dS_2 Supergravity

Beatrix Mühlmann

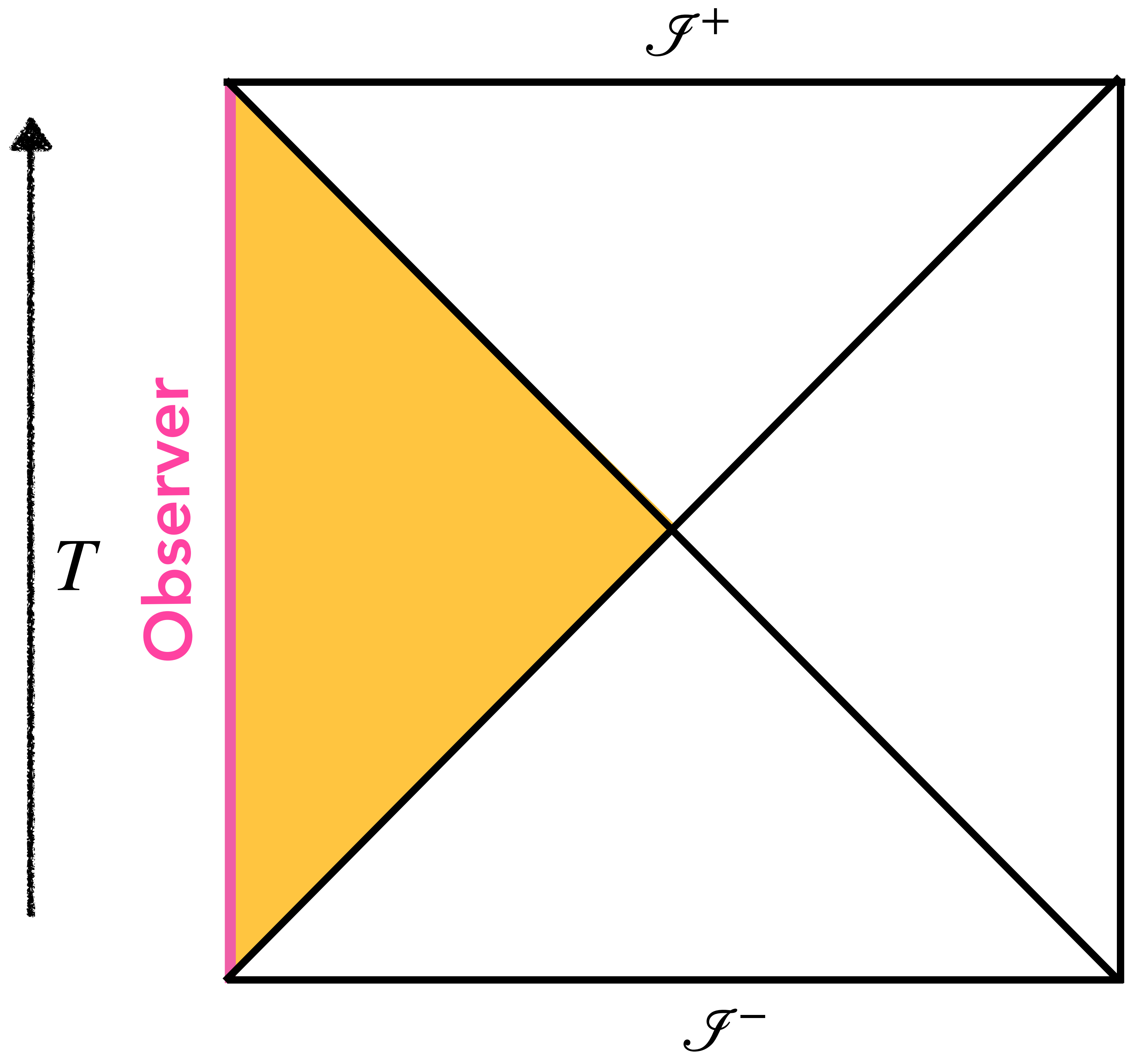
McGill University

Cosmology, Quantum Gravity, and Holography workshop @CERN

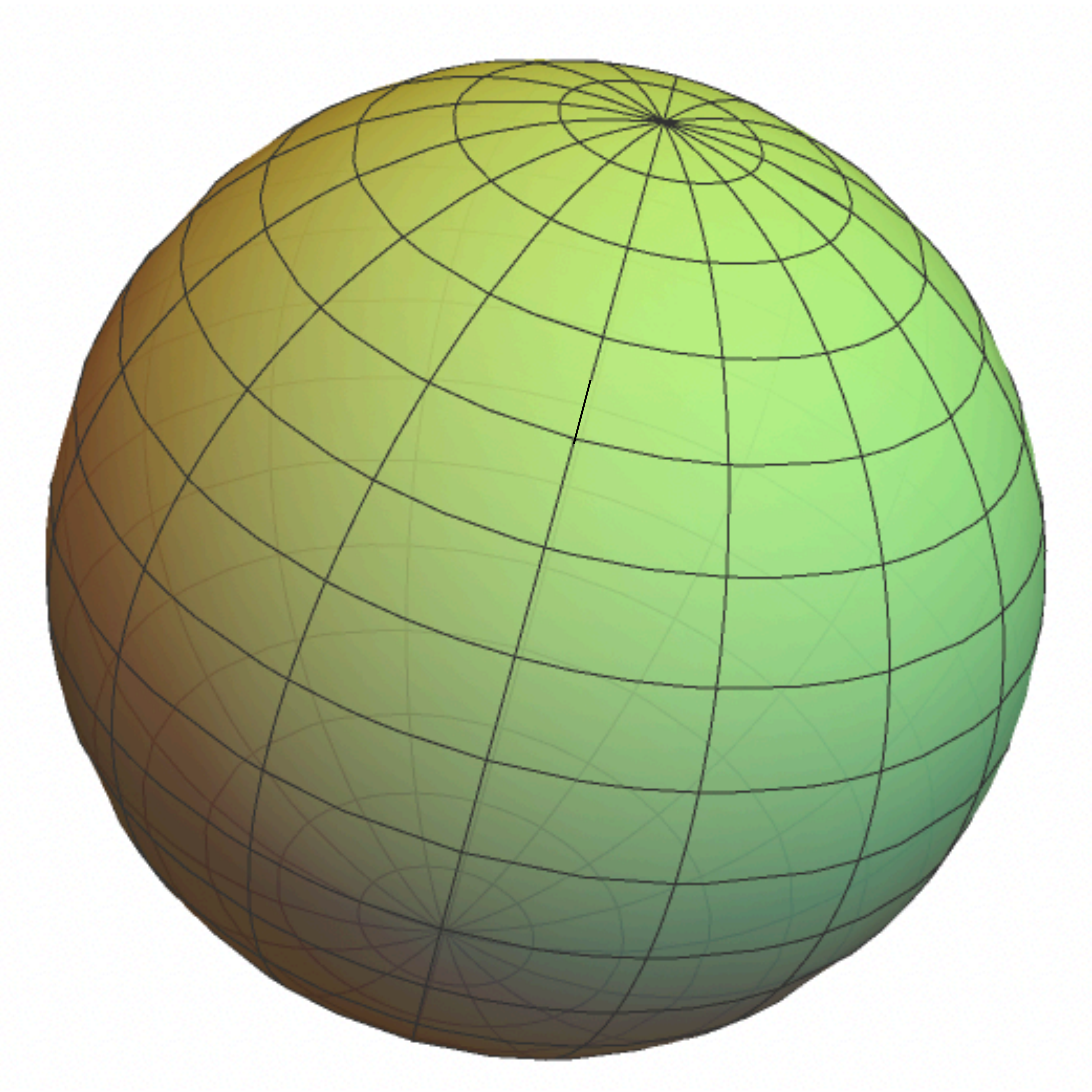
arXiv: 2309.02480 with Dionysios Anninos and Pietro Benetti Genolini

de Sitter space

Our Universe is expanding at an accelerated rate driven by positive cc → asymptotically dS_4 Universe



Penrose diagram of dS_4



Euclidean dS is the sphere

Why dS_2?

- Irreps of dS_2 isometry group $SO(2,1)$ similar to those of dS_4 isometry group $SO(4,1)$
- dS_2 has a non-trivial Gibbons-Hawking de Sitter entropy/ non-trivial path integral
- $dS_2 \times S^2$ (=Nariai geometry) is a solution of 4d gravity with positive cc
- Recent progress on euclidean 2d BH and AdS_2
  [Saad-Shenker-Stanford,…,Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini,…]

dS entropy?

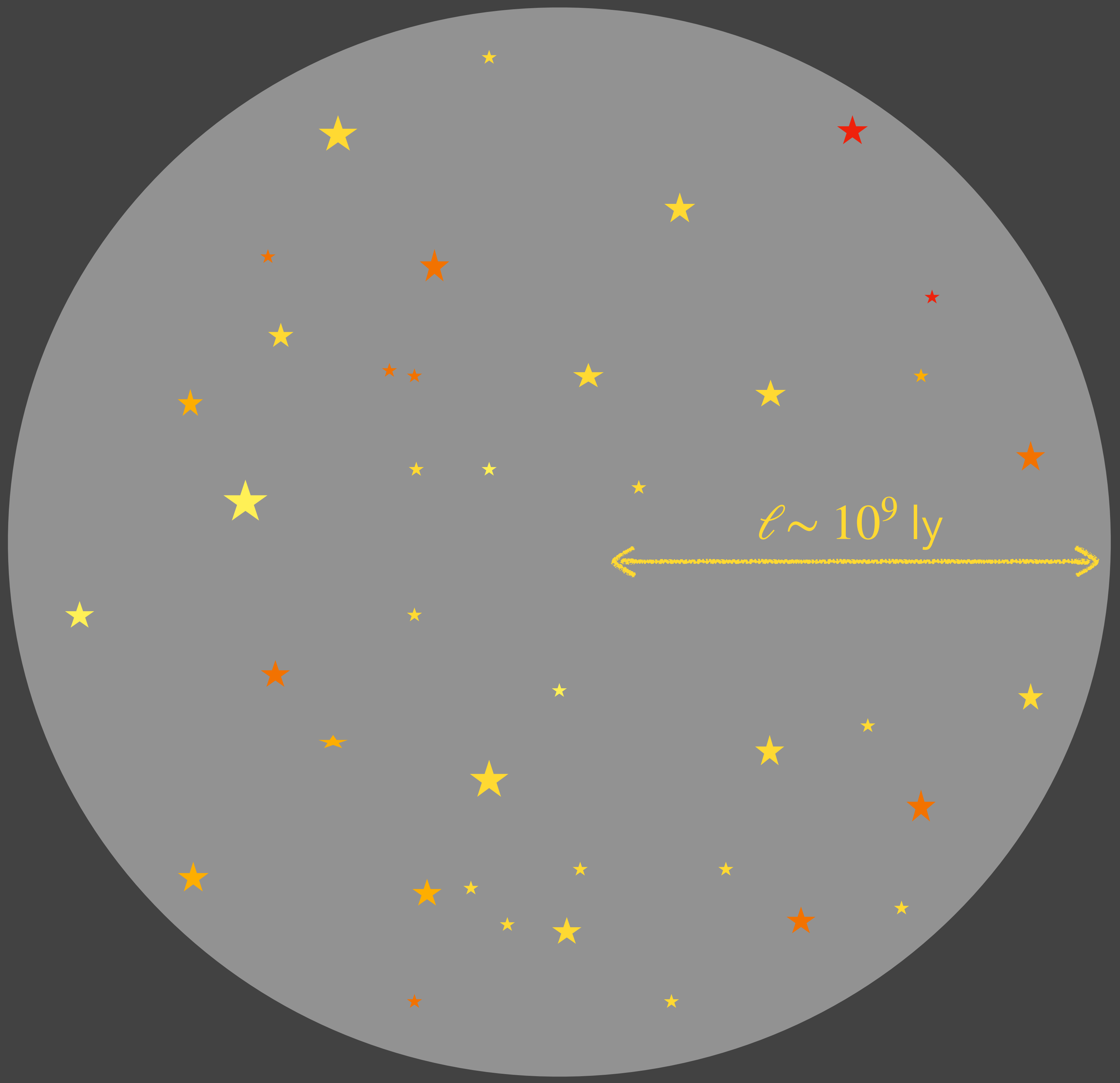
- Gibbons-Hawking conjecture $e^{S_{dS}} = \mathcal{Z}_{\text{grav}} = \sum_{\mathcal{M}\ compact} \int [\mathcal{D}g]\, e^{-S_{EH}[\Lambda,g,\mathcal{M}]} Z_{matter}[\mathcal{M}, g] \approx e^{\frac{A}{4G}+\dots}$

- For BH evidence that S_{BH} is a counting problem
  [Bekenstein-Hawking,…, Strominger-Vafa,…]

- Nature of S_{dS} ?

- SUSY leads to superstrings, AdS/CFT, BPS states for BH,…

- SUSY as a tool to understand dS entropy?



$$S_{dS} \approx \frac{A}{4G} + \dots \approx 10^{122}$$

SUGRA & dS

d>2

SUSY extension of $\mathfrak{so}(d,1)$ + unitary rep. does not exist
[Pilch-van Nieuwenhuizen-Sohnius, Lukierski-Novicki, …]

→ d=2

Susy extension of $\mathfrak{so}(2,1)$ (isometry algebra of AdS_2 and dS_2) + unitary rep. exist
[Lukierski-Novicki,…]

Super-conformal field theory okay with dS
[Hristov-Tomasiello-Zaffaroni, Anous-Freedman-Maloney,…]

→ Same holds in 2d

No-go's for classical embedding in superstrings
[Gibbons, Maldacena-Nuñez…]

→ Consider 2d as a UV finite theory on its own

Non-linearly realised dS-SUGRA (Volkov-Akulov)
[Bergshoeff-Freedman-Kallosh-van Proeyen,…]

→ Not known in 2d

Two concrete models

$\mathcal{N} = 1$ dS_2 supergravity: $\mathcal{N} = 1$ SUGRA + $\mathcal{N} = 1$ SCFT

$\mathcal{N} = 2$ dS_2 supergravity: $\mathcal{N} = 2$ SUGRA + $\mathcal{N} = 2$ SCFT

$\mathcal{N}=1$ dS_2 supergravity

- 2d $\mathcal{N}=1$ gravity multiplet $(\mathrm{e}^a_\mu, \chi_\mu, A)$: zweibein, spin 3/2 Majorana gravitino, real scalar

- $\mathcal{N}=1$ SUGRA + $\mathcal{N}=1$ SCFT on compact surface Σ_h of genus h

$$\mathcal{Z}^{\mathcal{N}=1}_{\text{grav}} = \sum_{h=0}^{\infty} e^{\vartheta(2-2h)} \int [\mathcal{D}\mathrm{e}^a_\mu][\mathcal{D}\chi_\mu][\mathcal{D}A]\, e^{\boldsymbol{\mu}\int_{\Sigma_h} d^2x\,\mathrm{e}\left(iA + \frac{i}{4}\bar{\chi}_\mu\gamma^{\mu\nu}\chi_\nu\right)} \times Z^{(h)}_{\text{SCFT}}[\mathrm{e}^a_\mu, \chi_\mu, A]$$

- $Z^{(h)}_{\text{SCFT}}[\mathrm{e}^a_\mu, \chi_\mu, A]$ is the genus h partition function of a SCFT with central charge c_m

- Absence of gravitino kinetic term $\bar{\chi}_\mu\gamma^{\mu\nu\rho}D_\nu\chi_\rho$ in 2d

- Focus on $h=0$ in this talk

super-Weyl gauge

- super-Weyl gauge: $\mathrm{e}_\mu^a = e^{b\varphi}\,\tilde{\mathrm{e}}_\mu^a\,, \qquad \chi_\mu = e^{\frac{1}{2}b\varphi}\gamma_\mu\psi\,, \qquad A = \frac{i}{b}e^{-b\varphi}F$
- $\tilde{\mathrm{e}}_\mu^a$ background metric on round S^2 with radius r
- Chiral multiplet (φ, ψ, F): real scalar, Majorana spin 1/2 fermion, real scalar F
- Complexified Euclidean multiplet $\longrightarrow$ real φ, F and Euclidean Majorana $\psi = \gamma_* C^{-1}\psi^*$

super-Liouville action

- $Z^{(0)}_{\text{SCFT}}$ is determined by superconformal anomaly ($\mathcal{N}=1$ analogue of Polyakov action)

- SUGRA with positive cc sector leads to $\mathcal{N}=1$ super-Liouville [Distler-Hlousek-Kawai,…]

$$\mathcal{Z}^{\mathcal{N}=1}_{\text{grav},(0)} = e^{2\vartheta} \times \left(\frac{r}{\ell_{\text{uv}}}\right)^{(c_m+c_{\mathfrak{bc}}+c_{\beta\gamma})/3} \times \int \frac{[\mathcal{D}\varphi][\mathcal{D}\psi]}{\text{vol}_{OSp(1|2;\mathbb{C})}} e^{-\mathcal{S}_L^{\mathcal{N}=1}}$$

$$\mathcal{S}_L^{\mathcal{N}=1} = \frac{1}{4\pi}\int_{S^2} d^2x\sqrt{\tilde{g}}\left(\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{Q\varphi}{r^2} + \Lambda e^{2b\varphi} - \frac{i}{2}\overline{\psi}\not{D}\psi + \frac{i}{2}\sqrt{\Lambda}\, b e^{b\varphi}\overline{\psi}\psi\right)$$

- $OSp(1\,|\,2,\mathbb{C})$ residual gauge group on S^2

$\mathcal{N}=1$ super-Liouville

$$\mathcal{S}_L^{\mathcal{N}=1} = \frac{1}{4\pi}\int_{S^2} d^2x\sqrt{\tilde{g}}\left(\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{i}{2}\overline{\psi}\not{D}\psi + \Lambda e^{2b\varphi} + \frac{Q\varphi}{r^2} + \frac{i}{2}\sqrt{\Lambda}\, b e^{b\varphi}\overline{\psi}\psi\right)$$

$\mathcal{N}=1$ super-Liouville is a 2d SCFT with $Q = b + b^{-1}$ and $c_L^{\mathcal{N}=1} = 3/2 + 3Q^2$

Anomaly cancellation: $c_L^{\mathcal{N}=1} + c_m + c_{\mathfrak{bc}} + c_{\beta\gamma} = 0$

- Positive cc
- For $c_m \to -\infty$ complex S^2 saddle
- For SCFT = super minimal model, non-perturbative completion known [Seiberg-Shih,…]

- Positive cc
- For $c_m \to \infty$ **real** S^2 saddle

$\mathcal{N} = 1$ timelike super-Liouville

Consider $\mathcal{N} = 1$ timelike super-Liouville + $\mathcal{N} = 1$ SCFT with $c_m \to \infty$

$$\mathcal{Z}_L = \int \frac{[\mathcal{D}\varphi][\mathcal{D}\psi]}{\text{vol}_{OSp(1|2;\mathbb{C})}} e^{-\mathcal{S}_{tL}^{\mathcal{N}=1}} \qquad \mathcal{S}_{tL}^{\mathcal{N}=1} = \frac{1}{4\pi}\int_{S^2} d^2x\sqrt{\tilde{g}}\left(-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{q\varphi}{r^2} + \Lambda\, e^{2\beta\varphi} + \frac{i}{2}\overline{\psi}\not{D}\psi + \frac{1}{2}\sqrt{\Lambda}\,\beta e^{\beta\varphi}\overline{\psi}\psi\right)$$

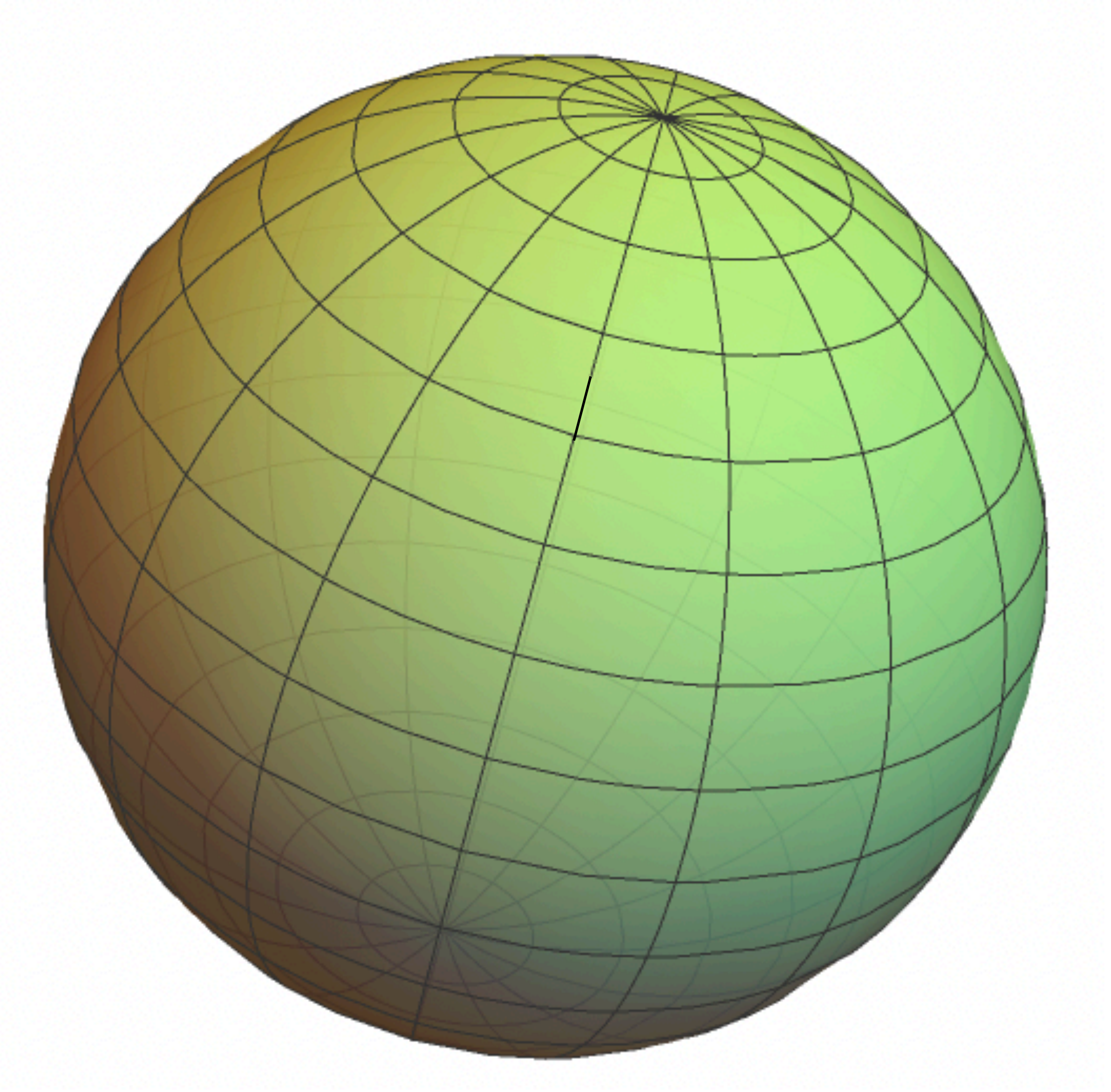
- Timelike super-Liouville is an $\mathcal{N} = 1$ superconformal field theory
- The action is invariant under $\mathcal{N} = 1$ supersymmetry transformations
- With regards to $\mathcal{N} = 1$ SUGRA TsL shares conformal mode ``problem" of higher d Euclidean gravity

dS$_2$ saddle

- The eom for φ admit a constant – $OSp(1|2,\mathbb{C})$ family – solution

$$-\nabla^2\varphi = \mu^2\beta e^{2\beta\varphi} - \frac{q}{r} + \frac{1}{2}\mu\,\beta^2 e^{\beta\varphi}\overline{\psi}\psi \quad \Rightarrow \quad \varphi_* = \frac{1}{2\beta}\log\frac{q}{r^2\mu^2\beta^2}$$

$\Rightarrow$ S^2 = Euclidean dS$_2$ saddle!! (Reminder: $g_{\mu\nu} = e^{2\beta\varphi}\tilde{g}_{\mu\nu}$)



Euclidean dS is the sphere

One-loop contribution

- Study fluctuations on top of S^2 saddle: $\varphi = \varphi_* + \delta\varphi$, $\psi = \psi_* + \delta\psi$

$$\log \int [\mathscr{D}\delta\varphi] e^{-\frac{1}{8\pi}\int_{S^2} dx^2 \sqrt{\tilde{g}}\, \delta\varphi\left(-\nabla^2 - \frac{2}{r^2}\right)\delta\varphi} = \int_0^\infty \frac{dt}{2t}\left[\frac{1+e^{-t}}{(1-e^{-t})}\left(2\chi_{\Delta=2}(t) + 5e^{-t} - 5e^{-2t} + 3e^{-3t}\right)\right]$$

$$\log \int [\mathscr{D}\delta\psi] e^{\frac{1}{8\pi}\int_{S^2} dx^2 \sqrt{\tilde{g}}\, \delta\overline{\psi}\left(i\not{D} + \frac{1}{r}\right)\delta\psi} = \int_0^\infty \frac{dt}{2t}\left[\frac{2e^{-\frac{t}{2}}}{(1-e^{-t})}\left(2\chi_{\Delta=3/2}(t) + \underbrace{2e^{-\frac{t}{2}} - e^{-\frac{3t}{2}}}_{\text{Account for zero modes}} + e^{-\frac{5t}{2}}\right)\right]$$

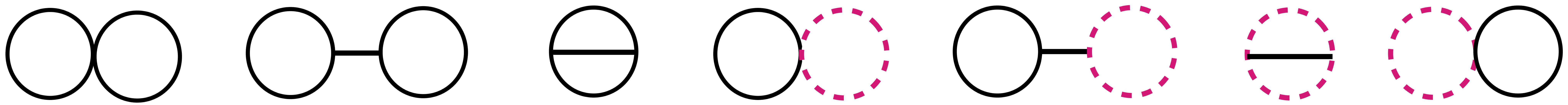
- Lorentzian Harish-Chandra character $\chi_\Delta(t) = \dfrac{e^{-\Delta t}}{(1-e^{-t})}$ for unitary discrete rep. of $SO(1,2) \cong PSL(2,\mathbb{R})$
  [Anninos-Denef-Law-Sun,…]

- Discrete series → massless fields

- Non-real Lorentzian action still seems to lead to unitary representations
  [Pilch-van Nieuwenhuizen-Sohnius,…,Letsios,…]

Non-Gaussian fluctuations

- Systematic higher loop on top of dS_2 saddle
- Bosonic and fermionic propagators $\delta\varphi$  $\delta\psi$
- At two-loop order we find



- Two-loop on S^2: all UV divergences cancel

$\mathcal{N} = 1$ timelike super-Liouville

$$Z_{tL}^{\mathcal{N}=1} \approx \pm i \left(\frac{\mu}{\beta}\right)^{-\frac{1}{\beta^2}+1} \Lambda_{uv}^{\frac{5}{4}-\beta^2} e^{-\frac{1}{2\beta^2}-\left(\frac{1}{\beta^2}-1\right)\log \beta^2} \times r^{\frac{c_{tL}^{\mathcal{N}=1}}{3}} \times \beta^{-1} \times (1 + \text{loops}\,\beta^2 + \ldots,)$$

- Unboundedness of conformal mode leads to $\pm i$ [Gibbons-Hawking-Perry,…]
- One-loop confirms central charge $c_{tL}^{\mathcal{N}=1} = \frac{3}{2} - 3(\beta^{-1} - \beta)^2$
- Fadeev-Popov gauge fixing of super-Moebius
- Systematic loop expansion with vanishing UV divergences

Question: Can we get this from structure constants of $\mathcal{N} = 1$ super-timeline Liouville?

$\mathcal{N} = 1$ SUGRA

$$\mathcal{Z}^{\mathcal{N}=1}_{\text{grav},(0)} = e^{2\vartheta} \times \left(\frac{r}{\ell_{\text{uv}}}\right)^{\frac{c_m + c_{\mathfrak{bc}} + c_{\beta\gamma}}{3}} \times \int \frac{[\mathcal{D}\varphi][\mathcal{D}\psi]}{\text{vol}_{OSp(1|2;\mathbb{C})}} e^{-\mathcal{S}^{\mathcal{N}=1}_{tL}} = e^{2\vartheta} \times \left(\frac{1}{\Lambda\,\ell_{\text{uv}}^2}\right)^{\left(\frac{c_m}{6} - \frac{7}{4} + \dots\right)} \times f_0(c_m)$$

- $\log \mathcal{Z}^{\mathcal{N}=1}_{\text{grav},0}$ has structure of 2d entanglement entropy
  [Cardy-Calabrese, Casini-Huerta-Myers, Holzhey-Larsen-Wilczek,…]

We constructed a 2d SUGRA coupled to matter SCFT such that we have

- linear SUSY transformations
- a positive cc
- Semiclassical dS_2 vacua for $c_m \to \infty$
- Systematic loop expansion with good UV properties

$\mathcal{N}$ =2 dS$_2$ supergravity

- A second model is $\mathcal{N} = 2$ SUGRA with $U(1)$ R-symmetry + $\mathcal{N} = 2$ SCFT
- Gravity multiplet is: zweibein, Dirac gravitino, $U(1)$-gauge field, complex scalar: $(e^a_\mu, \chi_{mu}, A_\mu, B)$
- Compared to $\mathcal{N} = 1$, contribution from the $U(1)$ gauge bundle
- For simplicity we consider the theory on S^2 and take vanishing $U(1)$-flux

super-Weyl gauge

- Super-Weyl gauge for $\mathcal{N} = 2$ $\quad e^a_\mu = e^{\frac{1}{2}b(\varphi+\widetilde{\varphi})}$, $\quad A^\mu = -\frac{i}{2}\epsilon^{\mu\nu}\partial_\nu(\varphi - \widetilde{\varphi}) + \dots$

- In super-Weyl gauge we obtain a theory of a chiral + anti chiral multiplet [Antoniadis-Bachas-Kounnas,…]

$$(\varphi, F, \psi) + (\widetilde{\varphi}, \widetilde{F}, \widetilde{\psi})$$

- φ and F are complex scalars, ψ is spin one-half Dirac spinor

- Unlike Lorentzian for Euclidean signature no a priori relation between tilded and untilded field

$\mathcal{N} = 2$ timelike super-Liouville

$$\mathcal{Z}^{\mathcal{N}=2}_{\text{grav},(0)} \approx e^{2\vartheta} \times \left(\frac{r}{\ell_{\text{uv}}}\right)^{(c_m + c_{\mathfrak{bc}} + 2c_{\beta\gamma} + c_{U(1)})/3} \times \int \frac{[\mathscr{D}\Phi][\mathscr{D}\widetilde{\Phi}]}{\text{vol}_{OSp(2|2;\mathbb{C})}} e^{-\mathcal{S}^{\mathcal{N}=2}_{tL}}$$

Super-Liouville: Theory of chiral+antichiral multiplet with exponential superpotential

$\lambda \in \mathbb{C}$, positive cc

Interaction with exponential superpotential

$$\mathcal{S}^{\mathcal{N}=2}_{tL} = \frac{1}{4\pi}\int_{S^2} d^2x\sqrt{\tilde{g}}\left(-\partial_\mu\varphi_1\partial^\mu\varphi_1 - \frac{2}{\beta r^2}\varphi_1 + \beta^2|\lambda|^2 e^{2\beta\varphi_1} - \partial_\mu\varphi_2\partial^\mu\varphi_2 + i\overline{\widetilde{\psi}}\not{D}\psi + \frac{\beta}{2}e^{\beta\varphi_1}\left(\lambda e^{i\beta\varphi_2}\overline{\psi}\psi + \lambda^* e^{-i\beta\varphi_2}\overline{\widetilde{\psi}}\widetilde{\psi}\right)\right),$$

$\varphi_1 = \frac{1}{2}(\varphi + \widetilde{\varphi})$ is the Weyl-factor for non-supersymmetric Liouville

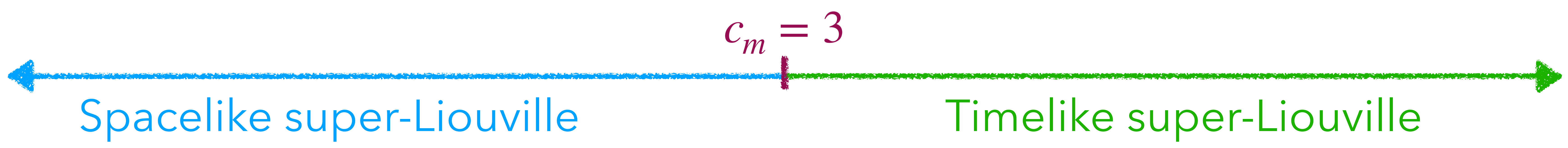
Encodes $U(1)$ gauge anomaly

super-Liouville

Timelike super-Liouville is a supersymmetric non-unitary SCFT with $c_{tL} = 3 - 6q^2$, $\quad q = 1/\beta$

Vanishing conformal anomaly: $q = \dfrac{1}{\beta} = \sqrt{\dfrac{c_m - 3}{6}}$

Absence of liner β term in q (Teschner's trick for DOZZ does not work)



Saddle+ fluctuations

- EOM of timelike super-Liouville admit dS_2 saddle
- One-loop contribution encode Lorentzian Harish-Chandra character for discrete series fields
- Systematic higher loops with vanishing UV divergences
- .. just many more diagrams than for the $\mathcal{N} = 1$ case :-)

Gravity path integral

With regards to the $\mathcal{N}=2$ SUGRA, TsL … … has a positive cc

… the EOM admit semiclassical dS$_2$ vacua for $c_m \to \infty$

… is well behaved in the UV

$$S_{dS} = \log \mathcal{Z}^{\mathcal{N}=2}_{\text{grav},(0)} = 2\theta - \left(\frac{c_m}{6} - \frac{3}{6}\right) \log\left(|\lambda|^2 \ell_{\text{uv}}^2\right) + f_0(c_m)\,, \quad f_0(c_m) \quad \text{are non-trivial}$$

Structure of 2d entanglement entropy [Cardy-Calabrese, Casini-Huerta-Myers, Holzhey-Larsen-Wilczek,…]

$$-\frac{3}{6} = \frac{(3_L - 26_{\mathfrak{bc}} + 2 \times 11_{\beta\gamma} - 2_{U(1)})}{6} \quad \text{gravity effect?}$$

SUSY localization

- $\mathcal{N} = 2$ theory on S^2 amenable to SUSY localization
  [Benini-Cremonesi, Doroud-Gomis-Le Floch-Lee,…]
- Combine Gibbons-Hawking proposal with SUSY localization?
  [Anninos-Galante-BM,…]
- Localization leads to an effective reduction in the dof that are integrated over
- Finiteness of dS Hilbert space?
  [Banks, Fischler, Bousso, Parikh-Verlinde, Susskind,…]

Localization for Liouville?

- Add a $\mathcal{Q}$-exact term $t\mathcal{Q}V$ with $\mathcal{Q}V|_{bos} > 0,\, t \in \mathbb{R}$

$$Z(t) = \int_{\mathscr{E}} [\mathscr{D}\Phi]\, e^{-S - t\mathcal{Q}V} \longrightarrow Z'(t) = -\int_{\mathscr{E}} [\mathscr{D}\Phi]\, \mathcal{Q}\left(V e^{-S-t\mathcal{Q}V}\right) \begin{cases} = 0 & \text{in most cases} \\ \neq 0 & \text{in the presence of bdy terms in field space} \end{cases}$$

- Independence of t allows to evaluate for $t \to \infty$ on BPS solutions of $\mathcal{Q}V = 0$

- Liouville action is $\mathcal{Q}$-exact and hence naive localization predicts trivial partition function

- $\mathcal{N} = 2$ Liouville localizes onto boundary terms in field space
  [Hori-Kapustin,…]

- Compatible with idea of Polchinski for non-supersymmetric Liouville (strings 1990)

Summary

$\mathcal{N} = 1$ SUGRA + $\mathcal{N} = 1$ SCFT

super-Weyl gauge

$\mathcal{N} = 1$ Liouville + $\mathcal{N} = 1$ SCFT

$\mathcal{N} = 2$ SUGRA + $\mathcal{N} = 2$ SCFT

super-Weyl gauge

$\mathcal{N} = 2$ Liouville + $\mathcal{N} = 2$ SCFT

spacelike regime

timelike regime

For $c_m \to \infty$ $\mathbf{dS}_2$ saddle

positive cc

Non-perturbative completion known

Localization onto bdy terms in field space

systematic loop expansion

Future directions

- ⋆ Structure constants for $\mathcal{N} = 1$ timelike super-Liouville
- ⋆ Structure constants for $\mathcal{N} = 2$ spacelike + timelike super-Liouville
- ⋆ dS+ super JT? … difficulties in non-supersymmetric case lifted?
- ⋆ Hartle-Hawking path integral… disk partition function
- ⋆ Higher topologies + non-zero flux… does the $\mathcal{N} = 2$ theory vanish for $h \geq 1$?
- ⋆ Lorentzian signature

THANK YOU!