The matter with $T\bar{T} + \Lambda_2$

G. Bruno De Luca - Stanford University

Work in preparation with Aguilera Damia, Batra, Freedman, Silverstein, Torroba, Yang
+ discussions with Banihashemi, Shaghoulian, Shyam, Soni…

CERN workshop: Cosmology, Quantum Gravity and Holography
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- Top-Down: describe dS and dS-like space-times in String/M-theory by compactifying extra dimensions

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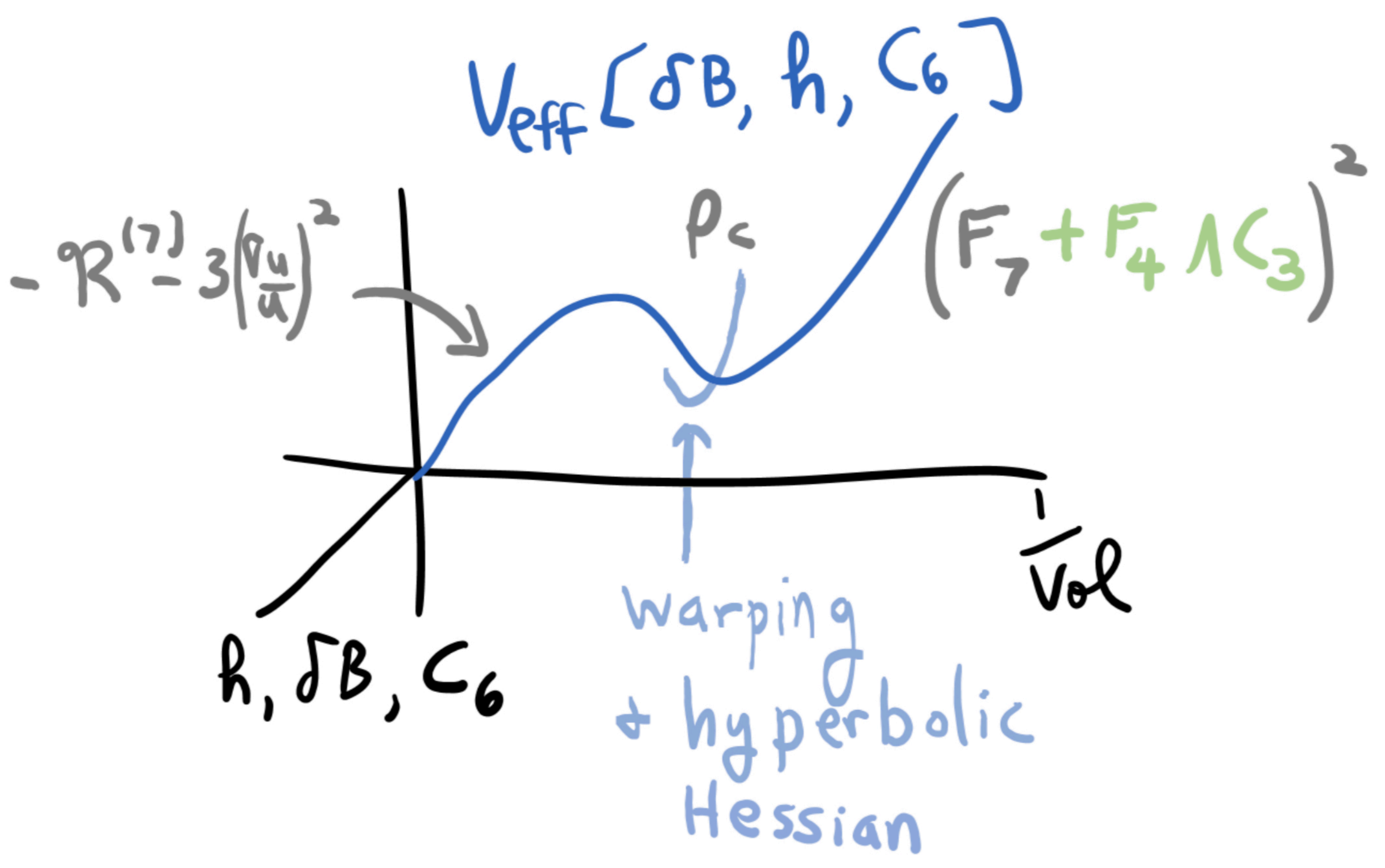
  - On negatively-curved spaces, use quantum effects (Casimir energies) for power-law stabilization. No moduli thanks to rigidity of hyperbolic spaces [GBDL, Silverstein, Torroba, ‘21]



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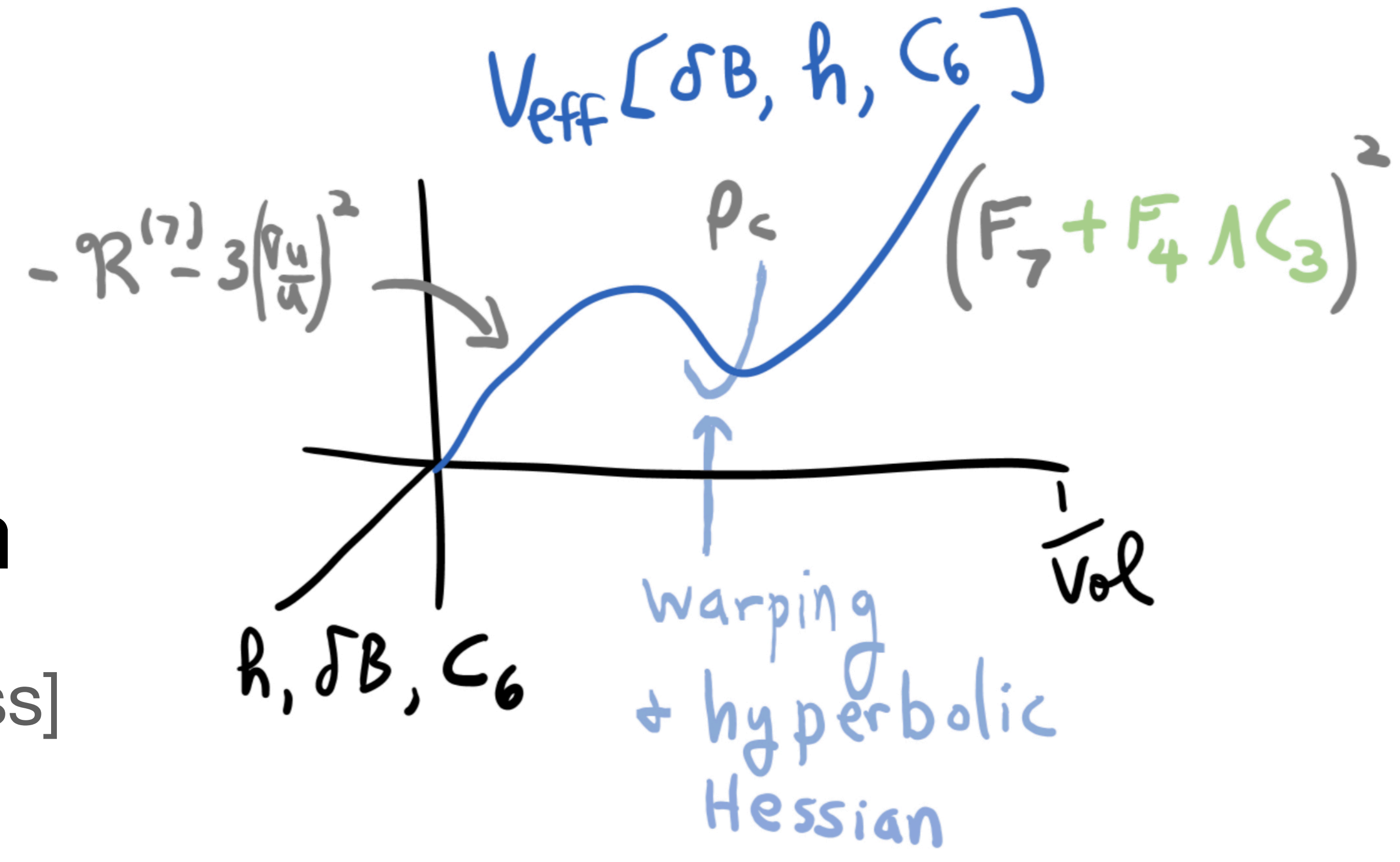

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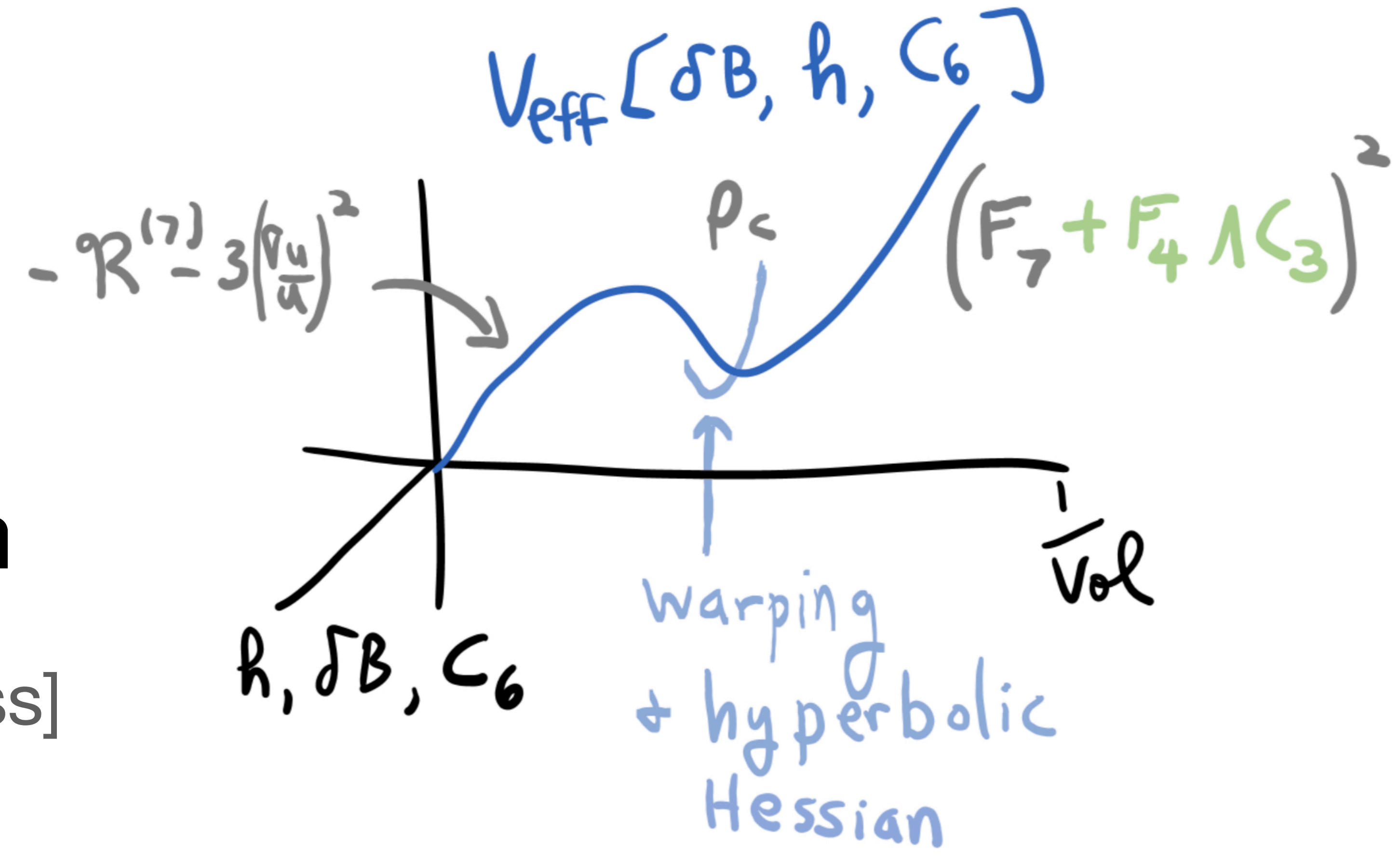

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  - Many other scenarios with interplay of classical and stringy effects

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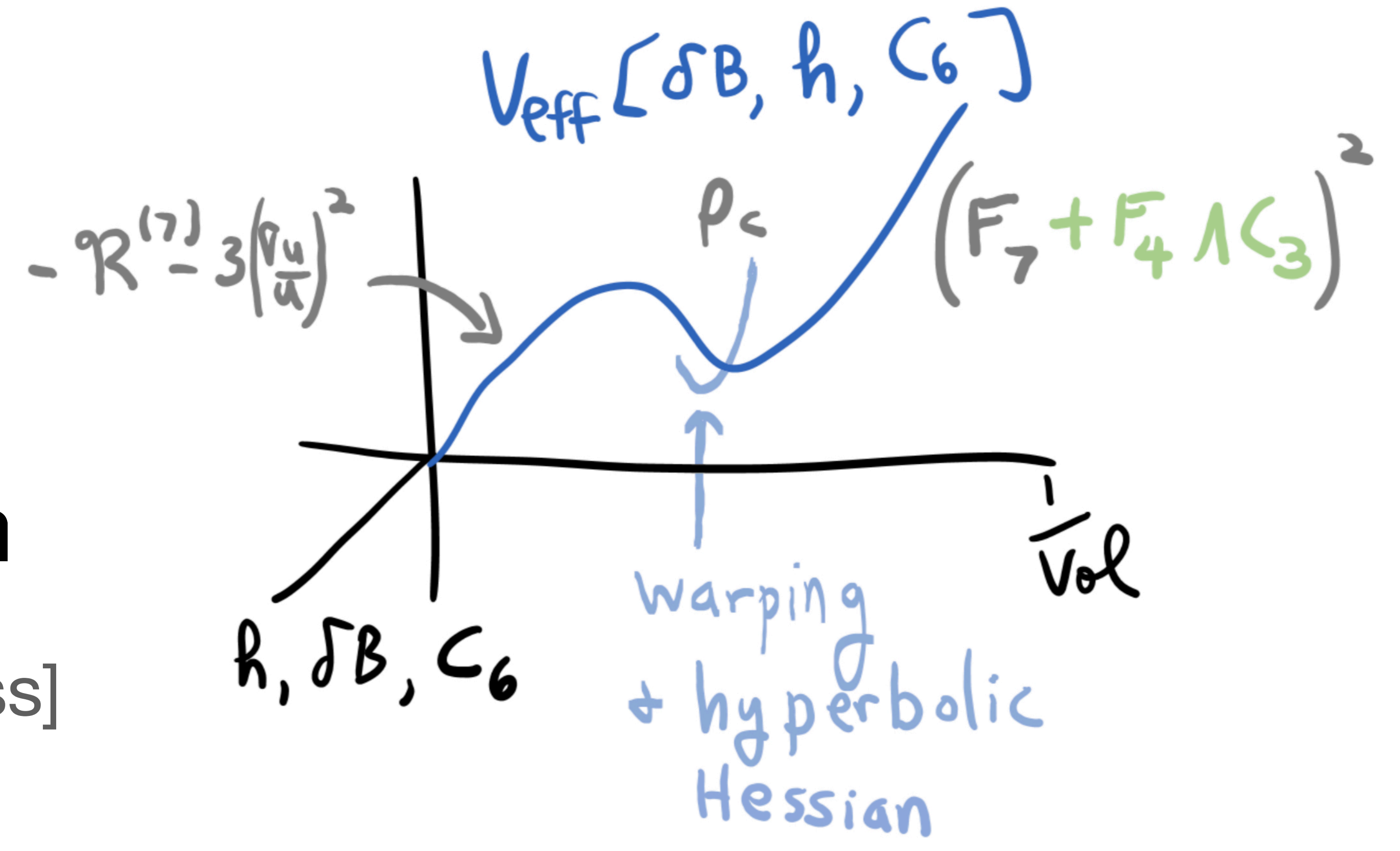

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- Lesson: 4d physics depends on the geometry of extra dimensions

- Bottom-up holography:
  - Various approaches to use holography to describe to describe accelerated expansions via lower-dimensional QFTs
    - dS/CFT, dS/dS, FRW/FRW, FRW/CFT, matrix models, …

[Strominger, ‘01; Anninos, Hartman, Strominger ‘17,
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    - Recently, the $T\bar{T} + \Lambda_2$ deformation of holographic CFTs suggests how to construct holographic descriptions of observer patches of dS_3, including entropy counts and finiteness of spectrum.

      [Zamolodchikov, ‘04; Smirnov, Zamolodchikov, ‘16 Cavaglia, Negro, Szecsenyi, Tateo, ‘16, McGough, Mezei, Verlinde, ‘16; Kraus, Liu, Marolf, ‘18; Gorbenko, Silverstein, Torroba, ‘18; Lewkowycz, Liu, SIlverstein, Torroba, ‘19; Shyam, ‘21; Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang, ‘21, …]

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  - Top-down and bottom-up are related by explicit uplifts of AdS/CFT, thermal mixing among internal spaces [Dong, Horn, Silverstein, Torroba, ‘10; Silverstein, ‘22]
- Today’s talk: review of this framework and extension to include local bulk matter.
  - [Time permitting, comment on extension of hyperbolic dS with ML]

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2) Integrability of the deformation: known deformation of the energy spectrum

- For a CFT on a cylinder with size L (and zero momentum)

$$y \equiv \frac{\lambda}{L^2} \qquad \pi y \varepsilon_n \partial_y \varepsilon_n - \partial_y \varepsilon_n + \frac{\pi}{2}\varepsilon_n^2 = 0$$

- On the gravity side, consider a bounded region of AdS_3

[McGough, Mezei, Verlinde, ‘16; Kraus, Liu, Marolf, ‘18]
[cf. Guica, Monten, ‘19]

$$S = \frac{1}{16\pi G_N}\int_{M_3} d^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right) - \frac{1}{8\pi G_N}\int_{\partial M_3} d^2x\sqrt{-g}\left(K - \frac{1}{\ell}\right)$$

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- If M_2 is a cylinder with radius-size L, and define the dimensionless energy

$$y \equiv \frac{\lambda}{L^2} \qquad \varepsilon \equiv -L\int d\theta\sqrt{g_{\theta\theta}}u^i u^j T_{ij}$$

2) Same equation for the energy levels as function of y (L changes).

$+\Lambda_2$

[Gorbenko, Silverstein, Torroba, ‘18; Lewkowycz, Liu, Silverstein, Torroba, ‘19; Shyam, ‘21; Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang, ‘21]

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- Differential equations for the energy levels (no momentum)

$$\pi y \varepsilon_n \partial_y \varepsilon_n - \partial_y \varepsilon_n + \frac{\pi}{2}\varepsilon_n^2 = \frac{1 - \eta}{2\pi y^2}$$

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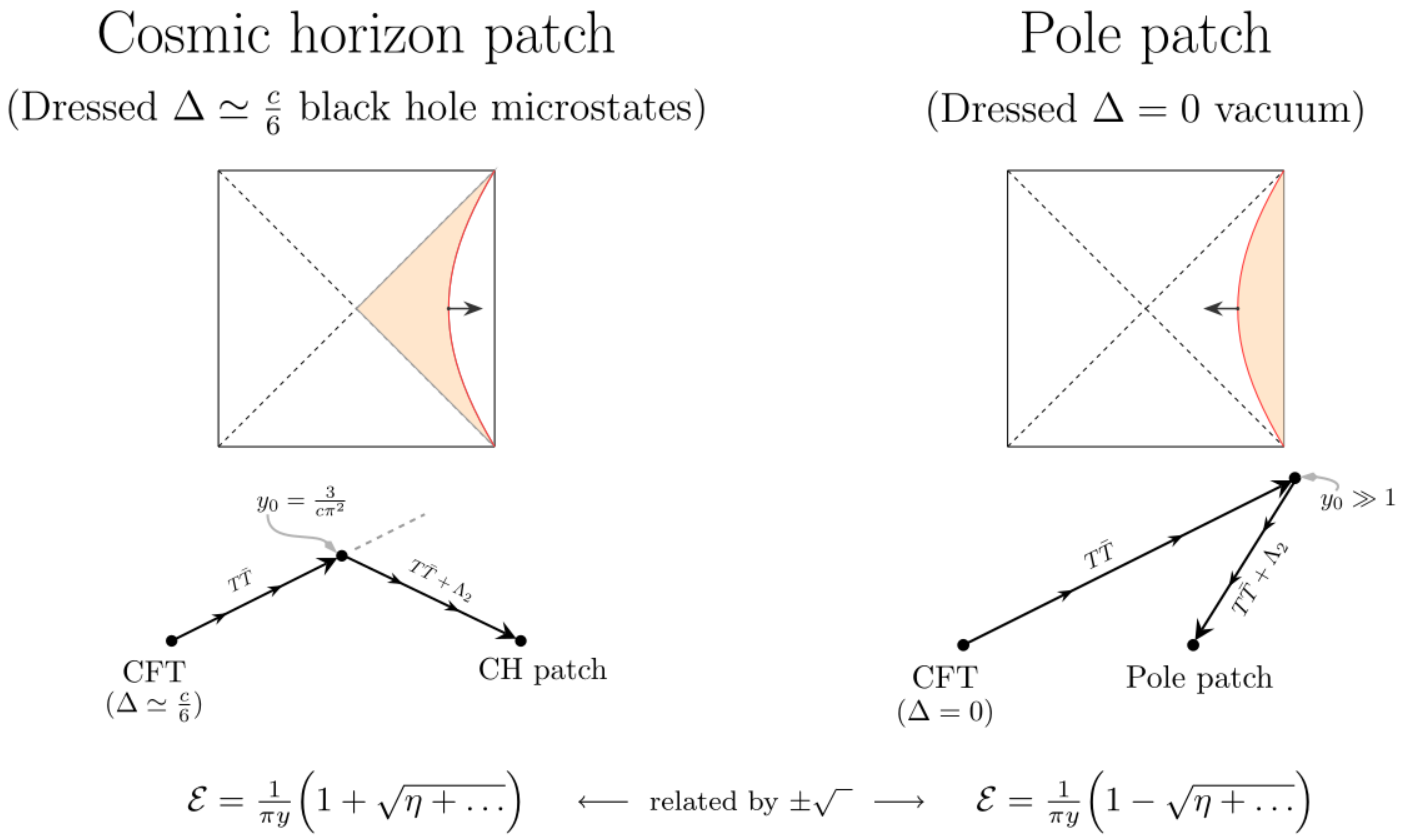
- For dS ($\eta = -1$), the trajectory cannot start at $y = 0$. Piece-wise trajectories starting from AdS:



[Picture from Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang, '21]

- How to build the Cosmic Horizon patch

[Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang, '21]

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- The horizon of a BTZ black hole with $r_h = \ell$ looks the same as the dS cosmic horizon

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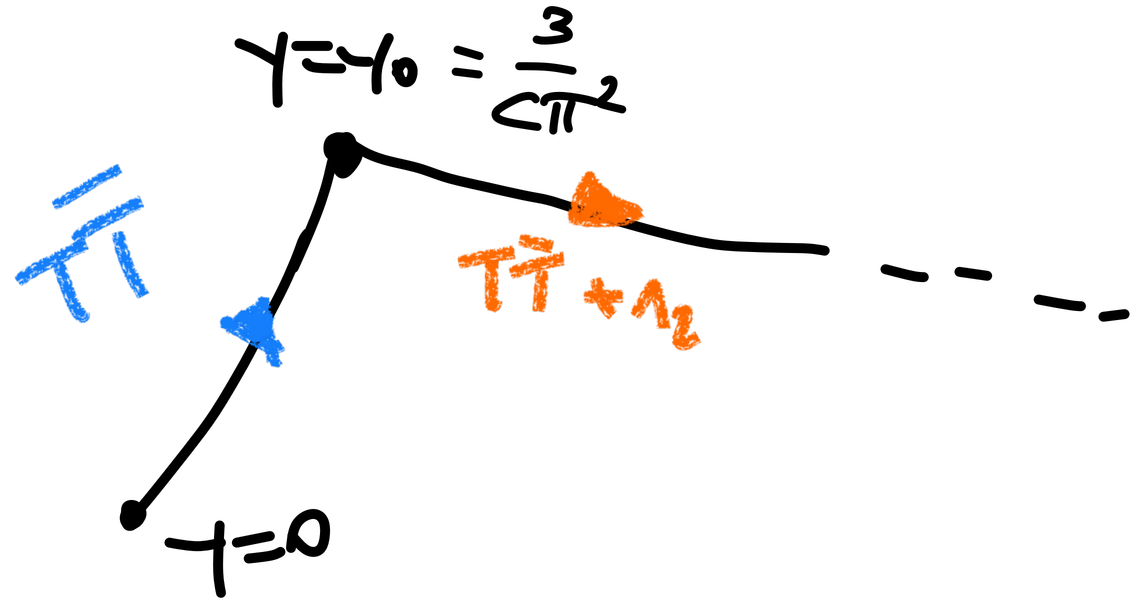

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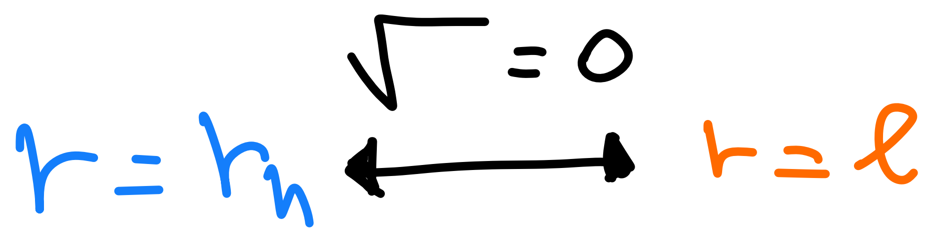

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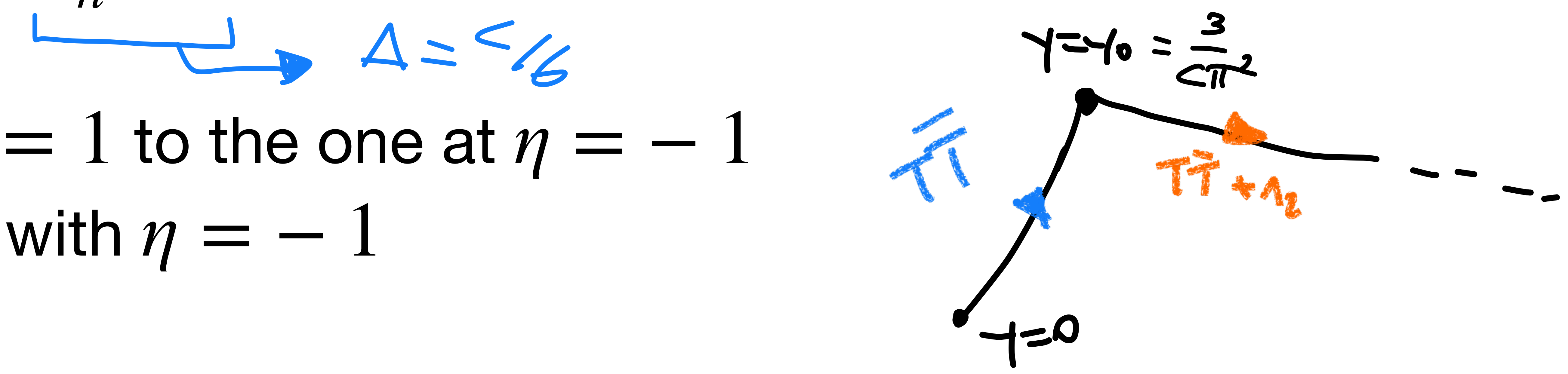

- States with $\Delta > \dfrac{c}{6} + \mathcal{O}(1)$ have formally complex energies, and are discarded from the theory
  - The theory has a finite number of states: discrete and bounded spectrum

- States with $\Delta < \dfrac{c}{6}$ are real but discontinuous

Recap so far

- $T\bar{T} + \Lambda_2$: way to deform holographic CFT to capture the physics of pure gravity on bounded regions of dS_3

[Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang, '21
Shyam, '21]

- It reproduces the Gibbons-Hawking entropy of dS, including logarithmic corrections

[Gibbons, Hawking '77, Anninos, Denef, Law, Sun, '20]

- Produces a finite theory (type I operator algebra)

[cf. Chandrasekaran, Longo, Pennington, Witten '22]

- Current limitations, only pure gravity; discontinuity the $\Delta \ll c/6$ levels
- Next: how to include bulk matter and address these issues

Adding matter at large c

[Hartman, Kruthoff, Shaghoulian, Tajdini, ‘18; Taylor, ‘18]

- Start from the gravity side. E.g. with a scalar field:

$$S = S_{\text{grav}} - \int \sqrt{-g} \left(\frac{1}{2} (\nabla \phi)^2 + V(\phi) \right) + \int_{\partial M_3} \sqrt{-g \,|_{\partial M_3}} B_{\text{ct}}(\phi)$$

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$$B_{ct}(\phi)T^i_i = -32\pi\ell G_N T\bar{T} - \frac{\eta - 1}{8\pi G_N} - \frac{\ell}{16\pi G_N}R^{(2)} - \underbrace{2\ell\mathcal{T}^{\phi}_{\mu\nu}\eta^\mu\eta^\nu}$$

$$\to \quad \ell\left(V(\phi) + \frac{1}{2}\left(\left(\nabla_{\parallel}\phi\right)^2 - \left(\nabla_{\perp}\phi\right)^2\right)\right)$$

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- Quadratic operator, not well-defined because of UV divergences at coincident points

- Our proposal: combine trajectories to avoid these ambiguities

Proposal for finiteness

- Deforming with pure $T\bar{T}$ gives a finite theory: spectrum discrete and truncated

[Smirnov, Zamolodchikov, '16]

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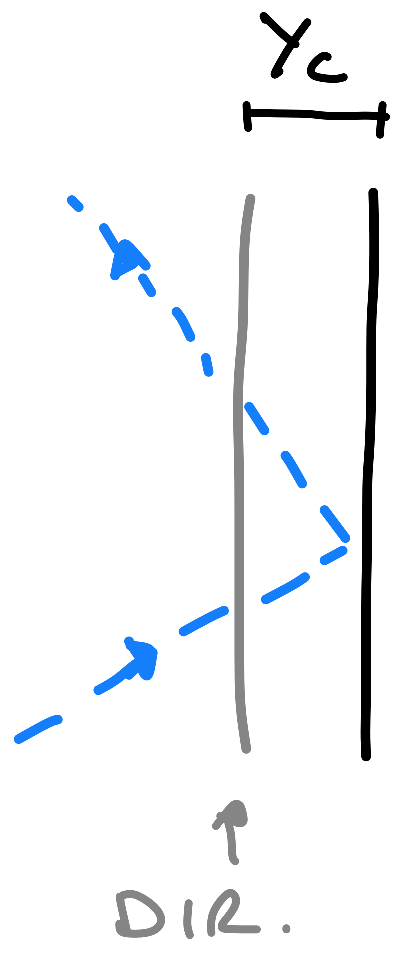

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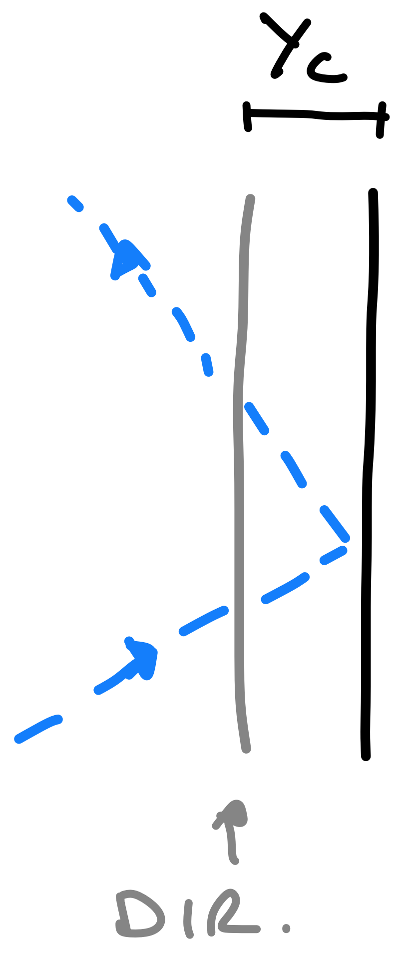

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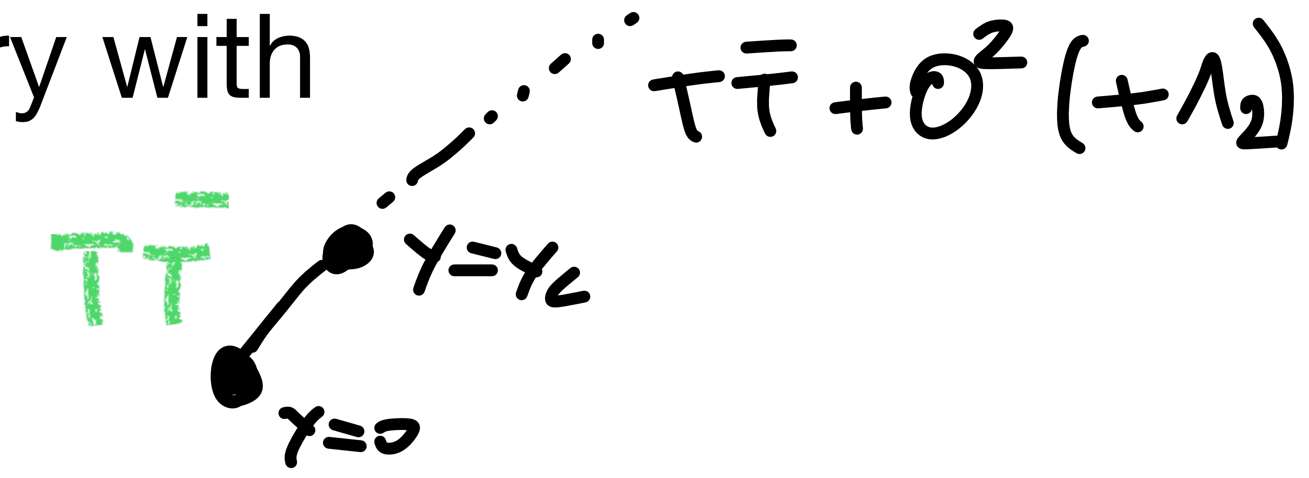

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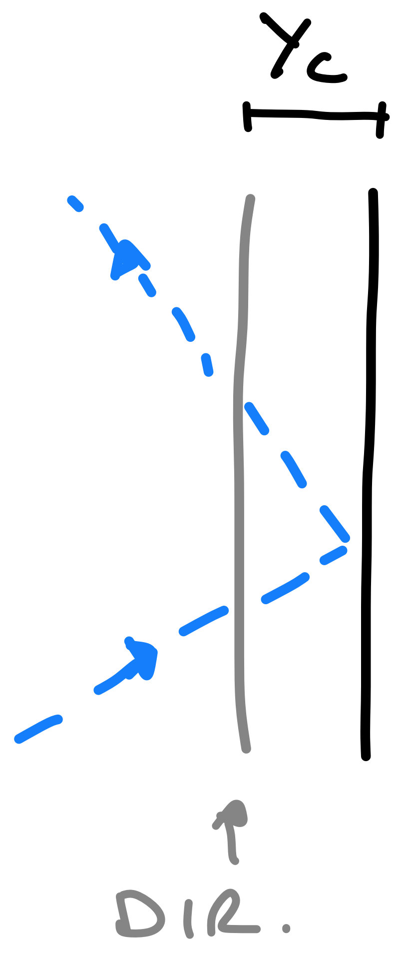

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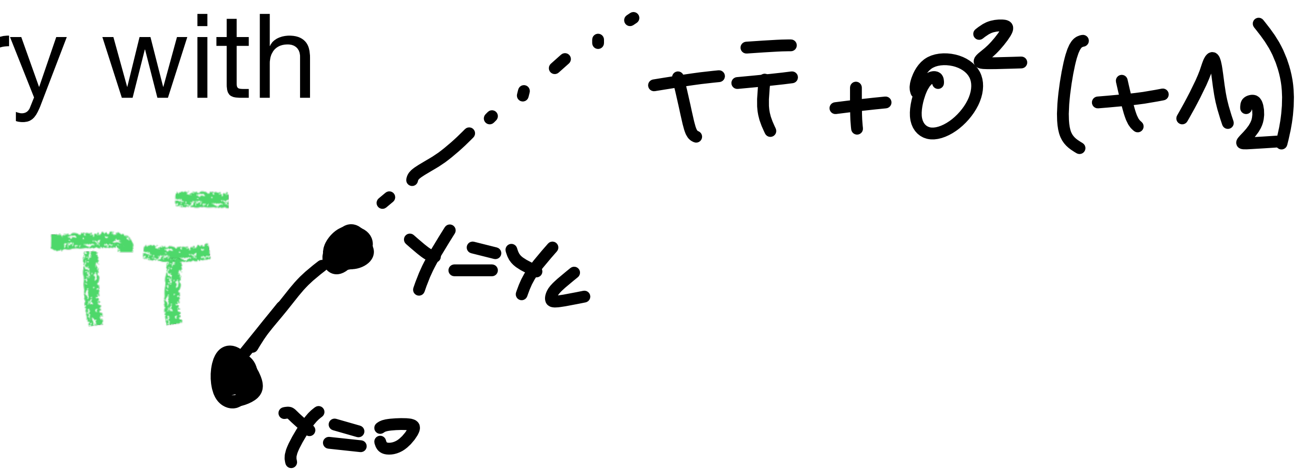

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- Should give a QG theory in (A)dS$_3$ which is well-defined and describes approx. local matter. Non-local effects at very high energies where QG effects become important.
- For dS, still need to address continuity of the $\Delta \ll c/6$ energy levels

Uplift sector and continuity

- Continuity is achieved for $\Delta = c/6$ because at the matching point the $\sqrt{}$ vanishes
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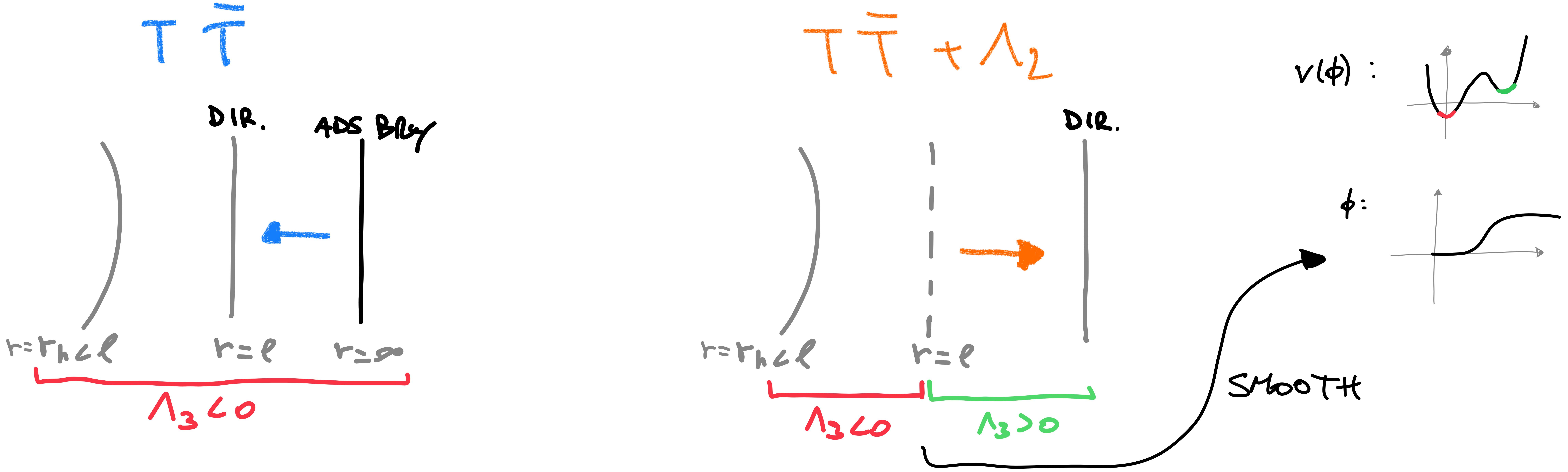

- Consider the ansatz [Bizon, Rostworowski, '11]

$$ds_3^2 = -\rho^2 A(t,\rho)e^{-2\delta(t,\rho)}dt^2 + A^{-1}(t,\rho)d\rho^2 + \ell^2 g(t,\rho)d\theta^2 \qquad \phi = \phi(\rho,t)$$

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$$\varepsilon = \frac{L^2}{16\pi^2\ell^2 G_N}\left(1 - \ell\sqrt{A}\frac{\partial_\rho g}{2g}\right)$$

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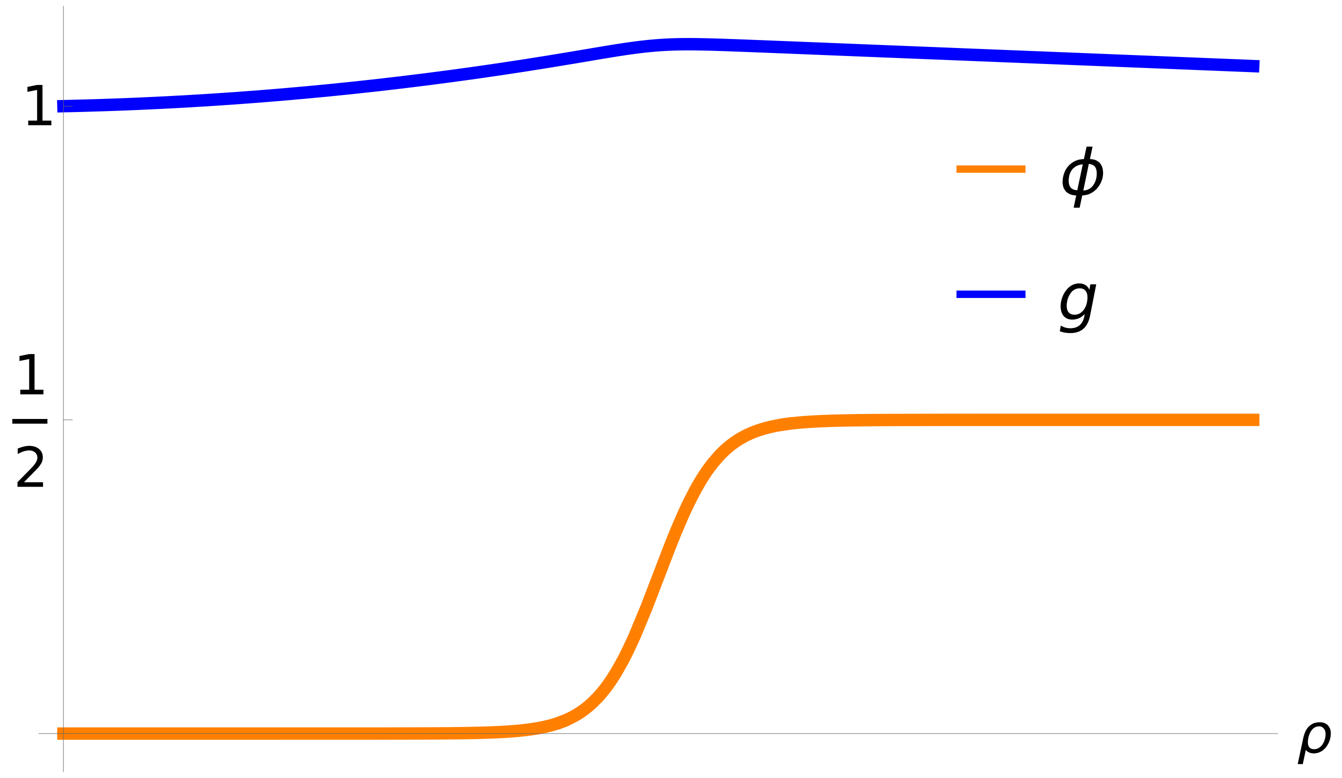
  - First check: at the linearized level a consistent solution of the constraint equations exists



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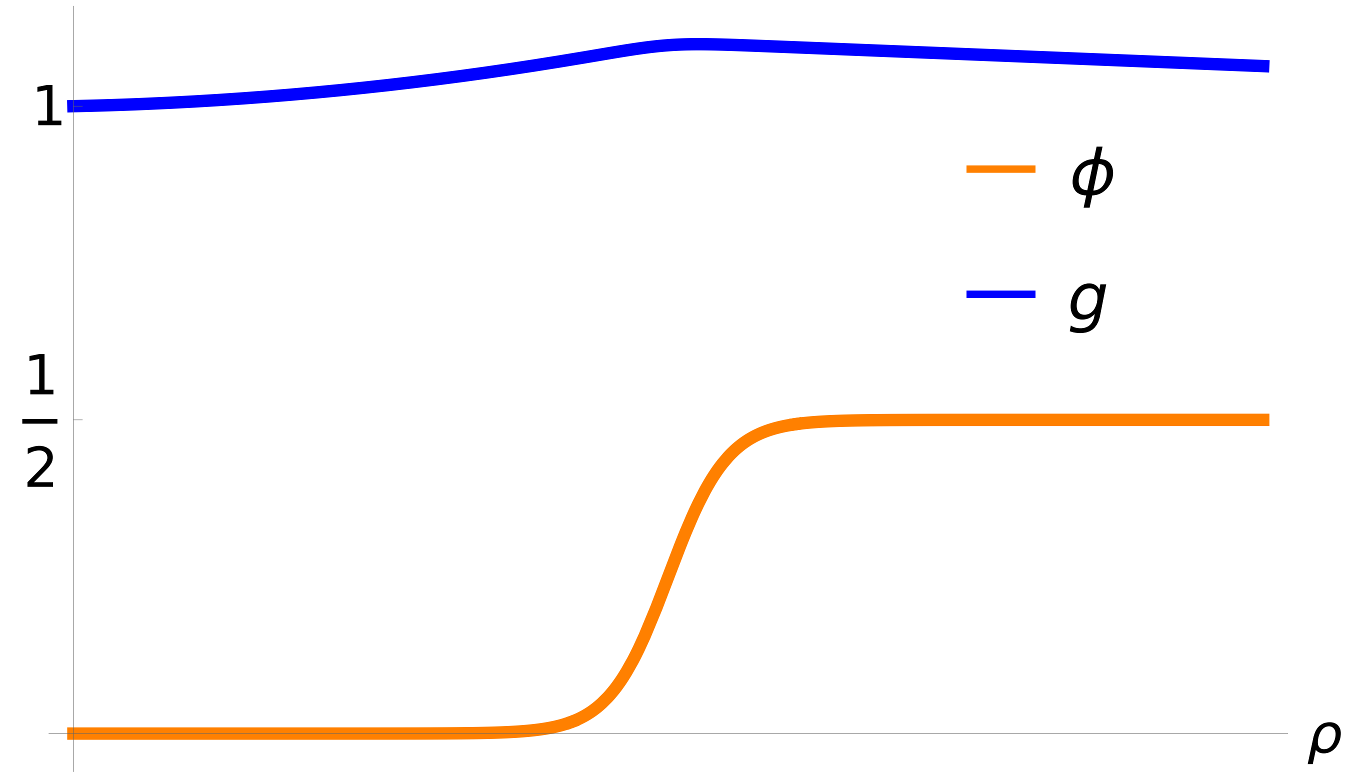

- The Dirichlet problem in gravity is not always well-posed

[e.g. An, Anderson '21]

- In progress: full solution of the non-linear problem

The complete prescription

- Putting together the initial small $T\bar{T}$ part of the trajectory and the inclusion of matter through the well-defined $\mathcal{O}^2$ terms, including the uplift sector

$$\frac{dW}{d\lambda} = -2\pi \int_{M_2} \sqrt{-\gamma} T\bar{T} \qquad y < y_c$$

$$\frac{dW}{d\lambda} = -2\pi \int_{M_2} \sqrt{-\gamma} \frac{1}{B_{ct}(\Phi_u(\lambda) + J_u)} \left(T\bar{T} + \mathcal{O}^2 - \gamma^{ij}\partial_i J \partial_j J \right) \qquad y > y_c$$

$$-2\pi \int_{M_2} \sqrt{-\gamma} \frac{1}{B_{ct}(\Phi_u(\lambda) + J_u)} \left(\lambda^{1/2}\Pi_u^2 - \gamma^{ij}\partial_i J_u \partial_i J_u - \lambda^{-1} B_{ct}(\Phi_u(\lambda) + J_u)^2 - V(\Phi_u(\lambda) + J_u) \right)$$

- Using the Hamiltonian path integral, we integrate only in regions of phase space with real energies

- The trajectory in theory space is piece-wise defined. At each value of the deformation λ we can use either the Lagrangian or the Hamiltonian formalism to compute the operators needed for $\lambda + \Delta\lambda$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} S \qquad \left\langle \mathcal{O}_{I_1} \dots \mathcal{O}_{I_n} \right\rangle = \frac{\delta}{\delta J^{I_1}} \dots \frac{\delta}{\delta J^{I_n}} W$$

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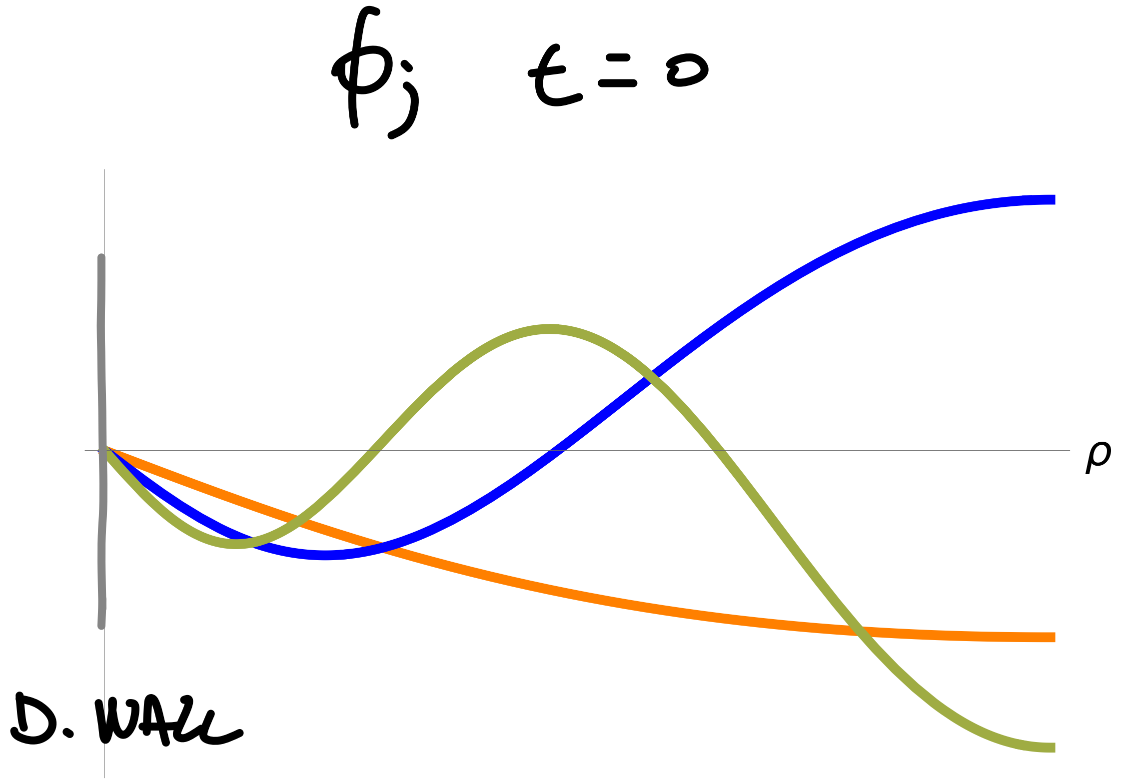
- At first non-trivial order in G_N the scalar sees the background geometry and backreacts on the metric:

$$A = 1 + G_N^2 \Delta A + \mathcal{O}(G_N^3) \qquad \delta = 0 + G_N^2 \delta_1$$

- Put a Dirichlet boundary cutting off the horizon. Explicit time-dependent solutions with correct BCs:



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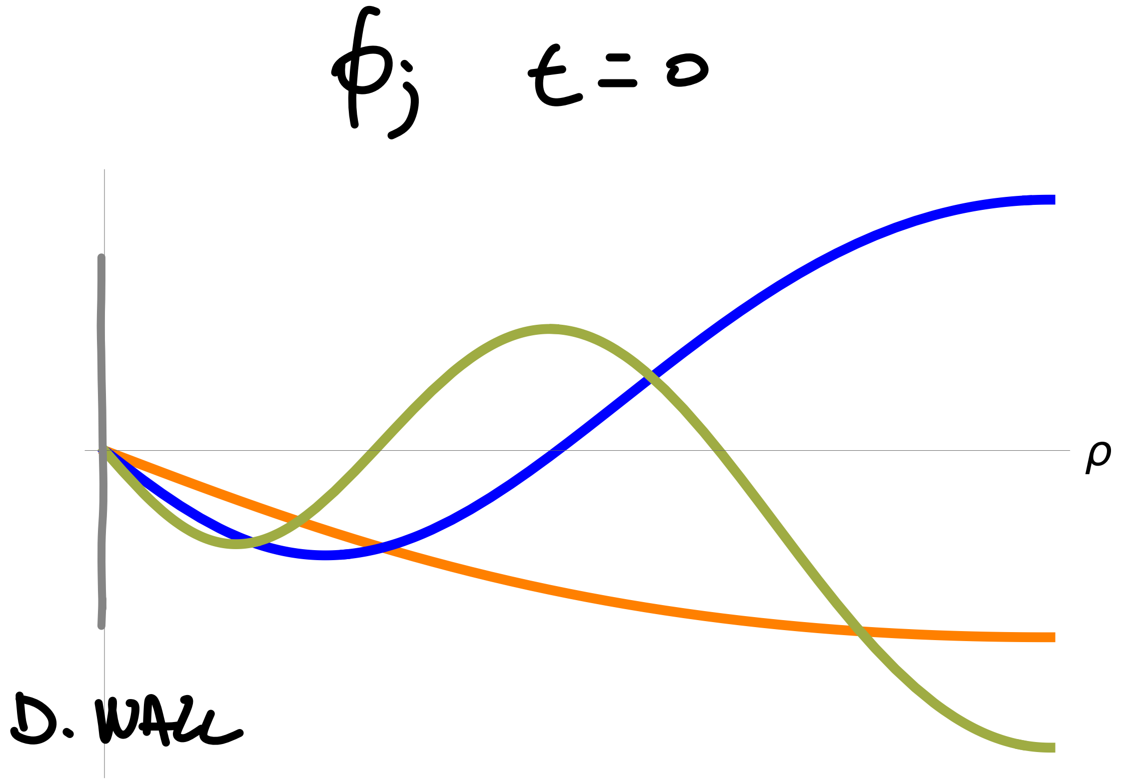

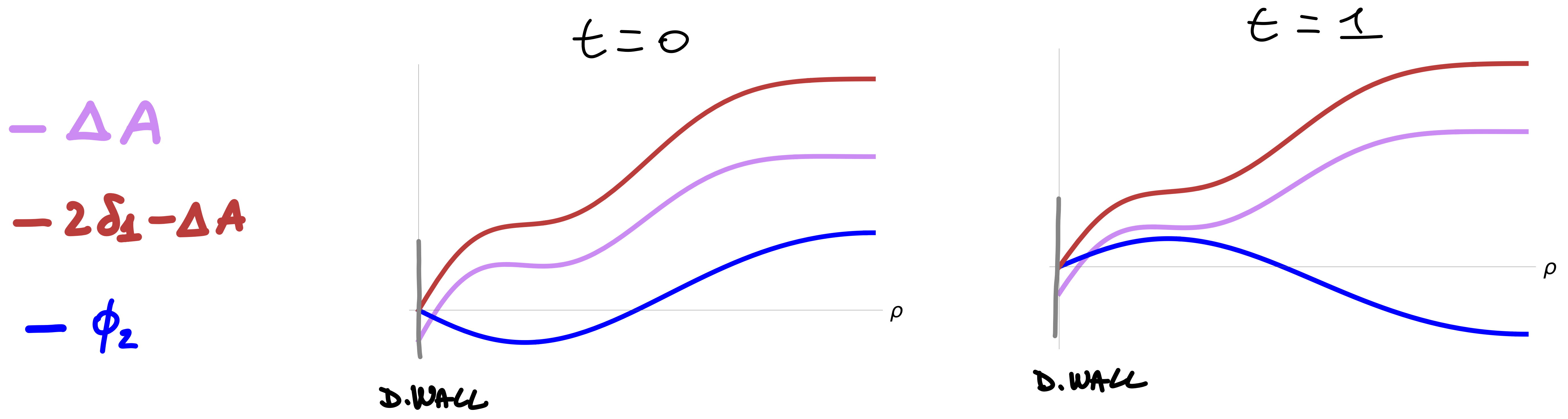

- QFT side: To a given mode in the bulk we can associate an effective field χ_j of mass ω_j on the bry

$$\phi = \sum_n \mathbf{a}_n f_n(\rho) e^{i\omega_n t} + \mathbf{a}_n^\dagger f_n^*(\rho) e^{-i\omega_n t}$$

$$\pi = \sum_n \mathbf{a}_n f_n'(\rho) e^{i\omega_n t} + \mathbf{a}_n^\dagger f_n'^*(\rho) e^{-i\omega_n t}$$

$$L_{\chi_j} = \frac{1}{2}\int dt d\theta \left(\dot{\chi}_j^2 - \omega_j^2 \chi^2\right) + \dots$$

$$\chi_j(t) = \mathbf{a}_j e^{i\omega_j t} + \mathbf{a}_j^\dagger e^{-i\omega_j t} \qquad \propto \pi(\rho_c)$$

- On an oscillating coherent state $|\psi\rangle$ of the field χ_j we can compute energy and pressure

  At first order:

$$\Delta T_t^t \propto \frac{1}{2}\left(\dot{\chi}^2 + \omega_j^2 \chi^2\right) \propto \omega_j^2 \qquad \Delta T_\theta^\theta(t) \propto \frac{1}{2}\left(\dot{\chi}_j^2 - \omega_j^2 \chi^2\right) \propto 1 - 2\sin^2(\omega_j t)$$

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- At first order the trace relation gives

$$\Delta T_t^t + \Delta T_\theta^\theta(t) = \pi\lambda\left(T_t^{t(0)} \Delta T_\theta^\theta(t) + T_\theta^{\theta(0)} \Delta T_t^t\right) - \lambda \mathcal{O}(t)^2$$

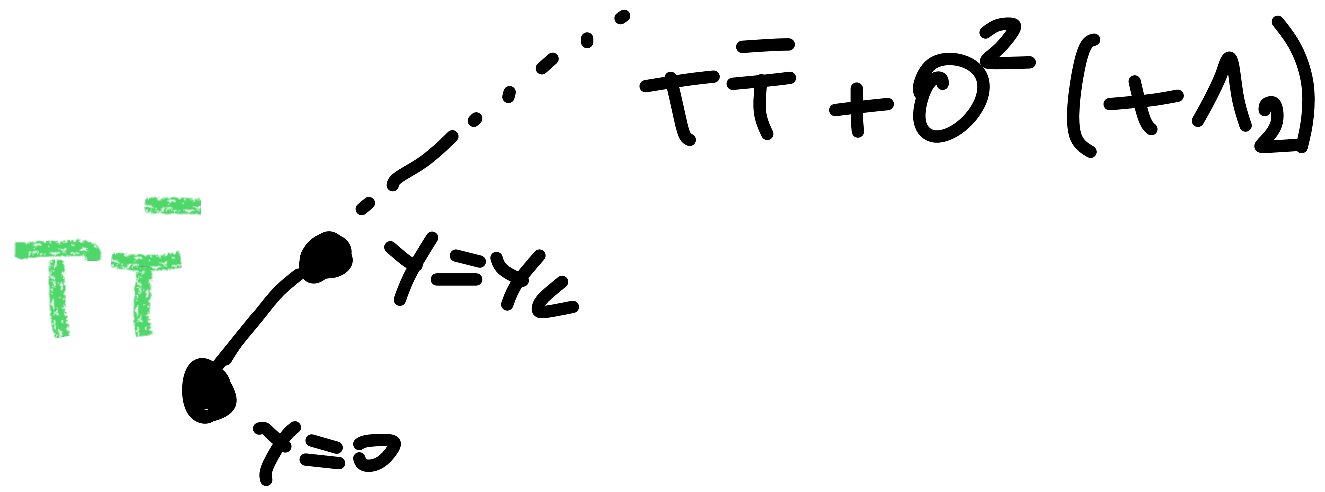
- By energy conservation the time-dependent terms cancel. This fixes $\mathcal{O}[\chi_j]$ and leaves

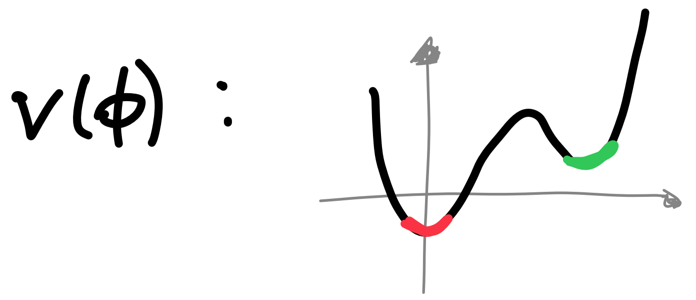
$$\Delta T_t^t \propto -\omega_j^2 \frac{1 - \pi\lambda T_t^{t(0)}}{1 - \pi\lambda T_\theta^{\theta(0)}}$$

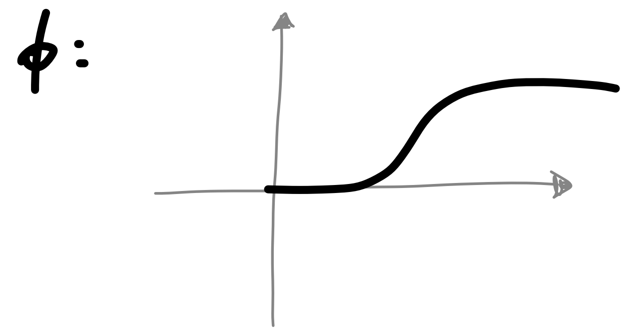
Recap

- The $T\bar{T} + \Lambda_2$ deformation of a CFT$_2$ describes 3-dimensional gravitational physics in patches of cosmological space-times
  - The original prescription correctly accounts for the entropy of the cosmic horizon, but only captures the “pure gravity” sector.
    - It is continuous only for the entropically dominant energy levels

- We are proposing how incorporate approx. local bulk matter at finite c
  - This involves piecewise-trajectories in theory space, where first the theory is rendered finite and then matter is added







- Thanks to matter, we can incorporate an “uplift” field that makes also the subdominant levels continuous

Thank you!