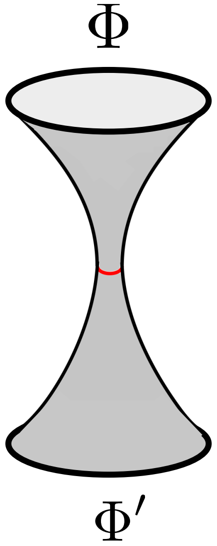
Cylinder: 

$$= \frac{i}{2\pi} \frac{\sqrt{\Phi\Phi'}}{\Phi - \Phi' + i\epsilon}$$

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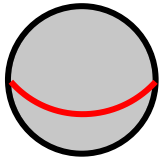
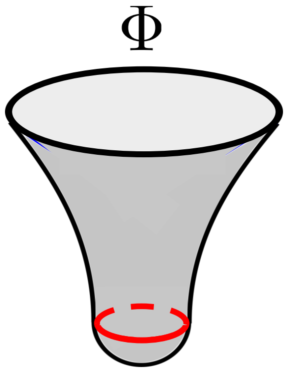
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

Disk: 

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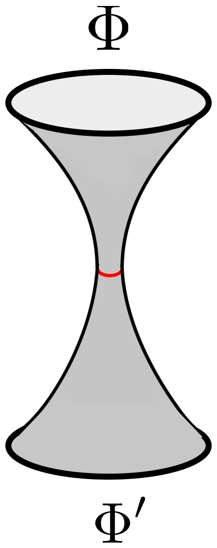
Cylinder: 

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dS JT amplitudes

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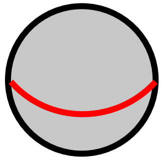
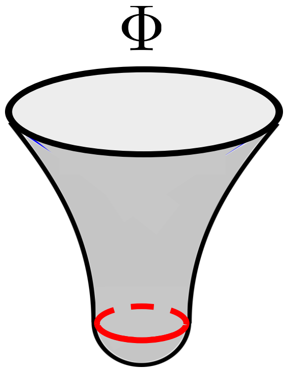
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

Disk: 

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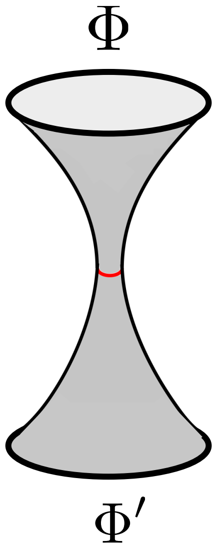
Cylinder: 

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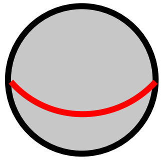
Sphere: 

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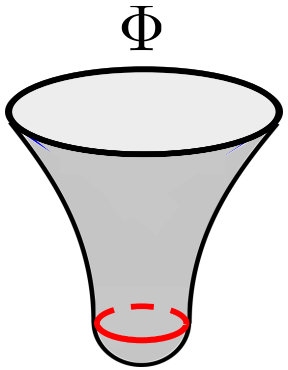
Disk: 

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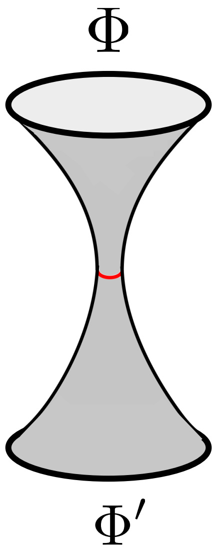
Cylinder: 

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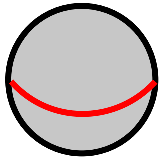
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

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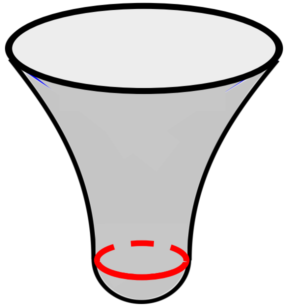
Disk:  $= \dfrac{(i\Phi)^{\frac{3}{2}}}{\sqrt{2\pi}} e^{S_0 + i\Phi}$

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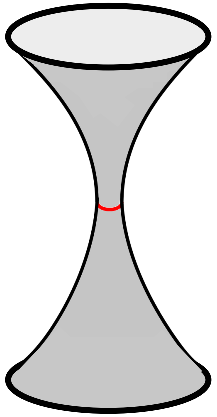
Cylinder:  $= \dfrac{i}{2\pi} \dfrac{\sqrt{\Phi\Phi'}}{\Phi - \Phi' + i\epsilon}$

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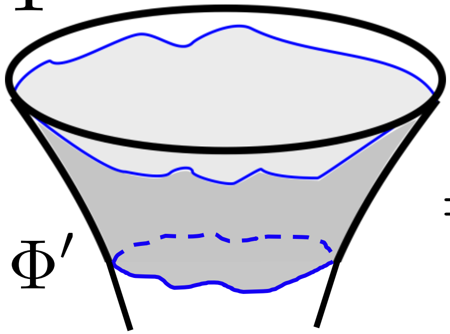 $= \langle \Phi | \Phi' \rangle$

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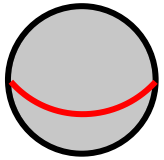
Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

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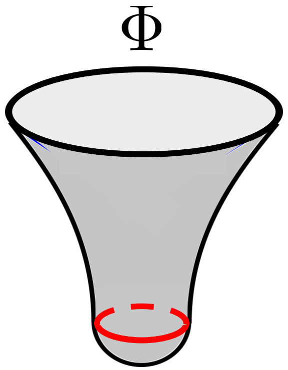

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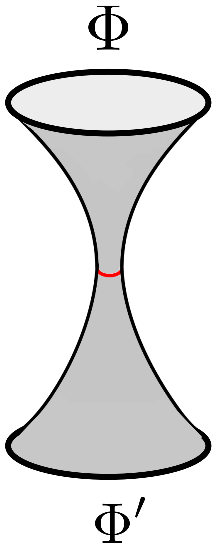

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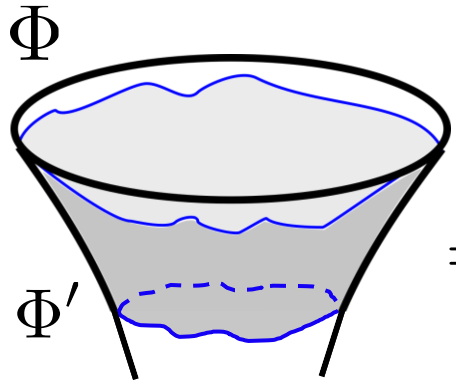

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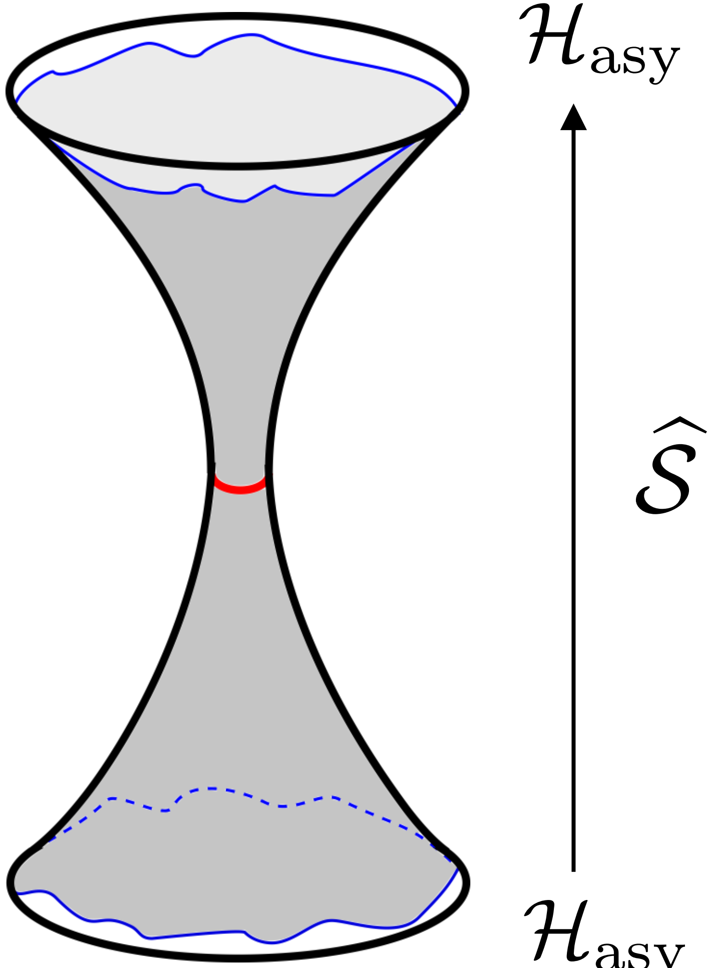

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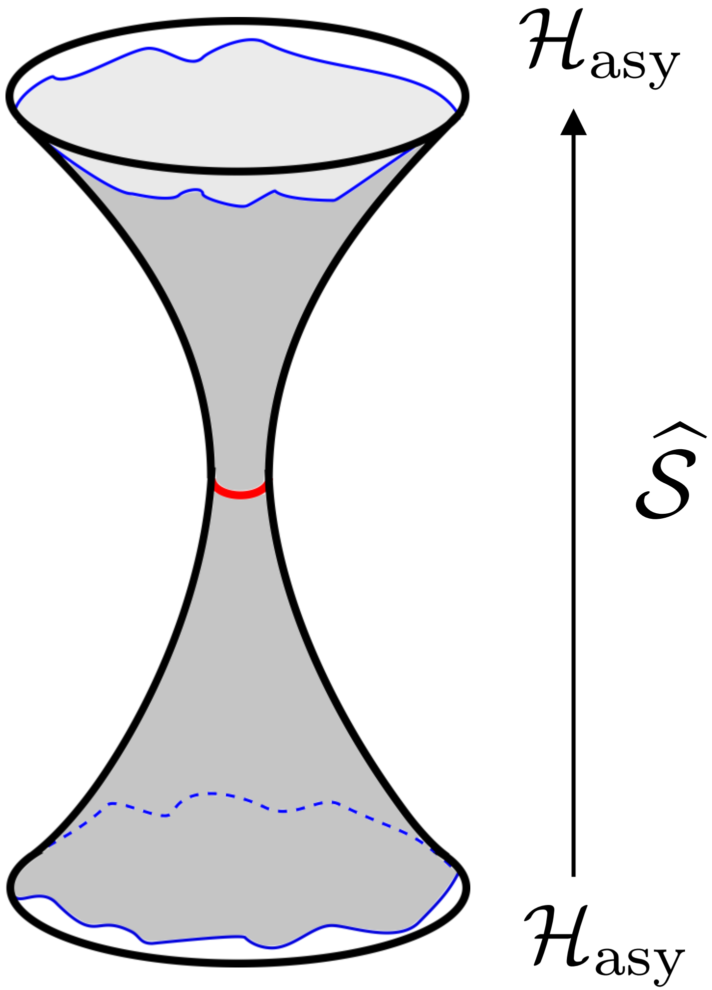
In dS JT gravity, the 1-to-1 S-matrix is a projector (!)



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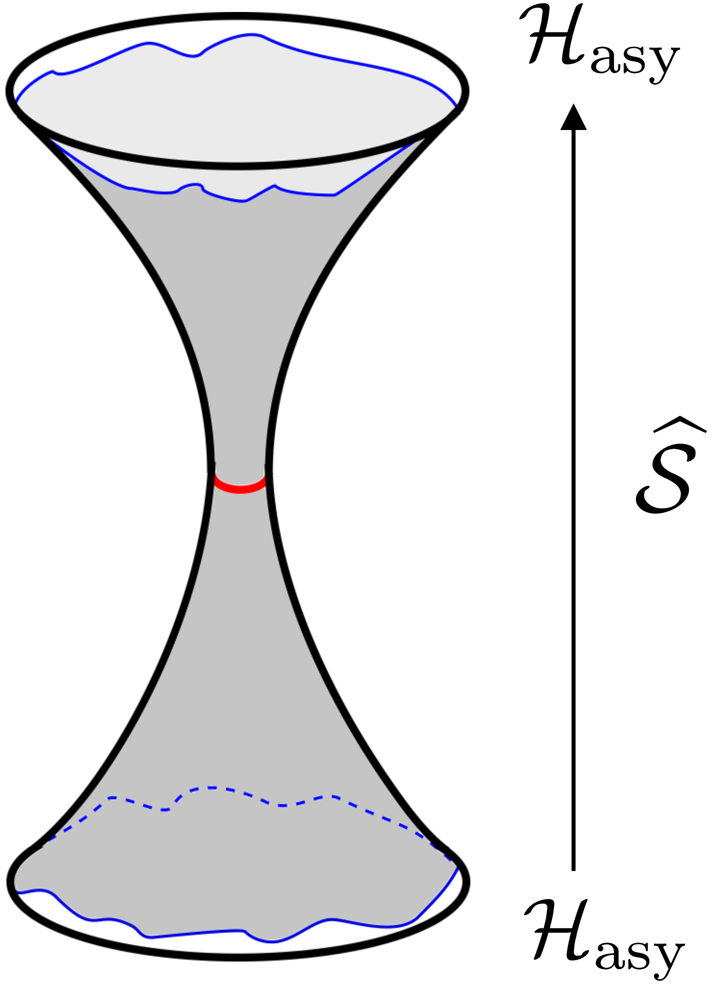
Can understand this by decomposing time evolution



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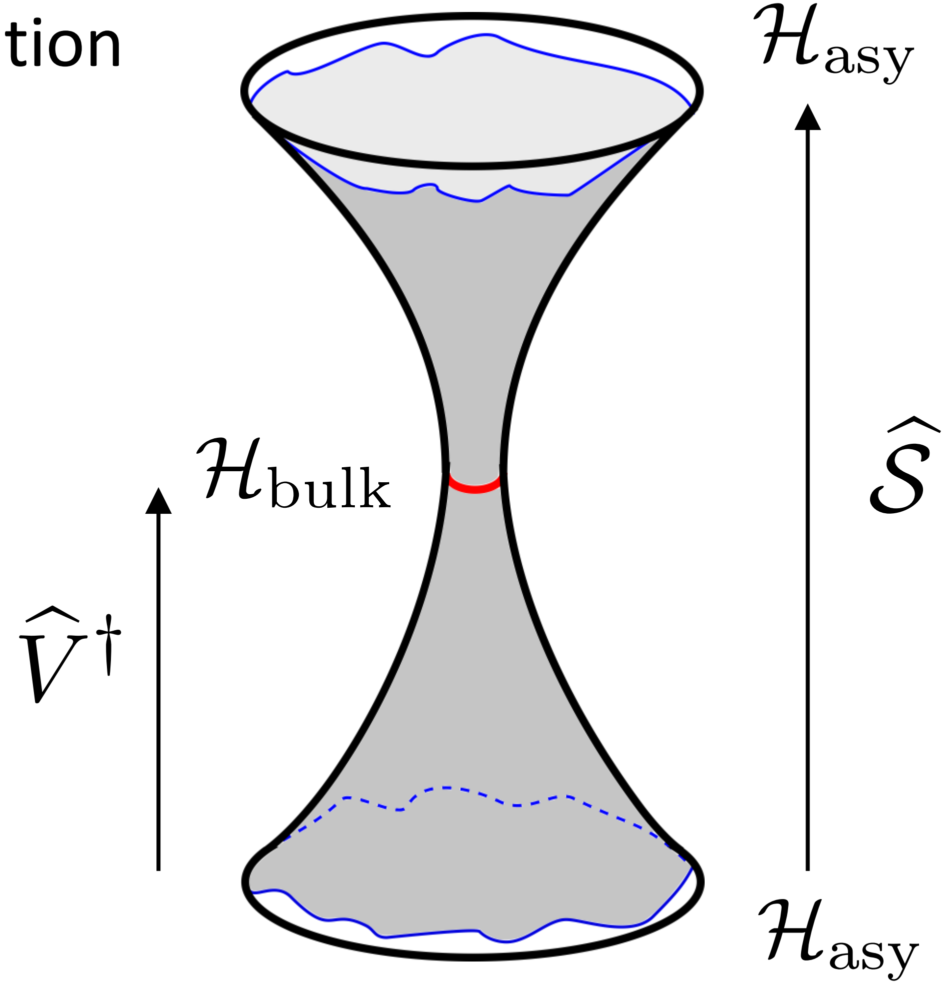

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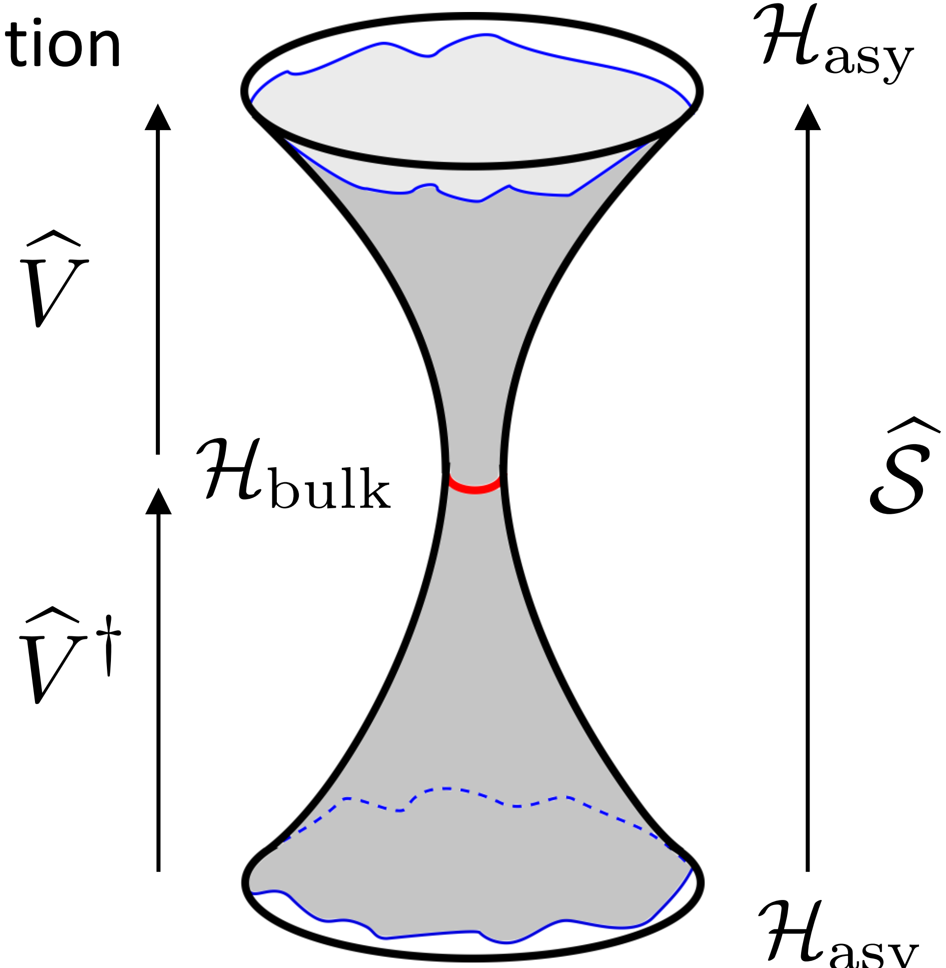

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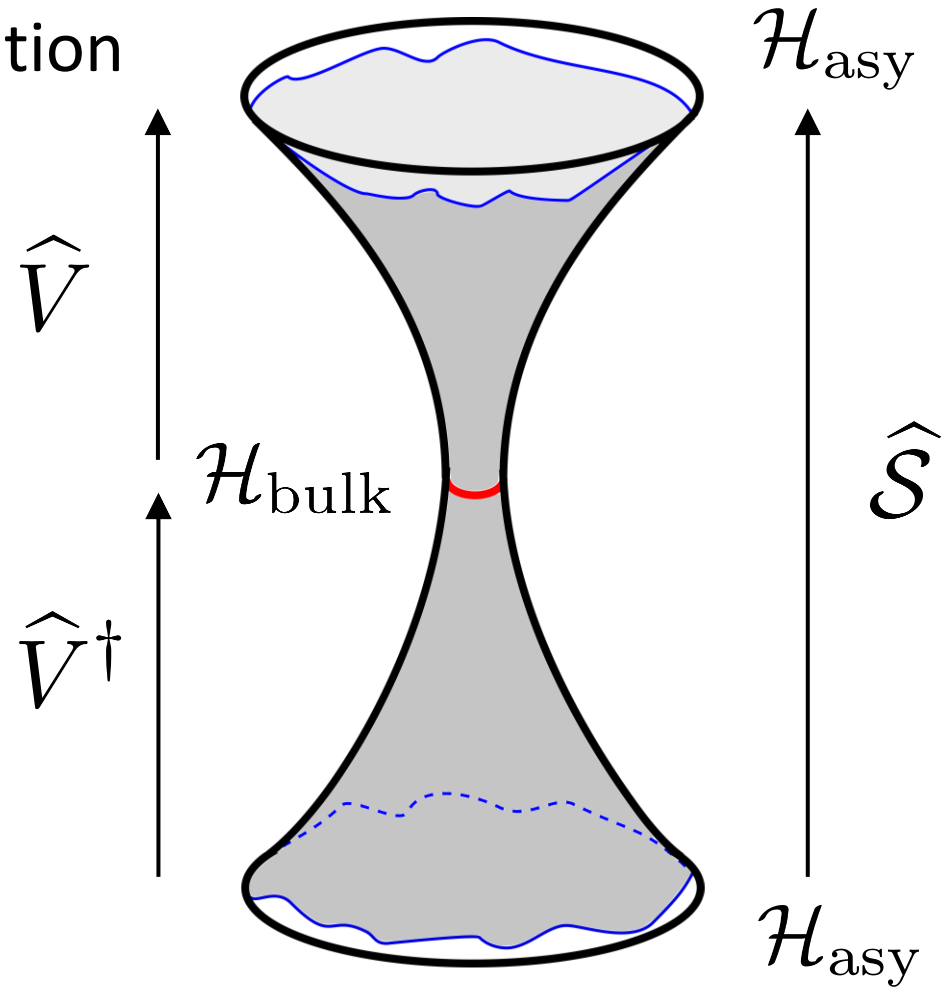

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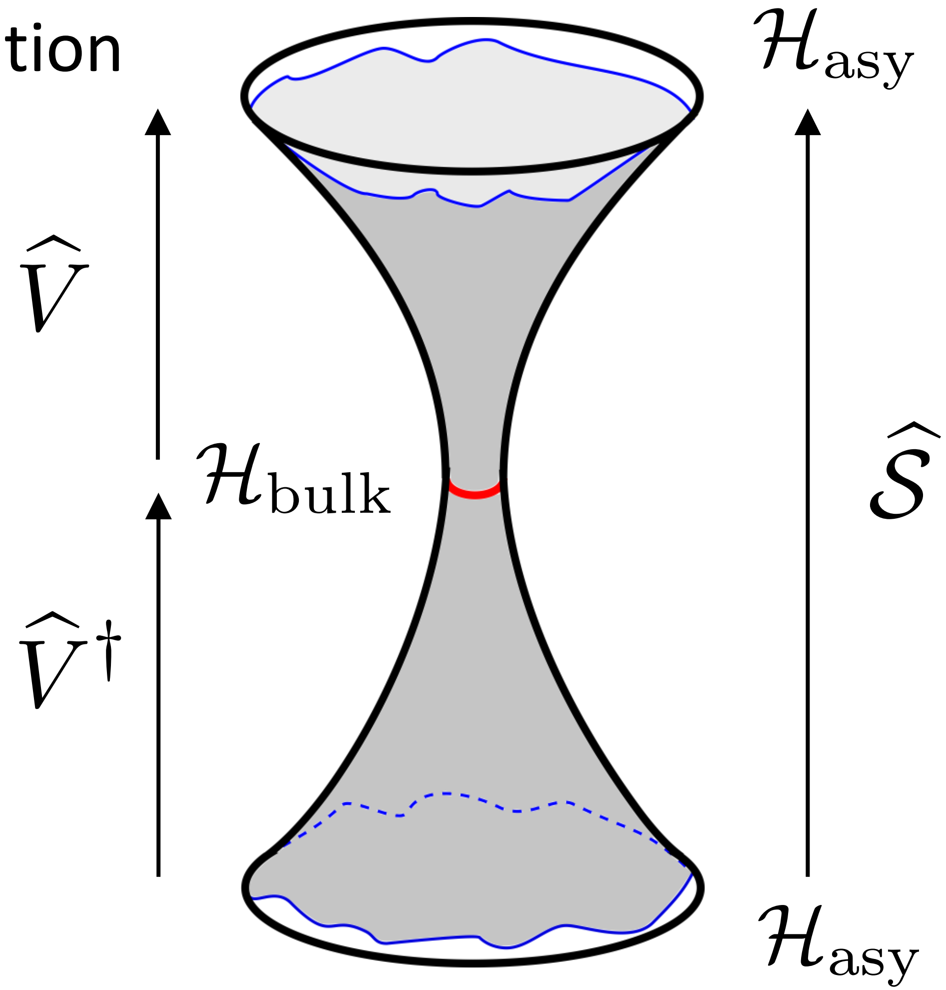

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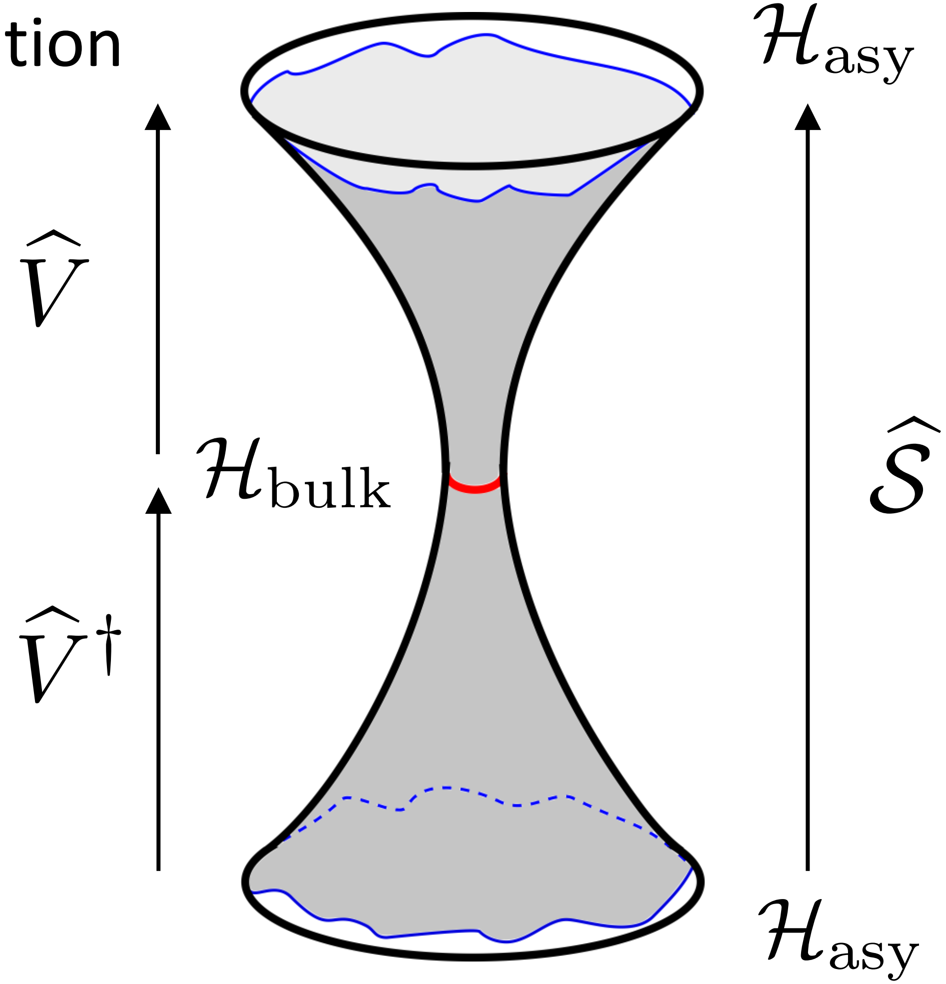

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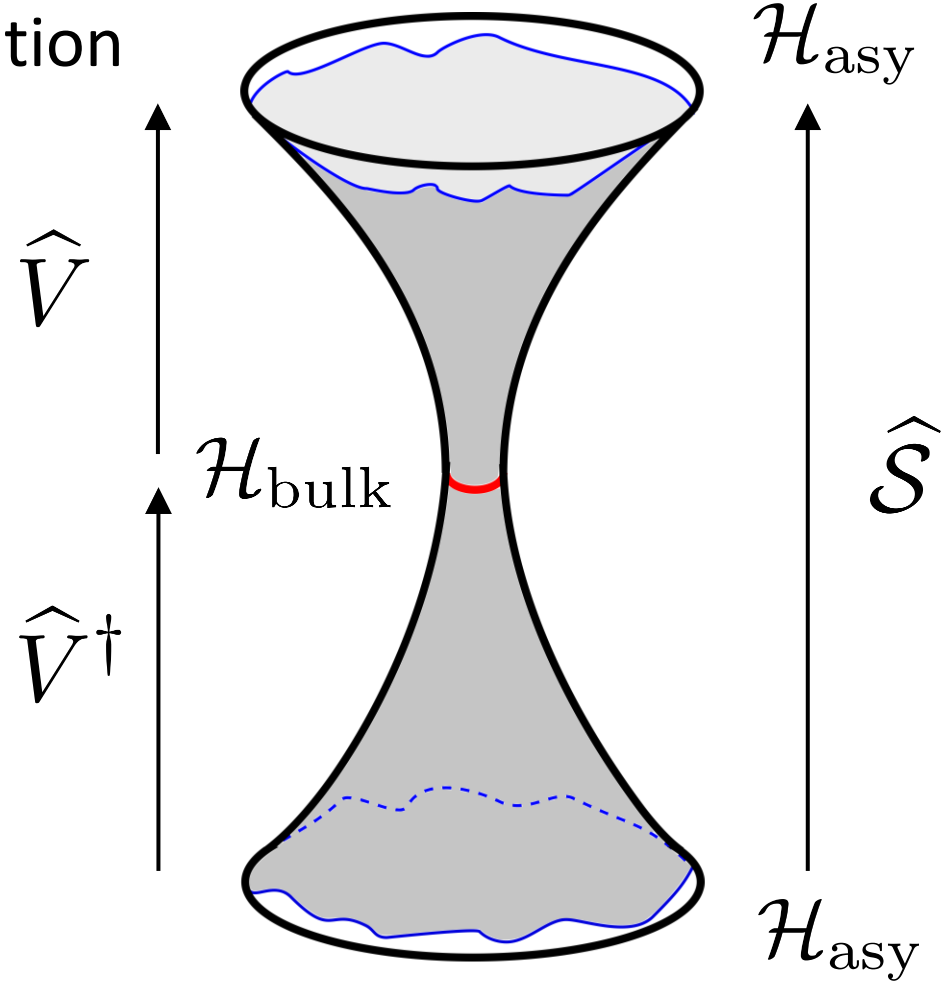

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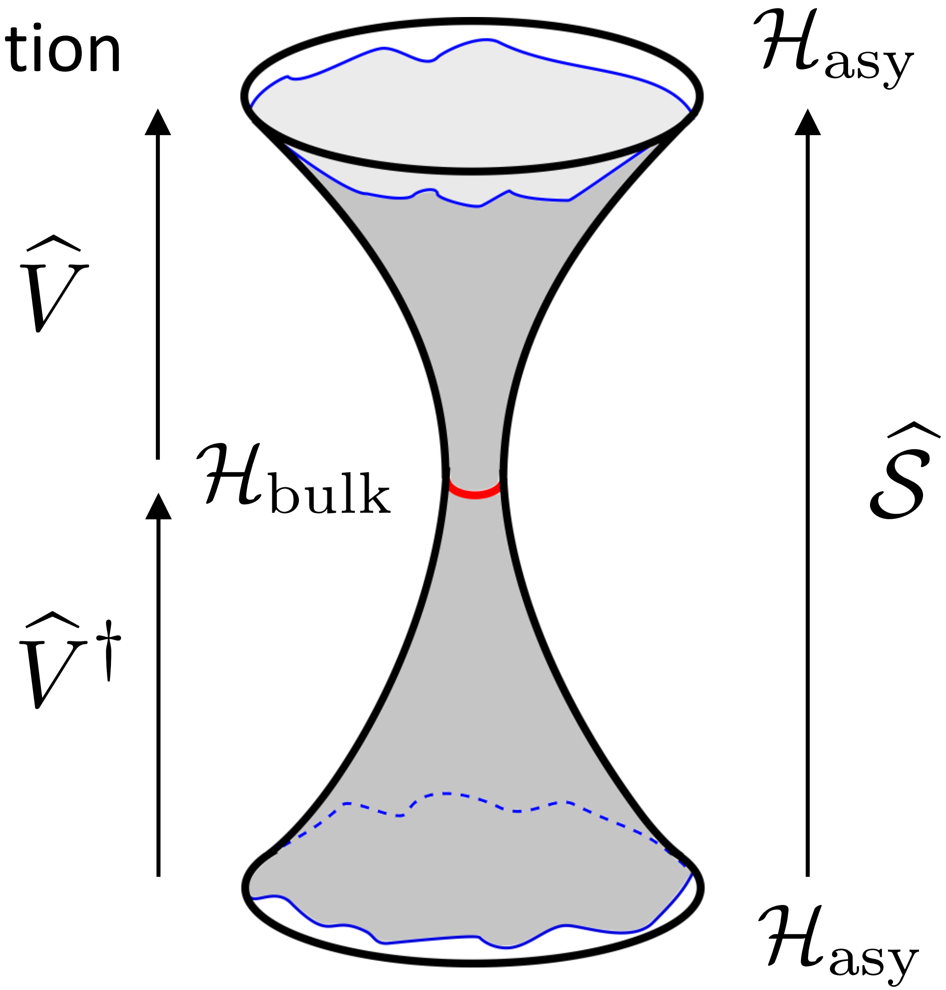

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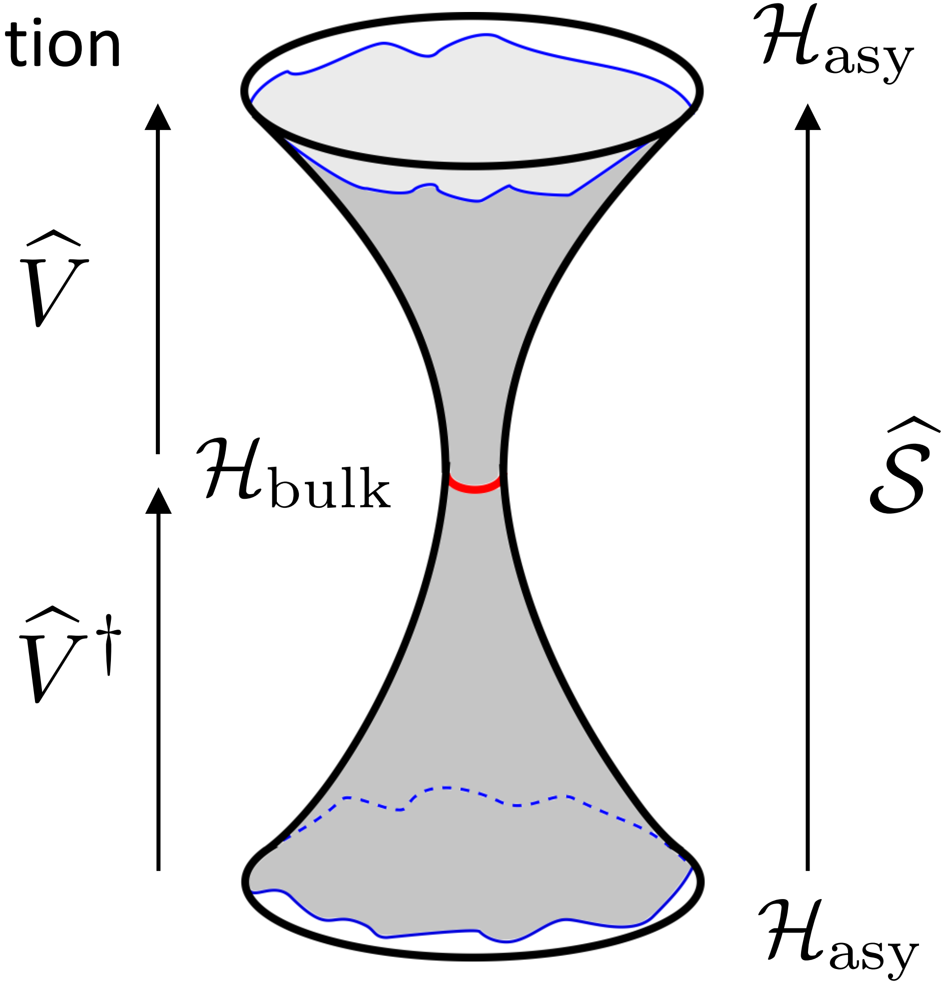
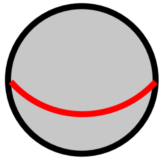
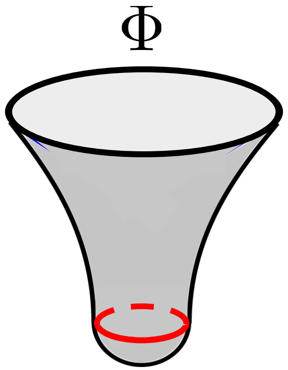


Anticipated by [Cotler, Strominger '22]

dS JT amplitudes

Sphere:  $= Z_{\mathbb{S}^2} = e^{2S_0} \times \infty$

Disk: 

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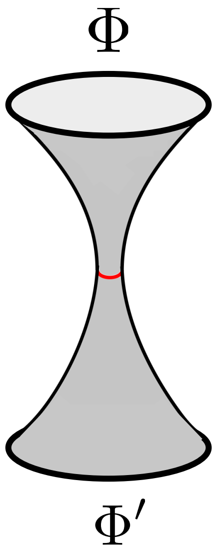
Cylinder: 

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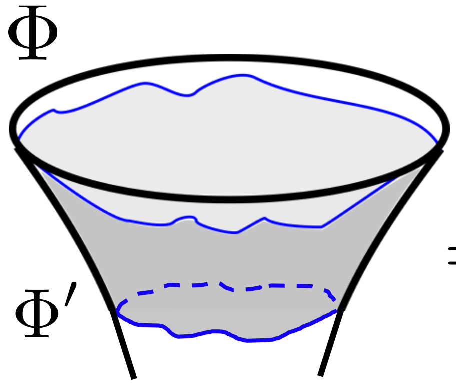

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Part II

Genus expansion

A puzzle

A puzzle

$$S_{\mathrm{JT}} = \int_{\mathcal{M}} d^2x\,\sqrt{-g}\,\phi(R-2) - 2\int_{\partial\mathcal{M}} dx\,\sqrt{h}\,\phi(K-1) - iS_0\chi$$

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There are no smooth Lorentzian $R = 2$ metrics on general Σ

Revisiting the disk amplitude

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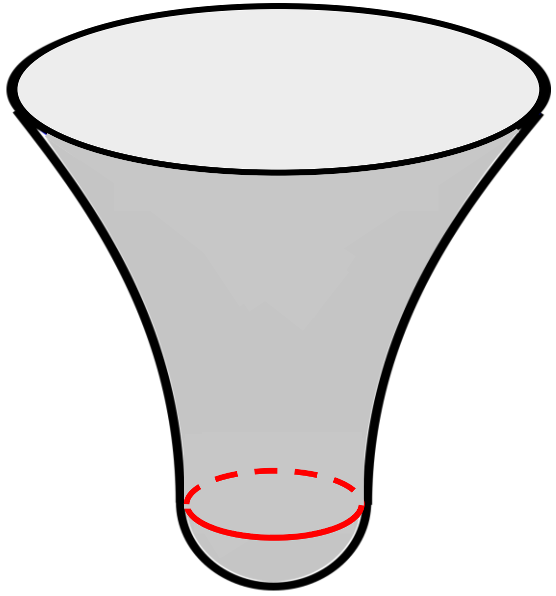
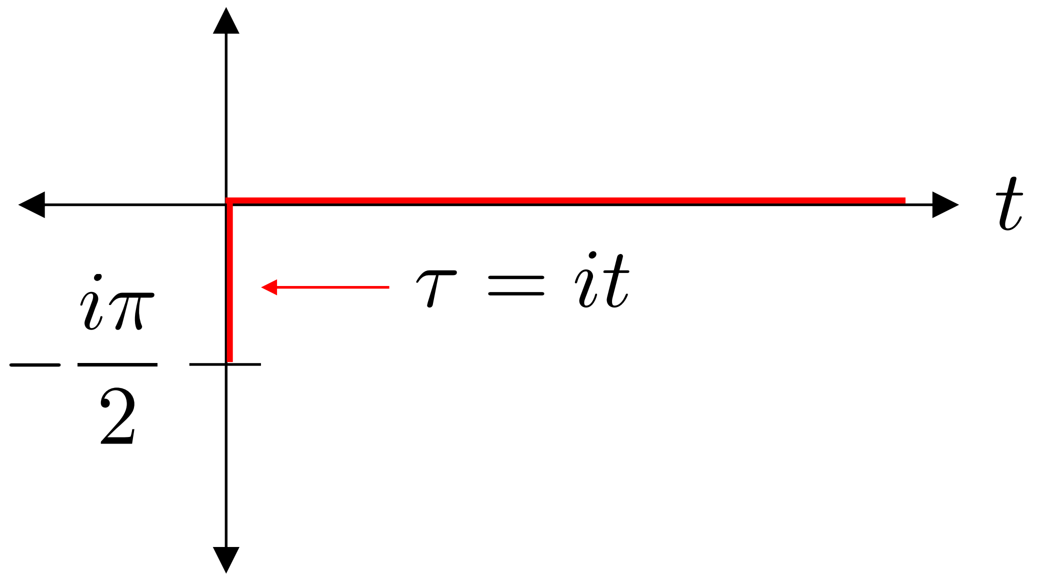
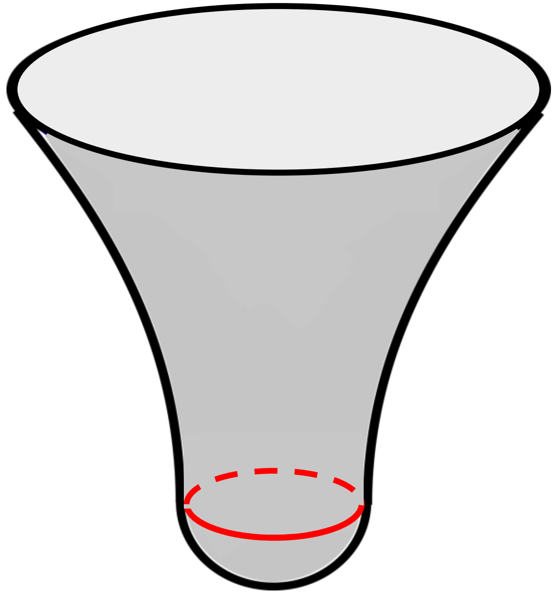


Revisiting the disk amplitude

$$\begin{cases} ds^2 = -dt^2 + \cosh^2(t)\, dx^2 \\ \phi = \frac{\Phi}{2\pi} \sinh(t) \end{cases}$$





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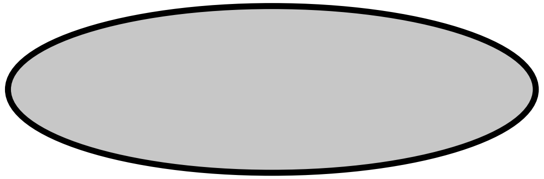
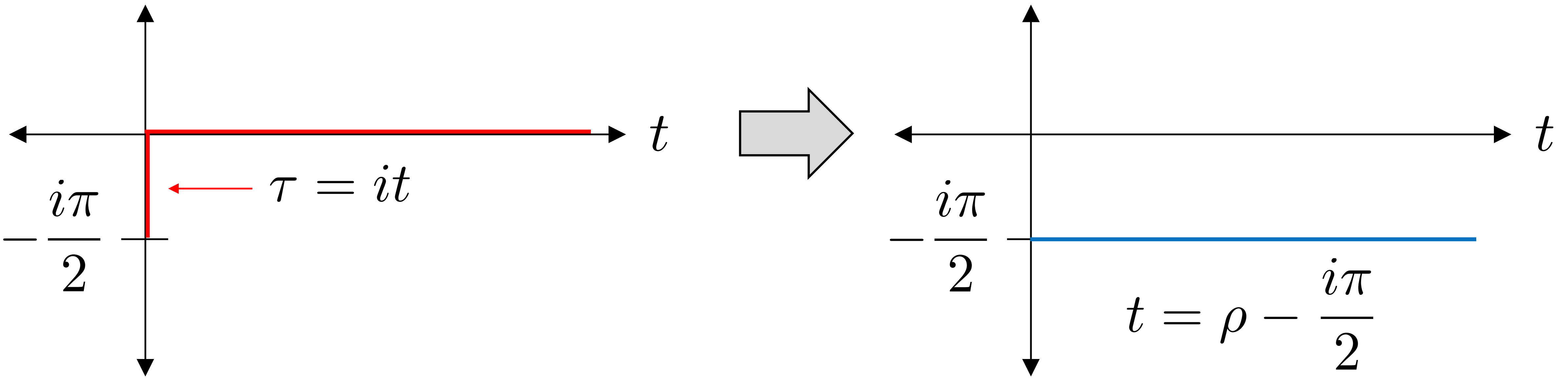
$$\begin{cases} ds^2 = -(d\rho^2 + \sinh^2(\rho)\, dx^2) \\ \phi = -i\,\frac{\Phi}{2\pi}\,\cosh(\rho) \end{cases}$$





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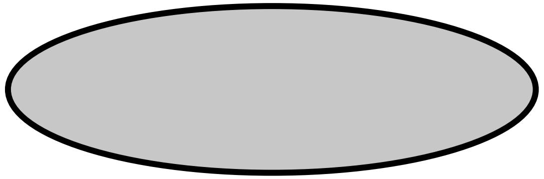

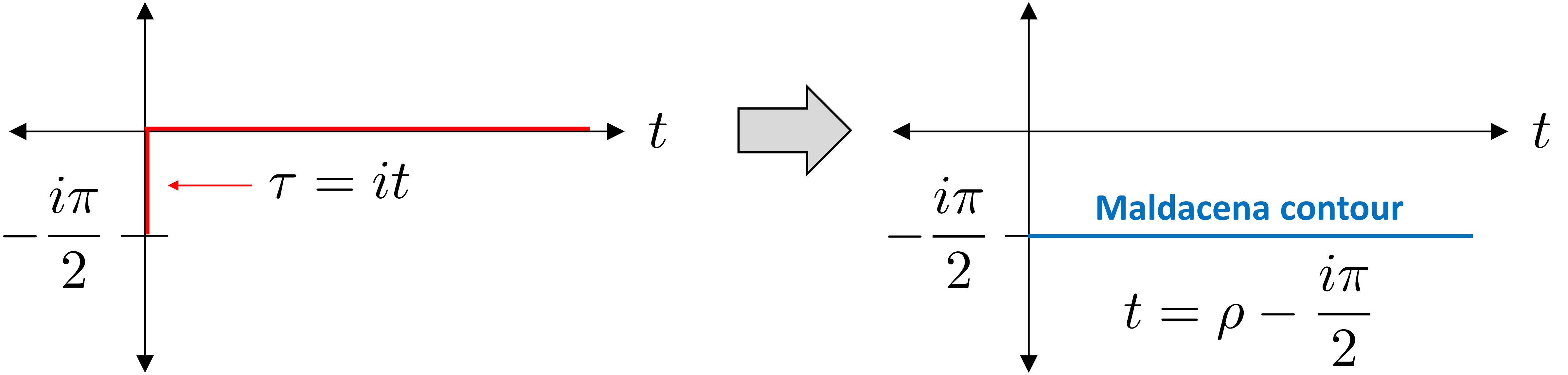

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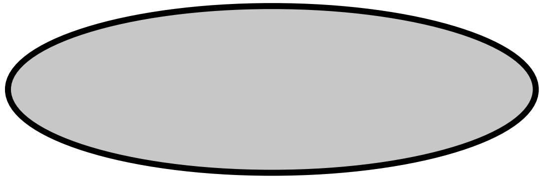
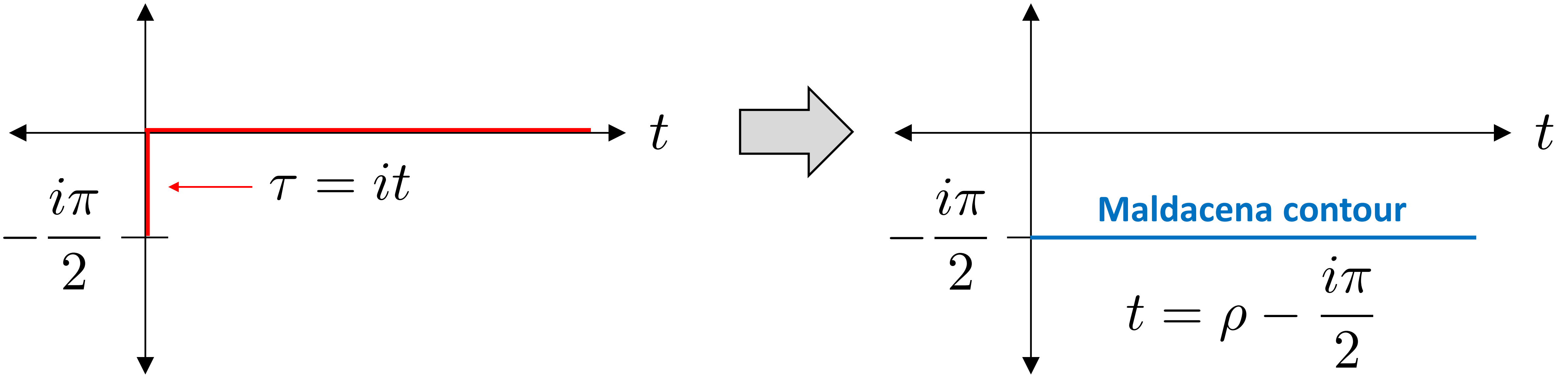
(−, −) signature
with R = 2





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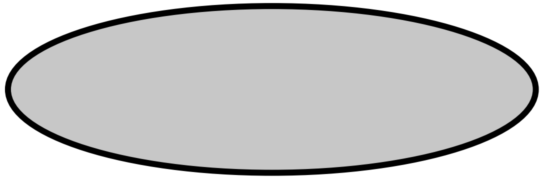
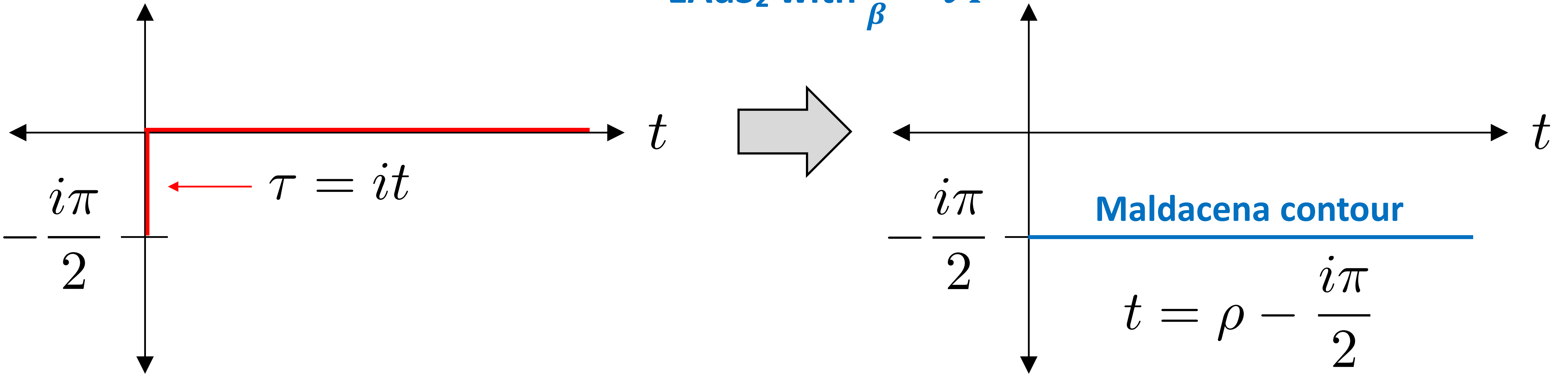


$(-,-)$ signature with R = 2

EAdS_2 with $\frac{1}{\beta} = i\Phi$



Generalization to arbitrary surfaces

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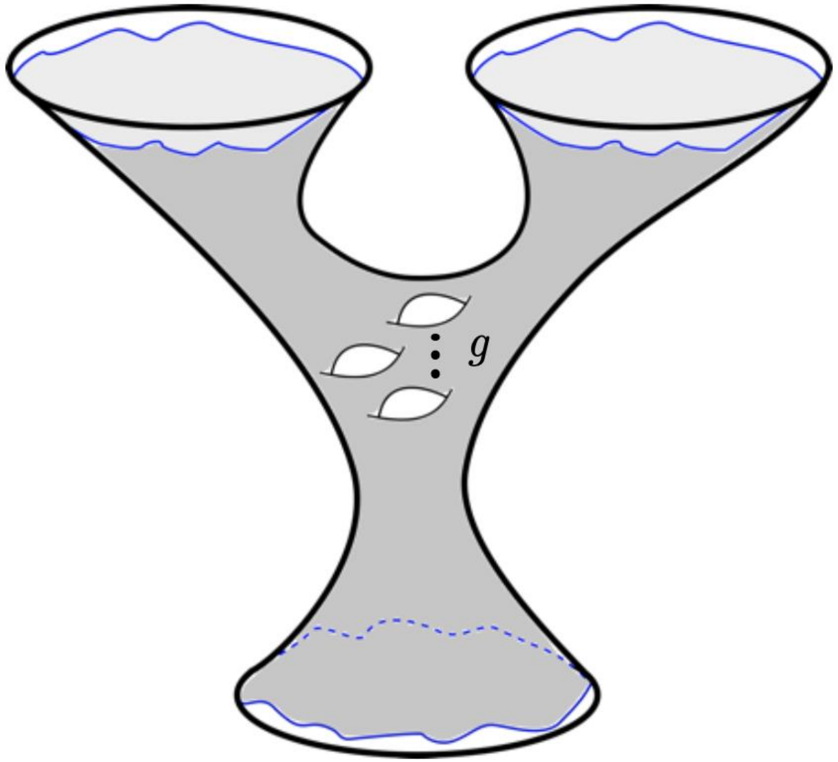
General hyperbolic metrics in $(-,-)$ signature have $R=2$



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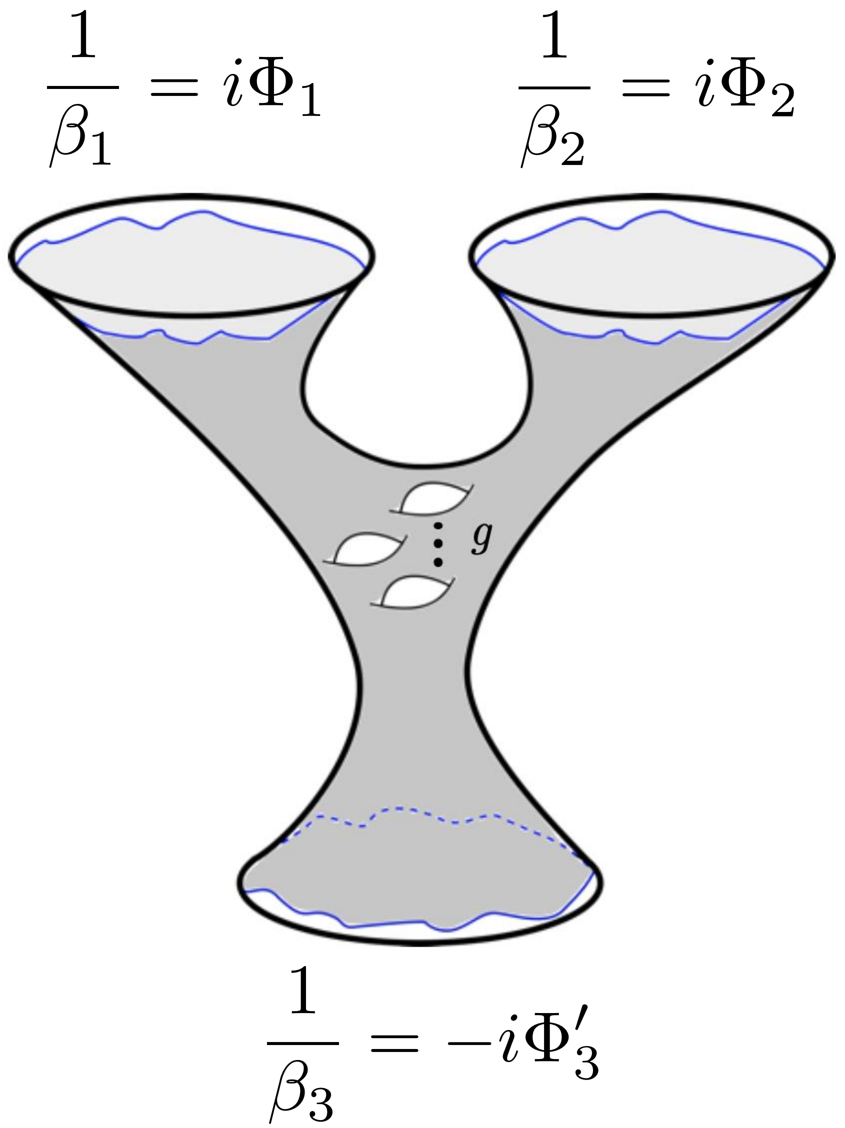
We find our desired surfaces using these metrics, in conjunction with analytically continuing the dilaton boundary conditions



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Two ingredients:

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Boundary conditions on circles have: $\begin{cases} \dfrac{1}{\beta_F} = i\Phi_F - \epsilon \\ \dfrac{1}{\beta_P} = -i\Phi_P - \epsilon \end{cases}$

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Note: $\mathrm{Re}(\beta) < 0$ instructs us to rotate the integration contour of the Schwarzian modes

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Two ingredients:

Path integral measure induced by $\mathrm{Pf}(-\Omega)$

Boundary conditions on circles have: $\begin{cases} \dfrac{1}{\beta_F} = i\Phi_F - \epsilon \\ \dfrac{1}{\beta_P} = -i\Phi_P - \epsilon \end{cases}$

Note: $\mathrm{Re}(\beta) < 0$ instructs us to rotate the integration contour of the Schwarzian modes

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$$Z_{g,n_F,n_P} = (-1)^{g+1} e^{S_0 \chi} \int_0^\infty \prod_{j=1}^{n=n_F+n_P} (-d\alpha_i)^2\, V_{g,n}(i\alpha_1, ..., i\alpha_n)\, Z_+(\alpha_1;\Phi_1)\cdots Z_+(\alpha_{n_F};\Phi_{n_F}) \\ \times Z_-(\alpha_{n_F+1};\Phi'_{n_F+1})\cdots Z_-(\alpha_n;\Phi'_n)$$

$$Z_\pm(\alpha;\Phi) = \sqrt{\frac{\pm i\Phi}{2\pi}}\, e^{\pm i\alpha^2\Phi}$$

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There can be additional doubly non-perturbative effects not captured by resurgence (e.g. eigenvalue instantons)

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$$\Longrightarrow \quad e^{\frac{2S_0}{3}} \left(\mathrm{Ai}'(-\xi)^2 + \xi\,\mathrm{Ai}(-\xi)^2 \right) \sim e^{S_0}\sqrt{-E}$$

Flips cut, so it only makes sense to probe model with $\beta < 0$

Part III

Discussion

Recap of results

Solved dS JT non-perturbatively in the genus expansion

Carefully treated the dS JT path integral measure

dS JT can be viewed as a subtle continuation of AdS JT

$$g_s = \pm\, i\, e^{-S_0}$$

Continuation of β's

Holographically dual matrix integral with $N_{\text{eff}} < 0$

Comments and speculations

dS JT gravity has isometric (and co-isometric) S-matrix evolution at leading order in the genus expansion, but this is broken at higher genus

The theory is UV-complete, but only *approximately* unitary / isometric

What happens doubly non-perturbatively?

Related to, but different than, the factorization problem in Euclidean AdS

Our analysis uncovers new features of the de Sitter holographic dictionary

More physics to mine out of the model