

# Quasi-it from quasi-qubit, entropyhedra and the QGT loopstrap

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[based on past work with Dio Anninos, Albert Law and Zimo Sun,  
and ongoing work with Bhavya Bandaru, Klaas Parmentier and Zimo Sun]

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- **part I: sketchy overview (slides)**
- **part II: elaborations (blackboard)**

## Outline

### Macroscopic data

dS static patch = semiclassical cosmological (quasi-)equilibrium state  
Quantum corrections = macroscopic data

### Constraining microscopic models

Spiritual analog: heat capacity of  $H_2$  and the structure of QM  
Ruling out microscopic models: cartoon example

### Program: to-do list and results

To-do list  
General exact solution 1-loop problem  
Interactions  
Loop integrals on the sphere  
Killing microscopic models

### Finding Microscopic Models

Quasi-it from large- $N$  emergent quasi-qubits

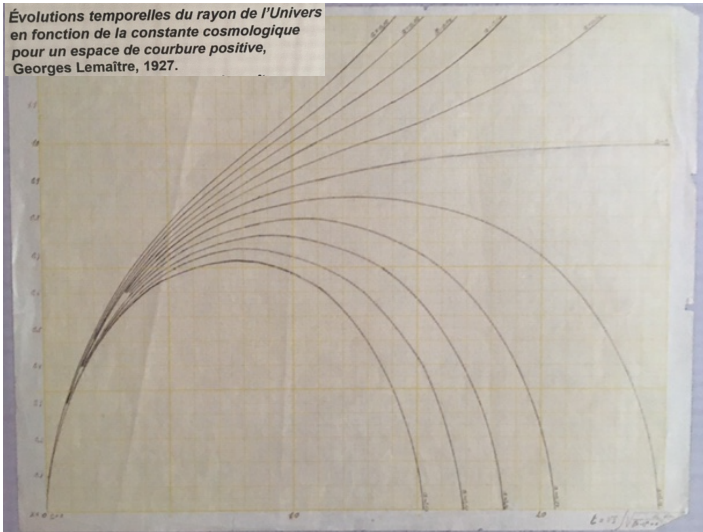
### Part II: elaborations

**Macroscopic data**

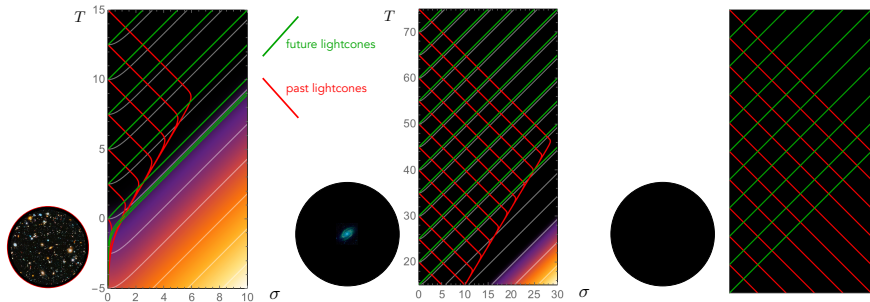
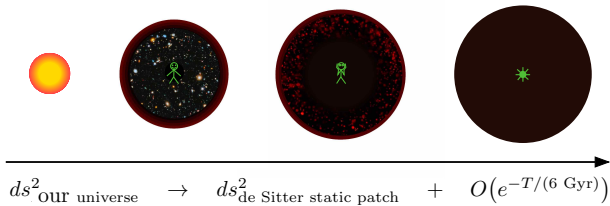


## (Non-)Universal Evolutions

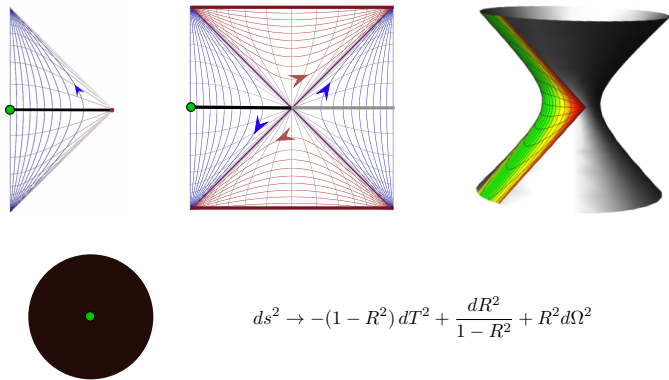
Évolutions temporelles du rayon de l'Univers en fonction de la constante cosmologique pour un espace de courbure positive, Georges Lemaître, 1927.



## Evolution of Ours



## Macroscopic quasi-equilibrium state = dS static patch geometry



$$ds^2 \rightarrow -(1 - R^2) dT^2 + \frac{dR^2}{1 - R^2} + R^2 d\Omega^2$$

[Gibbons-Hawking] Equilibrium = maximal entropy state:

$$S_{\text{EFT}} = \log Z_{\text{sphere}} = \frac{A}{4G} + \text{fine-structured loop corrections independent of UV completion}$$

## Microscopic origin?

$$S_{\text{EFT}} = \int Dg D\Phi \dots e^{-S_E[g, \Phi, \dots]} = \begin{cases} \log \dim \mathcal{H}_{\text{micro}} ? \\ -\text{Tr} \rho \log \rho ? \\ \text{something else ?} \\ \text{meaningless ?} \end{cases}$$

Challenges for mico-macro matching game: all symmetries are gauged, no charges, no S-matrix, no boundary, no anchor, no symmetry-based framework-independent asymptotic observables, tree-level  $S_{\text{EFT}} = \text{UV-sensitive renormalized coupling}$ ...

$\rightsquigarrow$  Without a priori assumptions about UV completion or additional structure: no framework-independent, gauge-invariant, field-redefinition invariant, macro-micro “matchables”?

There are! But they are necessarily *quantum*.

## Pure 3D gravity example

Tree level entropy = on-shell action of sphere saddle  $\Lambda > 0$  Euclidean gravity:

$$\mathcal{S}_0 \equiv \mathcal{S}_{\text{EFT}}^{(0)} = \frac{A}{4G}$$

$\mathcal{S}_0$  = UV-sensitive renormalized coupling constant = EFT *input* parameter.

In contrast to flat/AdS, **no UV-independent information content.**

In contrast to this: **loop-corrected entropy**, e.g. for pure 3D gravity

$$\mathcal{S} = \mathcal{S}_{\text{EFT}}^{(\text{all-loop})} = \mathcal{S}_0 - 3 \log \mathcal{S}_0 + 5 \log(2\pi) + \sum_{n \geq 1} \frac{1}{n} \frac{B_{2n}}{(2n)!} \left( \frac{4\pi^2}{\mathcal{S}_0} \right)^{2n}$$

$B_{2n}$  = Bernoulli numbers:  $B_2 = \frac{1}{6}$ ,  $B_4 = -\frac{1}{180}$ ,  $B_6 = \frac{1}{2835}$ , ...

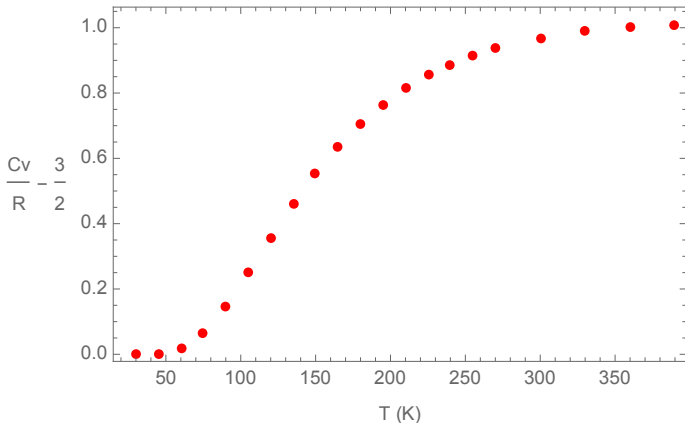
Only **even** powers of  $\mathcal{S}_0^{-1} \propto 1/\ell$  cannot be absorbed into local counterterms;  
UV-insensitive, unambiguous, **infinite macroscopic data set.**

## **Constraining microscopic models**

## Spiritual analog: heat capacity of H<sub>2</sub> and the structure of QM

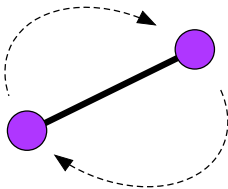
### Macroscopic data:

$T$	$c_V/R$
30.	1.50124
45.	1.50268
60.	1.51856
75.	1.56691
90.	1.64857
105.	1.7502
120.	1.85749
135.	1.9602
150.	2.05317
165.	2.13471
180.	2.20447
195.	2.26353
210.	2.31296
225.	2.35397
240.	2.38765
255.	2.41519
270.	2.43756
300.	2.46979
330.	2.48988
360.	2.50191
390.	2.50888



## Microscopic Model:

Two protons on a stick = rigid rotor:



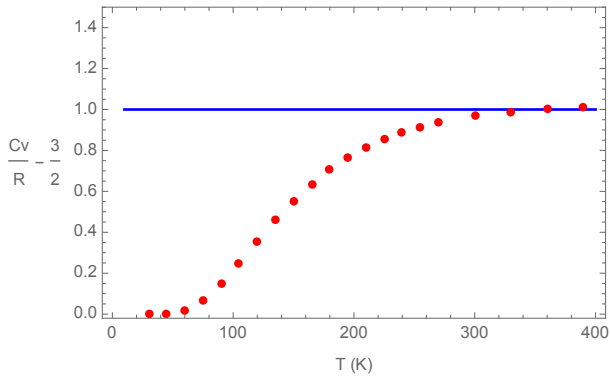
$$Z_{\text{rot}}(\beta) = \sum e^{-\beta L^2 / 2I}$$

$$c_V(\text{rot}) = \beta^2 \partial_\beta^2 \log Z_{\text{rot}}$$



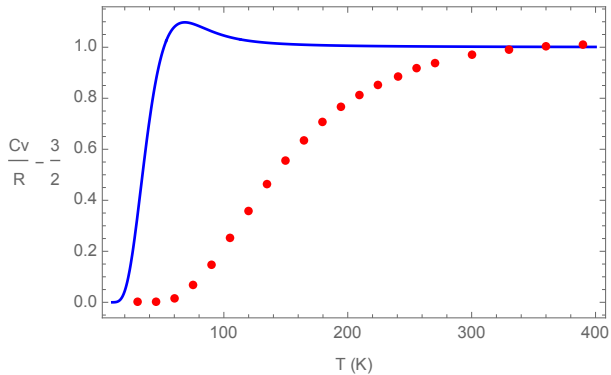
# Classical

$$Z_{\text{rot}} \propto \int_L e^{-\beta L^2/2I} \propto \frac{1}{\beta}$$



# Quantum v1

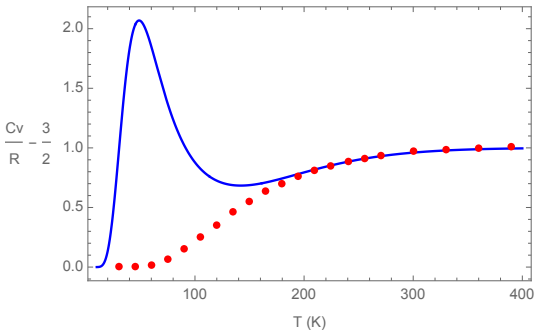
$$Z_{\text{rot}} = \sum_j (2j + 1) e^{-\beta j(j+1)/2I}$$



## Quantum 2.0: Fermi-Dirac statistics

$$Z(\text{spin singlet}) = \sum_{\text{even } j} (2j + 1) e^{-\beta j(j+1)/2I}, \quad Z(\text{spin triplet}) = \sum_{\text{odd } j} 3(2j + 1) e^{-\beta j(j+1)/2I}$$

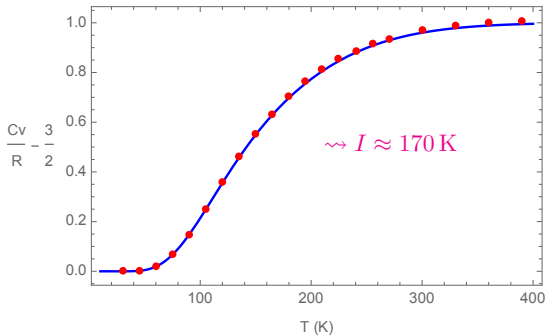
$$Z_{\text{rot}} = Z(\text{singlet}) + Z(\text{triplet})$$



## Quantum 2.0 + metastable equilibrium

Upon cooling from room  $T$ , time scale  $<$  weeks:

$$\log Z_{\text{rot}} = \frac{1}{4} \log Z(\text{singlet}) + \frac{3}{4} \log Z(\text{singlet})$$



## Spiritual lesson

$S_{H_2}(T)$  = unsung hero unraveling fundamental structure of QM

[Clayton A. Gearhart, "Astonishing Successes" and "Bitter Disappointment": The Specific Heat of Hydrogen in Quantum Theory, *Archive for History of Exact Sciences*, Vol. 64, No. 2, pp. 113-202]

Microscopic model (two protons on a stick) is grotesque oversimplification, yet getting precise match is tantamount to complete understanding of every aspect of the fundamental framework of quantum mechanics!

## Ruling out microscopic models: cartoon example

Recall pure 3D gravity macroscopic entropy:

$$\mathcal{S} = \mathcal{S}_0 - 3 \log \mathcal{S}_0 + 5 \log(2\pi) + \frac{4\pi^4}{3} \mathcal{S}_0^{-2} - \frac{8\pi^8}{45} \mathcal{S}_0^{-4} + \frac{128\pi^{12}}{2835} \mathcal{S}_0^{-6} + \dots \quad (*)$$

Say someone claims  $\mathcal{S} = \mathcal{S}_{\text{mic}} =$  entropy  $2N$ -spin 1D Ising model at  $E = 0$ :

$$\mathcal{S}_{\text{mic}} = \log \binom{2N}{N} = N\alpha - \frac{1}{2} \log N - \frac{1}{2} \log \pi - \frac{1}{8} N^{-1} + \frac{1}{192} N^{-3} - \frac{1}{640} N^{-5} + \dots$$

where  $\alpha \equiv \log 4$ . Exists *unique* identification  $\mathcal{S}_0 = N\alpha + \sum_k c_k N^{-k}$ ,

$$\mathcal{S}_0 = N\alpha - \frac{1}{8} N^{-1} + \frac{1}{192} N^{-3} - \frac{1}{640} N^{-5} + \dots,$$

such that expansion takes form of (\*) (only *even* powers  $1/\mathcal{S}_0$ ):

$$\mathcal{S}_{\text{mic}} = \mathcal{S}_0 - \frac{1}{2} \log \mathcal{S}_0 - \frac{1}{2} \log \frac{\pi}{\alpha} - \frac{\alpha}{16} \mathcal{S}_0^{-2} + \frac{2\alpha^3 + 9\alpha^2}{768} \mathcal{S}_0^{-4} - \frac{12\alpha^5 + 25\alpha^4 + 50\alpha^3}{15360} \mathcal{S}_0^{-6} + \dots$$

Coefficients do not match  $\Rightarrow$  claim ruled out.

**Program: to-do list and results**

## To-do list

- ① Find loop-corrected macroscopic  $\mathcal{S}$  for general EFTs of quantum gravity.
- ② Find matching microscopic models matching macroscopic data.

Easier said than done, but road so far remarkably prettier than feared.



## Exact 1-loop corrected $\mathcal{S}$ for arbitrary EFTs of gravity + anything

Particle spectrum  $\rightarrow$  bulk and edge **quasinormal mode** characters:

$$\chi = \text{Tr}_{\text{bulk QNM}} e^{-iHt} - \text{Tr}_{\text{edge QNM}} e^{-iHt}$$

$$\mathcal{S}^{(1)} = \log \prod_{a=0}^K \frac{(2\pi\gamma_a)^{\dim G_a}}{\text{vol } G_a} + \int_0^\infty \frac{dt}{2t} \left( \frac{1+q}{1-q} \chi_{\text{tot}}^{\text{bos}} - \frac{2\sqrt{q}}{1-q} \chi_{\text{tot}}^{\text{fer}} \right) + \mathcal{S}_{\text{ct}} \quad q \equiv e^{-t/\ell}.$$

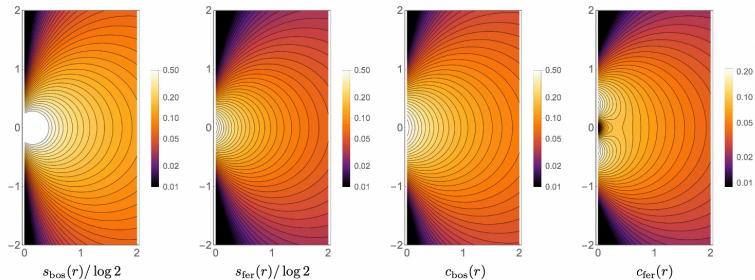
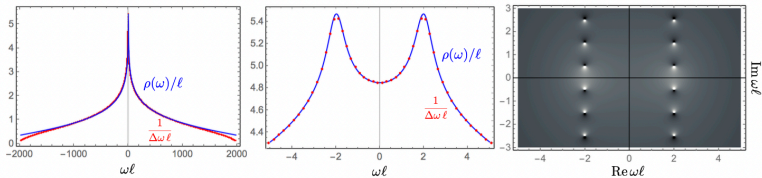
Gauge coupling constants (gravitational, Yang-Mills, HS) in log corrections:

$$\gamma_0 \equiv \sqrt{\frac{8\pi G_N}{A_{d-1}}} = \sqrt{\frac{2\pi}{\mathcal{S}^{(0)}}}, \quad \gamma_i \equiv \sqrt{\frac{g_i^2}{2\pi A_{d-3}}},$$

Examples explicit results:

content	$\mathcal{S}^{(1)}$
3D grav	$-3 \log \mathcal{S}^{(0)} + 5 \log(2\pi)$
3D $(s, m)$	$\frac{\pi}{3} (\nu^3 - (m\ell)^3) + \frac{3(s-1)^2}{2} m\ell - 2 \sum_{k=0}^2 \frac{\nu^k}{k!} \frac{\text{Li}_{3-k}(e^{-2\pi\nu})}{(2\pi)^{2-k}} - s^2 (\pi(m\ell - \nu) - \log(1 - e^{-2\pi\nu}))$
4D grav	$-5 \log \mathcal{S}^{(0)} - \frac{571}{45} \log(\ell/L) - \log \frac{8\pi}{3} + \frac{715}{48} - \frac{47}{3} \zeta'(-1) + \frac{2}{3} \zeta'(-3)$
5D su(4) ym	$-\frac{15}{2} \log(\ell/g^2) - \log \frac{256\pi^9}{3} + \frac{75 \zeta(3)}{16\pi^2} + \frac{45 \zeta(5)}{16\pi^4}$
5D $(\boxplus, m)$	$-15 \log(2\pi m\ell) + \frac{5 \zeta(5)}{8\pi^4} + \frac{65 \zeta(3)}{24\pi^2} \quad (m\ell \rightarrow 0), \quad \frac{5}{12} (m\ell)^4 e^{-2\pi m\ell} \quad (m\ell \rightarrow \infty)$
11D grav	$-33 \log \mathcal{S}^{(0)} + \log\left(\frac{4!6!8!10!}{2^4} (2\pi)^{63}\right) + \frac{1998469 \zeta(3)}{50400 \pi^2} + \frac{135619 \zeta(5)}{60480 \pi^4} - \frac{34463 \zeta(7)}{3840 \pi^6} + \frac{11 \zeta(9)}{6\pi^8} - \frac{11 \zeta(11)}{256 \pi^{10}}$
3D HS <sub>n</sub>	$-(n^2 - 1) \log \mathcal{S}^{(0)} + \log\left[\frac{1}{n} \left(\frac{n(n^2-1)}{6}\right)^{n^2-1} \mathbf{G}(n+1)^2 (2\pi)^{(n-1)(2n+1)}\right]$

# Quasinormal Quasi-Qubits



# Interactions: BRST gauge-fixed gravity action in first-order formalism

D=3 example:

In[ ]:= **LIII = Collect[LagIII + LGH, ε]**

$$\begin{aligned}
 \text{Out[ ]} = & \Lambda - \frac{R[\nabla]}{2} + \epsilon \left( \Lambda e^a{}_a + e^{ab} R[\nabla]_{ab} - \frac{1}{2} e^a{}_a R[\nabla] \right) + \\
 & \epsilon^2 \left( \Lambda \left( -\frac{1}{2} e_{ab} e^{ab} + \frac{1}{2} e^a{}_a e^b{}_b \right) - \frac{1}{2} \hat{\omega}^{abc} \hat{\omega}_{bac} + \bar{c}^a (C^b R[\nabla]_{ab} - \nabla_b C^a) + \right. \\
 & \quad \left. \bar{c}_{ab} \left( C^{ab} - \frac{1}{2} (\nabla^a C^b) + \frac{1}{2} (\nabla^b C^a) \right) - e^{ab} (\nabla_c \hat{\omega}_{ab}{}^c) \right) + \\
 & \epsilon^3 \left( \Lambda \left( \frac{1}{3} e_a{}^c e^{ab} e_{bc} - \frac{1}{2} e^a{}_a e_{bc} e^{bc} + \frac{1}{6} e^a{}_a e^b{}_b e^c{}_c \right) + e^{ab} \hat{\omega}_a{}^{cd} \hat{\omega}_{cbd} - \frac{1}{2} e^a{}_a \hat{\omega}^{bcd} \hat{\omega}_{cbd} + \right. \\
 & \quad \left. \bar{c}_{ab} \left( -\frac{1}{2} C^{bc} e^a{}_c + \frac{1}{2} C^{ac} e^b{}_c - \frac{1}{2} e^b{}_c (\nabla^a C^c) + \frac{1}{2} e^a{}_c (\nabla^b C^c) \right) + \bar{c}^a (C^{bc} \hat{\omega}_{bac} + \hat{\omega}_{bac} (\nabla^c C^b)) \right)
 \end{aligned}$$

**flds = GiveIndices /@ \$fields;**

**Qflds = dummies /@ (Q /@ flds);**

**{ flds, Qflds} // Transpose // TableForm**

\$Form=

$e_a{}^b$	$\theta^{c1} (-C^b{}_{c11} + \epsilon C^b{}_{c11} e_a{}^c + \nabla_a C^b{}_{c11} + \epsilon C^{cb} (\nabla_a C_{c11}) + \epsilon C^c{}_{c11} (\nabla_c e_a{}^b))$
$\omega_a{}^{bc}$	$\theta^{c1} (\epsilon C^{cd}{}_{c11} \omega_a{}^b{}_d - \epsilon C^{bd}{}_{c11} \omega_a{}^c{}_d + \epsilon \omega_d{}^{bc} (\nabla_a C^d{}_{c11}) - \nabla_a C^{bc}{}_{c11} + C^d{}_{c11} (-R[\nabla]_{ad}{}^{bc} + \epsilon (\nabla_d \omega_a{}^{bc})))$
$C^a$	$\epsilon C^b{}_{c11} \theta^{c1c2} (\nabla_b C^a{}_{c2})$
$C^{ab}$	$-\frac{1}{2} \epsilon \theta^{c1c2} (2 C^{ac}{}_{c11} C^b{}_{c2} - C^c{}_{c11} (C^d{}_{c2} R[\nabla]^{ab}{}_{cd} + 2 (\nabla_c C^{ab}{}_{c2})))$
$\bar{c}^a$	$i B^a$
$\bar{c}^{ab}$	$i B^{ab}$
$B^a$	0
$B^{ab}$	0
$\eta$	0
$\zeta$	0

## Loop integrals on the sphere: GKZ form

Universal GKZ Euler integral formula for arbitrary diagrams on  $S^{d+1}$

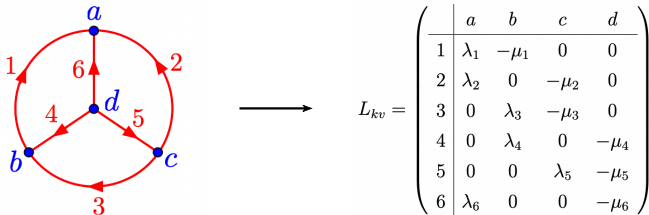
For example for diagram with  $V$  vertices and  $k$  lines  $k = 1, \dots, K$  propagating scalar fields of mass  $m_k^2 = \Delta_k(d - \Delta_k)$ :

$$\text{diagram} \propto \int_{\lambda, \mu} \prod_k \lambda_k^{\Delta_k} \mu_k^{(d-\Delta_k)} \mathcal{P}^{-(d+2)/2}$$

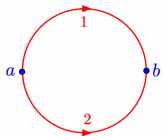
where  $\mathcal{P}$  = polynomial in  $\lambda, \mu$  given by

$$\mathcal{P} = \det(1 + L^T L)$$

with  $L$  a  $K \times V$  matrix fixed by diagram topology, e.g.



## Simple explicit example

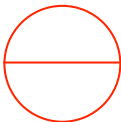


$$\rightarrow L = \begin{pmatrix} -\mu_1 & \lambda_1 \\ -\mu_2 & \lambda_2 \end{pmatrix} \rightarrow \mathcal{P} = 1 + \lambda_1^2 + \lambda_2^2 + \mu_1^2 + \mu_2^2 + (\lambda_1\mu_2 - \mu_1\lambda_2)^2$$

Then for  $m_k^2 = \Delta_k^+ \Delta_k^-$ ,  $\Delta_k^\pm = \frac{d}{2} \pm i\nu_k$ ,

$$\begin{aligned} \mathbb{I} &= \frac{1}{\Gamma(1 + i\frac{\nu_1 + \nu_2}{2}) \Gamma(1 - i\frac{\nu_1 + \nu_2}{2})} \int \frac{d\lambda_1}{\lambda_1} \frac{d\mu_1}{\mu_1} \frac{d\lambda_2}{\lambda_2} \frac{d\mu_2}{\mu_2} \frac{\lambda_1^{\frac{d}{2} + i\nu_1} \mu_1^{\frac{d}{2} - i\nu_1} \lambda_2^{\frac{d}{2} + i\nu_2} \mu_2^{\frac{d}{2} - i\nu_2}}{\left(1 + \lambda_1^2 + \lambda_2^2 + \mu_1^2 + \mu_2^2 + (\lambda_1\mu_2 - \mu_1\lambda_2)^2\right)^{\frac{d}{2} + 1}} \\ &= -\frac{1}{d} \frac{1}{\nu_1^2 - \nu_2^2} \int_0^1 \frac{dq}{q} \frac{q^{\frac{d}{2} + i\nu_1} + q^{\frac{d}{2} - i\nu_1} - q^{\frac{d}{2} + i\nu_2} - q^{\frac{d}{2} - i\nu_2}}{(1 - q)^d} \\ &= -\frac{\Gamma(1 - d)}{d(\nu_1^2 - \nu_2^2)} \left( \frac{\Gamma(\frac{d}{2} + i\nu_1)}{\Gamma(1 - \frac{d}{2} + i\nu_1)} + \frac{\Gamma(\frac{d}{2} - i\nu_1)}{\Gamma(1 - \frac{d}{2} - i\nu_1)} - \frac{\Gamma(\frac{d}{2} + i\nu_2)}{\Gamma(1 - \frac{d}{2} + i\nu_2)} - \frac{\Gamma(\frac{d}{2} - i\nu_2)}{\Gamma(1 - \frac{d}{2} - i\nu_2)} \right) \end{aligned}$$

## Less-simple explicit examples (3D)



$$I = \frac{\text{Log}[\mu]}{16 \pi^2} + \frac{\text{Csch}[\pi v 1] \text{Csch}[\pi v 2] \text{Csch}[\pi v 3] \text{Sinh}[\pi (v 1 + v 2 - v 3)] \text{S}\psi[0, 1, -v 1 - v 2 + v 3]}{64 \pi^2} +$$

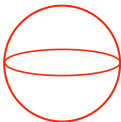
$$\frac{\text{Csch}[\pi v 1] \text{Csch}[\pi v 2] \text{Csch}[\pi v 3] \text{Sinh}[\pi (v 1 - v 2 + v 3)] \text{S}\psi[0, 1, v 1 - v 2 + v 3]}{64 \pi^2} -$$

$$\frac{\text{Csch}[\pi v 1] \text{Csch}[\pi v 2] \text{Csch}[\pi v 3] \text{Sinh}[\pi (v 1 - v 2 - v 3)] \text{S}\psi[0, 1, -v 1 + v 2 + v 3]}{64 \pi^2} -$$

$$\frac{\text{Csch}[\pi v 1] \text{Csch}[\pi v 2] \text{Csch}[\pi v 3] \text{Sinh}[\pi (v 1 + v 2 + v 3)] \text{S}\psi[0, 1, v 1 + v 2 + v 3]}{64 \pi^2}$$

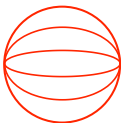
$$S\psi[0, n, v] = \frac{1}{2} \left( \psi^{(0)}\left(\frac{n+iv}{2}\right) + \psi^{(0)}\left(\frac{n-iv}{2}\right) \right)$$

$$= \frac{1}{2} \left( \text{PolyGamma}\left[0, \frac{n}{2} + \frac{iv}{2}\right] + \text{PolyGamma}\left[0, \frac{n}{2} - \frac{iv}{2}\right] \right)$$



$$\frac{v \text{Coth}[\pi v]}{64 \pi^3 \kappa} - \frac{v \text{Coth}[\pi v] (1 + \text{Log}[\mu])}{16 \pi^3} -$$

$$\frac{v \text{Coth}[\pi v] \text{Csch}[\pi v]^2 \text{S}\psi[0, 2, 2 v]}{32 \pi^3} + \frac{v \text{Csch}[\pi v]^4 \text{Sinh}[4 \pi v] \text{S}\psi[0, 2, 4 v]}{128 \pi^3}$$



$$\frac{v (-1 + v^2 + (1 + 9 v^2) \text{Cosh}[2 \pi v]) \text{Coth}[\pi v] \text{Csch}[\pi v]^2}{3072 \pi^5 \kappa} -$$

$$\frac{v (-1 + v^2 + (1 + 9 v^2) \text{Cosh}[2 \pi v]) \text{Coth}[\pi v] \text{Csch}[\pi v]^2 (11 + 6 \text{Log}[\mu])}{3072 \pi^5} +$$

$$\frac{5 v (1 + v^2) \text{Coth}[\pi v] \text{Csch}[\pi v]^4 \text{S}\psi[0, 4, 2 v]}{4096 \pi^5} -$$

$$\frac{v (1 + 4 v^2) (\text{Cosh}[\pi v] + \text{Cosh}[3 \pi v]) \text{Csch}[\pi v]^5 \text{S}\psi[0, 4, 4 v]}{1024 \pi^5} +$$

$$\frac{v (1 + 9 v^2) (\text{Cosh}[\pi v] + \text{Cosh}[3 \pi v] + \text{Cosh}[5 \pi v]) \text{Csch}[\pi v]^5 \text{S}\psi[0, 4, 6 v]}{4096 \pi^5}$$

## Killing microscopic models

A.I. view on the microscopic model graveyard:



[...]

## **Finding Microscopic Models**

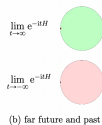
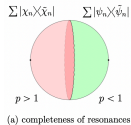
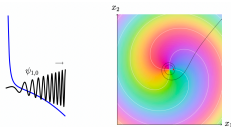
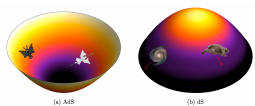


## Quasi hints

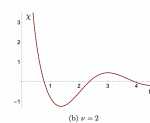
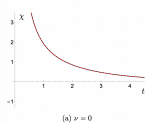
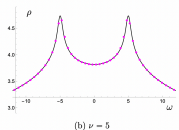
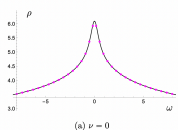
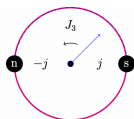
- Explicit expressions extremely complicated, but extremely simple and universal underlying structure: **quasinormal modes/resonances**
- Typical for hyperbolic systems, also in ordinary QM
- But if  $\dim \mathcal{H} < \infty$ : no QNMs/resonances...?!
- $\exists$  microscopic models with **emergent QNMs** in  $\dim \mathcal{H} \rightarrow \infty$  limit? **Yes!**

# dS<sub>2</sub> quasi-qubits from SU(2) spin qubits

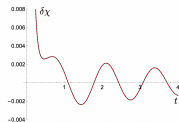
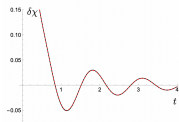
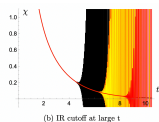
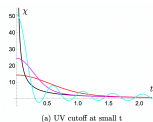
## Quasnormal Quantum Mechanics



## Quasnormal Emergence



## Quasi-quasnormality: 1/N corrections



## Part II:elaborations

continued on blackboard