de Sitter quantum gravity: advances and challenges

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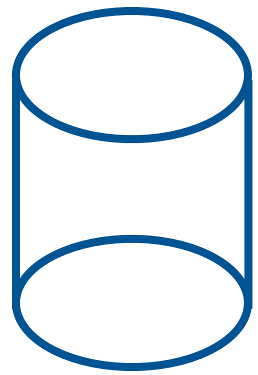
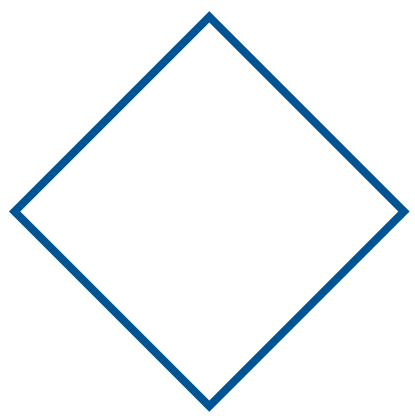
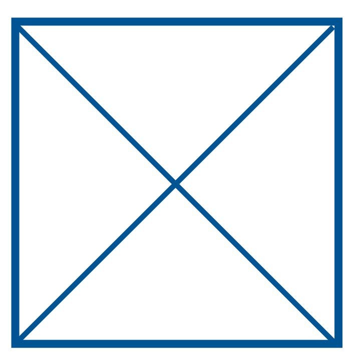
Classical and quantum gravity depend crucially on one parameter:
the cosmological constant Λ

c.c.	classical gravity		quantum gravity	
$\Lambda < 0$	AdS		AdS/CFT	
$\Lambda = 0$	Mink		celestial holography	
$\Lambda > 0$	dS		our Universe!	

$\Lambda \sim 10^{-122} M_{Planck}^2$ and inflationary period

Compared to the situation with negative c.c., dS cosmology and holography much harder and less developed!

- Difficult to obtain string theory solutions
- No asymptotic time-like boundary
- Observer-dependent horizons due to expansion
- Cosmological constant problem
- …

However: results and connections with Mathematics, QFT and GR generate new developments and opportunities

- Progress on hyperbolic geometry, well-suited for realizing de Sitter solutions.
- Powerful numerical methods based on machine learning.
- Operator algebras (type I, II, III) and gravity. Math version of "It from Qubit".
- Novel irrelevant deformations in QFT (TTbar), and relations to boundary conditions in General Relativity.

As with AdS/CFT, the understanding of dS quantum gravity comes from interplay between two directions:

- Microscopic solutions in String Theory:

Developments in SUGRA solutions, black branes, etc. paved the way for AdS/CFT. Explicit solutions allowed to extract general lessons, e.g.

$$N = 4\,\mathrm{SYM} \;\leftrightarrow\; AdS_5 \times S^5$$ [Maldacena]

Nonperturbative methods (SUSY, integrability), microstate counts, …

Can we find the analog of $AdS_5 \times S^5$ for dS gravity?

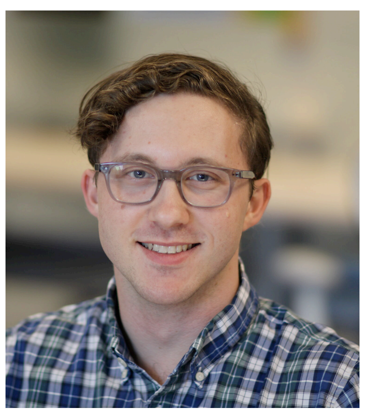
- Holographic methods:

Black hole thermodynamics [Bekenstein, Hawking, Gibbons, …];
holographic principle ['t Hooft, Susskind]
Quantum information theory and gravity [Ryu-Takayanagi, …].

Holography in finite regions?

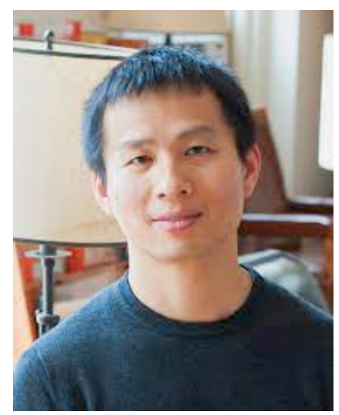
In this talk we will review efforts on both of these directions.

It is an extremely active area of research, with many different ideas and results. Very hard to review comprehensively.
Aside from some general remarks, I will focus on work with my collaborators. This is an on-going effort for more than 13 years, and I would like to thank them before starting:


E. Coleman


M. Dodelson

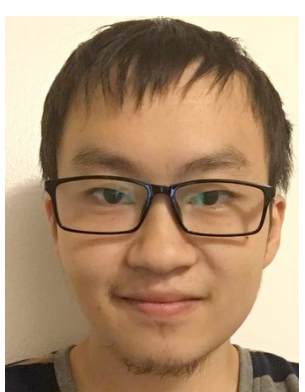

B. De Luca


Xi Dong


V. Gorbenko


B. Horn


A. Lewkowycz

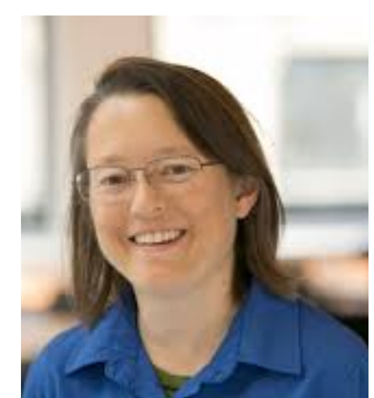

J. Liu


S. Matsuura


E. Mazenc


V. Shyam


R. Soni

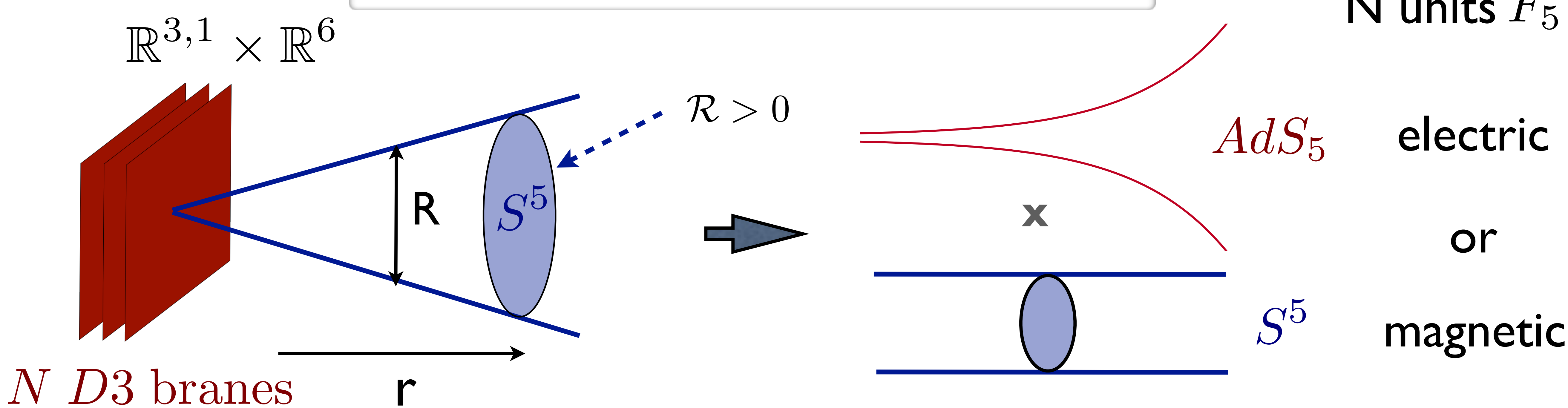

E. Silverstein


S. Yang

A. de Sitter in String Theory

The goal here is to construct explicit dS sols in String Theory. Let us first discuss the AdS case.

AdS sols and AdS/CFT



$$ds^2 = \frac{1}{\sqrt{1+\left(\frac{r_0}{r}\right)^4}}\eta_{\mu\nu}dx^\mu dx^\nu + \sqrt{1+\left(\frac{r_0}{r}\right)^4}(dr^2 + r^2 d\Omega_5^2)$$

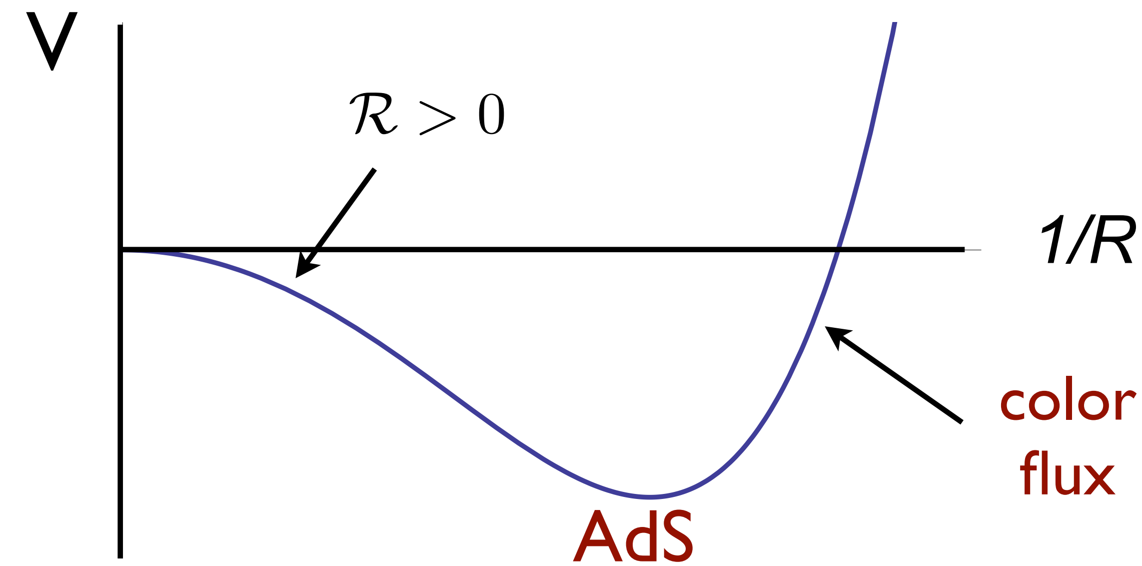
[Maldacena, '98]

$$\mathcal{N}=4\ \text{SUSY}\ \ SU(N)\ \text{YM}\ ,\ d=4 \Leftrightarrow AdS_5 \times S^5\ ,\ D=10\ \text{IIB supergravity}$$

Both sides are very special: maximal SUSY, large global symmetry, simple geometry … To extract general lessons, better to think in terms of energetics: dimensionally reduced effective potential

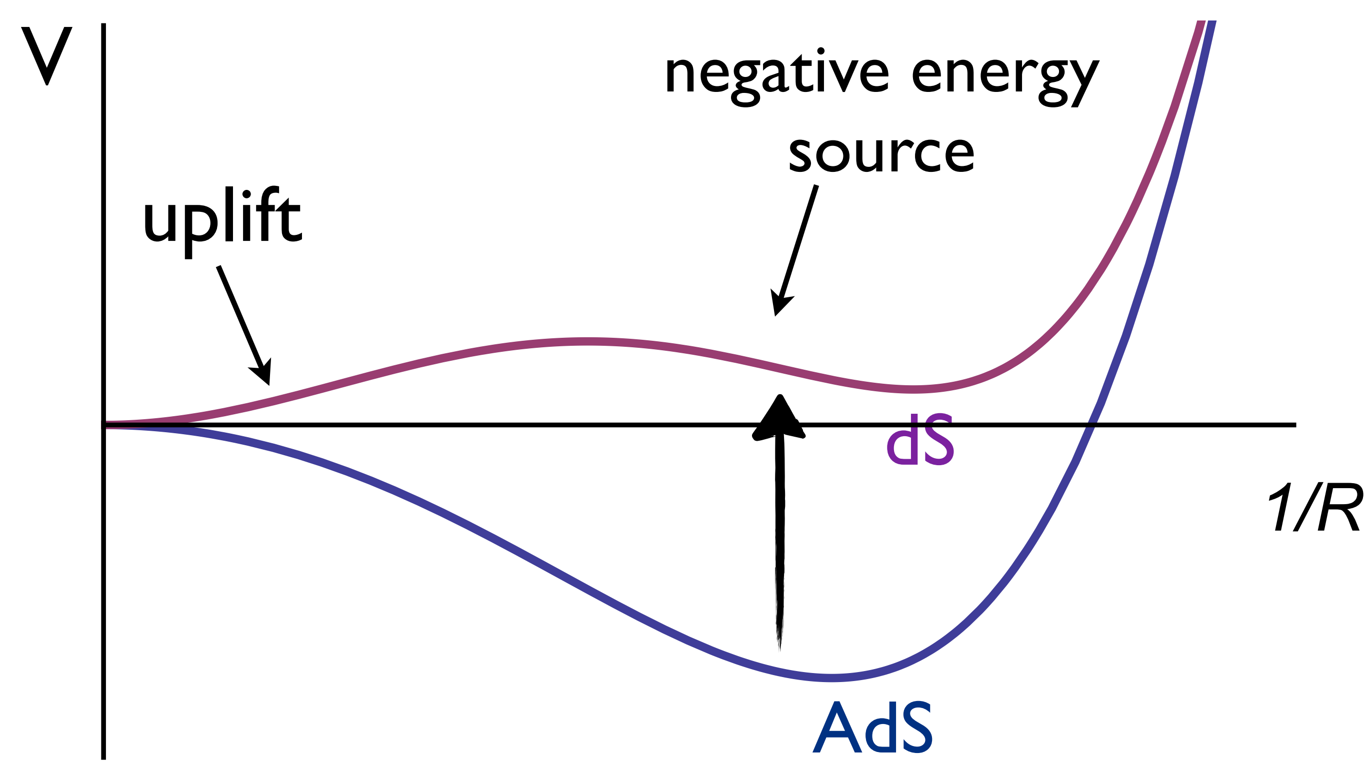
$$V_{\text{eff}} \sim \int_{B_5} \sqrt{g^{(5)}} \left(-R^{(5)} + |F_5|^2\right)$$

R: radius of internal sphere



Uplifting AdS to dS

So for dS we need at least a 3-term structure potential:



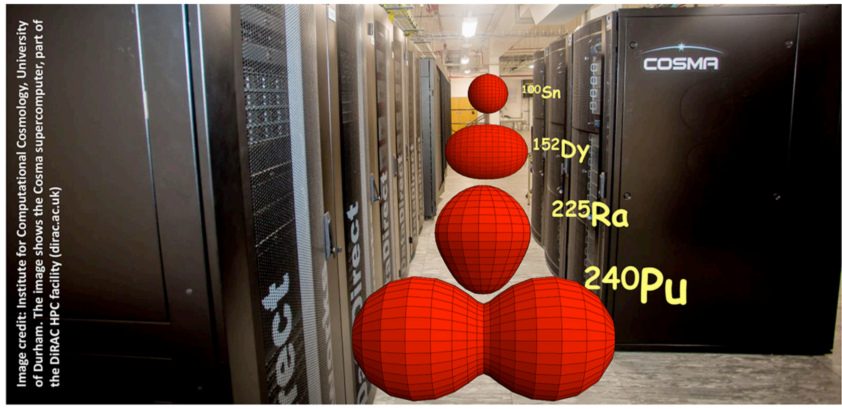
Many different mechanisms for uplift (antibranes, curvature, supercritical, U-branes, …) and negative energy contributions (orientifolds, nonpert. effects, α', Casimir, …)

Thinking in terms of energetics already reveals important lessons and useful no-go's. We see why dS and cosmological sols. are harder to find.

- no SUSY  … but perturbation theory

- limitations from metastability and negative energy sources

- absence of large global symmetries, no simple geometry. Numerics required. But building blocks fairly well understood.
Similar to other areas, such as Nuclear Physics



We will now review a specific proposal, based on [De Luca, Silverstein, GT, 2021]

Hyperbolic compactifications

Basic idea: replace internal sphere of AdS/CFT by negatively curved hyperbolic manifold. This provides the necessary uplift. Furthermore, these manifolds can have small circles, which contribute negative Casimir energy (given appropriate boundary conditions).

We consider 11d SUGRA/M-theory, with N units of F7 flux (M2 branes). With no more ingredients, it gives one of the original AdS/CFT examples: strongly coupled 3d CFT dual to $AdS_4 \times S^7$

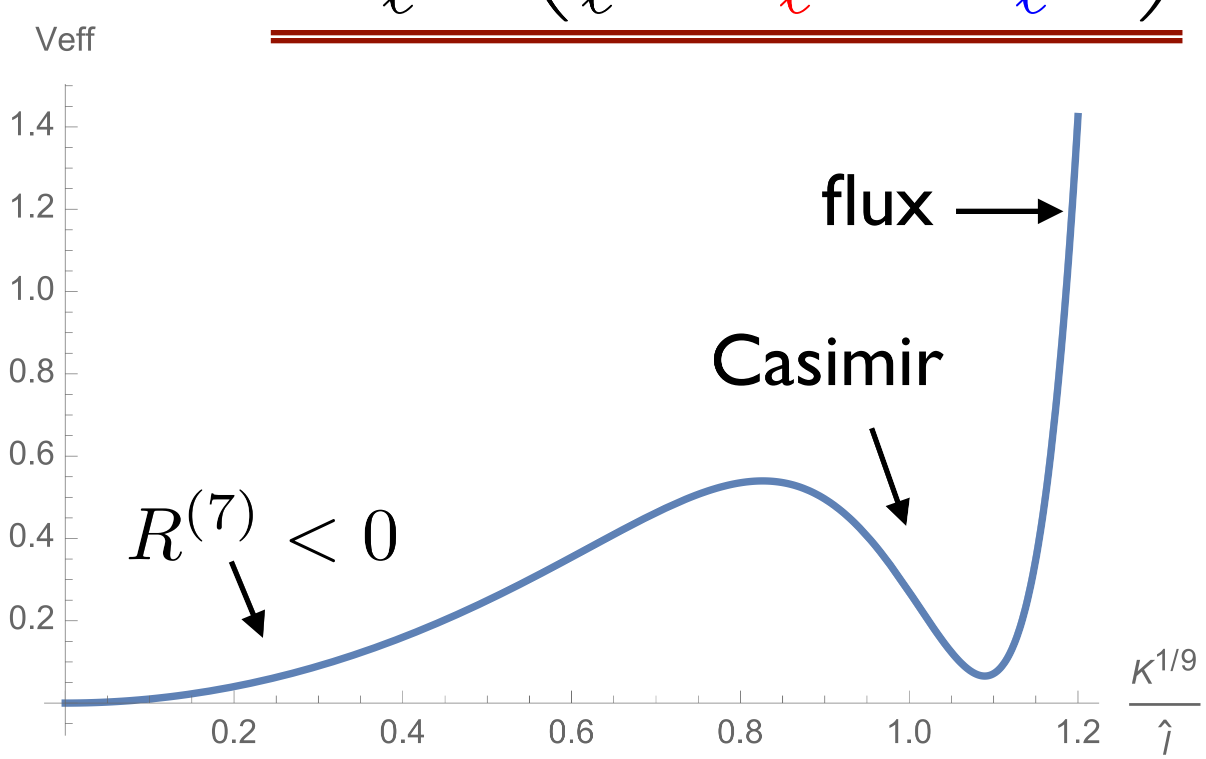
But now, compactify on 7-dim hyperbolic manifold, and negative Casimir energy from small circles (fermion antiperiodic b.c.)

$$S_{11} = \frac{1}{\ell_{11}^9} \int d^{11}x \sqrt{-g^{(11)}} \left(R^{(11)} - \frac{1}{2} |F_7|^2 \right) + S_{quantum}$$

$$\Rightarrow \ V \sim \frac{1}{\ell_{11}^9} \mathrm{Vol}_7 \left(\frac{a}{\ell^2} + \ell_{11}^{12} \frac{N^2}{\mathrm{Vol}_7^2} \right) + \int_{\mathbb{H}_7/\Gamma} d^7y \sqrt{g^{(7)}} \rho_C \longleftarrow \frac{1}{R_c^{11}}$$

$$\propto \frac{M_4^4}{\hat{\ell}^7} \left(\frac{a}{\hat{\ell}^2} - \frac{K}{\hat{\ell}^{11}} + \frac{N^2}{\hat{\ell}^{14}} \right)$$

$$\hat{\ell} \equiv \ell/\ell_{11} \qquad K \sim \left(\frac{\ell}{R_c} \right)^{11} \frac{\mathrm{Vol}_C}{\mathrm{Vol}_7}$$

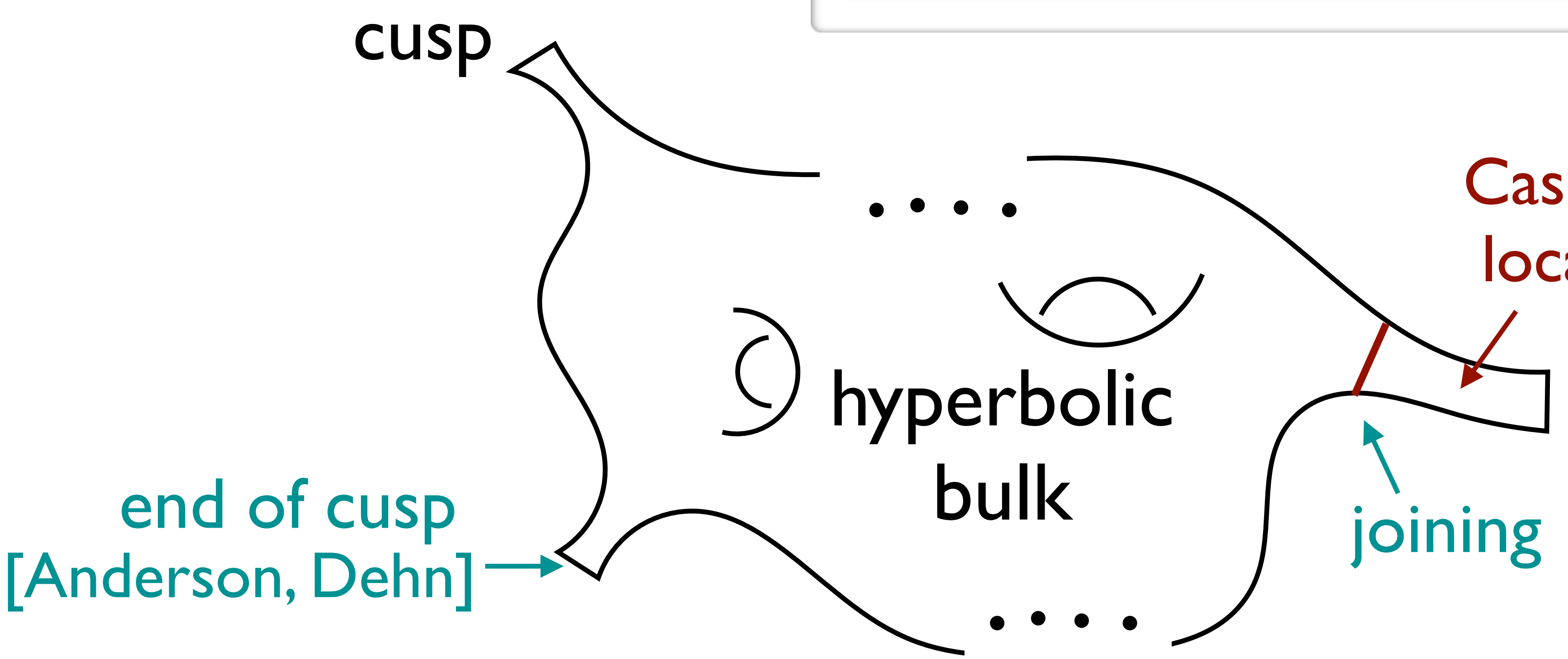


Supports a dS minimum

$$\hat{\ell} = \frac{\ell}{\ell_{11}} \sim \left(\frac{K}{a} \right)^{1/9} \gg 1$$

with $\ell_{11} \ll \ell \leq \ell_{dS}$

11d analysis



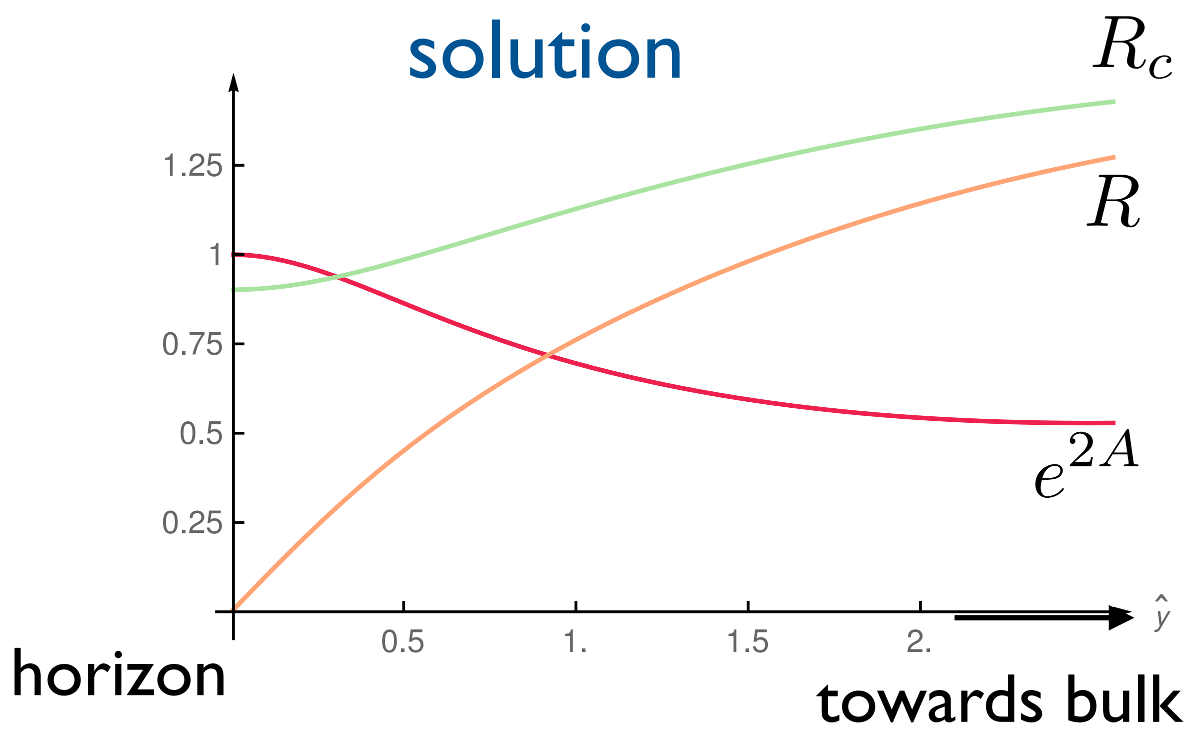
$$ds^2_{cusp} = dy^2 + e^{-2y/\ell} ds^2_{T^6}$$

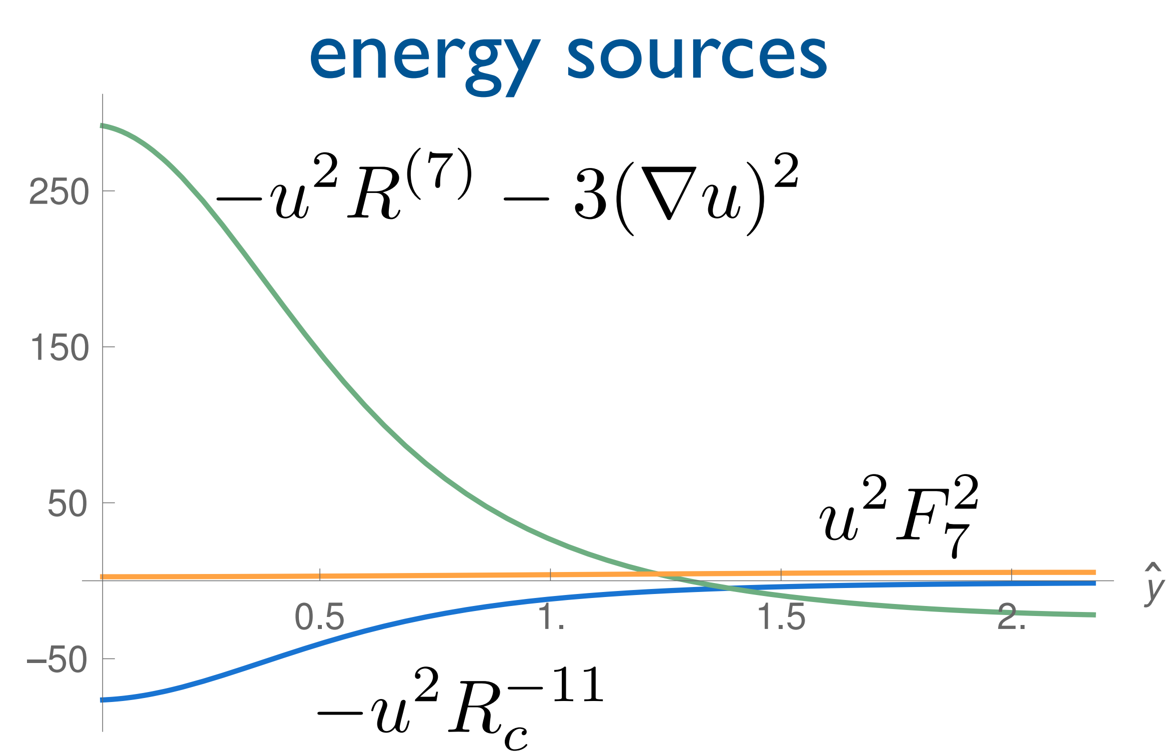
Solving the 11d equations requires turning un warp factor, volume mode, and additional internal perturbations

$$ds^2 = e^{2A(y)} ds^2_{dS_4} + e^{2B(y)} (g_{\mathbb{H}\,ij} + h_{ij}) dy^i dy^j$$

- Local analysis in cusp regions: predominant dependence on radial direction. Nonlinear ODEs.
- Then the joining introduces dependence on all internal variables (7). Nonlinear PDEs

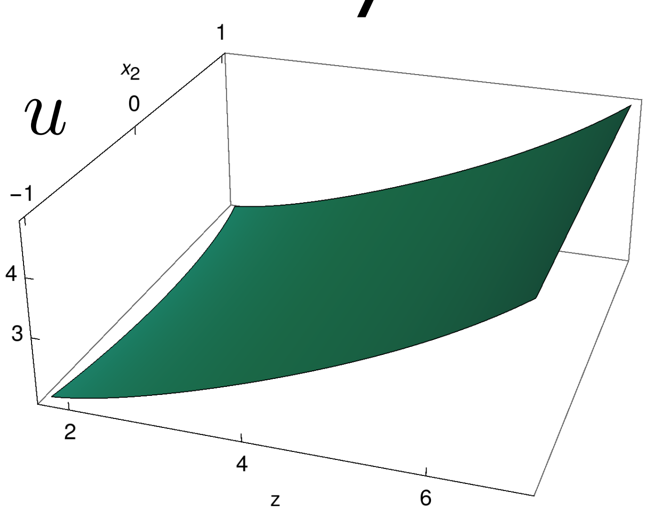
In cusp regions, we find numerical smooth solutions with dS4:

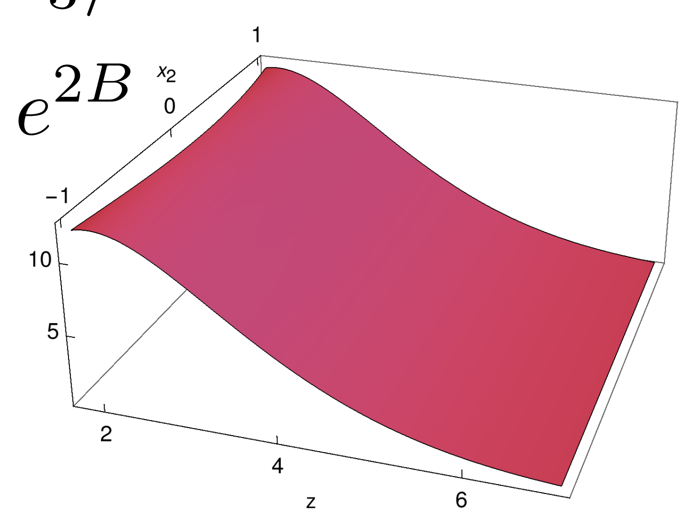


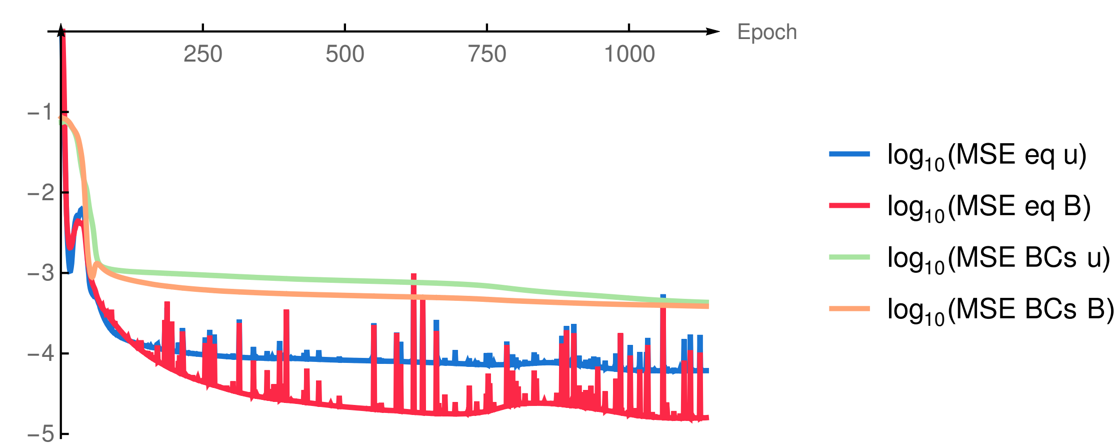


In bulk: perturbative expansion around homogeneous sol. Promising: use neural network methods to approximate PDE sol $\text{Loss} = \sum (\text{eqs})^2$

Preliminary results in $\mathbb{H}_3/\Gamma$







- Current analytic estimates and general constraints suggest a positive mass matrix

B. Holography for de Sitter

- A strong hint for holography comes from black hole thermodynamics, that is also shared by the de Sitter horizon. Entropy

[Bekenstein, Hawking, Gibbons, …]

$$S_{\text{horizon}} = \frac{\text{Area}}{4G_N}$$

holographic microstates ?!

This idea has been extended to more general regions via the Ryu-Takayanagi formula and entanglement entropy (also fruitful in dS)

- But a fundamental difference with AdS, is that dS does not have an asymptotic time-like boundary.

It would be useful to have a more realistic holography where the asymptotic boundary is not included. (related: quasi-local methods, holo RG, …)

Here we'll focus on recent progress from an unlikely direction: certain controlled irrelevant deformations in QFT, originally motivated from statistical mechanics.

The $T\bar{T}$ flow

Some 2d flows near IR fixed points are controlled by a universal operator, quadratic in the energy-momentum tensor. Zamolodchikov studied it,

$$(T\bar{T})(x) = \frac{1}{8}\left(T^{\mu\nu}T_{\mu\nu} - (T^{\mu}_{\mu})^2\right) \;, \quad \Delta_{T\bar{T}} = 4$$

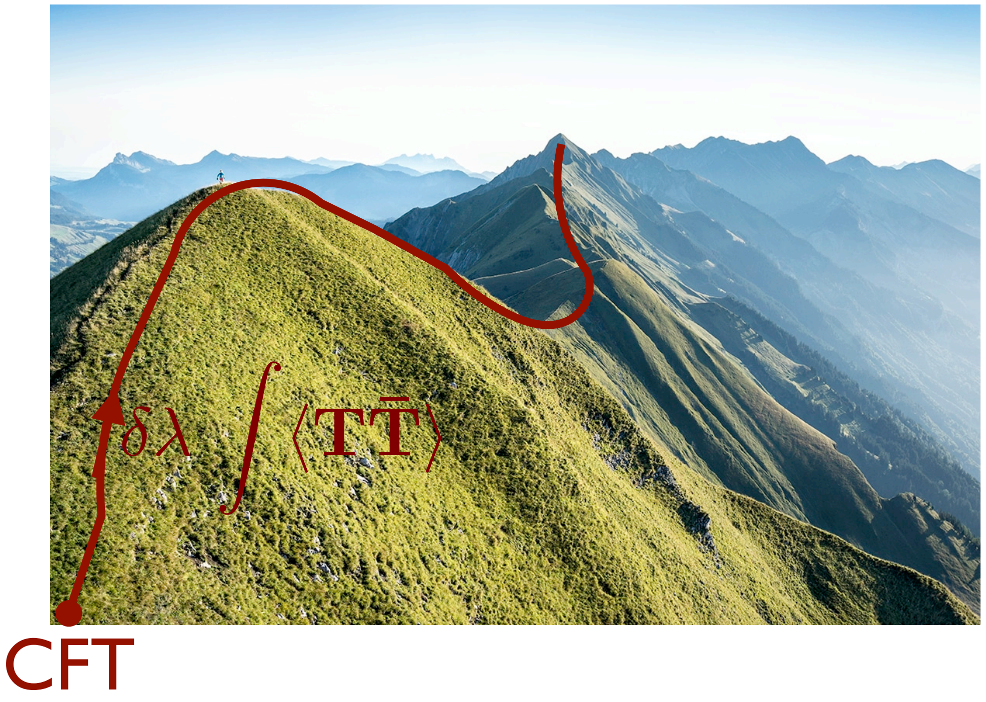
exact, d=2!

What happens if we add it to the action? Breaks renormalizability; QFT looses predictive power. Hard to go against the RG!

Surprisingly, there is a simple trajectory towards the UV that exhibits very special properties:

[Smirnov-Z]
[Cavaglia et al]

$$\partial_\lambda \log Z_\lambda = -2\pi \int d^2x \, \langle T\bar{T}\rangle_\lambda$$
$$T^{\mu}_{\mu} = -4\pi\lambda T\bar{T}$$

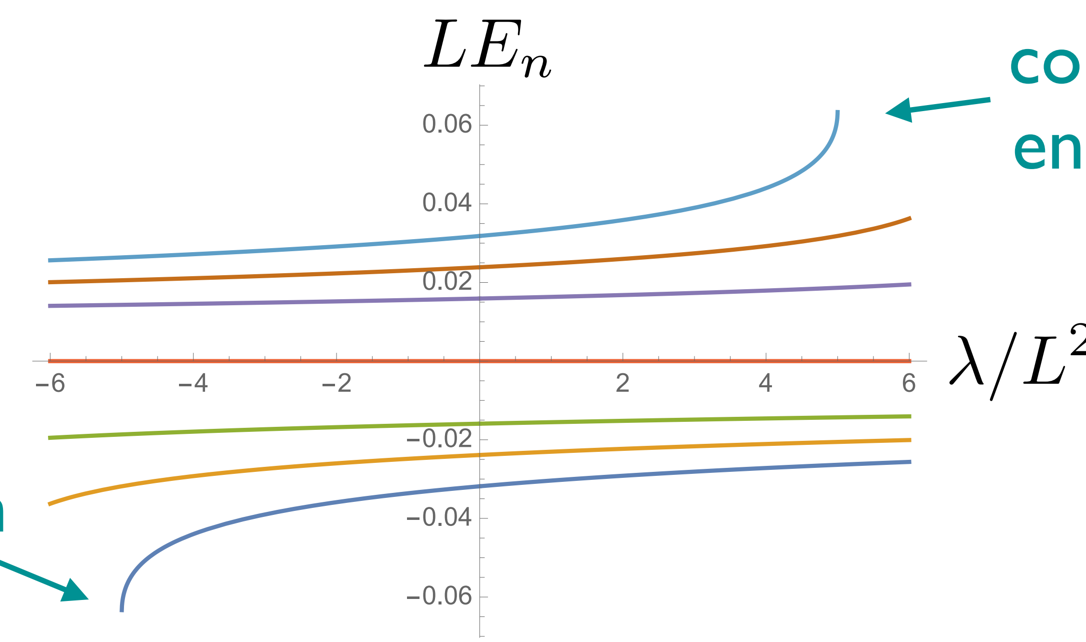


It will turn out this is precisely what we need for holography in finite regions

- A key result: put the CFT on spatial circle. The theory has discrete energy spectrum. Deformed energies can be computed non-perturbatively

recall $E_n^0 = \frac{2\pi}{L}\left(\Delta_n - \frac{c}{12}\right)$

$$E_n = \frac{L}{\pi\lambda}\left(1 - \sqrt{1 - \pi\frac{\lambda}{L}E_n^0}\right)$$



- UV behavior not fully understood, but various developments:
- gravitational dressing of S-matrix in Minkowski [Dubovsky, Gorbenko, Mirbabayi]
- equivalent description in terms of dynamical massive gravity [Cardy, …]
- nonlocalities similar to String Theory (NG action, softening, …)

- Different generalizations: addition of currents, double-trace deformations, etc. For our purpose, the following extension will be important

$$\frac{\partial}{\partial\lambda}\log Z = \int d^2x\left(-2\pi\langle T\bar{T}\rangle + \frac{c_2}{\lambda^2}\right)$$

Λ_2

equally universal as TTb and also solvable!

(needs a prior TTb deformation)

[Gorbenko, Silverstein, GT]

The gravity dual of TTbar

For simplicity, focus on pure 3d gravity (also valid in higher dimensions)

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}}\left(R^{(3)} + \frac{2\eta}{\ell^2}\right) + \frac{1}{8\pi G}\int_{\partial M}\left(K - \frac{1}{\ell}\right)\ , \ \eta = \pm 1$$

c.c. bdry. terms

Consider the spacetime with a radial cutoff, where we fix the metric

$$ds^2 = dr^2 + g_{\mu\nu}(r,x)dx^\mu dx^\nu\ , \ \ \text{cutoff}\ \ r < r_c \qquad K_{\mu\nu} = \frac{1}{2}\partial_r g_{\mu\nu}$$

This is the usual setup for AdS/CFT if $r_c \to \infty$. But now we want to keep cutoff it finite.

Basic entry of holographic dictionary: QFT stress tensor dual to Brown-York quasi-local tensor [Balasubramanian, Kraus]

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\text{on-shell}}}{\delta g^{\mu\nu}} = \frac{1}{8\pi G}\left(K_{\mu\nu} - K g_{\mu\nu} + \frac{1}{\ell}g_{\mu\nu}\right)$$

Imposing the radial Einstein equation

[McGough, Mezei, Verlinde]
[Kraus, Liu, Marolf]
[Gorbenko, Silverstein, GT]

$$E^r_r = \frac{1}{2}(K^2 - K^2_{ij}) - \frac{1}{2}R^{(2)} - \frac{\eta}{\ell^2} = 0$$

and rewriting $K_{\mu\nu}$ in terms of $T_{\mu\nu}$ gives

$$T^\mu_\mu = -\frac{c}{24\pi}R^{(2)} - 4\pi\lambda T\bar{T} - \frac{\overbrace{\eta - 1}^{\Lambda_2}}{\pi\lambda}$$

$\lambda = 8G\ell > 0$

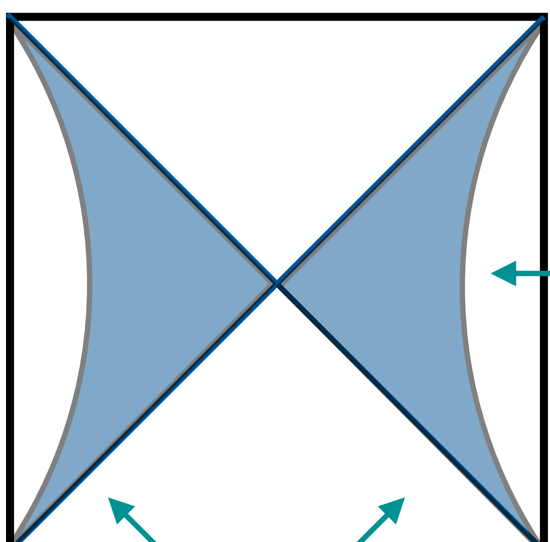
$c = \frac{3\ell}{2G}$

Weyl anomaly   TTbar def   nontrivial for dS!

This allows to formulate holography for (A)dS in finite patches!

2-side black hole in AdS

$$ds^2 = -\left(\frac{r^2}{\ell^2} - 8GM\right)dt^2 + \frac{dr^2}{\left(\frac{r^2}{\ell^2} - 8GM\right)} + r^2 d\phi^2$$



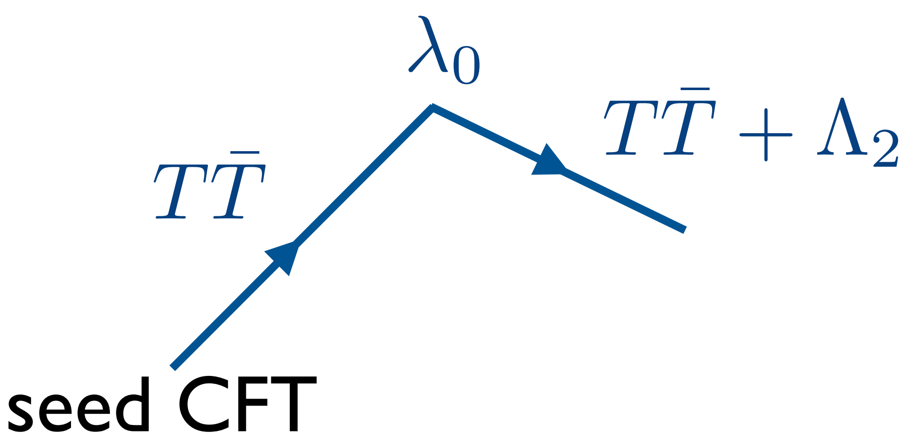
Quasilocal energy at cutoff surface:

$$T^t_t = \frac{1}{8\pi G\ell}\left(1 - \sqrt{1 - 8GM\frac{\ell^2}{r_c^2}}\right)$$

Agrees w/TTb. And $\sqrt{\cdots}$ captures the BH horizon

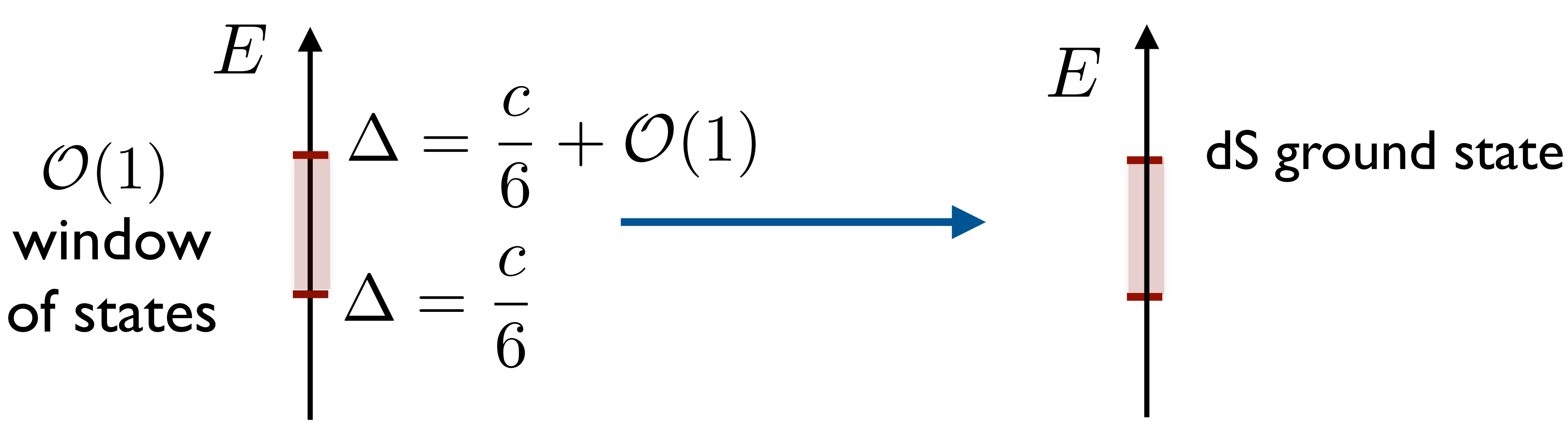
Applications to dS holography

Start from large N CFT w/gravity dual. Deform by $T\bar{T}$
Then turn on $T\bar{T} + \Lambda_2$ at some joining value λ_0



- λ : emergent radial direction that describes dS geometry
- different backgrounds for CFT allow to access different patches of dS
- also works in higher d at large N

‣ **dS microstates and entropy:** this def. preserves count of states



$$\Delta \geq \frac{c}{6} \; : \; S_{Cardy} = 2\pi\sqrt{\frac{c}{3}(\Delta - \frac{c}{12})}$$

$$S_{\Delta=c/6} = \frac{\pi c}{3}$$

$$S_{Gibbons-Hawking} = \frac{\pi c}{3}$$

dS microstates

Gravity side:

✓Bring in AdS BH bdry to horizon $r_h = \ell$

✓Bring out dS bdry from horizon $r_h = \ell$

[Coleman, Mazenc, Shyam, Silverstein, Soni, GT, Yang]

C. Summary

Exciting time for (quantum) gravity in cosmological space-times. We see important challenges, but also confluence of developments and new ideas in Math, QFT, machine learning, quantum information, gravity, etc.

We focused on two directions, but various other interesting developments. Early universe cosmology, inflation, correlators, concrete lower dim models, boundary conditions in GR, quantum info in cosmology, connections between micro/macro approaches …

See CERN TH workshop!