Non-perturbative analyticity from the sphere to de Sitter

Manuel Loparco (EPFL), 13 September 2023.

Based on 2309.xxxxx with Jiaxin Qiao and Zimo Sun

What are the QFTs on the sphere which continue to “good” QFTs on de Sitter?

For simplicity, focus on properties of scalar two-point functions

Flat space review

Euclidean flat space

Osterwalder-Schrader Axioms

- Temperedness
- Euclidean invariance
- Reflection positivity
- Spin-statistics

Positive Hamiltonian

Convergent Källén-Lehmann representation

Analyticity

Minkowski spacetime

Wightman Axioms

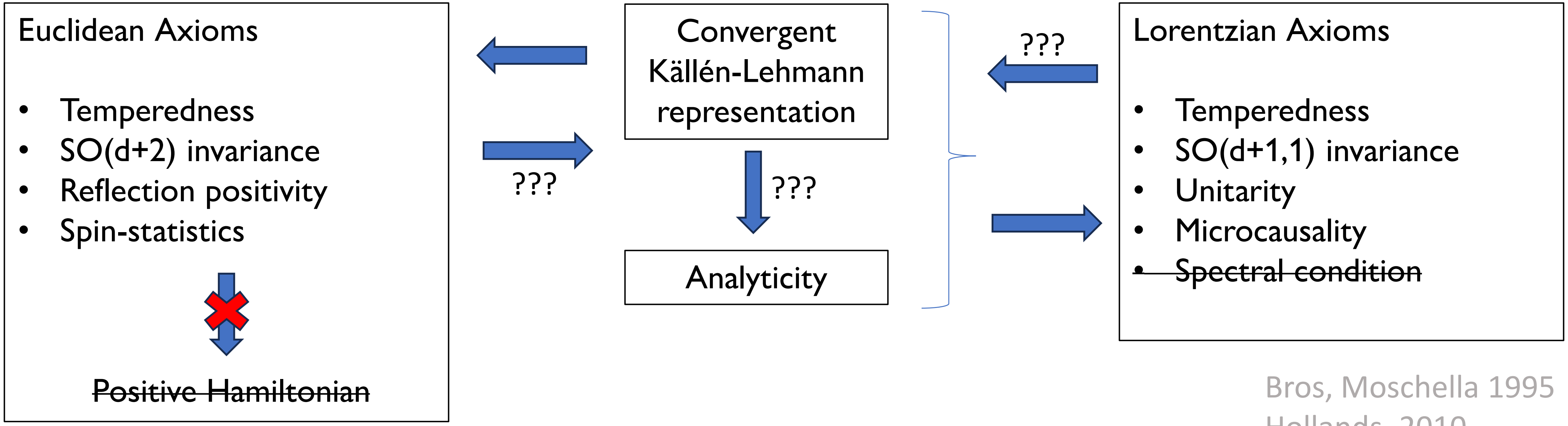
- Temperedness
- Poincaré invariance
- Unitarity
- Microcausality
- Spectral condition

References: K. Osterwalder, R. Schrader (1973, 1975)

The status in dS/Sphere

Sphere

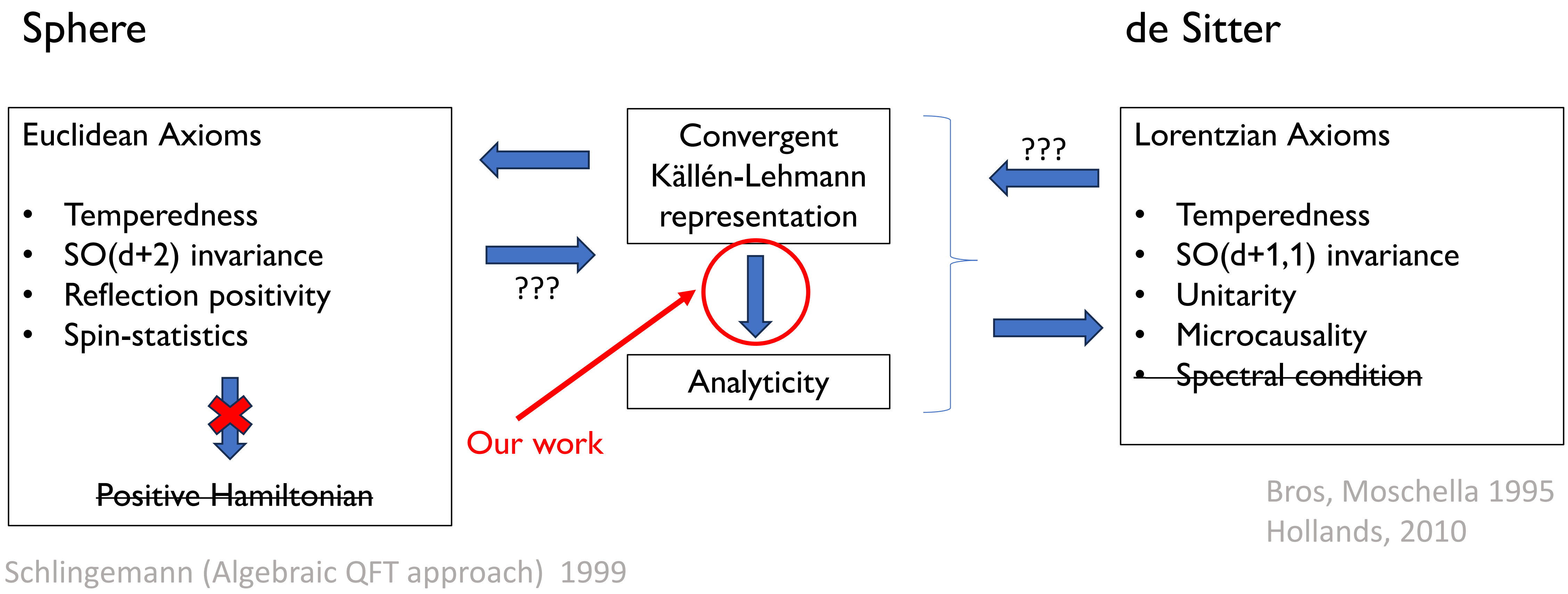
de Sitter



Bros, Moschella 1995
Hollands, 2010

Schlingemann (Algebraic QFT approach) 1999

The status in dS/Sphere



Starting point: Källén-Lehmann on the sphere

Bros, Moschella 1995
Penedones, Hogervorst, Vaziri 2021
Di Pietro, Gorbenko, Komatsu 2021
ML, Penedones, Sun, Vaziri 2023

Introduce the two-point invariant $\sigma \equiv \frac{X \cdot Y}{R^2}$

$$G_{\mathcal{O}}(\sigma) \equiv \langle \mathcal{O}(X)\mathcal{O}(Y) \rangle = \int_{\frac{d}{2}+i\mathbb{R}} d\Delta \ \varrho^{\mathcal{P}}(\Delta) G_\Delta(\sigma) + \int_0^d d\Delta \ \varrho^{\mathcal{C}}(\Delta) G_\Delta(\sigma)$$

Solutions to exceptional
casimir

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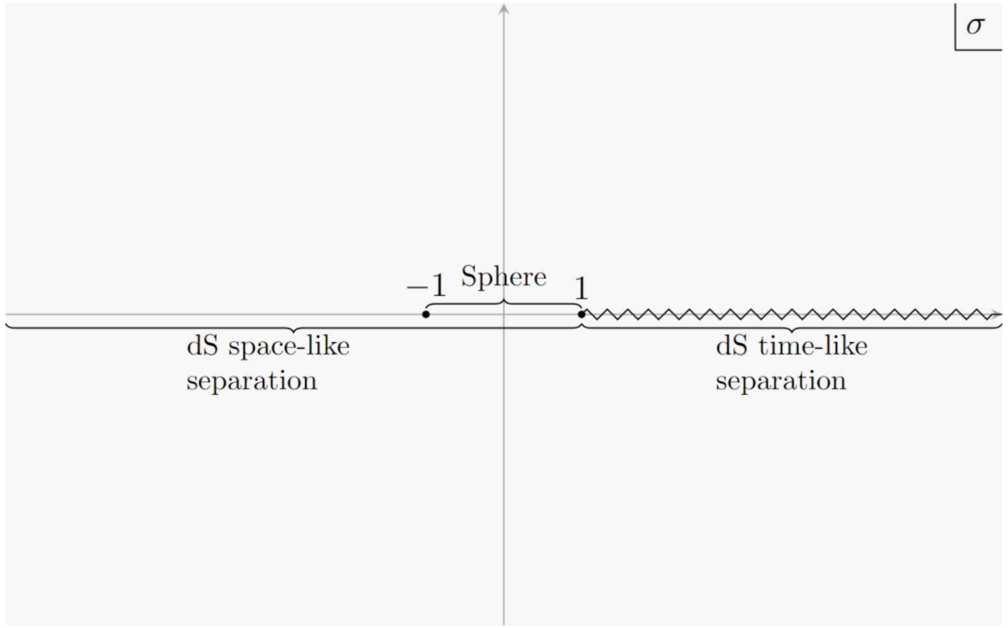
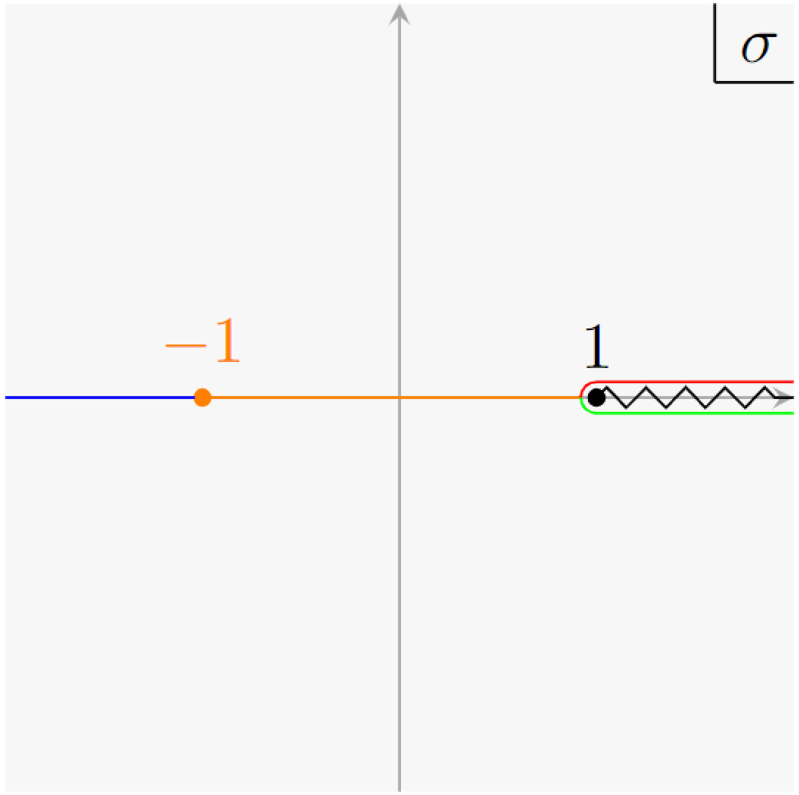
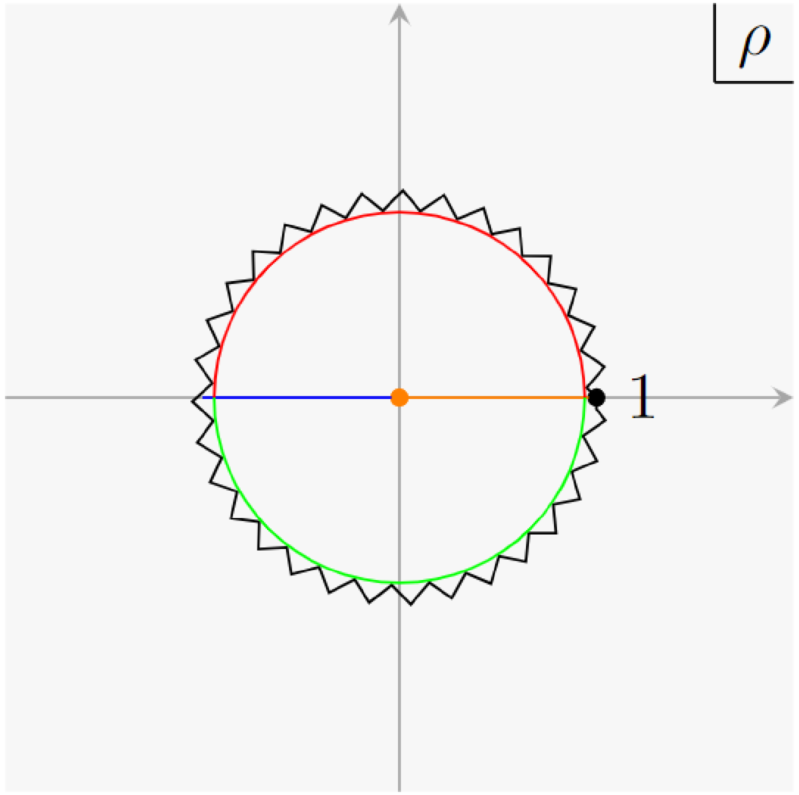


A useful change of variables



$$\frac{1+\sigma}{2} = \frac{4\rho}{(1+\rho)^2}$$



The argument for Analyticity

- Free propagators happen to have a positive series expansion in ρ (very nontrivial fact, bulk of our paper)

$$G_\Delta(\rho) = \sum_{n=0}^{\infty} b_n(\Delta)\rho^n\,, \qquad b_n(\Delta) \geq 0$$

- $G_{\mathcal{O}}(|\rho|)$ converges because $0 \leq |\rho| \leq 1$
- $$|G_{\mathcal{O}}(\rho)| \leq \int_{\frac{d}{2}+i\mathbb{R}} d\Delta\ \varrho^{\mathcal{P}}(\Delta) \sum_{n=0}^{\infty} b_n(\Delta)|\rho|^n + \int_0^d d\Delta\ \varrho^{\mathcal{C}}(\Delta) \sum_{n=0}^{\infty} b_n(\Delta)|\rho|^n = G_{\mathcal{O}}(|\rho|)$$
- We can swap sums and integrals

$$G_{\mathcal{O}}(\rho) = \sum_{n=0}^{\infty} c_n\rho^n\,, \qquad c_n \geq 0$$

Any two-point function with a convergent Källén-Lehmann decomposition on the sphere is analytic in the full complex cut domain, including paths traced for the Wick rotation to de Sitter and to EAdS

Some open questions

- Absence of exceptional series
- Euclidean Axioms ⟹ Källén-Lehmann
- Lorentzian Axioms ⟹ Källén-Lehmann