Cosmology and Naturalness

Sept 5th 2023

Cosmology, Quantum Gravity, and Holography
(Appreciate the Oxford comma...)

CERN

Matthew McCullough

Consider superconductors...

Ginzburg-Landau

The G-L Theory of superconductivity involves a complex scalar field and the photon (magnetic vector potential)

$$\Phi \qquad\qquad A$$

The Free energy for this theory is

$$F = |(\nabla + 2ieA)\Phi|^2$$

$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

Where the mass depends on the temperature.

Ginzburg-Landau

$$F = |(\nabla + 2ieA)\Phi|^2$$

$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

At high temperatures the mass-squared is positive:

Just a hot metal.

Ginzburg-Landau

$$F = |(\nabla + 2ieA)\Phi|^2$$

$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

At the critical temperature the mass-squared vanishes:

Strange theory with massless fluctuations.

Ginzburg-Landau

$$F = |(\nabla + 2ieA)\Phi|^2$$

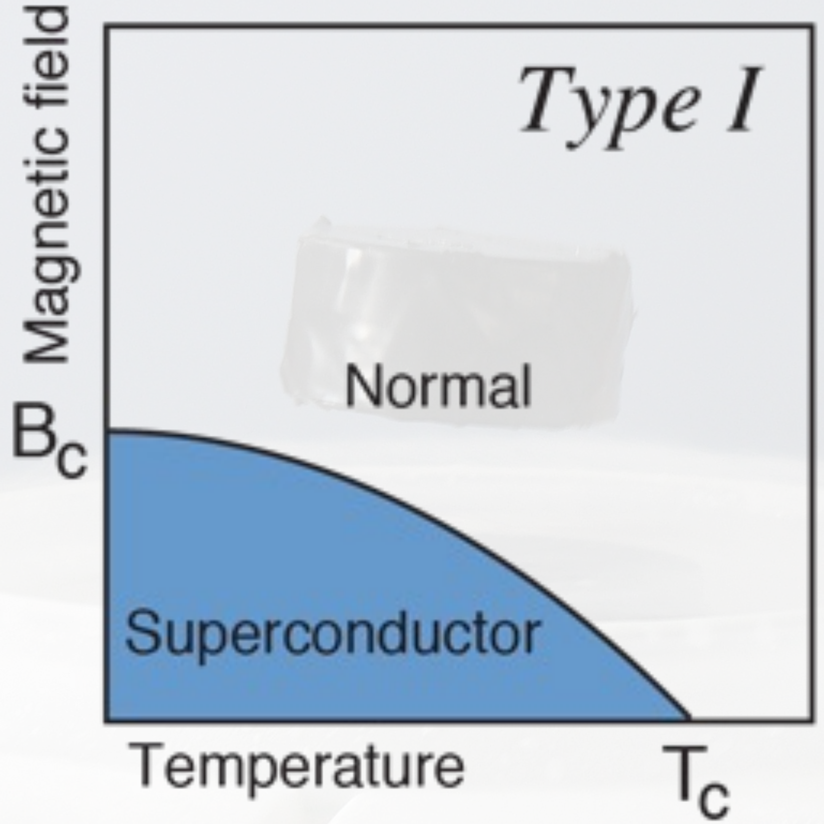
$$+m^2(T)|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

Below the critical temperature the mass-squared is negative:

Photon has become massive: $m_A \sim e\langle\Phi\rangle$

Other Background Parameters

The phase doesn't only depend on the temperature, but other background parameters such as an external magnetic field.

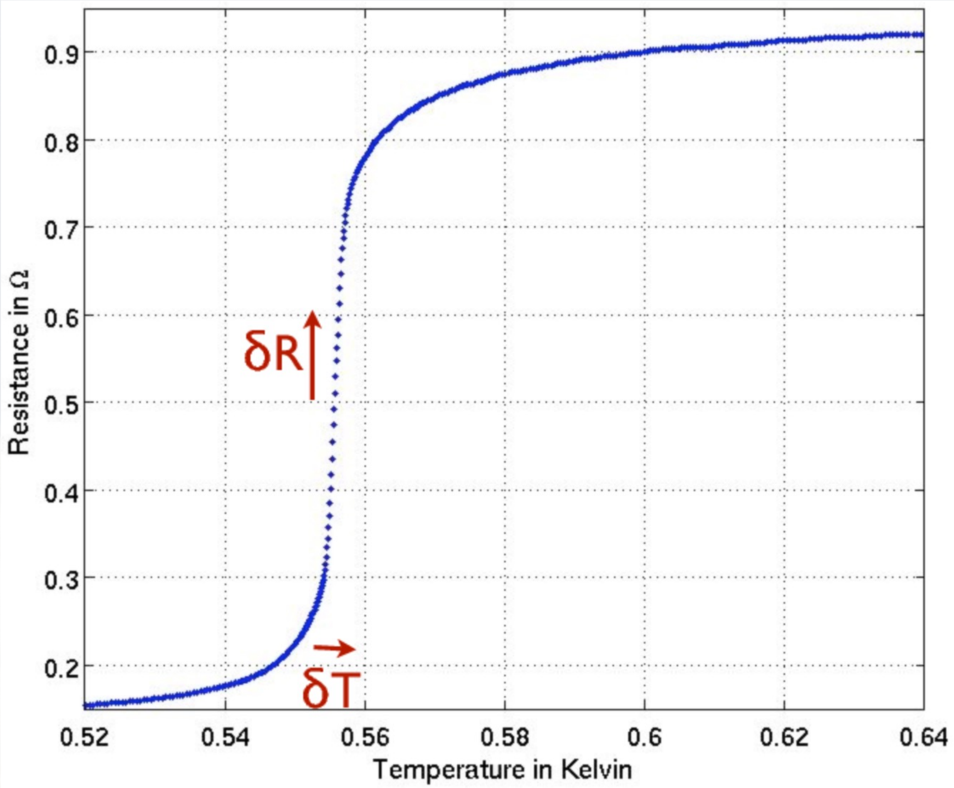


Taken from Hyperphysics

For a range of temperatures could tune B to sit arbitrarily close to the critical point.

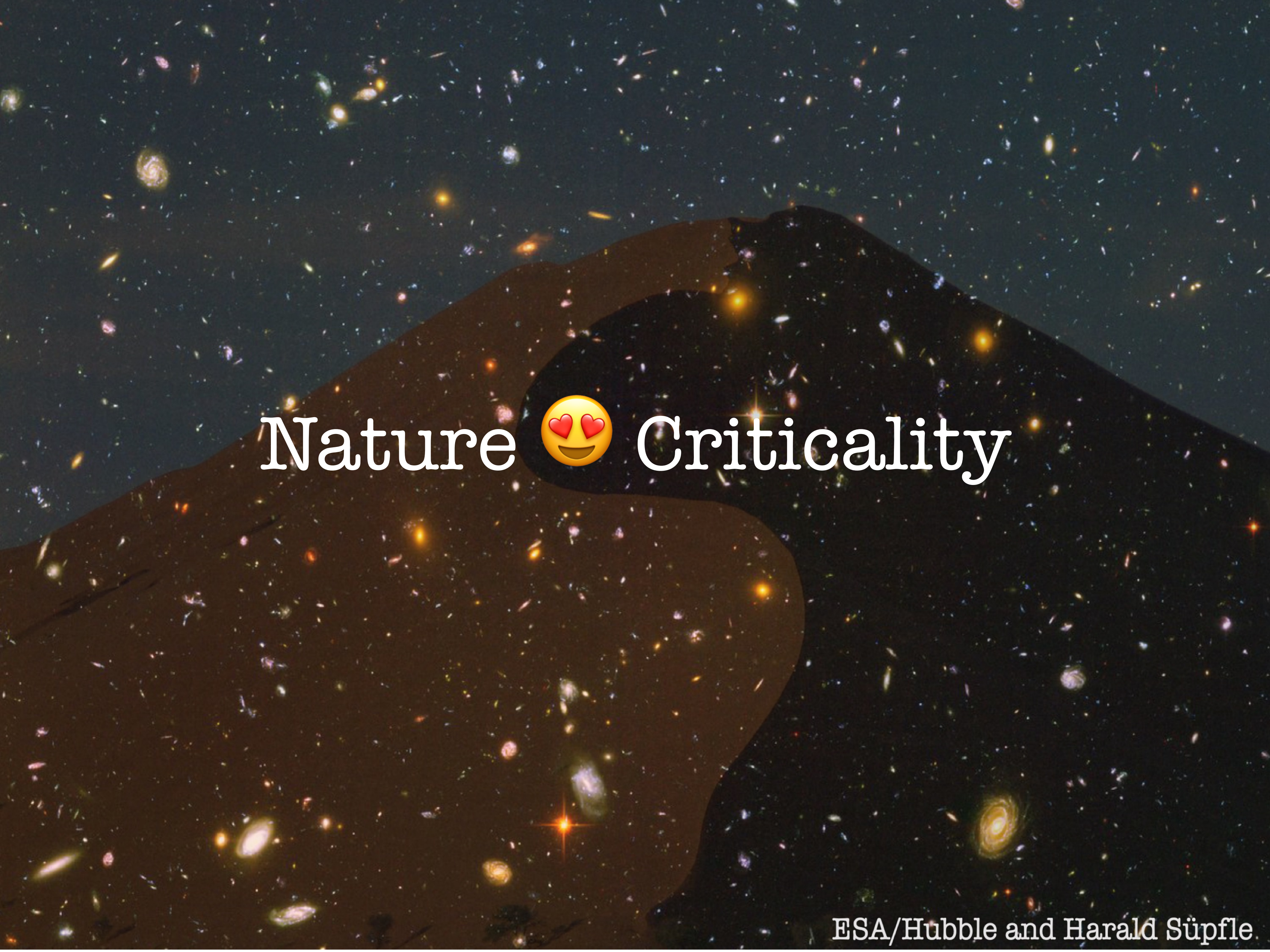
TES

In a transition edge sensor the temperature is fine-tuned, through a feedback loop, to sit at precisely the critical point...



Taken from 1309.5383

A big fluctuation gives a big change!


Nature 😍 Criticality
ESA/Hubble and Harald Süpfle

Puzzle 1

ESA/Hubble and Harald Süpfle

A Stable Universe?

It is as if there is some additional piece in the potential for the entire Universe that knew in advance, before the electroweak phase transition had even happened, to precisely cancel the Higgs/QCD… contribution

$$V = V_H + V_0$$

where

$$V \ll V_H$$

The Universe is delicately and calmly balanced between two violent phases. Why?

A Stable Universe?

It is as if there is some additional piece in the potential for the entire Universe that knew in advance, before the electroweak phase t[illegible]ition had even happened, to precisely cance[illegible] (QCD[illegible]) contribution

The cosmological constant problem.

$$V = V_H + \ldots$$ [illegible]

where

$$V \ll V_H$$

The Universe is delicately and calmly balanced between two violent phases. Why?

Puzzle 2

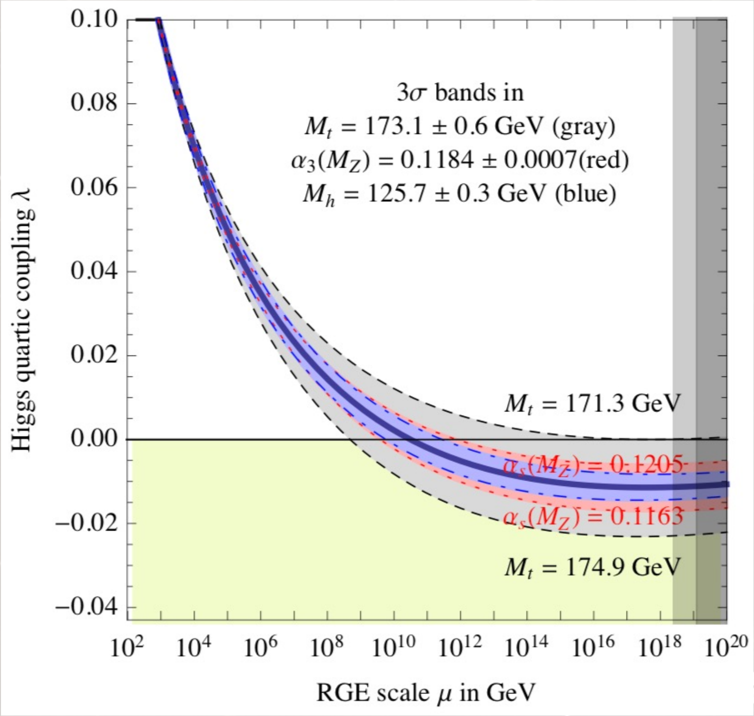
ESA/Hubble and Harald Süpfle

A Metastable Universe?

The Higgs potential also depends on the Higgs itself, due to quantum mechanics:

$$V_H = M^2(H)|H|^2 + \lambda(H)|H|^4$$

The Higgs quartic interaction effectively turns negative at large field values:



Taken from 1205.6497

A/Hubble and Harald Süpfle

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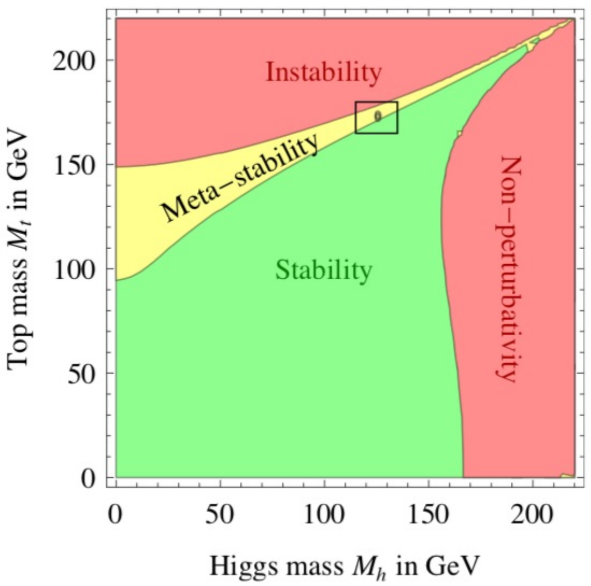
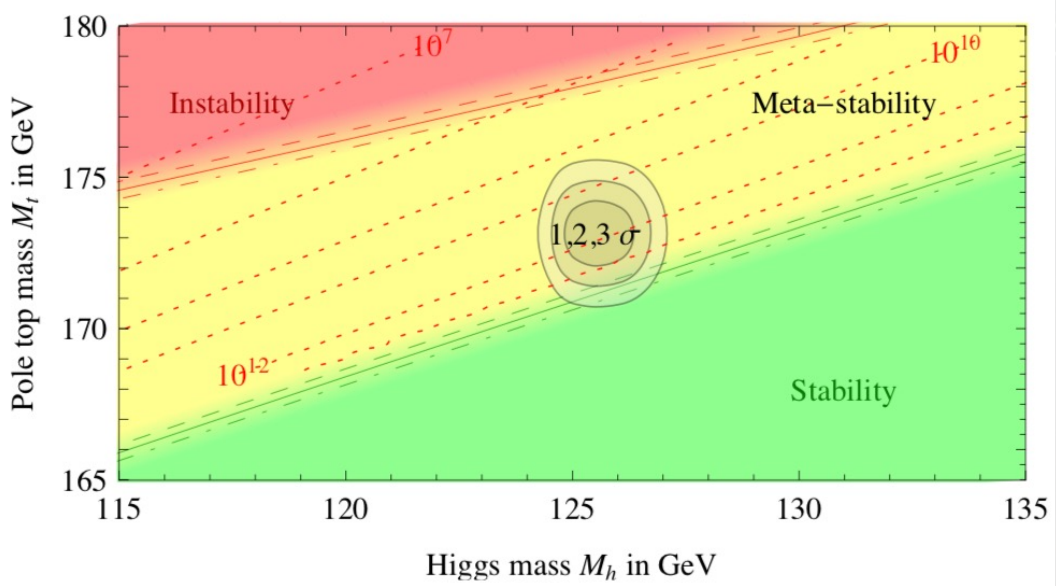
ESA/Hubble and Harald Süpfle

A Metastable Universe?

This means that the vacuum we are in, as in the pictures, is just local, but there is a deeper one out at large field values.





The fundamental parameters we have measured in our "room" imply that nature is delicately balanced at a critical point where two Higgs phases may coexist.

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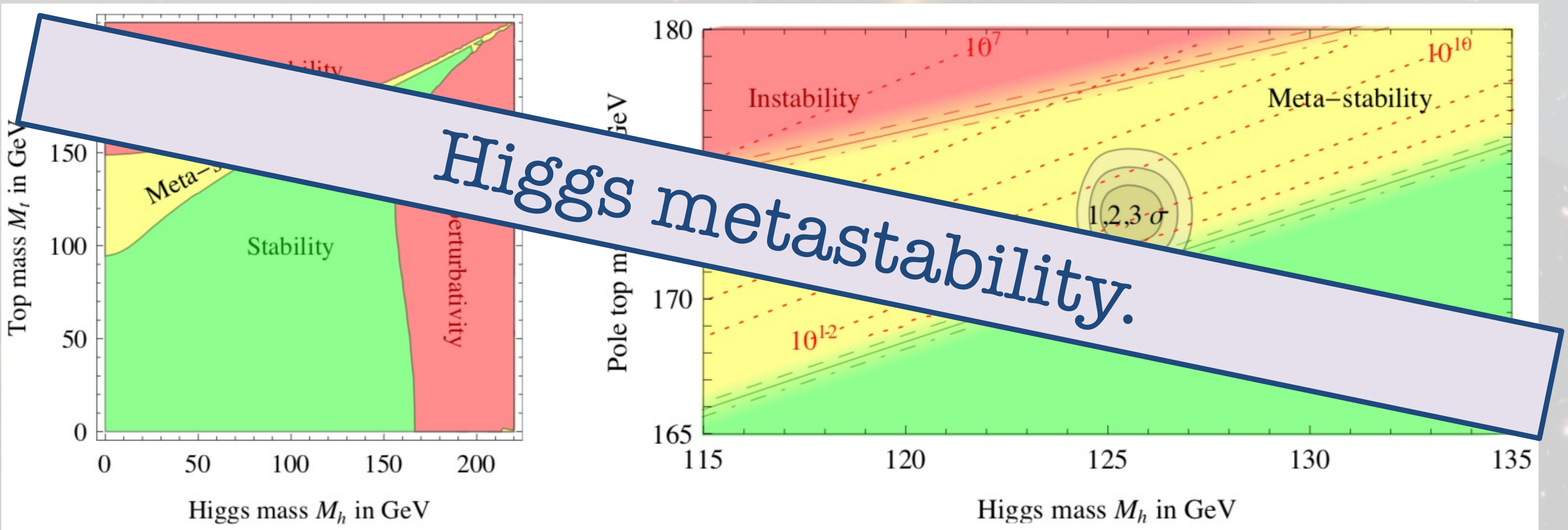

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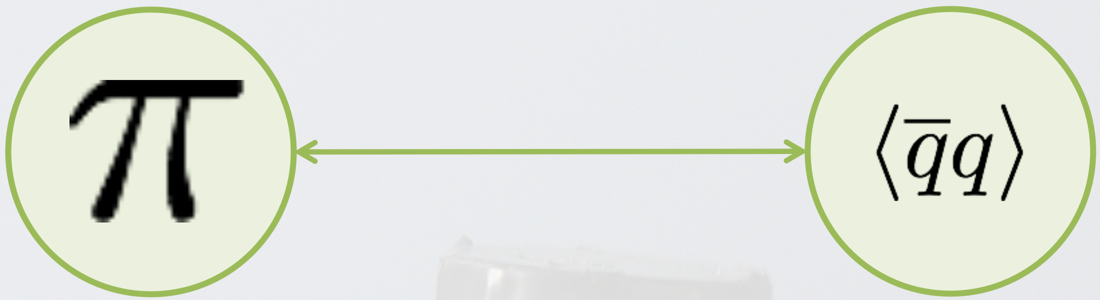
ESA/Hubble and Harald Süpfle

Puzzle 3

Critical Higgs

Consider pions:



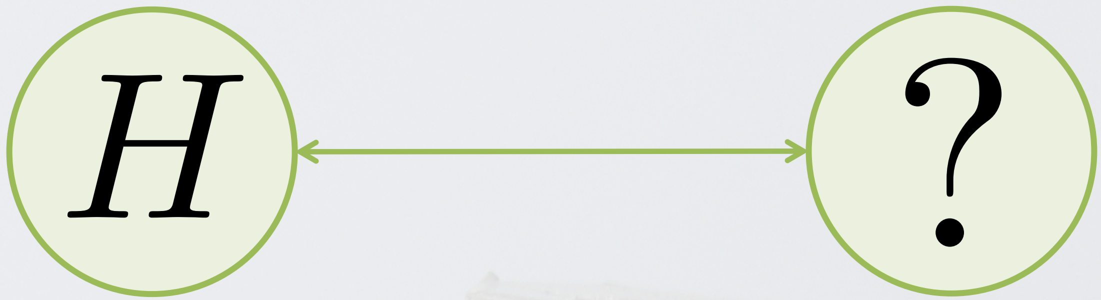
The order parameter for the condensate and the pion mass are both calculable in terms of microscopic theory

$$f_\pi \sim \frac{\Lambda_{\rm QCD}}{g_\star} \qquad\qquad m_\pi^2 \sim m_q \Lambda_{\rm QCD}$$

and both follow typical symmetries + scales.

Critical Higgs

What about the Higgs?

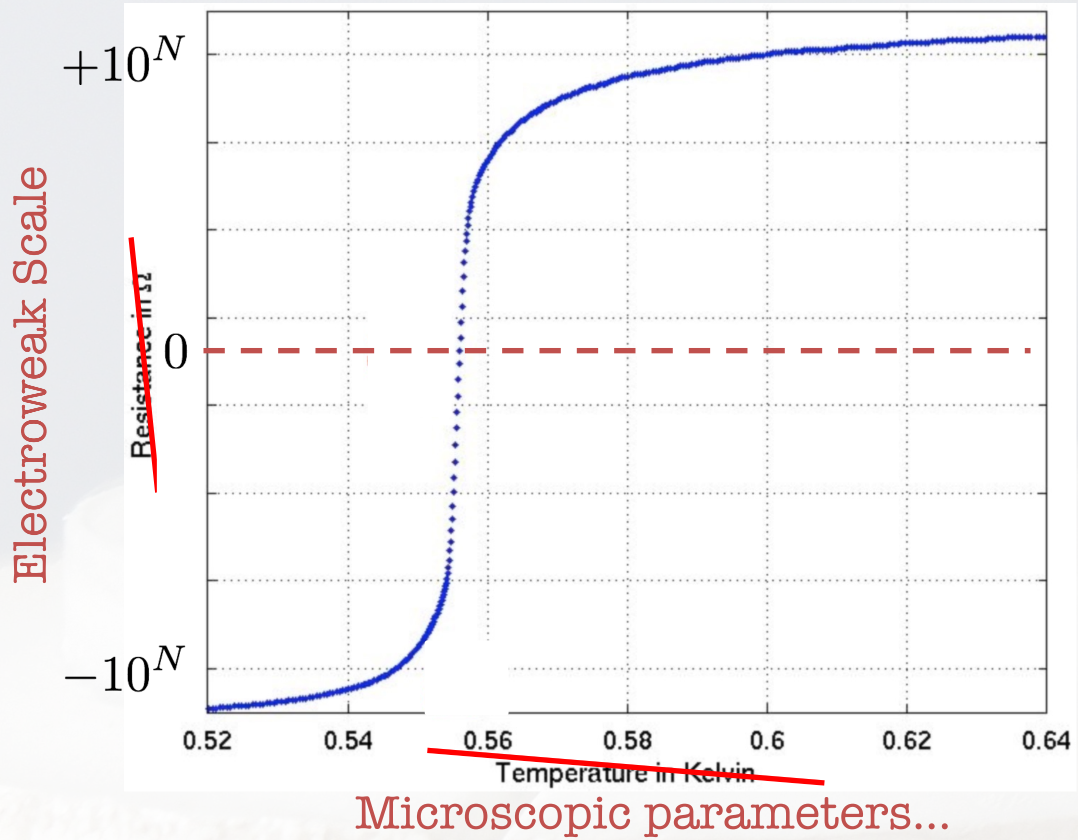


If there is some scale at which the electroweak scale (order parameter) and Higgs mass become calculable in terms of the microscopic theory then the LHC is telling us that:

$$v \ll \frac{\Lambda}{g_\star} \qquad\qquad m_h^2 \ll \hbar \lambda^2 \Lambda^2$$

Critical Higgs

Essentially, it seems like the Universe is just like a Transition Edge Sensor:



- Taken from 1309.5383

Cosmology 😍 Criticality?

ESA/Hubble and Harald Süpfle

Hypothesis

Could cosmological evolution of a light field have created (tuned) a hierarchy?

A light scalar on its own: pNGB only option...

$$\mathcal{L}_{\mathrm{SM},\phi} = \ldots + \Lambda^2 |H|^2 \sum c_n \left(\frac{\phi}{f}\right)^n + \ldots$$

c_n: $\mathcal{O}(\sim 1)$

Perform an unphysical shift to give simple ansatz:

$$V \approx -\Lambda^2 \frac{\phi}{f} |H|^2 + g_\epsilon^2 \left(f^3 \phi + c_2 f^2 \phi^2 + \ldots \right)$$

Hypothesis

When Higgs effective bilinear, including all quantum corrections, is positive we have:

$$V \approx g_\epsilon^2 \left(f^3 \phi + c_2 f^2 \phi^2 + \ldots \right)$$

And when it is negative we have:

$$V \approx -\frac{\Lambda^4}{\lambda} \left(\frac{\phi}{f} \right)^2 + g_\epsilon^2 \left(f^3 \phi + c_2 f^2 \phi^2 + \ldots \right)$$

But can at most have $\Lambda^4 \lesssim g_\epsilon^2 \lambda f^4$ otherwise virtual corrections from Higgs dominate the potential.

Hypothesis

When Higgs effective bilinear, including all quantum corrections, is positive we have:

$$\ldots^2 \left(f^3\phi + c_2 f^2 \phi^2 + \ldots\right)$$

The critical point is <u>not generally</u> an extremum of the scalar potential unless one fine-tunes the potential parameters!

$$V \approx -\frac{\Lambda^4}{\lambda}\left(\frac{\phi}{f}\right)\ldots + g_\epsilon \left(\ldots\right)$$

But can at most have $\Lambda^4 \lesssim g_\epsilon^2 \lambda f^4$ otherwise virtual corrections from Higgs dominate the potential.

Necessary Ingredients

a) Some reason for the critical point to be special for an evolving scalar.

b) Some way to get to, and stay at, the critical point.

The Relaxion

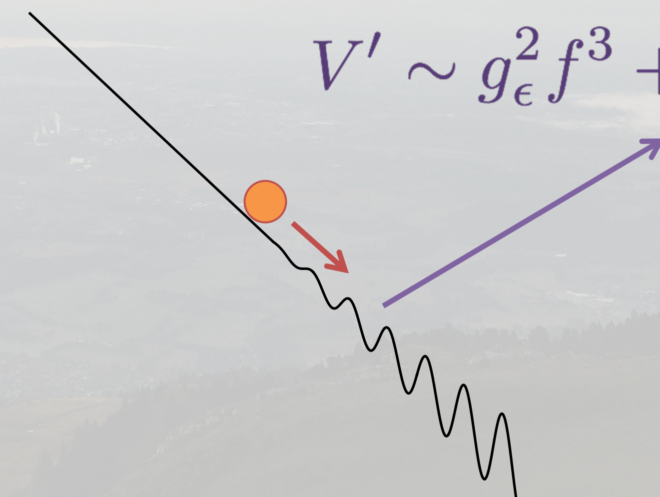
Graham, Kaplan, Rajendran, 2015

a) Some reason for the critical point to be special for an evolving scalar:

$$\mathcal{L}_{\rm Int} = \frac{g^2\phi}{32\pi^2 F}\widetilde{G}G \Rightarrow m_\pi^2 f_\pi^2 \frac{\langle h \rangle}{v} \cos\frac{\phi}{F}$$

b) Some way to get to, and stay at, the critical point:

$$V' \sim g_\epsilon^2 f^3 + m_\pi^2 \frac{f_\pi^2}{F}\frac{\langle h \rangle}{v}\cos\frac{\phi}{F}$$



The Relaxion

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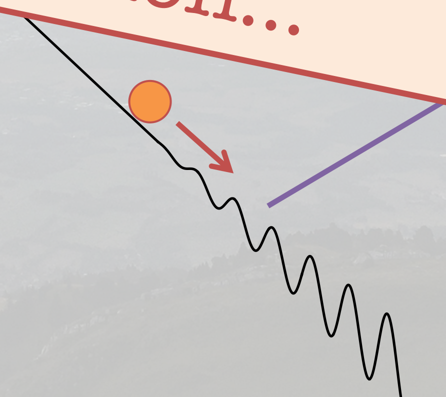
a) Some reason for the critical point to be ...ial for an evolving scalar:

Footnote: Doesn't work for QCD coupling. New gauge group, new EW states needed.

Also tension with anthropics for CC solution...



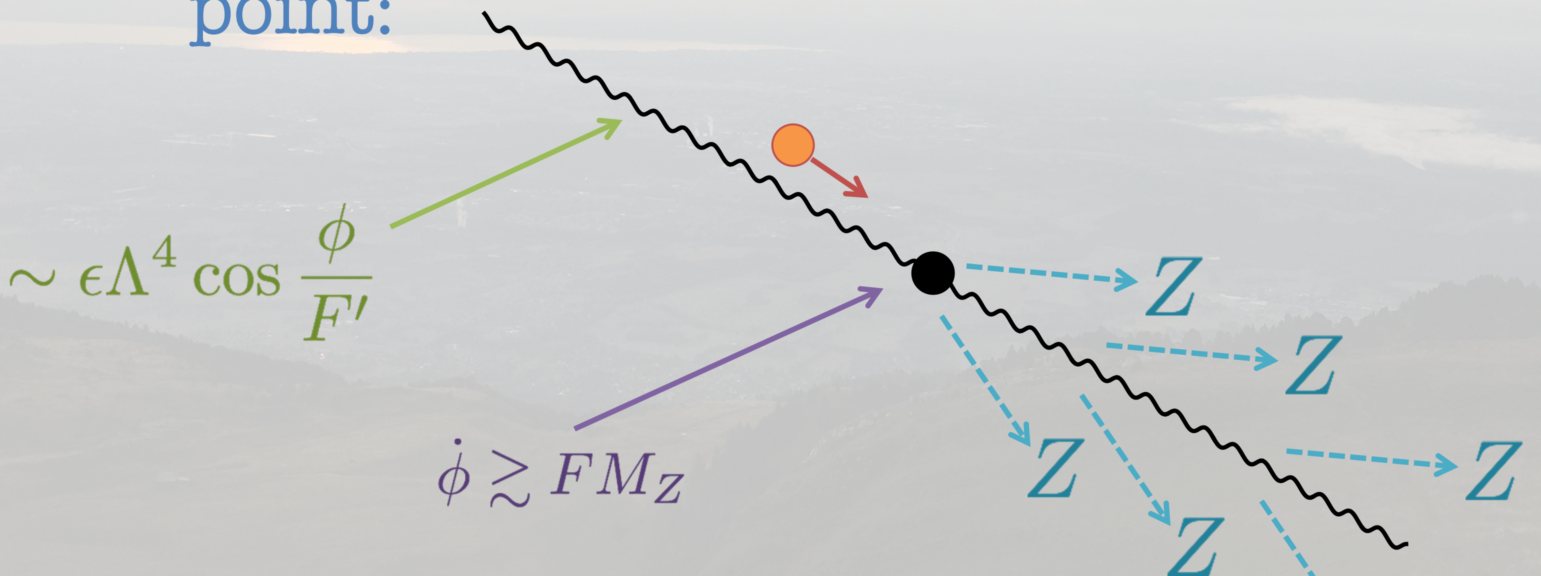
Particle Production

Hook, Marques-Tavares, 2016

a) Some reason for the critical point to be special for an evolving scalar:

$$\mathcal{L}_{\rm Int} = \frac{\phi}{8\pi F}\left(\alpha_Y \widetilde{B}B - \alpha_W \widetilde{W}W\right)$$

b) Some way to get to, and stay at, the critical point:



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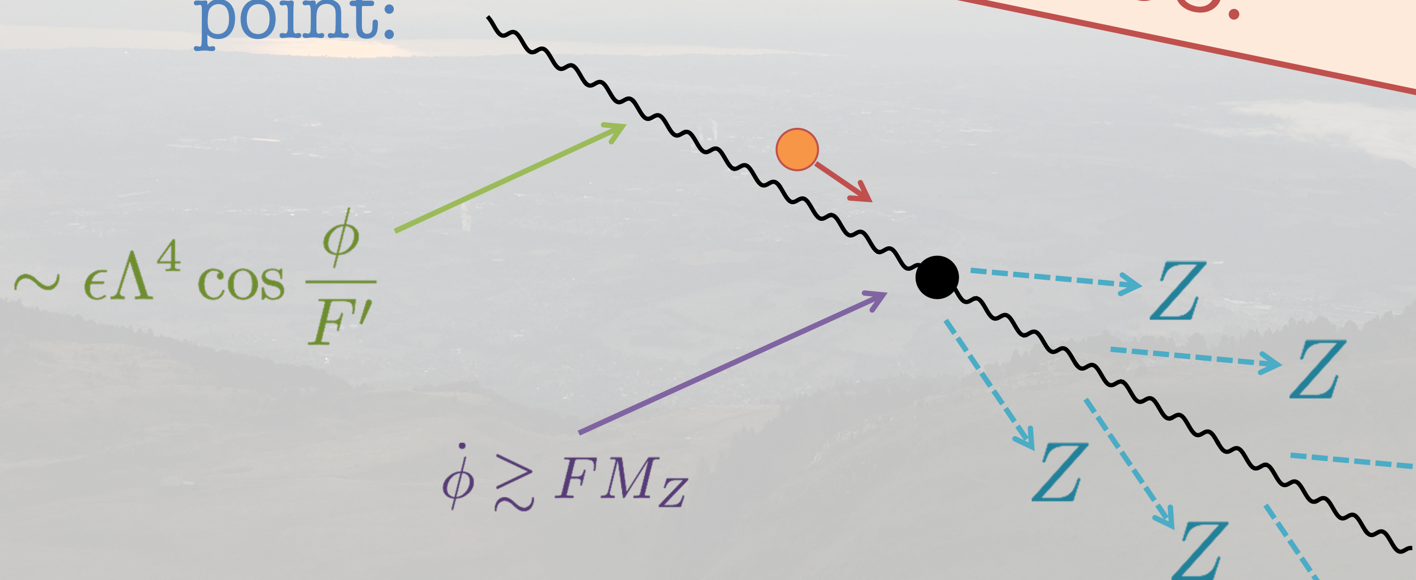
a) Some reason for the critical point to be special for an evolving scalar:

$$\frac{\phi}{\cdots}\left(\alpha_Y \widetilde{B}B - \alpha_W \widetilde{W}W\right)$$

Footnote: Beware Schwinger production.
See 2108.11295.

b) Some way to g… point:



Inflating to the Weak Scale

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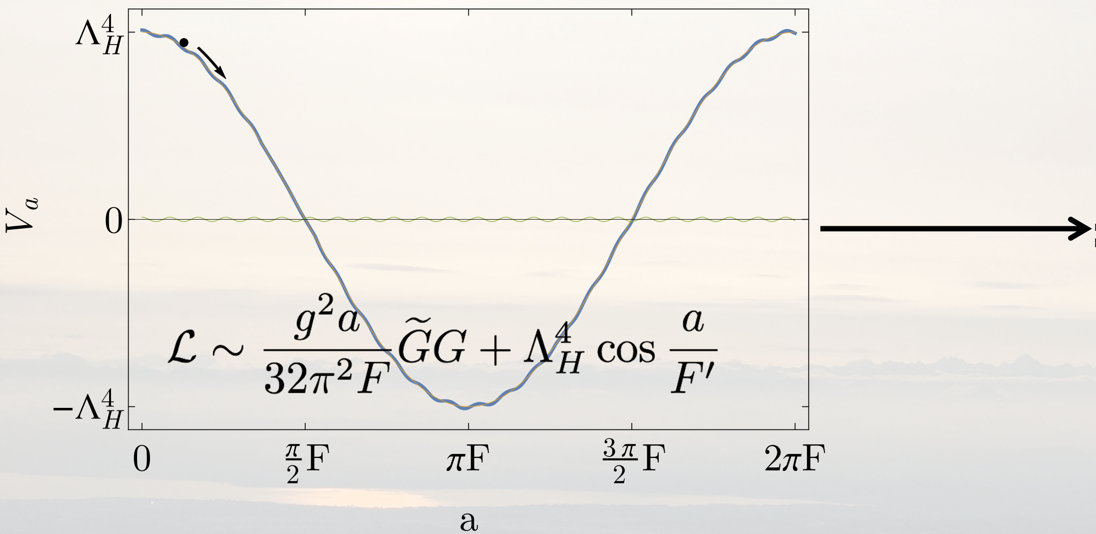

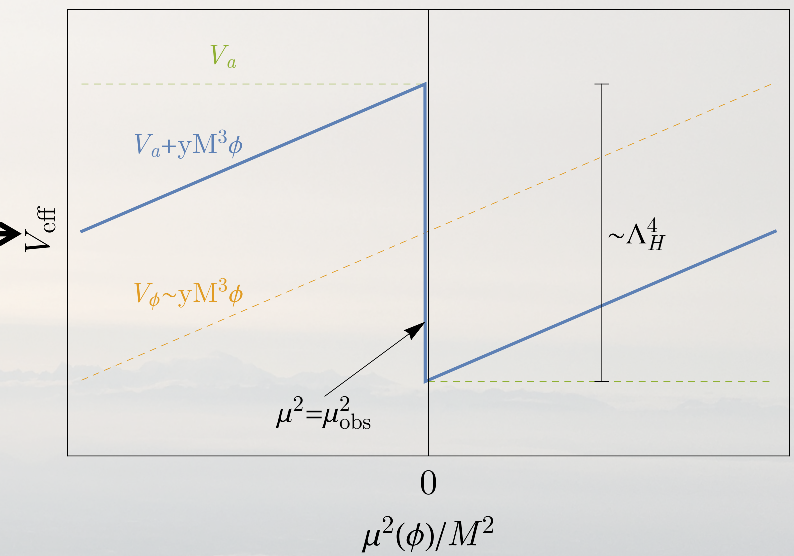

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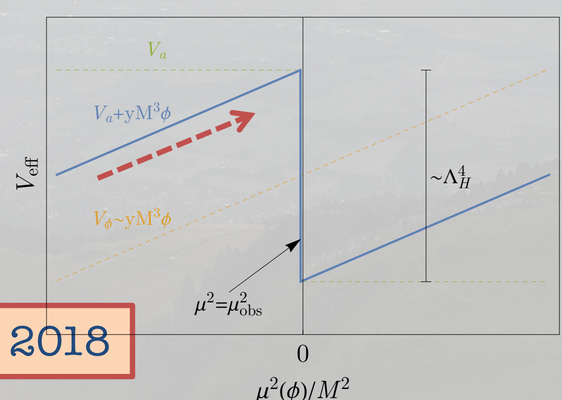

Volume effects in eternal inflation drive volume-weighted scalar distribution to diffuse up potential.

Geller, Hochberg, Kuflik, 2018

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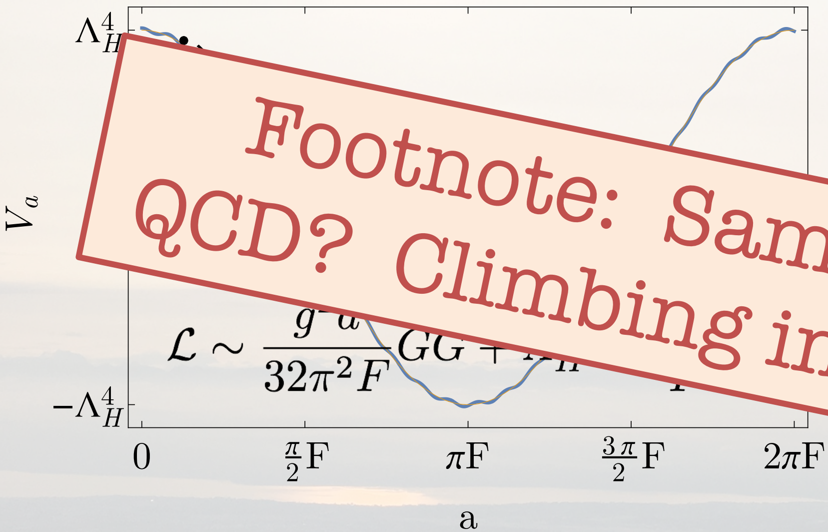

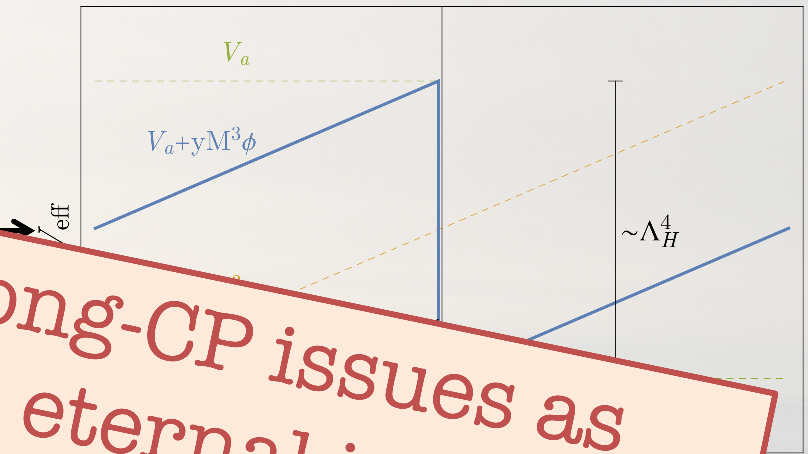

Footnote: Same Strong-CP issues as QCD? Climbing implies eternal inflation.

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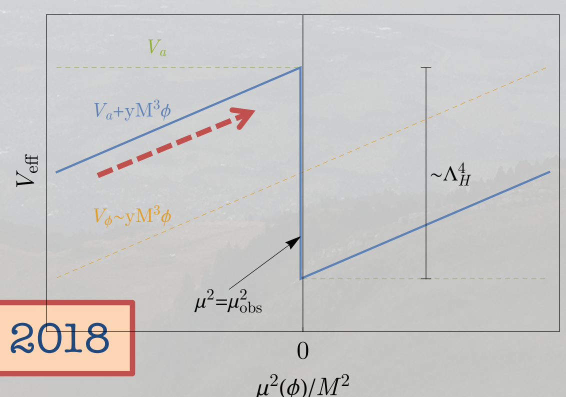

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Crunching Dilaton

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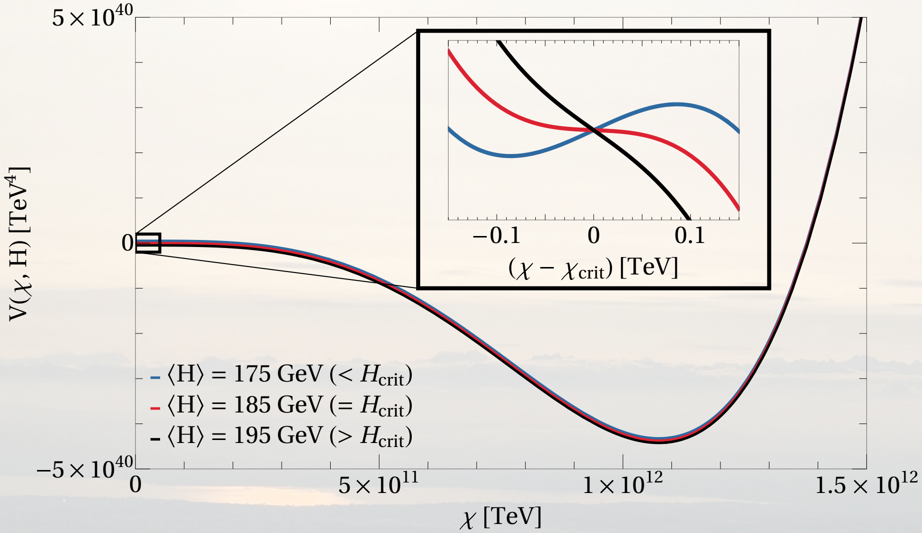

Higgs mixes with dilaton of a CFT sector. Only for small weak scale does a metastable vacuum exist.

b) Some way to get to, and stay at, the critical point: Landscape scans Higgs bilinear. All values outside weak scale range are crunched.

Csaki, D'Agnolo, Geller, Ismail, 2020

Crunching Dilaton

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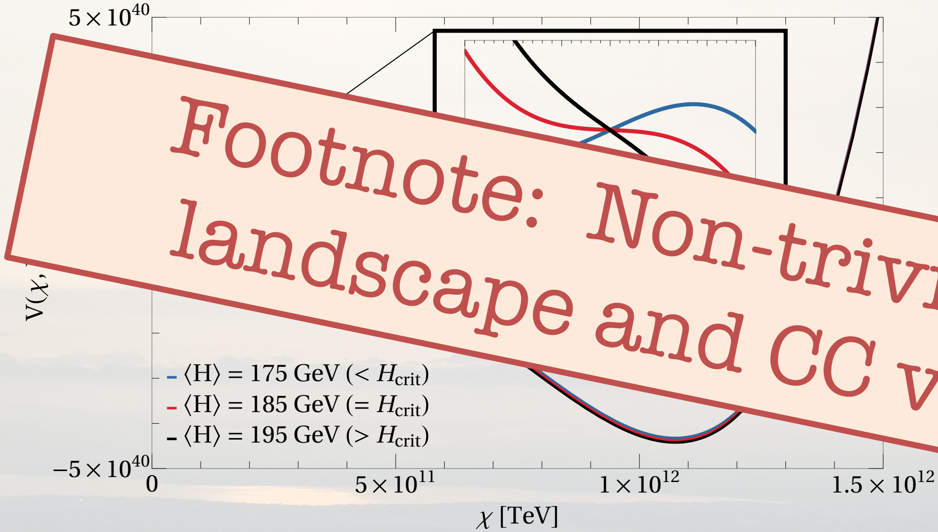

Higgs mixes with dilaton of a CFT sector. Only for small weak scale does a metastable ...um exist.

Footnote: Non-trivial interplay with landscape and CC values/scanning.

b) Some way to get to, and stay at, the critical point: Landscape scans Higgs bilinear. All values outside weak scale range are crunched.

Csaki, D'Agnolo, Geller, Ismail, 2020

Outlier: NNaturalness

Suppose there are many sectors. If randomly distributed some of them will be light:

Nima ArkaNi-Hamed, Tim CoheN, Raffaele Tito D'AgNolo, ANson Hook, HyuNg Do Kim, David PiNNer, 2016

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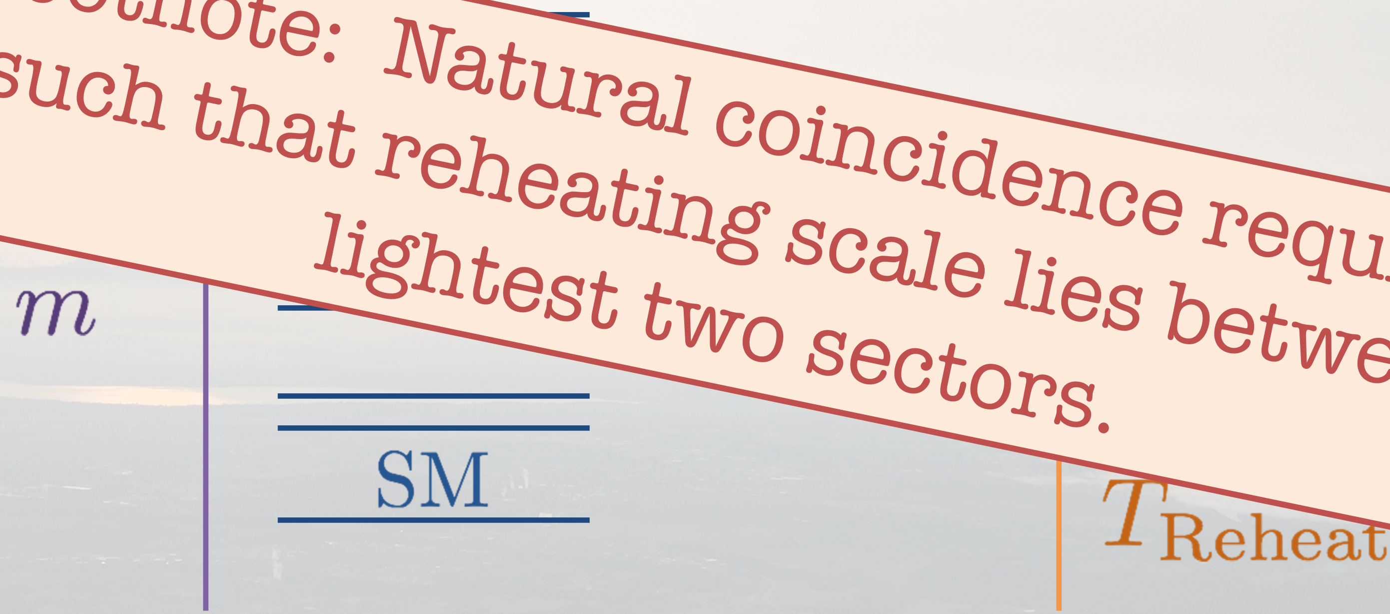
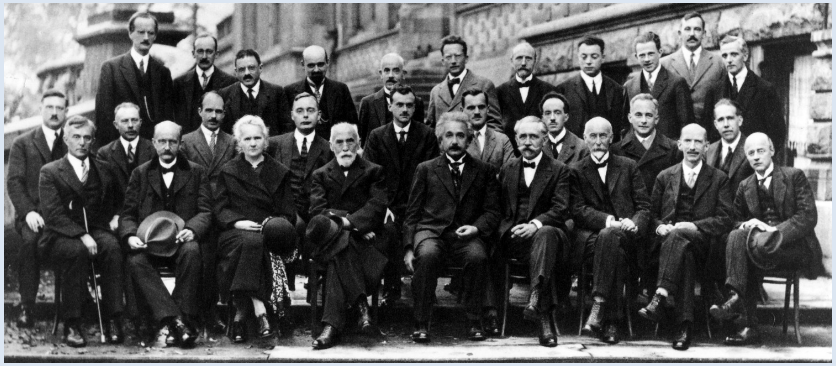


Footnote: Natural coincidence required such that reheating scale lies between lightest two sectors.

Nima ArkaNi-Hamed, Tim CoheN, Raffaele Tito D'AgNolo, ANson Hook, HyuNg Do Kim, David PiNNer, 2016

Personal View

Past breakthroughs were made by venturing into incomplete frameworks, often even involving unregulated infinities. QM, QFT...



Given the limited success of symmetry-based approaches in taking us beyond the SM, perhaps we ought to spend more time in uncharted territory?

“Self-Organised Localisation”.

Giudice, MM, You, 2021

ESA/Hubble and Harald Süpfle

Stochastic Inflation

The stochastic approach to inflation developed by Starobinsky and others has been a useful guide.

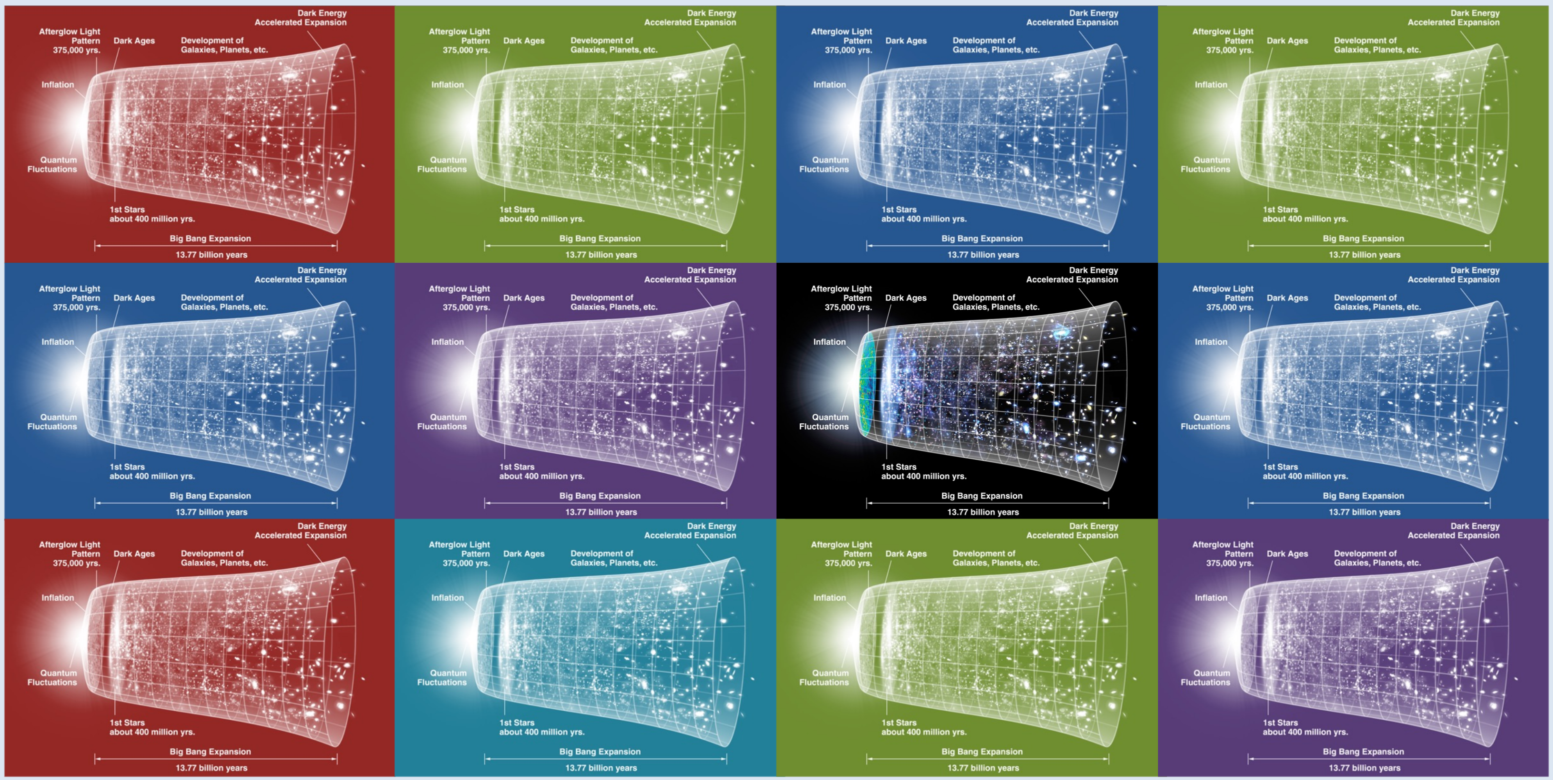
Quantum calculations have confirmed some aspects of this approach as a leading order picture, in the pre-eternal regime.

(See papers by Gorbenko, Senatore, and Cohen, Green, Premkumar, Ridgway!)

Still, however, much work remaining...

Inflation

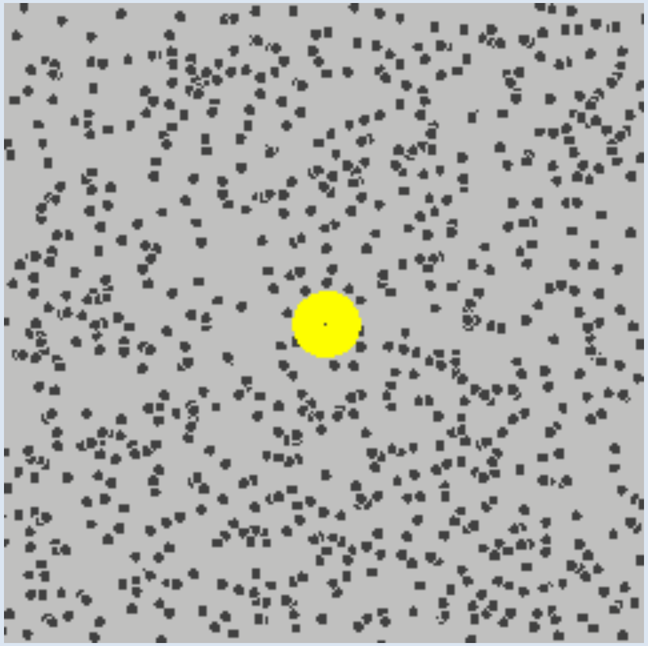
Eternal inflation leads to a multiverse of different Universes. We are but one...



In each one, different parameters, forces...

Inflation

Suppose you have a box of gas and you measure the velocity of one atom, once.



Is that value of velocity likely, or unlikely?

Inflation

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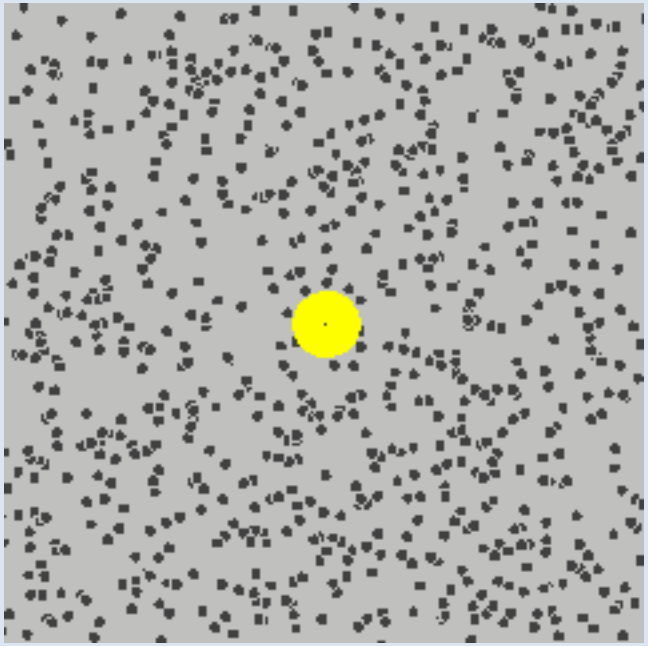
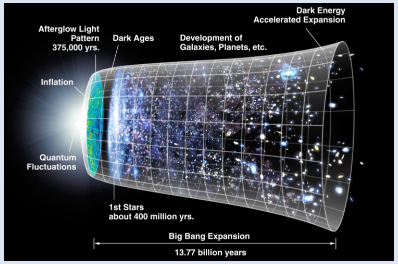


If we know the properties of the statistical ensemble at equilibrium, we have context.

Inflation

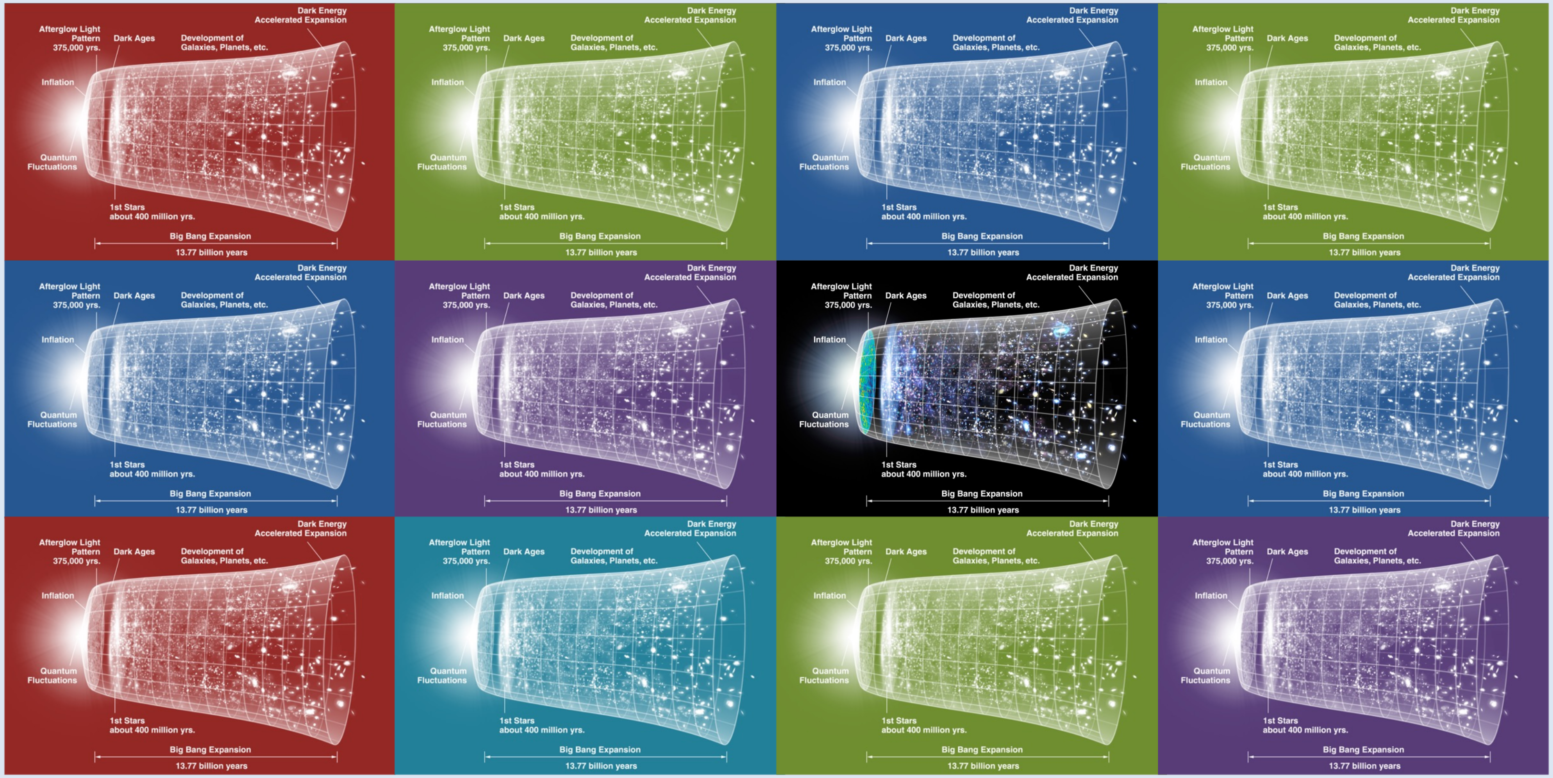
If there is a multiverse in which parameters, forces, etc are scanned then by measuring SM parameters...



how can we know if they are likely, unlikely, tuned, etc? Anthropics...?

Inflation

We need to know the macroscopic properties of the statistical ensemble



to assign context to parameters.


Towards quasi-statistics...
ESA/Hubble and Harald Süpfle

Random-ish Walks

The semi-classical stochastic approach offers possibility of "estimating" scalar field "distributions".

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3 P_{\mathrm{FP}})}{\partial\phi}+\frac{V' P_{\mathrm{FP}}}{3H}\right]=\frac{\partial P_{\mathrm{FP}}}{\partial t}$$

"Quantum" Gaussian Noise — EOM Drift — Time Evolution

Light scalar fields follow a Langevin-like trajectory. Average of trajectories described by a Fokker-Planck equation.

Turning the Volume Up

We're interested in the volume distribution

$$\frac{\partial}{\partial \phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial (H^3 P)}{\partial \phi} + \frac{V' P}{3H}\right] + 3HP = \frac{\partial P}{\partial t}$$

"Quantum" Gaussian Noise | EOM Drift | Volume expansion | Time Evolution

Light scalar fields follow a Langevin-like trajectory. Volume average of trajectories described by a Fokker-Planck-like equation.

Turning the Volume Up

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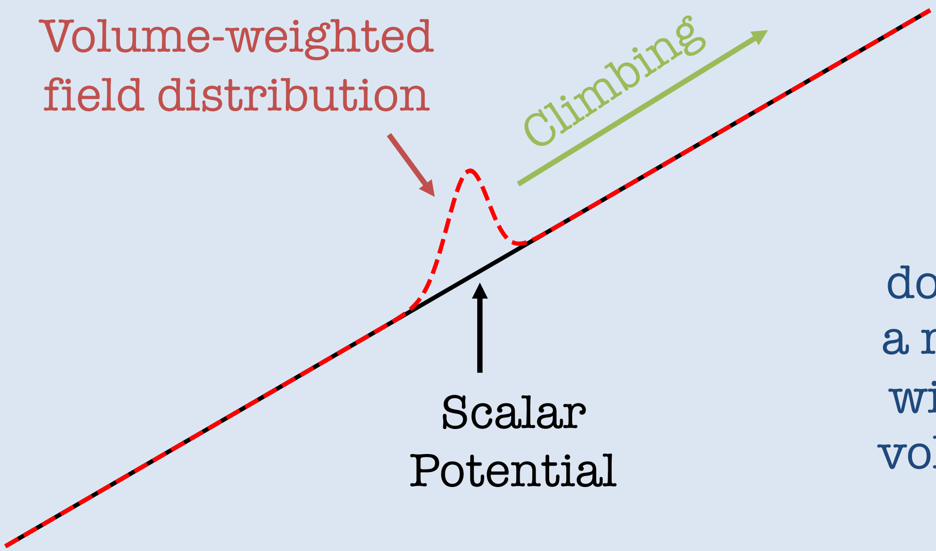
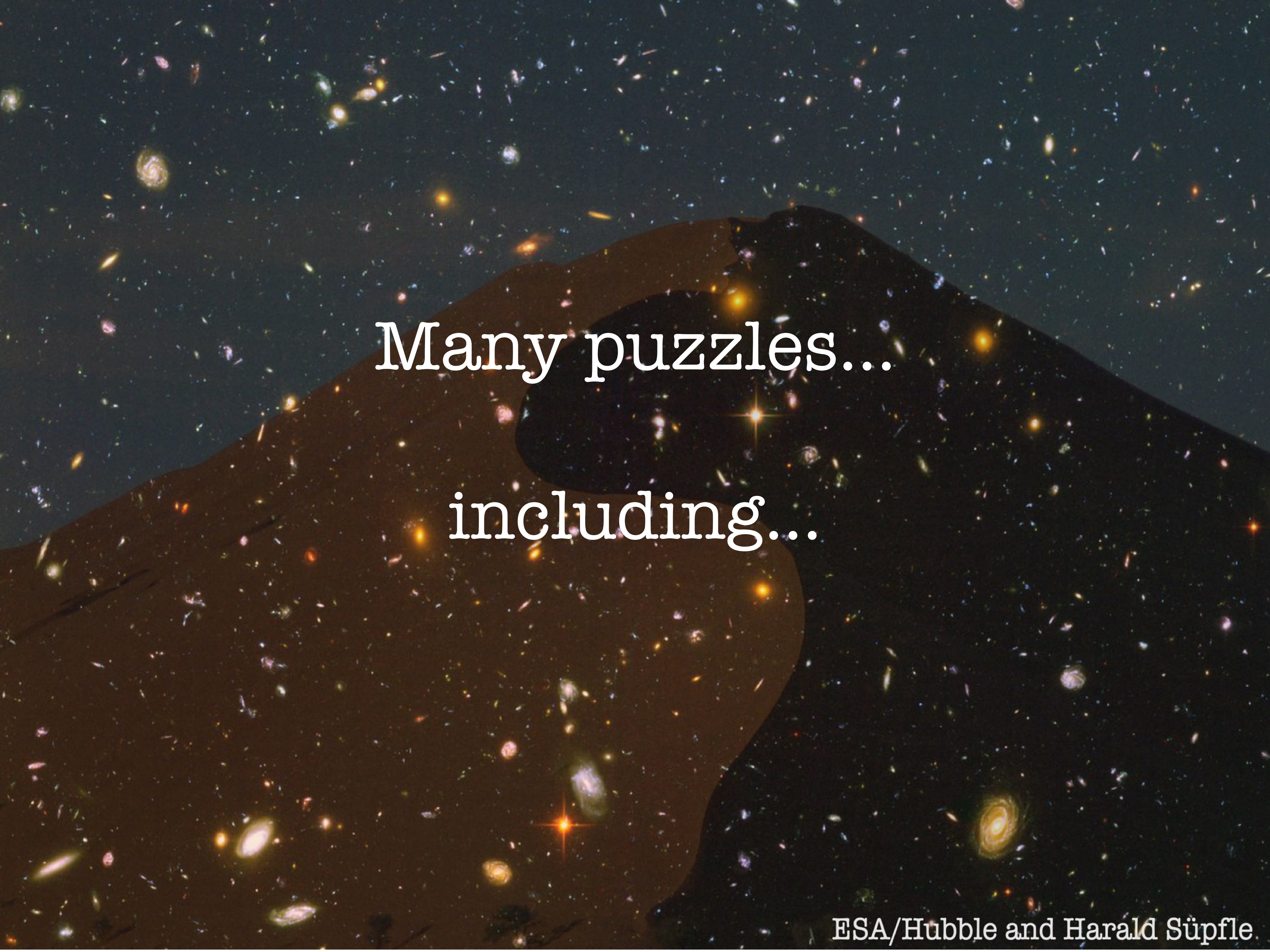
$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3\,P)}{\partial\phi}+\frac{V'\,P}{3H}\right]+3H\,P=\frac{\partial P}{\partial t}$$



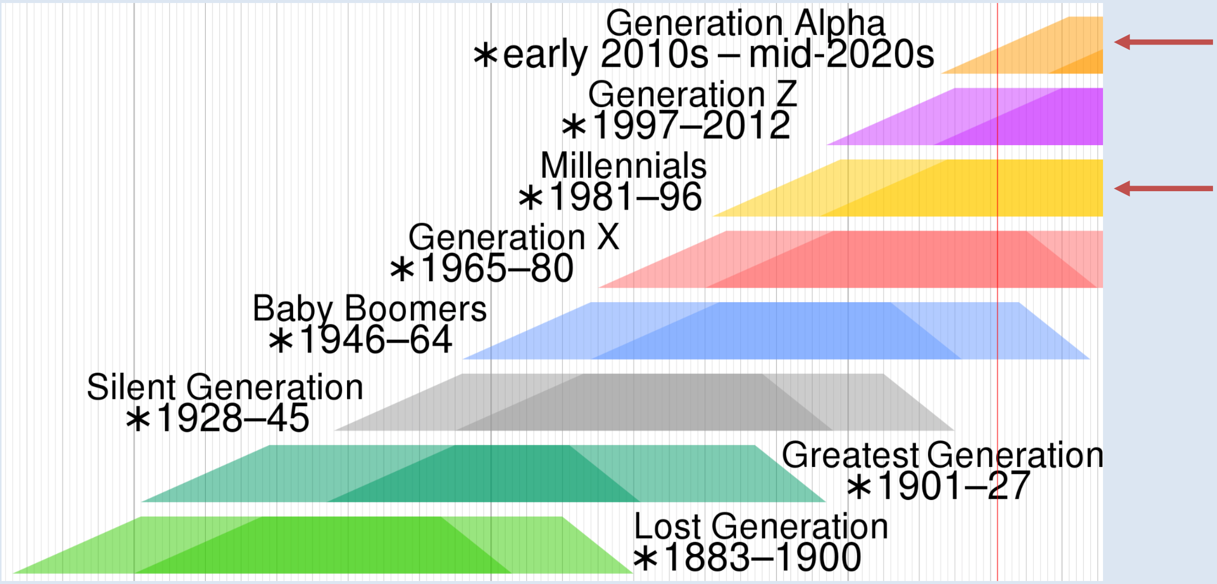
Even if a scalar field wants to roll down the scalar potential, on average, a rare upward fluctuation is rewarded with exponential growth. As a result, volume-weighted field distribution can climb up a potential!



ESA/Hubble and Harald Süpfle

Possible Biggest Question

The youngness paradox is severe.
Emphasised to us by Andrei Linde.



Universe should be much younger and hotter if proper time cutoff naively extrapolated to our time.

Criticality and Eternal Inflation...

Our Setup

Suppose the background parameters are controlled by some scalar field. In general language:

$$V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi) \ , \qquad \omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!}\varphi^n \ , \qquad \varphi \equiv \frac{\phi}{f} \ , \qquad \omega(0) = 0$$

When scalar potential is a small perturbation across the field range we can expand perturbatively to find:

$$\frac{\alpha}{2}\frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial\left(\omega' P\right)}{\partial \varphi} + \beta \omega P = \frac{\partial P}{\partial T}$$

Where:

$$\alpha = \frac{3\hbar H_0^4}{4\pi^2 g_\epsilon^2 f^4} \ , \qquad \beta = \frac{3\xi f^2}{2M_P^2} \ , \qquad T = \frac{t}{t_R} \ , \qquad t_R = \frac{3H_0}{g_\epsilon^2 f^2}$$

Quantum | Range | Clock | Timescale

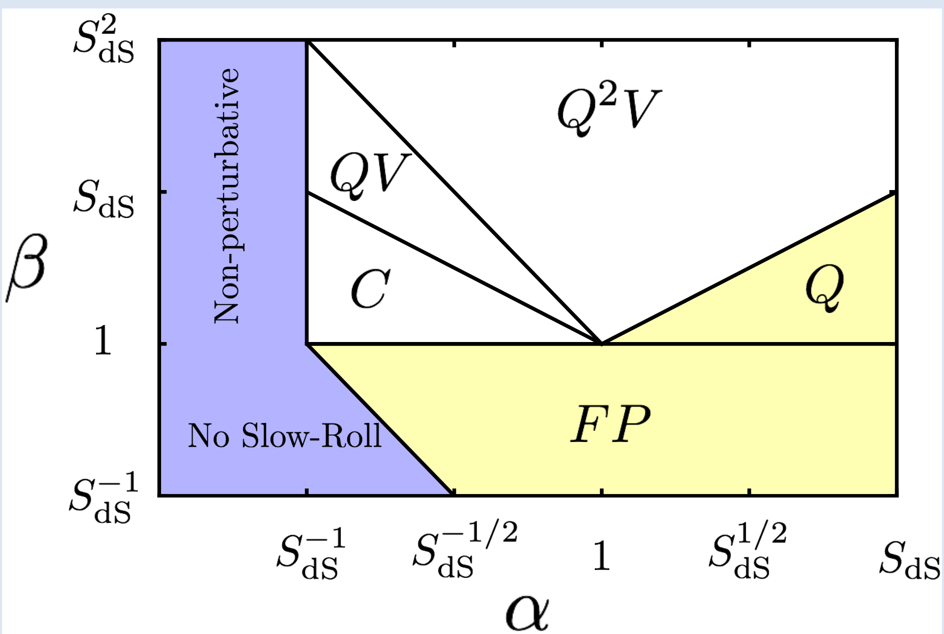
In Stasis

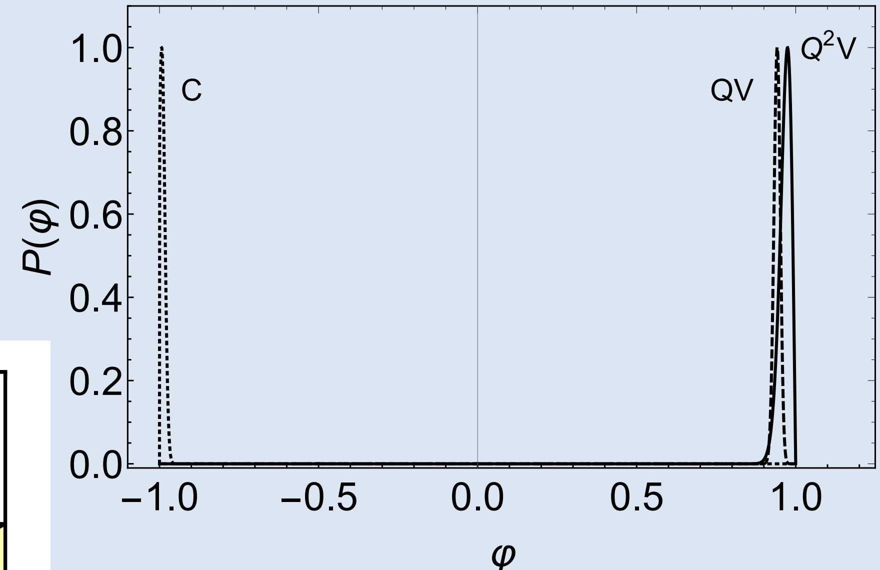
On a linear slope we identify three distinct parameter regimes:

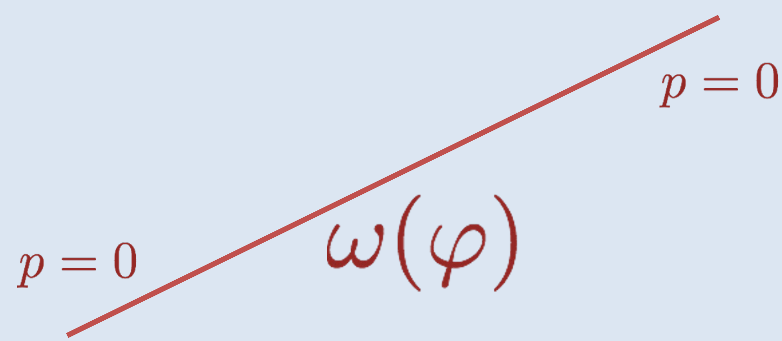
$$C : \alpha\beta < 1$$

$$QV : \alpha\beta > 1$$

$$Q^2V : \alpha^2\beta > 1$$







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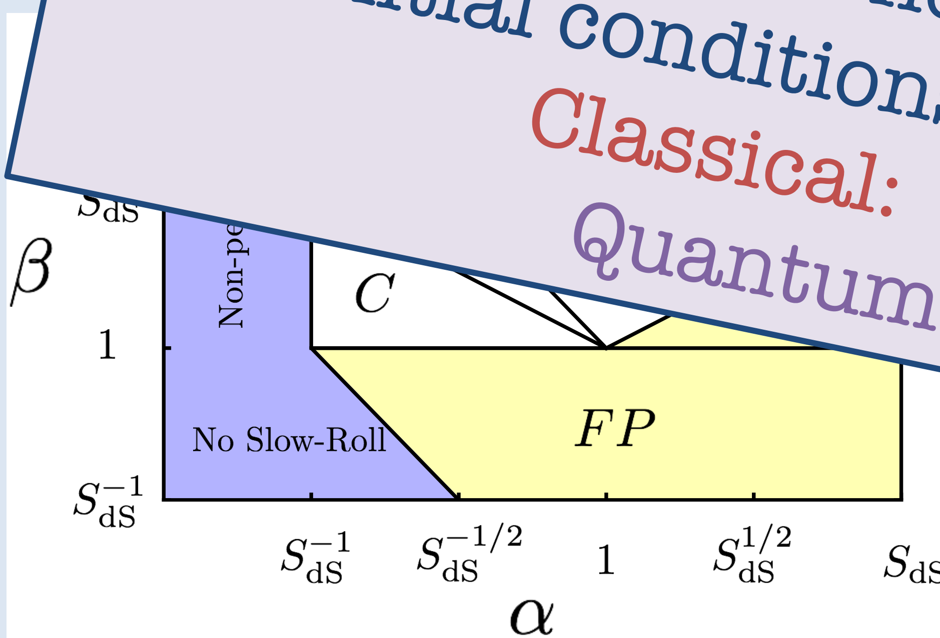
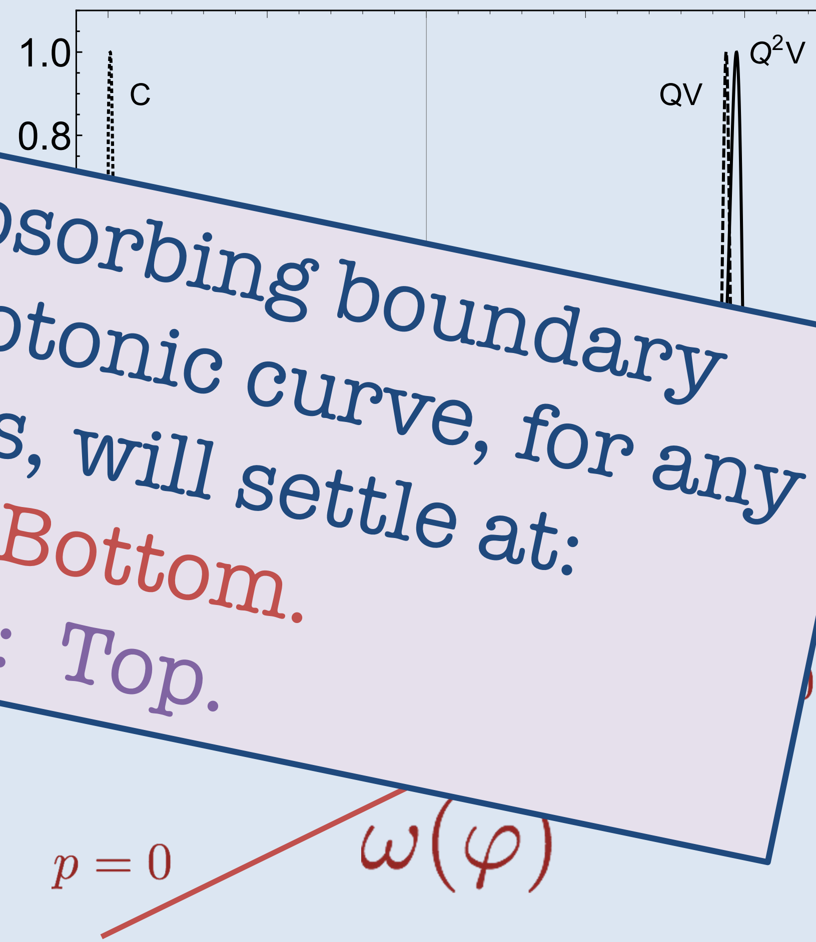




Punchline: For absorbing boundary conditions on a monotonic curve, for any initial conditions, will settle at:

Classical: Bottom.

Quantum: Top.

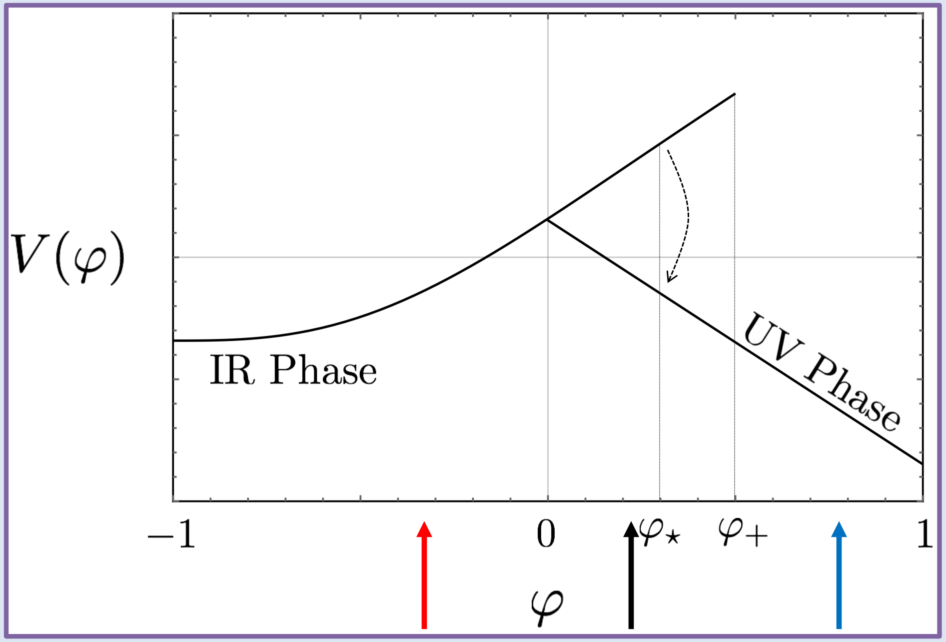
Putting this to use…

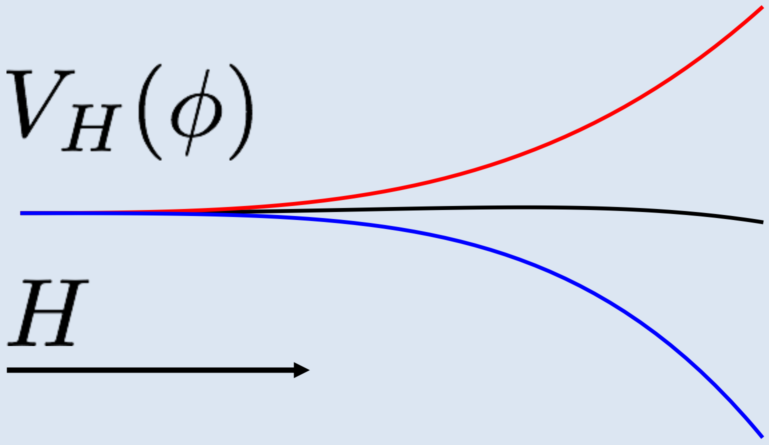
Application: SM Quartic

Suppose the scalar is scanning the SM parameters. In particular, the Higgs quartic, consistent with EFT

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) + \frac{\lambda(\varphi, h)}{4}\left(h^2 - v^2\right)^2$$

Scalar potential is:



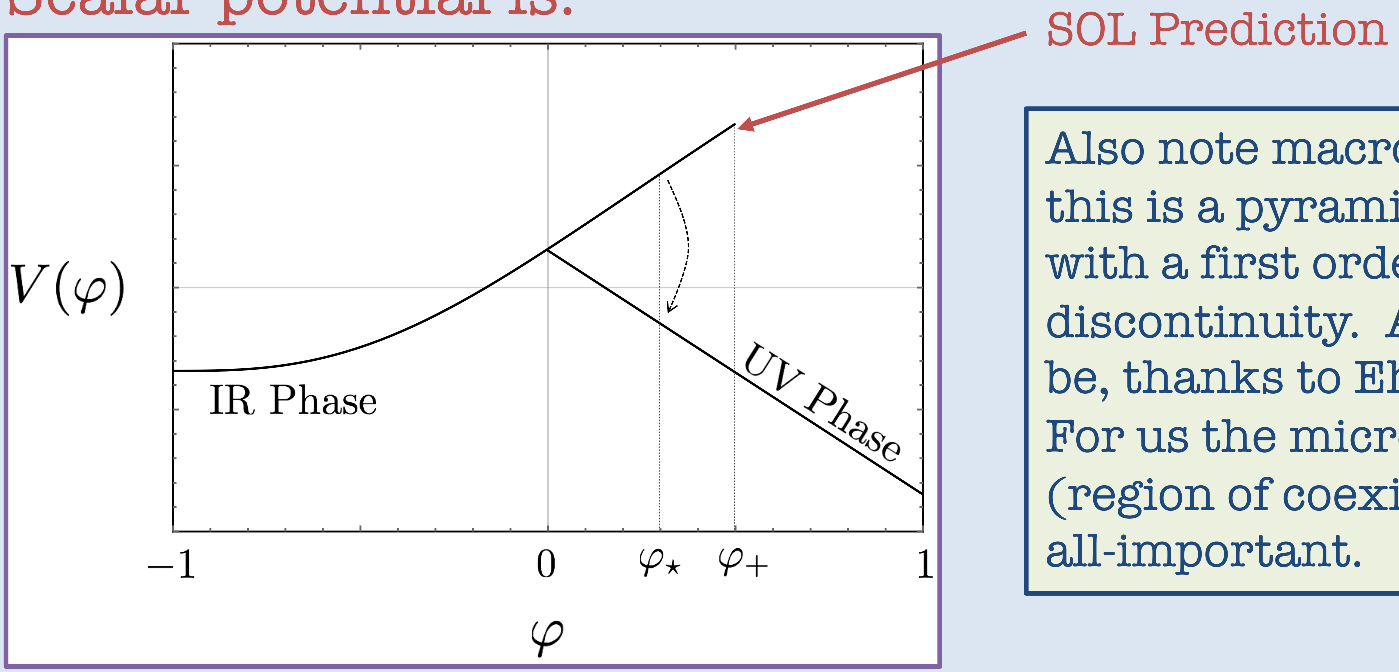


Application: SM Quartic

Suppose the scalar is scanning the SM parameters. In particular, the Higgs quartic, consistent with EFT

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) + \frac{\lambda(\varphi, h)}{4}\left(h^2 - v^2\right)^2$$

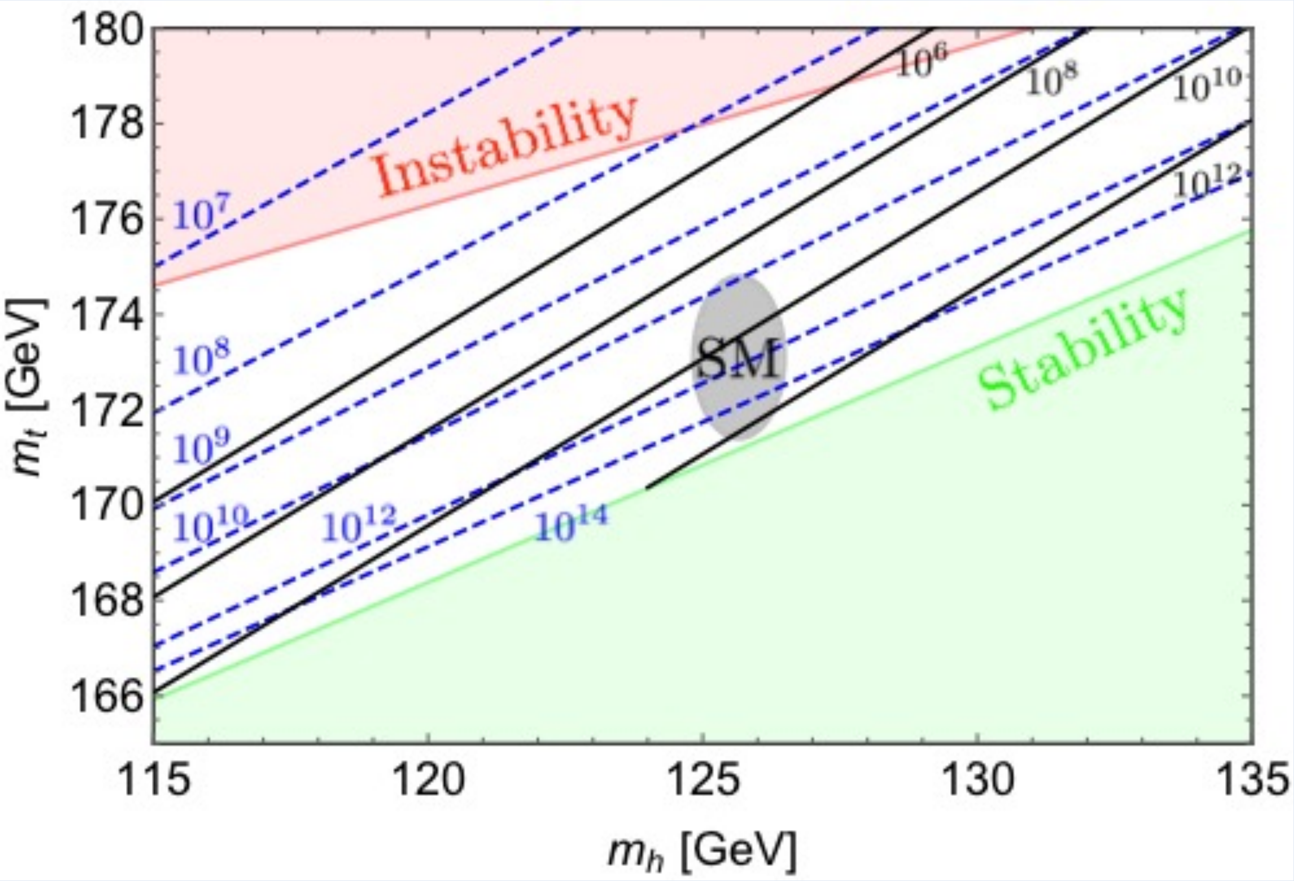
Scalar potential is:



Also note macroscopically this is a pyramid potential with a first order discontinuity. As it should be, thanks to Ehrenfest. For us the microscopics (region of coexistence) is all-important.

Application: SM Quartic

Prediction is metastability region, since top of potential:



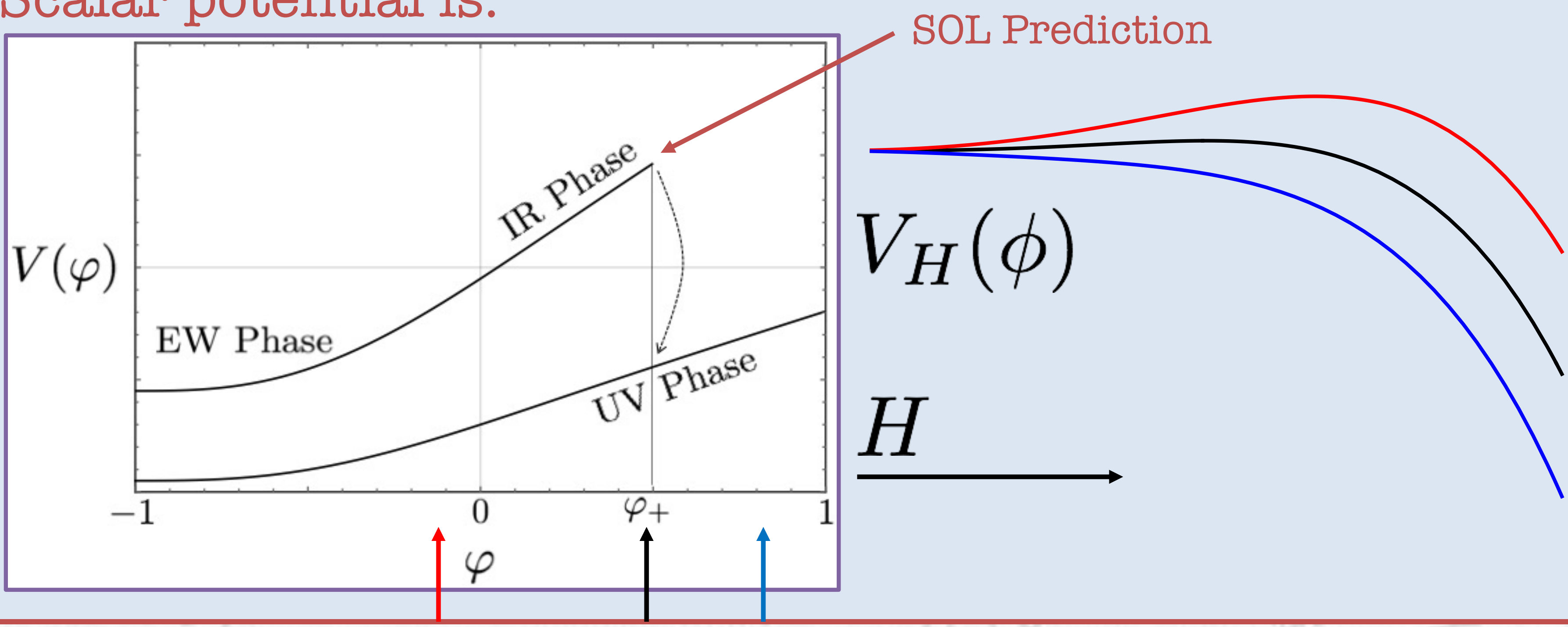
Blue is instability scale, black fixed Hubble contours.

Application: SM Naturalness

Consider the same Higgs instability question, but with a field-dependent mass bilinear:

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h)\,h^4}{4}$$

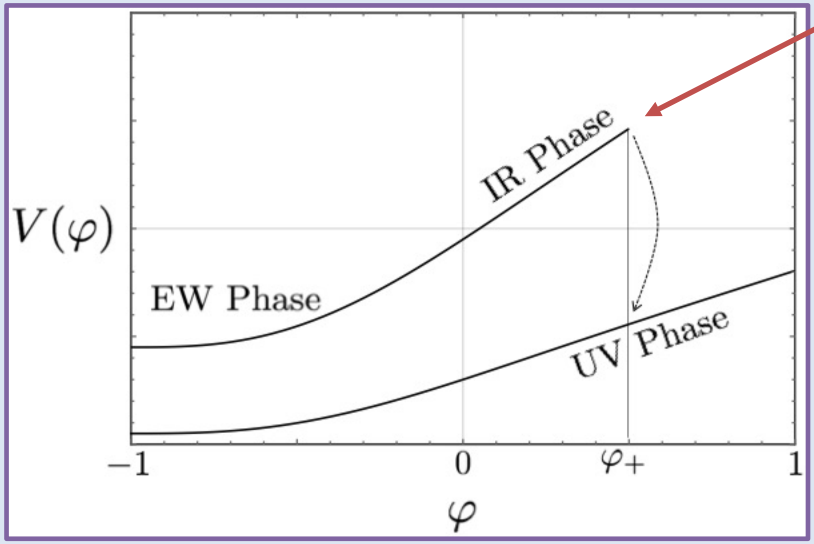
Scalar potential is:



Application: SM Naturalness

Within the SM alone the SOL prediction is the SM instability scale. This is remarkable: Quartic running generates an exponential scale separation between cutoff and instability scale.

SOL Prediction



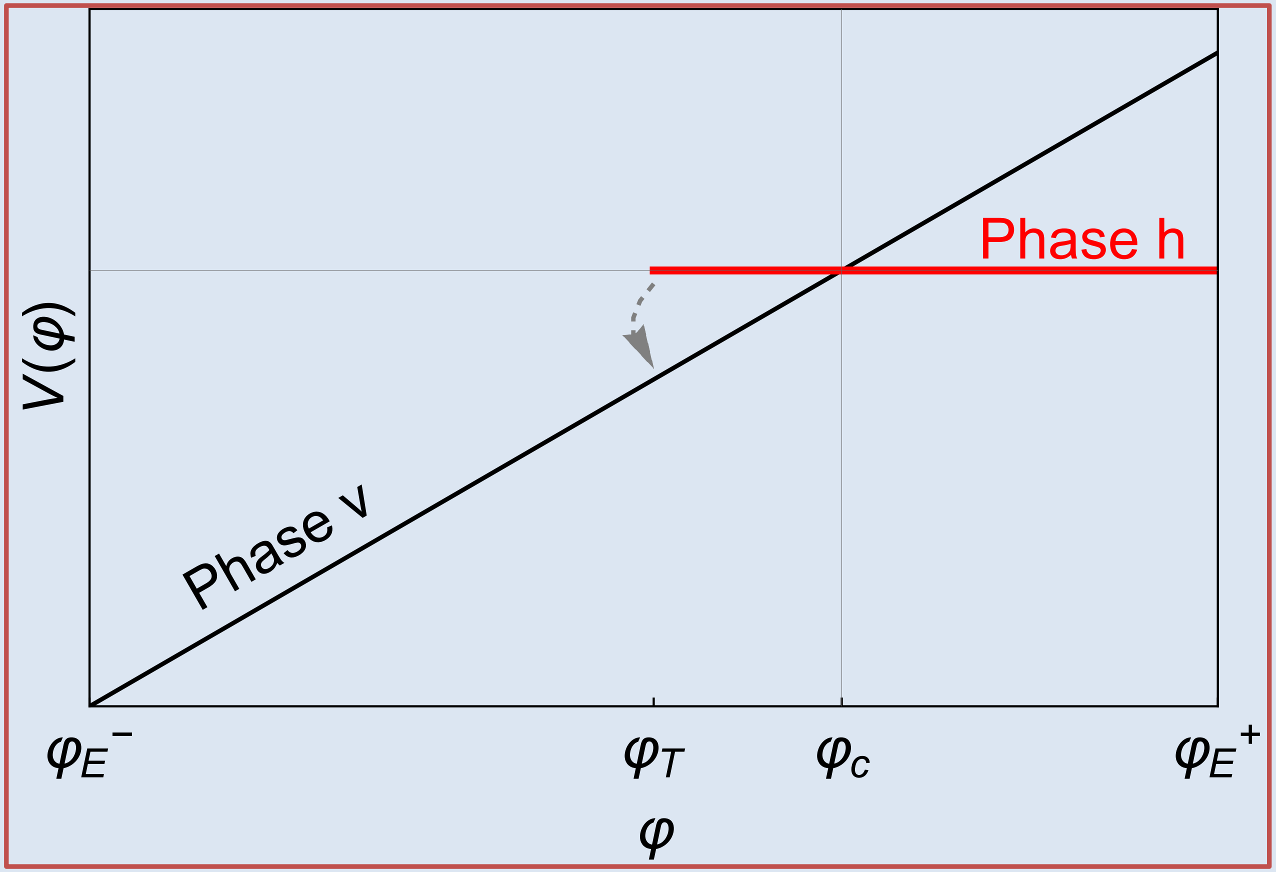
But the instability scale is still way above the weak scale...

$$\Lambda_{\text{Inst}} \sim 10^{10}\ \text{GeV}$$

From quite speculative, to spectacularly speculative...

Waterfalls

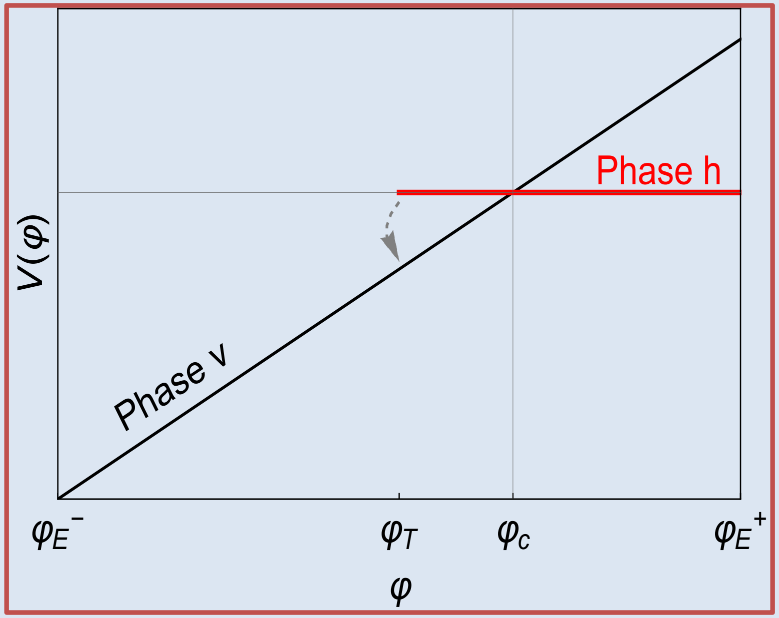
Consider the following "Waterfall" scalar potential:

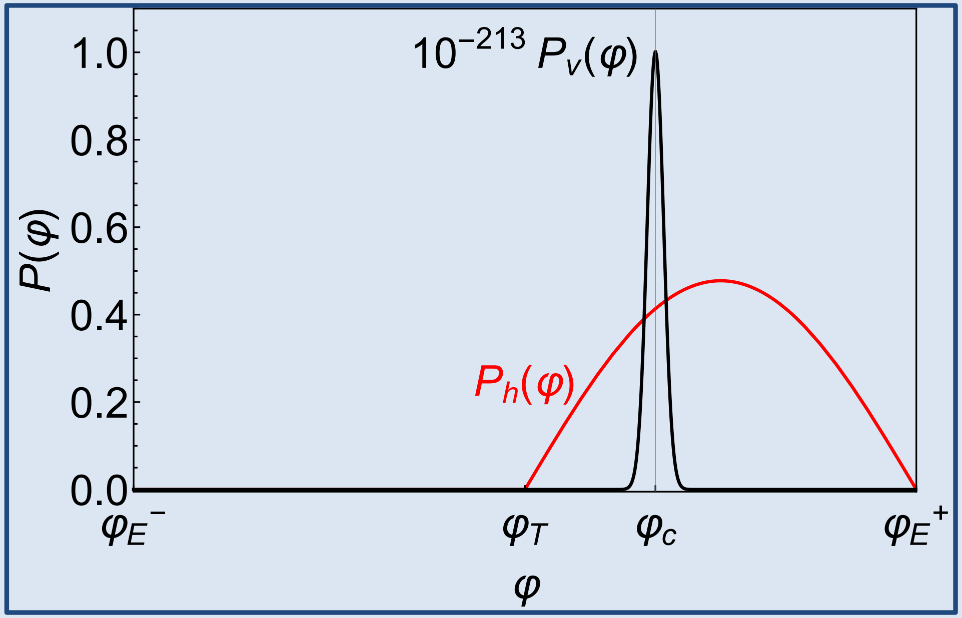


What does the field distribution look like in steady state?

Waterfalls

Consider the following "Waterfall" scalar potential:

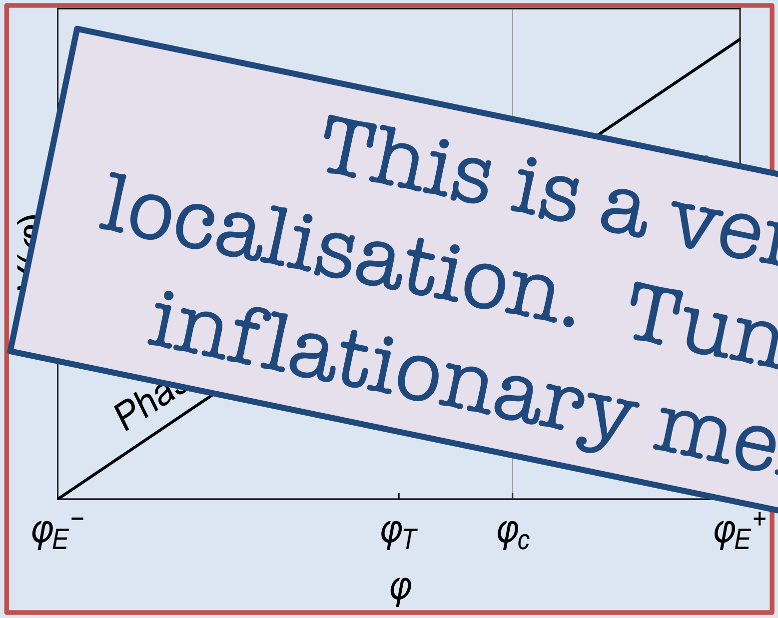


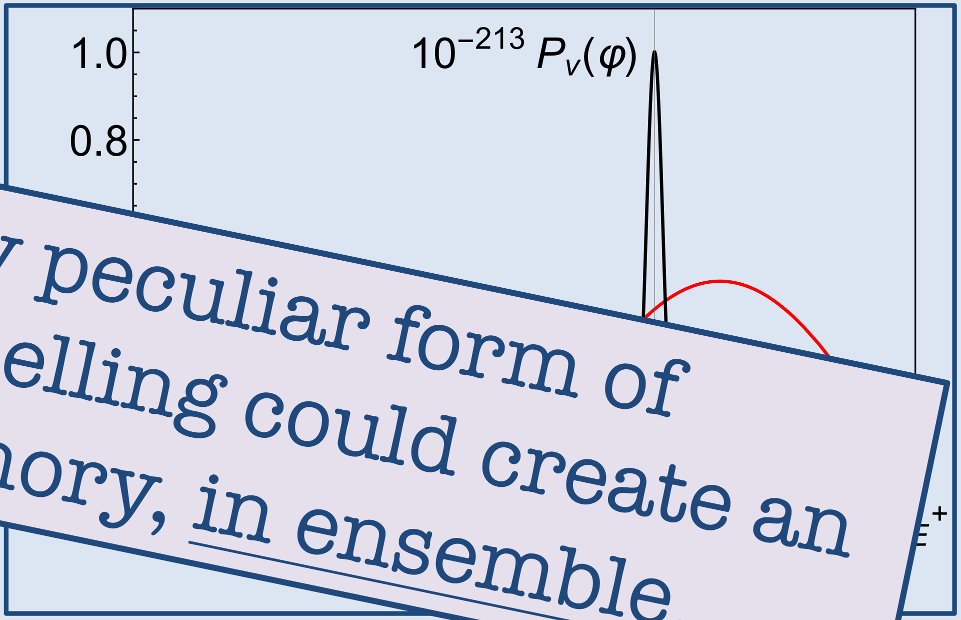


In steady state the solution on the "v" branch is localised at the point where it crosses the same height at the "h" branch, even though tunneling far away.

Waterfalls

Consider the following "Waterfall" scalar potential:





This is a very peculiar form of localisation. Tunnelling could create an inflationary memory, in ensemble....

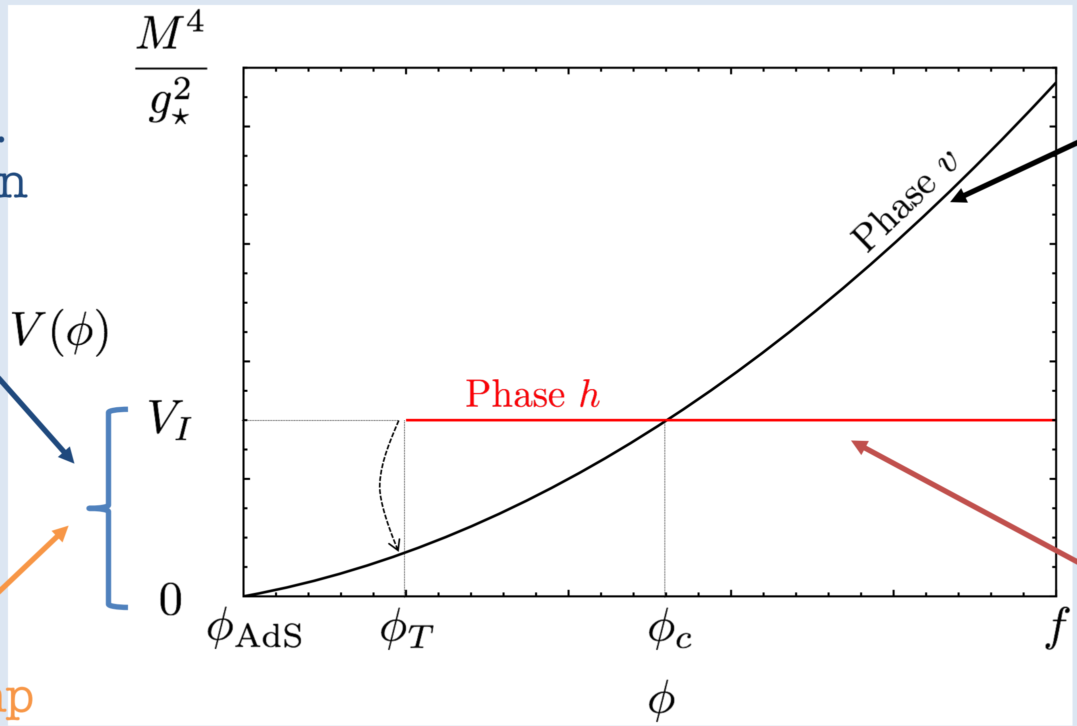
- "v" has non-zero flux injected at the top.
- "h" branch also has flux.
- (No significant sensitivity to these BCs though.)

Application...

Waterfalls

Consider the following "Waterfall" scalar potential in a SUSY setup:

Inflationary sector secluded. this contribution vanishes after reheating.



SUSY and R-symmetry broken, large symmetry breaking.

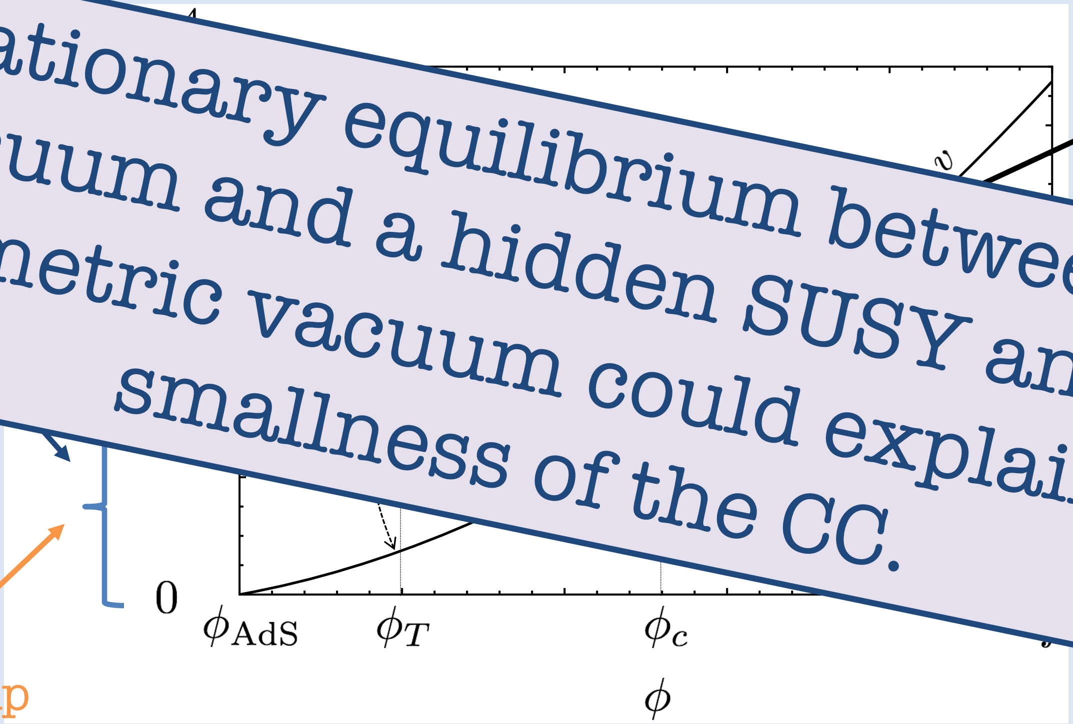
SUSY vacuum, scalar a pNGB with small explicit breaking. No R-breaking.

For C regime max reheat temp is 25 MeV or so.

Waterfalls

Consider the following "Waterfall" scalar potential in a SUSY setup:

Inf
se
th
v
n



SUSY and R-symmetry broken, large symmetry

For C regime max reheat temp is 25 MeV or so.

n,
B
explicit breaking. No R-breaking.

Inflationary equilibrium between our vacuum and a hidden SUSY and R-symmetric vacuum could explain the smallness of the CC.

Summary

Growing selection of possibilities linking criticality to cosmology.

In my view, no Goldilocks proposal yet. Lots of puzzles, particularly concerning eternal inflation.

But perhaps, as in previous revolutions, these puzzles just require a change of perspective...

ESA/Hubble and Harald Süpfle

O.T'. R.U. ATOME? $\iint \text{Sphere}^2 dS$ was done in the most general form in 1867. I have now bagged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(s)})^2 dS$ in 4 lines, thus verifying T+T''s value of $\iint (\vartheta_i^{(s)})^2 dS$

Your plan seems indepᵗ of T+T' or of me. Publish! I am busy supplying the physical necessities of scientific life at 11 Scroope Terrace, Cambridge. Prooves have got as far as grooves, corrugated plates, gratings & ... rings. If you have time for criticism then ...

$$\iint (Y_i^{(s)})^2 dS = \frac{8\pi a^2}{2i+1} \frac{\lfloor i+s \; \lfloor i-s}{2^{2s} \lfloor i \; \lfloor i}$$

except when s = 0 when $\iint (Q_i)^2 dS = \frac{4\pi a^2}{2i+1}$

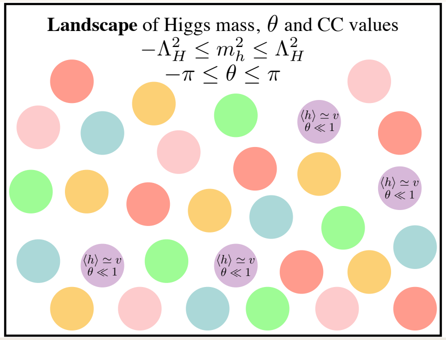
Hence

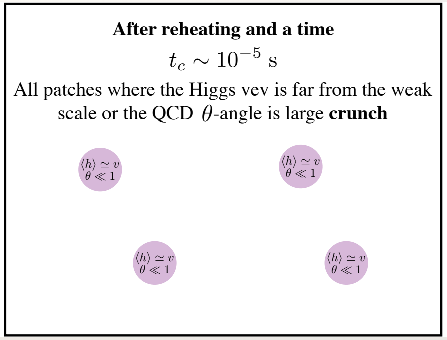
$$\int_{-1}^{+1} (\vartheta_i^{(s)})^2 d\mu = \frac{2}{2i+1} \frac{2^{2s} \lfloor i-s \; \lfloor s \; \lfloor s}{\lfloor i+s}$$

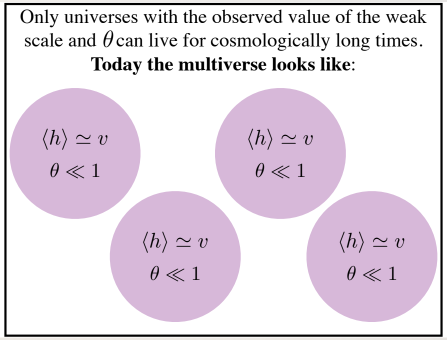
without exception ... $\frac{dp}{dt}$

Sliding Naturalness

a) Some reason for the critical point to be special for an evolving scalar:



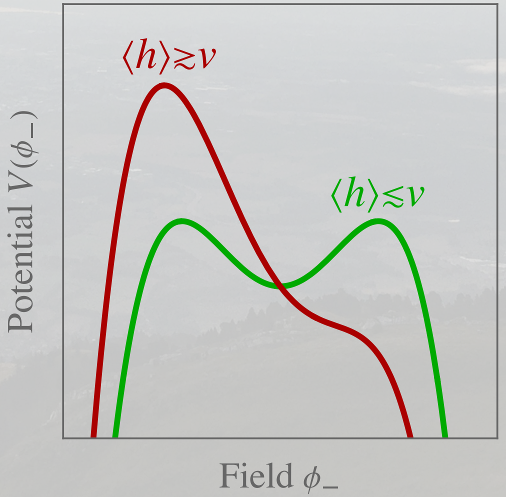


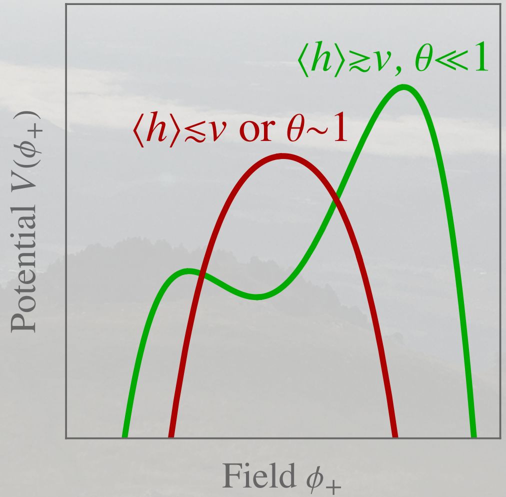


b) Some way to get to, and stay at, the critical point:

$$V_\phi = \mp \frac{m_{\phi_\pm}^2}{2}\phi_\pm^2 - \frac{m_{\phi_\pm}^2}{4M_\pm^2}\phi_\pm^4$$

$$V_{\phi H} = -\frac{\alpha_s}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta\right)\tilde{G}G$$

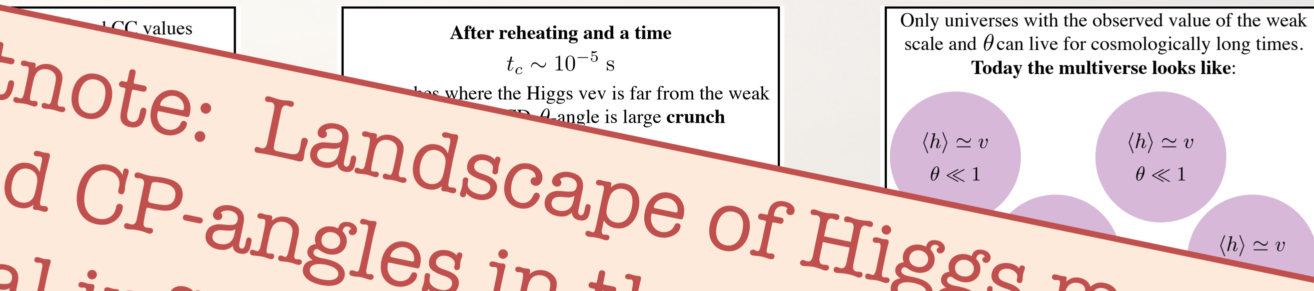




D'Agnolo, Teresi, 2021

Sliding Naturalness

a) Some reason for the critical point to be special for an evolving scalar:



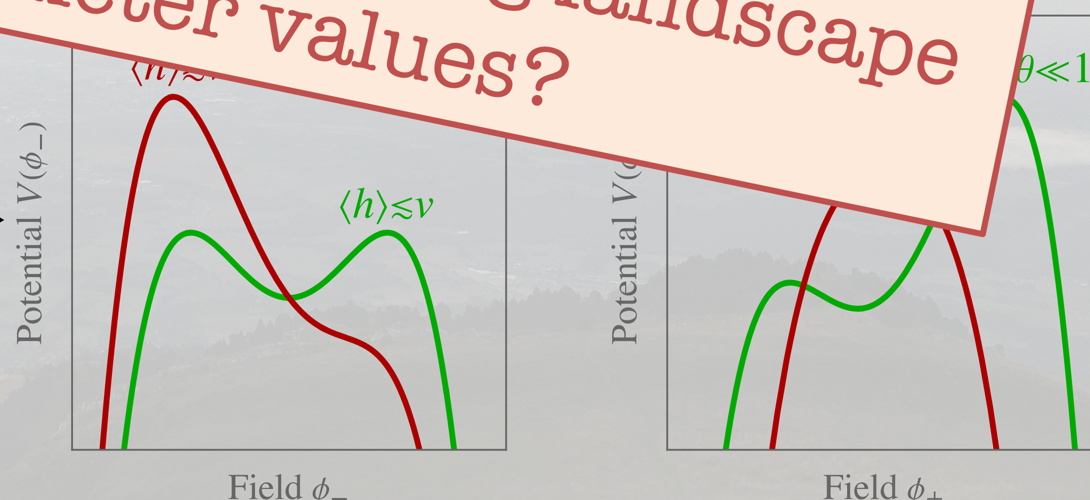
Footnote: Landscape of Higgs masses and CP-angles in the background. Eternal inflation for realising landscape of parameter values?

b)

point:

$$V_\phi = \mp \frac{m_{\phi_\pm}^2}{2}\phi_\pm^2 - \frac{m_{\phi_\pm}^2}{4M_\pm^2}\phi_\pm^4$$

$$V_{\phi H} = -\frac{\alpha_s}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta\right)\tilde{G}G$$



D'Agnolo, Teresi, 2021