Towards a microscopic formulation of the no-boundary wave function

CERN Cosmo Workshop

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Wave function of the Universe

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(Received 29 July 1983)

The quantum state of a spatially closed universe can be described by a wave function which is a functional on the geometries of compact three-manifolds and on the values of the matter fields on these manifolds. The wave function obeys the Wheeler-DeWitt second-order functional differential equation. We put forward a proposal for the wave function of the "ground state" or state of minimum excitation: the ground-state amplitude for a three-geometry is given by a path integral over all compact positive-definite four-geometries which have the three-geometry as a boundary. The requirement that the Hamiltonian be Hermitian then defines the boundary conditions for the Wheeler-DeWitt equation and the spectrum of possible excited states. To illustrate the above, we calculate the ground and excited states in a simple minisuperspace model in which the scale factor is the only gravitational degree of freedom, a conformally invariant scalar field is the only matter degree of freedom and $\Lambda > 0$. The ground state corresponds to de Sitter space in the classical limit. There are excited states which represent universes which expand from zero volume, reach a maximum size, and then recollapse but which have a finite (though very small) probability of tunneling through a potential barrier to a de Sitter-type state of continual expansion. The path-integral approach allows us to handle situations in which the topology of the three-manifold changes. We estimate the probability that the ground state in our minisuperspace model contains more than one connected component of the spacelike surface.

Cet instant unique, qui n'avait pas
Georges Lemaitre (1894 -

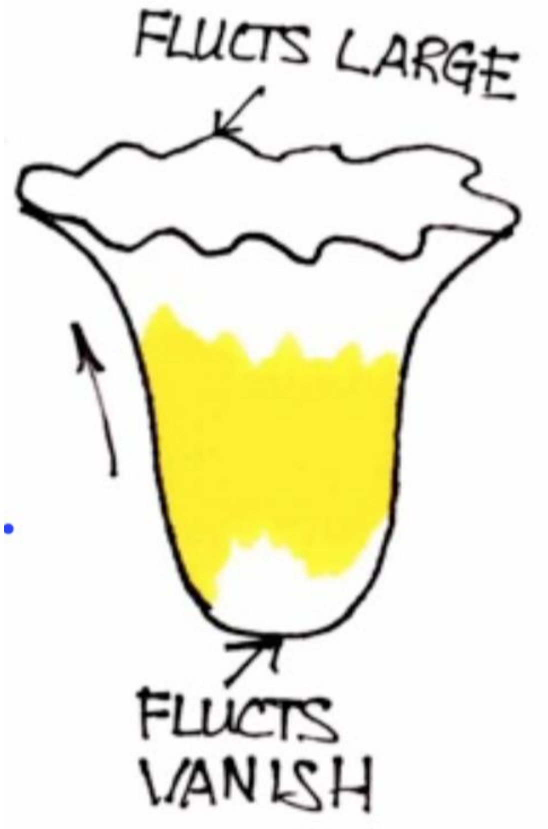
A Quantum Universe

If the universe is a quantum mechanical system it has a quantum state. What is it?

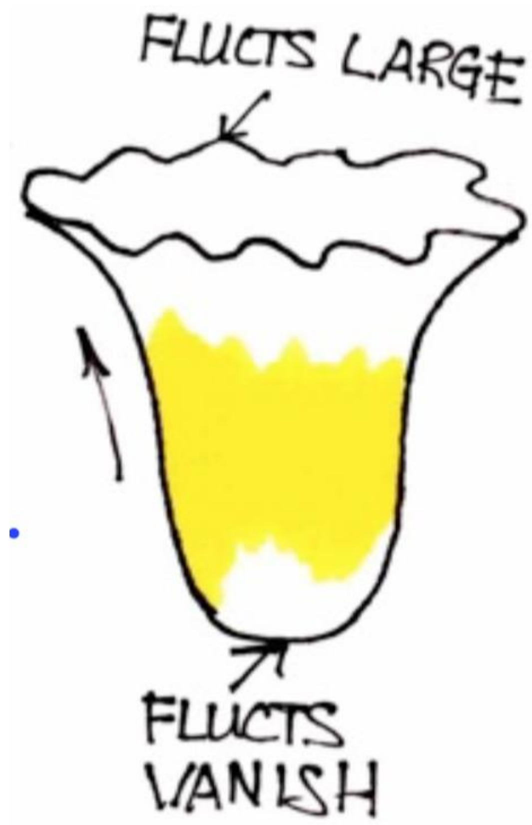
Ψ

That is the problem of Quantum Cosmology.

A theory of the
quantum state
of the universe
is as much a part of a
final theory
as a theory of dynamics.


FLUCTS LARGE
FLUCTS
VANISH

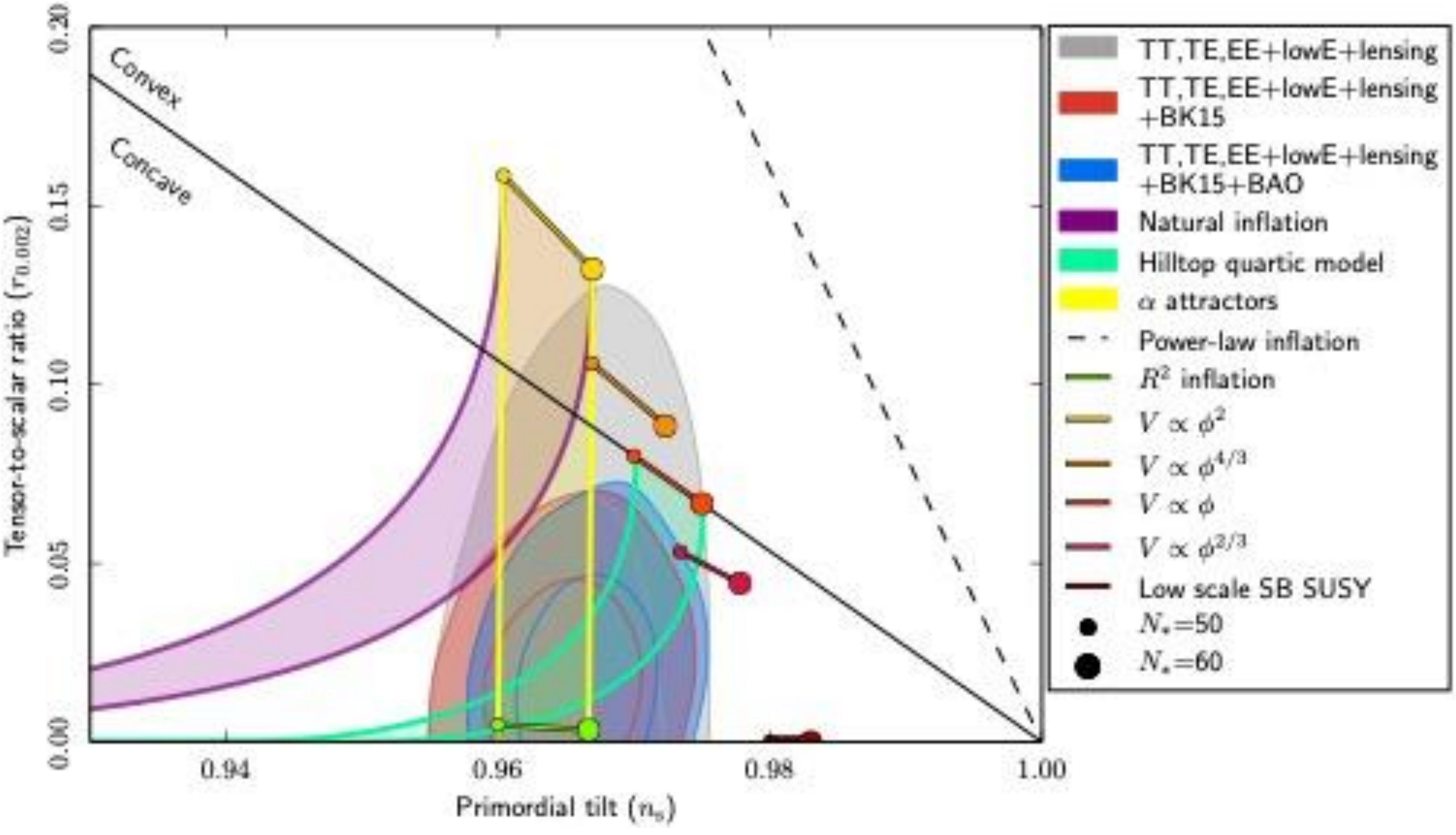
- Classical cosmological evolution emerges
- through a dS-like phase of inflation.



- Fluctuations are initially small,
- giving rise to a physical arrow of time.

[Hartle, Hawking, TH, 2008]

A Prior for Planck?



[Planck 2018 results: Constraints on inflation, 1807.06211]

A Prior for Planck?

[Submitted on 27 May 2013 (v1), last revised 10 Feb 2014 (this version, v3)]

Predicting a Prior for Planck

Thomas Hertog

The quantum state of the universe combined with the structure of the landscape potential implies a prior that specifies predictions for observations. We compute the prior for CMB related observables given by the no-boundary wave function (NBWF) in a landscape model that includes a range of inflationary patches representative of relatively simple single-field models. In this landscape the NBWF predicts our classical cosmological background emerges from a region of eternal inflation associated with a plateau-like potential. The spectra of primordial fluctuations on observable scales are characteristic of concave potentials, in excellent agreement with the Planck data. By contrast, alternative theories of initial conditions that strongly favor inflation at high values of the potential are disfavored by observations in this landscape.

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- From wave function to observables
- Observables must be observed
- Saddle point form only an approximation

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- From wave function to observables
- Observables must be observed
- Saddle point form only an approximation

→ Can the no-boundary theory be refined -> its predictions strengthened?

Towards a microscopic formulation of the no-boundary wave function

1. Allowability -> Saddle selection

   hep-th, 2305.15440 [w/ Joel Karlsson, Oliver Janssen]

2. Holography -> microscopic dS entropy

   hep-th, 2211.05907 [w/ N. Bobev, Junho Hong, Joel karlsson, Valentine Reys]

3. Observer -> Page-like transition

   hep-th, 1009.2525 [w/ Hartle, Hawking]

Saddle selection

Kontsevich- Segal: consider only those (complex) saddle backgrounds on which an arbitrary QFT can be defined [2105.10161]

$$\mathrm{Re}\left(\sqrt{g}\, g^{\mu_1\nu_1}\cdots g^{\mu_p\nu_p} F_{\mu_1\cdots\mu_p} F_{\nu_1\cdots\nu_p}\right) > 0$$

Witten: elevate this to a selection principle of saddles in gravitational path integrals [2111.06514]

Some evidence: the criterion eliminates pathological wormholes but allows e.g. for complexified Kerr

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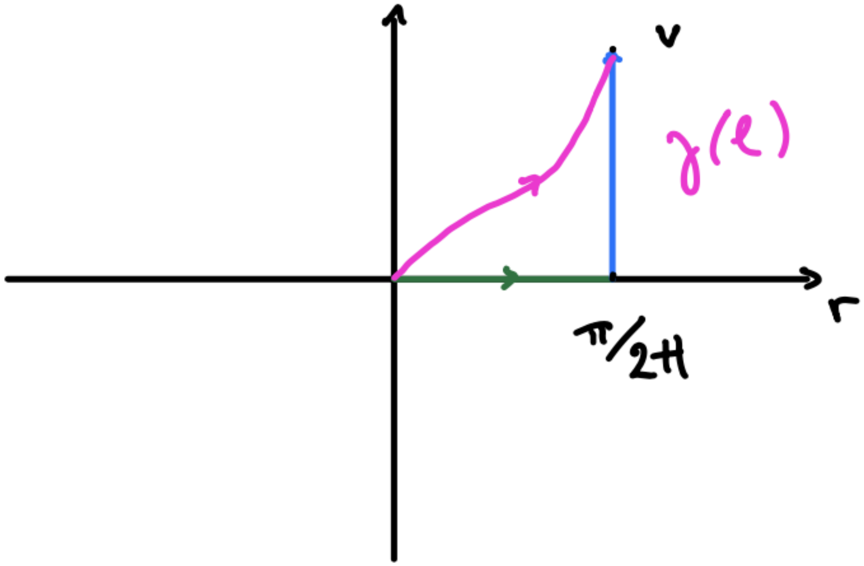
Do all no-boundary saddles that describe
the origin of inflation satisfy the KSW criterion?

Saddle selection

What can go wrong?

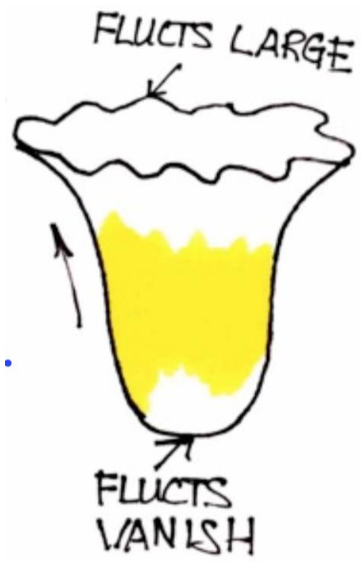


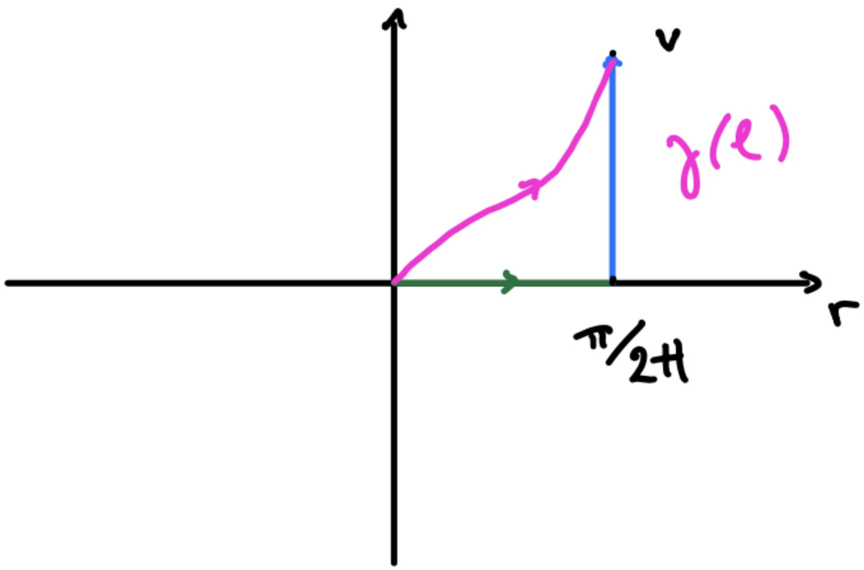
Saddle selection

#	V/Λ	allowable	disallowable
①	$1+\cos(\phi/f)$	$[2, 6.09)$	$[6.09, 10]$
②	$1-\phi^2/\mu^2$	$[10^{1/2}, 10^4]$	
③	$1-\phi^4/\mu^4$	$[10^{-1}, 10^2]$	
④	$1-\exp(-q\phi)$	$[10^{-3}, 10^3]$	
⑤	$1-\mu^2/\phi^2$	$[10^{-6}, 10^3]$	
⑥	$1+\alpha\log\phi$	$[10^{-3}, 10]$	
⑦	$\left[1-\exp\left(-\sqrt{2}\phi/\sqrt{3\alpha}\right)\right]^2$	$[10^{-1}, 93.9)$	$[93.9, 10^4]$
⑧	ϕ^p	$[1/2, 1.05)$	$[1.05, 4]$

Saddle selection

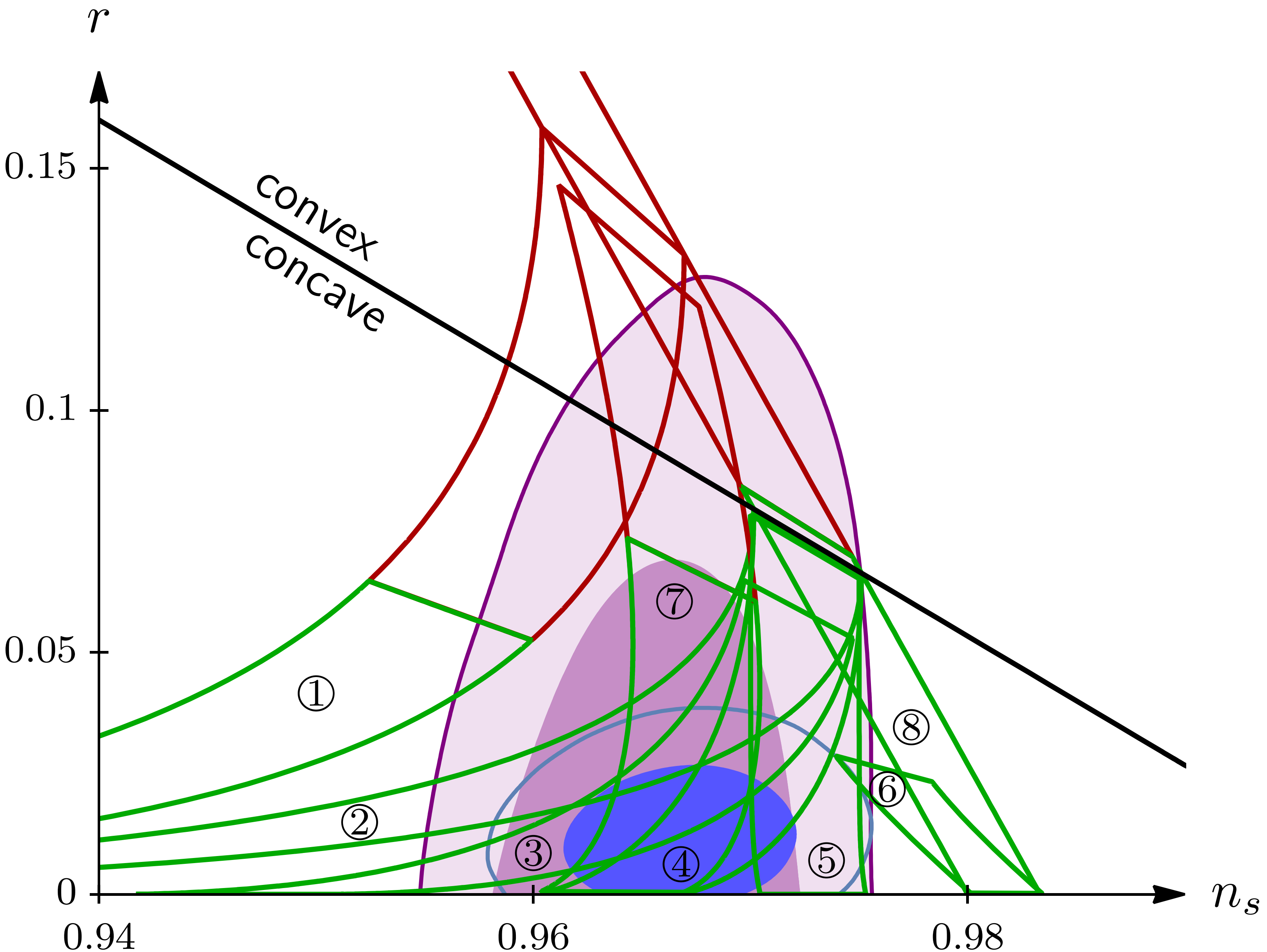
What can go wrong?





→ The KS(W)-criterion selects those no-boundary saddles in which the universe emerges on a concave patch of the scalar slow-roll potential.

Observational implications



The KS-criterion constrains inflation in the no-boundary state

Thomas Hertog[♠], Oliver Janssen[♣] and Joel Karlsson[♠]

[♠] *Institute for Theoretical Physics, KU Leuven, Celestijnenlaan 200D, 3001 Leuven, Belgium*
[♣] *International Centre for Theoretical Physics, Strada Costiera 11, 34151 Trieste, Italy and Institute for Fundamental Physics of the Universe, Via Beirut 2, 34014 Trieste, Italy*

We show that the Kontsevich–Segal (KS) criterion, applied to the complex saddles that specify the semiclassical no-boundary wave function, acts as a selection mechanism on inflationary scalar field potentials. In this context the KS-criterion effectively bounds the tensor-to-scalar ratio of cosmic microwave background fluctuations to be less than 0.08, in line with current observations. We trace the failure of complex saddles to meet the KS-criterion to the development of a tachyon in their spectrum of perturbations.

hep-th, 2305.15440

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- Quid eternal inflation?
- This result resonates with swampland mantra..
- Quid Quintessence?

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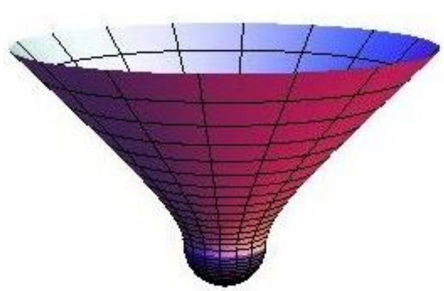
de Sitter saddle

[Hartle, TH ’11; Harlow, Stanford ‘11]

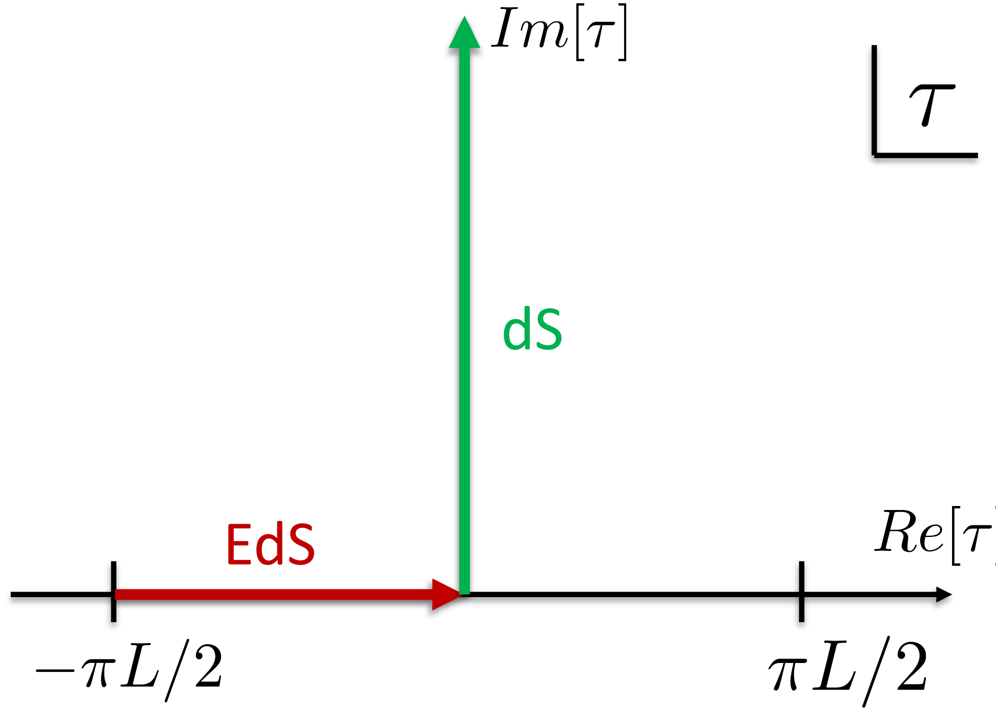
$$ds^2 = \left[-dt^2 + L^2 \cosh^2(t/L)\, d\Omega_3^2\right]$$

$$t \longrightarrow i\tau$$

$$ds^2 = \left[d\tau^2 + L^2 \cos^2(\tau/L)\, d\Omega_3^2\right]$$



$$\Psi \sim e^{-I_{EdS}/2} e^{iI_{dS}}$$



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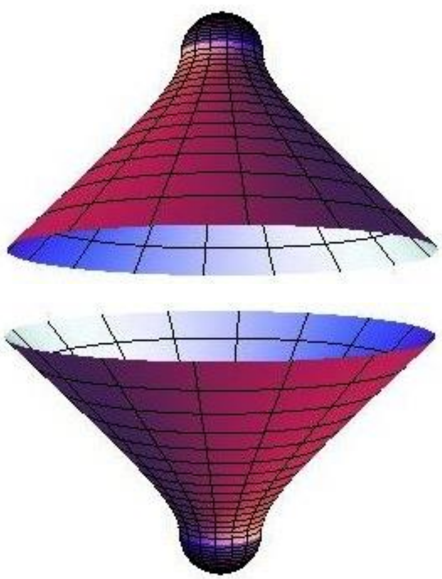
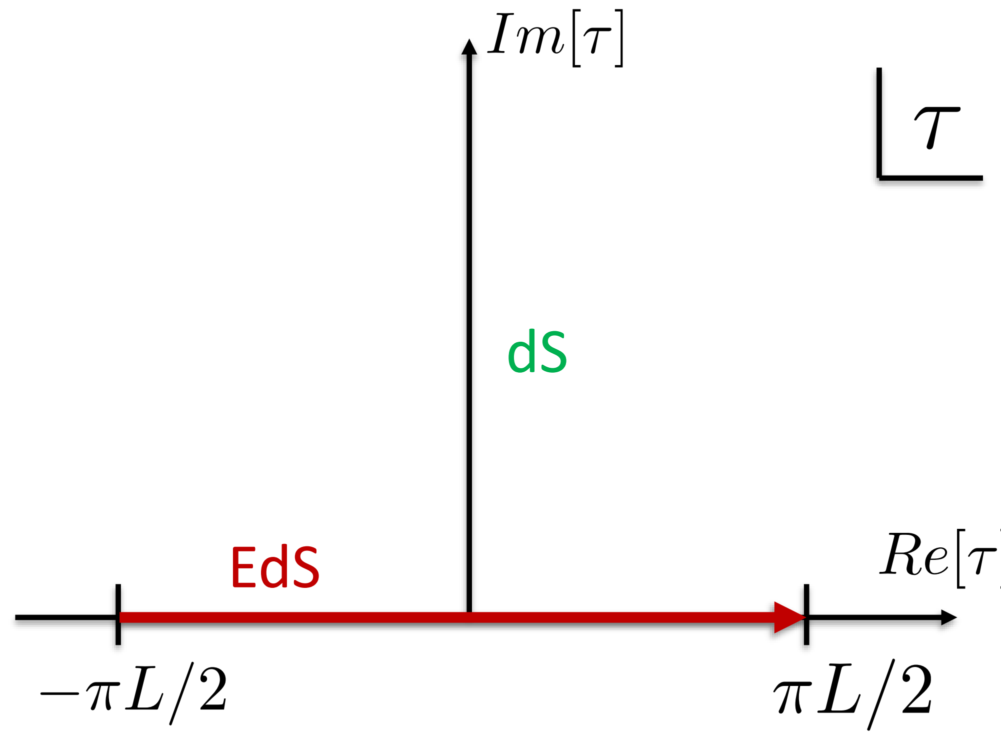


$$\Psi^*\Psi \sim e^{-I_{EdS}}$$



de Sitter entropy

$$ds^2 = \left[-dt^2 + L^2 \cosh^2(t/L)\, d\Omega_3^2\,\right]$$

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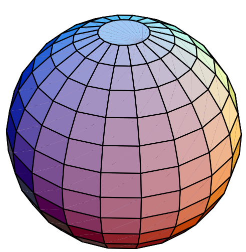

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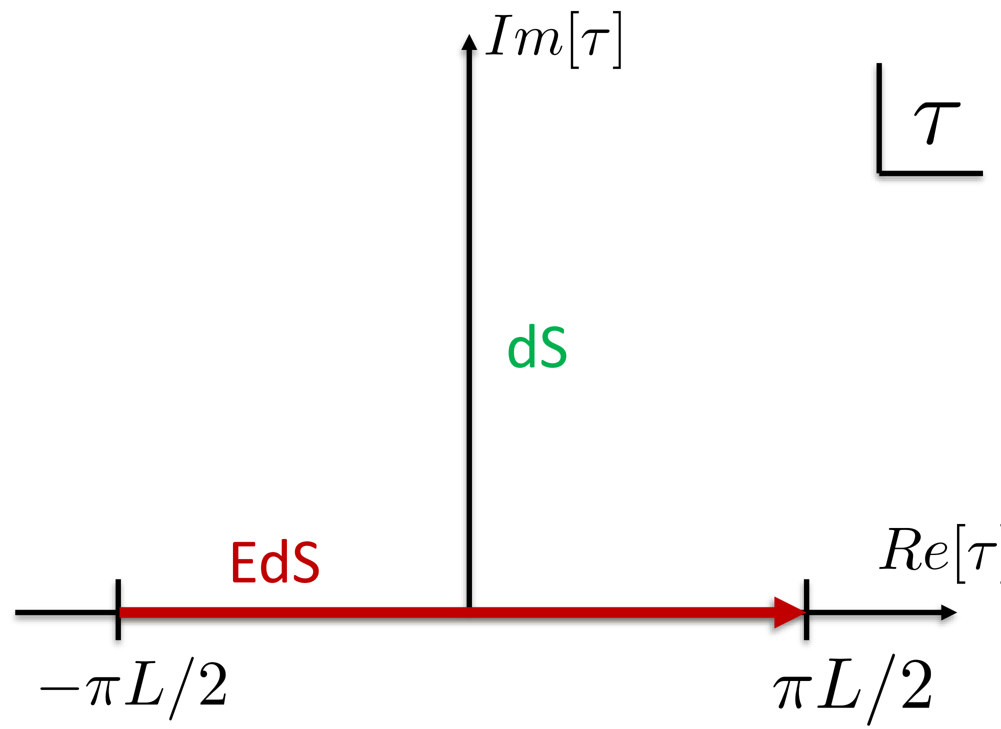

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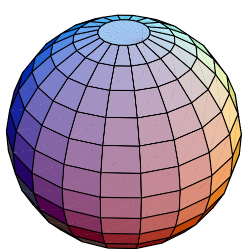

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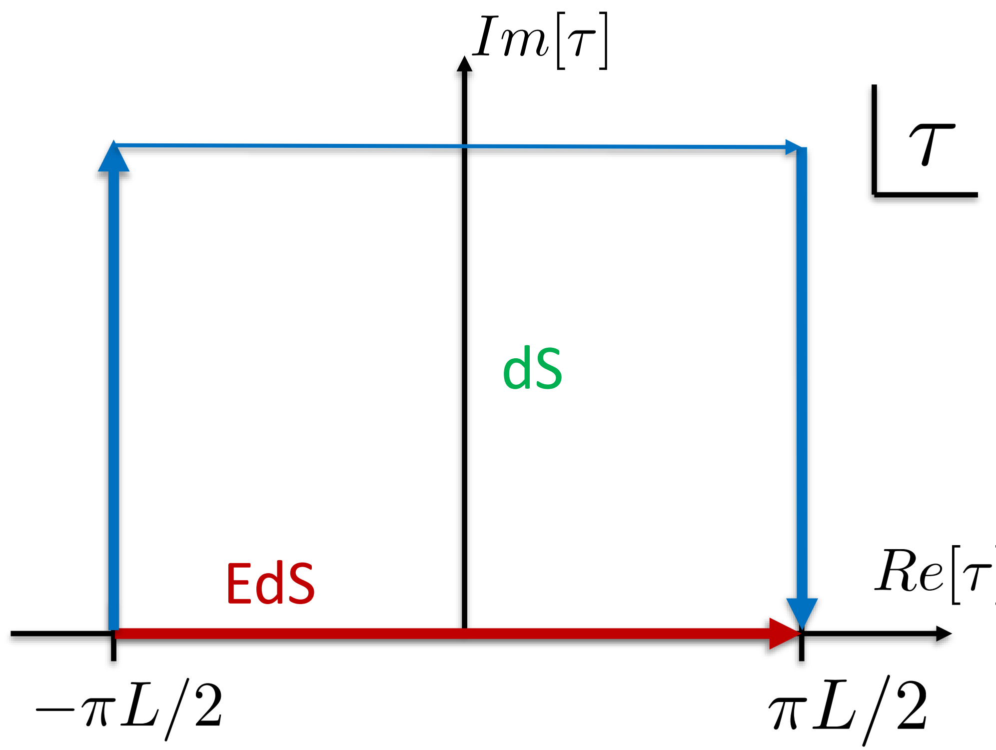

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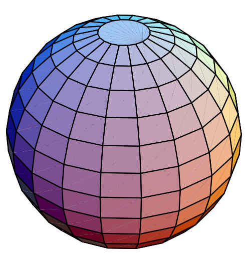
$$ds^2 = \left[d\tau^2 + L^2 \cos^2(\tau/L)\, d\Omega_3^2\,\right]$$

$$\tau = -\pi L/2 + ir$$

$$ds^2 = \left[-dr^2 - L^2 \sinh^2(r/L)\, d\Omega_3^2\,\right]$$



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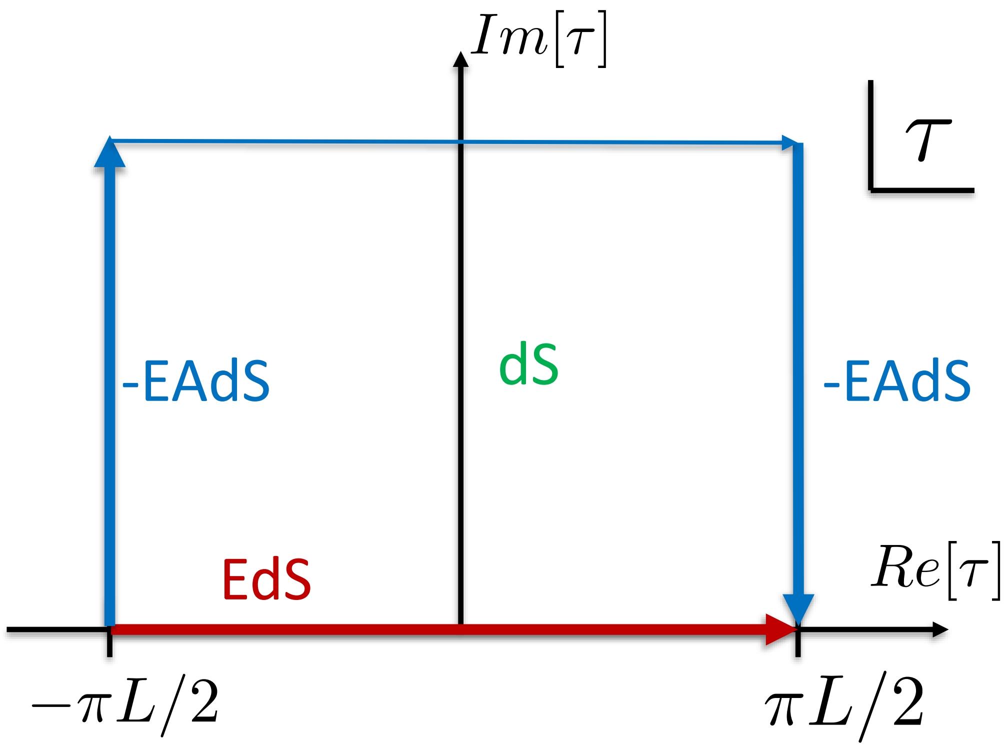


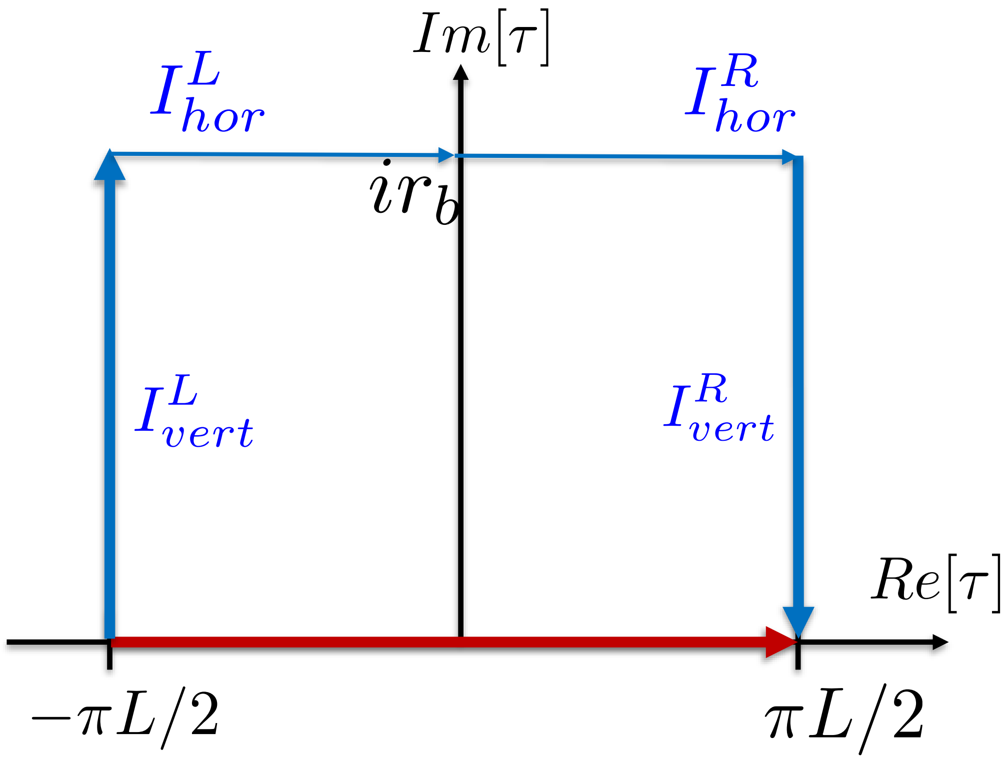
de Sitter entropy

$$I^L_{vert} = -I^{\rm reg}_{\rm EAdS} - I_{ct} + \mathcal{O}(e^{-r_b/L})$$

$$I^L_{hor} = +I_{ct} - iI_{ct} + \mathcal{O}(e^{-r_b/L})$$

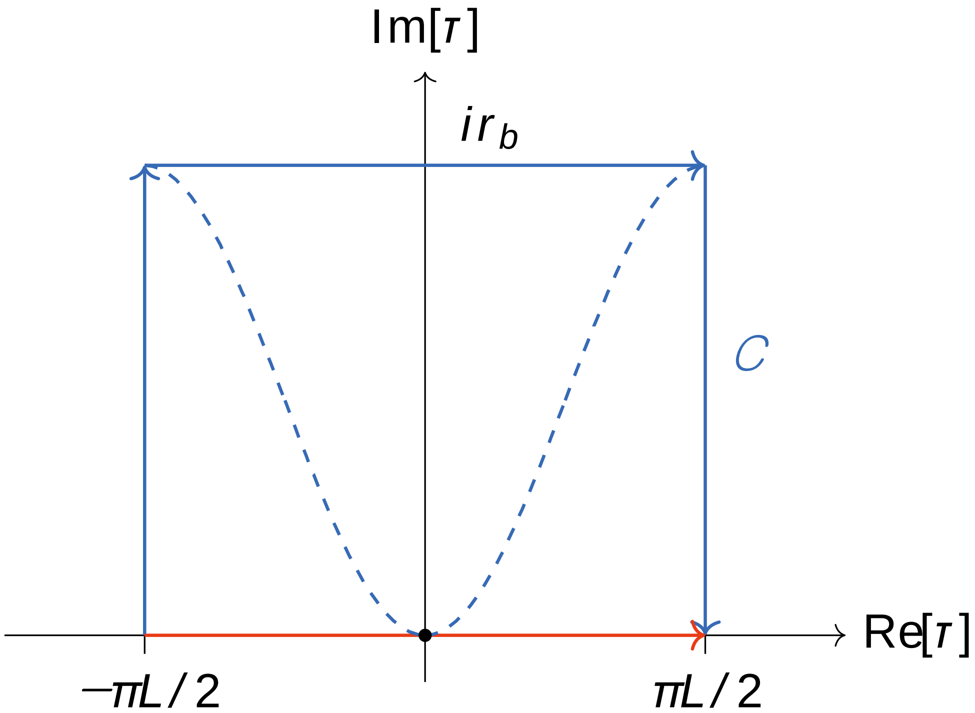
$$I^R = (I^L)^*$$

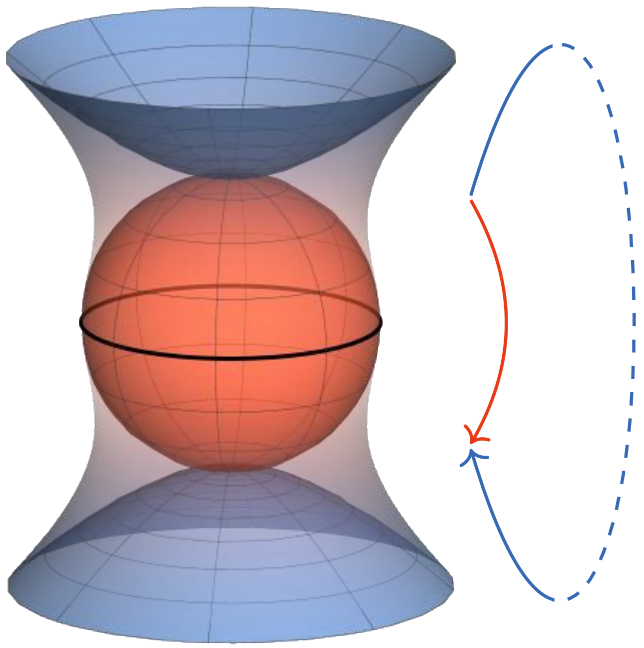
$$I_{EdS} = -2\, I^{\rm reg}_{\rm EAdS}$$



de Sitter entropy

$$S_{dS} = -I_{EdS} = 2I^{\mathrm{reg}}_{\mathrm{EAdS}}$$



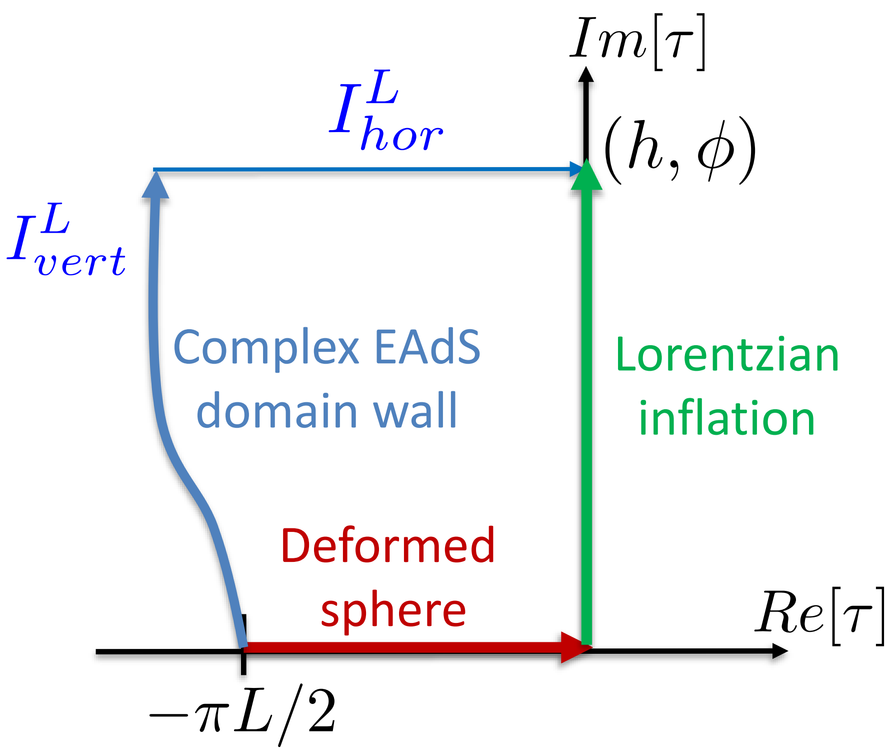


Digression: saddles with sources

[Hartle, TH '11]

$$\Psi_{HH} = \mathcal{A}_{sp} e^{iS}$$

$$\log \mathcal{A}_{sp} = I^{\mathrm{reg}}_{as\mathrm{EAdS}}$$



de Sitter entropy: microscopics

$$S_{dS} = -I_{EdS} = 2I_{\mathrm{EAdS}}^{\mathrm{reg}}$$

- AdS/CFT *EAdS*$_4$ *x S*^{7} : $-I_{\mathrm{EAdS}}^{\mathrm{reg}} + \cdots = \log Z_{S^3}^{\mathrm{CFT}}$

- Conjecture: $\boxed{S_{dS} = -2\log Z_{S^3}^{\mathrm{CFT}}}$

de Sitter entropy: microscopics

- AdS/CFT $EAdS_4 \times S^7$:

$$S_{dS} = -I_{EdS} = 2I^{\text{reg}}_{\text{EAdS}} = -2\log Z^{\text{ABJM}}_{S^3}$$

- Leading term matches !

- **Q:** What about quantum corrections?

de Sitter entropy: microscopics

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- **Q:** What about quantum corrections?

$$\mathcal{S}_{\mathrm{dS}} = \frac{2\pi\sqrt{2k}}{3}\,N^{3/2} - \frac{\pi(k^2+8)}{12\sqrt{2k}}\,N^{1/2} + \frac{1}{2}\log N + \mathcal{O}(N^0)$$

Quid bulk?

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Quid bulk?

- Leading correction ←→ higher-derivative terms. Match !
- Log correction ←→ one-loop det. in 11d Eucl SUGRA on $-S^4 \times S^7/\mathbb{Z}_k$

  [Bhattacharyya, Grassi, Marino, Sen '12]

`11d quantm supergravity’

[Bhattacharyya, Grassi, Marino, Sen ‘12]

- Consider 11d Euclidean SUGRA on $-S^4 \times S^7/\mathbb{Z}_k$
- One-loop determinants generate log corrections to the free energy
- Odd dimensions: only zero modes contribute
- Massless 11d fields: metric, gravitino and three-form
- Ghosts are important!
- Metric and gravitino have no zero mode because S^4 is compact.
- Logarithmic correction due to a p-form:

$$\Delta F = \sum_j (-1)^j \left(\beta_{p-j} - j - 1\right) n^0_{\Delta_{p-j}} \log L/l_P, \qquad \beta_k = \frac{D-2k}{2}$$

- → $\Delta S_{dS} = 3 \log L/l_P$ $\qquad \mathcal{S}_{\mathrm{dS}} \overset{\mathrm{V}}{=} -2\log Z^{\mathrm{ABJM}}_{S^3}$

So.. what does S_{dS} count?

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- Not quite count microstates, since there is no time, no Hamiltonian.
- In effect, exp (S_{dS}) not an integer for low (N,k)
- But, being given by a (QFT) path integral, the microscopic entropy does represent some sort of measure of degrees of freedom.

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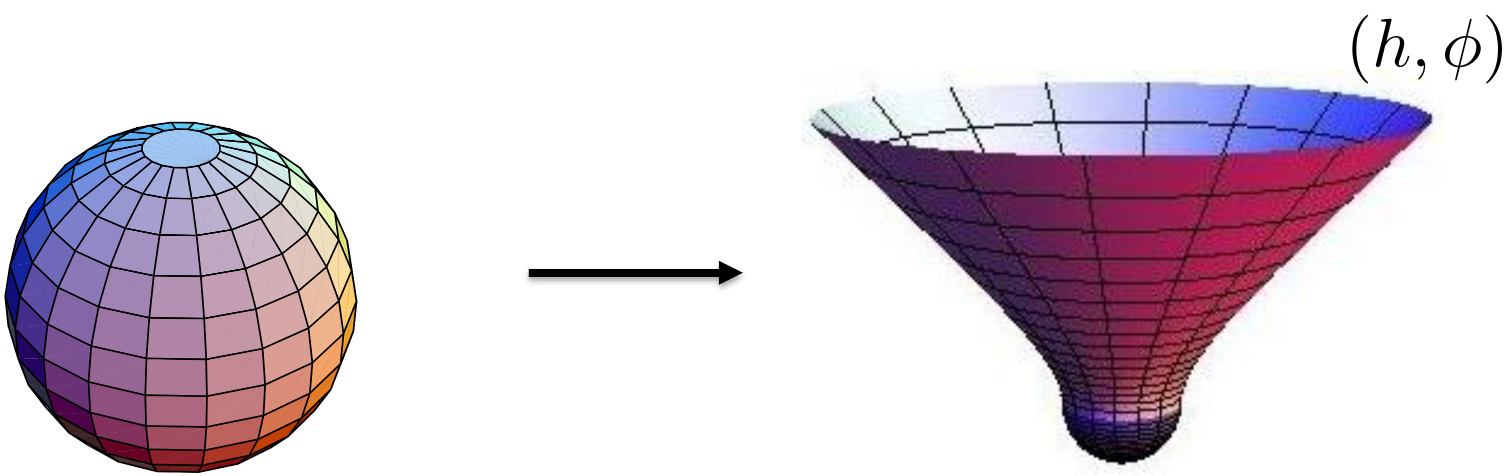
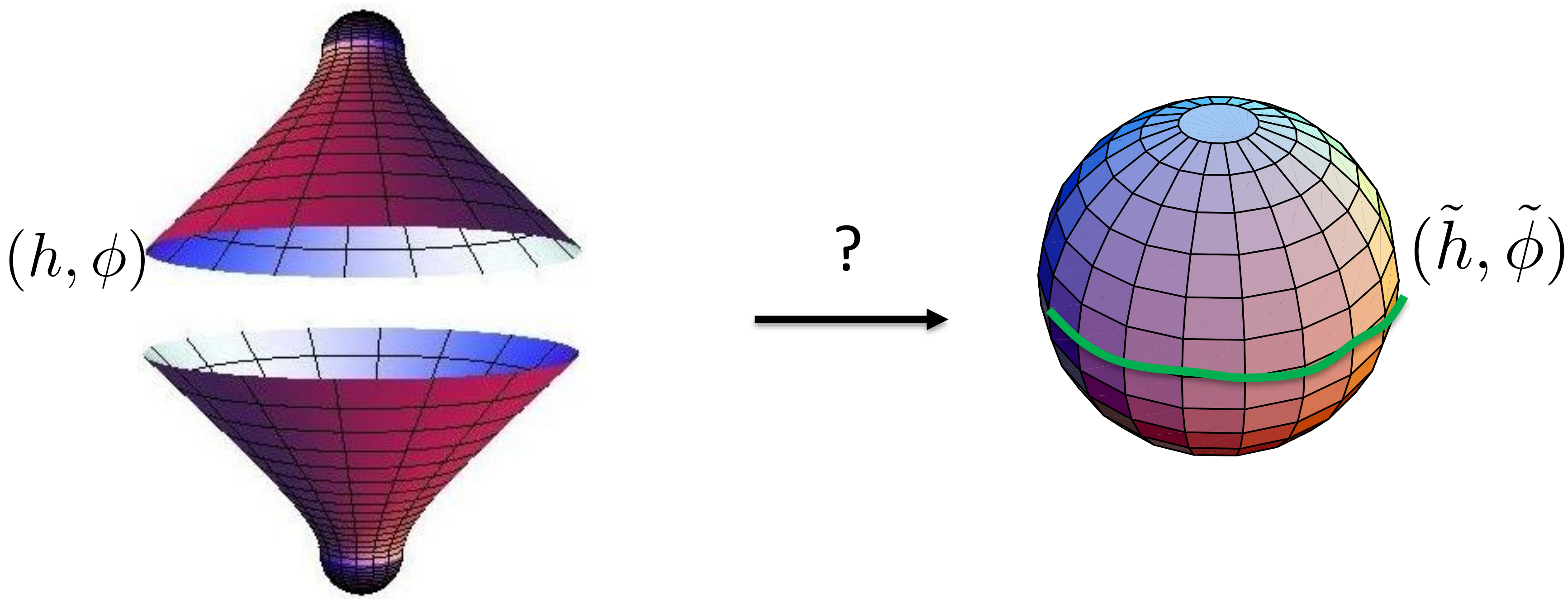


Quantum cosmology on the (deformed) sphere?

Observables ~ $\int \Psi^* \mathcal{O} \Psi$



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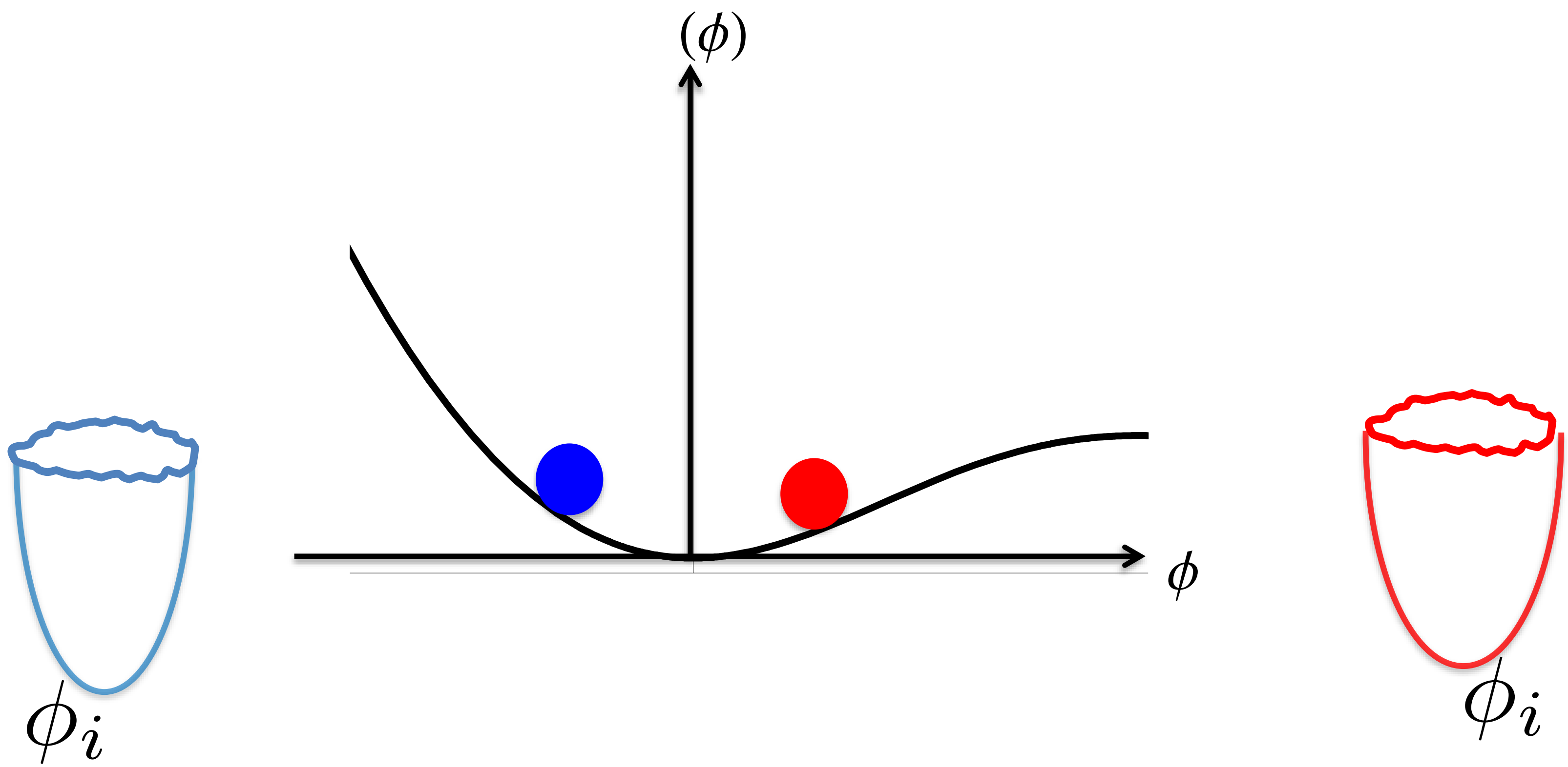
   hep-th, 1009.2525 [w/ Hartle, Hawking]

No-boundary prior

$$|\Psi_{HH}|^2 \sim e^{1/V(|\phi_i|)} \sim e^{3\pi/m^2 N(|\phi_i|)}$$



→ minimum amount of inflation

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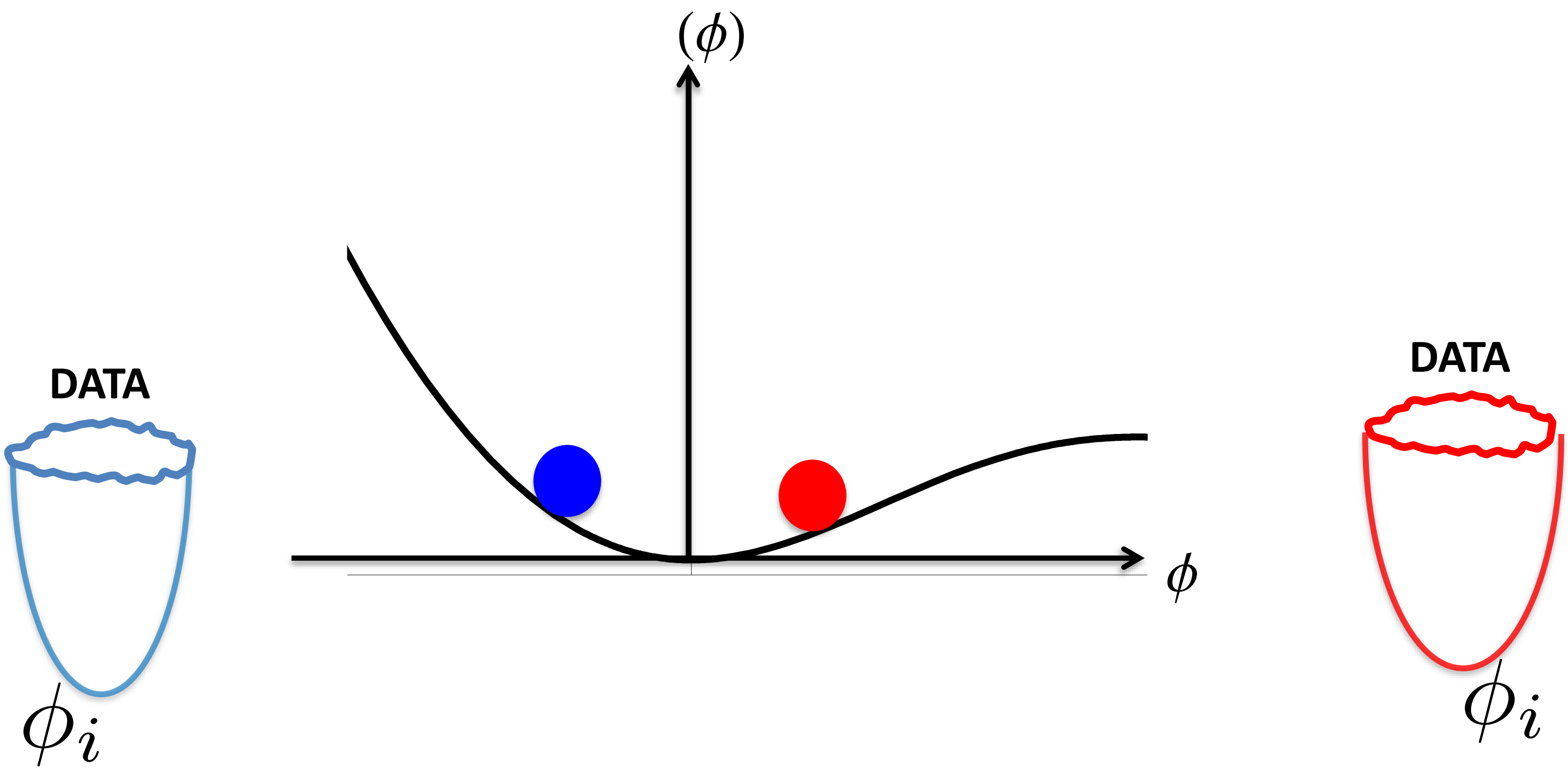

→ minimum amount of inflation

DATA

Conditional probabilities

Consider P(N|D) for given boundary data D, treated quantum mechanically, i.e. data with a probability p_H (D) to occur in any H-volume Where p_H is determined by the wave function itself (for fluctuations).

$$P(N|D^{\geq 1}) \sim \left(1-[1-p_H(D)]^{N_H(N)}\right)\exp[3\pi/m^2N]$$

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$$P(N|D^{\geq 1}) \sim \left(1 - [1 - p_H(D)]^{N_H(N)}\right) \exp[3\pi/m^2 N]$$

- Wave function at face value: no data D → low amount of inflation

$$P(N) = |\Psi|^2 \sim e^{3\pi/m^2 N}$$

$$N_H \geq 1/p_H$$

- Conditioned on few data D: $p_H \ll 1/N_H$ → low amount inflation

DATA

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$$P(N|D^{\geq 1}) \sim \left(1 - [1 - p_H(D)]^{N_H(N)}\right) \exp[3\pi/m^2 N]$$

Conditioned on lots of data D: $p_H \ll 1/N_H$

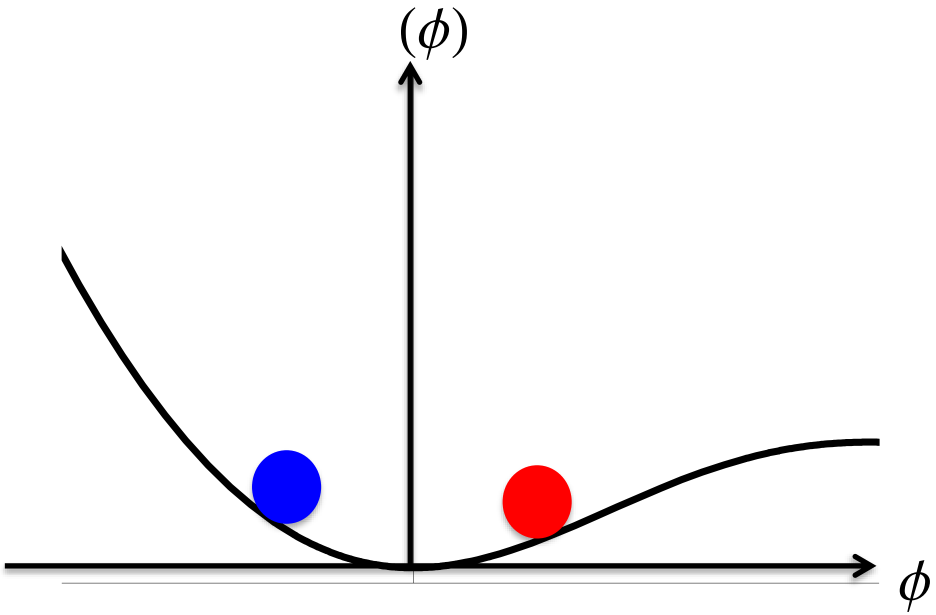
$$P(N|D^{\geq 1}) \sim p_H(D) N_H(N) \exp[3\pi/m^2 N]$$

This distribution exhibits a Page-like transition at the threshold of eternal inflation, where a new saddle becomes dominant.

-> large amount of inflation

No-boundary prior

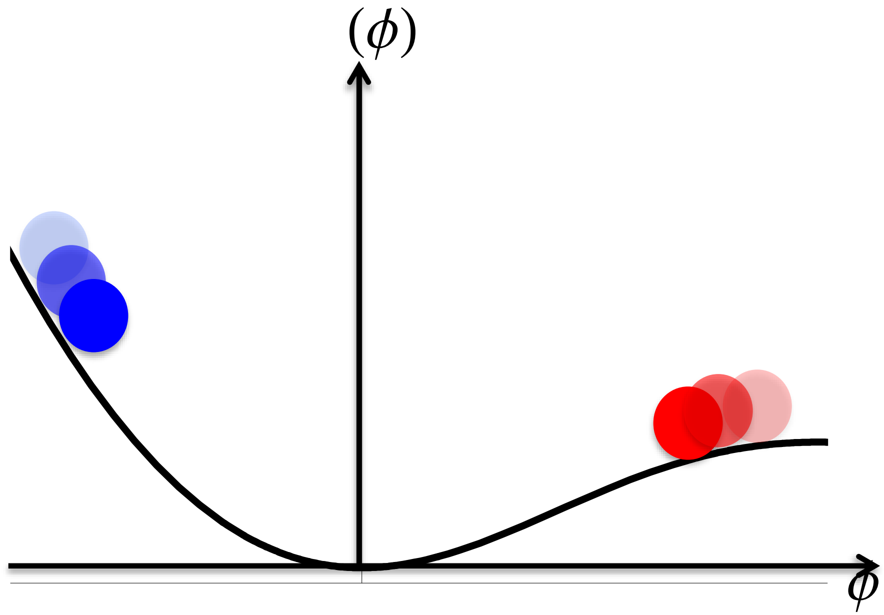
$$P(N) \sim e^{3\pi/m^2 N(|\phi_i|)}$$



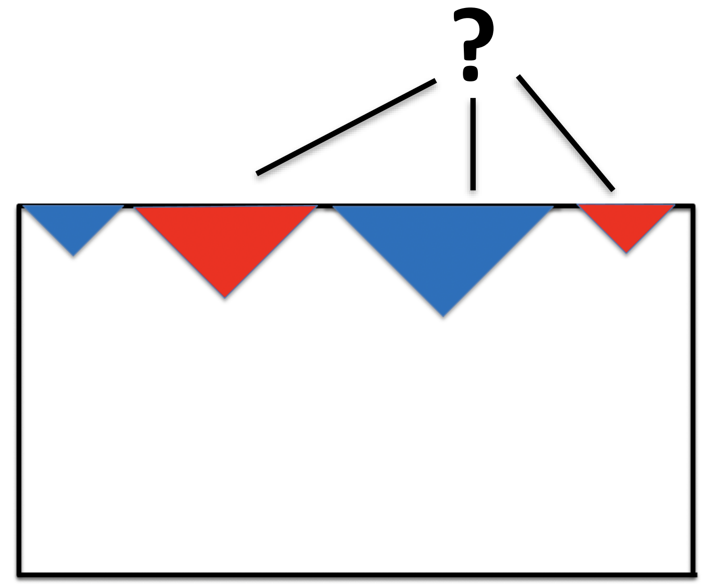
→ minimum amount of inflation

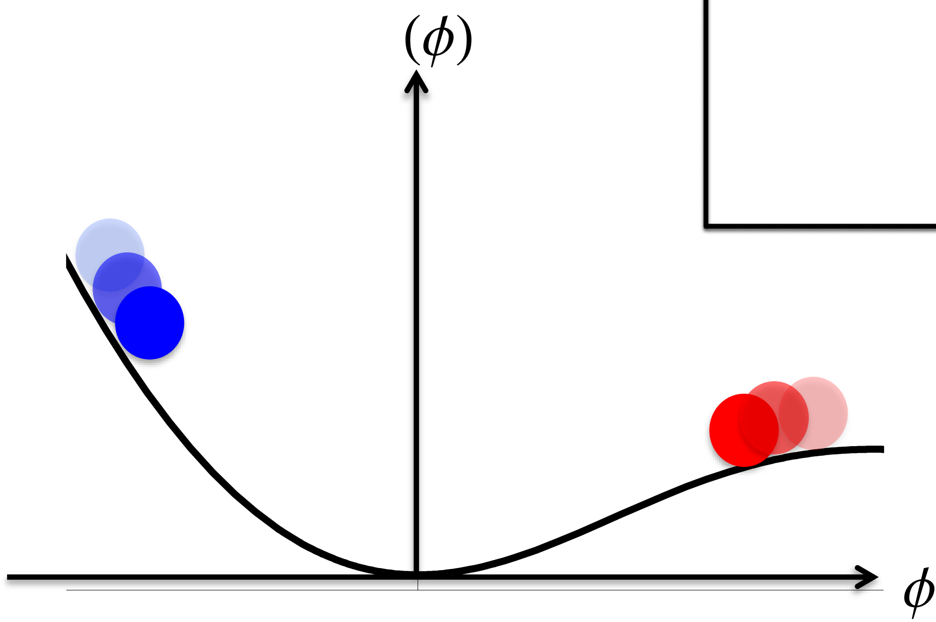
No-boundary prior 2.0

$$P(N|D^{\geq 1})$$



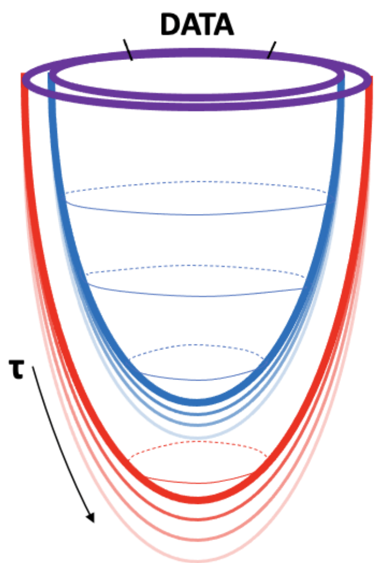
Validity saddle pt approximation?

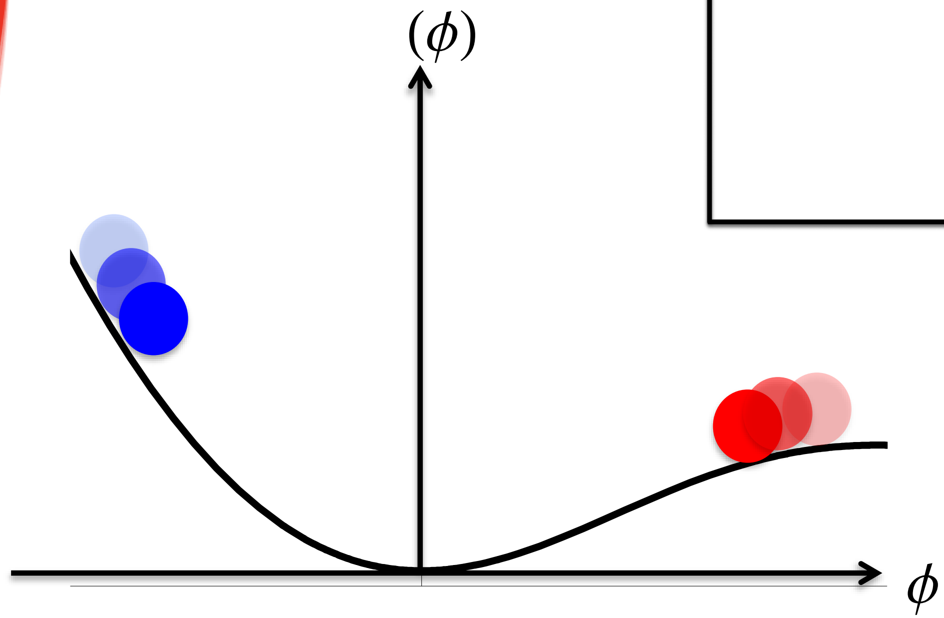


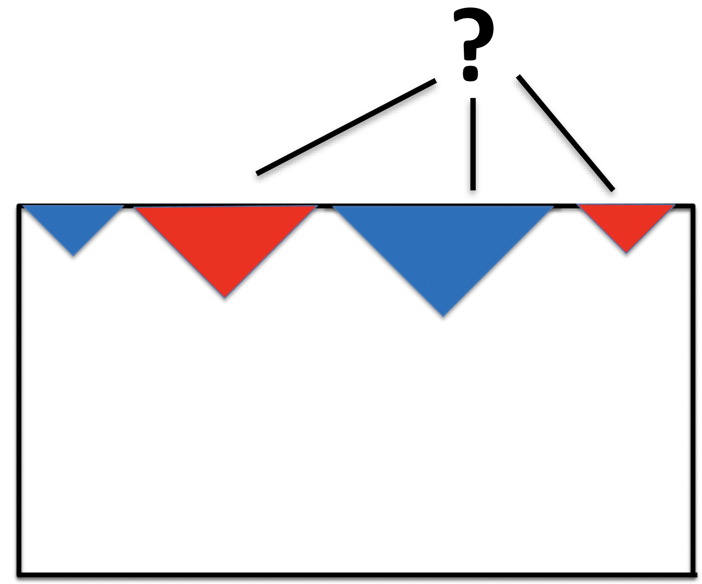


[Hartle, Hawking, TH, 1009.2525]

Validity saddle pt approximation?







[Hartle, Hawking, TH, 1009.2525]

Towards a microscopic formulation of the no-boundary wave function

1. Allowability -> Saddle selection

   hep-th, 2305.15440 [w/ Joel Karlsson, Oliver Janssen]

2. Holography -> microscopic dS entropy

   hep-th, 2211.05907 [w/ N. Bobev, Junho Hong, Joel karlsson, Valentine Reys]

3. Observer -> Page-like transition

   hep-th, 1009.2525 [w/ Hartle, Hawking]