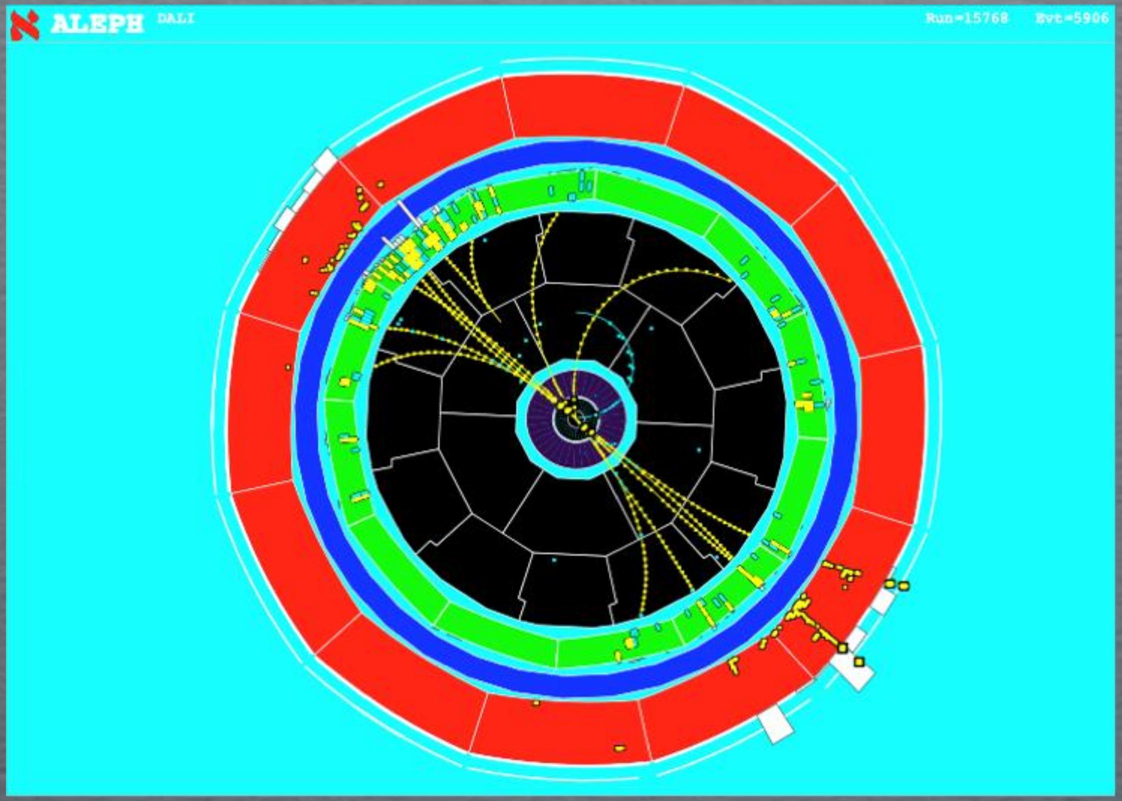
The Lund plane: history and resummations



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First Lund Plane Jet Institute – 3 July 2023 – CERN

Outline

- Brief historical introduction to the Lund plane
- Lund plane and rIRC safe resummations
- Non-global and collinear logarithms
- Three-jet observables in the two-jet limit

This is a very personal selection of topics. However, the principles discussed here can be applied unaltered to many more observables, including subjet multiplicities, jet substructure, jet-radius resummations, hadronisation corrections, etc

Historical introduction

The Lund plane appears

Coherence effects in deep inelastic scattering

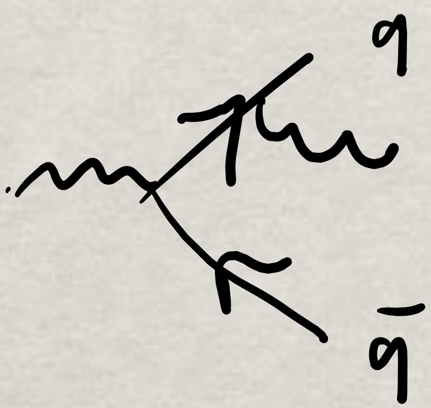
B. Andersson, G. Gustafson, L. Lönnblad, U. Pettersson
Department of Theoretical Physics, University of Lund, Sölvegatan 14A, S-223 62 Lund, Sweden

Received 23 January 1989

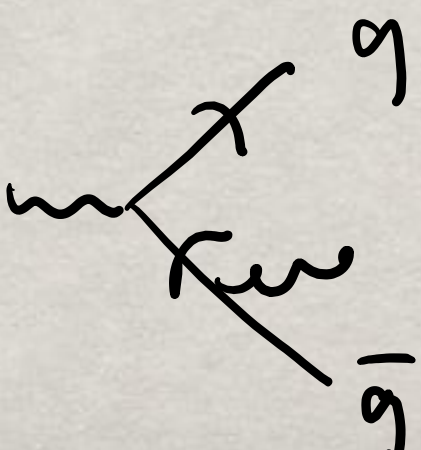
[*Z.Phys.C* 43 (1989) 625]

- Describe kinematical distribution of hadrons in DIS using a dipole-model of QCD radiation

[Gustafson *Nucl.Phys.B* 306 (1988) 746]



$$d\sigma = \frac{3\alpha_s}{4\pi} \cdot \frac{x_1^{a_1} + x_3^{a_3}}{(1-x_1)(1-x_3)}\, dx_1\, dx_3 \simeq \frac{\alpha_0}{\ln(k_T^2/\Lambda^2)} \frac{dk_T^2}{k_T^2}\, dy$$



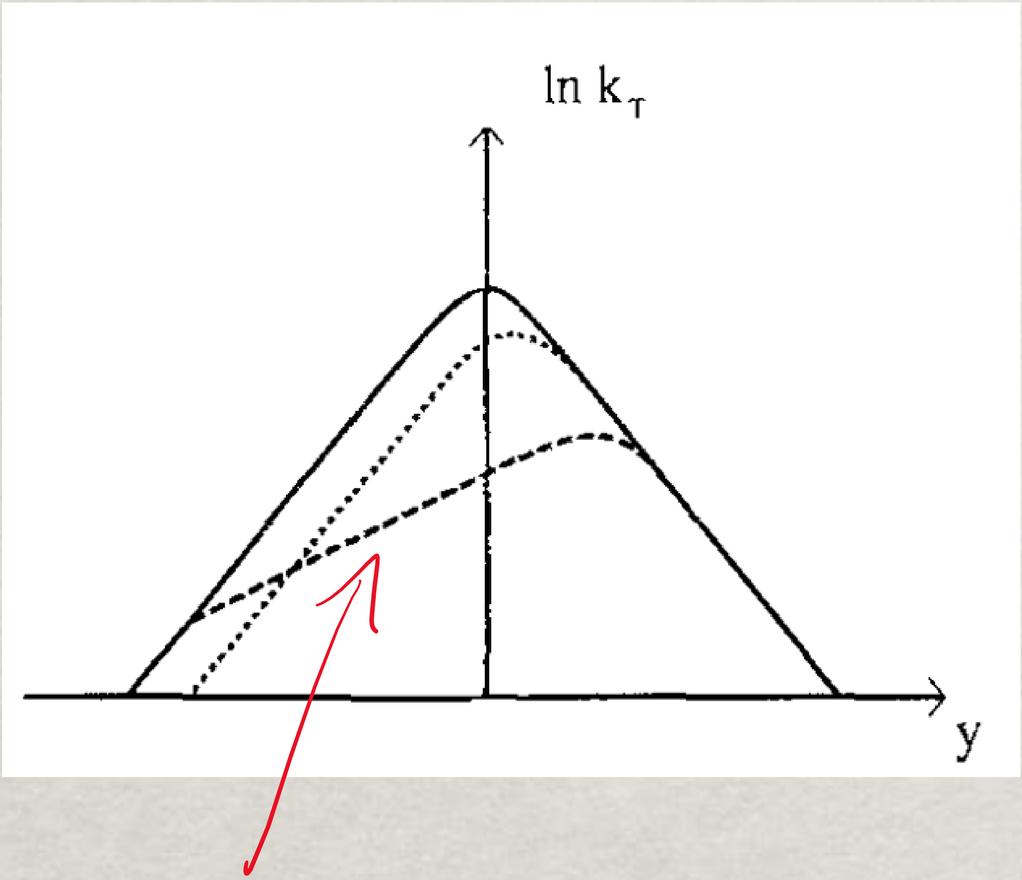
$$k_T^2 = \frac{s_{12}s_{23}}{s}$$

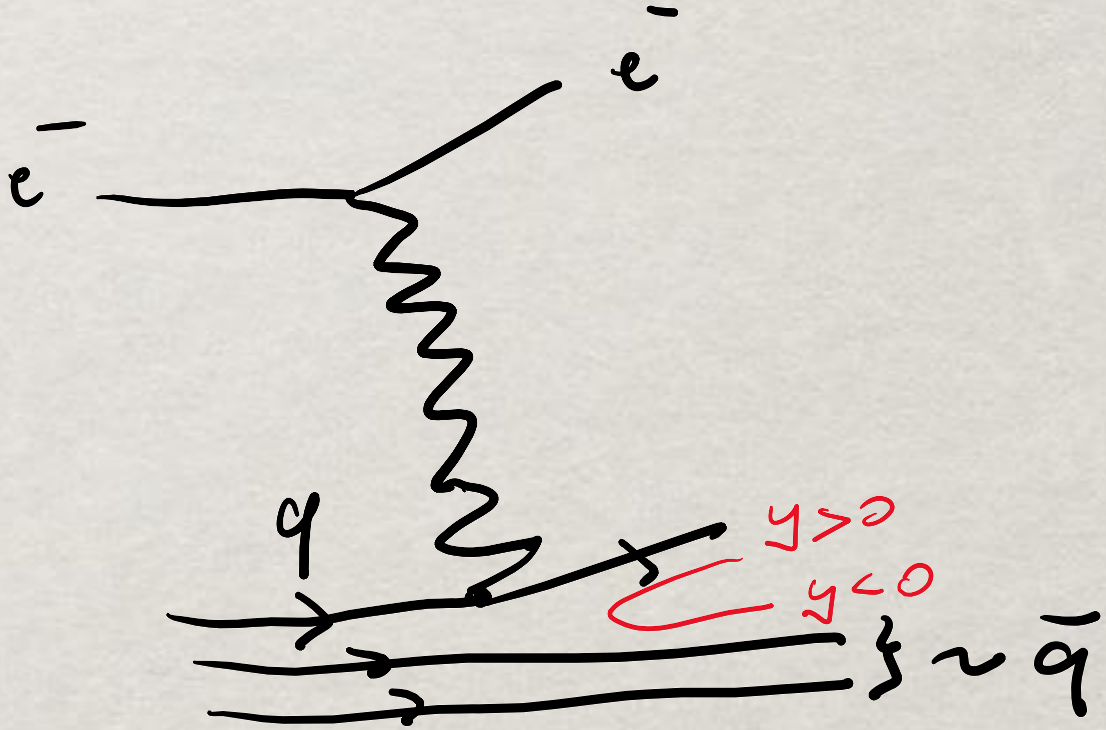
$$y = \tfrac{1}{2}\ln(s_{12}/s_{23}).$$

$$|y| \lesssim \ln\left(\frac{\sqrt{s}}{k_T}\right)$$

The Lund plane appears

- The dipole considered by AGLP was that formed by a quark hit by the virtual photon and the proton remnant





- Suppression of radiation in the direction of the proton remnant due to bound-state effects

- The $y - \ln k_T$ plane allowed an intuitive visualisation of the phase space of emitted gluons

Secondary Lund planes

- The dipole radiation model was used to simulate gluon radiation in the generator ARIADNE, as well as for analytical calculations

Fluctuations and anomalous dimensions in QCD cascades

B. Andersson*, G. Gustafson, A. Nilsson, C. Sjögren

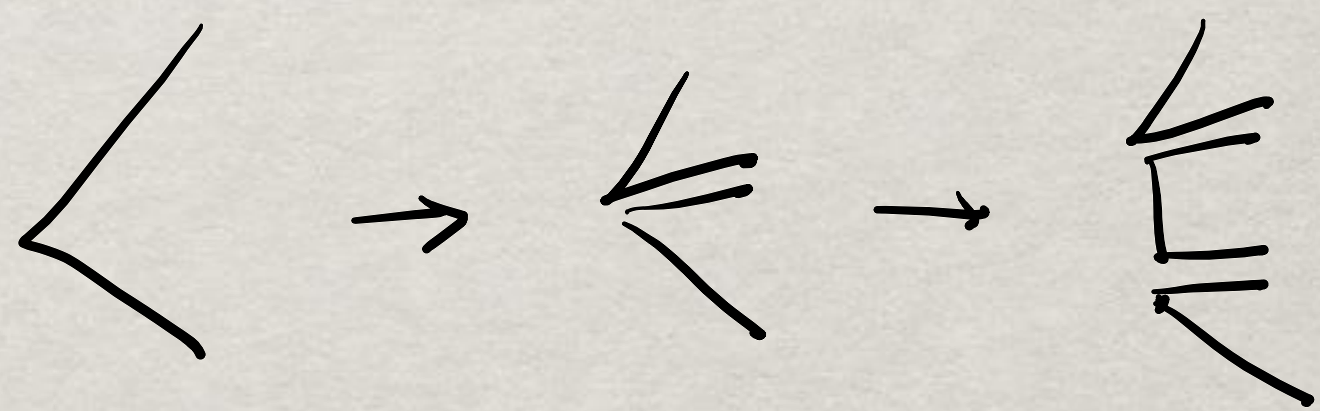
Department of Theoretical Physics, University of Lund, Sölvegatan 14A, S-223 62 Lund, Sweden

Received 5 June 1990

[*Z.Phys.C* 49 (1991) 79]

- The emission of a gluon from a dipole creates two new dipoles, with a span in rapidity given by

$$\Delta y = \ln\left(\frac{s_{12}}{k_{\perp 2}^2}\right) + \ln\left(\frac{s_{23}}{k_{\perp 2}^2}\right)$$
$$= \ln s + \ln k_{\perp 1}^2 - 2\ln k_{\perp 2}^2 .$$



- The increase of the available rapidity (in the dipole rest frame) correspond to the addition of a new triangle to the original one, a secondary Lund plane

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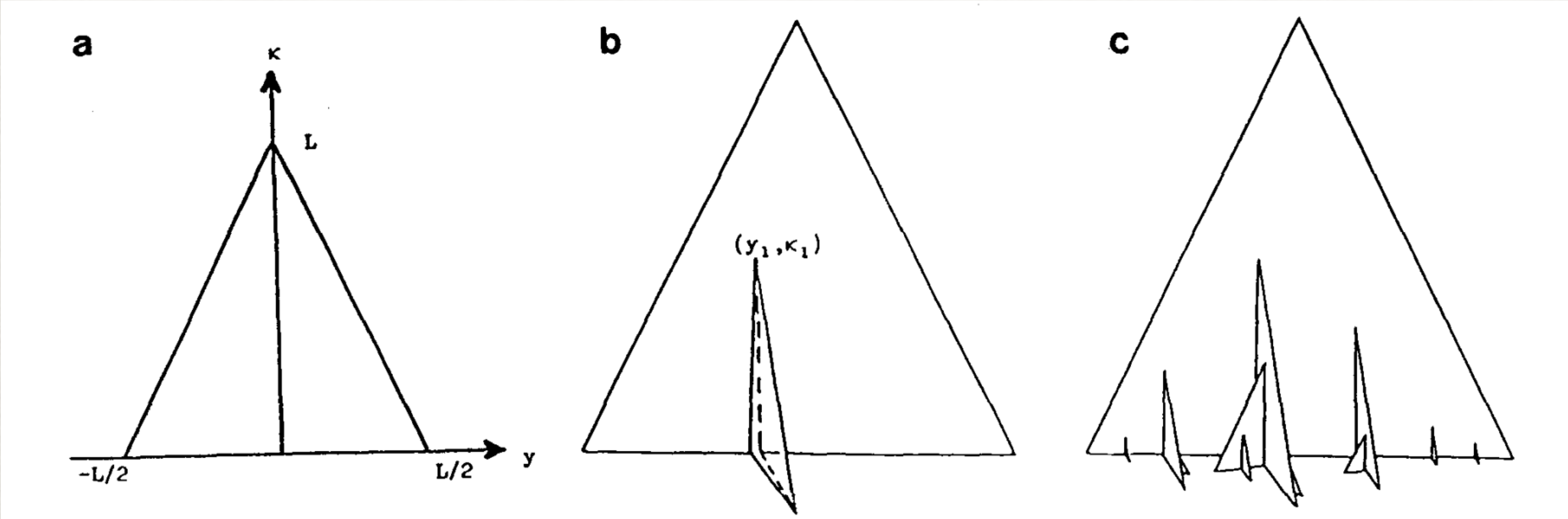
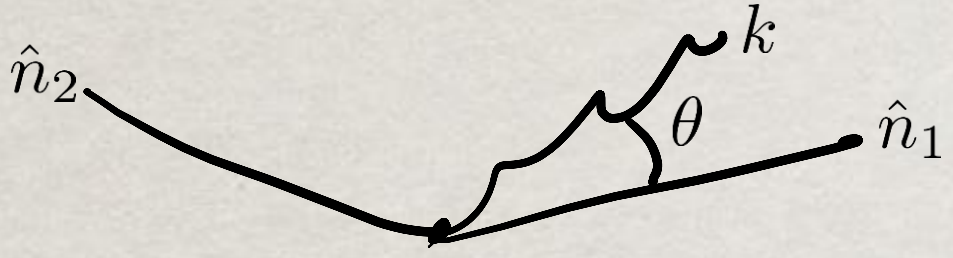


The Lund plane

- Soft-collinear emissions can be visualised as points in the Lund plane



Sudakov decomposition

$$P_1 = E_1(1, \hat{n}_1) \qquad P_2 = E_2(1, \hat{n}_2)$$

$$k = z_1 P_1 + z_2 P_2 + \kappa$$

- Useful kinematical variables

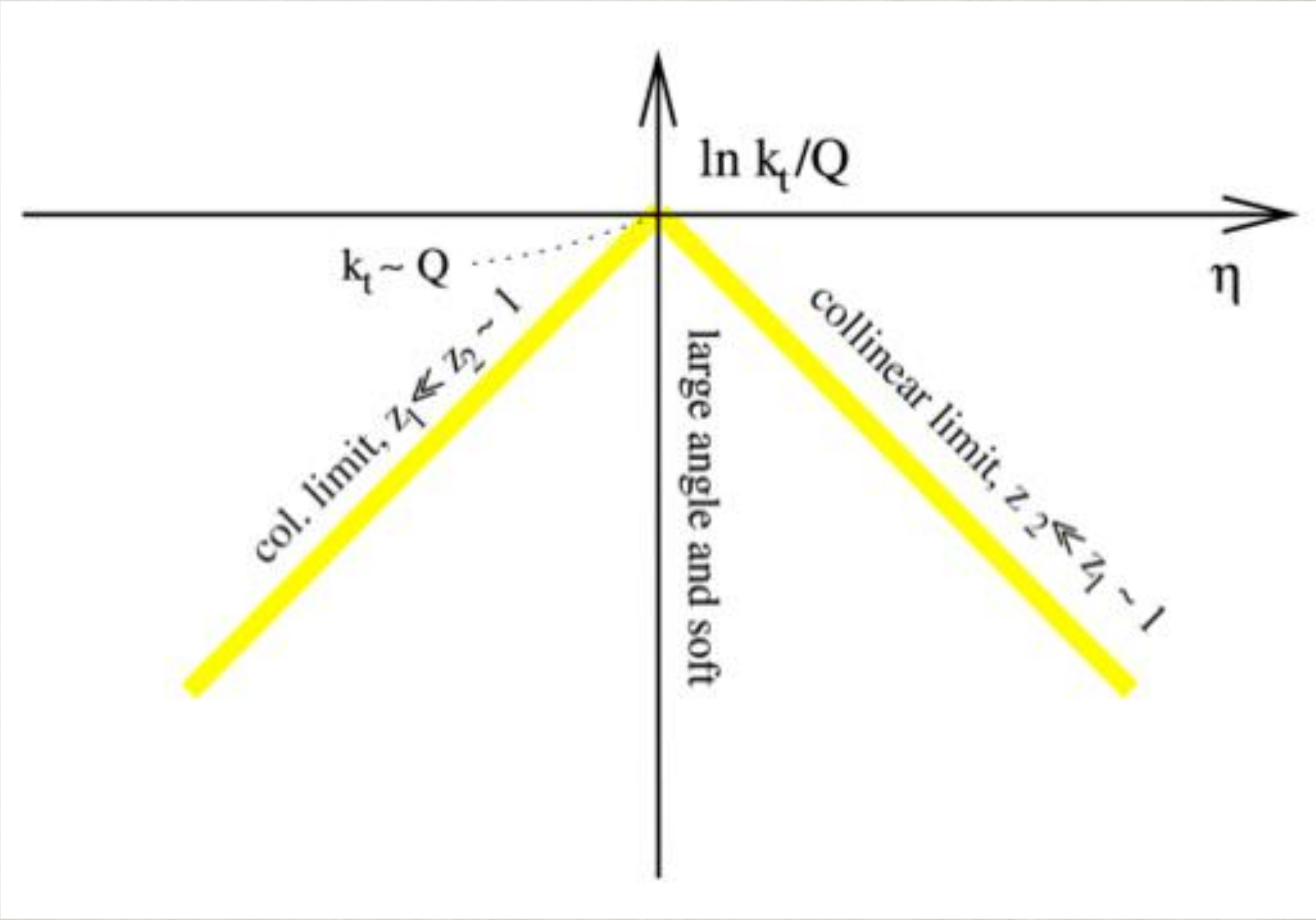
$$Q^2 = Q_{12}^2 = 2 P_1 \cdot P_2$$

$$k_t^2 \equiv -\kappa \cdot \kappa$$

$$\eta \equiv \frac{1}{2} \ln \left(\frac{z_1}{z_2} \right) \simeq \ln \frac{1}{\theta} \quad z_1 \gg z_2$$

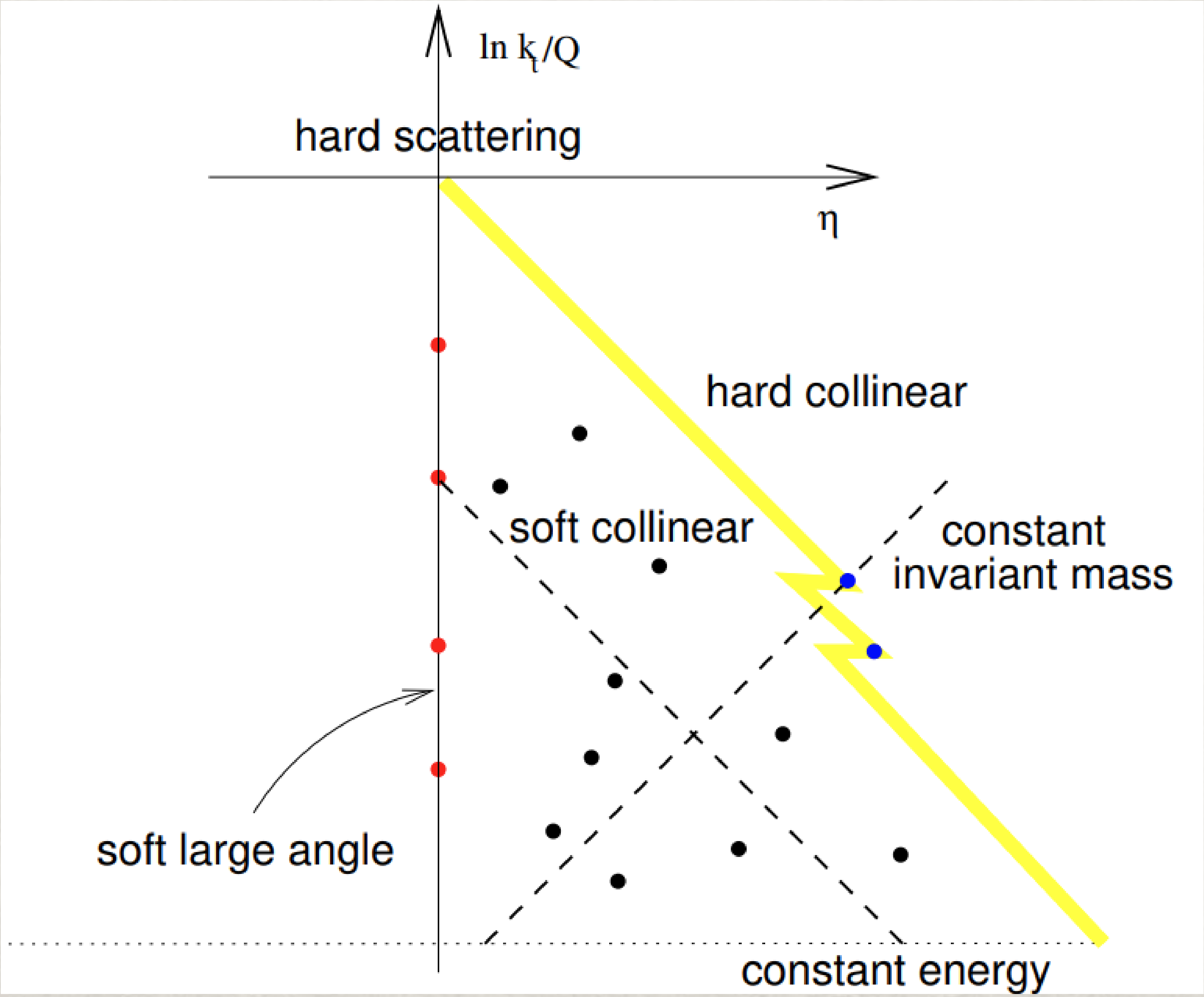
- Collinear limit

$$z_1, z_2 < 1 \implies |\eta| < \ln \left(\frac{Q}{\kappa_t} \right)$$



Lund plane guidelines

- In the Lund plane, it is useful to identify lines corresponding to constant energy and constant invariant mass



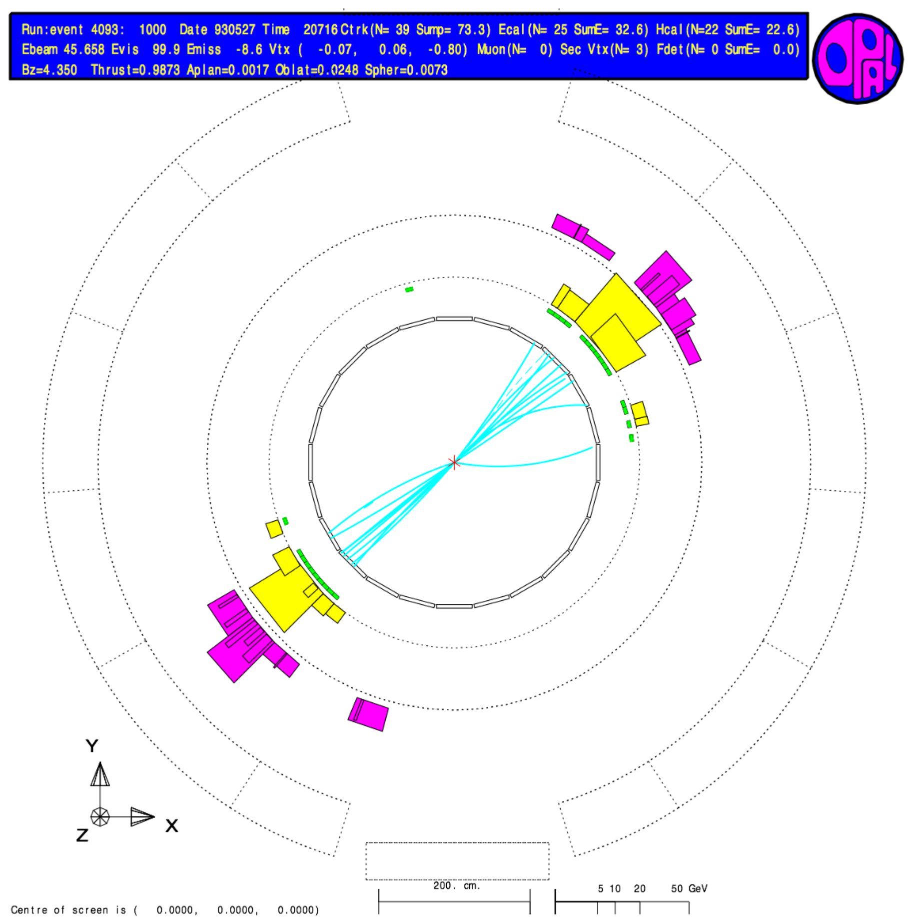
rIRC safe resummations

Final-state jet observables

- We consider a generic IRC safe final-state jet observable, a function $V(p_1, \ldots, p_n)$ of all final-state momenta $p_1, \ldots, p_n$

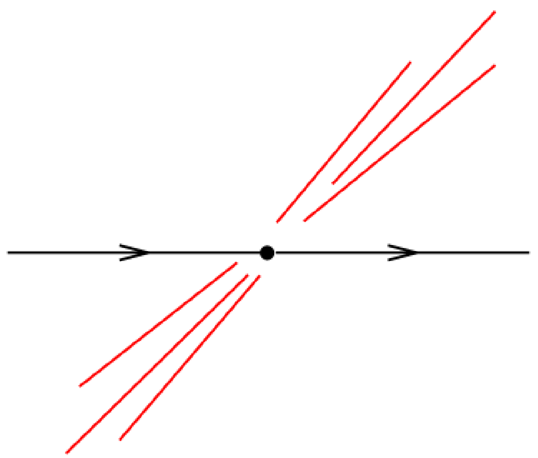
- Example: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \text{hadrons}$

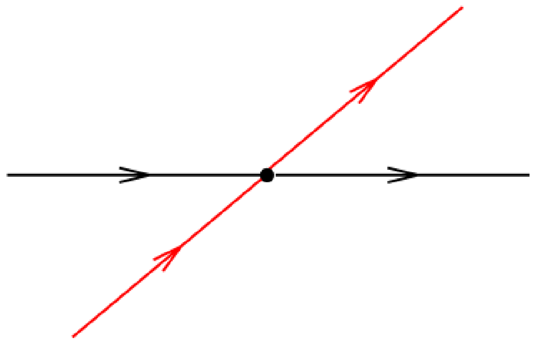
$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \mathrm{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$



$$\Sigma(v) = \mathrm{Prob}[V(p_1, \ldots, p_n) < v]$$

$v \ll 1$





$\Sigma(v)$ quantifies the departure from the Born limit

All-order resummation

- Close to the Born limit, distributions in final-state observables exhibit large logarithms that need to be resummed at all orders in QCD perturbation theory
- For many observables, it is possible to reorganise the perturbative series in the region $\alpha_s L \sim 1$ with $L = \ln(1/v)$

$$\Sigma(v) \simeq e^{\overbrace{Lg_1(\alpha_s L)}_{\mathrm{LL}}} \left(\underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots \right)$$

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- In resummations, scaling is everything $\Longrightarrow$ consider the limit

$$\alpha_s \to 0, v \to 0 \quad \text{with} \quad \alpha_s L \quad \text{fixed}$$

- This limit corresponds to extremely energetic partons surrounded by extremely soft and/or collinear emissions

Soft emissions in the Lund plane

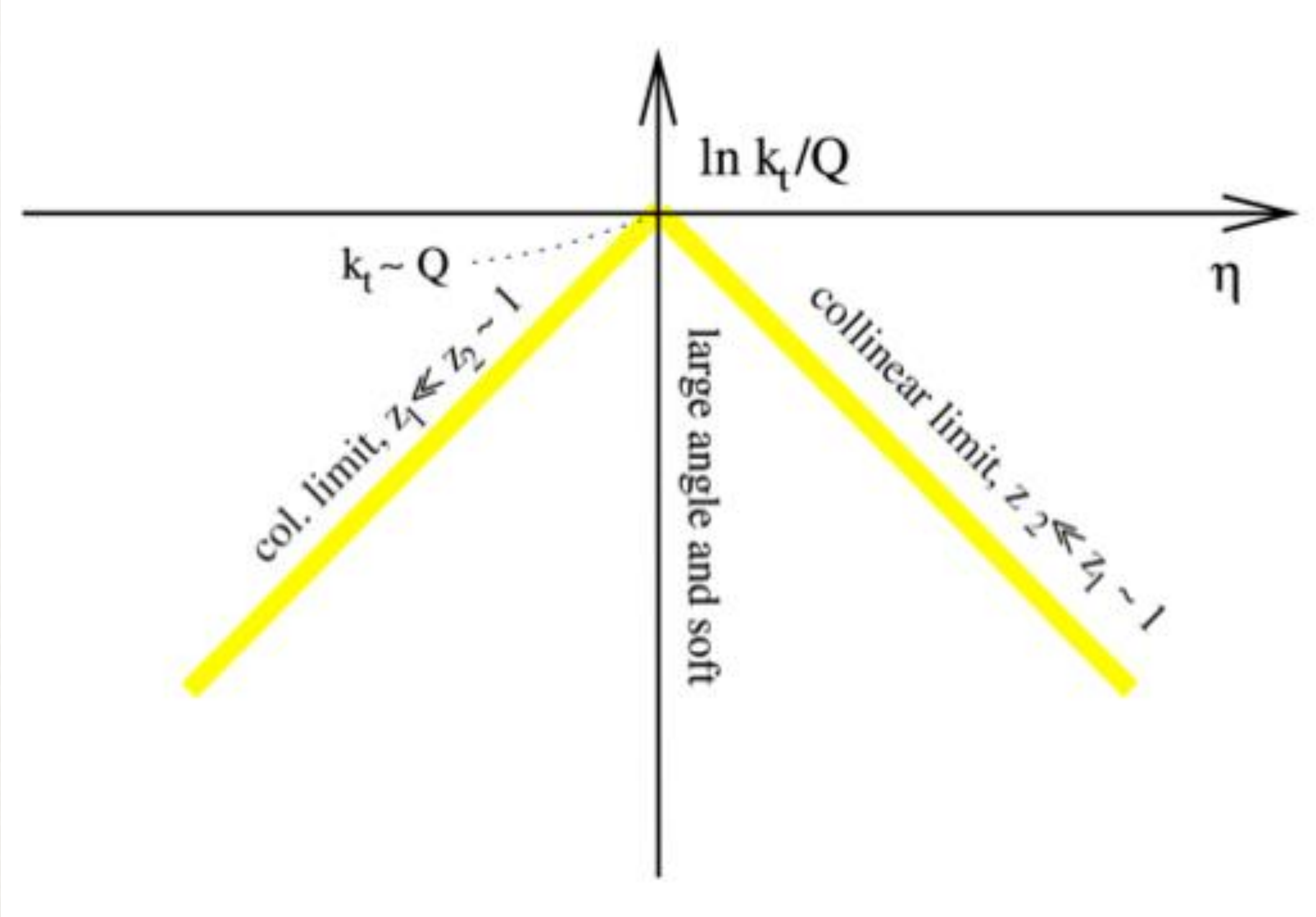
- Soft gluon emission from a dipole is uniform in rapidity

$$[dk]M^2(k) = -2(\vec{T}_1 \cdot \vec{T}_2)\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}d\eta\frac{d\phi}{2\pi}$$

- For fixed coupling, areas correspond to double logarithms, lines to single logarithms, dots to contributions of relative order α_s

- For observables whose LL exponentiate, the running of the coupling does not alter the hierarchy of logarithms

$$\alpha_s(k_t) = \alpha_s(Q)\times$$
$$\times\left(1 - 2\beta_0\alpha_s(Q)\ln\left(\frac{Q}{k_t}\right) + \dots\right)$$

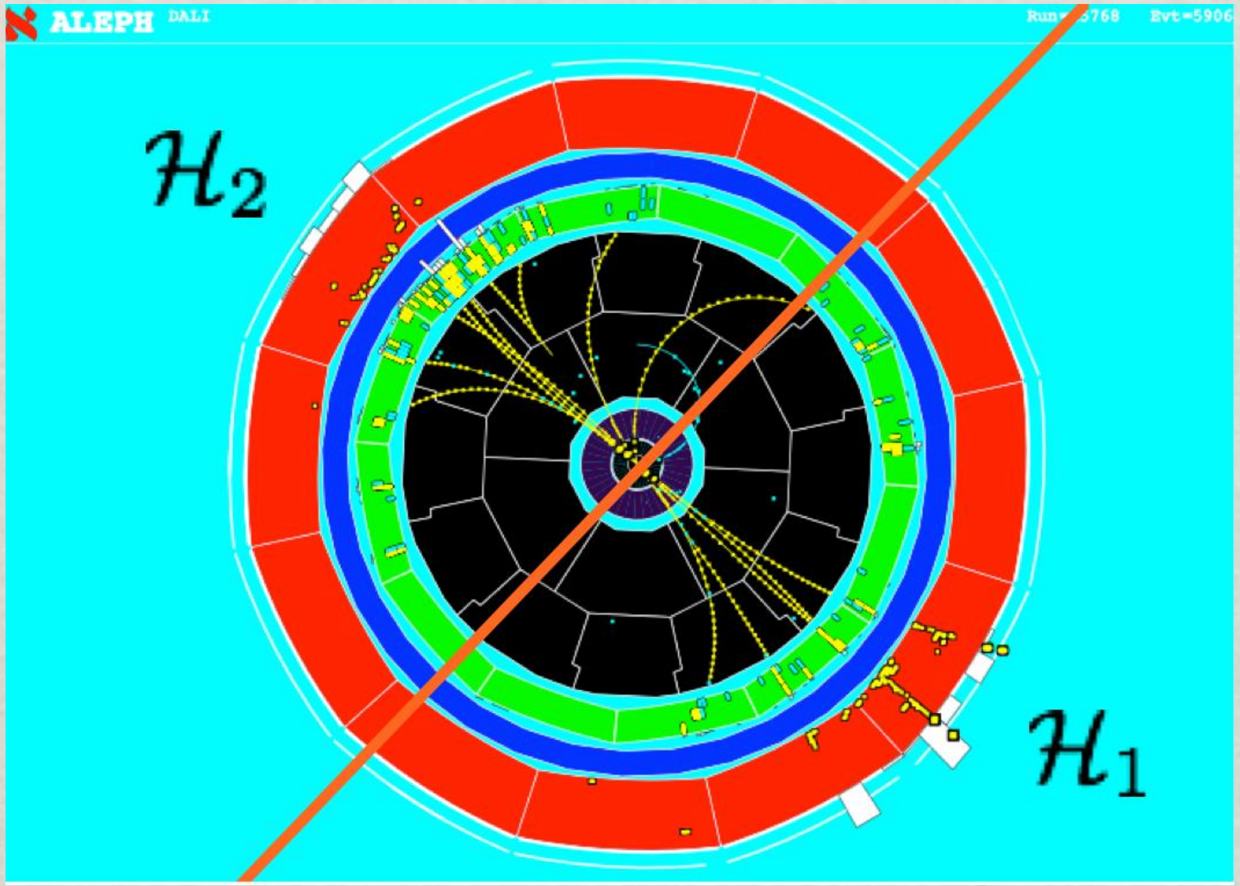


The thrust in the Lund plane

- Behaviour of the thrust in the soft-collinear limit

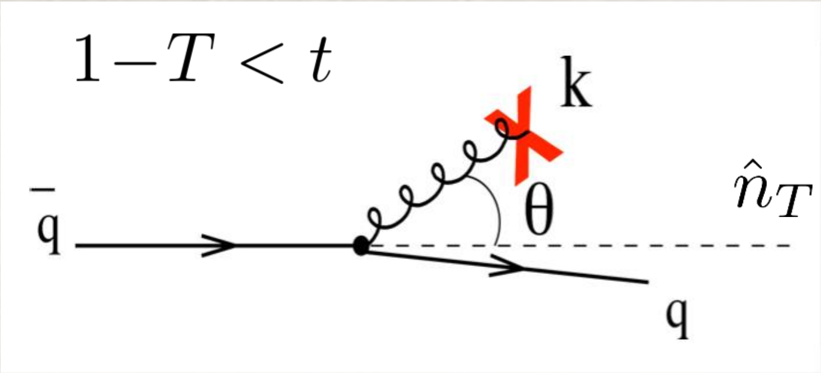
recoiling $q\bar{q}$ pair

$$1-T\left(\{\tilde{p}\},k_1,\ldots,k_n\right) \simeq \sum_i \frac{k_{ti}}{Q}e^{-|\eta_i|} + \sum_{\ell=1,2}\frac{1}{Q^2}\frac{\left|\sum_{i\in\mathcal{H}_\ell}\vec{k}_{ti}\right|^2}{1-\sum_{i\in\mathcal{H}_\ell}z_\ell^{(i)}}$$



The thrust in the Lund plane

- The Lund plane can be used to study the behaviour of the final-state observables in the soft-collinear limit [Dasgupta Salam hep-ph/0208073]



Sudakov decomposition

$$P_1 = \frac{Q}{2}(1, \hat{n}_T) \qquad P_2 = \frac{Q}{2}(1, -\hat{n}_T)$$

- Soft and collinear

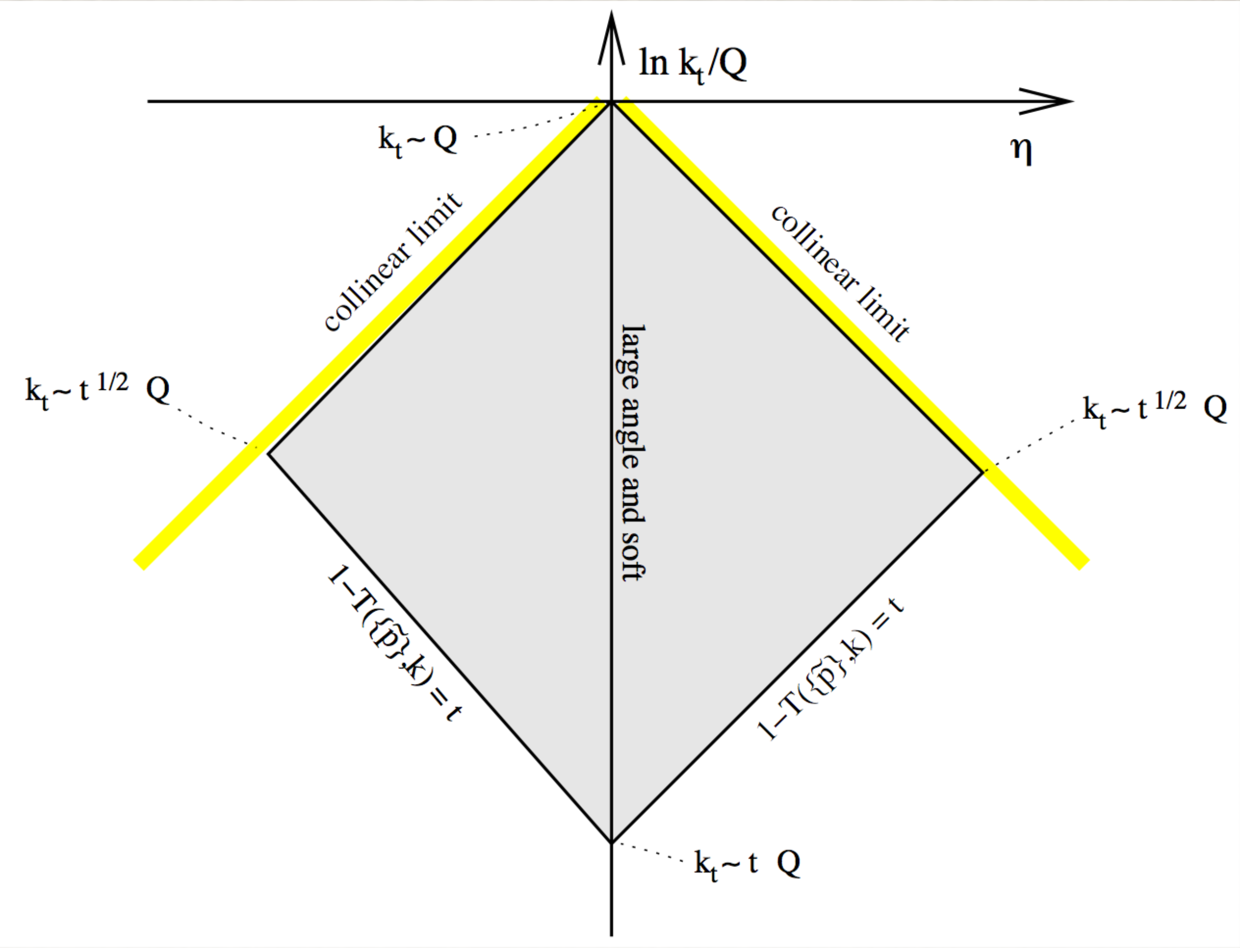
$$1 - T(\{\tilde{p}\}, k) \simeq \frac{k_t}{Q} e^{-|\eta|}$$

- Soft and large angle

$$1 - T(\{\tilde{p}\}, k) \sim k_t$$

- Hard and collinear

$$1 - T(\{\tilde{p}\}, k) \sim k_t^2$$



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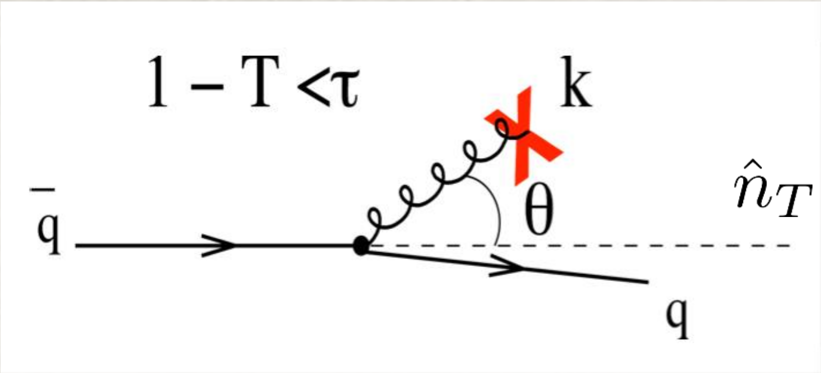

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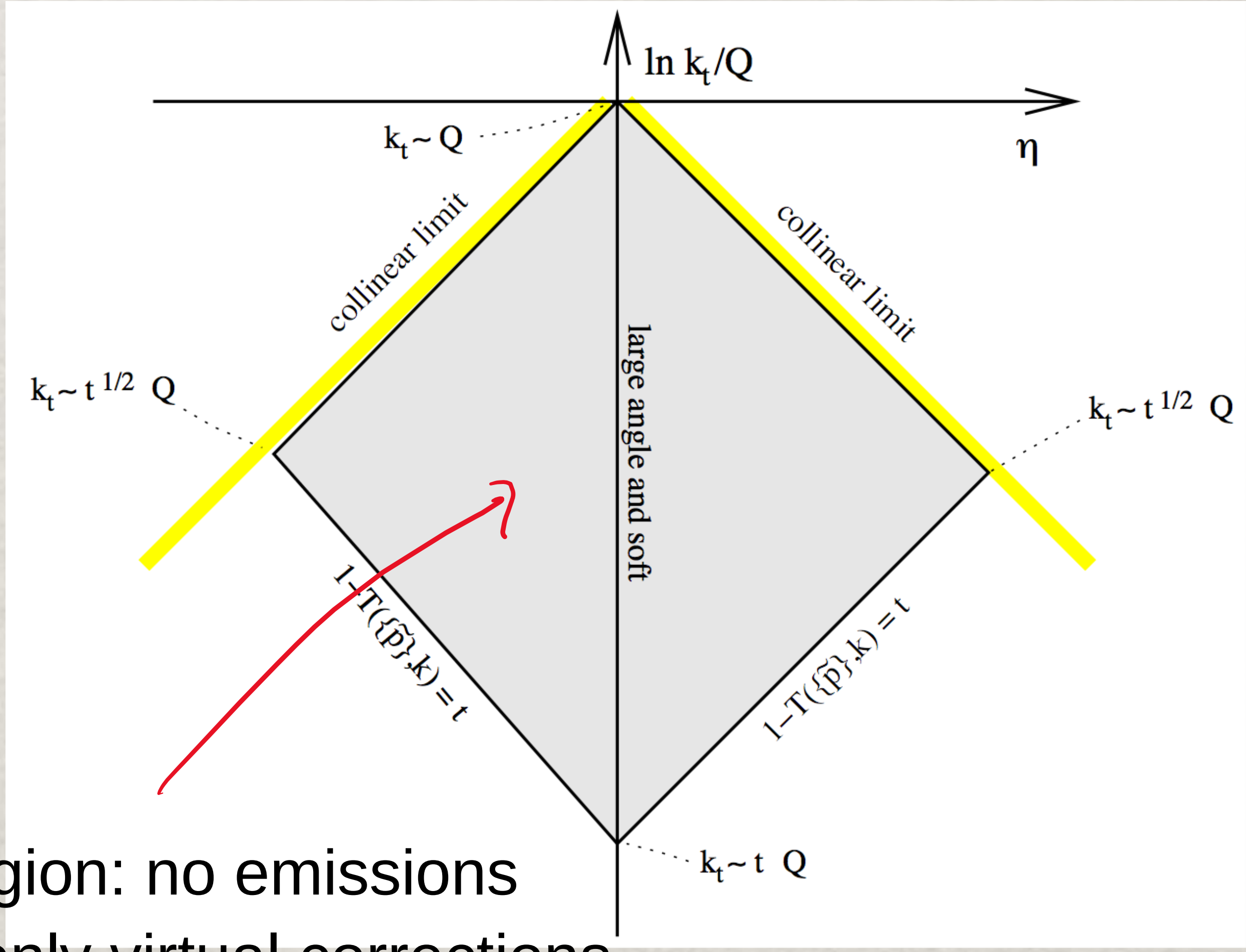
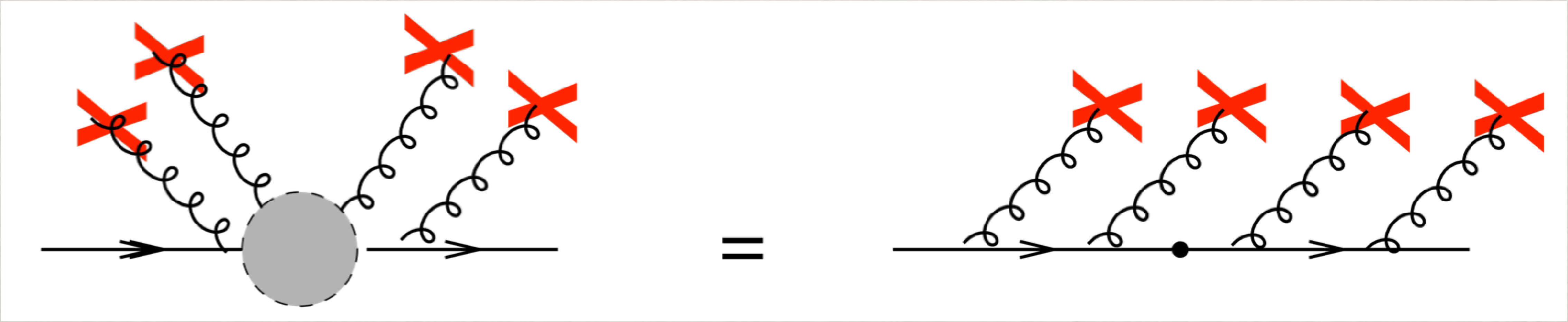


vetoed region: no emissions allowed, only virtual corrections

Multiple soft-collinear emissions

- For a generic observable, one has to consider the contribution of multiple soft-collinear emissions
- Due to QCD coherence, the probability of emitting multiple soft gluons widely separated in angle factorises into the product of single-emission probabilities

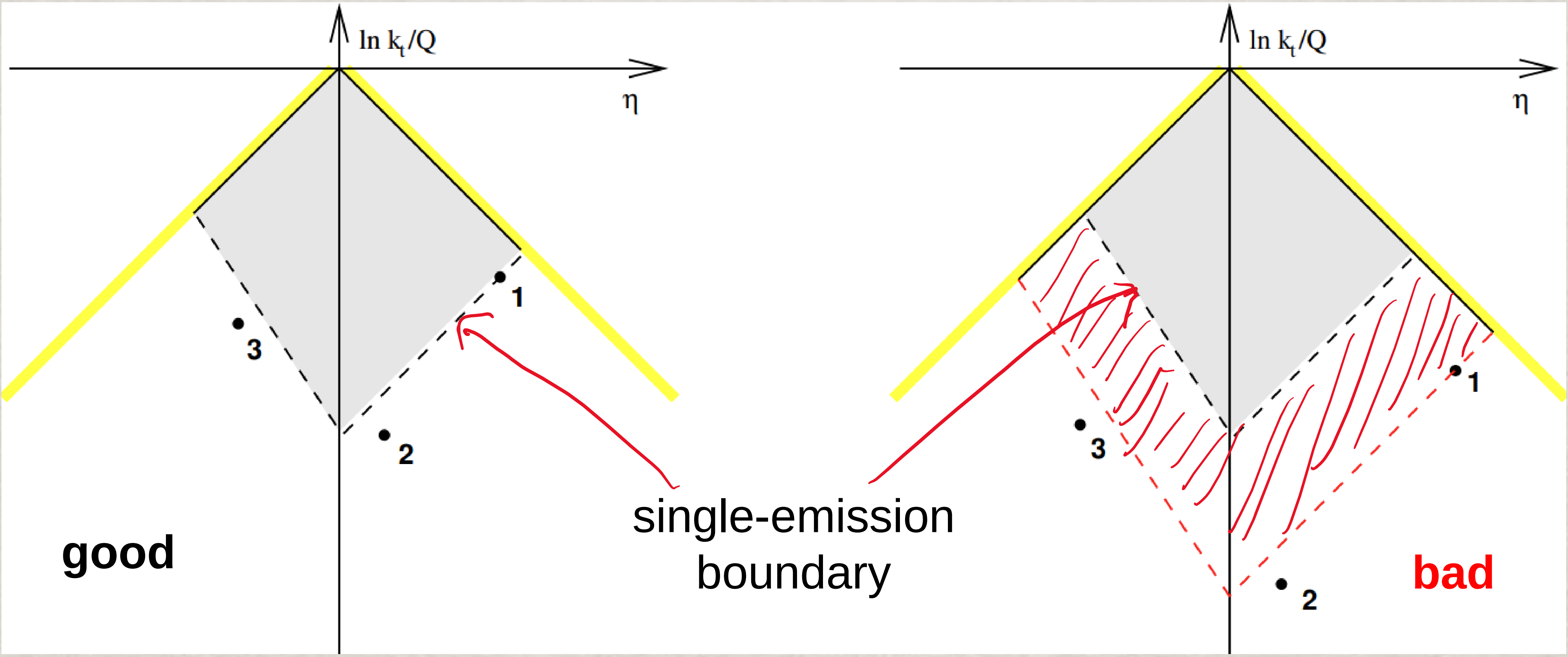


- Observables that are mainly sensitive to those emissions have the greatest chance to be resummed at an arbitrary logarithmic accuracy
- The Lund plane greatly helps identify what conditions an observable must satisfy for such resummation to be feasible

Recursive IRC safety condition 1

- Suppose we have many emissions with $V(\{\tilde{p}\}, k_i) \sim v$. What can we say about $V(\{\tilde{p}\}, k_1, \ldots, k_n)$? [AB Salam Zanderighi hep-ph/0407286]

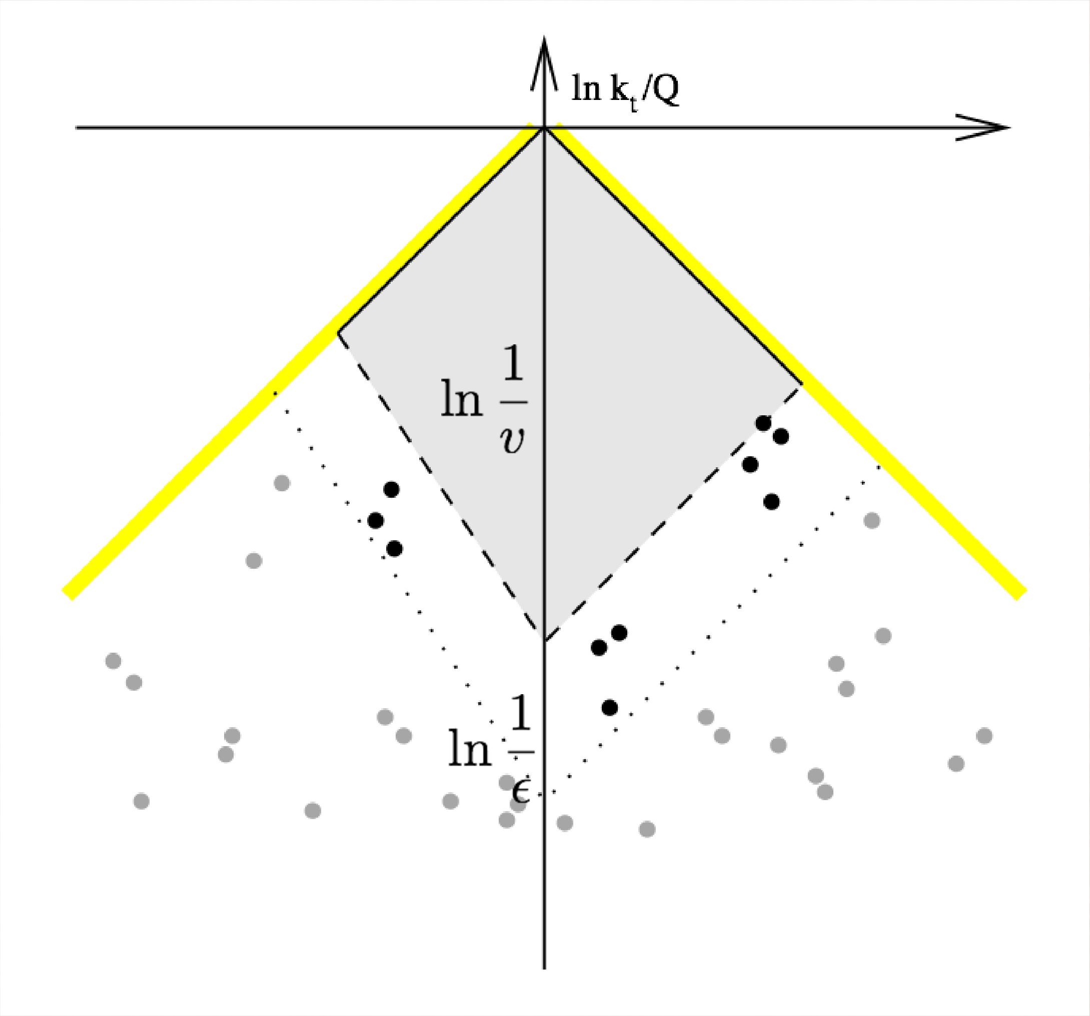
- If $V(\{\tilde{p}\}, k_1, \ldots, k_n) \sim v$ the region where emissions are vetoed is stable with respect to the number of emissions



- No double logarithms are created at the boundary of the allowed region for real emissions $\Rightarrow$ the observable satisfies rIRC safety condition 1

Recursive IRC safety condition 2a

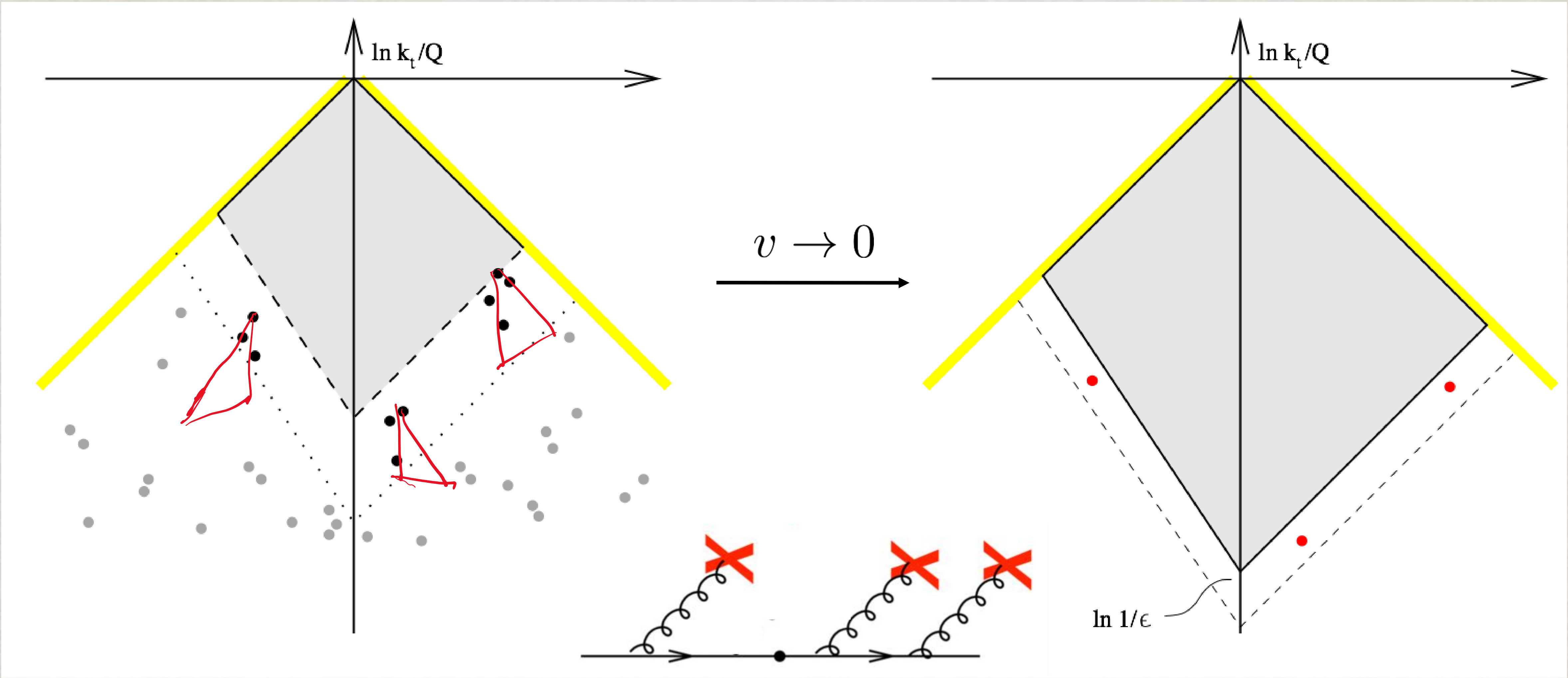
- Suppose that, for $v \to 0$, we can neglect all emissions with $V(\{\tilde{p}\}, k_i) < \epsilon v$ and $\epsilon \gg v$ [AB Salam Zanderighi hep-ph/0407286]



- For $v \to 0$ all relevant emissions are pushed towards the boundary of the vetoed region $\Rightarrow$ the observable satisfies rIRC safety condition 2a

Recursive IRC safety condition 2b

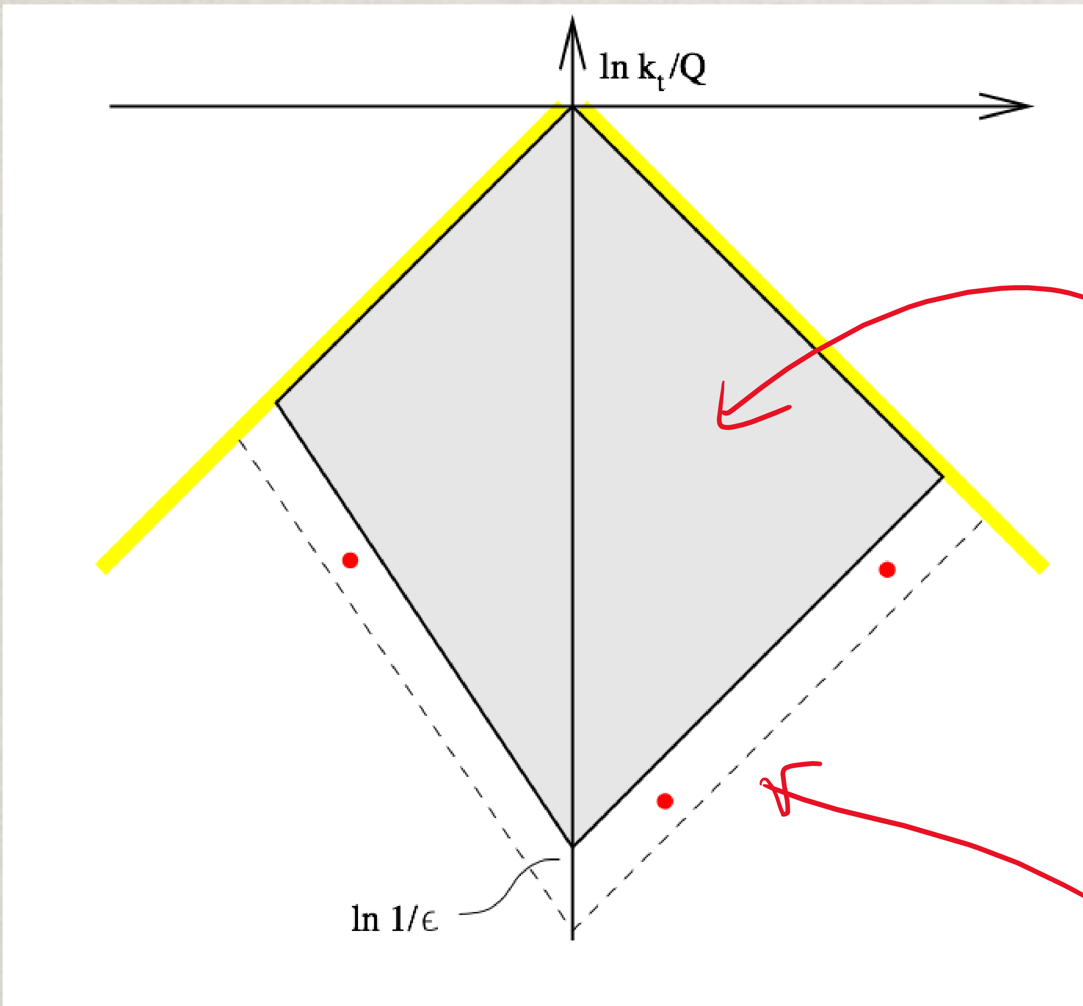
- Correlated emissions give rise to secondary Lund planes. If these shrink to a point for $v \to 0$ the observable satisfies rIRC safety condition 2b



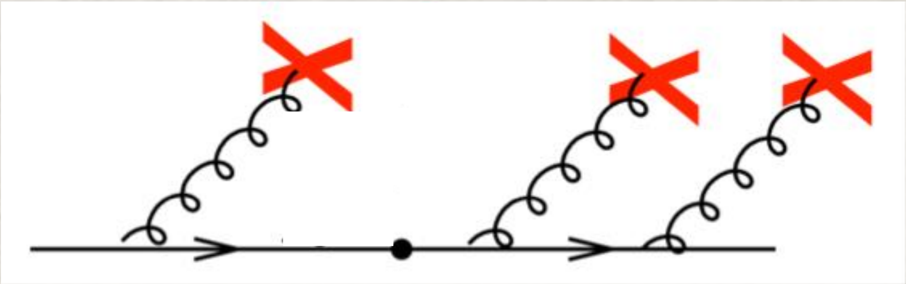
- At NLL accuracy, relevant emissions are soft and collinear clusters, widely separated in angle, and in a strip of size $\ln v \times \ln \epsilon$ [AB Salam Zanderighi hep-ph/0407286]
- The strip is a line in the Lund plane, hence an NLL contribution

Resummation and rIRC safety

- rIRC safety is a sufficient condition for exponentiation of double logarithms



vetoed region: virtual corrections only, giving the LL Sudakov exponent

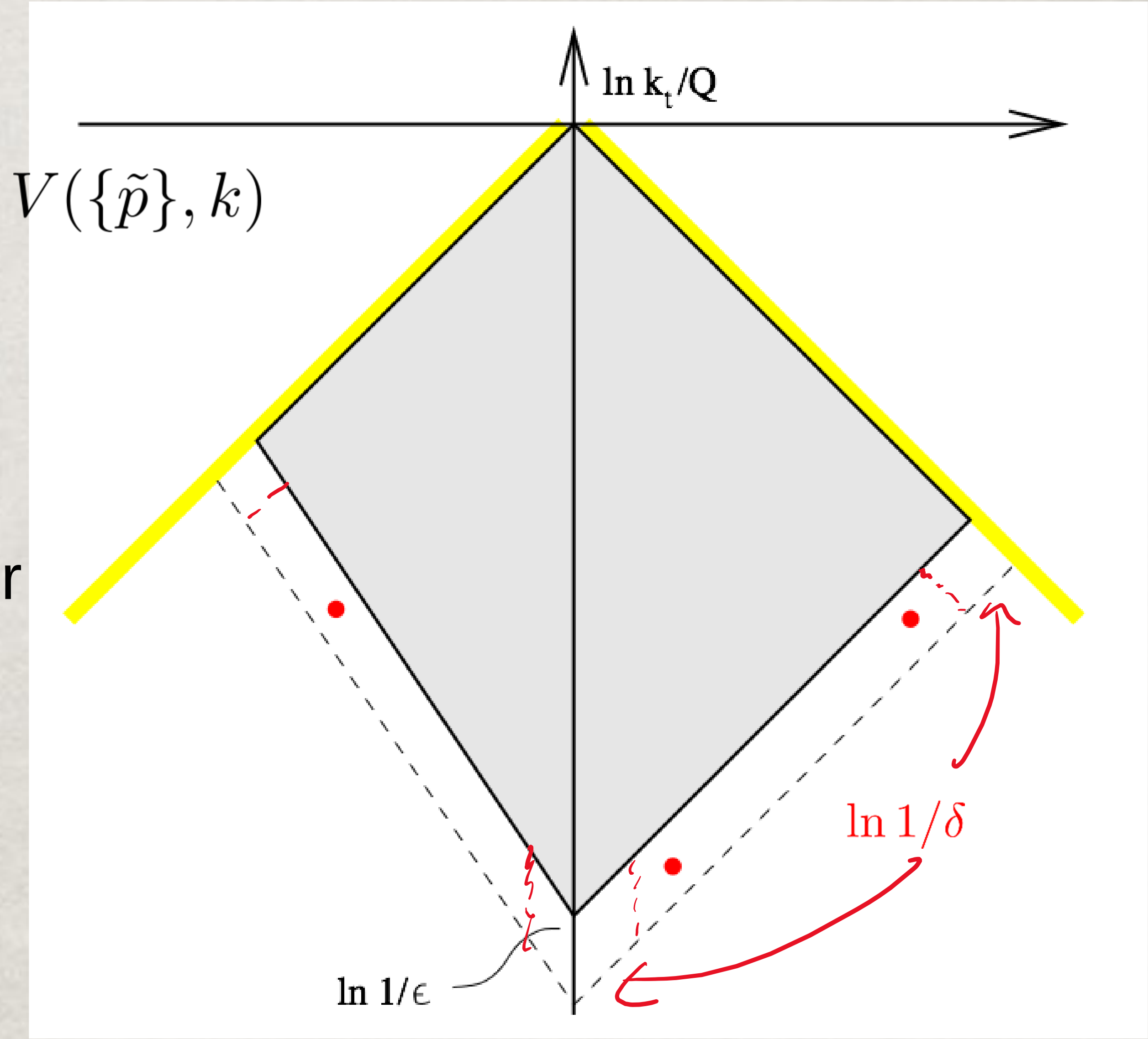


widely separated soft and collinear clusters, contributing at most at NLL accuracy

- Beyond NLL accuracy, the Lund plane becomes less effective, as one needs to resolve all the dots, i.e. work out the exact calculation of real and virtual corrections [AB Monni Salam Zanderighi, Becher Schwartz Neubert Bell, Stewart Tackmann, …]

Fully numerical resummations

It is possible to generate soft and collinear emissions fully numerically, including exact energy-momentum conservation, provided



- the value of v is small enough that $V(\{\tilde{p}\}, k)$ is given by its soft-collinear parameterisation
- a rapidity buffer $\delta \gg v$ prevents emission to become too collinear or too large-angle
- Automated NLL resummation implemented in CAESAR

[AB Salam Zanderighi hep-ph/0407286]

- Beyond NLL accuracy, one needs to carefully extrapolate the observable in the collinear and large-angle, beyond the rapidity buffer

Non-global and collinear logarithms

Corners in the Lund plane

- Soft and collinear to leg $\ell = 1, 2$

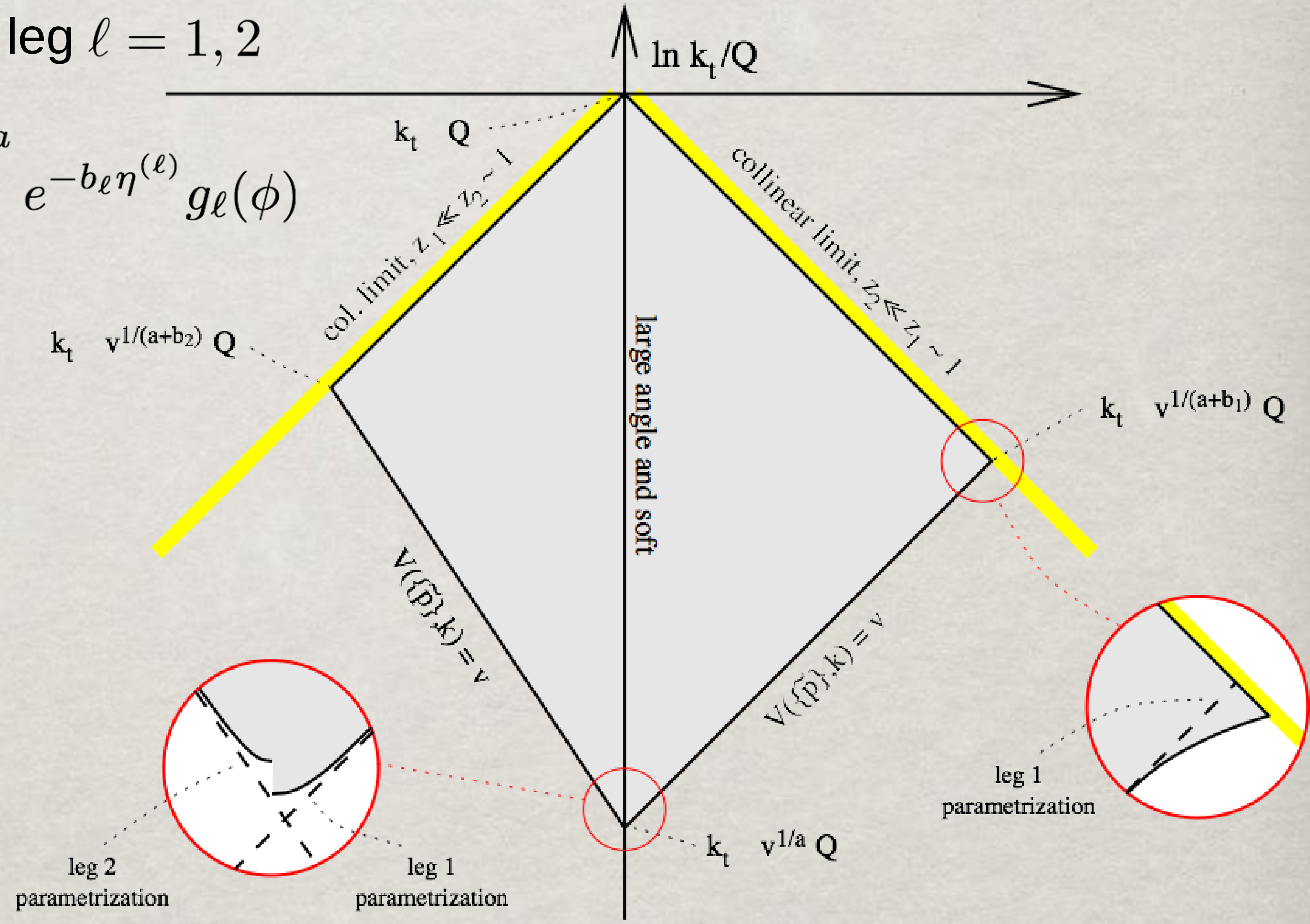
$$V(\{\tilde{p}\}, k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi)$$

- Soft and large angle

$$V(\{\tilde{p}\}, k) \sim k_t^a$$

- Hard and collinear

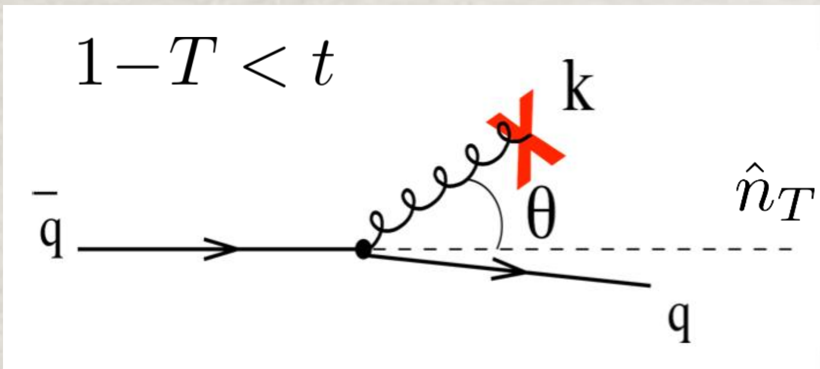
$$V(\{\tilde{p}\}, k) \sim k_t^{a+b_\ell}$$



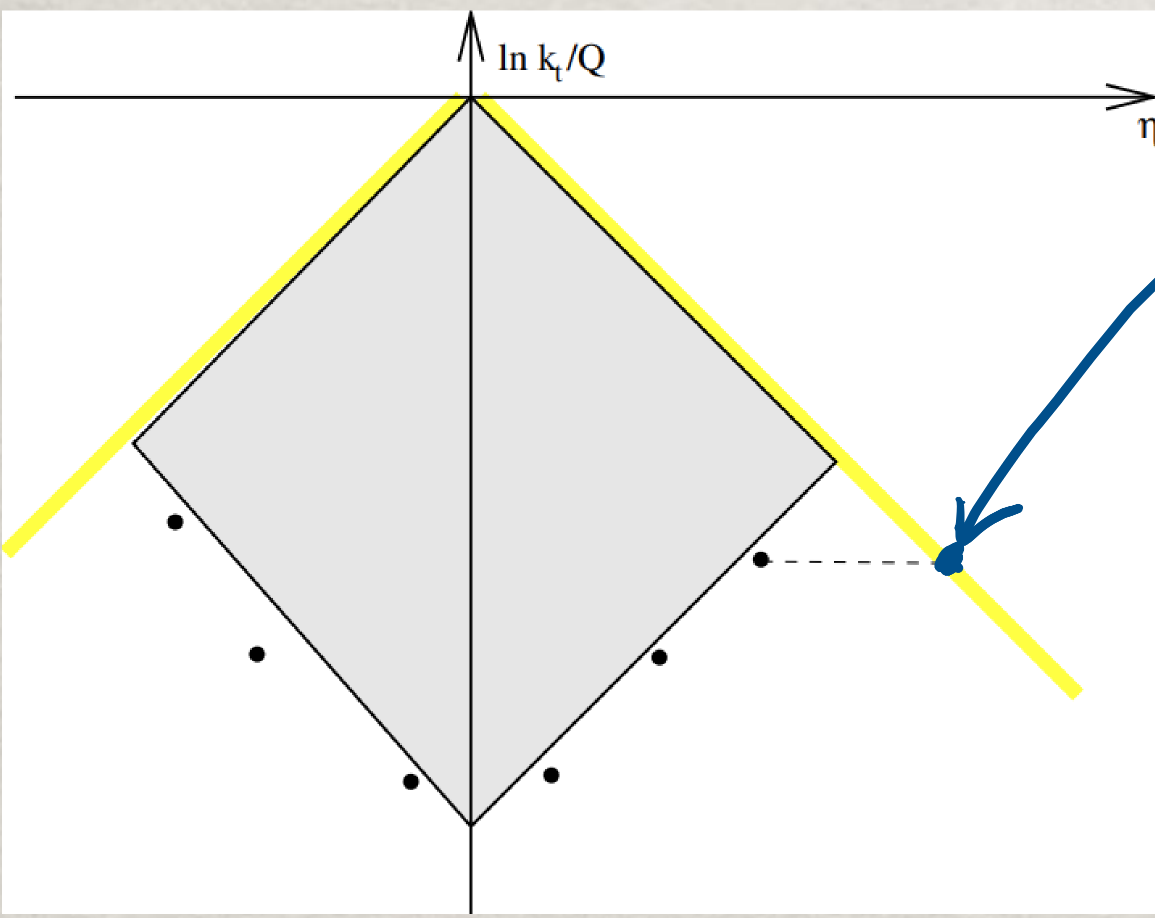
- For rIRC safe observables, the region in which the observable differs from the above parametrisation must also shrink to a point for $v \to 0$

Sensitivity to recoil

- Hard emitting partons recoil again soft-collinear partons. It is their recoiled momenta that enter the calculation of the observable



$$\vec{p}_{t,\ell} = -\sum_{i\in\mathcal{H}_\ell} \vec{k}_{ti}$$



$$\vec{p}_{t,\ell} \simeq -\max_{i\in\mathcal{H}_\ell} \vec{k}_{ti}$$

- Hard partons recoiling against soft and collinear emissions gives a negligible contribution to a jet's invariant mass

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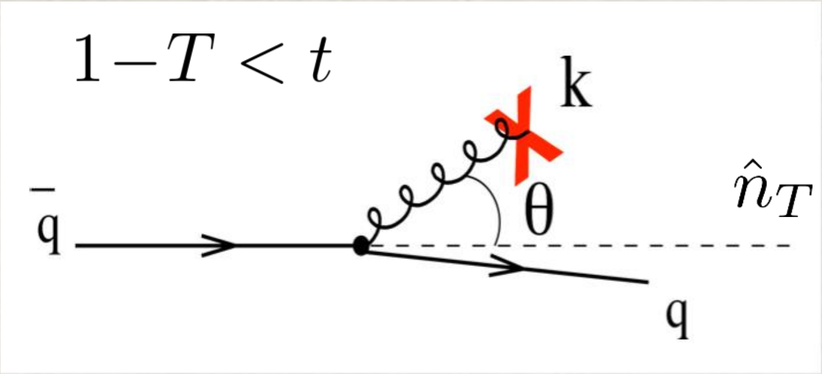

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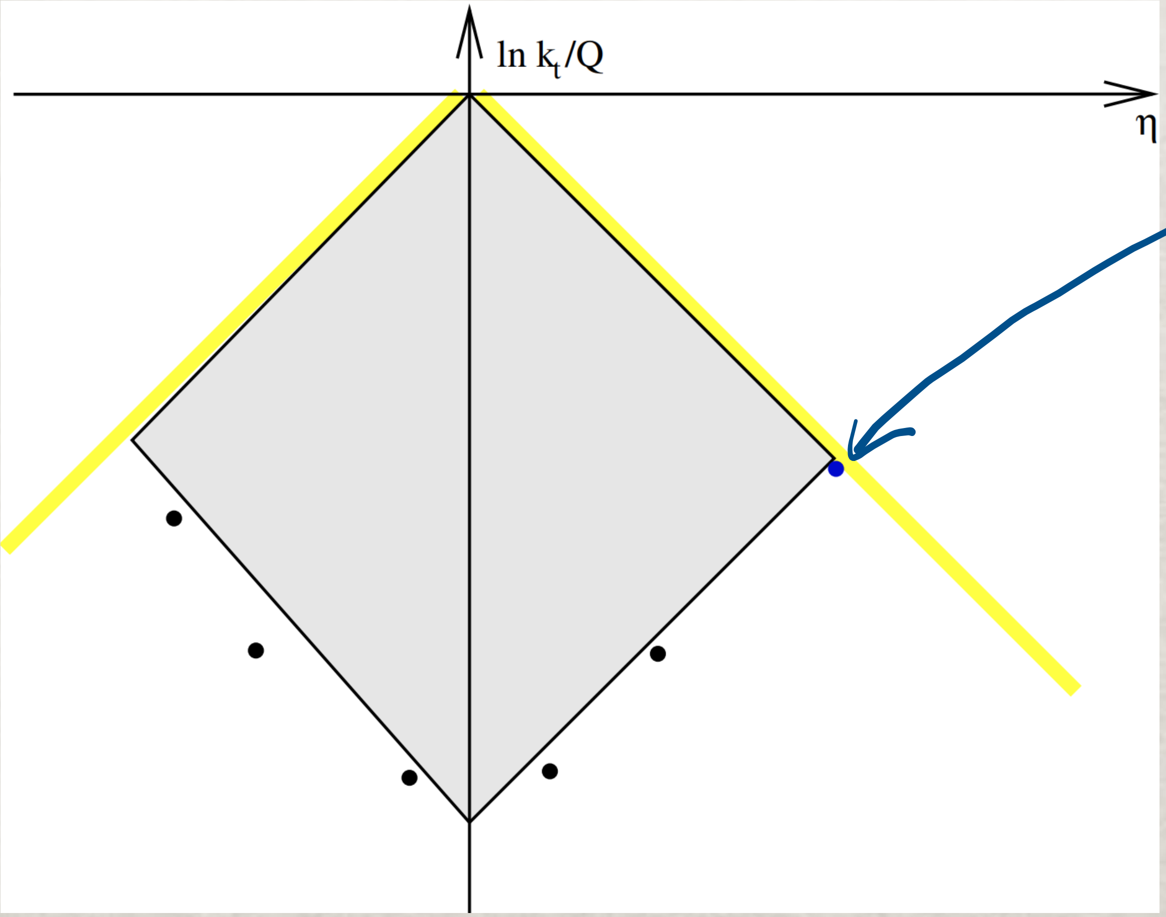


$$\vec{p}_{t,\ell} \simeq -\max_{i\in\mathcal{H}_\ell} \vec{k}_{ti}$$

- Hard partons recoiling against soft and collinear emissions gives a negligible contribution to a jet's invariant mass
- Recoil is non-negligible only if it's against a hard collinear gluon $\Longrightarrow$ NNLL

[AB McAslan Monni Zanderighi 1412.2126]

NON-GLOBAL LOGARITHMS

- If the boundaries of the soft large-angle region do not shrink to a point, we can have NLL contributions from soft emissions close to the boundary

[Dasgupta Salam hep-ph/0208073]

These emissions are not widely separated in angle

$\Downarrow$

Secondary emissions can never be neglected

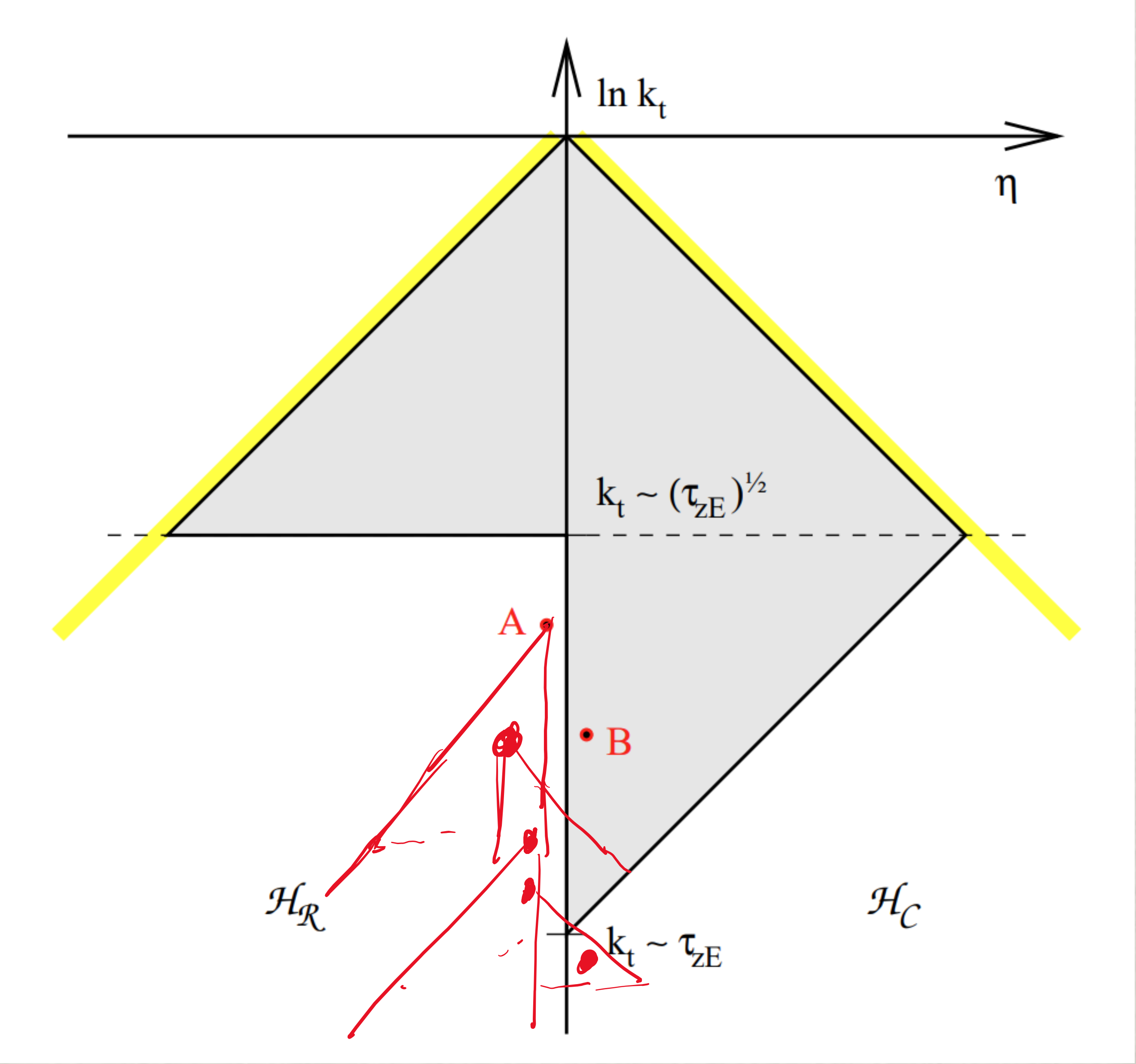
$\Downarrow$

Gluon dynamics is intrinsically non-linear

$\Downarrow$

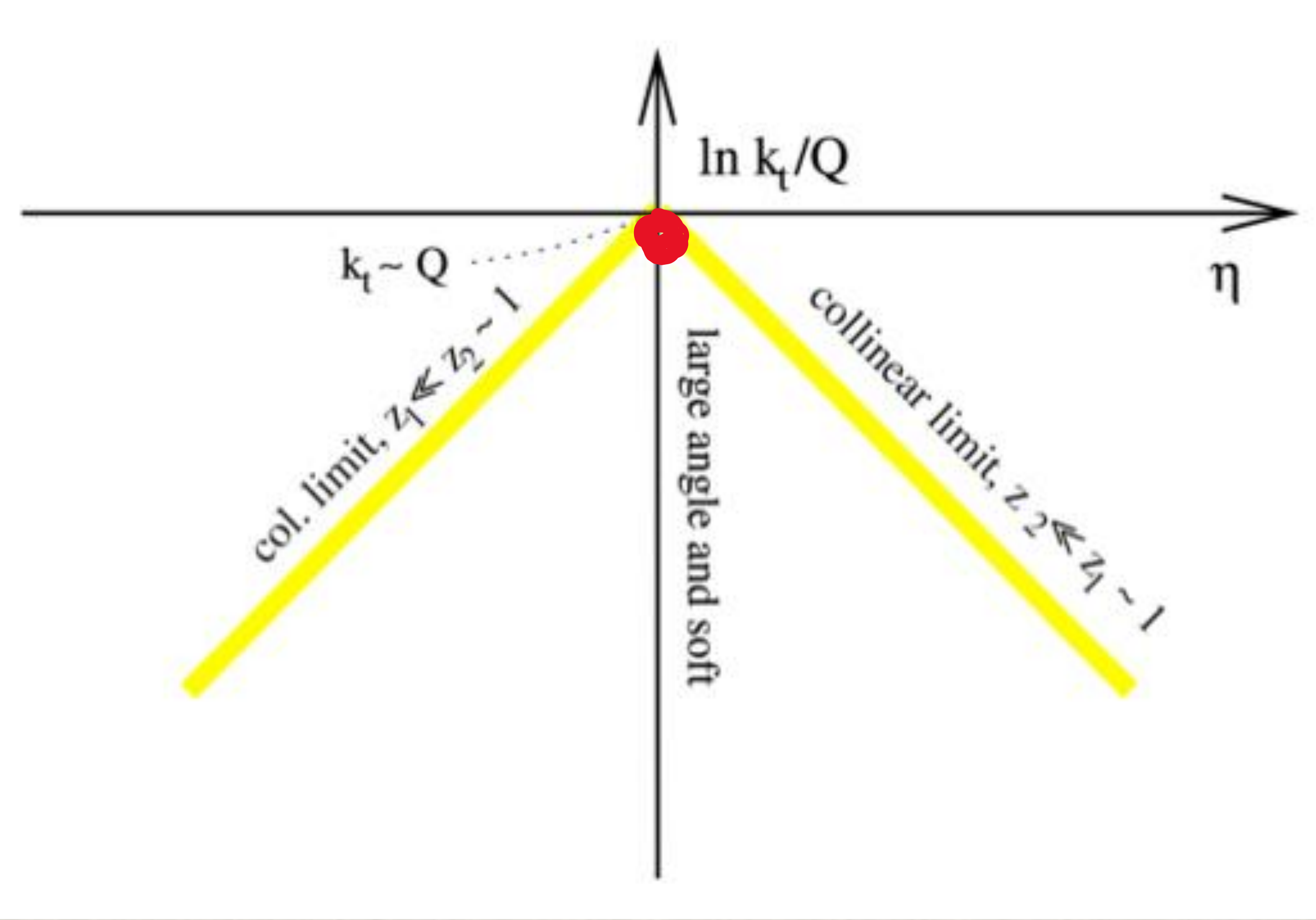
Non-global logarithms

[Dasgupta Salam hep-ph/0104277]



Inclusive observables

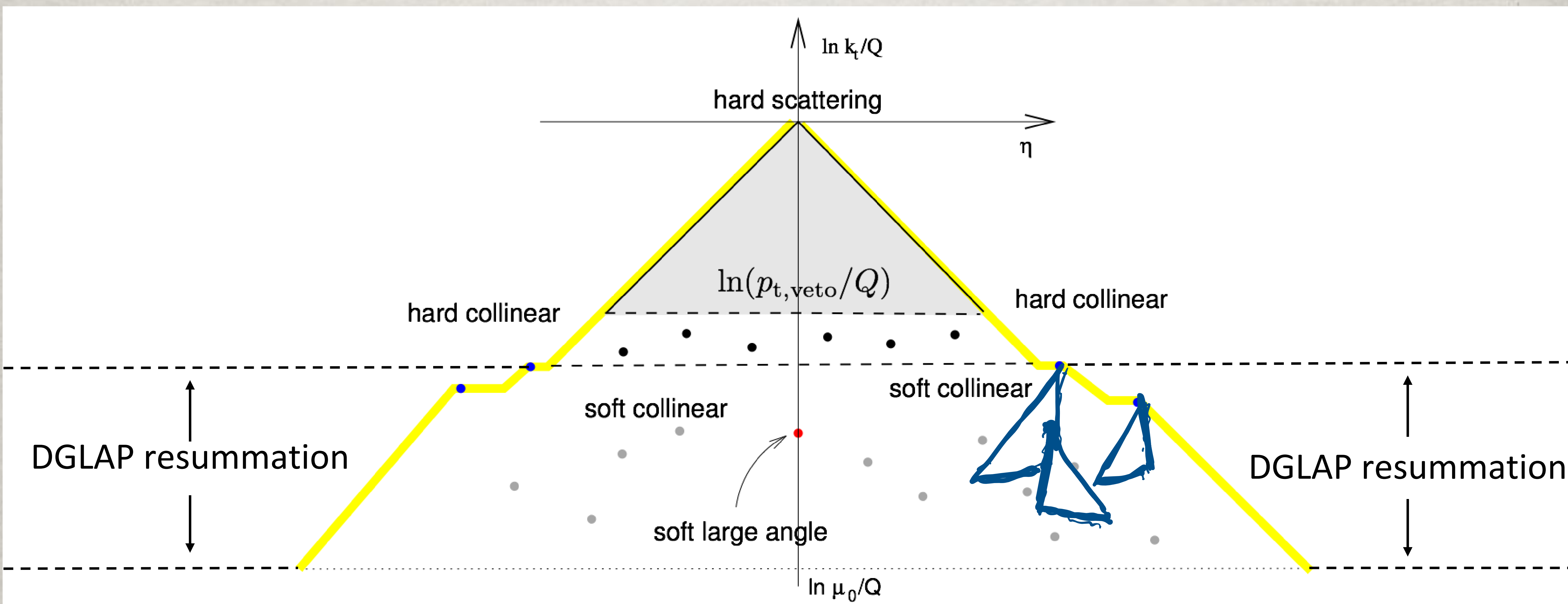
- Inclusive observables have essentially no veto region $\Longrightarrow$ real and virtual contributions cancel up to the hard scale Q



- Relevant emissions are confined to a corner of the Lund plane $\Longrightarrow$ fixed order contribution starting at order α_s

DGLAP RESUMMATION

- Each hard and collinear emission also gives rise to a secondary Lund plane

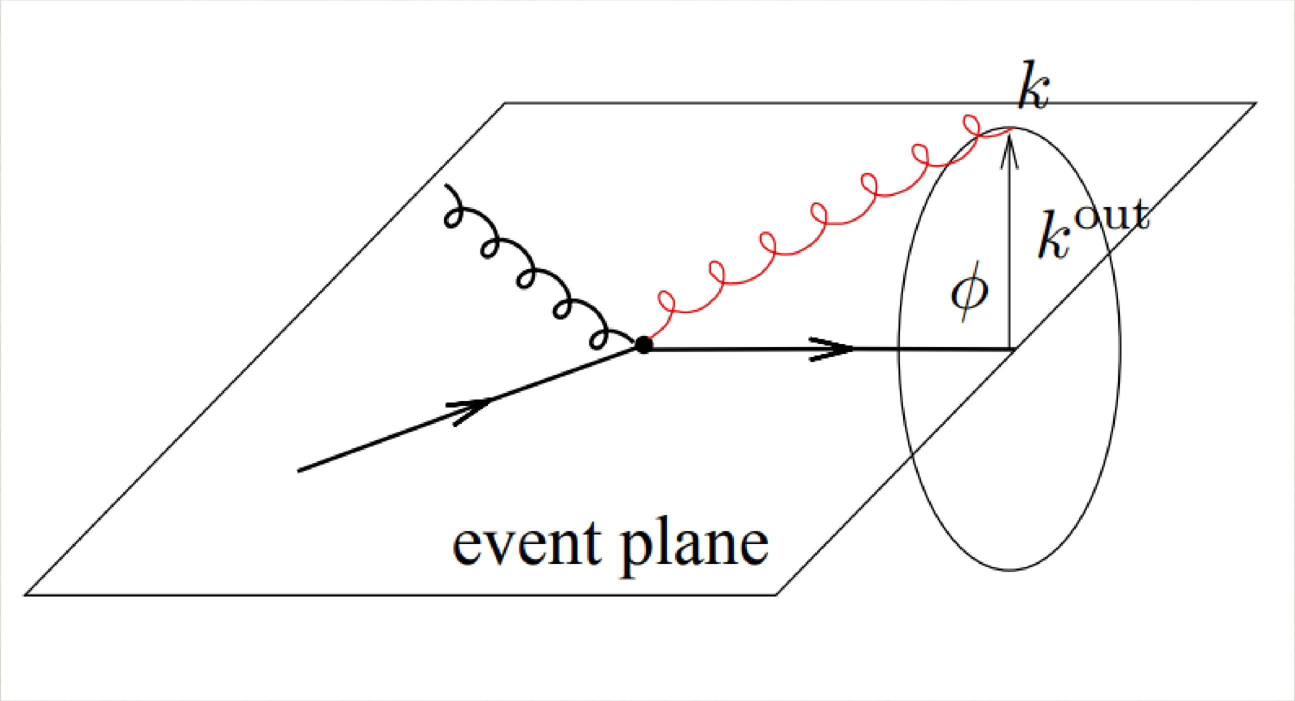


- rIRC safe observables are sensitive only to emissions close to the vetoed region $\Longrightarrow$ inclusiveness with respect to all secondary emissions
- Secondary Lund planes shrink to a point $\Longrightarrow$ single logarithmic DGLAP resummation

Three-jet observables in the two-jet limit

Near-to–planar event shapes

- Out-of-plane event shapes, e.g. D-parameter, give access to properties of QCD radiation such as coherence, spin correlations, that are not immediately accessible for two-jet observables



$$D = \frac{27}{Q^3} \sum_{i<j<k} \frac{[\vec{p}_i \cdot (\vec{p}_j \times \vec{p}_k)]^2}{E_i E_j E_k}$$

- In the tree-jet region, resummation can be performed at NNLL accuracy

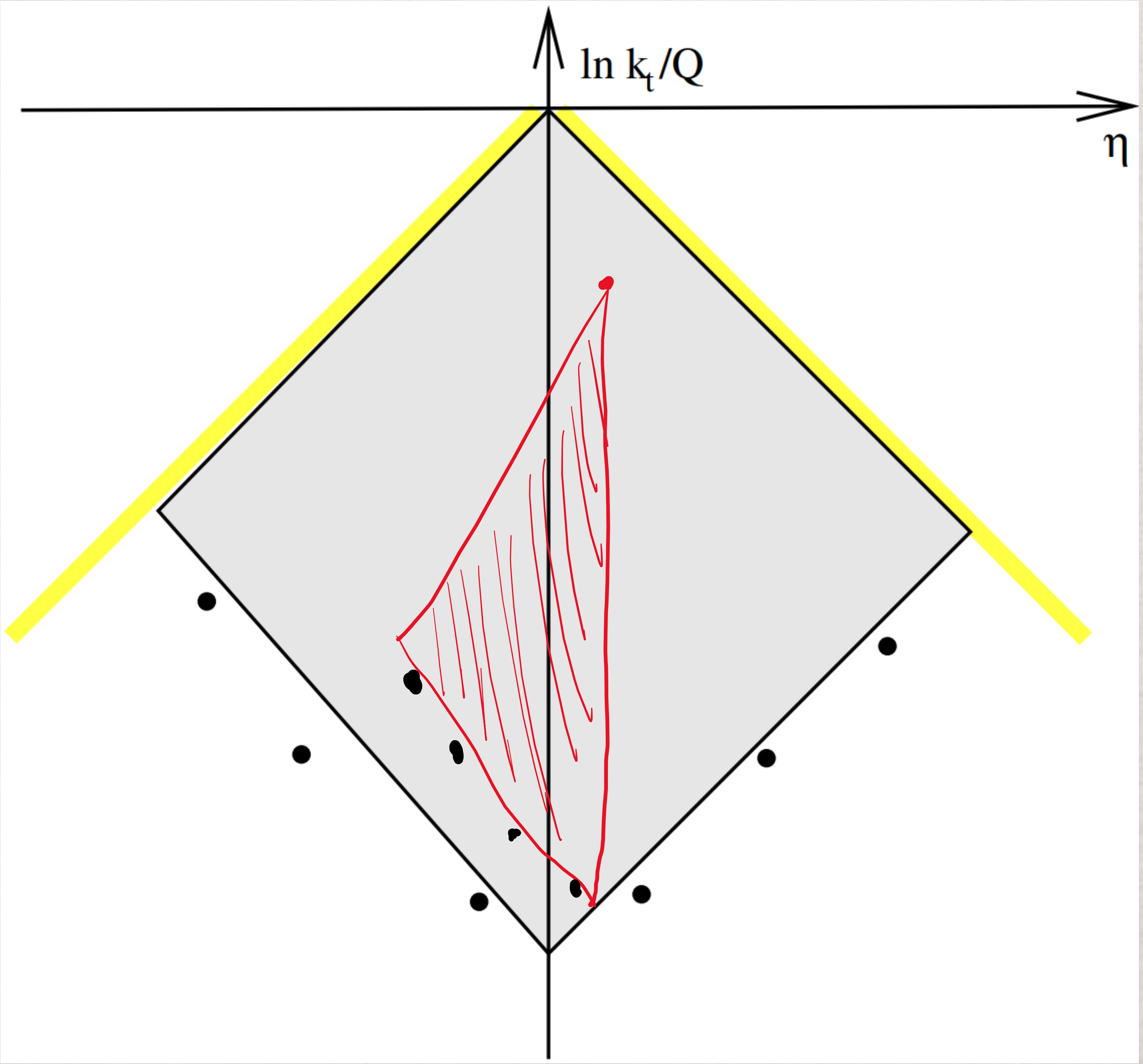
[Arpino AB El-Menoufi 1912.09341]

- The two-jet region gives important information on the properties of secondary emissions

[Larkoski Procita 1810.06563]

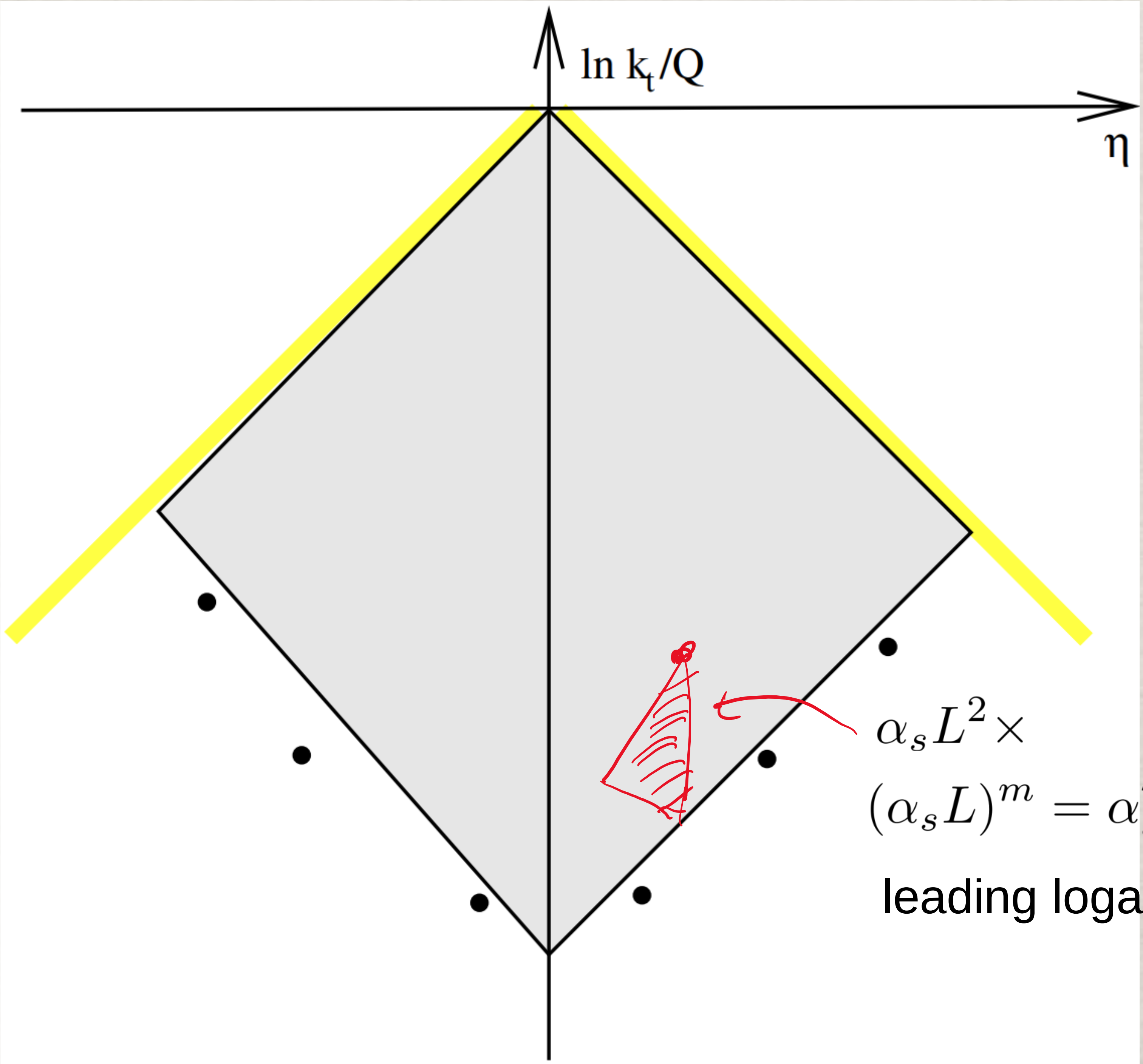
D-parameter in three-jets

- In the three-jet region, we have a large secondary Lund plane starting at the transverse momentum of the emitted gluon $\Longrightarrow$ resummation guaranteed



D-PARAMETER IN TWO JETS

- The secondary gluon can be soft and collinear, but still inside the vetoed region $\Longrightarrow$ mixing of logarithmic orders [Larkoski Procita 1810.06563]



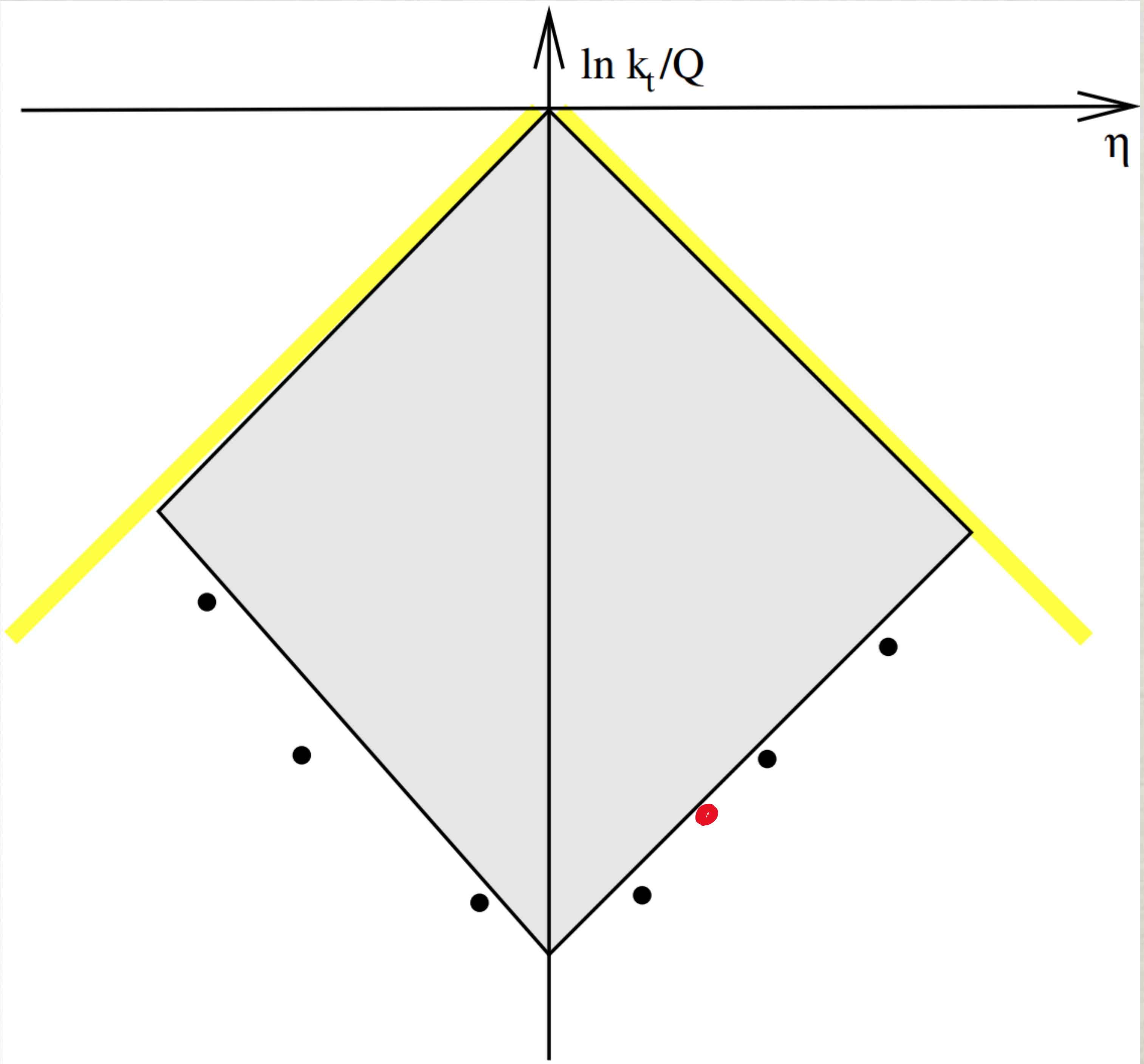
$\alpha_s L^2 \times$

$(\alpha_s L)^m = \alpha_s^n L^{n+1}$

leading logarithm?

D-parameter in two jets

- All secondary emissions are below the vetoed region $\Longrightarrow$ no secondary Lund plane $\Longrightarrow$ multiple emissions give an NLL contribution



Concluding remarks

- The Lund plane gives an intuitive visualisation of the phase space available to gluons

- Extremely useful to identify double logarithmic and single logarithmic contributions $\Longrightarrow$ visual NLL resummations

- Beyond NLL accuracy, one needs to carefully consider the exact phase space of secondary emissions, and the details of the cancellation between real and virtual contributions

- Most results used the properties of the primary Lund plane. It is time to consider observables that probe secondary Lund planes

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Thank you for your attention

Extra

Spare Lund plane