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Two different applications of QFT techniques to Cosmology

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Positivity bounds on effective field theories with spontaneously broken Lorentz

with Creminelli, Janssen **JHEP 2022**

EFT's & Positivity bounds

– EFT's are the common framework to describe phenomena below a certain energy.

– Given a set of DOF, write down all operators allowed by the symmetries

– Is every operator possible? With arbitrary prefactor?

– The seminal work of Allan Adams et al, **2006** showed that, by assuming unitarity, locality and Lorentz invariance of the UV completion, there are bounds on some coefficients.

  – This is very interesting theoretically and experimentally.

  – Much much work has followed since then, and is happening today.

e.g. Caron Hout and Van Duong **2020**

EFT's & Positivity bounds

- Is it possible to extend such a program to theories with Lorentz invariance, and in particular boosts, are spontaneously broken?
  - Typical regime for Cosmology and Condensed matter
- Why that would be interesting?
  - Cosmology:
    - Not so many data
    - Peculiar looking theories:
      - Galileons, Ghost Condensate
        - » While strange behaviors in Lorentz invariant limit, not clear the broken phase can be ruled out.
  - Condensed Matter
- One could perhaps argue that these kinds of Lagrangians are much more numerous to probe experimentally.

EFT's & Positivity bounds

– Using that the Lorentz-breaking EFT is originating from a Lorentz preserving one is not easy.

– Normal bounds are based on $2 \to 2$ scattering. But in Lorentz breaking background operators with many legs become relevant.

$$(\partial\phi)^n \to (\dot{\phi}_0)^{n-2}(\partial\delta\phi)^2$$

– not much is known about scattering $n \to m$

– Sometimes it is very hard to connect the Lorentz preserving and Lorentz breaking theories: e.g. fluids. There is no straightforward limit.

– Therefore, try to study directly the broken phase.

Review of Lorentz Invariant case

–Useful/needed properties. The S-matrix:

1. It is a physically well-defined function for all real s.
2. It is field redefinition independent.
3. It has an analytic continuation to the upper and lower half complex s-planes, with singularities residing only on the real axis, including unitarity cuts for energies $|s| > 4m^2$ where m is the mass gap in the theory, which is assumed to be non-zero. This property is a consequence of locality and Lorentz invariance.
4. The discontinuity across the cut on the positive real axis is $i \times$ a positive number. This is a consequence of unitarity.
5. It satisfies a crossing symmetry: $\mathcal{M}(s)^* = \mathcal{M}(4m^2 - s^*)$. This is a consequence of locality and Lorentz invariance.
6. It decays as $|\mathcal{M}(s)|/s^2 \to 0$ as $|s| \to \infty$. This property follows from the minimal requirements to derive the Froissart bound [16].

Review of Lorentz Invariant case

– The S-matrix in an EFT, in the forward limit, will take the following form

$$\hat{\mathcal{M}}(\hat{s}) = c_0 + c_2\frac{\hat{s}^2}{\Lambda^4} + c_4\frac{\hat{s}^4}{\Lambda^8} + \dots \ ,$$

– Then $\oint d\hat{s}\, \frac{\hat{\mathcal{M}}(\hat{s})}{\hat{s}^3} = 2\pi i \frac{c_2}{\Lambda^4}\ .$

– Deform contour by analyticity

  – Circle at infinity negligible

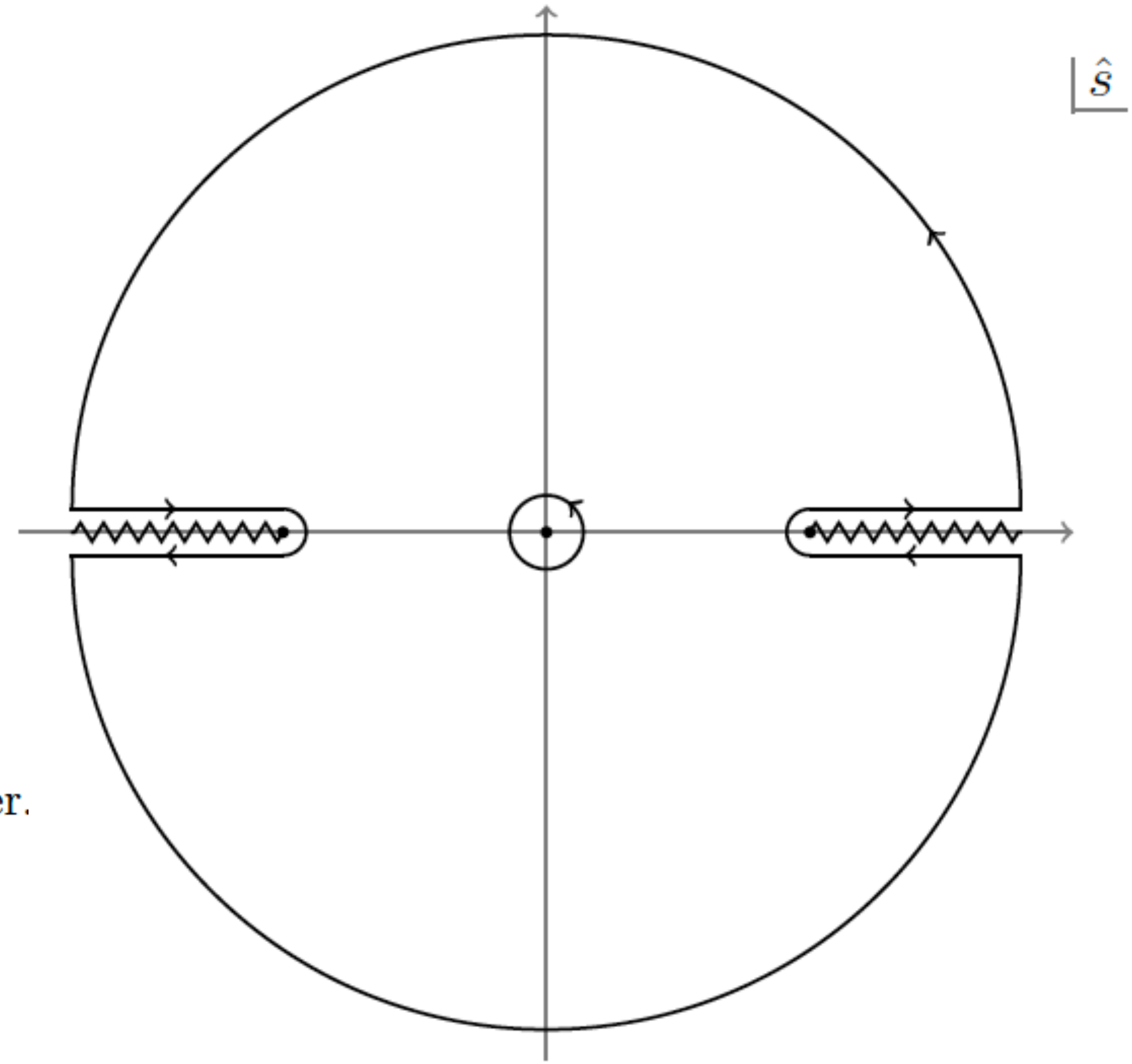
  – Integral along negative cut

    • =along positive cut

  – integral along positive cut=

    $i \times c_+$, with c_+ a non-negative number.

  – $\Rightarrow$ $c_2 \geq 0$.



Doing the same for Lorentz breaking EFT's

– Many difficulties

  – Most important: with boosts, the *in* and *out* states, no matter how energetic, can be mapped to the same state. So, they are defined no matter what the center of mass energy s is. So S-matrix is defined at all s.

    - Without boosts, this cannot be done. It is clearly impossible to scatter a 1 TeV phonon, because it simply does not exists (as there is a privileged reference frame).

  – Other difficulties relate to analyticity, crossing, etc.. But the one above seems just a show stopper.

  – Explorations with assumptions made in e.g.

Grall and Melville **2021**
Baumann, Green and Porto **2015**

- Let us try to find the *same ingredients* that we use for the S-matrix, but controlled.

UV/IR control

– Something that we control both in the UV and IR

– Idea: correlation functions of conserved currents (or the stress tensor), as they are defined at all energies.

– In the UV, we *assume* the theory goes to a conformal fixed point, a CFT. Currents are primary operators and their 2-point function is fixed:

$$\langle J^\mu(-k) J^\nu(k)\rangle = c_J (k^\mu k^\nu - \eta^{\mu\nu} k^2) k^{d-4} \,,$$

– Also, they are *field-redefinition independent*

• Which correlation function to study?

– Since we expect causality to play a role, choose ret. or adv. Green's functions:

$$G_R^{\mu\nu}(x-y) = i\theta(x^0 - y^0) \langle 0|[J^\mu(x), J^\nu(y)]|0\rangle \,,$$
$$G_A^{\mu\nu}(x-y) = -i\theta(y^0 - x^0) \langle 0|[J^\mu(x), J^\nu(y)]|0\rangle \,.$$

Analyticity

– . $$\tilde{G}^{\mu\nu}_{R,A}(\omega, \boldsymbol{p}) = \int_{\mathbb{R}^d} \mathrm{d}^d x \, e^{-ip\cdot x} G^{\mu\nu}_{R,A}(x)\,.$$

– $G^{\mu\nu}_R(x) = 0$ for $x^0 < 0$ and for $x^2 > 0$

– $\Rightarrow$ Integration region restricted to (FLC): $x^0 > 0, x^2 < 0$

– Consider complex four-momentum p : convergence for

$$\mathrm{Re}(-ip\cdot x) < 0 \text{ or } p^{\mathsf{Im}}\cdot x < 0 \text{ as } |x| \to \infty$$

– or: $p^{\mathsf{Im}} \in \mathrm{FLC}$

– So, for $p^{\mathsf{Im}} \in \mathrm{FLC}$, $\tilde{G}^{\mu\nu}_R(\omega, \boldsymbol{p})$ is analytic.

– Analogously, $\tilde{G}^{\mu\nu}_A(\omega, \boldsymbol{p})$ is analytic in backward light cone.

Analiticity

– We explore this region by choosing: $$\boldsymbol{p} = \boldsymbol{k}_0 + \omega \boldsymbol{\xi}$$

– where $\boldsymbol{k}_0, \boldsymbol{\xi} \in \mathbb{R}^{d-1}$, $|\boldsymbol{\xi}| \equiv \xi < 1$, and

$$\omega^{\mathsf{Im}} > 0 \text{ for } \tilde{G}_R \text{ and } \omega^{\mathsf{Im}} < 0 \text{ for } \tilde{G}_A$$

– Let us now define:
$$\tilde{G}^{\mu\nu}(\omega) = \begin{cases} \tilde{G}^{\mu\nu}_R(\omega, \boldsymbol{p}) & \text{if } \omega^{\mathsf{Im}} \geq 0\,, \\ \tilde{G}^{\mu\nu}_A(\omega, \boldsymbol{p}) & \text{if } \omega^{\mathsf{Im}} < 0\,, \end{cases}$$

– This function is analytic on $\mathbb{C} \setminus \{(-\infty, -m) \cup (m, \infty)\}$

Analiticity

–. $$\tilde{G}^{\mu\nu}(\omega) = \begin{cases} \tilde{G}_R^{\mu\nu}(\omega, \boldsymbol{p}) & \text{if } \omega^{\mathsf{Im}} \geq 0\,, \\ \tilde{G}_A^{\mu\nu}(\omega, \boldsymbol{p}) & \text{if } \omega^{\mathsf{Im}} < 0\,, \end{cases} \qquad \mathbb{C} \setminus \{(-\infty, -m) \cup (m, \infty)\}$$

– Consider $\omega \in \mathbb{R}$:

$$\lim_{\varepsilon \to 0} \left(\tilde{G}^{\mu\nu}(\omega + i\varepsilon) - \tilde{G}^{\mu\nu}(\omega - i\varepsilon) \right) = i \int_{\mathbb{R}^d} \mathrm{d}^d x \, e^{-ip\cdot x} \langle 0 | [J^\mu(x), J^\nu(0)] | 0 \rangle \tag{8}$$

$$= i \int_{\mathbb{R}^d} \mathrm{d}^d x \, e^{-ip\cdot x} \langle 0 | J^\mu(x) \left(\sum_n |P_n\rangle \langle P_n| \right) J^\nu(0) | 0 \rangle - (\mu \leftrightarrow \nu, x \leftrightarrow 0)$$

$$= i \int_{\mathbb{R}^d} \mathrm{d}^d x \, e^{-ip\cdot x} \langle 0 | e^{-i\hat{P}\cdot x} J^\mu(0) e^{i\hat{P}\cdot x} \left(\sum_n |P_n\rangle \langle P_n| \right) J^\nu(0) | 0 \rangle - (\mu \leftrightarrow \nu, x \leftrightarrow 0)$$

$$= i(2\pi)^d \sum_n \{ \delta^{(d)}(p - P_n) \langle 0 | J^\mu(0) | P_n \rangle \langle P_n | J^\nu(0) | 0 \rangle - \delta^{(d)}(p + P_n) \langle 0 | J^\nu(0) | P_n \rangle \langle P_n | J^\mu(0) | 0 \rangle$$

– Assuming a mass gap: $P_n^0 > m > 0$, the difference vanish in $|\omega| < m$, so function is analytic except for the two cuts.

– Analiticity ok

Positivity along cut

– Since we aim for a contour argument similar to S-matrix one, we need positivity along the cuts.

$$\lim_{\varepsilon\to 0}\left(\tilde{G}^{\mu\nu}(\omega+i\varepsilon)-\tilde{G}^{\mu\nu}(\omega-i\varepsilon)\right)=$$

$$i(2\pi)^d\sum_n\{\delta^{(d)}(p-P_n)\langle 0|J^\mu(0)|P_n\rangle\langle P_n|J^\nu(0)|0\rangle-\delta^{(d)}(p+P_n)\langle 0|J^\nu(0)|P_n\rangle\langle P_n|J^\mu(0)|0\rangle\}$$

– Contract with a real $V^\mu V^\nu$, divide by ω^ℓ and integrate along the positive cut. Only one $\delta-$function contributes:

$$\frac{1}{(2\pi)^d}\int_{(m,\infty)\ \mathrm{cut}}\frac{\mathrm{d}\omega}{\omega^\ell}\tilde{G}^{\mu\nu}(\omega)V_\mu V_\nu=i\int_m^\infty\frac{\mathrm{d}\omega}{\omega^\ell}\sum_n\delta^{(d)}(p-P_n)\ \ |\langle P_n|J^\mu(0)V_\mu|0\rangle|^2\ ,$$

– this is $i\times(\text{positive})$

- Similarly for negative cut:

$$\frac{1}{(2\pi)^d}\int_{(-\infty,-m)\ \mathrm{cut}}\frac{\mathrm{d}\omega}{\omega^\ell}\tilde{G}^{\mu\nu}(\omega)V_\mu V_\nu=-i\int_{-\infty}^{-m}\frac{\mathrm{d}\omega}{\omega^\ell}\sum_n\delta^{(d)}(p+P_n)\ \ |\langle P_n|J^\mu(0)V_\mu|0\rangle|^2\ ,$$

- for odd ℓ, this is $i\times(\text{positive})$. Positivity ok.

Crossing Symmetry

– Useful, though not necessary, property:

$$\begin{aligned}\tilde{G}^{\nu\mu}_A(-p) &= -i\int_{\mathbb{R}^d} \mathrm{d}^d x\, e^{ip\cdot x}\theta(-x^0)\,\langle 0|[J^\nu(x), J^\mu(0)]|0\rangle \\ &= -i\int_{\mathbb{R}^d} \mathrm{d}^d x\, e^{-ip\cdot x}\theta(x^0)\,\langle 0|[J^\nu(-x), J^\mu(0)]|0\rangle \\ &= -i\int_{\mathbb{R}^d} \mathrm{d}^d x\, e^{-ip\cdot x}\theta(x^0)\,\langle 0|[J^\nu(0), J^\mu(x)]|0\rangle = \tilde{G}^{\mu\nu}_R(p)\,,\end{aligned}$$

– In particular: $\tilde{G}^{\mu\nu}(\omega) = \tilde{G}^{\nu\mu}(-\omega)$ when $\boldsymbol{k}_0 = \mathbf{0}$

– Reality of Green's function: $\tilde{G}^{\mu\nu}_R(p) = \tilde{G}^{\mu\nu}_R(-p^*)^*$

– Combining:

$$\tilde{G}^{\mu\nu}_R(p) = \tilde{G}^{\nu\mu}_A(p^*)^*$$

Gauging the symmetry

– UV-IR connection

  – Need to be sure we are computing, in the IR, with EFT, the same quantity that in the UV has the CFT scaling.

  – Integrated-out heavy modes generate contact terms at low energies. These are not encoded in the Noether current constructed from the EFT. Therefore, neglecting them would give IR-UV mismatch.

- To keep track of contact terms: gauge the symmetry & interpret the correlation functions of currents as functional derivatives with respect to the non-dynamical gauge bosons.

- Let us be explicit. Notice

$$G_R^{\mu\nu}(x-y) = i\theta(x^0-y^0)\langle 0|[J^\mu(x),J^\nu(y)]|0\rangle = i\langle 0|\mathrm{T}\{J^\mu(x)J^\nu(y)\}|0\rangle - i\langle 0|J^\nu(y)J^\mu(x)|0\rangle \ .$$

  – The last term does not produce contact terms, as only low-energy states contribute:

$$i\int_{\mathbb{R}^d} \mathrm{d}^d x\, e^{-ip\cdot x}\langle 0|J^\nu(0)J^\mu(x)|0\rangle = i(2\pi)^d\sum_n \delta^{(d)}(p+P_n)\langle 0|J^\nu(0)|P_n\rangle\langle P_n|J^\mu(0)|0\rangle$$

  – but time-ordering has a convolution and so they contribute

Gauging the symmetry

$$G_R^{\mu\nu}(x-y) = i\theta(x^0-y^0)\langle 0|[J^\mu(x), J^\nu(y)]|0\rangle = i\langle 0|\mathrm{T}\{J^\mu(x)J^\nu(y)\}|0\rangle - i\langle 0|J^\nu(y)J^\mu(x)|0\rangle$$

– Time-ordered part:

$$\langle 0|\mathrm{T}\{J^\mu(x)J^\nu(y)\}|0\rangle = \frac{1}{Z}\int \mathcal{D}\phi\, e^{i\int_{\mathbb{R}^d} \mathrm{d}^d x\, \mathcal{L}(\phi)} J^\mu(x)J^\nu(y)$$

$$Z = \int \mathcal{D}\phi\, e^{i\int_{\mathbb{R}^d} \mathrm{d}^d x\, \mathcal{L}(\phi)}$$

– Non-ordered part:

– Go to Shroedinger picture:

$$\langle 0|J^\nu(y)J^\mu(x)|0\rangle = \langle 0|U(+\infty, y^0)J^\nu_{(s)}(\boldsymbol{y})U(y^0, x^0)J^\mu_{(s)}(\boldsymbol{x})U(x^0, -\infty)|0\rangle$$

– Inserting unity

$$\mathbb{1} = \int \mathcal{D}\phi(\tilde{\boldsymbol{x}})\, |\phi(\tilde{\boldsymbol{x}})\rangle\langle\phi(\tilde{\boldsymbol{x}})|$$

– and time evolution: $\langle\phi(y^0, \tilde{\boldsymbol{y}})|U(y^0, x^0)|\phi(x^0, \tilde{\boldsymbol{x}})\rangle = \int_{\phi(\tilde{\boldsymbol{x}})}^{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi\, e^{i\int_{x^0}^{y^0} \mathrm{d}^d x\, \mathcal{L}(\phi)}$

- We get:

$$\langle 0|J^\nu(y)J^\mu(x)|0\rangle = \frac{1}{Z}\int \mathcal{D}\phi(\tilde{\boldsymbol{x}})\int \mathcal{D}\phi(\tilde{\boldsymbol{y}})\; J^\nu(\phi(y^0, \boldsymbol{y}))J^\mu(\phi(x^0, \boldsymbol{x}))\int_{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_3\, e^{i\int_{y^0}^{+\infty} \mathrm{d}^d x\, \mathcal{L}(\phi_3)} \times$$
$$\int_{\phi(\tilde{\boldsymbol{x}})}^{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_2\, e^{i\int_{x^0}^{y^0} \mathrm{d}^d x\, \mathcal{L}(\phi_2)} \int^{\phi(\tilde{\boldsymbol{x}})} \mathcal{D}\phi_1\, e^{i\int_{-\infty}^{x^0} \mathrm{d}^d x\, \mathcal{L}(\phi_1)}\,. \tag{26}$$

Gauging the symmetry

– So we can write, gauging the symmetry:

$$
\begin{aligned}
&G_R^{\mu\nu}(x,y) = \\
&= \frac{i}{Z}\Bigg(\int \mathcal{D}\phi_0 \; e^{i \int_{\mathbb{R}^d} \mathrm{d}^d x \, \mathcal{L}\left(\phi_0, A_\mu^{(0)}\right)} \; J^\mu\left(\phi_0(x)\right) J^\nu\left(\phi_0(y)\right)\Big|_{A_\mu^{(0)}=0} + \\
&\int \mathcal{D}\phi(\tilde{\boldsymbol{x}}) \int \mathcal{D}\phi(\tilde{\boldsymbol{y}}) \; J^\nu(\phi(y^0, \boldsymbol{y})) J^\mu(\phi(x^0, \boldsymbol{x})) \int_{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_3 \; e^{i \int_{y^0}^{+\infty} \mathrm{d}^d x \, \mathcal{L}\left(\phi_3, A_\mu^{(3)}\right)} \times \\
&\int_{\phi(\tilde{\boldsymbol{x}})}^{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_2 \; e^{i \int_{x^0}^{y^0} \mathrm{d}^d x \, \mathcal{L}\left(\phi_2, A_\mu^{(2)}\right)} \quad \int^{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_1 \; e^{i \int_{-\infty}^{x^0} \mathrm{d}^d x \, \mathcal{L}\left(\phi_1, A_\mu^{(1)}\right)} \Big|_{A_\mu^{(1,2,3)}=0} \Bigg) \; .
\end{aligned}
$$

– or equivalently as functional derivative:

$$
\begin{aligned}
G_R^{\mu\nu}(x,y) = \frac{i}{Z}\Bigg(&- \frac{\delta^2}{\delta A_\mu^{(0)}(x) \delta A_\nu^{(0)}(y)} \int \mathcal{D}\phi \; e^{i \int_{\mathbb{R}^d} \mathrm{d}^d x \, \mathcal{L}\left(\phi_0, A_\mu^{(0)}\right)} \Big|_{A_\mu^{(0)}=0} - \\
&\frac{\delta^2}{\delta A_\mu^{(1)}(x) \; \delta A_\nu^{(3)}(y)} \int \mathcal{D}\phi(\tilde{\boldsymbol{x}}) \int \mathcal{D}\phi(\tilde{\boldsymbol{y}}) \; \int_{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_3 \; e^{i \int_{y^0}^{+\infty} \mathrm{d}^d x \, \mathcal{L}\left(\phi_3, A_\mu^{(3)}\right)} \; \times \\
&\int_{\phi(\tilde{\boldsymbol{x}})}^{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_2 \; e^{i \int_{x^0}^{y^0} \mathrm{d}^d x \, \mathcal{L}\left(\phi_2, A_\mu^{(2)}\right)} \int^{\phi(\tilde{\boldsymbol{x}})} \mathcal{D}\phi_1 \; e^{i \int_{-\infty}^{x^0} \mathrm{d}^d x \, \mathcal{L}\left(\phi_1, A_\mu^{(1)}\right)} \Big|_{A_\mu^{(1,2,3)}=0} \Bigg) \; .
\end{aligned}
$$

Gauging the symmetry

– . $$G_R^{\mu\nu}(x,y) = \frac{i}{Z}\left(- \frac{\delta^2}{\delta A_\mu^{(0)}(x)\delta A_\nu^{(0)}(y)} \int \mathcal{D}\phi\; e^{i\int_{\mathbb{R}^d} \mathrm{d}^d x\, \mathcal{L}\left(\phi_0, A_\mu^{(0)}\right)}\Big|_{A_\mu^{(0)}=0} - \right.$$
$$\frac{\delta^2}{\delta A_\mu^{(1)}(x)\;\delta A_\nu^{(3)}(y)} \int \mathcal{D}\phi(\tilde{\boldsymbol{x}}) \int \mathcal{D}\phi(\tilde{\boldsymbol{y}}) \int_{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_3\; e^{i\int_{y^0}^{+\infty} \mathrm{d}^d x\, \mathcal{L}\left(\phi_3, A_\mu^{(3)}\right)} \times$$
$$\left. \int_{\phi(\tilde{\boldsymbol{x}})}^{\phi(\tilde{\boldsymbol{y}})} \mathcal{D}\phi_2\; e^{i\int_{x^0}^{y^0} \mathrm{d}^d x\, \mathcal{L}\left(\phi_2, A_\mu^{(2)}\right)} \int^{\phi(\tilde{\boldsymbol{x}})} \mathcal{D}\phi_1\; e^{i\int_{-\infty}^{x^0} \mathrm{d}^d x\, \mathcal{L}\left(\phi_1, A_\mu^{(1)}\right)}\Big|_{A_\mu^{(1,2,3)}=0} \right) .$$

– This is the expression in the UV. In the IR, $e^{iS_{\mathrm{EFT}}(\phi_\ell, A_\mu)} = \int \mathcal{D}\phi_h\, e^{iS_{\mathrm{EFT}}(\phi_h,\phi_\ell,A_\mu)}$

– and we generate contact terms. They are captured by the gauge bosons dependence and therefore by the functional derivatives:

  – only from the T-ordered part, because contain the *same* gauge boson.

• UV and analyticity control.

Contour argument

– Consider, for example:

$$\tilde{G}^{00}(\omega) = \mu^{d-2}\left[c_1 \frac{1}{1-c_s^2\xi^2} + \frac{\omega^2}{\Lambda^2}\left(\frac{c_2}{(1-c_s^2\xi^2)^2} + d_1\right) + \mathcal{O}\left(\frac{\omega^4}{\Lambda^4}\right)\right]$$

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non-relativistic speed

cutoff

contact terms

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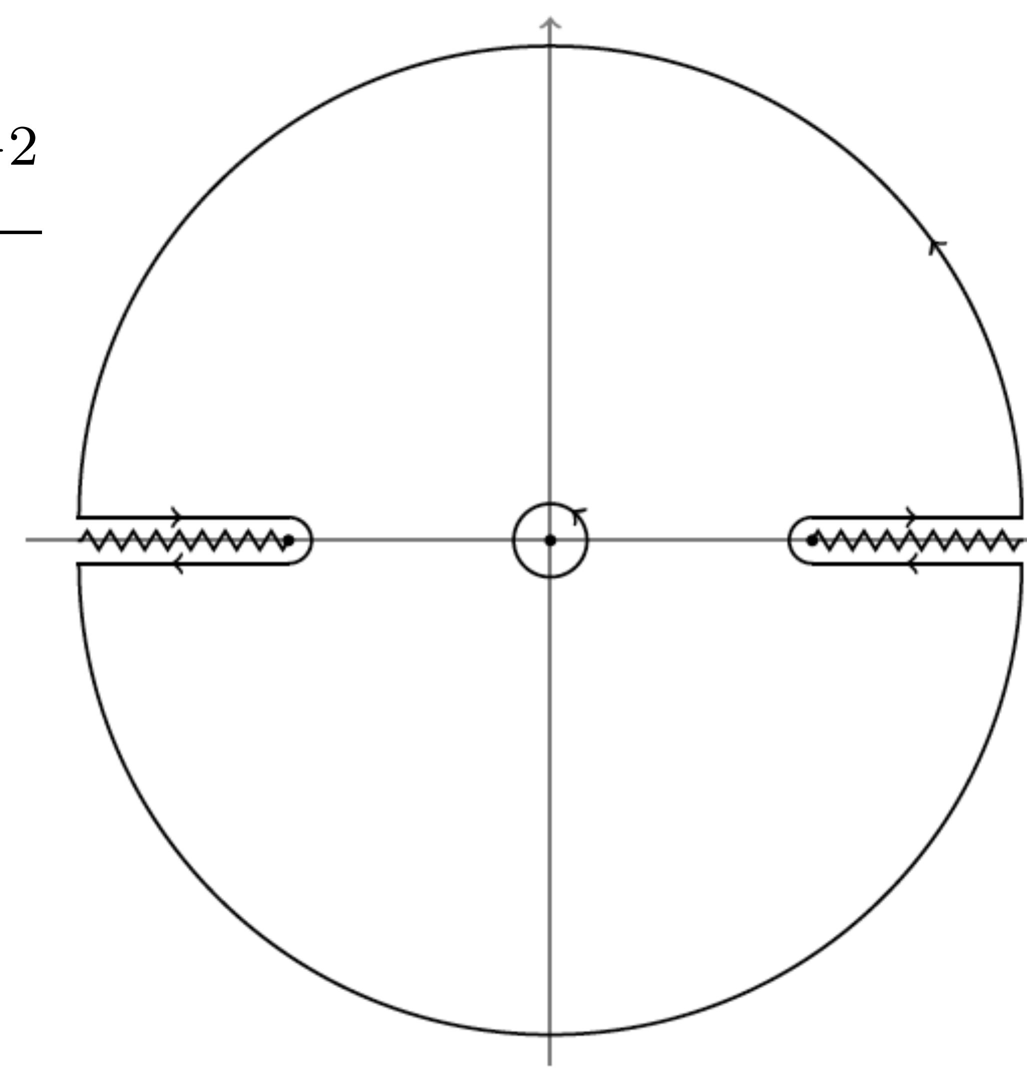
non-relativistic speed

cutoff

contact terms

–. $$\oint \mathrm{d}\omega \frac{\tilde{G}^{00}(\omega)}{\omega^3} = 2\pi i \left(\frac{c_2}{(1-c_s^2\xi^2)^2} + d_1\right)\frac{\mu^{d-2}}{\Lambda^2}$$



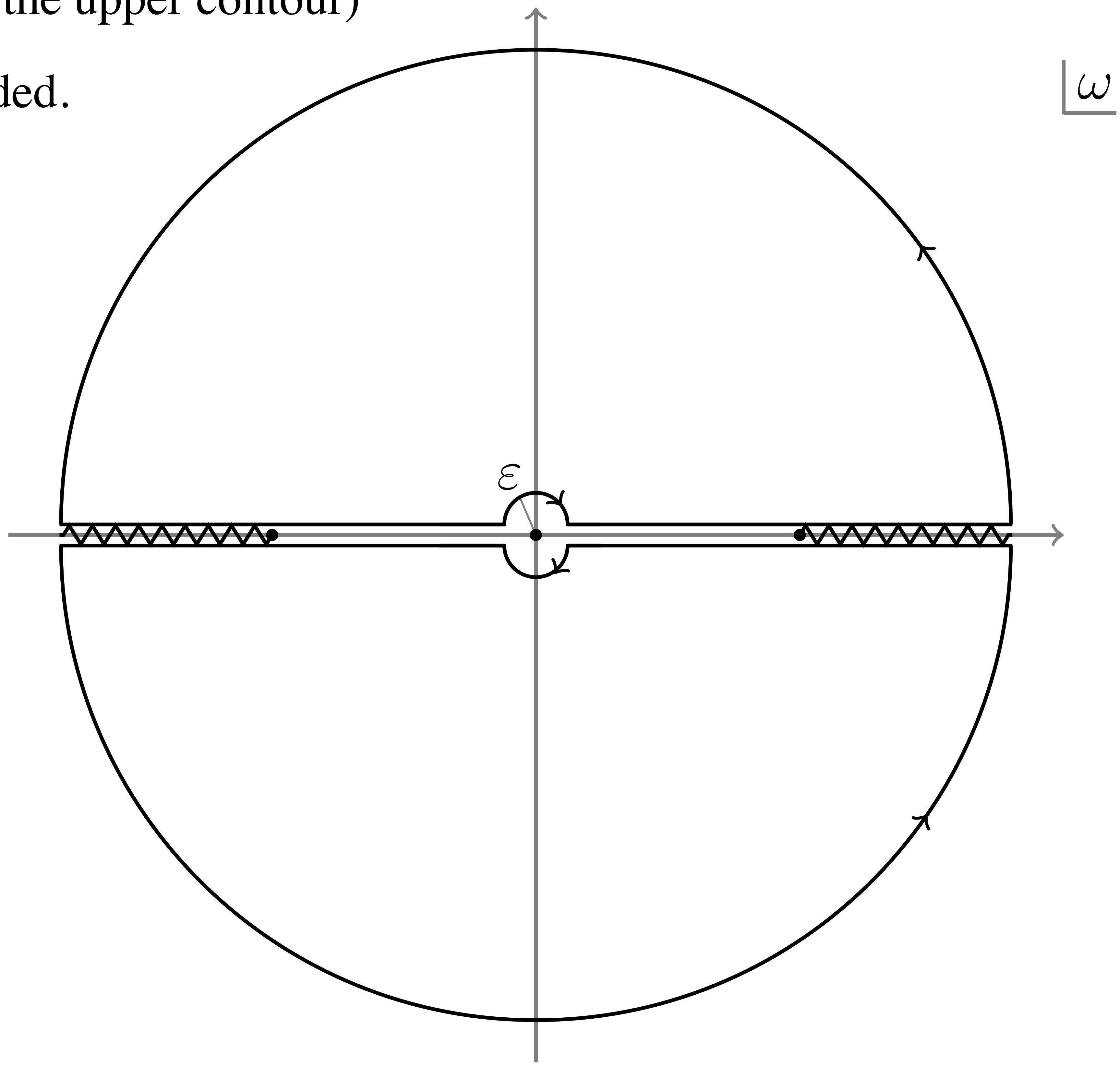
– For $d = 3$, $\tilde{G}^{00}(\omega) \sim \omega$ for $\omega \to \infty$

– circle negligible

•. $$\Rightarrow \quad \frac{c_2}{(1-c_s^2\xi^2)^2} + d_1 \geq 0$$

Without mass gap

- At loop level, the cut extends all the way to origin. One can use this contour (or, using crossing symmetry, just the upper contour)
  - So, no mass gap needed.



An example

Conformal Superfluids

– Apply setup to example of the EFT by Hellerman et al, **2015** Monin et al, **2017**

– Motivated by CFT studies, they match an operator at large charge with a state (at large charge): correlation functions of large charge operators can be computed with an EFT around this state. This state spontaneously breaks the symmetry, and also breaks, due to finite chemical potential, also time translations.

– An EFT can be constructed, using the non-linear realization of symmetries. The full symmetry is (could be an inflationary model!)

$SO(d,2) \times U(1)$ broken to $\text{rotations and spacetime translations}$

– Simplest construction: Cuomo, **2021**

– Write diff. invariant action with Wyel invariant metric: $\hat{g}_{\mu\nu} \equiv g_{\mu\nu} |g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi|$

– and $\chi = \mu t + \pi(t,x)$

– (we will Gauge it)

- Leading operator: $S^{(1)} = \frac{c_1}{6}\int d^3x\sqrt{-\hat{g}} = \frac{c_1}{6}\int d^3x\sqrt{-g}|\partial\chi|^3$

JJ calculation

– The EFT action reads, at NLO:

$$\mathcal{L} = \frac{c_1}{6}|\nabla\chi|^3 - 2c_2\frac{(\partial|\nabla\chi|)^2}{|\nabla\chi|} + c_3\left(2\frac{(\nabla^\mu\chi\partial_\mu|\nabla\chi|)^2}{|\nabla\chi|^3} + \partial_\mu\left(\frac{\nabla^\mu\chi\nabla^\nu\chi}{|\nabla\chi|^2}\right)\partial_\nu|\nabla\chi|\right)$$

$$-\frac{b}{4}\frac{F_{\mu\nu}F^{\mu\nu}}{|\nabla\chi|} + \frac{d}{2}\frac{F^\mu{}_i F^{\nu i}}{|\nabla\chi|^3}\nabla_\mu\chi\nabla_\nu\chi\,,$$

$$\nabla_\mu\chi \equiv \partial_\mu\chi - A_\mu\,,$$

$$|v| \equiv \sqrt{-v_\mu v^\mu}\,.$$

– Gauge symmetry: $\pi(x) \to \pi(x) + \Lambda(x),\, A_\mu(x) \to A_\mu(x) + \partial_\mu\Lambda(x)$

– Several contact terms.

– Expanding to quadratic order:

$$\mathcal{L}_{(2)} = \frac{c_1\mu^3}{6} + \frac{\mu c_1}{2}\left[(\dot\pi + A^0)^2 - \frac{1}{2}(\partial_i\pi - A_i)^2 + \mu\left(\dot\pi + A^0\right)\right] + \frac{2c_2}{\mu}\left[-\pi\Box\ddot\pi + 2A^0\Box\dot\pi + A^0\Box A^0\right]$$

$$+\frac{2c_3}{\mu}\left[-\pi\Box\ddot\pi + 2A^0\Box_{c_s}\dot\pi - A^i\partial_i\ddot\pi + (\dot A^0)^2 + \dot A^0\partial_i A^i\right] +$$

$$+\frac{(b+d)}{2\mu}\left[(\partial_i A^0)^2 + (\partial_0 A_i)^2 + 2\dot A^0(\partial_i A_i)\right] - \frac{b}{4\mu}(\partial_i A_j - \partial_j A_i)^2\,,$$

JJ calculation

– Noether current:

$$J_N^0 = -\frac{\mu^2 c_1}{2} - \mu c_1 \dot{\pi} - \frac{4c_2}{\mu}\Box\dot{\pi} - \frac{4c_3}{\mu}\Box_{c_s}\dot{\pi} \,,$$

$$J_N^i = \frac{\mu c_1}{2}\partial_i \pi - \frac{2c_3}{\mu}\partial_i \ddot{\pi} \,,$$

– We compute the correlation functions of the Noether currents, using

$$\mathcal{L}_{(2),A=0} = \frac{\mu c_1}{2}\pi\Box_{c_s}\pi - \frac{2(c_2+c_3)}{\mu}\pi\Box\ddot{\pi}$$

– and add the contact terms, as prescribed by the path integral formula:

$$\frac{1}{Z}\int \mathcal{D}\phi \; e^{i\int_{\mathbb{R}^3} \mathrm{d}^3x \; \mathcal{L}\left(\phi_0, A_\mu^{(0)}\right)} \; \frac{\delta^2 \mathcal{L}\left(\phi_0, A_\mu^{(0)}\right)}{\delta A_\mu^{(0)}(x)\,\delta A_\nu^{(0)}(x)}\Bigg|_{A_\mu^{(0)}=0}$$

JJ conservation

– We notice that it is true that

$$k_\mu \langle J^\mu(-k) J^\nu(k) \rangle = 0$$

– without any contact terms.

- Proof: consider $\mathcal{K} = \int \mathcal{D}\phi \; e^{i \int \mathrm{d}^d x \; \mathcal{L}(\phi, A_\mu)}$ and change variables $\phi' = e^{-i\alpha(x)}\phi$, and use $\mathcal{D}\phi' = \mathcal{D}\phi$, to get $\mathcal{K} = \int \mathcal{D}\phi \; e^{i \int \mathrm{d}^d x \; \mathcal{L}(\phi'(\phi), A_\mu)}$

- Gauge invariance $\mathcal{L}\left(\phi'(\phi), A_\mu - \partial_\mu \alpha\right) = \mathcal{L}\left(\phi, A_\mu\right)$

$$\Rightarrow \mathcal{L}\left(\phi'(\phi), A_\mu\right) = \mathcal{L}\left(\phi, A_\mu + \partial_\mu \alpha\right)$$

- So:

$$\mathcal{K} = \int \mathcal{D}\phi \; e^{i \int \mathrm{d}^d x \; \mathcal{L}(\phi, A_\mu + \partial_\mu \alpha)} = \int \mathcal{D}\phi \; e^{i \int \mathrm{d}^d x \; \mathcal{L}(\phi, A_\mu)} \left(1 + i \int \mathrm{d}^d x \; \partial_\mu \alpha(x) \frac{\delta S}{\delta A_\mu(x)}\right)$$

JJ conservation

– So:

$$0 = \int \mathcal{D}\phi \, e^{i \int \mathrm{d}^d x \, \mathcal{L}(\phi, A_\mu)} \int \mathrm{d}^d x \, \partial_\mu \alpha(x) \frac{\delta S}{\delta A_\mu(x)}$$

$$= - \int \mathrm{d}^d x \, \alpha(x) \, \partial_{x^\mu} \int \mathcal{D}\phi \, e^{i \int \mathrm{d}^d x' \, \mathcal{L}(\phi(x'), A_\nu(x'))} \frac{\delta S}{\delta A_\mu(x)}$$

$$= i \int \mathrm{d}^d x \, \alpha(x) \, \partial_{x^\mu} \frac{\delta}{\delta A_\mu(x)} \int \mathcal{D}\phi \, e^{i \int \mathrm{d}^d x' \, \mathcal{L}(\phi(x'), A_\nu(x'))} \, .$$

$$\Rightarrow \quad 0 = \partial_{x^\mu} \frac{\delta}{\delta A_\mu(x)} \int \mathcal{D}\phi \, e^{i \int \mathrm{d}^d x' \, \mathcal{L}(\phi(x'), A_\nu(x'))}$$

– Take a second derivative:

$$\Rightarrow \quad 0 = \partial_{x^\mu} \frac{\delta^2}{\delta A_\mu(x) \delta A_\nu(y)} \int \mathcal{D}\phi \, e^{i \int \mathrm{d}^d x' \, \mathcal{L}(\phi(x'), A_\rho(x'))} \Big|_{A_\sigma = 0}$$

– This is our functional form. But notice that it includes the contact terms.

JJ conservation

– So: $k_\mu \langle J^\mu(-k) J^\nu(k) \rangle = 0$

– $\Longrightarrow$ it has 2 tensorial structures (non relativistic theory):

$$i\langle J^\mu(-k) J^\nu(k) \rangle = \mathsf{A}\left(k^\mu k^\nu - \eta^{\mu\nu} k^2\right) + \mathsf{B}\left(k^i k^j - \delta^{ij} \boldsymbol{k}^2\right)$$

– Wordking in d=3:

$$\mathsf{A} = -\frac{\mu c_1}{2\left(\omega^2 - c_s^2 \boldsymbol{k}^2\right)} + \frac{c_2}{\mu}\frac{\left(\omega^2 - \boldsymbol{k}^2\right)\boldsymbol{k}^2}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} - \frac{c_3}{\mu}\frac{\omega^2\boldsymbol{k}^2}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} + \frac{b}{\mu} + \frac{d}{\mu}\,,$$

$$\mathsf{B} = \frac{\mu c_1}{4\left(\omega^2 - c_s^2 \boldsymbol{k}^2\right)} + \frac{c_2}{\mu}\frac{\left(\omega^2 - \boldsymbol{k}^2\right)^2}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} - \frac{c_3}{\mu}\frac{\omega^2(\omega^2 - \boldsymbol{k}^2)}{\left(\omega^2 - c_s^2\boldsymbol{k}^2\right)^2} - \frac{d}{\mu}\,.$$

Positivity bounds from JJ

– There is a rich kinematical structure. Consider:

$$\tilde{f}(\omega) = \tilde{G}^{\mu\nu}(k)V_\mu(k)V_\nu(k)\Big|_{k=(\omega,\boldsymbol{k}_0+\omega\boldsymbol{\xi})},$$

– Take $\boldsymbol{k}_0 = \boldsymbol{0}$ (as it does not change the result)

– Take the most general $V(\omega) = \alpha(\omega)\hat{K} + \beta(\omega)\hat{E} + \gamma(\omega)\hat{F}$

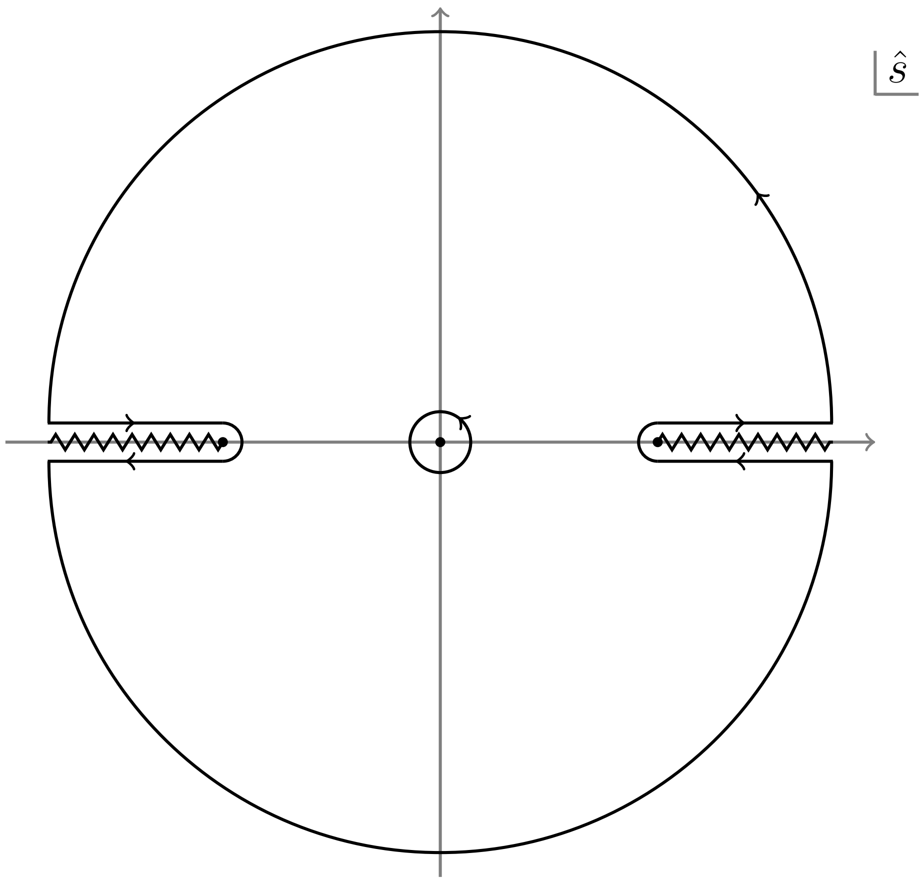
– (expanded in a base)

– Get: $\tilde{f}(\omega) = \mathsf{A}\omega^2(1-\xi^2)(\beta^2+\gamma^2) - \mathsf{B}\xi^2\omega^2\gamma^2$

– Contour argument:

– $\oint \mathrm{d}\omega \frac{\tilde{f}(\omega)}{\omega^3} = i\pi\tilde{f}''(0)$

– $\tilde{f}''(0) \geq 0$



Positivity bounds from JJ

– $\tilde{f}''(0) \geq 0$:

$$c_2 \frac{\xi^2(1-\xi^2)}{(1-\xi^2/2)^2}\beta^2 - c_3 \frac{\xi^2}{(1-\xi^2/2)^2}\beta^2 + b(\beta^2+\gamma^2) + d\left(\beta^2 + \frac{\gamma^2}{1-\xi^2}\right) \geq 0$$

–. $\xi \to 1$ with $\gamma \neq 0$ we obtain $\boxed{d \geq 0}$

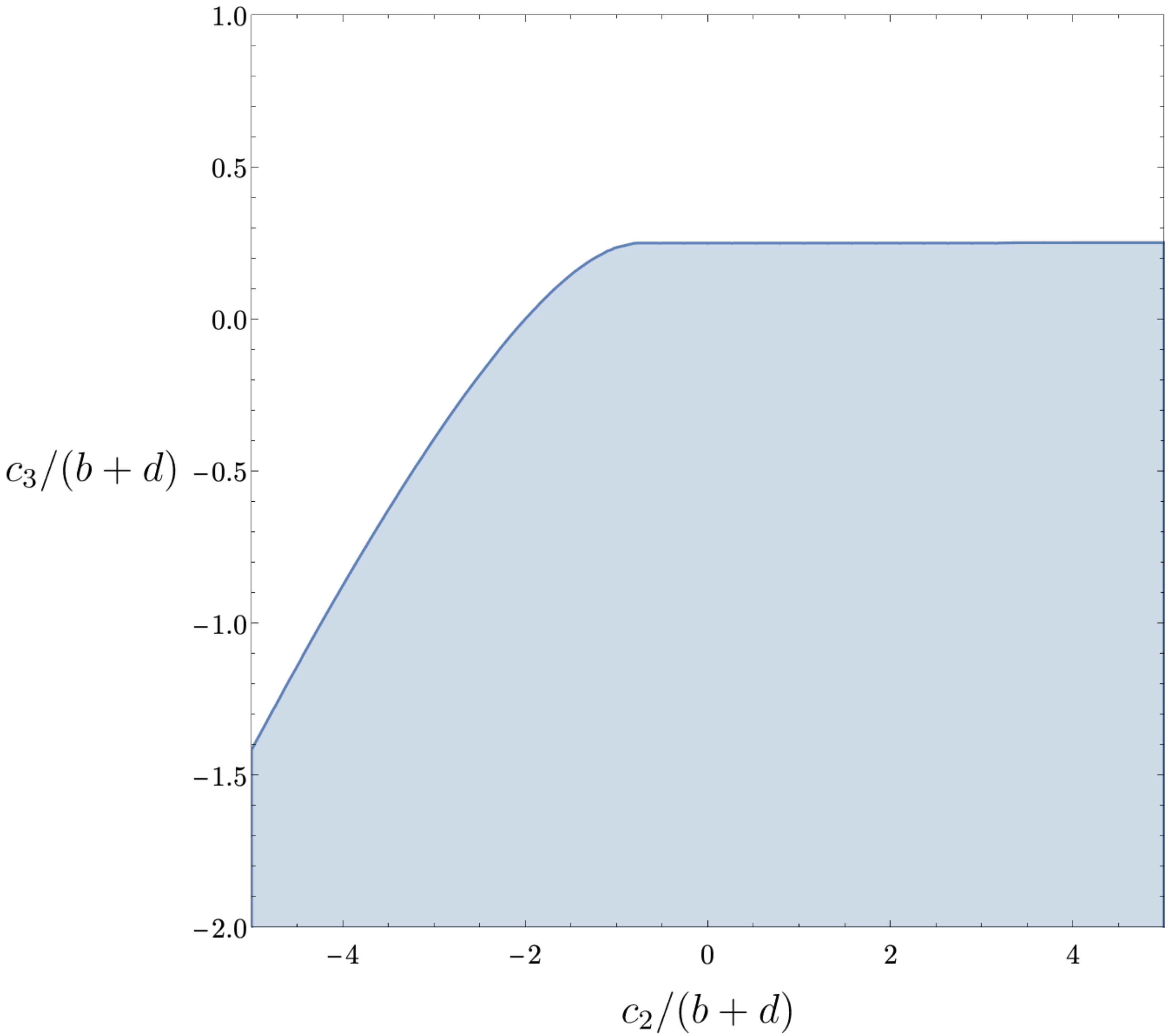
–. letting $\xi \to 0$ we get $\boxed{b + d \geq 0}$

–. Look at terms in γ^2 : most stringent is for $\gamma = 0$:

$$\boxed{\frac{c_2}{b+d}(1-\xi^2) - \frac{c_3}{b+d} \geq -\frac{(1-\xi^2/2)^2}{\xi^2}\,.} \qquad \xi \in [0,1)$$

Positivity bounds from JJ

– bound : $$\boxed{\frac{c_2}{b+d}(1-\xi^2) - \frac{c_3}{b+d} \geq -\frac{(1-\xi^2/2)^2}{\xi^2}\,.} \qquad \xi \in [0,1)$$



TT calculation

– We need to go to NNLO. It is possible to classify all the operators, and at quadratic order, there are only 3 independent ones:

$$S = \int \mathrm{d}^3x \sqrt{-\hat{g}} \left(\frac{c_1}{6} - c_2 \hat{R} + c_3 \hat{R}^{\mu\nu} \hat{\partial}_\mu \chi \hat{\partial}_\nu \chi + c_4 \hat{R}^2 + c_5 \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + c_6 \hat{R}^0_\mu \hat{R}^{\mu 0} \right)$$

$$\hat{R}^0_\mu \equiv \hat{R}^\lambda{}_\mu \partial_\lambda \chi$$

– We consider $\langle T^{\mu\nu}(-k) T^{\rho\sigma}(k) \rangle$, again, defined through path integral

– Conservation constraints the form: $i\langle T^{\mu\nu}(-k)T^{\rho\sigma}(k)\rangle_{\mathsf{subl.}} = \mathsf{C}(k)\,\Pi^{\mu\nu\rho\sigma}(k) + \mathsf{D}(k)\,\tilde{\Pi}^{\mu\nu\rho\sigma}(k)$

with

$$\Pi^{\mu\nu\rho\sigma} = \frac{1}{2}\left(\pi^{\mu\rho}\pi^{\nu\sigma} + \pi^{\mu\sigma}\pi^{\nu\rho}\right) - \frac{1}{d-1}\pi^{\mu\nu}\pi^{\rho\sigma}\,,$$
$$\tilde{\Pi}^{\mu\nu\rho\sigma} = \frac{1}{4}\left(\pi^{\mu\rho}\tilde{\pi}^{\nu\sigma} + \pi^{\mu\sigma}\tilde{\pi}^{\nu\rho} + \pi^{\nu\sigma}\tilde{\pi}^{\mu\rho} + \pi^{\nu\rho}\tilde{\pi}^{\mu\sigma}\right) - \frac{1}{d-2}\tilde{\pi}^{\mu\nu}\tilde{\pi}^{\rho\sigma}\,,$$

where

$$\pi^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\,,$$
$$\tilde{\pi}^{\mu\nu} = \delta^{mn} - \frac{k^m k^n}{\boldsymbol{k}^2}\,.$$

TT conservation

– Similar to current:

$$0 = -i\nabla_{x^\mu} \int \mathcal{D}\phi \; e^{i\int \mathrm{d}^d x' \sqrt{-g}\,\mathcal{L}(\phi(x'),g_{\rho\sigma}(x'))} \left(\frac{1}{\sqrt{-g(x)}} \frac{\delta S}{\delta g_{\mu\nu}(x)} \right) =$$

$$= \nabla_{x^\mu} \left(\frac{1}{\sqrt{-g(x)}} \frac{\delta}{\delta g_{\mu\nu}(x)} \int \mathcal{D}\phi \; e^{i\int \mathrm{d}^d x' \sqrt{-g}\,\mathcal{L}(\phi(x'),g_{\rho\sigma}(x'))} \right) .$$

– when act with second derivative, we hit the Christoffell:

$$0 = \frac{1}{\sqrt{-g(y)}} \frac{\delta}{\delta g_{\rho\sigma}(y)} \nabla_{x^\mu} \left(\frac{1}{\sqrt{-g(x)}} \frac{\delta}{\delta g_{\mu\nu}(x)} \int \mathcal{D}\phi \; e^{i\int \mathrm{d}^d x' \sqrt{-g}\,\mathcal{L}\left(\phi(x'),g_{\alpha\beta}(x')\right)} \right)$$

$$= \nabla_{x^\mu} \left(\frac{1}{\sqrt{(-g(x)(-g(y))}} \frac{\delta^2}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(y)} \int \mathcal{D}\phi \; e^{i\int \mathrm{d}^d x' \sqrt{-g}\,\mathcal{L}\left(\phi(x'),g_{\alpha\beta}(x')\right)} \right)$$

$$+ \frac{1}{\sqrt{-g(y)}} \frac{\delta}{\delta g_{\rho\sigma}(y)} \left(\frac{1}{\sqrt{-g(x)}} \Gamma^{\nu}_{\theta\gamma}(x) \right) \left(\frac{\delta}{\delta g_{\theta\gamma}} \int \mathcal{D}\phi \; e^{i\int \mathrm{d}^d x' \sqrt{-g}\,\mathcal{L}\left(\phi(x'),g_{\alpha\beta}(x')\right)} \right)$$

TT conservation

– At $g_{\mu\nu} = \eta_{\mu\nu}$,

$$0 = \partial_{x^\mu} \left(\frac{\delta^2}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(y)} \int \mathcal{D}\phi \; e^{i \int \mathrm{d}^d x' \, \sqrt{-g} \, \mathcal{L}\left(\phi(x'), g_{\alpha\beta}(x')\right)} \right)\Bigg|_{g_{\alpha\beta}=\eta_{\alpha\beta}}$$

$$+ \frac{1}{\sqrt{-g(x)}} \frac{\delta \Gamma^{\nu}_{\theta\gamma}(x)}{\delta g_{\rho\sigma}(y)} \Big|_{g_{\mu\nu}=\eta_{\mu\nu}} \cdot \left(\frac{\delta}{\delta g_{\theta\gamma}} \int \mathcal{D}\phi \; e^{i \int \mathrm{d}^d x' \, \sqrt{-g} \, \mathcal{L}\left(\phi(x'), g_{\alpha\beta}(x')\right)} \right)\Bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

– The second term is proportion to $\delta^{(d)}(x-y)$ and to the vev of the stress tensor. (for us it is proportional to c_1)

TT calculation

– We need to go to NNLO. It is possible to classify all the operators, and at quadratic order, there are only 3 independent ones:

$$S = \int \mathrm{d}^3x \sqrt{-\hat{g}} \left(\frac{c_1}{6} - c_2 \hat{R} + c_3 \hat{R}^{\mu\nu} \hat{\partial}_\mu \chi \hat{\partial}_\nu \chi + c_4 \hat{R}^2 + c_5 \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + c_6 \hat{R}^0_\mu \hat{R}^{\mu 0} \right)$$

$$\hat{R}^0_\mu \equiv \hat{R}^\lambda{}_\mu \partial_\lambda \chi$$

– We consider $\langle T^{\mu\nu}(-k) T^{\rho\sigma}(k) \rangle$, again, defined through path integral

– Conservation constraints the form: $i\langle T^{\mu\nu}(-k)T^{\rho\sigma}(k)\rangle_{\mathsf{subl.}} = \mathsf{C}(k)\,\Pi^{\mu\nu\rho\sigma}(k) + \mathsf{D}(k)\,\tilde{\Pi}^{\mu\nu\rho\sigma}(k)$

with

$$\Pi^{\mu\nu\rho\sigma} = \frac{1}{2}\left(\pi^{\mu\rho}\pi^{\nu\sigma} + \pi^{\mu\sigma}\pi^{\nu\rho}\right) - \frac{1}{d-1}\pi^{\mu\nu}\pi^{\rho\sigma}\,,$$

$$\tilde{\Pi}^{\mu\nu\rho\sigma} = \frac{1}{4}\left(\pi^{\mu\rho}\tilde{\pi}^{\nu\sigma} + \pi^{\mu\sigma}\tilde{\pi}^{\nu\rho} + \pi^{\nu\sigma}\tilde{\pi}^{\mu\rho} + \pi^{\nu\rho}\tilde{\pi}^{\mu\sigma}\right) - \frac{1}{d-2}\tilde{\pi}^{\mu\nu}\tilde{\pi}^{\rho\sigma}\,,$$

where

$$\pi^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\,,$$

$$\tilde{\pi}^{\mu\nu} = \delta^{mn} - \frac{k^m k^n}{\boldsymbol{k}^2}\,.$$

TT calculation

–.

$$\mathsf{C} = -\frac{\mu}{2}\frac{\omega^2(\omega^2-\boldsymbol{k}^2)^2}{(\omega^2-c_s^2\boldsymbol{k}^2)^2}(c_2+c_3) + \frac{1}{\mu}\frac{\boldsymbol{k}^4(\omega^2-\boldsymbol{k}^2)^2}{(\omega^2-c_s^2\boldsymbol{k}^2)^2}c_4 + \frac{1}{2\mu}\frac{(\omega^2-\boldsymbol{k}^2)^2\left(\omega^2(\omega^2-\boldsymbol{k}^2)+\boldsymbol{k}^4\right)}{(\omega^2-c_s^2\boldsymbol{k}^2)^2}c_5$$

$$+\frac{1}{4\mu}\frac{\boldsymbol{k}^2\omega^2(\omega^2-\boldsymbol{k}^2)^2}{(\omega^2-c_s^2\boldsymbol{k}^2)^2}c_6 - \frac{1}{2\mu}\frac{(c_2+c_3)^2}{c_1}\frac{\boldsymbol{k}^4\omega^2(\omega^2-\boldsymbol{k}^2)^2}{(\omega^2-c_s^2\boldsymbol{k}^2)^3}\,, \qquad (7$$

$$\mathsf{D} = -\frac{\mu}{4}\frac{\boldsymbol{k}^4(\omega^2-\boldsymbol{k}^2)}{(\omega^2-c_s^2\boldsymbol{k}^2)^2}(c_2+c_3) - \frac{1}{\mu}\frac{\boldsymbol{k}^4(\omega^2-\boldsymbol{k}^2)^2}{(\omega^2-c_s^2\boldsymbol{k}^2)^2}\left(2c_4+\frac{3}{4}c_5\right) + \frac{1}{8\mu}\frac{\boldsymbol{k}^6(\omega^2-\boldsymbol{k}^2)}{(\omega^2-c_s^2\boldsymbol{k}^2)^2}c_6$$

$$+\frac{1}{\mu}\frac{(c_2+c_3)^2}{c_1}\frac{\boldsymbol{k}^4\omega^2(\omega^2-\boldsymbol{k}^2)^2}{(\omega^2-c_s^2\boldsymbol{k}^2)^3}\,. \qquad (7$$

–Contract with general symmetric 2-tensor: $\langle T^{\mu\nu}T^{\rho\sigma}\rangle A_{\mu\nu}A_{\rho\sigma}$

$$A_{\mu\nu} = \alpha\hat{K}_\mu\hat{K}_\nu + \beta\hat{E}_\mu\hat{E}_\nu + \gamma\hat{F}_\mu\hat{F}_\nu + \tilde{\alpha}\left(\hat{K}_\mu\hat{E}_\nu + \hat{K}_\nu\hat{E}_\mu\right) + \tilde{\beta}\left(\hat{K}_\mu\hat{F}_\nu + \hat{K}_\nu\hat{F}_\mu\right) + \tilde{\gamma}\left(\hat{E}_\mu\hat{F}_\nu + \hat{E}_\nu\hat{F}_\mu\right)$$

–We get the bound:

$$i\langle T^{\mu\nu}T^{\rho\sigma}\rangle_{\mathsf{subl.}}A_{\mu\nu}A_{\rho\sigma} = \frac{\mathsf{C}}{2}\left[(\beta-\gamma)^2+4\tilde{\gamma}^2\right] + \mathsf{D}\tilde{\gamma}^2$$

TT positivity

–Explicitly

$$4\xi^4\delta^2 c_4 + 2\left[(2-\xi^2)^2\tilde{\gamma}^2 + (1-\xi^2+\xi^4)\delta^2\right] c_5 + \xi^2\left(\frac{(2-\xi^2)^2}{1-\xi^2}\tilde{\gamma}^2 + \delta^2\right) c_6$$

$$\geq \frac{4\xi^4\delta^2}{2-\xi^2}\frac{(c_2+c_3)^2}{c_1}$$

–Not hard to show that the most stringent bounds are:

$$\boxed{c_5 \geq 0} \text{ and } \boxed{c_6 \geq 0},$$

$$\boxed{4c_4 + 2c_5 + c_6 \geq 4(c_2+c_3)^2/c_1}.$$

Summary of the bounds

–By working at NLO and NNLO, we obtained:

$$
\begin{aligned}
c_1 &\geq 0 \quad \text{(for healthy fluctuations)}, \\
\frac{c_2}{b+d}(1-\xi^2) - \frac{c_3}{b+d} &\geq -\frac{(1-\xi^2/2)^2}{\xi^2}\,, \\
d &\geq 0\,, \\
b+d &\geq 0\,, \\
4c_4 + 2c_5 + c_6 &\geq 4(c_2+c_3)^2/c_1\,, \\
c_5 &\geq 0\,, \\
c_6 &\geq 0\,.
\end{aligned}
$$

Loop corrections?

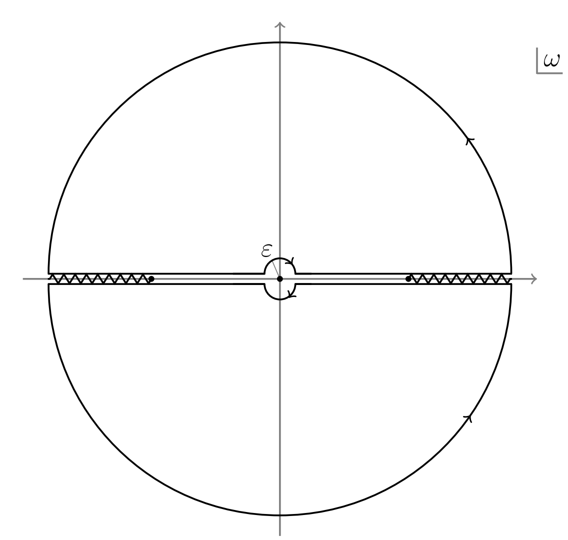
– So far, we worked at tree-level. In this particular case, up to NNLO d=3, there are no-loop corrections. In fact, in canonical normalization:

$$\mathcal{L} = \\ = \frac{1}{2}\left[\dot{\pi}_c{}^2 - \frac{1}{2}(\partial_i\pi_c)^2\right] + \frac{1}{c_1^{1/2}\mu^{3/2}}\dot{\pi}_c^3 + \frac{1}{c_1\mu^3}\dot{\pi}_c^4 + \frac{c_{2;3}}{c_1\mu^2}\partial^2\pi_c\partial^2\pi_c + \frac{c_{2;3}}{c_1^{3/2}\mu^3}\partial^2\pi_c\partial^2\pi_c\dot{\pi}$$

$$+ \frac{c_{4;5;6}}{c_1\mu^4}\partial^3\pi_c\partial^3\pi_c + \ldots$$

– and combinations of $c_{2;3}$ and $c_{4;5;6}$ do not have the right μ-dependence to make these coefficient run (it will happen at higher order).

– In general, however, no problem: one can do the loop with this contour, and use a finite radius:



Conclusions

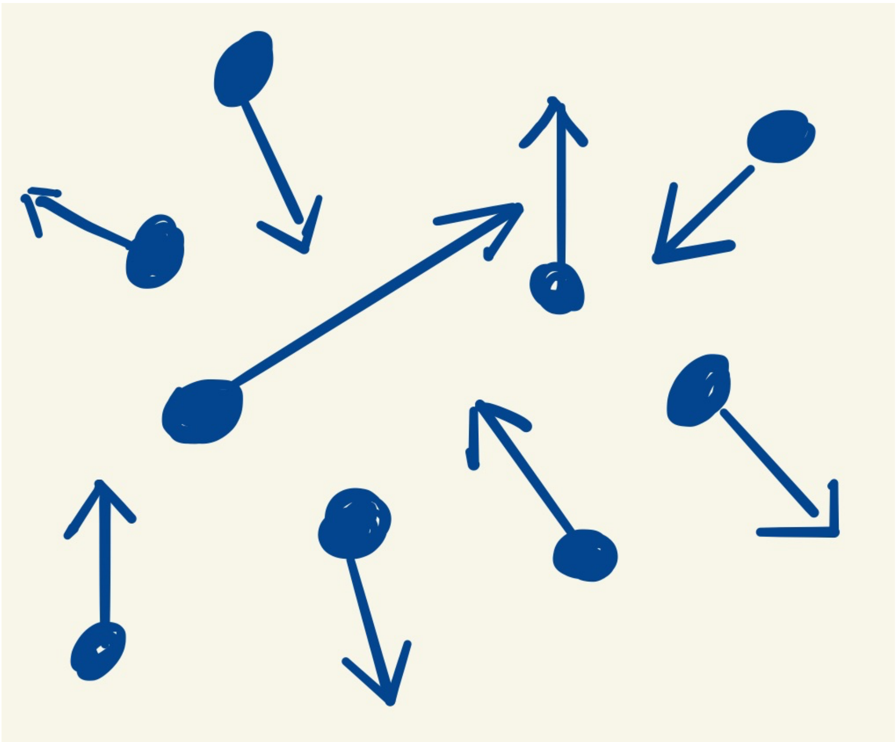
- We have constructed a method to derive robust bound on coefficients of operators where Boosts are spontaneously broken.
- Method based on 2-point functions of conserved current and stress tensor.
  - proved that they have the right analytic properties and also controlled UV behavior thanks to CFT UV assumption
  - then argument similar to S-matrix derived.

- Many applications:
  - Light in Material
  - QCD at finite μ
  - Inflation
- Limitations:
  - need to go to high order to ensure convergence
  - presence of the contact terms
- …Perhaps, we just started… perhaps…

Leonardo Senatore (ETH)

On the Effective Field Theory of Large Scale Structure

What is a fluid?



wikipedia: credit
National Oceanic and Atmospheric Administration/
Department of Commerce

$$\partial_t \rho_\ell + \partial_i \left(\rho_\ell v_\ell^i \right) = 0$$

$$\partial_t v_\ell^i + v_\ell^j \partial_j v_\ell^i + \frac{1}{\rho_\ell} \partial_i p_\ell = \text{viscous terms}$$

– From short to long

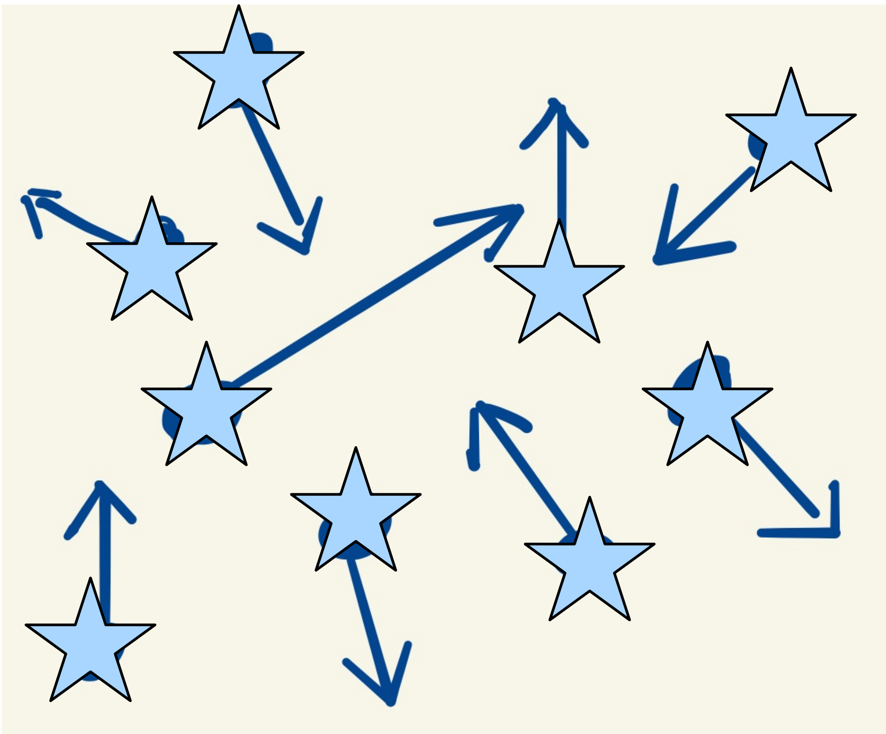
– The resulting equations are simpler

– Description arbitrarily accurate

  – construction can be made without knowing the nature of the particles.

– short distance physics appears as a non trivial stress tensor for the long-distance fluid

Do the same for matter in our Universe



credit NASA

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**
with Carrasco and Hertzberg **JHEP 2012**

$$\nabla^2\Phi_\ell = H^2\left(\delta\rho_\ell/\rho\right)$$
$$\partial_t\rho_\ell + H\rho_\ell + \partial_i\left(\rho_\ell v_\ell^i\right) = 0$$
$$\partial_t v_\ell^i + v_\ell^j\partial_j v_\ell^i + \partial_i\Phi_\ell = \partial_j\tau^{ij}$$

– From short to long

– The resulting equations are simpler

– Description arbitrarily accurate

  – construction can be made without knowing the nature of the particles.

– short distance physics appears as a non trivial stress tensor for the long-distance fluid

$$\tau_{ij} \sim \delta_{ij}\,\rho_{\rm short}\,\left(v^2_{\rm short} + \Phi_{\rm short}\right)$$

Dealing with the Effective Stress Tensor

- For long distances: expectation value over short modes (integrate them out)

$$\langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} = f_{\text{very complicated}}\left(\{H, \Omega_m, \ldots, m_{\text{dm}}, \ldots, \rho_\ell(x)\}_{\text{past light cone}}\right)$$

At *long* wavelengths $\Downarrow$ Taylor Expansion

$$\langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} = \int^t dt' \left[c(t,t')\,\frac{\delta\rho_\ell}{\rho}(\vec{x}_{\text{fl}},t') + \mathcal{O}\left((\delta\rho_\ell/\rho)^2\right)\right]$$

- Equations with only long-modes

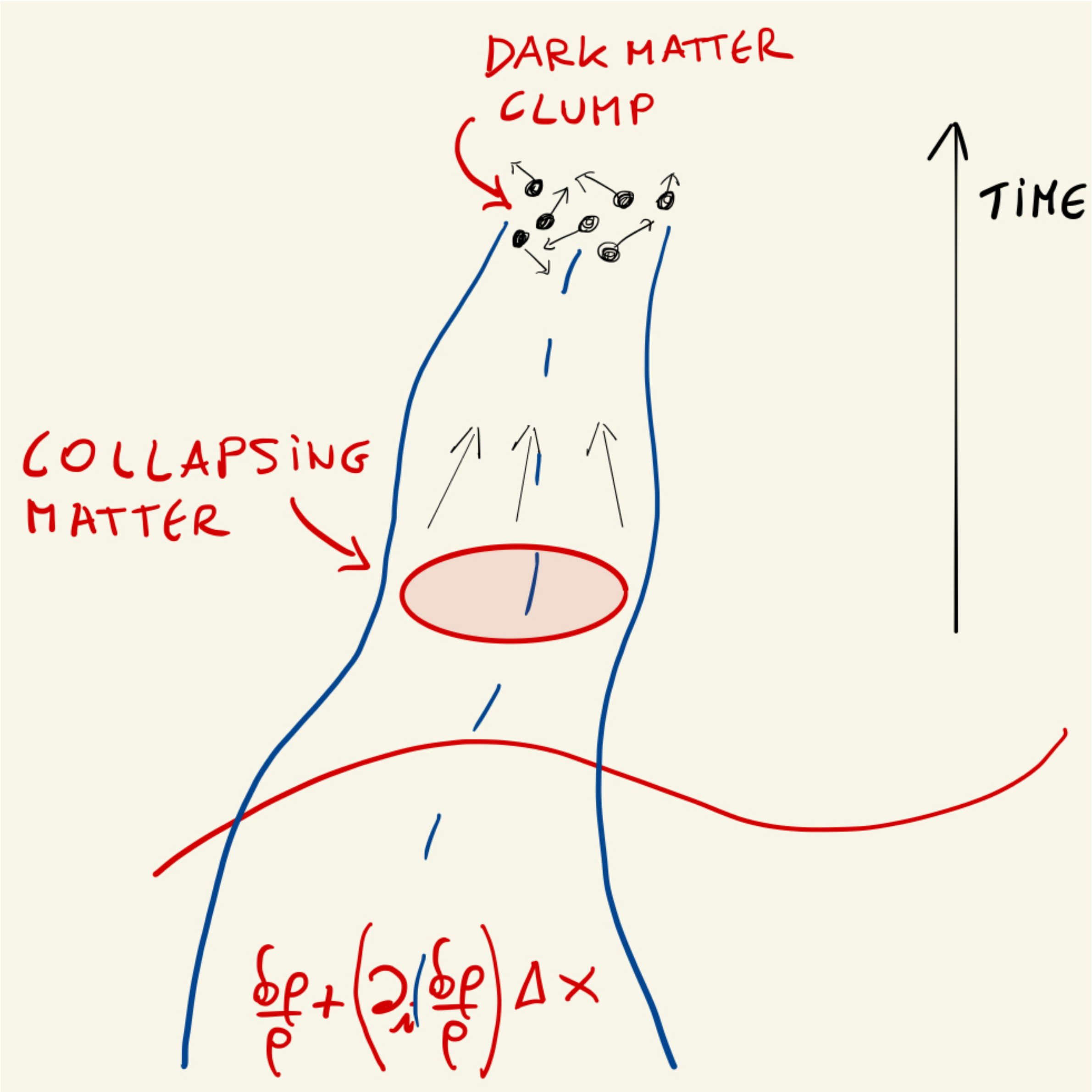
$$\partial_t v_\ell^i + v_\ell^j \partial_j v_\ell^i + \partial_i \Phi_\ell = \partial_j \tau^{ij}$$

$$\tau_{ij} \sim \delta\rho_\ell/\rho + \ldots$$

every term allowed by symmetries

- each term contributes as factor of

$$\frac{\delta\rho_l}{\rho} \sim \frac{k}{k_{\text{NL}}} \ll 1$$



Perturbation Theory within the EFT

- In the EFT we can solve iteratively $\delta_\ell, v_\ell, \Phi_\ell \ll 1$, where $\delta_\ell = \frac{\delta\rho_\ell}{\rho}$

$$\nabla^2\Phi_\ell = H^2\left(\delta\rho_\ell/\rho\right)$$
$$\partial_t\rho_\ell + H\rho_\ell + \partial_i\left(\rho_\ell v_\ell^i\right) = 0$$
$$\partial_t v_\ell^i + v_\ell^j\partial_j v_\ell^i + \partial_i\Phi_\ell = \partial_j\tau^{ij}$$

$$\tau_{ij} \sim \delta\rho_\ell/\rho + \ldots$$

- Two scales:

$$k\,[\text{Mean Free Path Scale}] \sim k\left[\left(\frac{\delta\rho}{\rho}\right)\sim 1\right] \sim k_{\rm NL}$$

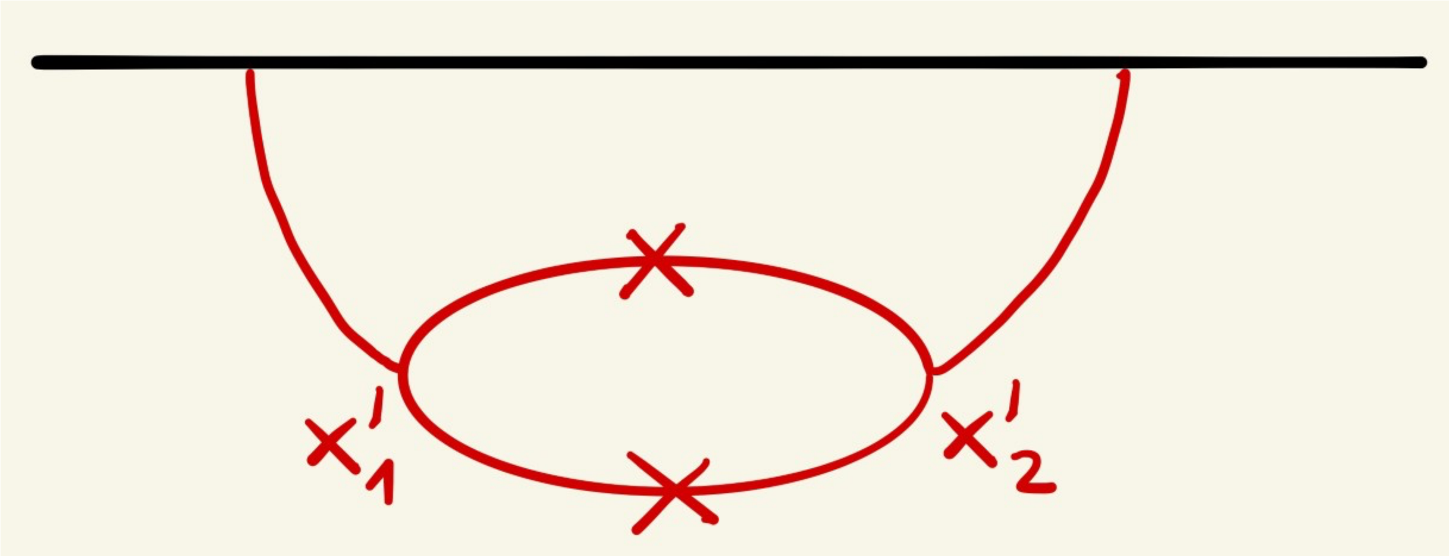
Perturbation Theory within the EFT

- Solve iteratively some non-linear eq. $\delta_\ell = \delta_\ell^{(1)} + \delta_\ell^{(2)} + \ldots \ll 1$
- Second order:

$$\partial^2 \delta_\ell^{(2)} = \left(\delta_\ell^{(1)}\right)^2 \quad \Rightarrow \quad \delta_\ell^{(2)}(x) = \int d^4x' \ \text{Greens}(x,x') \ \left(\delta_\ell^{(1)}(x')\right)^2$$

- Compute observable:

$$\langle \delta_\ell(x_1)\delta_\ell(x_2)\rangle \supset \langle \delta_\ell^{(2)}(x_1)\delta_\ell^{(2)}(x_2)\rangle \sim \int d^4x_1' d^4x_2' \ (\text{Green's})^2 \ \langle \delta_\ell^{(1)}(x_1')^2 \delta_\ell^{(1)}(x_2')^2\rangle$$



- We obtain Feynman diagrams
- Sensitive to short distance

$$x_2' \to x_1'$$

  - Need to add counterterms from $\tau_{ij} \supset c_s^2 \delta_\ell$ to correct
- Loops and renormalization applied to galaxies

Perturbation Theory within the EFT

Pajer and Zaldarriaga **2013**

- Regularization and renormalization of loops (no-scale universe) $P_{11}(k) = \frac{1}{{k_{\rm NL}}^3}\left(\frac{k}{k_{\rm NL}}\right)^n$

  – evaluate with cutoff:

$$P_{1-\rm loop} = c_1^{\Lambda}\left(\frac{\Lambda}{k_{\rm NL}}\right)\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

$$\langle \left(\frac{\delta\rho}{\rho}\right)^2_k \rangle$$

  – divergence (we extrapolated the equations where they were not valid anymore)

Perturbation Theory within the EFT

Pajer and Zaldarriaga **2013**

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–evaluate with cutoff:

$$P_{1-\text{loop}} = c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\text{finite}}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

$$\langle \left(\frac{\delta\rho}{\rho}\right)_k^2 \rangle$$

– divergence (we extrapolated the equations where they were not valid anymore)

– we need to add effect of stress tensor $\tau_{ij} \supset c_s^2 \delta_\ell$

$$P_{11,\,c_s} = c_s \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} \quad , \text{ choose} \qquad c_s = -c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right) + c_{s,\,\text{finite}}$$

$$\Longrightarrow P_{1-\text{loop}} + P_{11,\,c_s} = c_{s,\,\text{finite}}\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\text{finite}}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

–we just re-derived renormalization

–after renormalization, result is finite and small for $\frac{k}{k_{\rm NL}} \ll 1$

Perturbation Theory within the EFT

Pajer and Zaldarriaga **2013**

- Regularization and renormalization of loops (no-scale universe) $P_{11}(k) = \frac{1}{{k_{\rm NL}}^3}\left(\frac{k}{k_{\rm NL}}\right)^n$

–evaluate with cutoff:

$$P_{1-\text{loop}} = c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

$$\langle\left(\frac{\delta\rho}{\rho}\right)^2_k\rangle$$

– divergence (we extrapolated the equations where they were not valid anymore)

– we need to add effect of stress tensor $\tau_{ij} \supset c_s^2 \delta_\ell$

$$P_{11,\,c_s} = c_s\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} \quad \text{choose} \quad c_s = -c_1^\Lambda\left(\frac{\Lambda}{k_{\rm NL}}\right) + c_{s,\,\rm finite}$$

$$\Rightarrow P_{1-\text{loop}} + P_{11,\,c_s} = c_{s,\,\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\rm NL}}$$

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…. lots of work ….

Galaxy Statistics

Senatore **1406**
with Lewandowsky *et al* **1512**
with Perko *et al*. **1610**

Galaxies in the EFTofLSS

Senatore **1406**

- On galaxies, a long history before us, summarized by McDonald, Roy **2010** .
  - Senatore **1406** provided first complete parametrization.

- Nature of Galaxies is very complicated

$$n_{\rm gal}(x) = f_{\rm very\ complicated}\left(\{H, \Omega_m, \ldots, m_e, g_{ew}, \ldots, \rho(x)\}_{\rm past\ light\ cone}\right)$$

Galaxies in the EFTofLSS

Senatore **1406**

$$n_{\rm gal}(x) = f_{\text{very complicated}}\Big(\{H, \Omega_m, \ldots, m_e, g_{ew}, \ldots, \rho(x)\}_{\text{past light cone}}\Big)$$

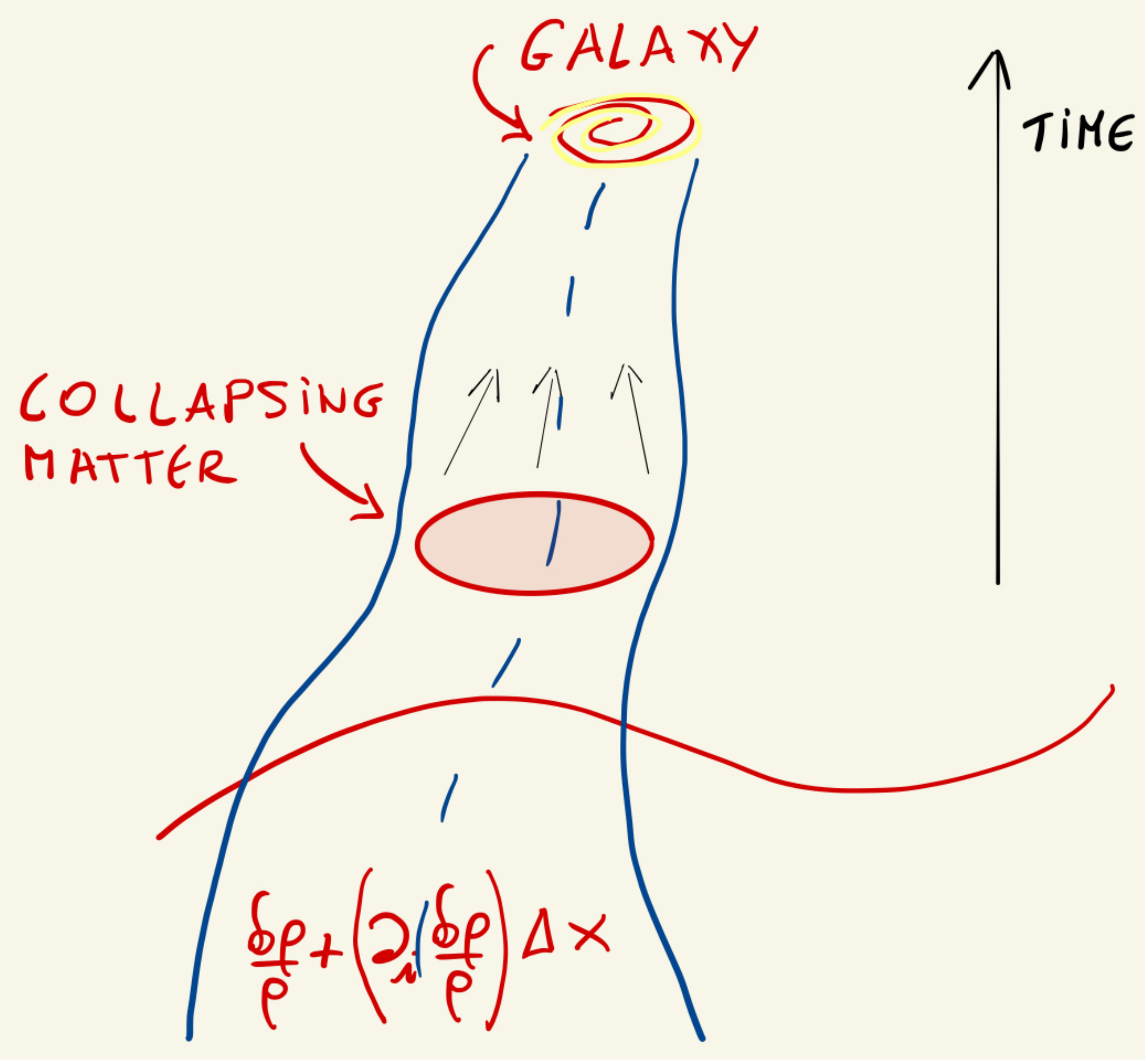
At *long* wavelengths $\Downarrow$ Taylor Expansion

$$\left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x) \sim \int^t dt' \left[c(t,t')\ \left(\frac{\delta\rho}{\rho}\right)(\vec{x}_{\rm fl}, t') + \ldots\right]$$

- all terms allowed by symmetries
- all physical effects included
  - e.g. assembly bias
- .

$$\left\langle \left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x)\left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(y)\right\rangle =$$

$$= \sum_n \text{Coeff}_n \cdot \langle \text{matter correlation function}\rangle_n$$



It is familiar in dielectric E&M

- Polarizability:

$$\vec{P}(\omega) = \chi(\omega)\vec{E}(\omega) \quad \Rightarrow \quad \vec{P}(t) = \int dt' \chi(t - t')\vec{E}(t')$$

- and in fact, also the EFT of Non-Relativistic binaries Goldberger and Rothstein **2004**

  is non-local in time.

Consequences of non-locality in time

with Carrasco, Foreman, Green **1310**
Senatore **1406**

- The EFT is non-local in time $\Longrightarrow$ $\langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} \sim \int^t dt' \; K(t,t') \; \delta\rho(\vec{x}_{\text{fl}},t') + \dots$
- Perturbative Structure has a decoupled structure

$$\delta\rho(x,t') = D(t')\delta\rho(\vec{x})^{(1)} + D(t')^2\delta\rho(\vec{x})^{(2)} + \dots$$

- A few coefficients for each counterterm:

$$\Rightarrow \quad \langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} \sim \int^t dt' \; K(t,t') \; \left[D(t')\delta\rho(\vec{x})^{(1)} + D(t')^2\delta\rho(\vec{x})^{(2)} + \dots\right] \simeq$$

$$\simeq c_1(t)\;\delta\rho(\vec{x})^{(1)} + c_2(t)\;\delta\rho(\vec{x})^{(2)} + \dots$$

- where $c_i(t) = \int dt' \; K(t,t') \; D(t')^i$

- Difference:

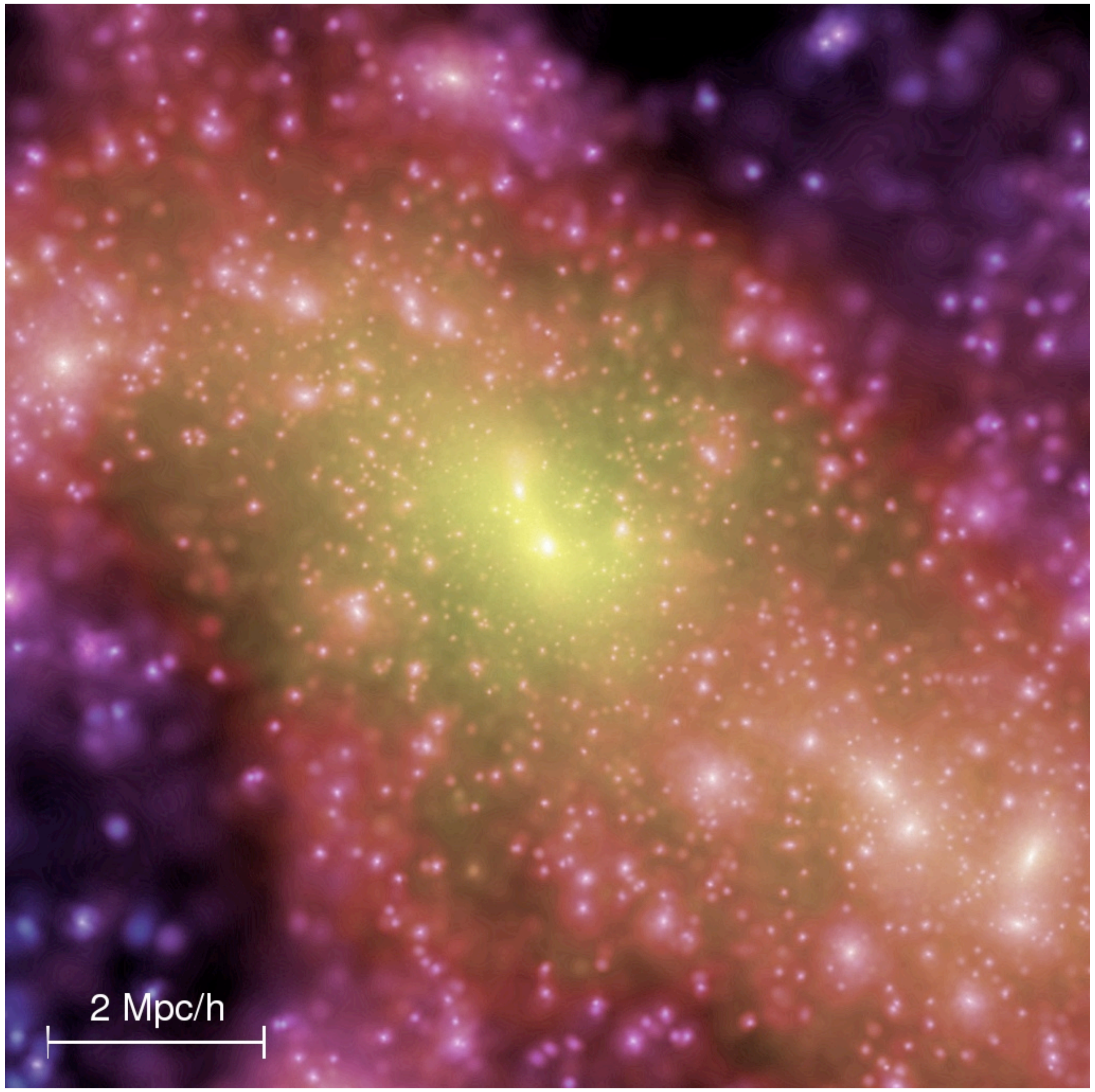
  Time-Local QFT: $c_1(t)\left[\delta\rho(\vec{x})^{(1)} + \delta\rho(\vec{x})^{(2)} + \dots\right]$

  Non-Time-Local QFT: $c_1(t)\;\delta\rho(\vec{x})^{(1)} + c_2(t)\delta\rho(\vec{x})^{(2)} + \dots$

  - More terms, but not a disaster

Baryonic effects

- When stars explode, baryons behave differently than dark matter



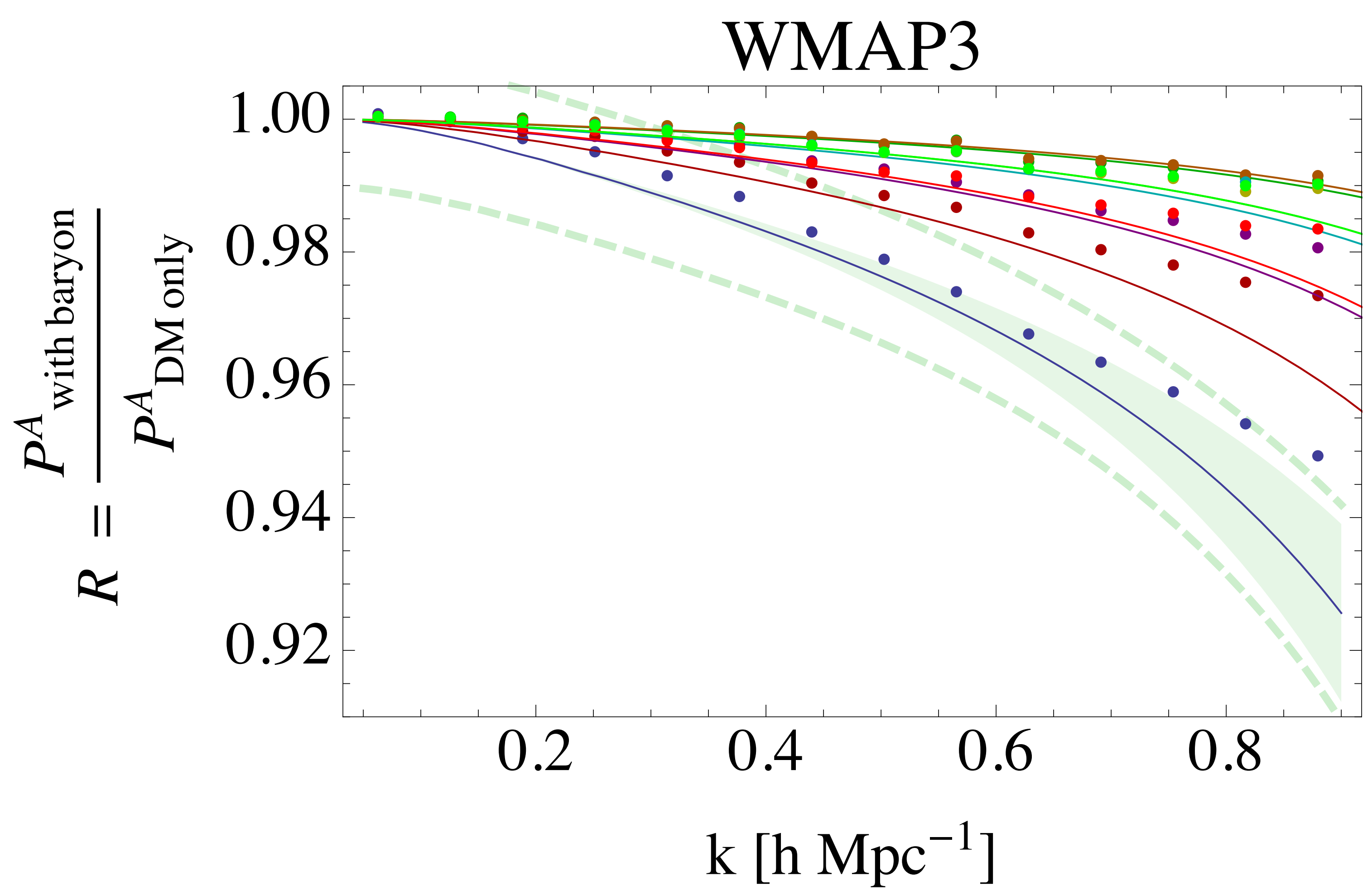
credit: Millenium Simulation,
Springel *et al*. (2005)

- They cannot be reliably simulated due to large range of scales

Baryons

- Idea for EFT for dark matter:
  - Dark Matter moves $1/k_{\rm NL} \sim 10\,{\rm Mpc}$
    - $\Longrightarrow$ an effective fluid-like system with mean free path ~ $1/k_{\rm NL}$
- Baryons heat due to star formation, but move the same:
  - Universe with CDM+Baryons $\Longrightarrow$ EFTofLSS with 2 specie

$$\Delta P_b(k) \simeq c_\star^2 \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}^A(k)$$



Baryons

- EFT Equations:

$$\text{Continuity:} \quad \dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i \pi^i_\sigma = 0 \ ,$$

$$\text{Momentum:} \quad \dot{\pi}^i_c + 4H\pi^i_c + a^{-1}\partial_j\left(\frac{\pi^i_c\pi^j_c}{\rho_c}\right) + a^{-1}\rho_c\partial_i\Phi = +a^{-1}\gamma^i - a^{-1}\partial_j\tau^{ij}_c \ ,$$

$$\dot{\pi}^i_b + 4H\pi^i_b + a^{-1}\partial_j\left(\frac{\pi^i_b\pi^j_b}{\rho_b}\right) + a^{-1}\rho_b\partial_i\Phi = -a^{-1}\gamma^i - a^{-1}\partial_j\tau^{ij}_b \ .$$

Baryons

- EFT Equations:

$$\text{Continuity:} \quad \dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i \pi^i_\sigma = 0 \ ,$$

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$$\dot{\pi}^i_b + 4H\pi^i_b + a^{-1}\partial_j \left(\frac{\pi^i_b \pi^j_b}{\rho_b} \right) + a^{-1}\rho_b \partial_i \Phi = -a^{-1}\gamma^i - a^{-1}\partial_j \tau^{ij}_b \ .$$

effective force

dynamical friction

- Counterterms:

$$\gamma^i \propto v^i_{\rm rel}$$

no derivative: marginal operator

A relevant operator

- Dynamical friction term is indeed needed for renormalization of the theory, i.e. it is generated.

- Dynamical friction is a relevant operator: i.e. it cannot be treated perturbatively: it is an essential part of the linear *equations:*

$$a^2\delta_I^{(1)\prime\prime}(a,\vec{k})+\left(2+\frac{a\mathcal{H}'(a)}{\mathcal{H}(a)}\right)a\delta_I^{(1)\prime}(a,\vec{k})=\int^a da_1 g(a,a_1)a_1\delta_I^{(1)\prime}(a_1,\vec{k})\ .$$

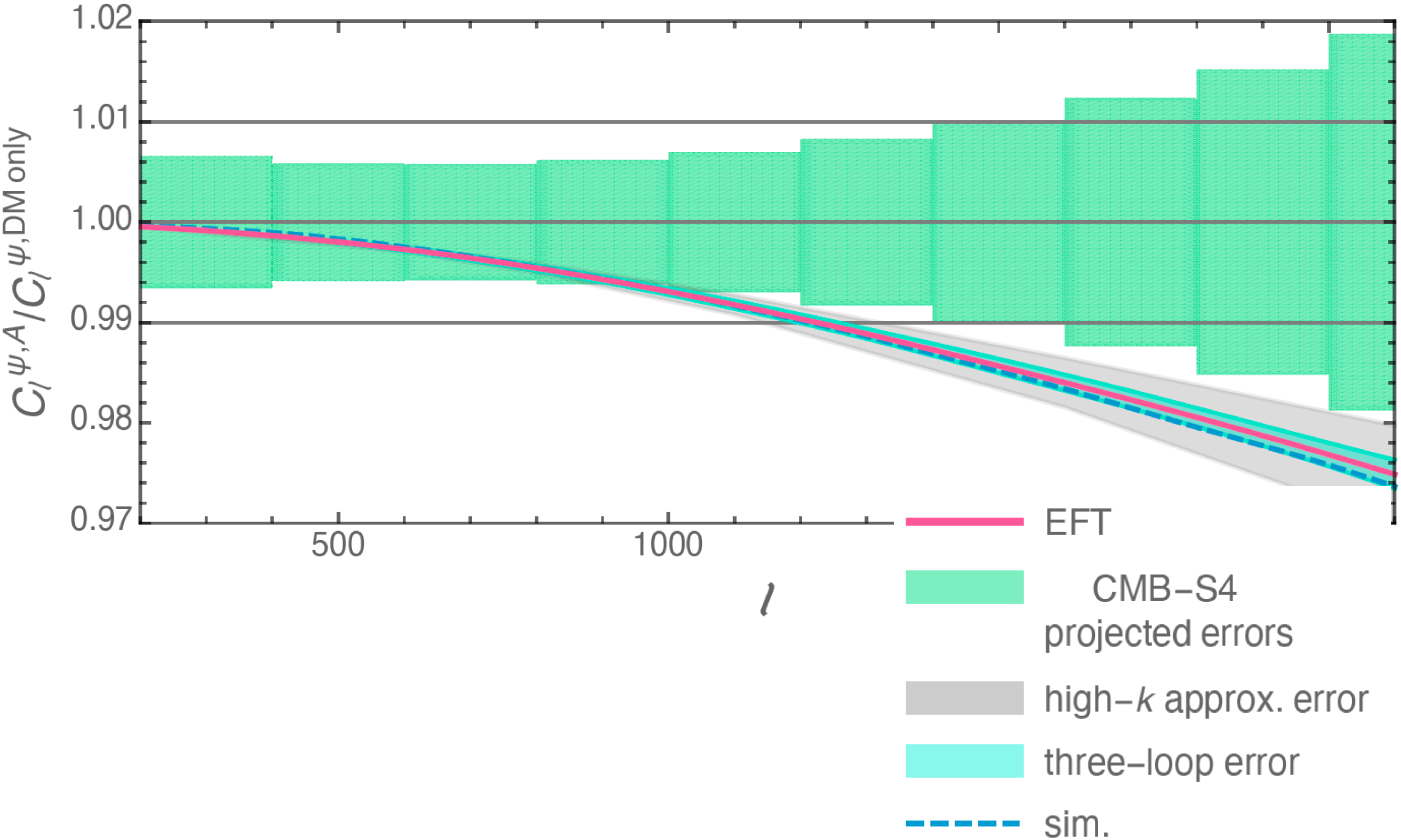
  – due to the time-translation breaking and actually even non-locality, very very very very very very hard to handle consistently.

    - we can make some guesses

- Luckily: it only affect the decaying mode of the isocurvature, which is very very very very very small.

Predictions for CMB Lensing

- Baryon corrections are detectable in next CMB S-4 experiments. But we can predict it:



Bispectrum at one loop

with D'Amico, Donath, Lewandowski, Zhang **2206**

Bispectrum

- The tree level bispectrum had been already used for cosmological parameter analysis in

  with Guido D'Amico, Jerome Gleyzes,
  Nickolas Kockron, Dida Markovic, Pierre Zhang, Florian Beutler, Hector Gill-Marin **1909.05271**

  Philcox, Ivanov **2112**

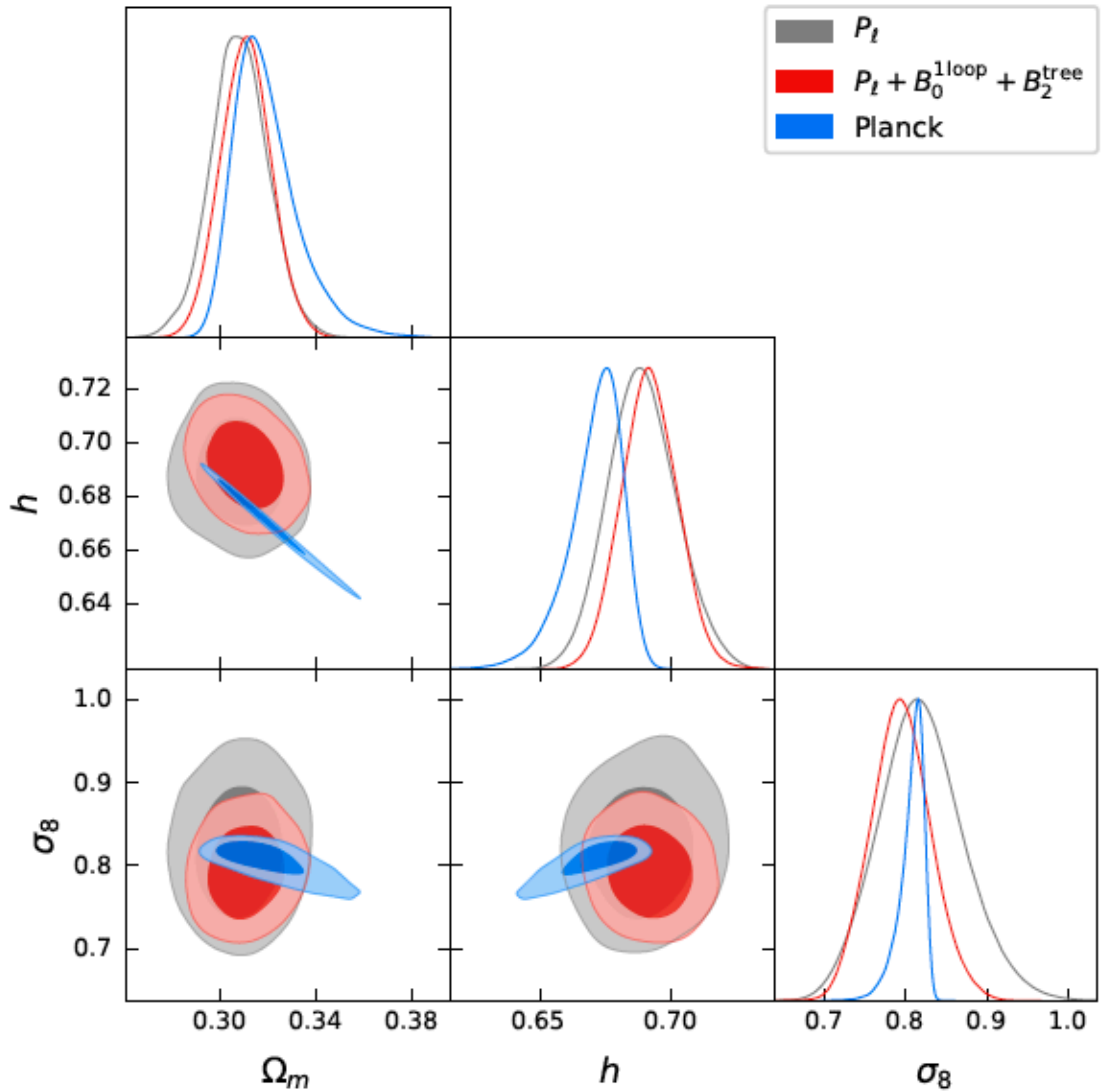
  - ~10% improvement on A_s

- Time to move to one-loop:
  - Large effort:
    - data analysis with D'Amico, Donath, Lewandowski, Zhang **2206**
    - theory model with D'Amico, Donath, Lewandowski, Zhang **2211**
    - theory integration with Anastasiou, Braganca, Zheng **2212**

Data Analysis

with D'Amico, Donath, Lewandowski, Zhang **2206**

- Main result:
  - Improvements:
  - 30% on σ_8
  - 18% on h
  - 13% on Ω_m



- Compatible with Planck
  – no tensions
- Often Planck Comparable

Theory Model

with D'Amico, Donath, Lewandowski, Zhang **2206**

- We add all the relevant biases (4th order) and counterterms (2nd order):

$$P_{11}^{r,h}[b_1]\ ,\quad P_{13}^{r,h}[b_1,b_3,b_8]\ ,\quad P_{22}^{r,h}[b_1,b_2,b_5]\ ,$$
$$B_{211}^{r,h}[b_1,b_2,b_5]\ ,\quad B_{321}^{r,h,(II)}[b_1,b_2,b_3,b_5,b_8]\ ,\quad B_{411}^{r,h}[b_1,\dots,b_{11}]\ ,$$
$$B_{222}^{r,h}[b_1,b_2,b_5]\ ,\quad B_{321}^{r,h,(I)}[b_1,b_2,b_3,b_5,b_6,b_8,b_{10}]\ ,$$

$$P_{13}^{r,h,ct}[b_1,c_{h,1},c_{\pi,1},c_{\pi v,1},c_{\pi v,3}]\ ,\quad P_{22}^{r,h,\epsilon}[c_1^{\text{St}},c_2^{\text{St}},c_3^{\text{St}}]\ ,$$
$$B_{321}^{r,h,(II),ct}[b_1,b_2,b_5,c_{h,1},c_{\pi,1},c_{\pi v,1},c_{\pi v,3}]\ ,\quad B_{321}^{r,h,\epsilon,(I)}[b_1,c_1^{\text{St}},c_2^{\text{St}},\{c_i^{\text{St}}\}_{i=4,\dots,13}]\ ,$$
$$B_{411}^{r,h,ct}[b_1,\{c_{h,i}\}_{i=1,\dots,5},c_{\pi,1},c_{\pi,5},\{c_{\pi v,j}\}_{j=1,\dots,7}]\ ,\quad B_{222}^{r,h,\epsilon}[c_1^{(222)},c_2^{(222)},c_5^{(222)}]\ .$$

- IR-resummation:
  - For the power spectrum, we use the correct and controlled IR-resummation.
  - For the bispectrum, we use the wiggle/no-wiggle approximation   Ivanov and Sibiryakov **2018**

$$B_{211}^{r,h}=2K_1^{r,h}(\vec{k}_1;\hat{z})K_1^{r,h}(\vec{k}_2;\hat{z})K_2^{r,h}(\vec{k}_1,\vec{k}_2;\hat{z})P_{\text{LO}}(k_1)P_{\text{LO}}(k_2)+\ 2\text{ perms.}\ ,$$
$$P_{\text{LO}}(k)=P_{\text{nw}}(k)+(1+k^2\Sigma_{\text{tot}}^2)e^{-\Sigma_{\text{tot}}^2}P_{\text{w}}(k)$$

    - For the loop, we just use $P_{\text{NLO}}(k)=P_{\text{nw}}(k)+e^{-\Sigma_{\text{tot}}^2}P_{\text{w}}(k)\ ,$ in the non-integrated power spectra

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang
2211

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang **2211**

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Renormalization of velocity
  - In the EFTofLSS, the velocity is a composite operator $v^i(x) = \frac{\pi^i(x)}{\rho(x)}$, so, it needs to be renormalized:

$$[v^i]_R = v^i + \mathcal{O}_v^i \ ,$$

  - Under a diffeomorphisms:

$$v^i \to v^i + \chi^i \quad \Rightarrow \quad \mathcal{O}_v^i \text{ is a scalar}$$

  - In redshift space, we have local product of velocities, which need to be renormalized but have non-trivial transformations under diff.s:

$$[v^i v^j]_R \to [v^i v^j]_R + [v^i]_R \chi^j + [v^j]_R \chi^i + \chi^i \chi^j$$

  - To achieve this, one can do: (so must include products $v^i \cdot \mathcal{O}_v^i$)

$$[v^i v^j]_R = [v^i]_R [v^j]_R + \mathcal{O}_{v^2}^{ij} \ , \quad \text{where } \mathcal{O}_{v^2}^{ij} \text{ is a scalar}$$

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang **2211**

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Non-local-contributing counterterm.
  - This is a normal effect, just strange-looking in the EFTofLSS context.
  - Normally, counterterms are local, but, contributing through non-local Green's functions, they contribute non-locally.

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang
2211

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Non-local-contributing counterterm.
  - In the EFTofLSS, the Green's function is simple: $\dfrac{1}{\partial^2}$
  - Counterterms typically come with $\partial^2 \mathcal{O}_{\rm local} \quad \Rightarrow \quad \delta_{\rm counter} \sim \dfrac{1}{\partial^2}\partial^2 \mathcal{O}_{\rm local} \sim \mathcal{O}_{\rm local}$
    - result almost trivial
  - But at second order, and for velocity fields, contracted along the line of sight, the derivative do not cancel, so we get

$$\delta_{\rm counter}(\vec{x}) \sim \hat{z}^i \hat{z}^j \partial_i \pi^j_{(2)}(\vec{x}) \quad \sim \quad \hat{z}^i \hat{z}^j \frac{\partial_i \partial_j \partial_k \partial_m}{\partial^2} \mathcal{O}_{\rm local}$$

$$\sim \quad \hat{z}^i \hat{z}^j \frac{\partial_i \partial_j \partial_k \partial_m}{\partial^2} \left(\frac{\partial_k \partial_l}{H^2} \Phi(\vec{x}) \frac{\partial_l \partial_m}{H^2} \Phi(\vec{x}) \right)$$

  - This is truly non-locally contributing, truly non-trivial.

  - We check that all these terms are *needed and sufficient* for renormalization

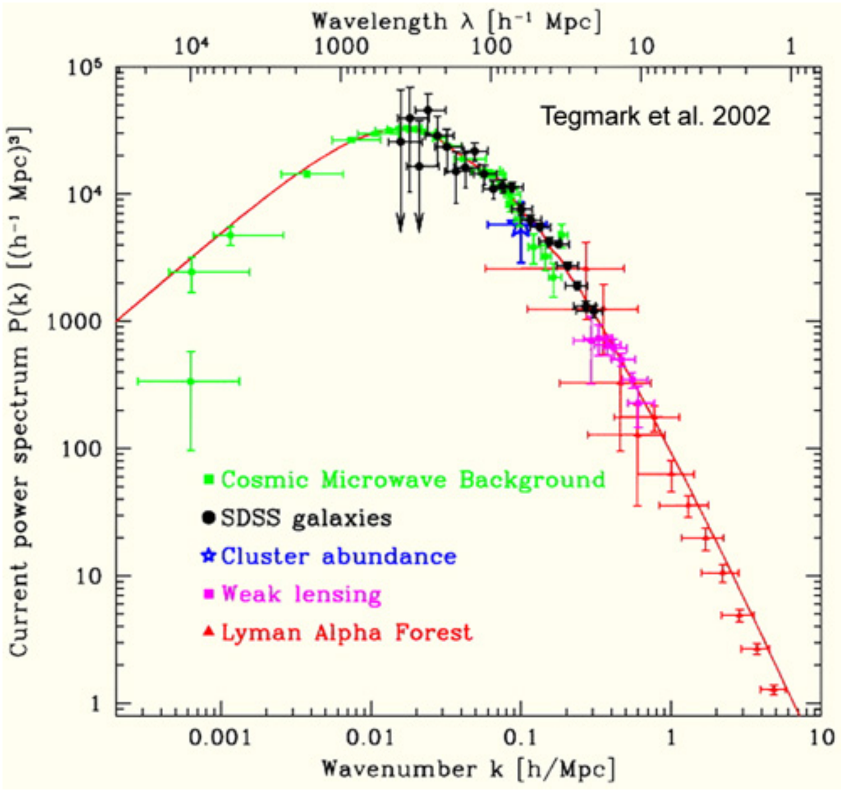
Evaluational/Computational Challenge

with Anastasiou, Braganca, Zheng **2212**

The best approach so far

Simonovic, Baldauf, Zaldarriaga, Carrasco, Kollmeier **2018**

- Nice trick for fast evaluation of the loops integrals
- The power spectrum is a numerically computed function
- Decompose linear power spectrum



$$P_{11}(k) = \sum_n c_n \, k^{\mu + i\alpha\, n}$$

- Loop can be evaluated analytically

$$P_{1-\text{loop}}(k) = \int_{\vec{q}} K(\vec{q},\vec{k}) \; P_{11}(k-q)\; P_{11}(q) =$$

$$= \sum_{n_1,n_2} c_{n_1} c_{n_2} \left(\int_{\vec{q}} K(\vec{q},\vec{k}) \; k^{\mu+i\alpha n_1} \; k^{\mu+i\alpha n_2} \right) = \sum_{n_1,n_2} c_{n_1} c_{n_2} M_{n_1,n_2}(k)$$

–using quantum field theory techniques

– $M_{n_1 n_2}$ is cosmology independent $\Rightarrow$ so computed once

Computational Challenge

Philcox, Ivanov, Cabass, Simonovic, Zaldarriaga **2022**

- Two difficulties:

$$P_{1-\text{loop}}(k) = \int_{\vec{q}} K(\vec{q}, \vec{k})\ P_{11}(k-q)\ P_{11}(q) =$$

$$= \sum_{n_1, n_2} c_{n_1} c_{n_2} \left(\int_{\vec{q}} K(\vec{q}, \vec{k})\ k^{\mu + i\alpha n_1}\ k^{\mu + i\alpha n_2} \right) = \sum_{n_1, n_2} c_{n_1} c_{n_2} M_{n_1, n_2}(k)$$

  - integrals are complicated due to fractional, complex exponents
  - many functions needed, the matrix $M_{n_1 n_2 n_3}$ for bispectrum is about 50Gb, so, ~impossible to load on CPT for data analysis

- In order to ameliorate (solve) these issues, we use a different basis of functions.

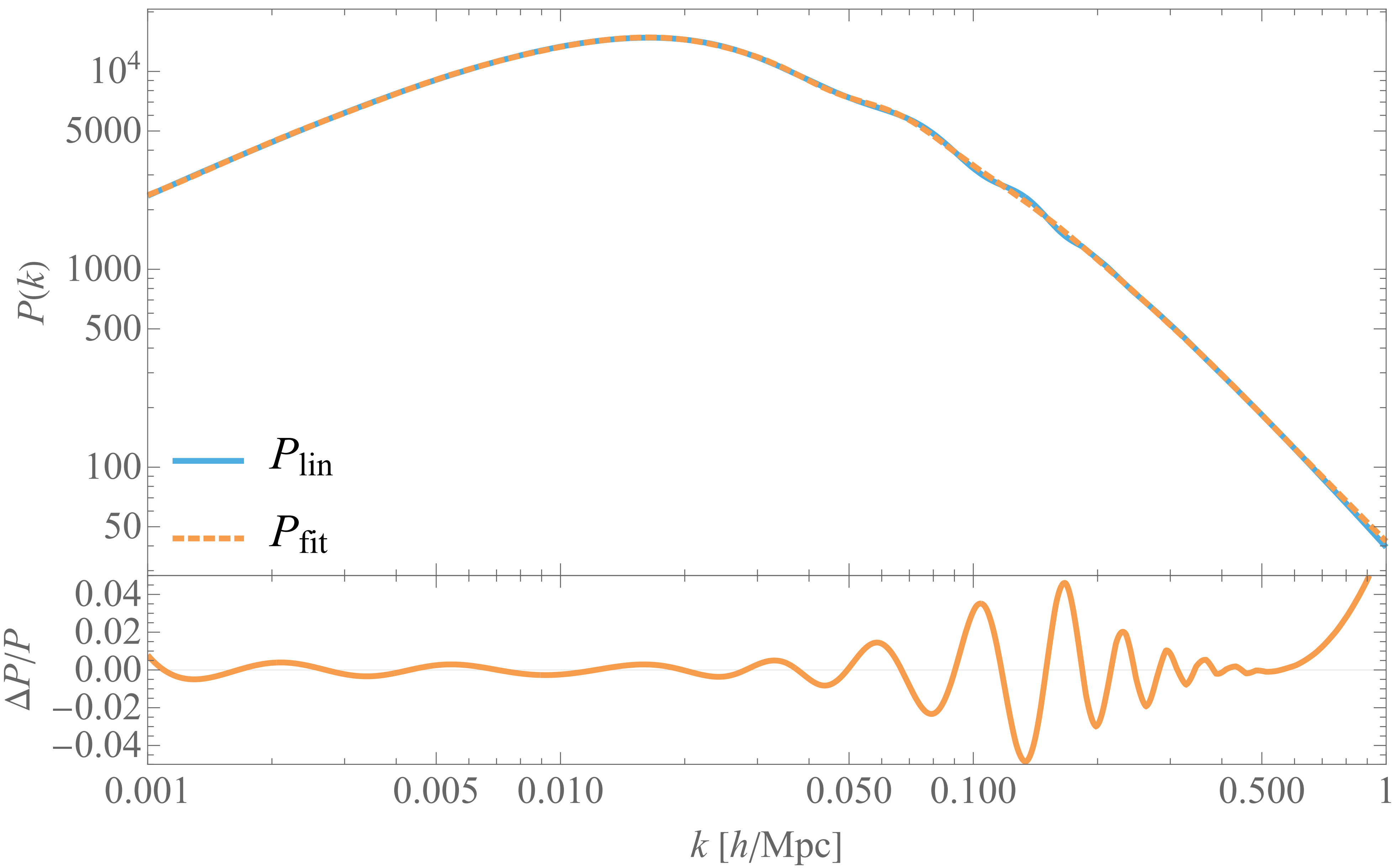
Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- Use as basis:

$$f(k^2, k^2_{\rm peak}, k^2_{\rm UV}, i, j) \equiv \frac{\left(k^2/k_0^2\right)^i}{\left(1+\frac{(k^2-k^2_{\rm peak})^2}{k^4_{\rm UV}}\right)^j},$$

- With just 16 functions:



Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- This basis is equivalent to massive propagators to integer powers

$$\frac{1}{\left(1+\frac{(k^2-k_{\rm peak}^2)^2}{k_{\rm UV}^4}\right)^j}=\frac{k_{\rm UV}^{4j}}{\left(k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2\right)^j\left(k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2\right)^j},$$

$$\frac{k_{\rm UV}^2}{\left(k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2\right)\left(k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2\right)}=-\frac{i/2}{k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2}+\frac{i/2}{k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2}$$

- So, each basis function:

$$f(k^2,k_{\rm peak}^2,k_{\rm UV}^2,i,j)=\sum_{n=1}^{j}k_{\rm UV}^{2(n-i)}\,k^{2i}\left(\frac{\kappa_n}{(k^2+M)^n}+\frac{\kappa_n^*}{(k^2+M^*)^n}\right)$$

Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- This basis is equivalent to massive propagators to integer powers

$$\frac{1}{\left(1+\frac{(k^2-k_{\rm peak}^2)^2}{k_{\rm UV}^4}\right)^j} = \frac{k_{\rm UV}^{4j}}{\left(k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2\right)^j\left(k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2\right)^j},$$

$$\frac{k_{\rm UV}^2}{\left(k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2\right)\left(k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2\right)} = -\frac{i/2}{k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2}+\frac{i/2}{k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2}$$

Complex-Mass propagator

- So, each basis function:

$$f(k^2,k_{\rm peak}^2,k_{\rm UV}^2,i,j) = \sum_{n=1}^{j} k_{\rm UV}^{2(n-i)}\,k^{2i}\left(\frac{\kappa_n}{(k^2+M)^n}+\frac{\kappa_n^*}{(k^2+M^*)^n}\right)$$

Complex-Masses Propagators

with Anastasiou, Braganca, Zheng **2212**

- We end up with integral like this:

$$L(n_1,d_1,n_2,d_2,n_3,d_3) = \int_q \frac{(\boldsymbol{k}_1-\boldsymbol{q})^{2n_1}\boldsymbol{q}^{2n_2}(\boldsymbol{k}_2+\boldsymbol{q})^{2n_3}}{((\boldsymbol{k}_1-\boldsymbol{q})^2+M_1)^{d_1}(\boldsymbol{q}^2+M_2)^{d_2}((\boldsymbol{k}_2+\boldsymbol{q})^2+M_3)^{d_3}}$$

- with integer exponents.
- First we manipulate the numerator to reduce to:

$$T(d_1,d_2,d_3) = \int_q \frac{1}{((\boldsymbol{k}_1-\boldsymbol{q})^2+M_1)^{d_1}(\boldsymbol{q}^2+M_2)^{d_2}((\boldsymbol{k}_2+\boldsymbol{q})^2+M_3)^{d_3}},$$

- Then, by integration by parts, we find (i.e. QCD teaches us how to) recursion relations

$$\int_q \frac{\partial}{\partial q_\mu}\cdot\left(q_\mu t(d_1,d_2,d_3)\right) = 0$$

$$\Rightarrow \quad (3-d_{1223})\hat{0} + d_1 k_{1s}\widehat{1^+} + d_3(k_{2s})\widehat{3^+} + 2M_2 d_2\widehat{2^+} - d_1\widehat{1^+}\widehat{2^-} - d_3\widehat{2^-}\widehat{3^+} = 0$$

- relating same integrals with raised or lowered the exponents (easy terminate due to integer exponents).

Complex-Masses Propagators

with Anastasiou, Braganca, Zheng **2212**

- We end up to three master integrals:
- Tadpole:

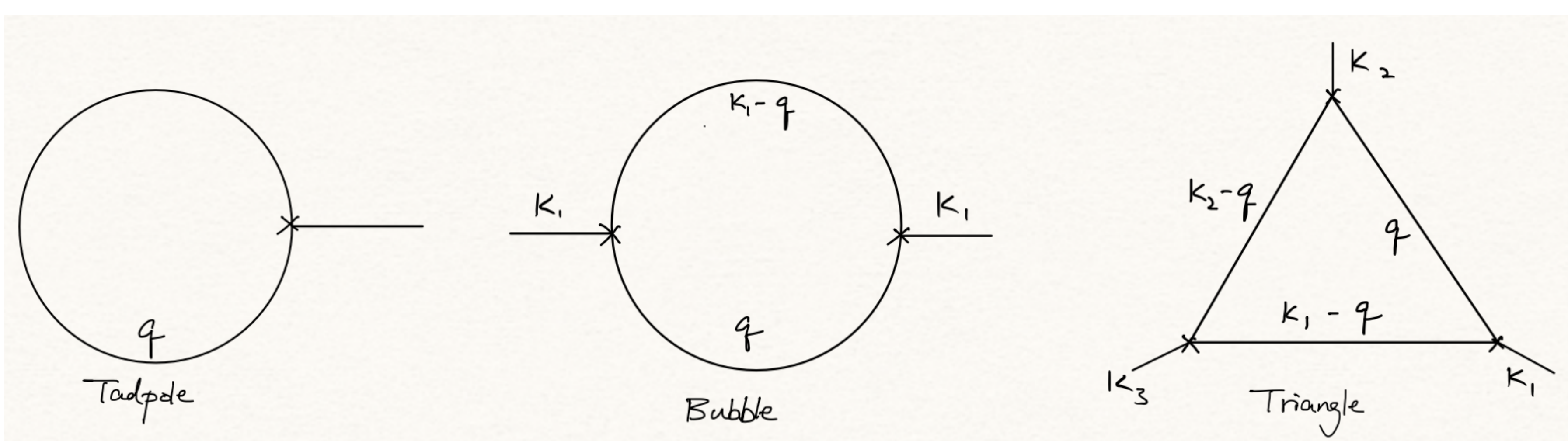
$$\text{Tad}(M_j, n, d) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{(\boldsymbol{p}_i^2)^n}{(\boldsymbol{p}_i^2 + M_j)^d}$$

- Bubble:

$$B_{\text{master}}(k^2, M_1, M_2) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\boldsymbol{k} - \boldsymbol{q}|^2 + M_2)}$$

- Triangle:

$$T_{\text{master}}(k_1^2, k_2^2, k_3^2, M_1, M_2, M_3) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\boldsymbol{k}_1 - \boldsymbol{q}|^2 + M_2)(|\boldsymbol{k}_2 + \boldsymbol{q}|^2 + M_3)},$$



Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- The master integrals are evaluated with Feynman parameters, but with great care of branch cut crossing, which happens because of complex masses.

- Bubble Master:

$$B_{\text{master}}(k^2, M_1, M_2) = \frac{\sqrt{\pi}}{k} i\left[\log\left(A(1,m_1,m_2)\right) - \log\left(A(0,m_1,m_2)\right) - 2\pi i H(\operatorname{Im} A(1,m_1,m_2)) H(-\operatorname{Im} A(0,m_1,m_2))\right],$$

$$A(0,m_1,m_2) = 2\sqrt{m_2} + i(m_1 - m_2 + 1),$$
$$A(1,m_1,m_2) = 2\sqrt{m_1} + i(m_1 - m_2 - 1),$$
$$m_1 = M_1/k^2 \text{ and } m_2 = M_2/k^2$$

- Triangle Master:

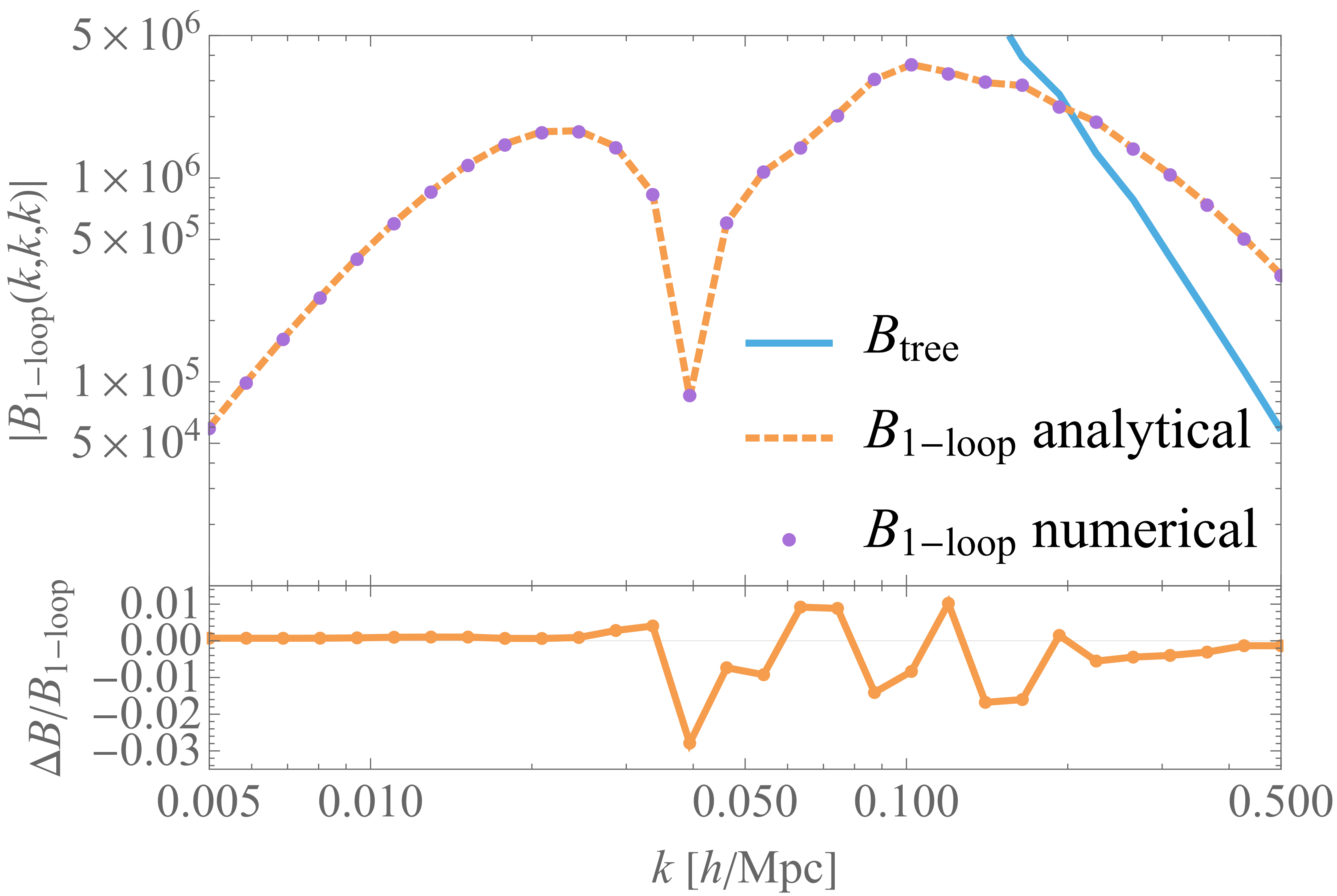
$$F_{\text{int}}(R_2, z_+, z_-, x_0) = s(z_+, -z_-) \frac{\sqrt{\pi}}{\sqrt{|R_2|}} \left. \frac{\arctan\left(\frac{\sqrt{z_+ - x}\sqrt{x_0 - z_-}}{\sqrt{x_0 - z_+}\sqrt{x - z_-}}\right)}{\sqrt{x_0 - z_+}\sqrt{x_0 - z_-}} \right|_{x=0}^{x=1}.$$

- Very simple expressions with simple rule for branch cut crossing.

Result of Evaluation

with Anastasiou, Braganca, Zheng **2212**

- All automatically coded up.
- For BOSS analysis, evaluation of matrix is 2.5CPU hours and 800 Mb storage, very fast matrix contractions.
- Accuracy with 16 functions:

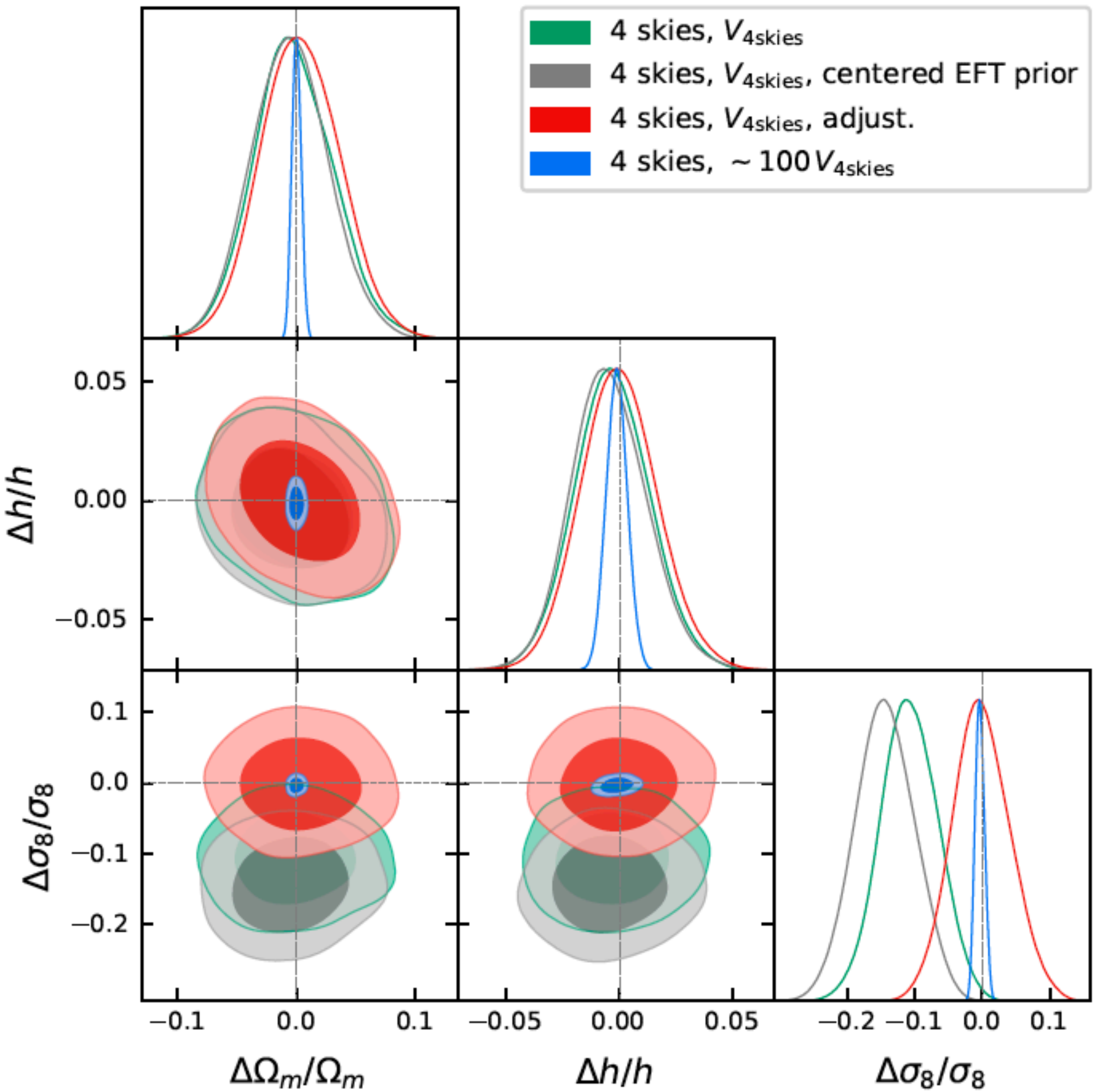


Back to data-analysis: Pipeline Validation

with D'Amico, Donath, Lewandowski, Zhang **2206**

Measuring and fixing phase space

- We consider synthetic data, i.e. data made out of the model, and analyze them:
  - Green: biased.
- Why?
  - –Priors centered on zero?
    - Grey: biased
  - –Bug in pipeline?
    - Test by reducing covar.
    - Red: non-biased
- It must be phase space projection
- But the grey line offers
  - –an honest measurement of it.
-

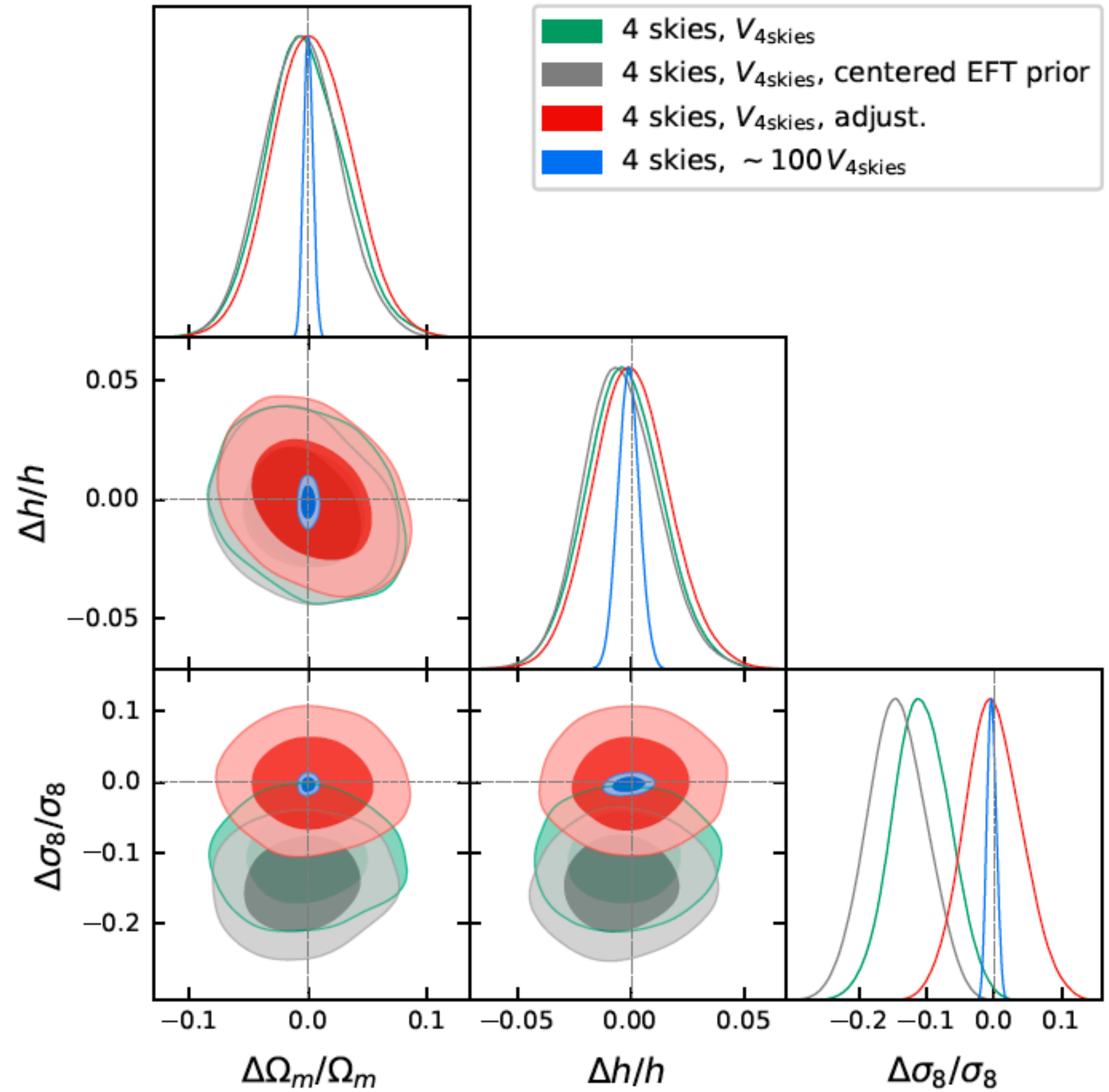


Measuring and fixing phase space

- We add:

$$\ln \mathcal{P}_{\rm pr}^{\rm ph.\ sp.\ 4sky} = -48\left(\frac{b_1}{2}\right) + 32\left(\frac{\Omega_m}{0.31}\right) + 48\left(\frac{h}{0.68}\right) \ ,$$

$\sigma_{\rm proj}/\sigma_{\rm stat}$	Ω_m	h	σ_8	ω_{cdm}
1 sky, $\sim 100\, V_{\rm 1sky}$	-0.1	-0.14	-0.21	-0.2
1 sky, $V_{\rm 1sky}$, adjust.	0.13	0.06	0.04	0.15
4 skies, $V_{\rm 4skies}$, adjust.	0.1	0.	-0.05	0.07



- no more proj. effect.

Scale cut from NNLO

with D'Amico, Donath, Lewandowski, Zhang **2206**

- We can estimate the $k_{\rm max}$ without the use of simulations, by adding NNLO terms, and seeing when they make a difference on the posteriors.

$$P_{\rm NNLO}(k,\mu) = \frac{1}{4} c_{r,4} b_1^2 \mu^4 \frac{k^4}{k_{\rm NL,R}^4} P_{11}(k) + \frac{1}{4} c_{r,6} b_1 \mu^6 \frac{k^4}{k_{\rm NL,R}^4} P_{11}(k)\,,$$

$$B_{\rm NNLO}(k_1,k_2,k_3,\mu,\phi) = 2 c_{\rm NNLO,1} K_2^{r,h}(\vec{k}_1,\vec{k}_2;\hat{z}) K_1^{r,h}(\vec{k}_2;\hat{z}) f \mu_1^2 \frac{k_1^4}{k_{\rm NL,R}^4} P_{11}(k_1) P_{11}(k_2)$$

$$+ c_{\rm NNLO,2} K_1^{r,h}(\vec{k}_1;\hat{z}) K_1^{r,h}(\vec{k}_2;\hat{z}) P_{11}(k_1) P_{11}(k_2) f \mu_3 k_3 \frac{(k_1^2+k_2^2)}{4 k_1^2 k_2^2 k_{\rm NL,R}^4} \Big[-2 \vec{k}_1 \cdot \vec{k}_2 (k_1^3 \mu_1 + k_2^3 \mu_2)$$

$$+ 2 f \mu_1 \mu_2 \mu_3 k_1 k_2 k_3 (k_1^2 + k_2^2) \Big] + \text{perm.}\,, \quad (4$$

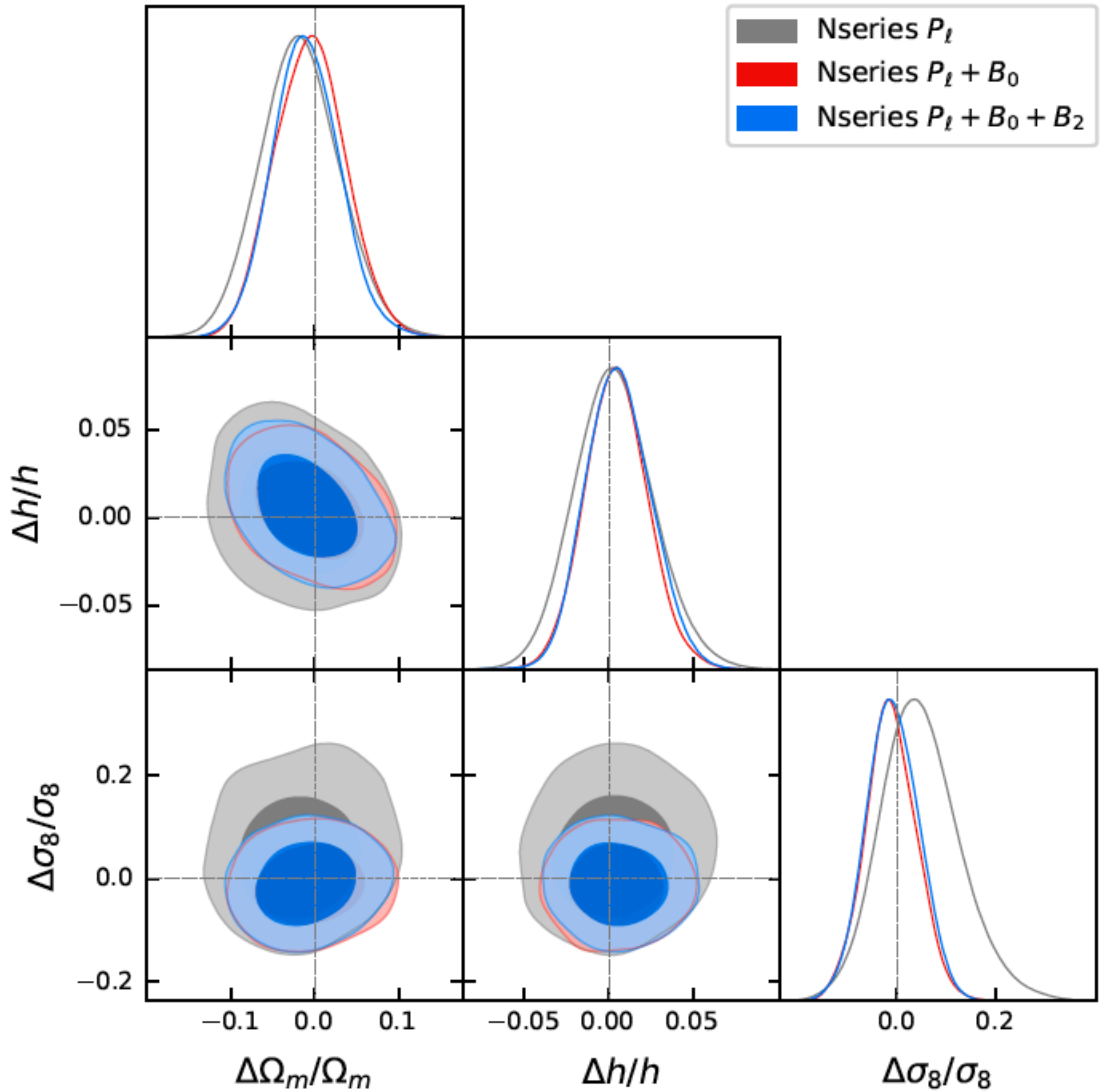
- For our $k_{\rm max}$, we find the following shifts, which are ok:

$\Delta_{\rm shift}/\sigma_{\rm stat}$	Ω_m	h	σ_8	ω_{cdm}	$\ln(10^{10} A_s)$	S_8
$P_\ell + B_0$: base - w/ NNLO	-0.03	-0.09	-0.03	-0.1	0.05	-0.04

Scale-cut from simulations

with D'Amico, Donath, Lewandowski, Zhang **2206**

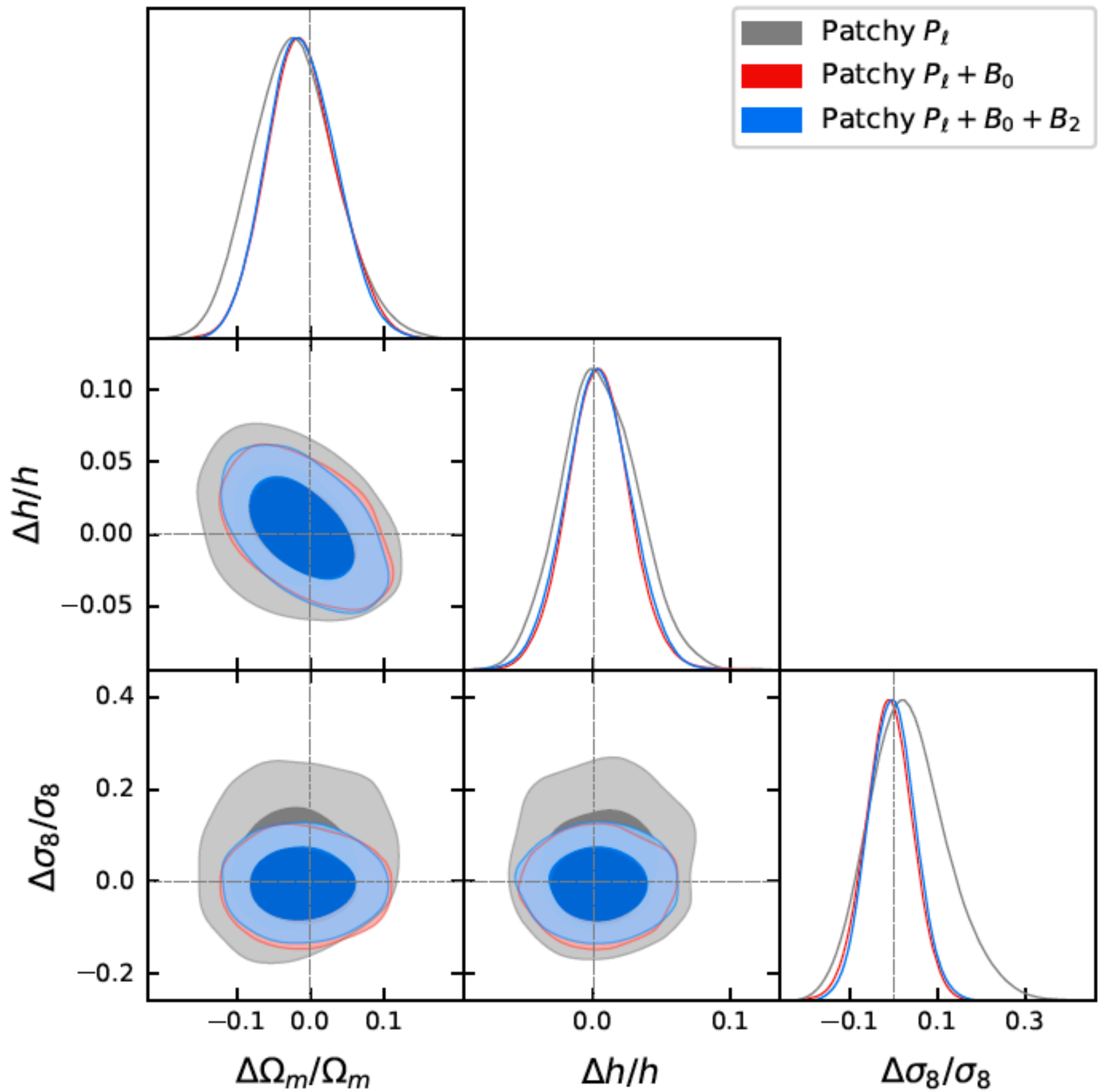
- N-series
  - Volume ~80 BOSS
  - safely within $\sigma_{\rm data}/3$
- After phase-space correction



Scale-cut from simulations

with D'Amico, Donath, Lewandowski, Zhang **2206**

- Patchy:
  - Volume ~2000 BOSS
  - safely within $\sigma_{\rm data}/3$
- After phase-space correction

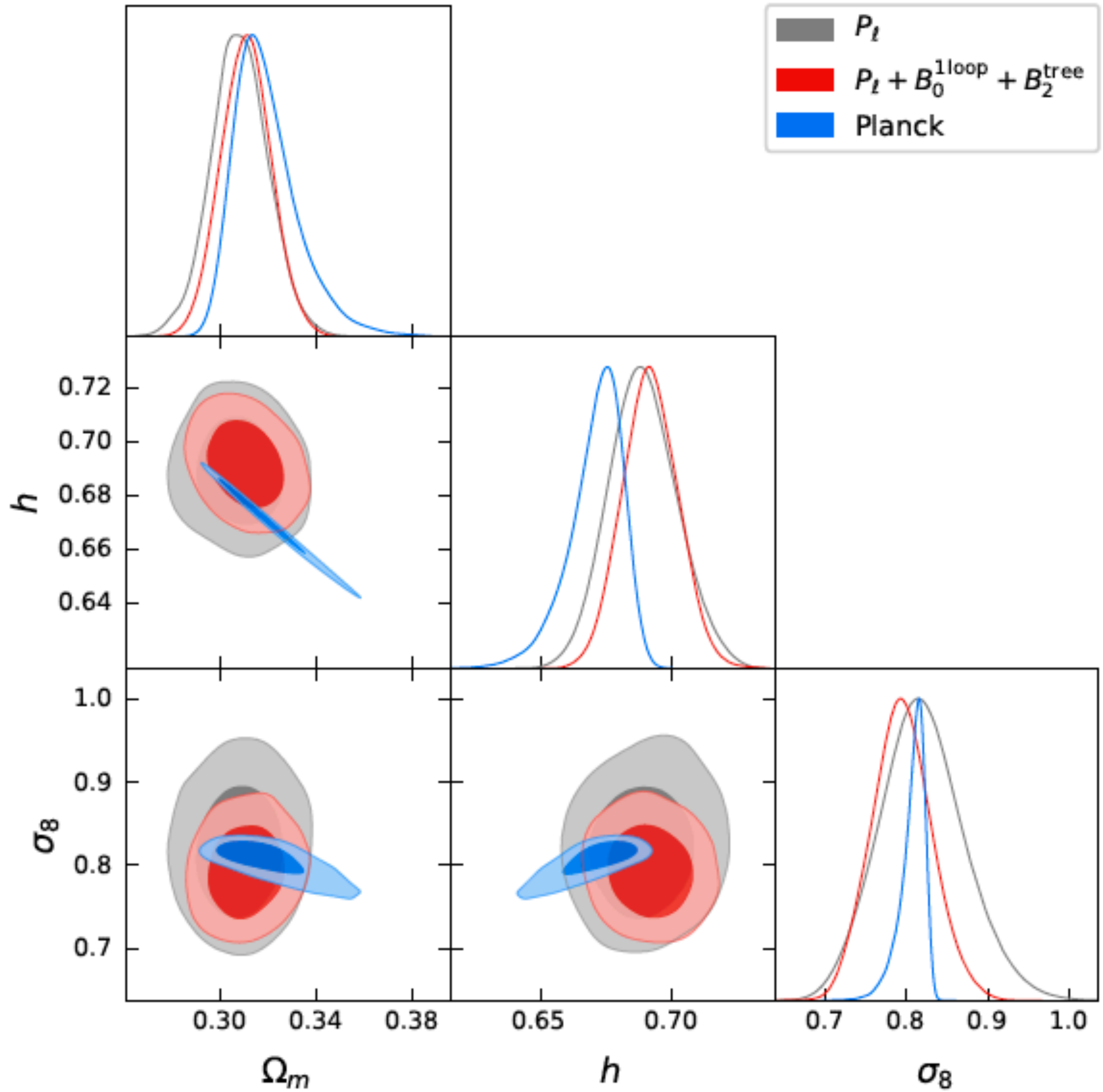


BOSS data

Data Analysis

with D'Amico, Donath, Lewandowski, Zhang **2206**

- Main result:
  - Improvements:
  - 30% on σ_8
  - 18% on h
  - 13% on Ω_m



- Compatible with Planck
  - no tensions
- Remarkable consistency
  - of observables

Direct Measurement of formation time of galaxies

with Donath and Lewandowski **2307**

Galaxies in the EFTofLSS

Senatore **1406**
Mirbabayi et al. **1412**

$$n_{\rm gal}(x) = f_{\rm very\ complicated}\left(\{H, \Omega_m, \ldots, m_e, g_{ew}, \ldots, \rho(x)\}_{\rm past\ light\ cone}\right)$$

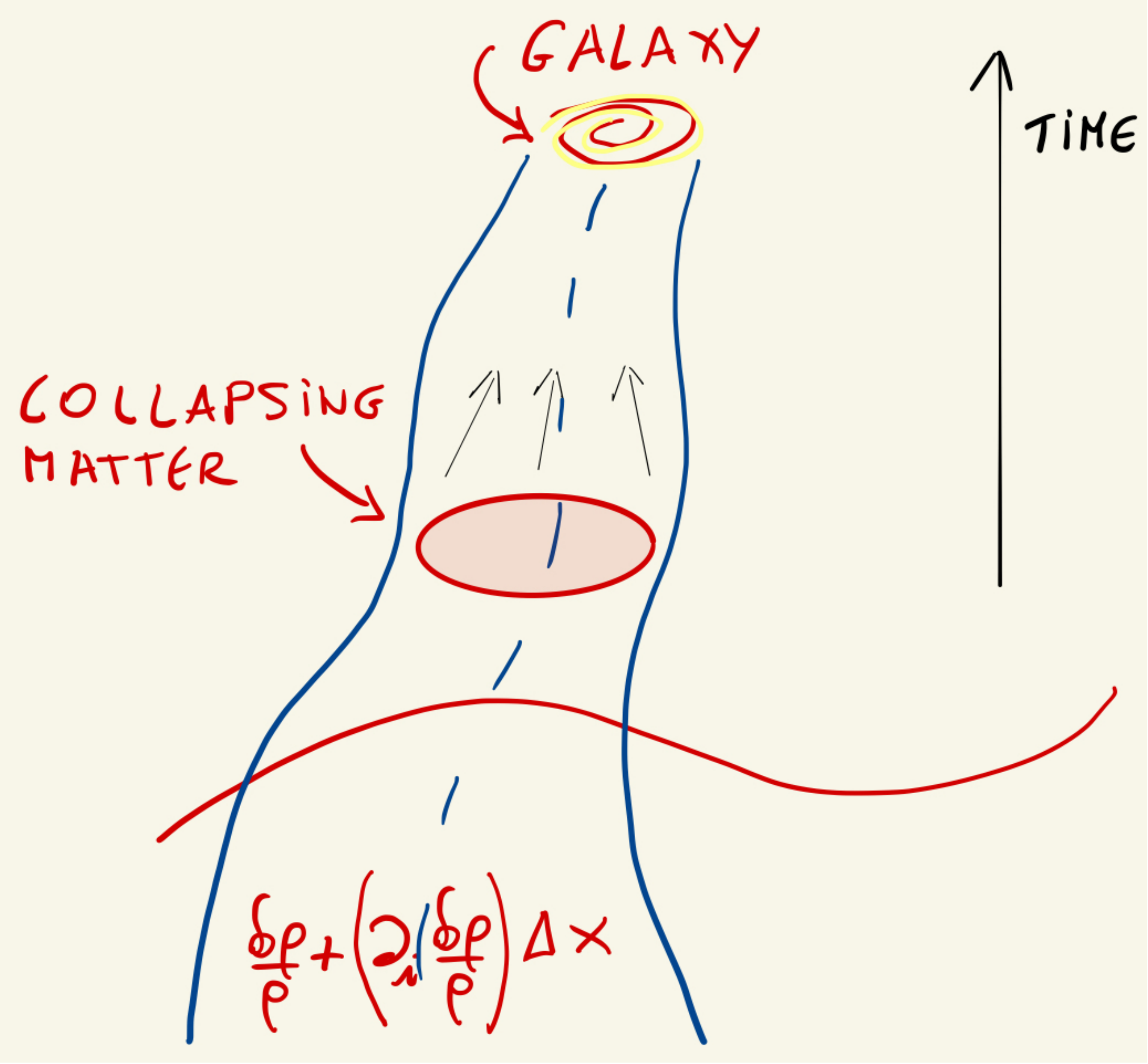
At *long* wavelengths $\Downarrow$ Taylor Expansion

$$\left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x) \sim \int^t dt' \left[c(t,t') \left(\frac{\delta\rho}{\rho}\right)(\vec{x}_{\rm fl}, t') + \ldots\right]$$

- all terms allowed by symmetries
- all physical effects included
  - e.g. assembly bias
- .

$$\left\langle \left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x) \left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(y) \right\rangle =$$

$$= \sum_n {\rm Coeff}_n \cdot \langle \text{matter correlation function} \rangle_n$$



Consequences of non-locality in time

- This means that one *does not* get the same terms as in the local-in-time expansion

- If we could measure one of these terms, we could *measure* that Galaxies take an Hubble time to form. We have never measured this: we take pictures of different galaxies at different stages of their evolution. But we have never *seen* a galaxy form in an Hubble time.
  - This would be the first direct evidence that the universe lasted an Hubble time.

- So, detecting a non-local-in-time bias would allow us to measure that, and from the size, the formation time. Unfortunately, so far, not yet.

Consequences of non-locality in time

- Mathematics again:
  - non-local in time:

$$\delta_g^{(n)}(\vec{x},t) = \sum_{\mathcal{O}_m} \int^t dt' H(t') c_{\mathcal{O}_m}(t,t') \times [\mathcal{O}_m(\vec{x}_{\rm fl}(\vec{x},t,t'),t')]^{(n)} \ ,$$

$$\mathcal{O}_{m=3} \supset \delta^2\theta, \delta^3, \dots$$

  - local in time:

$$\Rightarrow \quad \delta_{g,\rm loc}^{(n)}(\vec{x},t) = \sum_{\mathcal{O}_m} c_{\mathcal{O}_m}(t)\, \mathcal{O}_m^{(n)}(\vec{x},t) \ ,$$

  - more non local in time:

$$[\mathcal{O}_m(\vec{x}_{\rm fl}(\vec{x},t,t'),t')]^{(n)} = \sum_{\alpha=1}^{n-m+1} \left(\frac{D(t')}{D(t)}\right)^{\alpha+m-1} \mathbb{C}_{\mathcal{O}_m,\alpha}^{(n)}(\vec{x},t)$$

$$\Rightarrow \quad \delta_g^{(n)}(\vec{x},t) = \sum_{\mathcal{O}_m} \sum_{\alpha=1}^{n-m+1} c_{\mathcal{O}_m,\alpha}(t)\, \mathbb{C}_{\mathcal{O}_m,\alpha}^{(n)}(\vec{x},t)$$

Consequences of non-locality in time

$$\delta^{(n)}_{g,\rm loc}(\vec x,t)=\sum_{\mathcal{O}_m} c_{\mathcal{O}_m}(t)\,\mathcal{O}^{(n)}_m(\vec x,t)\ , \qquad \delta^{(n)}_{g}(\vec x,t)=\sum_{\mathcal{O}_m}\sum_{\alpha=1}^{n-m+1} c_{\mathcal{O}_m,\alpha}(t)\,\mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec x,t)$$

- it turns out that up to 4th order, the two basis of operators were identical.
- but at 5th order they are not!
  - out of 29 independent operators, 3 cannot be written as local in time ones.
- $\Rightarrow$ By looking at, eg,

$$\langle\delta^{(5)}_{g_1}(\vec x_1)\delta^{(1)}_{g_2}(\vec x_2)\delta^{(1)}_{g_3}(\vec x_3)\delta^{(1)}_{g_4}(\vec x_4)\delta^{(1)}_{g_5}(\vec x_5)\delta^{(1)}_{g_6}(\vec x_6)\rangle$$

- we can detect these biases, and, from their size, determine:
  - the order of magnitude of the formation time of galaxies
  - direct evidence that the universe lasted 13 Billion years

Consequences of non-locality in time

- more on time-non-locality:
    - if formation time is fast, $1/\omega$, we can Taylor expand the Kernels:

$$c_{\mathcal{O}_m,\alpha}(t) \approx c_{\mathcal{O}_m}(t)\left(1 + g_{\mathcal{O}_m,\alpha}(t)\frac{H}{\omega} + \dots\right) \ ,$$

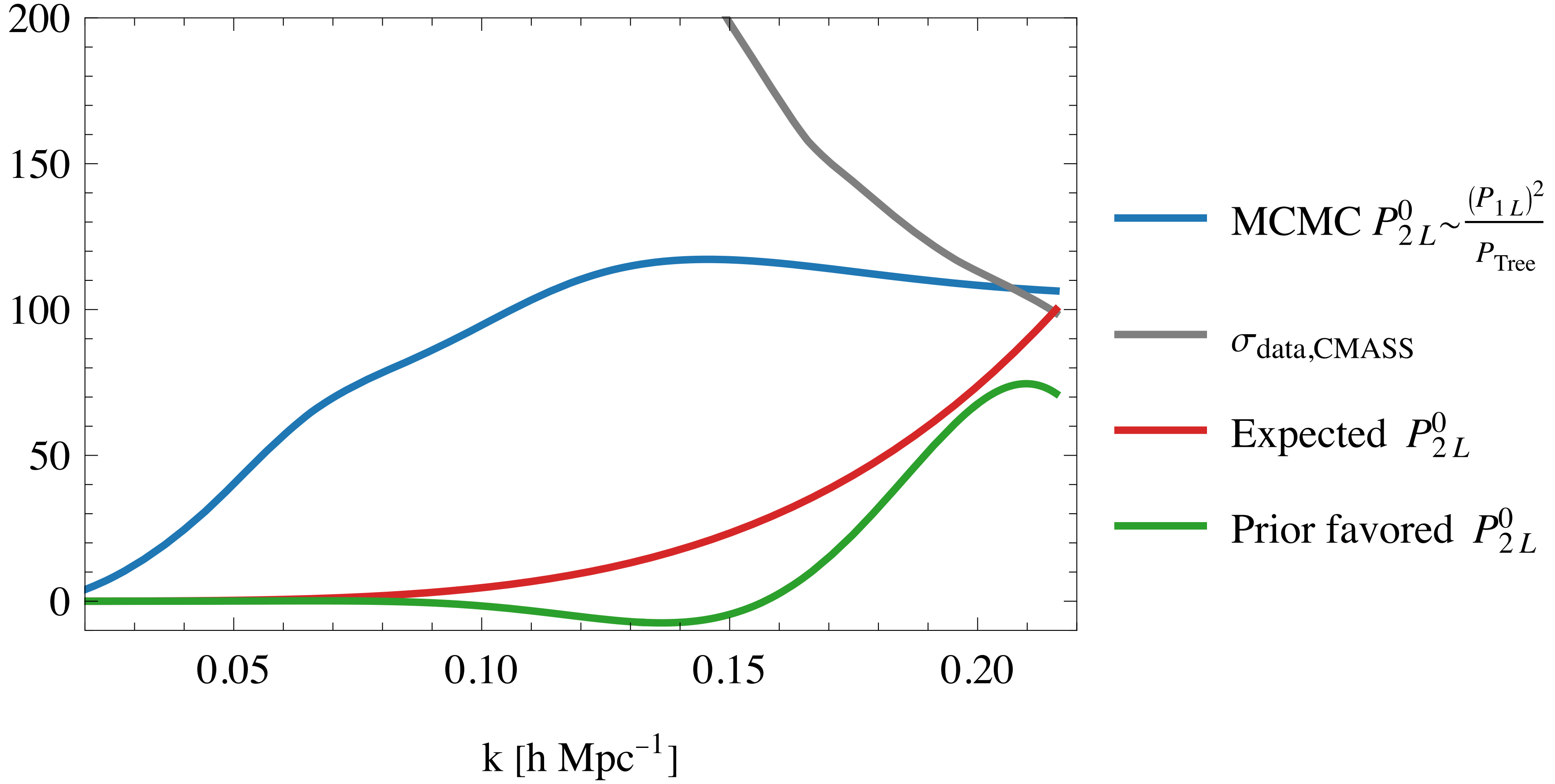
    - so these terms would be suppressed, and we could therefore determine a fast formation time.

Peeking into the next Decade

with Donath, Bracanga and Zheng **2307**

Next Decade

- After validating our technique against the MCMC's on BOSS data, we Fisher forecast for DESI and Megamapper
- Prediction of one-loop Power Spectrum and Bispectrum
- We introduce a `*perturbativity prior*': impose expected size and scaling of loop



- Also a `*galaxy formation prior*' , 0.3 in each EFT-parameter

Results: Non-Gaussianities

BOSS: $\sigma(\cdot)$	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P+B_{\rm Tree}$	37	357	142
$P+B$	23	253	67
$P+B$+p.p.	17	228	62
$P+B$+p.p.+g.p.	15	163	49

DESI: $\sigma(\cdot)$	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P+B_{\rm Tree}$	3.61	142	71.5
$P+B$	3.46	114	30.2
$P+B$+p.p.	3.26	91.5	27.0
$P+B$+p.p.+g.p.	3.19	77.0	21.8

MMo: $\sigma(\cdot)$	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P+B_{\rm Tree}$	0.29	23.4	8.7
$P+B$	0.27	17.7	4.6
$P+B$+p.p.	0.26	16.0	4.2
$P+B$+p.p.+g.p.	0.26	12.6	3.4

- Just using perturbativity prior, potentially a factor of $20, 3, 6$ over Planck!!

Results: Curvature and Neutrinos

DESI: $\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	Ω_k
$P{+}B$	0.004	0.035	0.002	0.011	0.013
$P+B$+p.p.	0.004	0.032	0.002	0.008	0.012
$P{+}B$+p.p.+g.p.	0.004	0.025	0.002	0.007	0.009

MMo: $\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	Ω_k
$P{+}B$	0.002	0.0052	0.0003	0.002	0.0015
$P+B$+p.p.	0.002	0.0046	0.0003	0.002	0.0012
$P{+}B$+p.p.+g.p.	0.002	0.0044	0.0003	0.001	0.0011

- Just using perturbativity prior, potentially factor of 5 over Planck!
    - Important for the landscape of string theory.
- Neutrinos: guaranteed evidence/detection:

$$2\sigma \quad \text{DESI}, \qquad 14\sigma \quad \text{MegaMapper}$$

Where can we make better?

- Shot noise and EFT-parameters:

$\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P{+}B$	0.0042	0.020	0.0022	0.010	3.5	114	30
$P+B+g.p.:$	0.0042	0.018	0.0022	0.009	3.4	83	23
$P+B:$ bias fixed	0.0037	0.010	0.0016	0.004	2.0	21	11
$P+B: n_b \to \infty$	0.0035	0.011	0.0009	0.005	1.7	67	17

DESI

$\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P{+}B$	0.0021	0.0047	0.00034	0.0017	0.27	18	4.6
$P+B+g.p.:$	0.0020	0.0045	0.00033	0.016	0.26	13	3.6
$P+B:$ bias fixed	0.0016	0.0034	0.00021	0.0010	0.17	3.6	1.7
$P+B: n_b \to \infty$	0.00019	0.00045	0.000029	0.00017	0.11	5.4	1.5

MegaMapper

Summary

- After the initial, successful, application to BOSS data:
  - measurement of cosmological parameters
  - new method to measure Hubble
  - perhaps fixing tension
- the EFTofLSS is starting to look ahead to
  - higher-order and higher-n point functions
  - enlightening what next surveys could do, and how to design them
  - learning about some astrophysics, qualitative facts on the universe

Consequences of non-locality in time

- Nice recursion relations for these operators:

- . $$[\mathcal{O}_m(\vec{x}_{\rm fl}(\vec{x},t,t'),t')]^{(n)} = \sum_{\alpha=1}^{n-m+1} \left(\frac{D(t')}{D(t)}\right)^{\alpha+m-1} \mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec{x},t)$$

$$\Rightarrow \qquad \mathcal{O}^{(n)}_m(\vec{x},t) = \sum_{\alpha=1}^{n-m+1} \mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec{x},t) \ ,$$

equal-time completeness relation

fluid recursion

- . $$\Rightarrow \ \mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec{x},t) = \sum_{q=m}^{n-1} \frac{1}{n-\alpha-m+1} \partial_i \mathbb{C}^{(q)}_{\mathcal{O}_m,\alpha}(\vec{x},t) \frac{\partial_i}{\partial^2}\theta(\vec{x},t)^{(n-q)},$$

- Easy higher order:

$\Rightarrow$