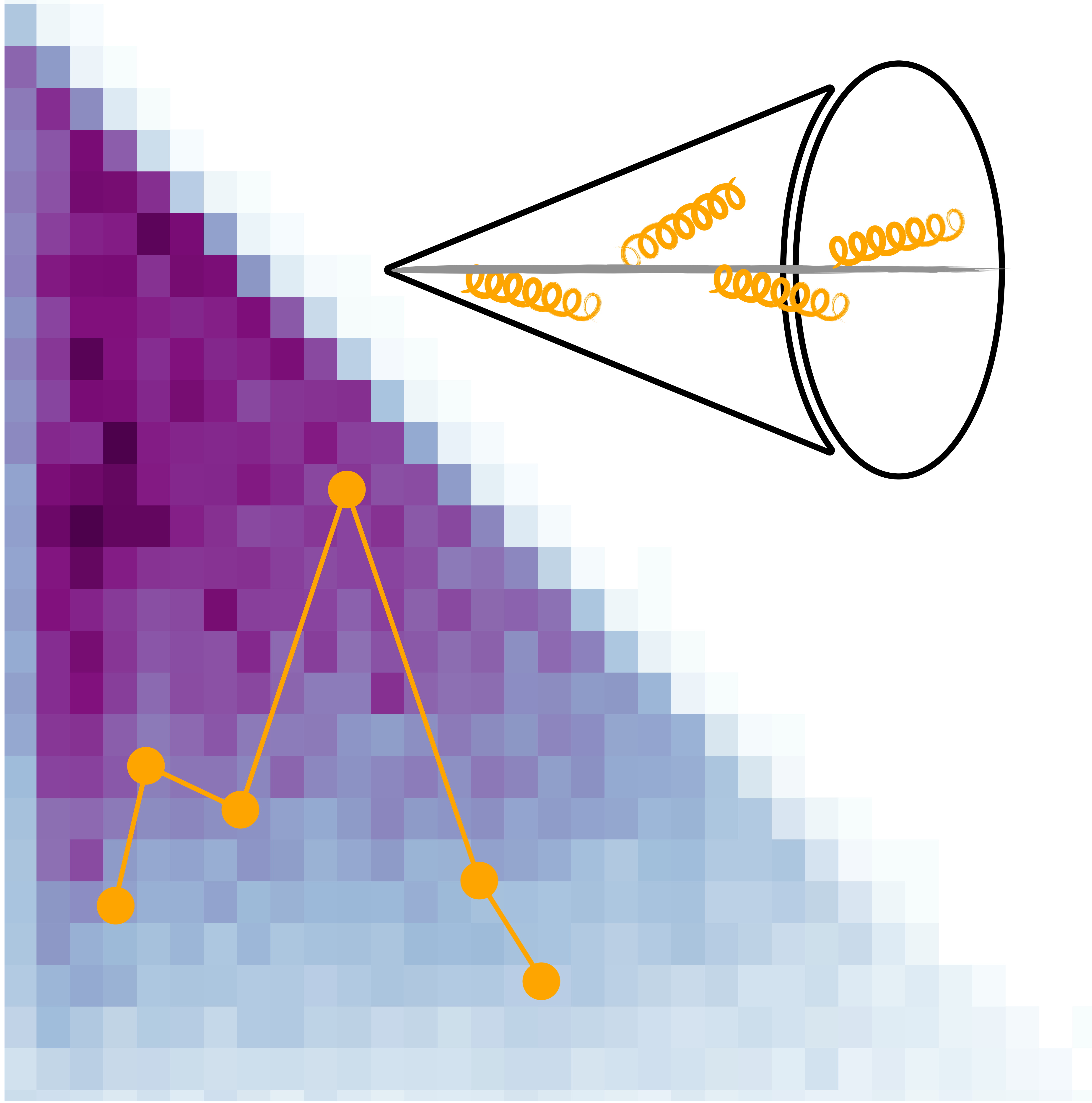
Counting jets in the Lund plane

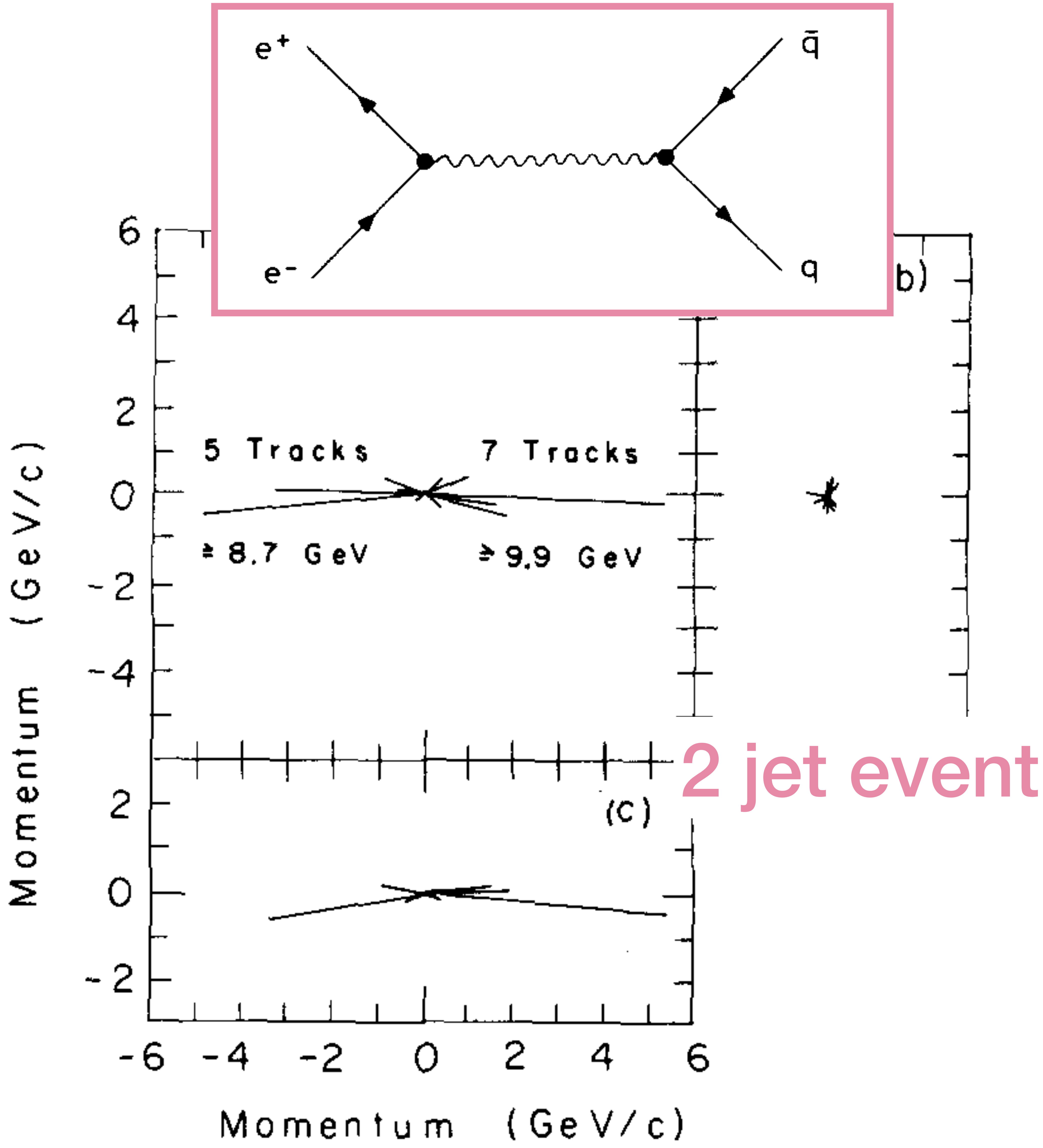


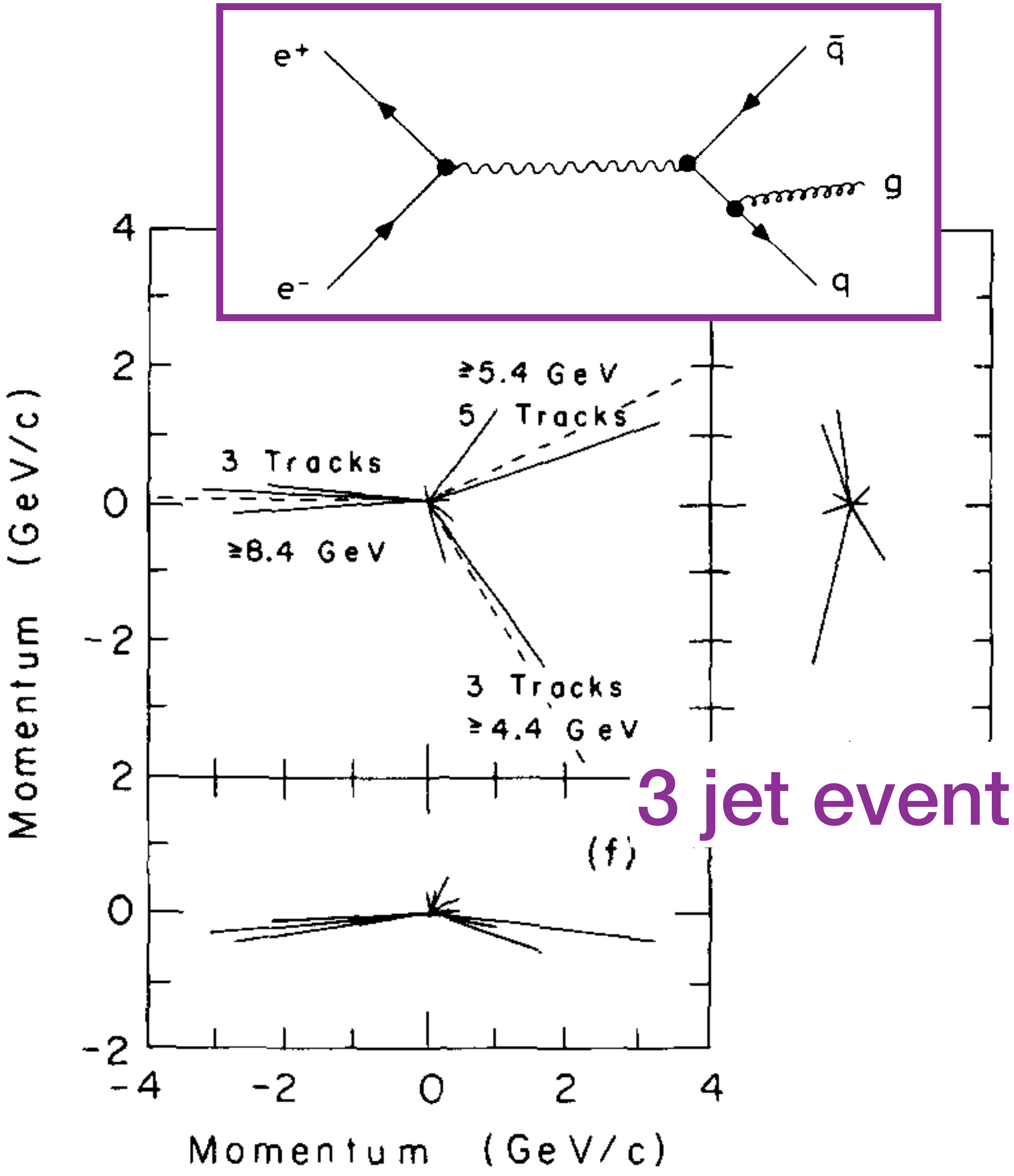
Alba Soto-Ontoso

1st Lund jet plane Institute

CERN, 3rd July, 2023

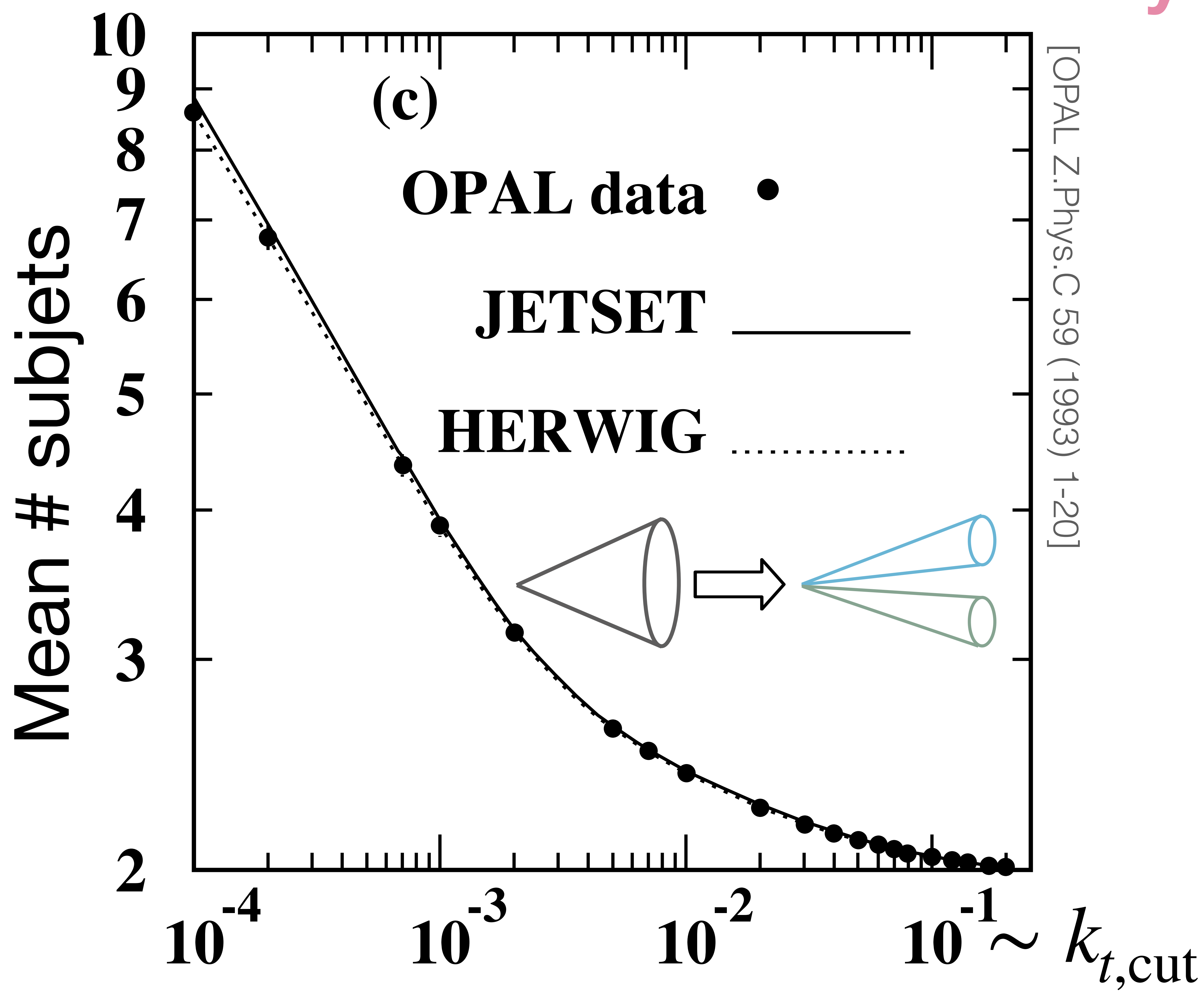
Why counting jets? e.g. to discover the gluon



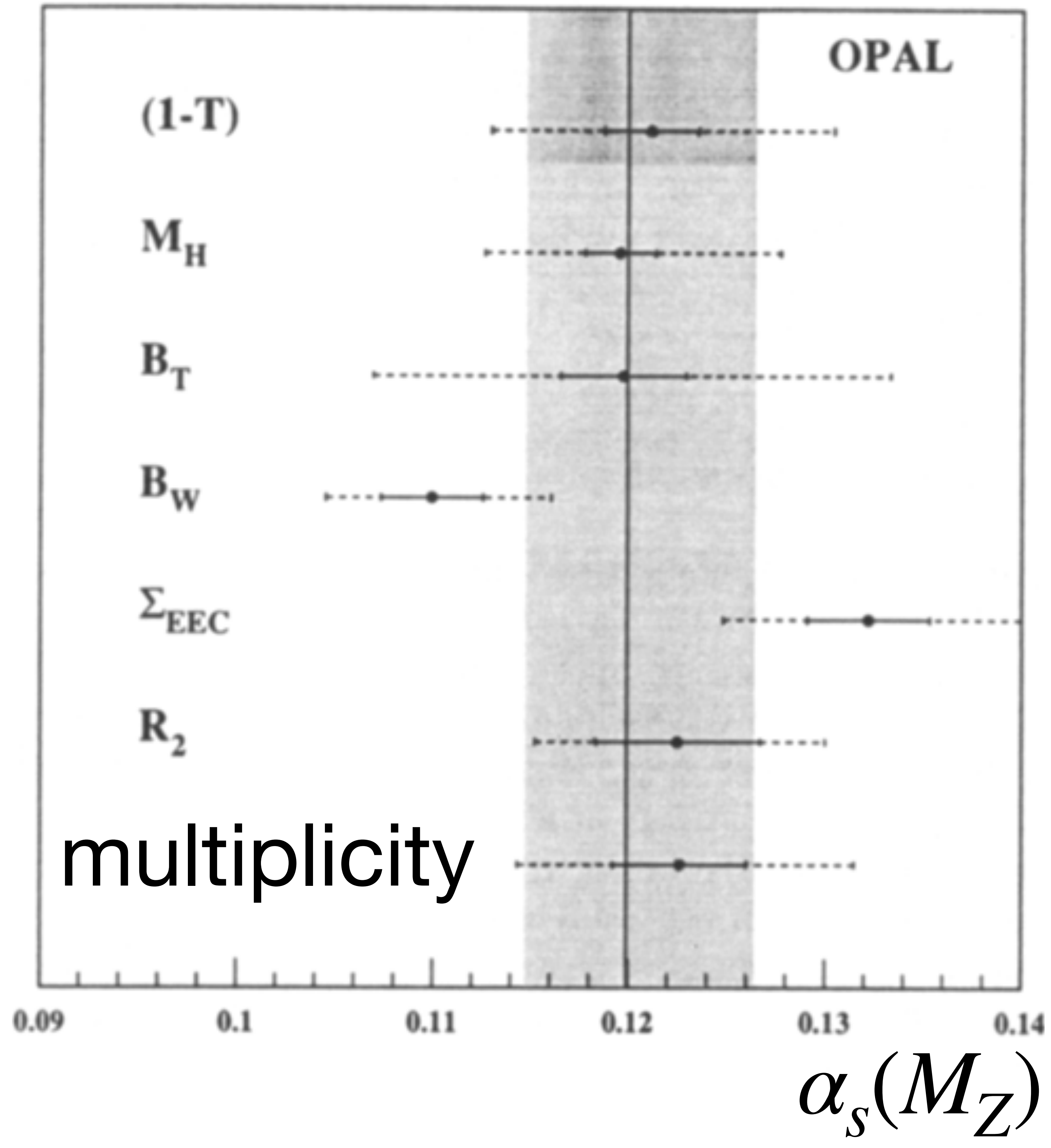


Why counting jets? e.g. to extract α_s in e^+e^-@LEP

early 90's



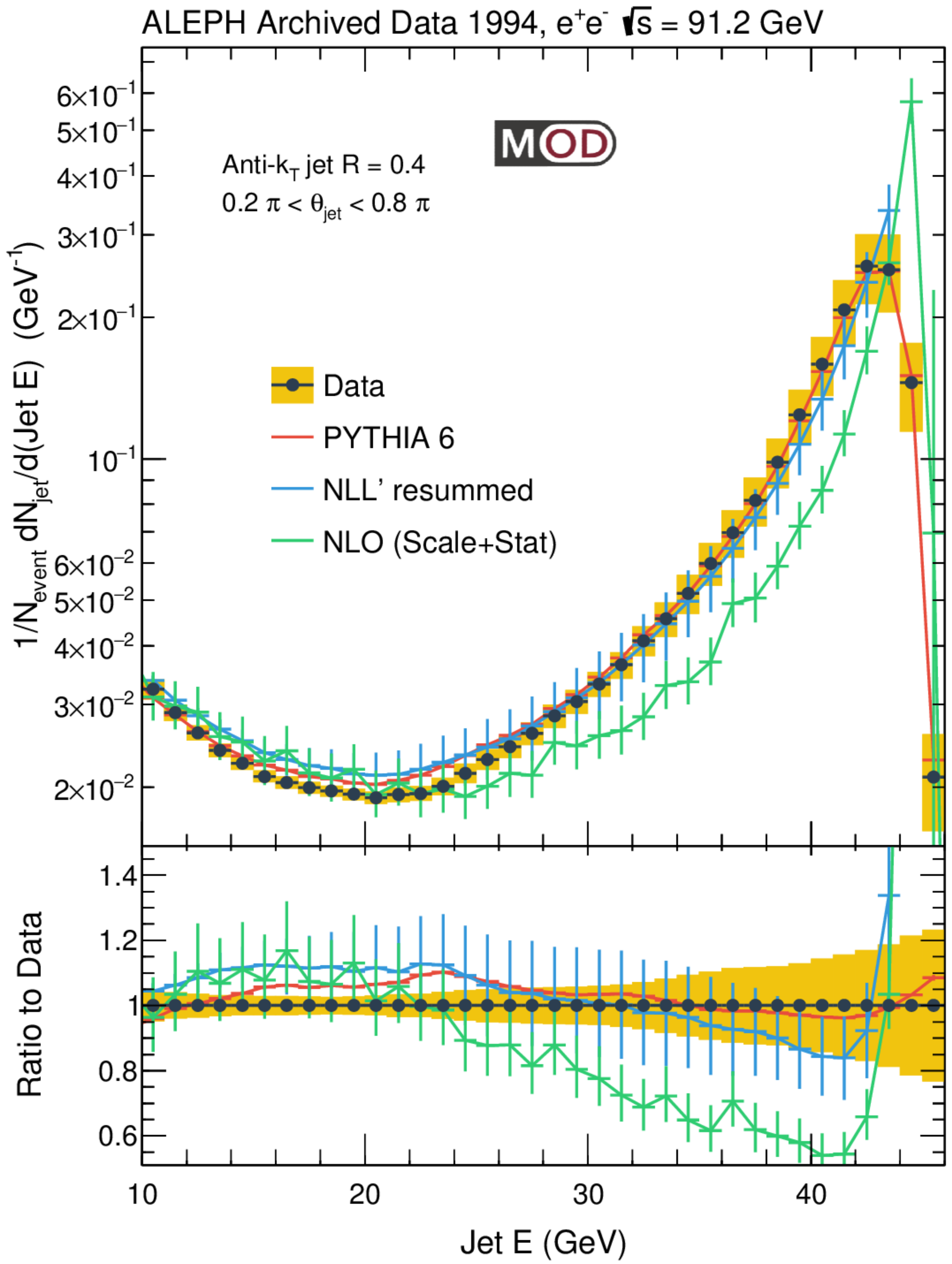
[OPAL Z.Phys.C 59 (1993) 1-20]

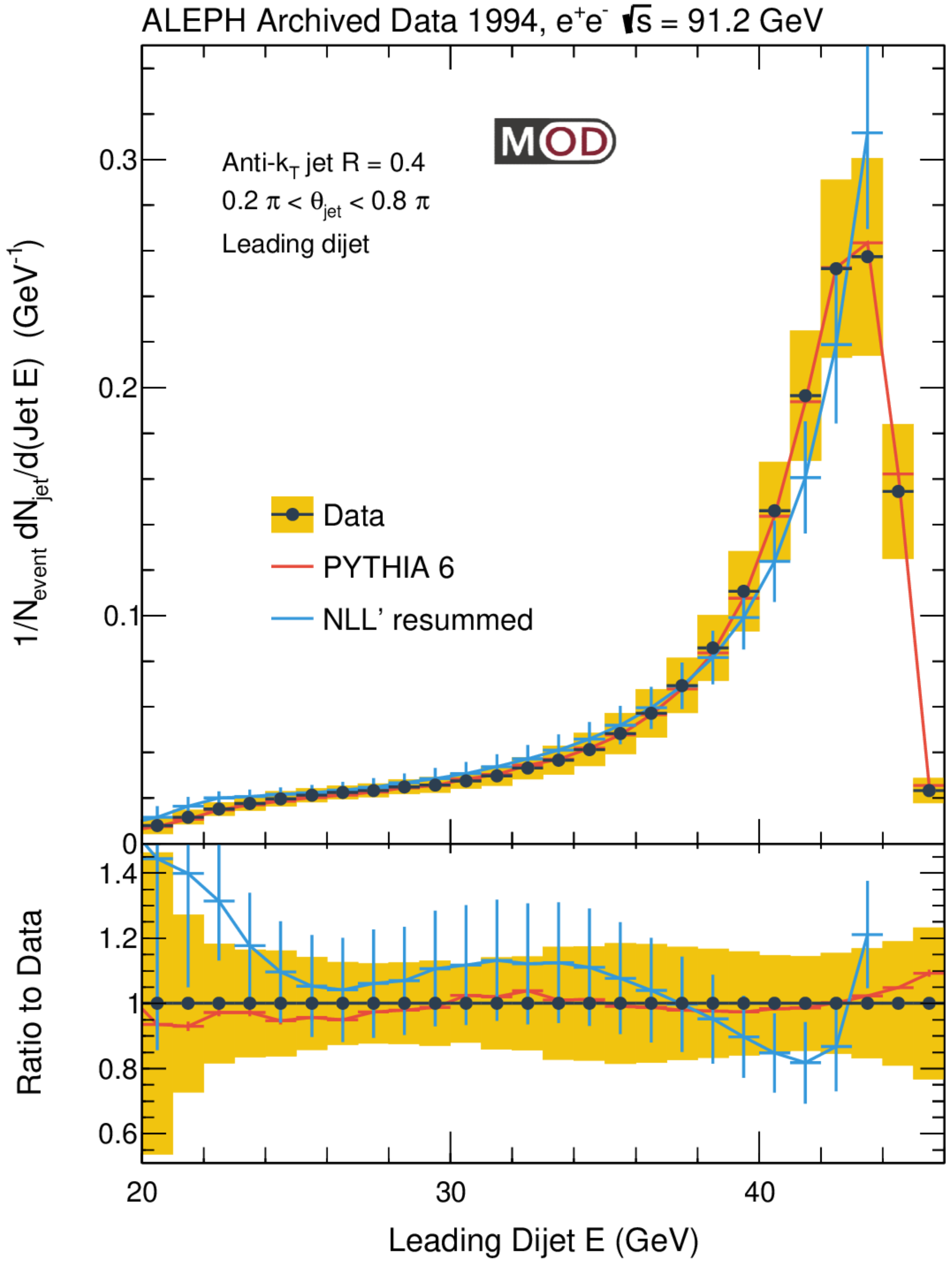


[Catani et al. NPB 377 (1992) 445-460]

Why counting jets? e.g. to extract α_s in e^+e^-@LEP

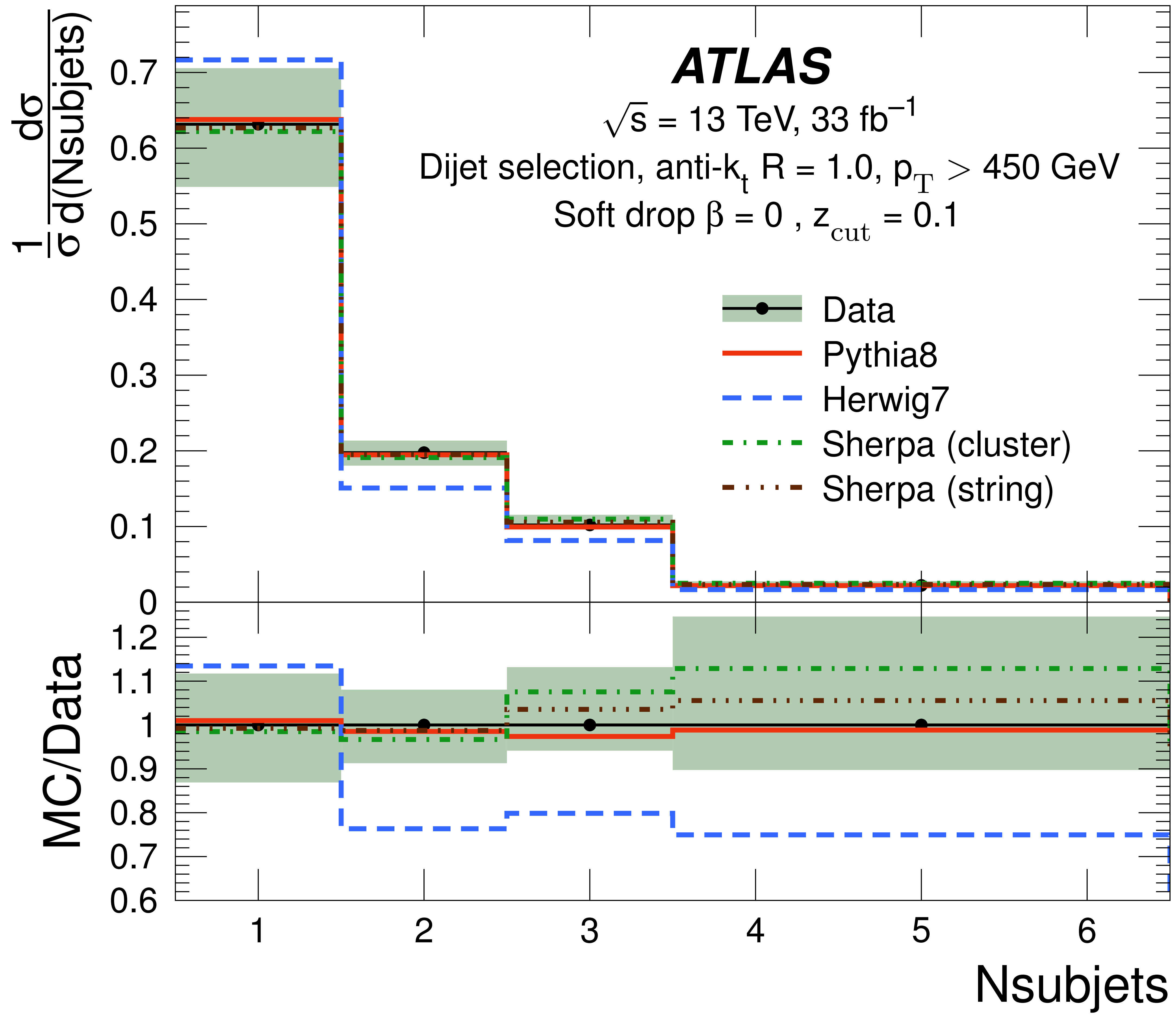
early 2020's



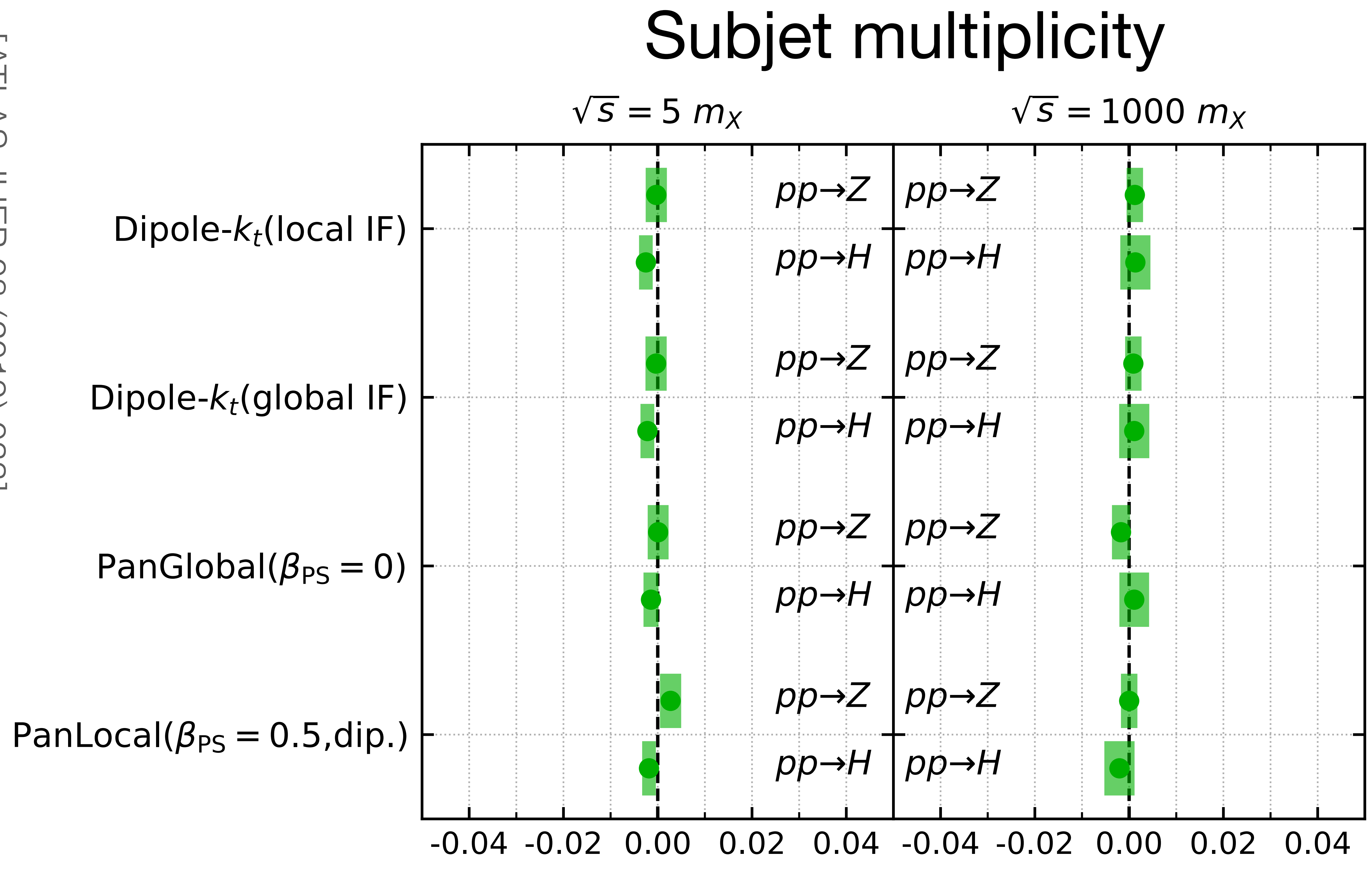


[Yi Chen et al. JHEP 06 (2022) 008]

Why counting jets? e.g. to test parton showers



[ATLAS JHEP 08 (2019) 033]



green == shower matches NDL analytics

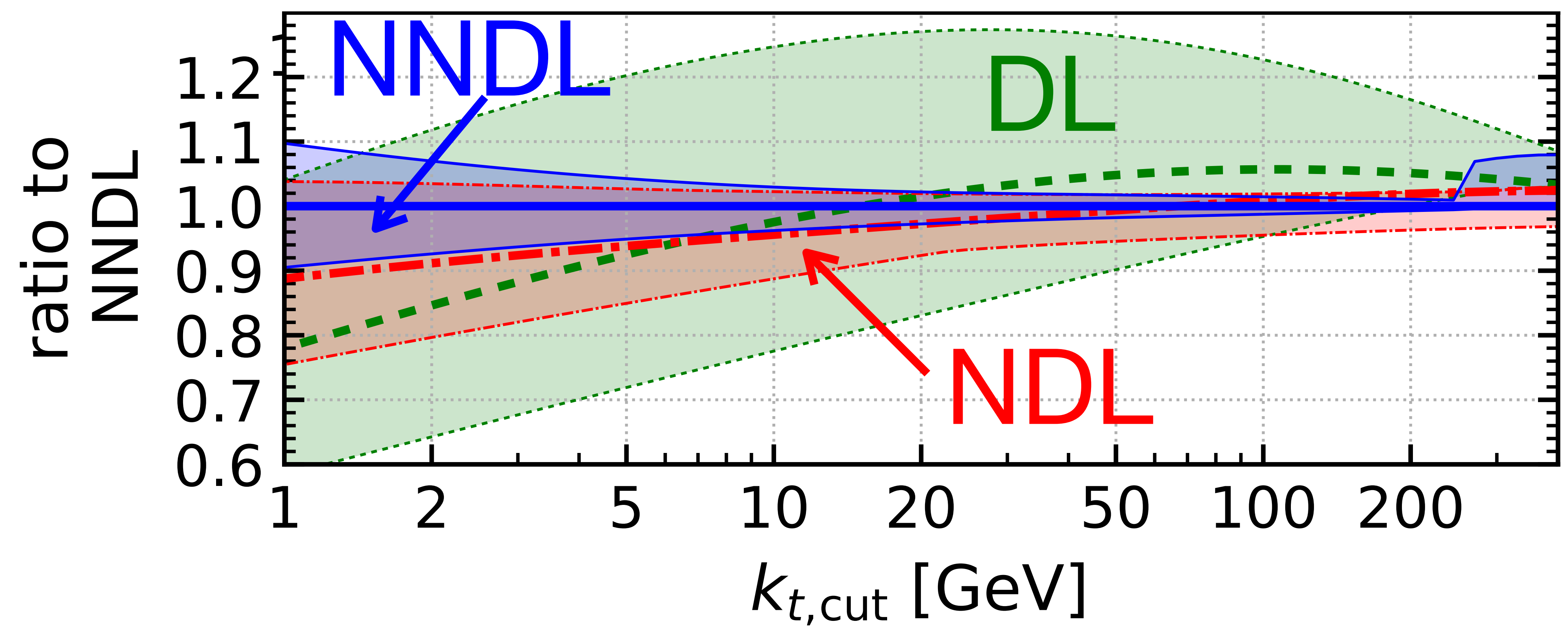
[van Beekveld, ASO et al. JHEP 11 (2022) 019, JHEP 11 (2022) 020]

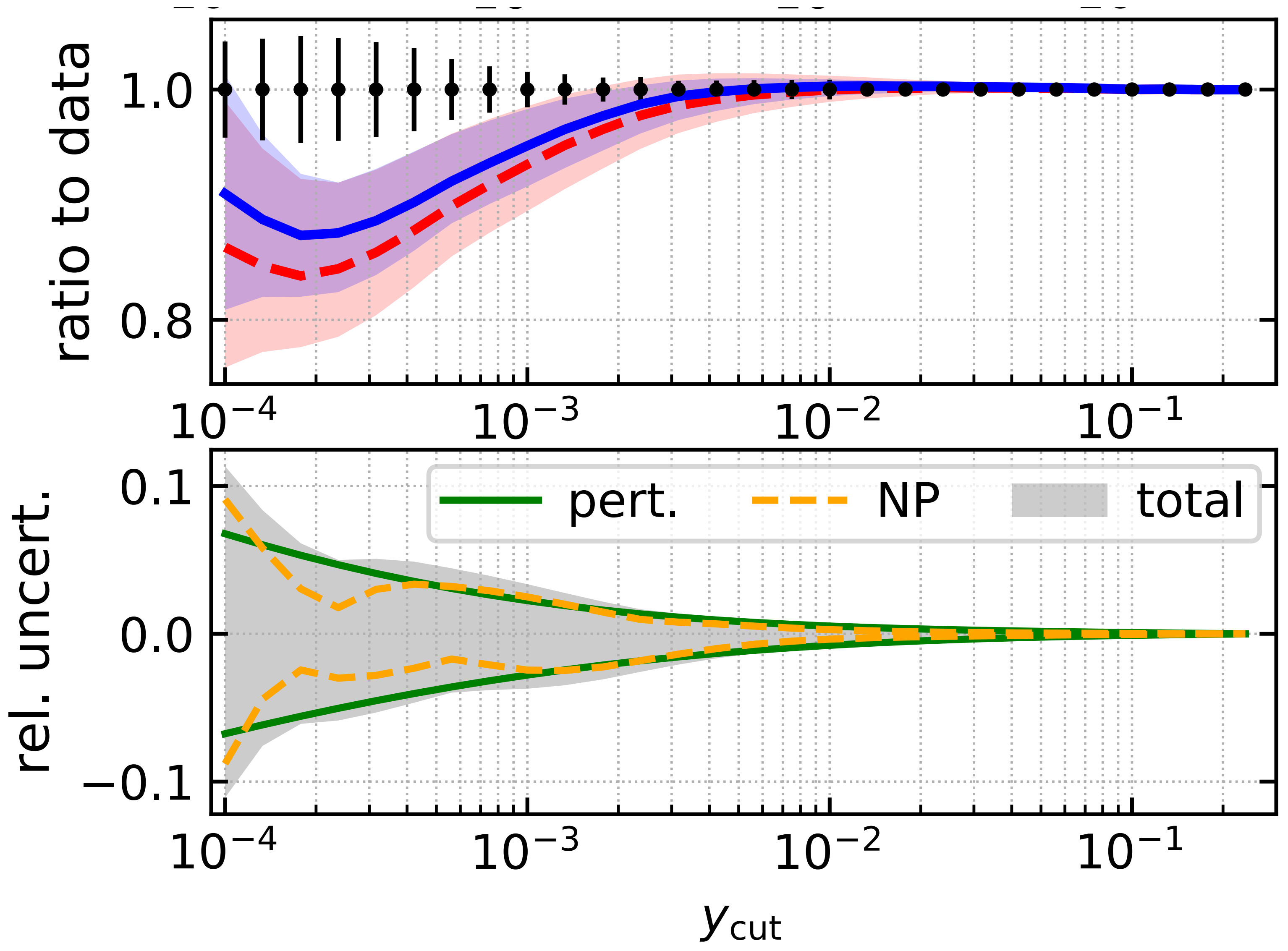
Outline of this talk

1 Multiplicity definition: charged particle, jets, subjets?

2 Lund multiplicity: resummation structure in both e^+e^- and pp

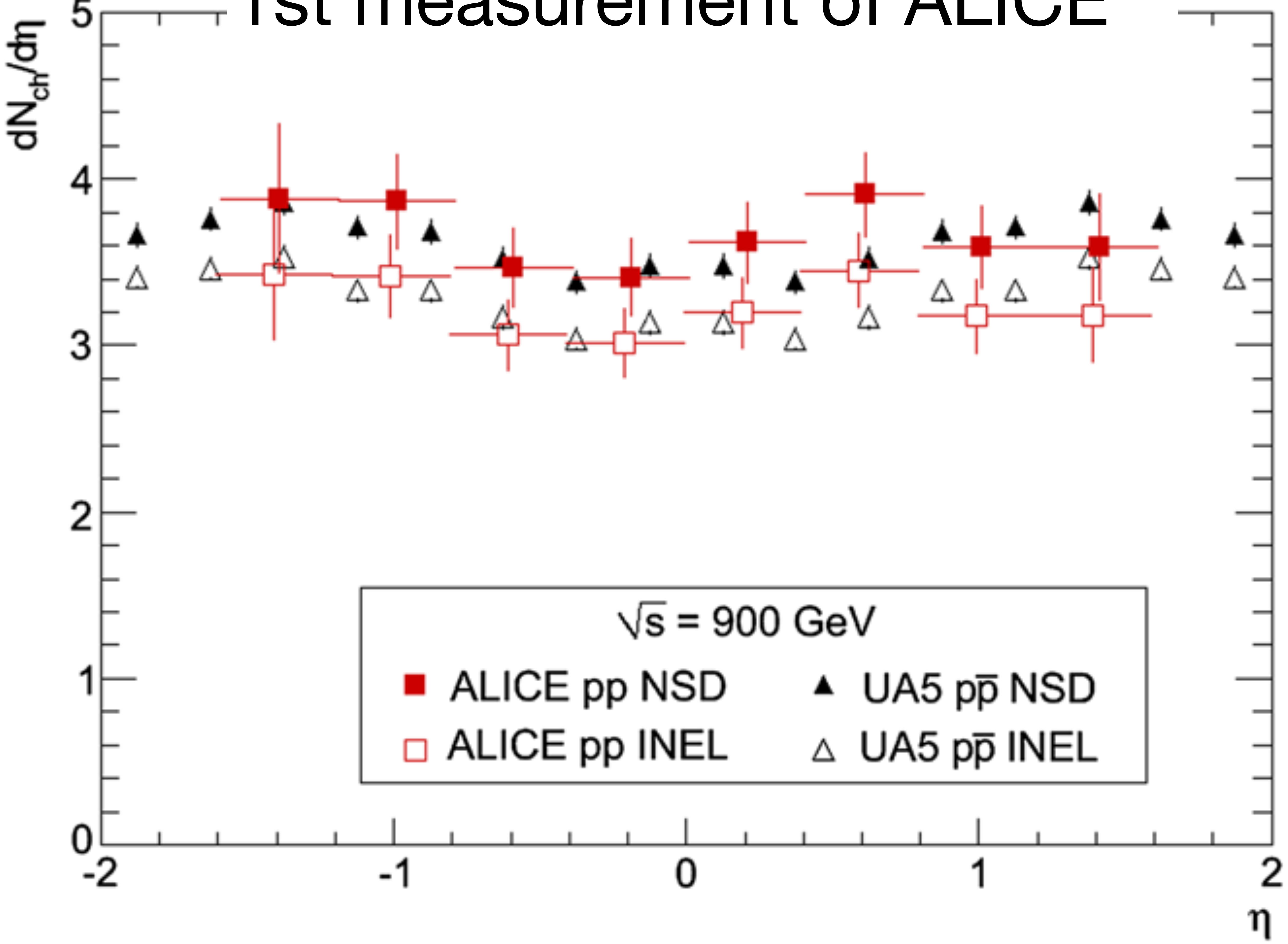
3 Phenomenological studies: NLO+NNDL+NP





Why counting jets and not particles?

1st measurement of ALICE



[ALICE EPJC 65 (2010)]

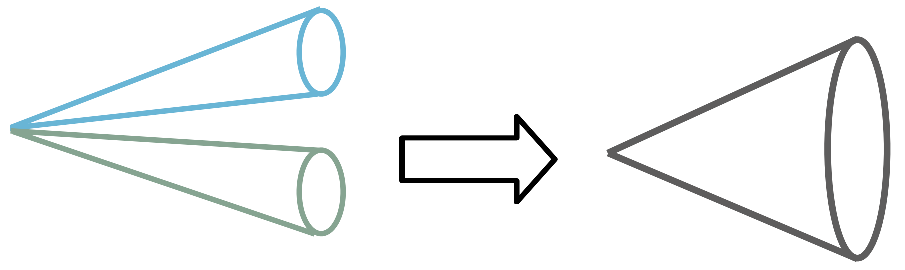
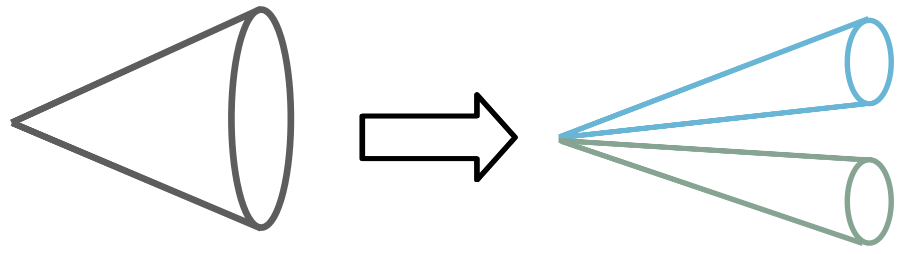
Particle multiplicities are non-perturbative objects

Jet algorithms ⟺ multiplicity

1. Choose a jet algorithm: k_t-like or angular ordered

2. Choose a resolution parameter: e.g. $k_{t,\mathrm{cut}}$

3. Count how many pairs of subjets satisfy $k_t > k_{t,\mathrm{cut}}$

- either during the clustering 
- or during the de-clustering 

Choices impact calculability of the observable

Analytic structure of the average subjet multiplicity

The perturbative expansion of the average subjet multiplicity reads

$$\langle N(\alpha_s; L)\rangle = \Big[\underbrace{h_1(\alpha_s L^2)}_{\mathrm{DL}} + \underbrace{\sqrt{\alpha_s}\, h_2(\alpha_s L^2)}_{\mathrm{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\mathrm{NNDL}\sim 10\%} + \ldots\Big] \quad L = \ln \frac{Q}{k_{t,\mathrm{cut}}}$$

$\alpha_s \ll 1$
$\alpha_s L^2 \sim 1$

where $\mathrm{N}^k\mathrm{DL}$ accuracy implies control over $\alpha_s^n L^{2n-k}$ terms with $0 < n < \infty$

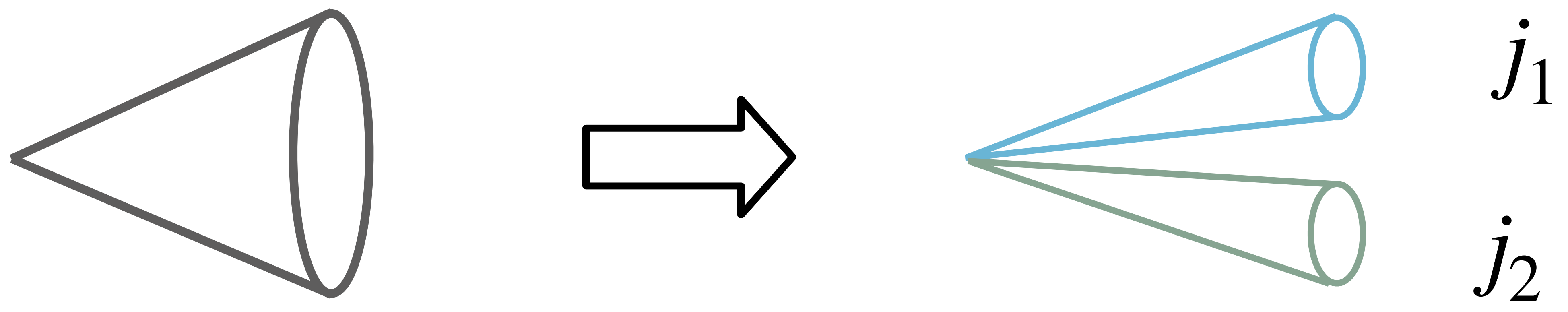
Jet algorithm and/or exact definition of $k_t > k_{t,\mathrm{cut}}$ matter at NNDL

Lund-based definition of subjet multiplicity

[Medves, ASO, Soyez, JHEP 10 (2022) 156, JHEP 04 (2023) 104]

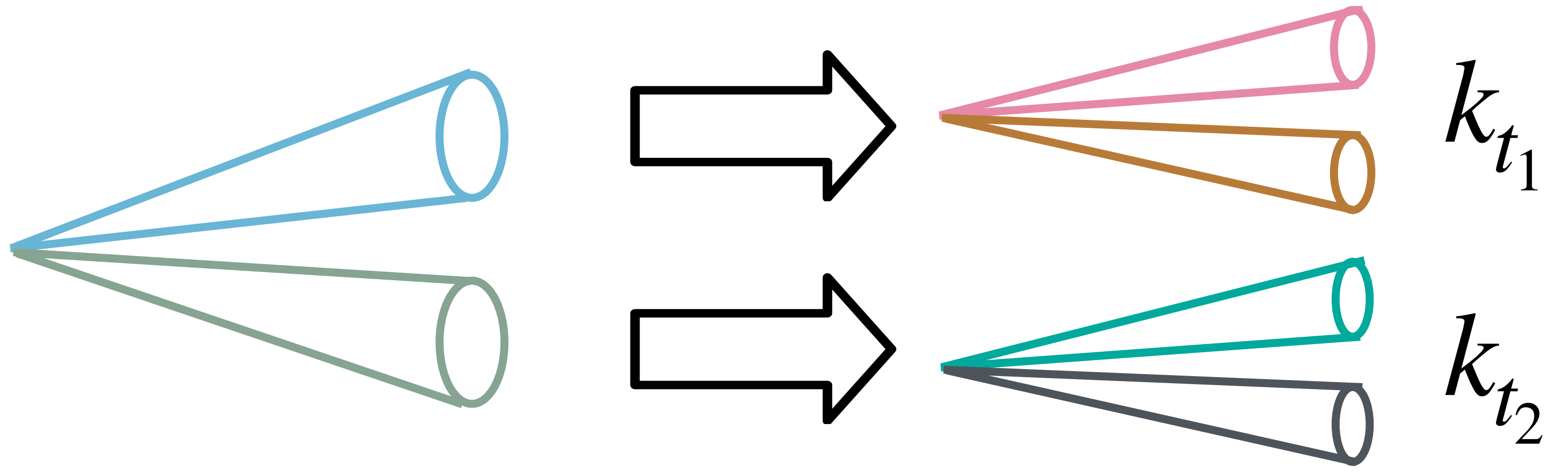
1. Recluster a jet with the Cambridge/Aachen algorithm

2. Traverse backwards the angular ordered sequence

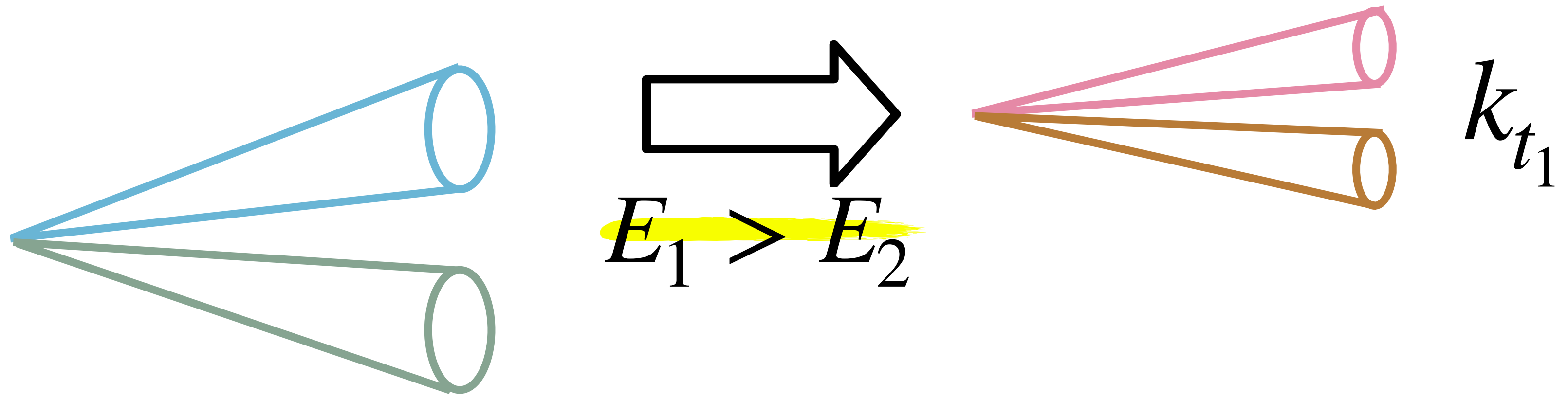


$$k_t^{e^+e^-} = \min(E_1, E_2)\sin\theta_{12} \qquad\qquad k_t^{pp} = \min(p_{t1}, p_{t2})\Delta R_{12}$$

a. If $k_t > k_{t,\mathrm{cut}} \Longrightarrow N^{\mathrm{LP}} = N^{\mathrm{LP}} + 1$ and



b. If $k_t < k_{t,\mathrm{cut}} \Longrightarrow$

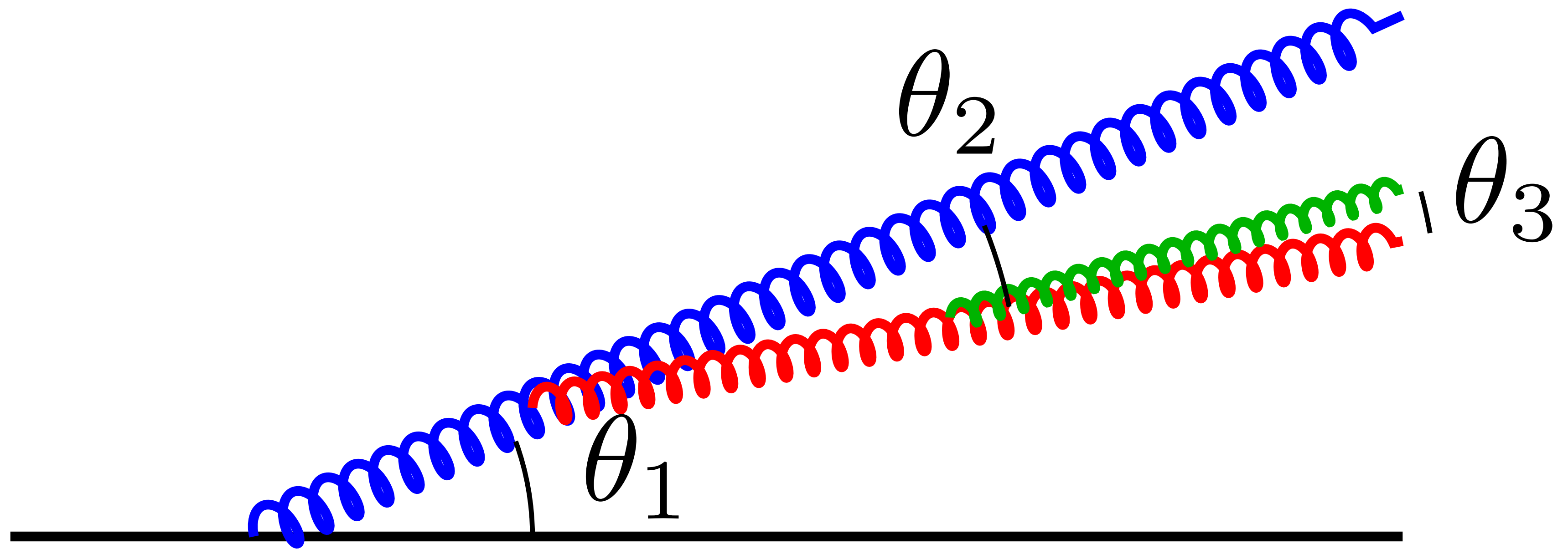


Lund multiplicity: all orders (DL) $(\alpha_s L^2)^n$

$$\langle N\rangle_{\rm DL} = 1 + \frac{C_i}{C_A}\sum_{n=1}^{\infty}\bar{\alpha}^n \underbrace{\int_0^\infty d\eta_1 \int_{\eta_1}^\infty d\eta_2 \ldots \int_{\eta_{n-1}}^\infty d\eta_n}_{\text{angular-ordering}} \underbrace{\int_0^1 \frac{dx_1}{x_1}\int_0^{x_1}\frac{dx_2}{x_2}\ldots\int_0^{x_{n-1}}\frac{dx_n}{x_n}}_{\text{energy-ordering}} \underbrace{\Theta(x_n e^{-\eta_n} > e^{-L})}_{k_t > k_{t,\rm cut}}$$

$$Q \gg E_1 \gg E_2 \gg E_3 \gg k_{t,\rm cut}$$

$$1 \gg \theta_1 \gg \theta_2 \gg \theta_3$$



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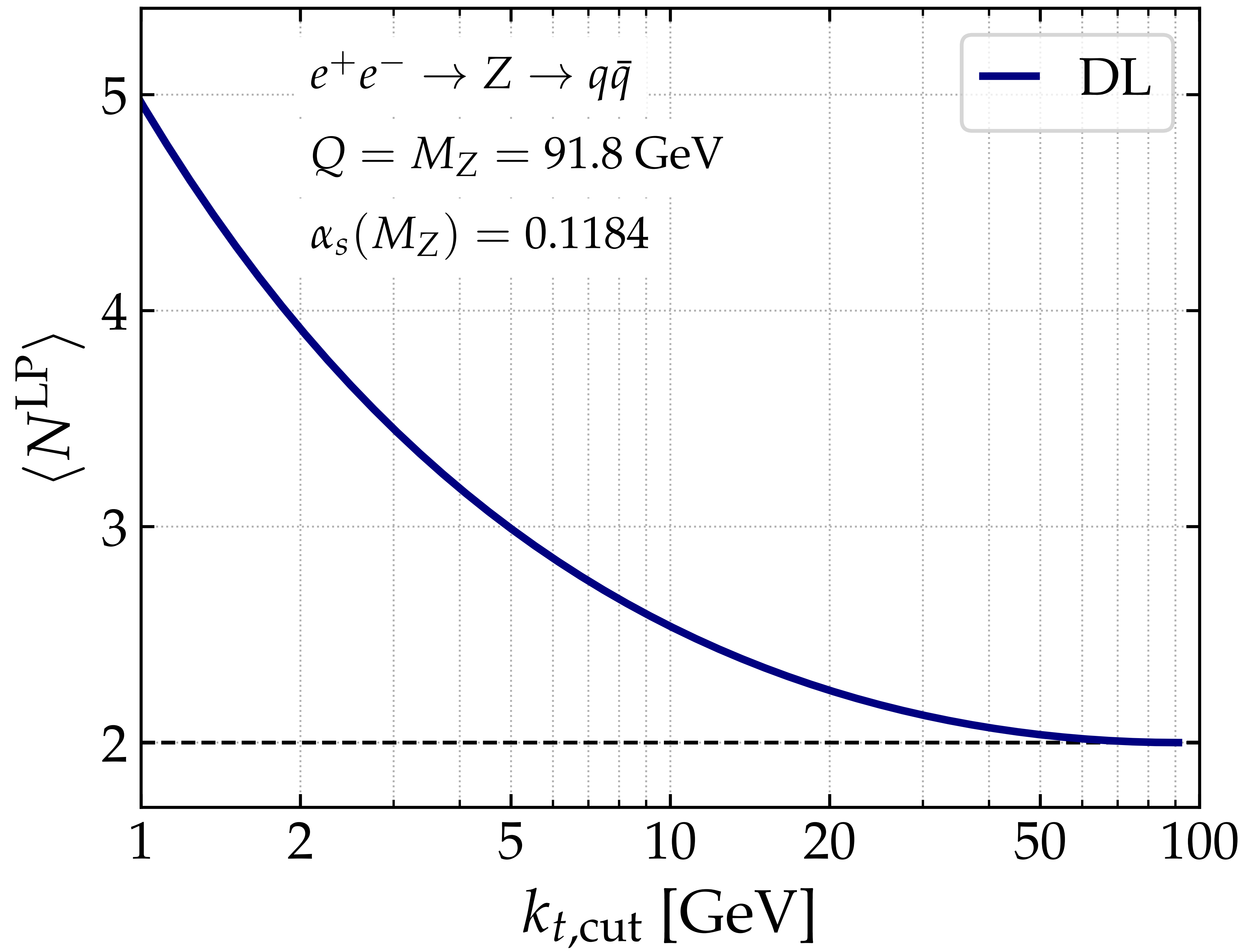
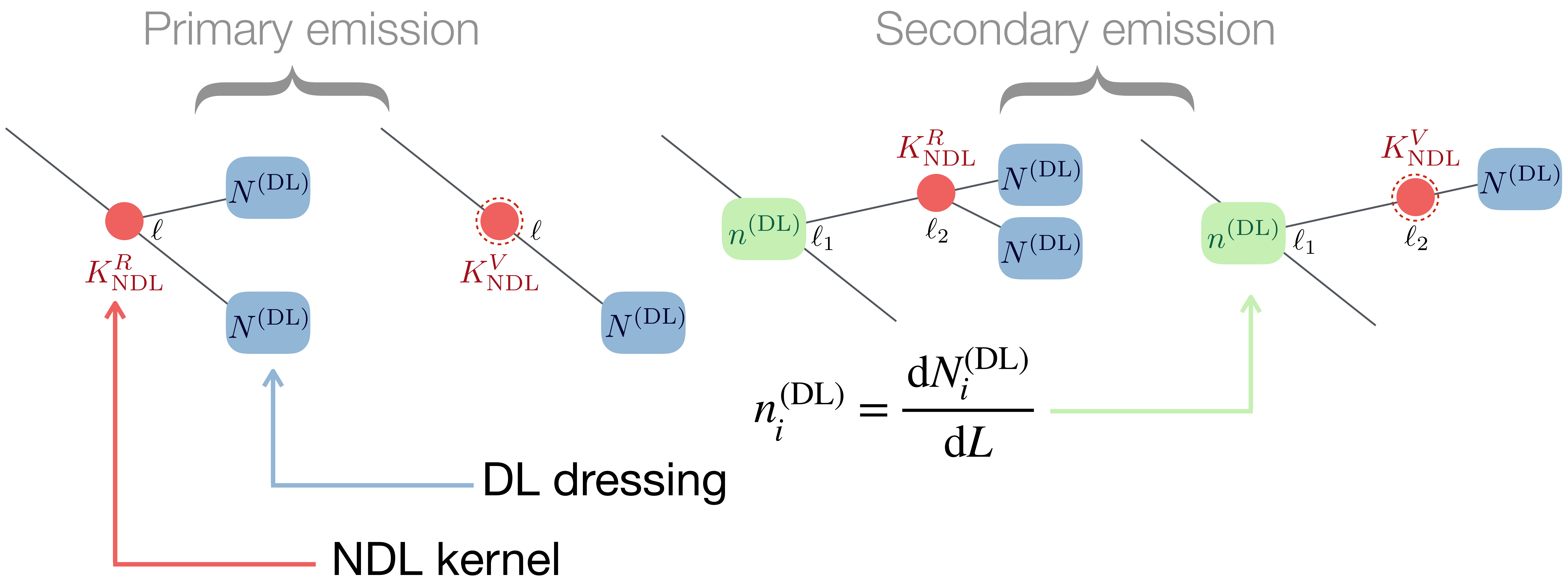
$$\boxed{\langle N\rangle_{\rm DL} = 1 + \frac{C_i}{C_A}\left[\cosh\nu - 1\right]}$$



$$\nu = \sqrt{2\alpha_s C_A L^2/\pi}$$

Subjet multiplicity: NDL resummation strategy

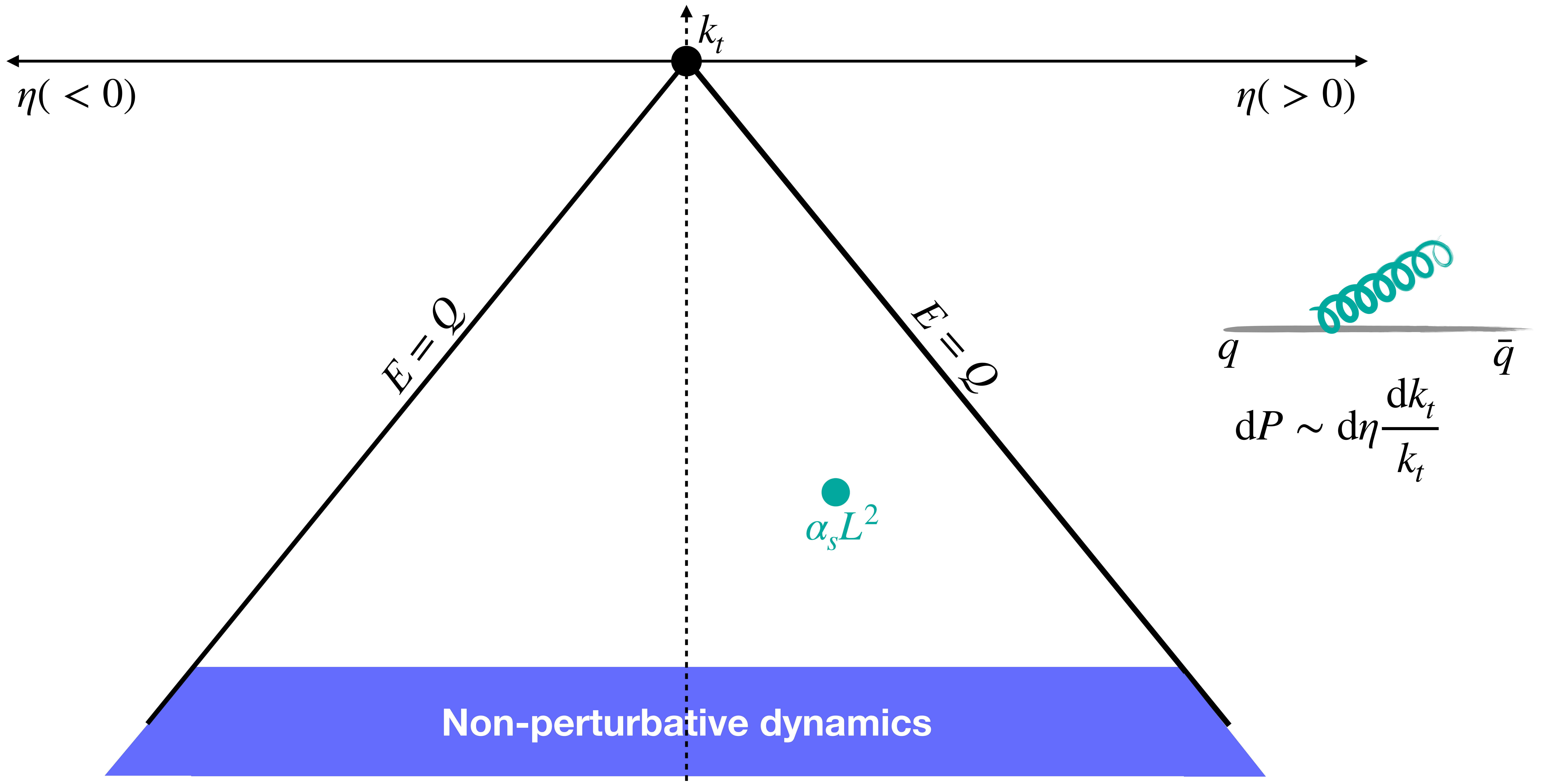
NDL == one NDL-like emission in the chain at a momentum scale ℓ



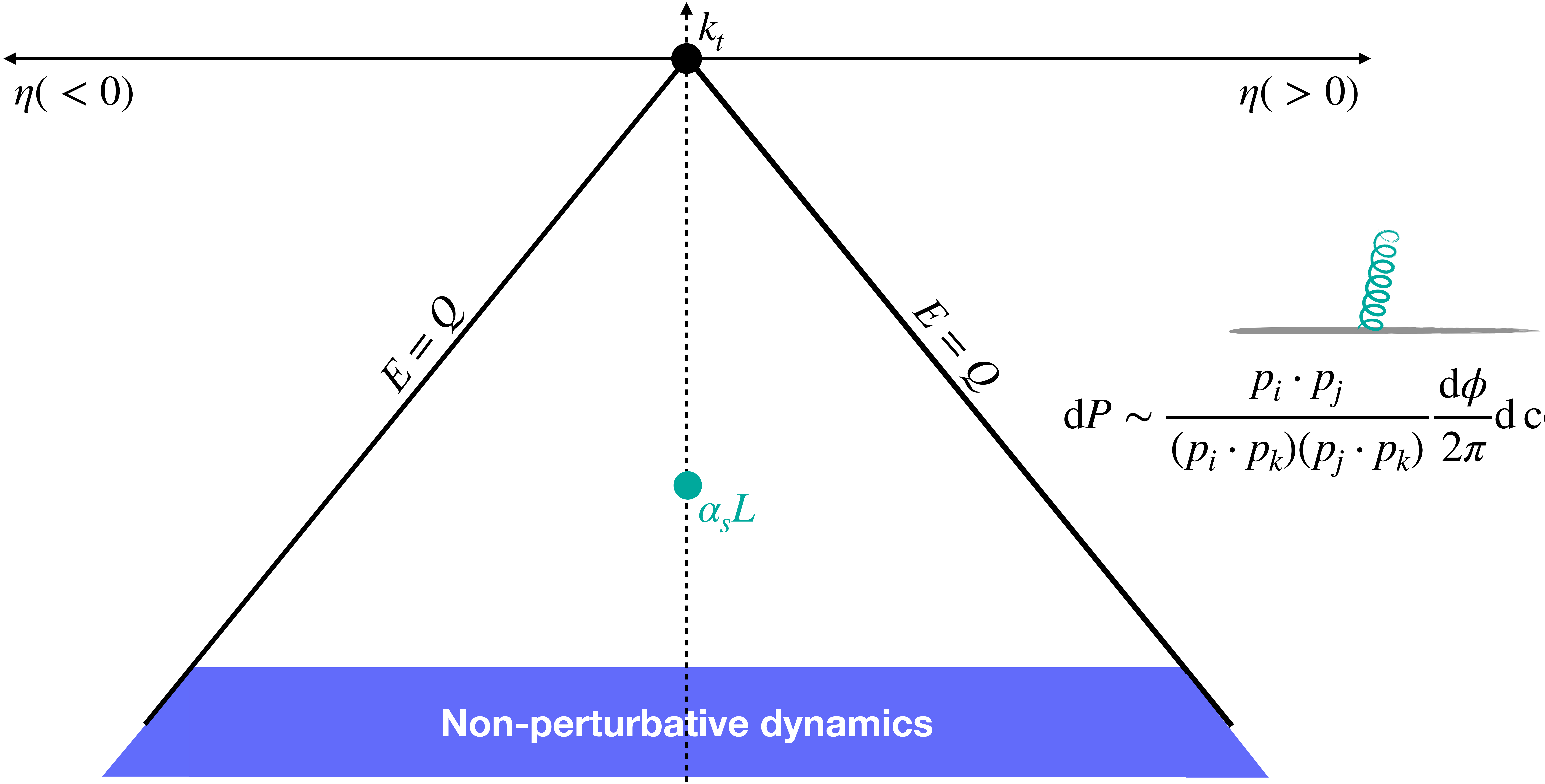
Idea: use the Lund plane to systematically identify all NDL terms

Interlude: counting logs in the Lund jet plane

[Dreyer, Salam, Soyez JHEP 12 (2018) 064]

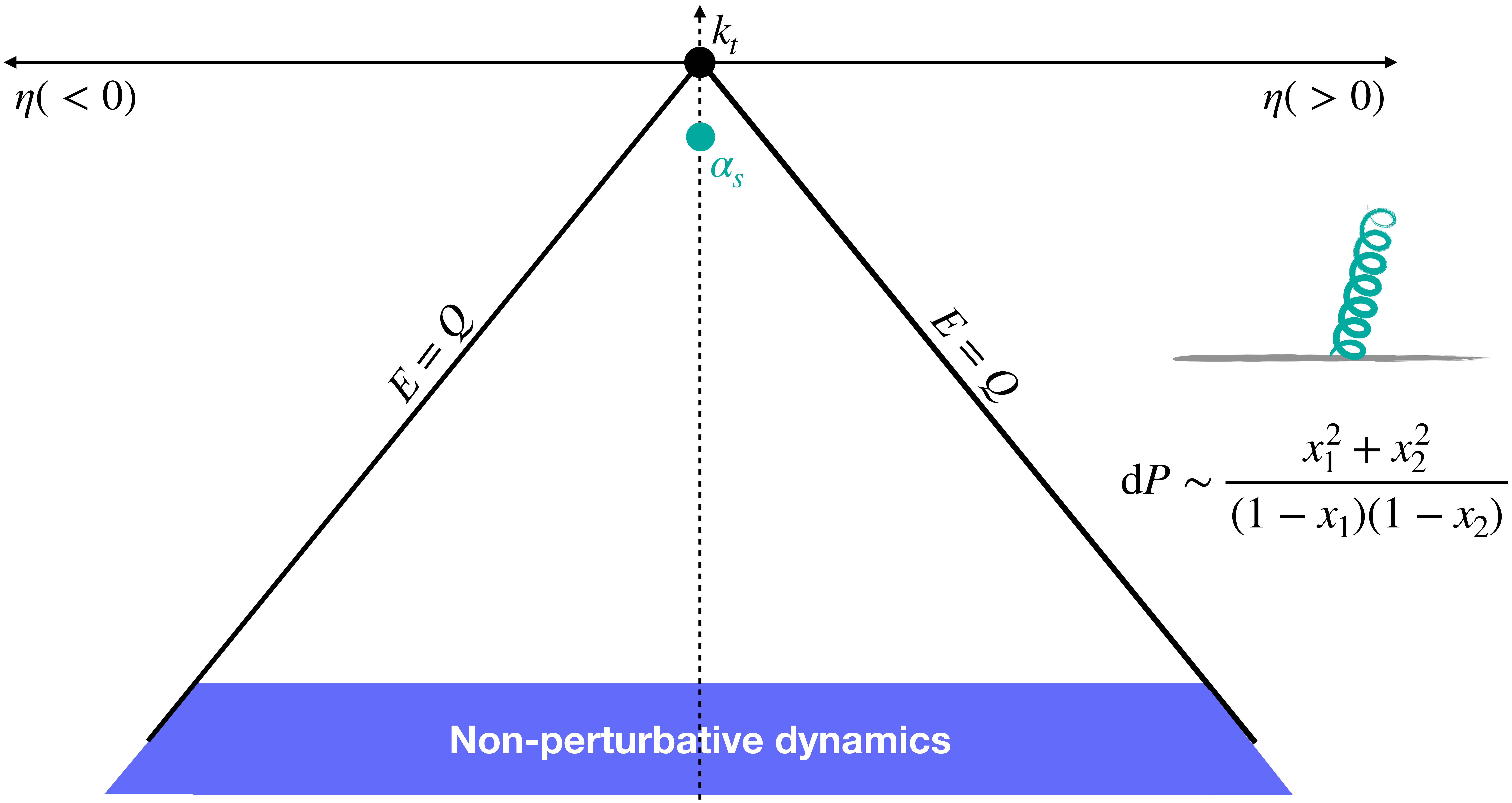


Interlude: counting logs in the Lund jet plane [Dreyer, Salam, Soyez JHEP 12 (2018) 064]



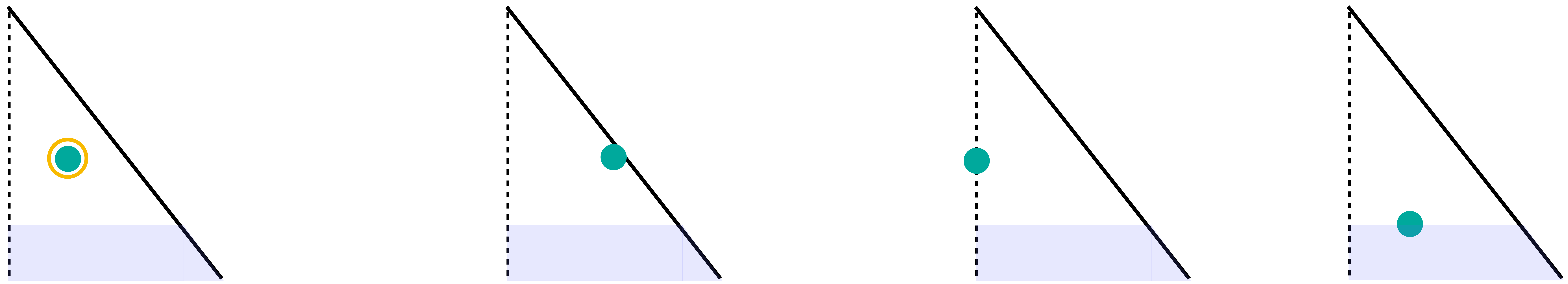
$$\mathrm{d}P \sim \frac{p_i \cdot p_j}{(p_i \cdot p_k)(p_j \cdot p_k)} \frac{\mathrm{d}\phi}{2\pi} \mathrm{d}\cos\theta$$

Interlude: counting logs in the Lund jet plane [Dreyer, Salam, Soyez JHEP 12 (2018) 064]



$$\mathrm{d}P \sim \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Lund multiplicity: NDL resummation $\alpha_s L(\alpha_s L^2)^n$



Running coupling

$$\boxed{\alpha_s \to \alpha_s - 2\alpha_s^2 \beta_0 \ell + \mathcal{O}(\alpha_s^3)}$$

with $\ell \equiv \ln(k_t/Q)$

Hard-collinear

$$\boxed{\frac{1}{z} \to C_F\left(\frac{1-z}{z} + \frac{z}{2}\right)}$$

Large-angle

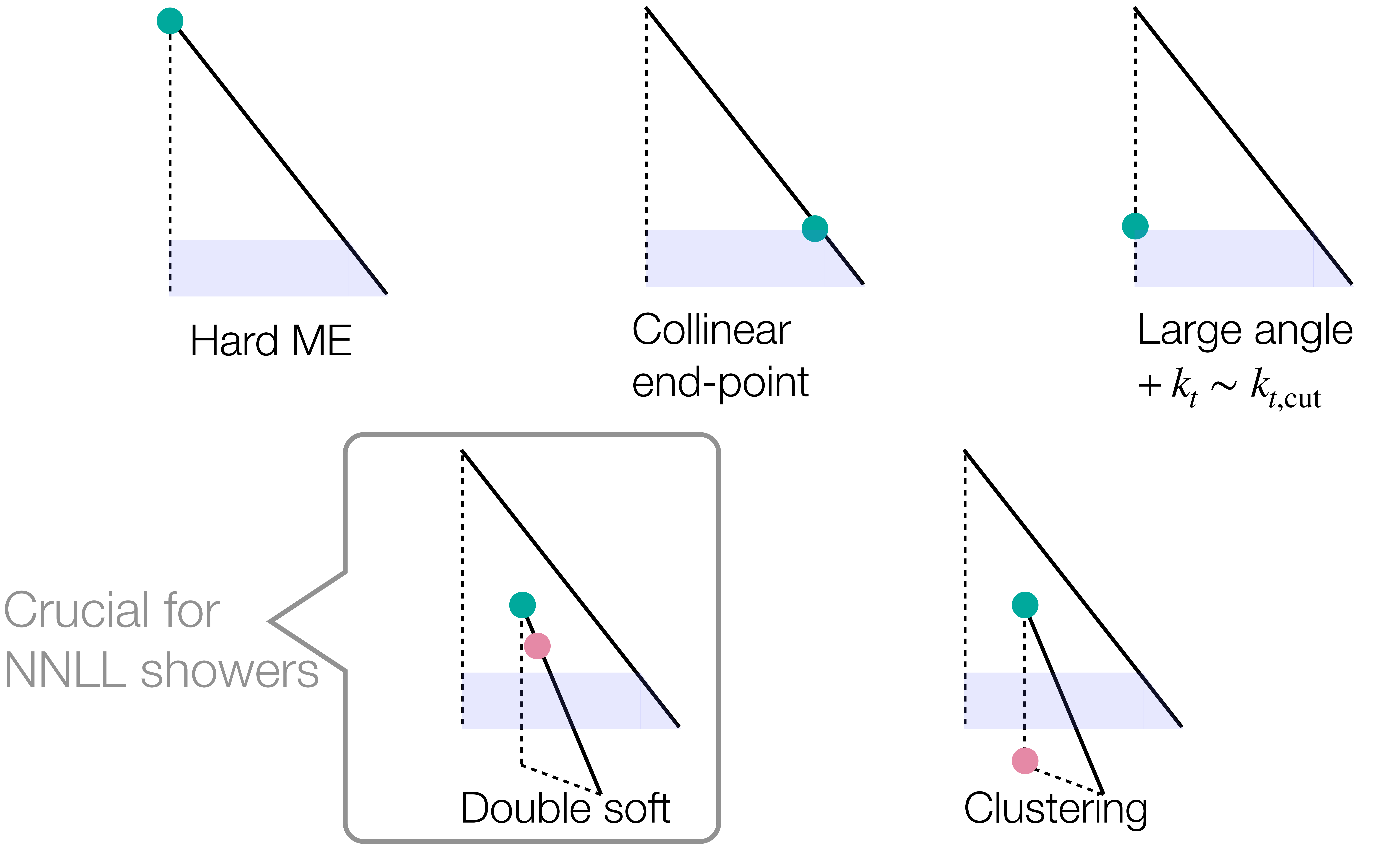
$$\frac{\mathrm{d}z}{z}\mathrm{d}\eta$$

$$\downarrow$$

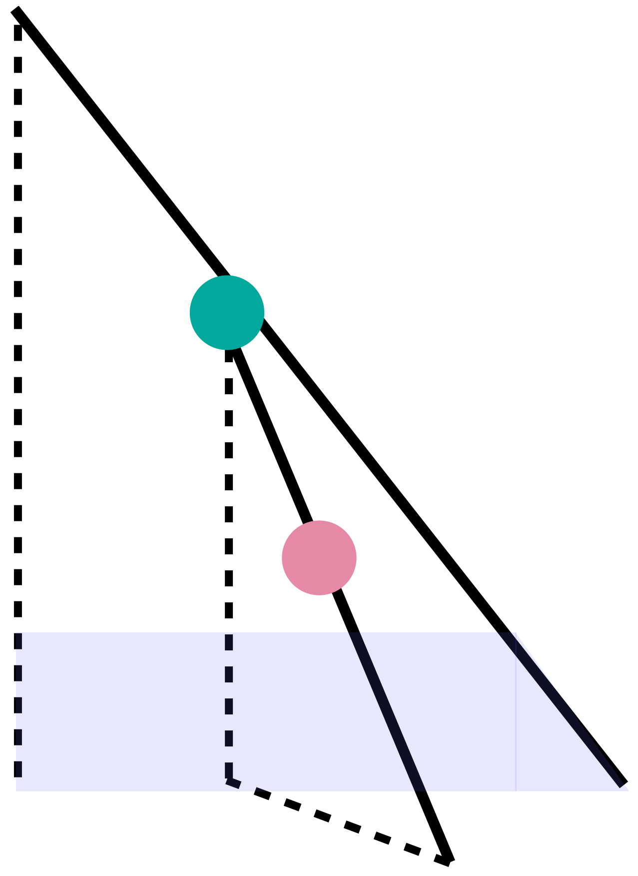
$$\boxed{\frac{p_i \cdot p_j}{p_i \cdot p_k p_j \cdot p_k}\frac{\mathrm{d}\phi}{2\pi}\mathrm{d}\cos\theta}$$

$k_t \sim k_{t,\mathrm{cut}}$

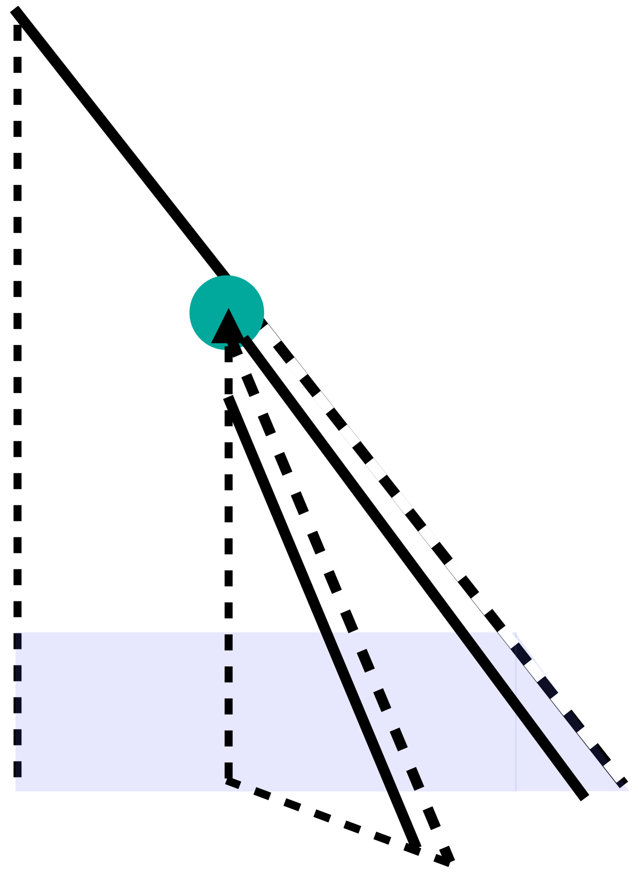
Lund multiplicity: NNDL resummation $\alpha_s(\alpha_s L^2)^n$



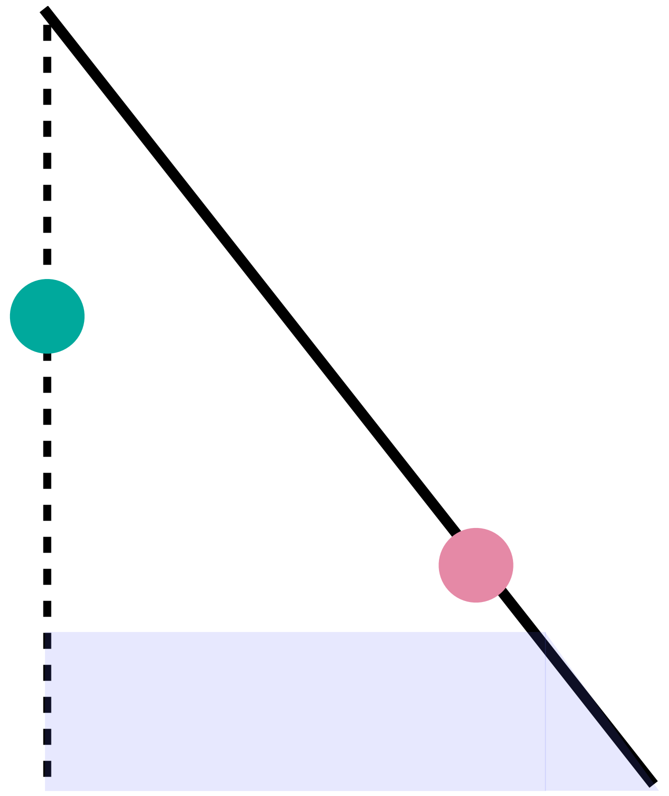
Lund multiplicity: NNDL resummation $(\alpha_s L)(\alpha_s L)(\alpha_s L^2)^n$ ~ (NDL)²



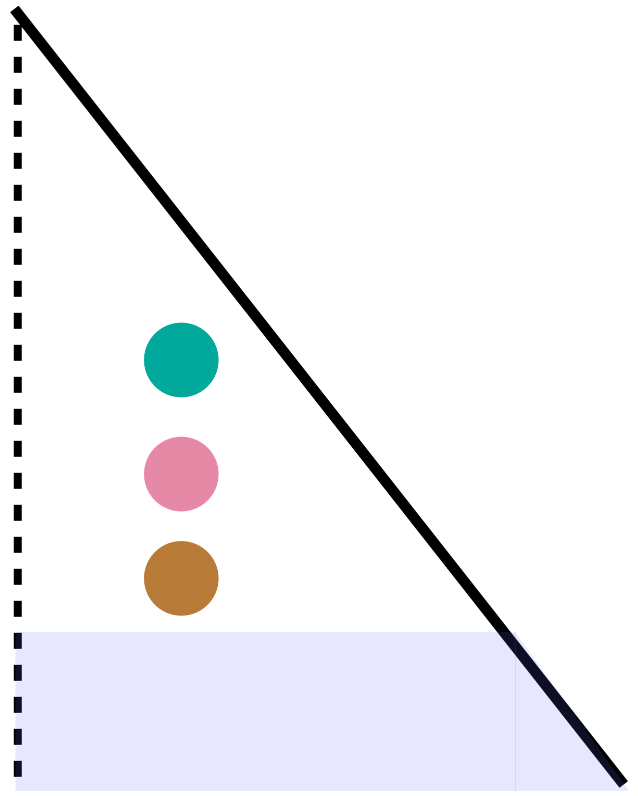
2 hard collinear



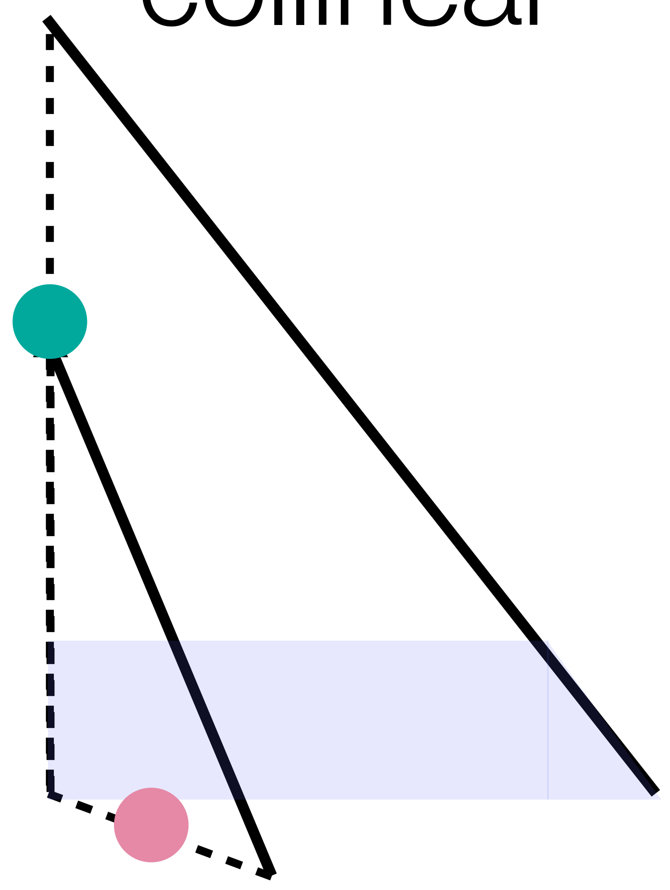
Energy loss



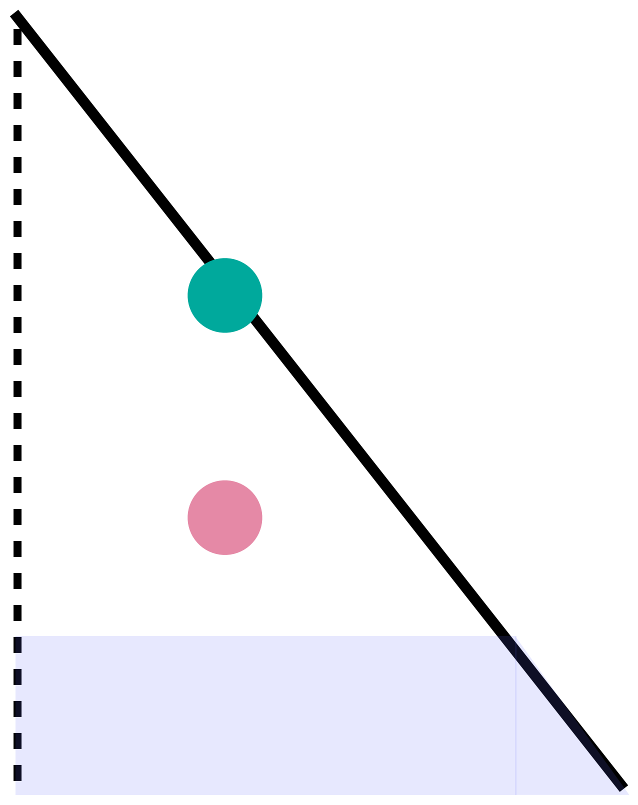
Large angle + hard-collinear



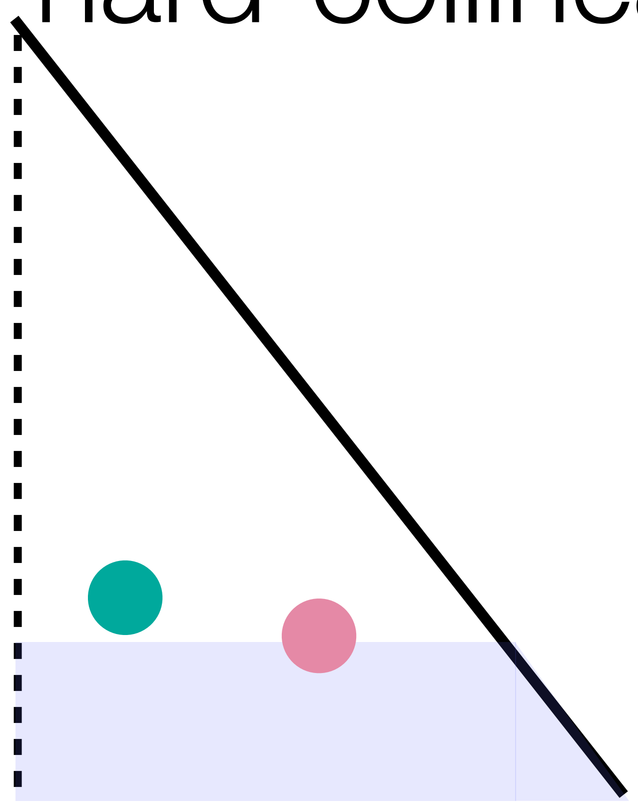
3 commensurate angles



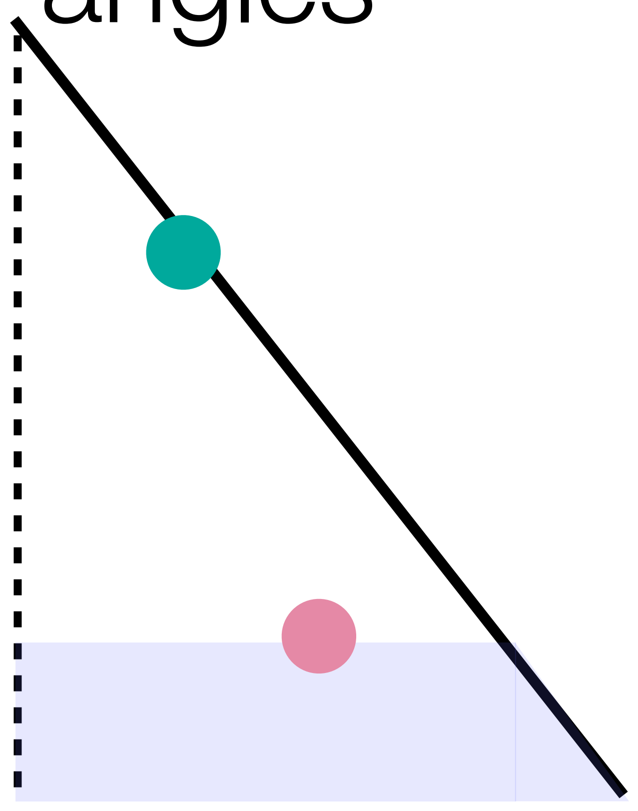
Large angle + $k_t \sim k_{t,\mathrm{cut}}$



Hard coll. + commensurate η

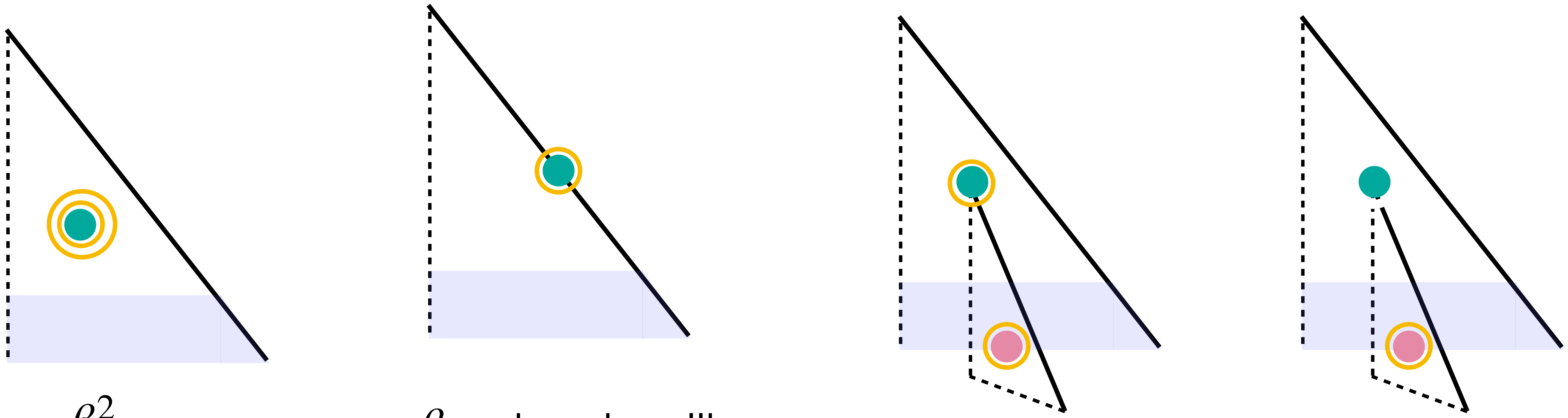


2 $k_t \sim k_{t,\mathrm{cut}}$



Hard coll. + $k_t \sim k_{t,\mathrm{cut}}$

Lund multiplicity: NNDL resummation (running coupling)



β_0^2 β_0 + hard-collinear $\beta_0 \times \beta_0$ $\beta_0 \times$hard-collinear

Lund multiplicity: NNDL result

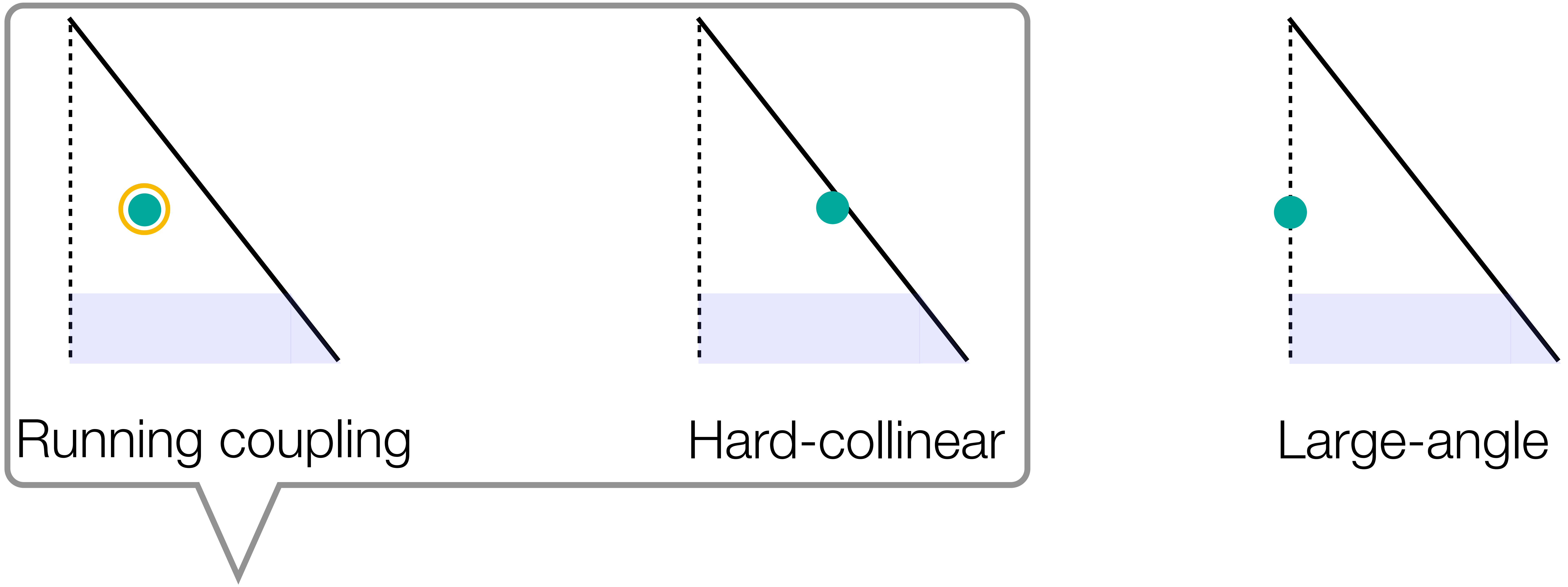
NNDL:
$$2\pi h_3^{(q)} = D_{\text{end}}^{q\to qg} + \left(D_{\text{end}}^{g\to gg} + D_{\text{end}}^{g\to q\bar{q}}\right)\frac{C_F}{C_A}(\cosh\nu - 1) + D_{\text{hme}}^{qqg}\cosh\nu$$
$$+ \frac{C_F}{C_A}\left[(1-c_\delta)D_{\text{pair}}^{q\bar{q}}(\cosh\nu - 1) + \left(K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar{q}}\right)\frac{\nu}{2}\sinh\nu\right]$$
$$+ C_F\left[\left(\cosh\nu - 1 - \frac{1-c_\delta}{4}\nu^2\right)D_{\text{clust}}^{(\text{prim})} + (\cosh\nu - 1)D_{\text{clust}}^{(\text{sec})}\right]$$
$$+ \frac{C_F}{C_A}\left[D_{\text{e-loss}}^{g}\frac{\nu}{2}\sinh\nu + (D_{\text{e-loss}}^{q} - D_{\text{e-loss}}^{g})(\cosh\nu - 1)\right]$$
$$+ \frac{C_F}{2}\Big\{(B_{gg} + c_\delta B_{gq})^2\nu^2\cosh\nu + 8\left[2c_\delta B_{gg} - 2c_\delta B_q - (1-3c_\delta^2)B_{gq}\right]B_{gq}\cosh\nu$$
$$+ [4B_q(B_{gg} + (2c_\delta + 1)B_{gq}) - (B_{gg} + c_\delta B_{gq})(B_{gg} + 9c_\delta B_{gq})]\,\nu\sinh\nu$$
$$+4(1-c_\delta^2)B_{gq}^2\nu^2 + 8\left[2c_\delta B_q - 2c_\delta B_{gg} + (1-3c_\delta^2)B_{gq}\right]B_{gq}\Big\}$$
$$+ \frac{C_F}{C_A}\frac{\pi\beta_0}{2}\Big\{(B_{gg} + c_\delta B_{gq})\nu^3\sinh\nu + [2B_q - 2B_{gg} + (6-8c_\delta)B_{gq}]\,\nu\sinh\nu$$
$$+2(B_q + B_{gg} + B_{gq})\nu^2\cosh\nu - 4(1-c_\delta)B_{gq}(2\cosh\nu - 2 + \nu^2)\Big\}$$
$$+ \frac{C_F}{C_A}\frac{\pi^2\beta_0^2}{8C_A}[3\nu(2\nu^2 - 1)\sinh\nu + (\nu^4 + 3\nu^2)\cosh\nu]$$

@ full colour

Extension to QCD jets: DL and NDL

DL: $h_{1,pp}(\xi) = f_q h^{(q)}_{1,e^+e^-}(\xi) + f_g h^{(g)}_{1,e^+e^-}(\xi)$

NDL:

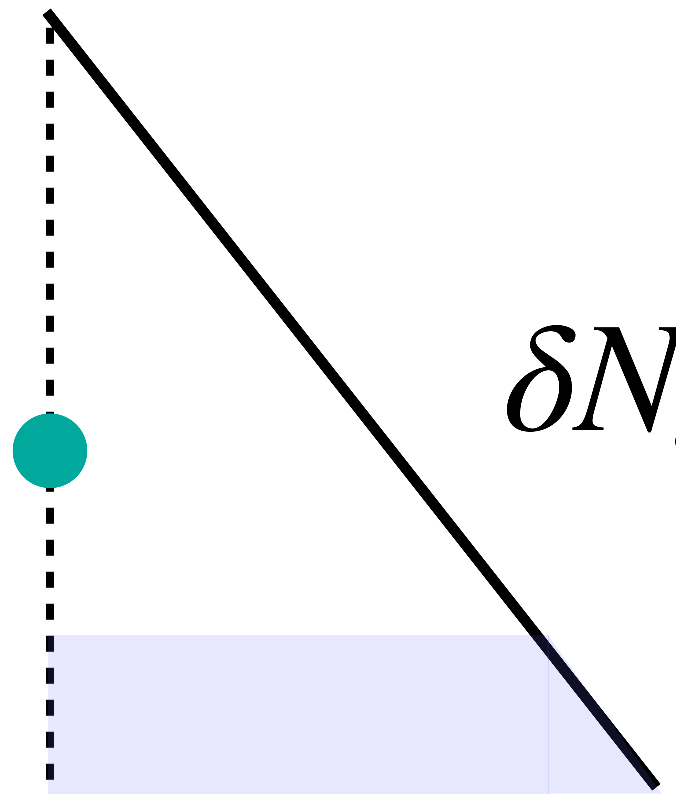


Collinear physics is universal: recompute only wide-angle diagrams

Extension to QCD jets: DL and NDL

DL: $$h_{1,pp}(\xi) = f_q h^{(q)}_{1,e^+e^-}(\xi) + f_g h^{(g)}_{1,e^+e^-}(\xi)$$

NDL:



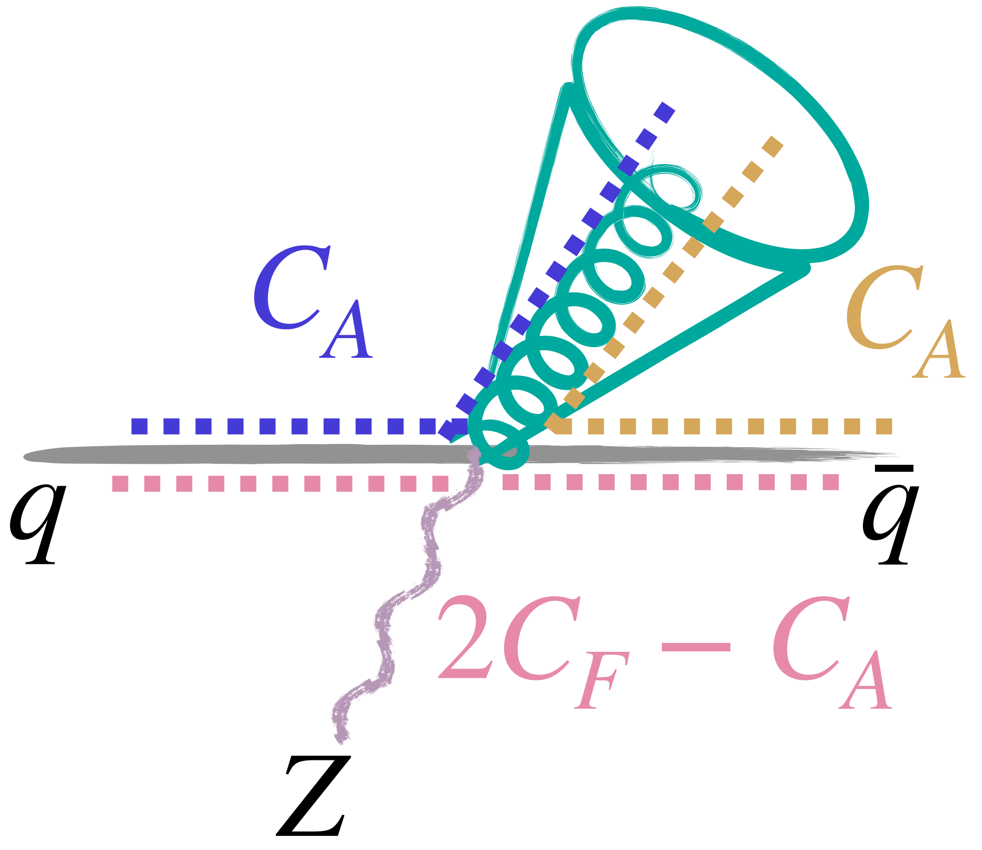
Large-angle

$$\int [dk] \frac{p_a p_b}{(p_a p_k)(p_b p_k)} \Theta(\Delta_k < R)$$

$$\delta N^{(\mathrm{NDL})}_{\mathscr{C},(ab),\mathcal{O}(\alpha_s)} = \frac{\alpha_s}{\pi} L \omega^{\mathscr{C}}_{ab} D^{\mathrm{la}}_{ab}$$

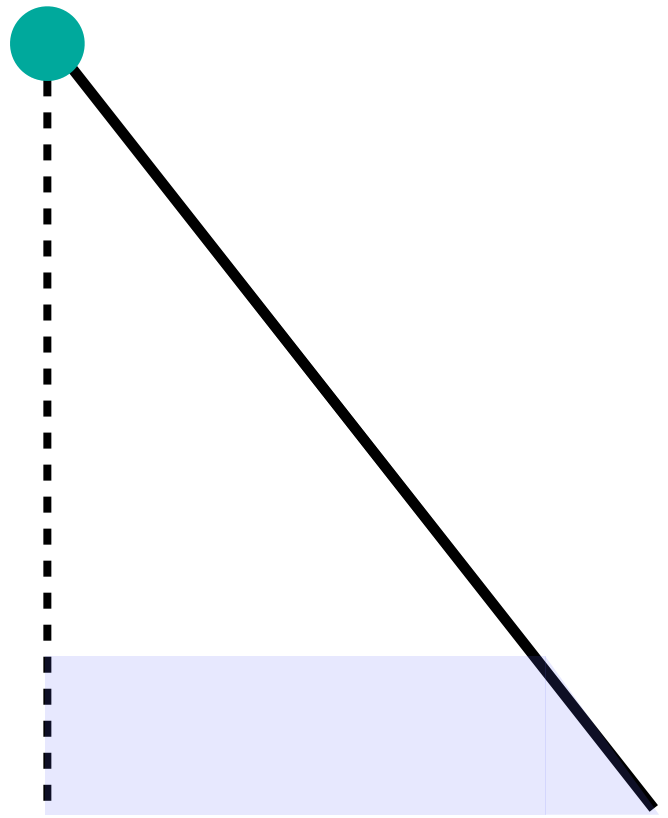
e.g.

$$\omega^{\mathscr{C}}_{ab} = (-2\mathbf{T}_a \cdot \mathbf{T}_b)$$



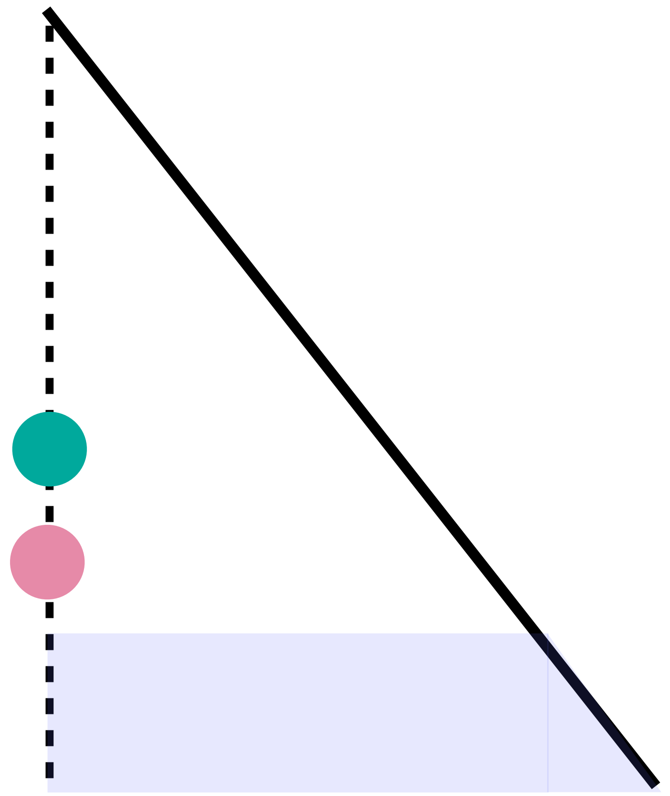
Extension to QCD jets: NNDL

NNDL:

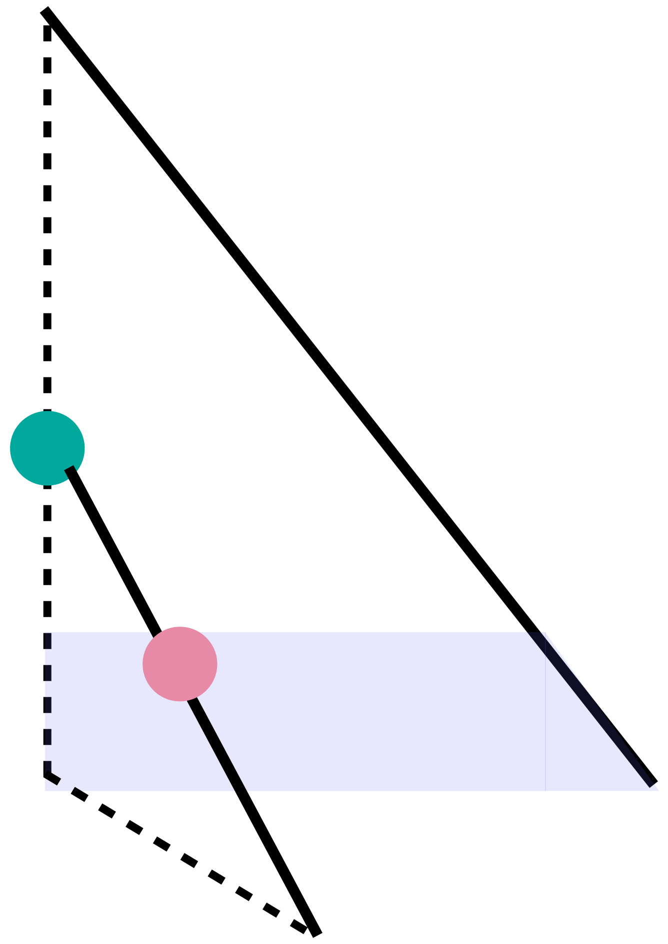


Hard ME

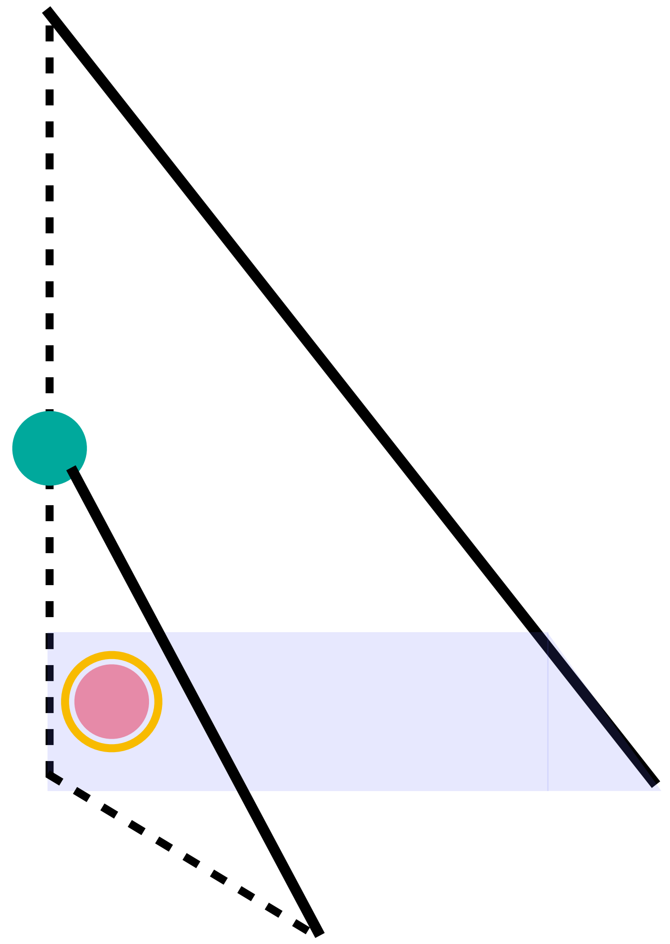
$\alpha_s(\alpha_s L^2)^n$



2 large-angle



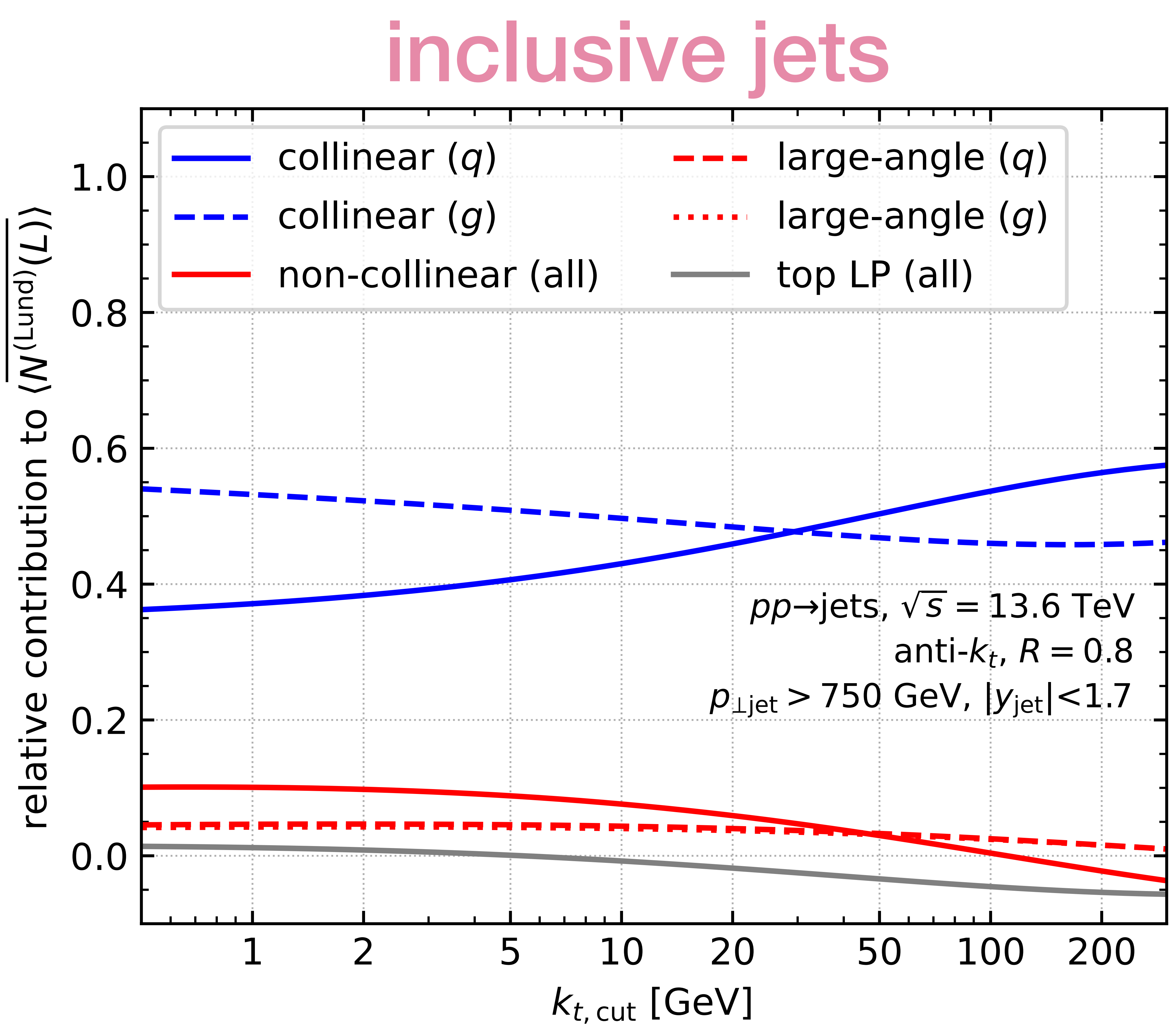
Large angle x hard-coll

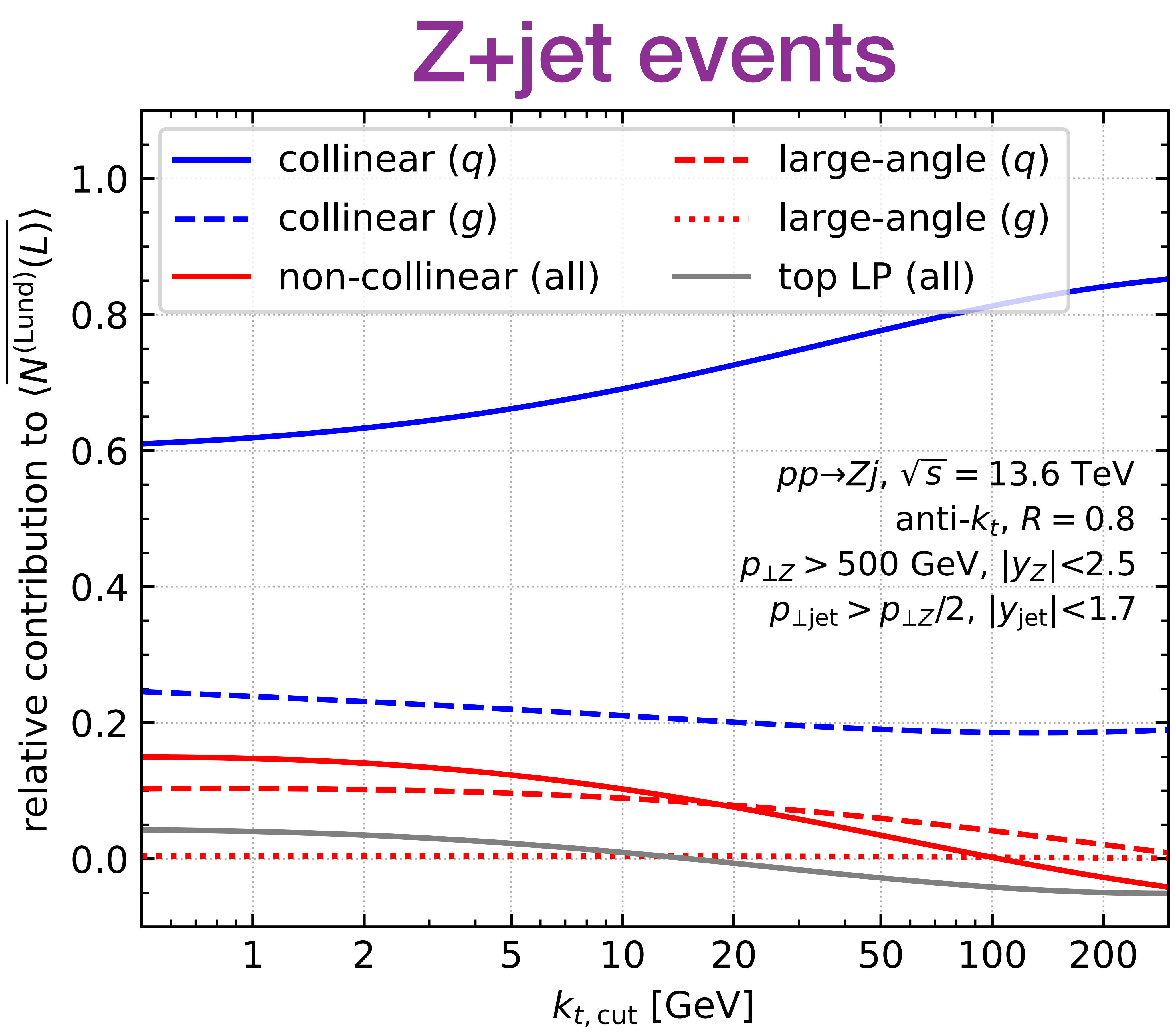


Large angle x β_0

(NDL)^2 = $(\alpha_s L)(\alpha_s L)(\alpha_s L^2)^n$

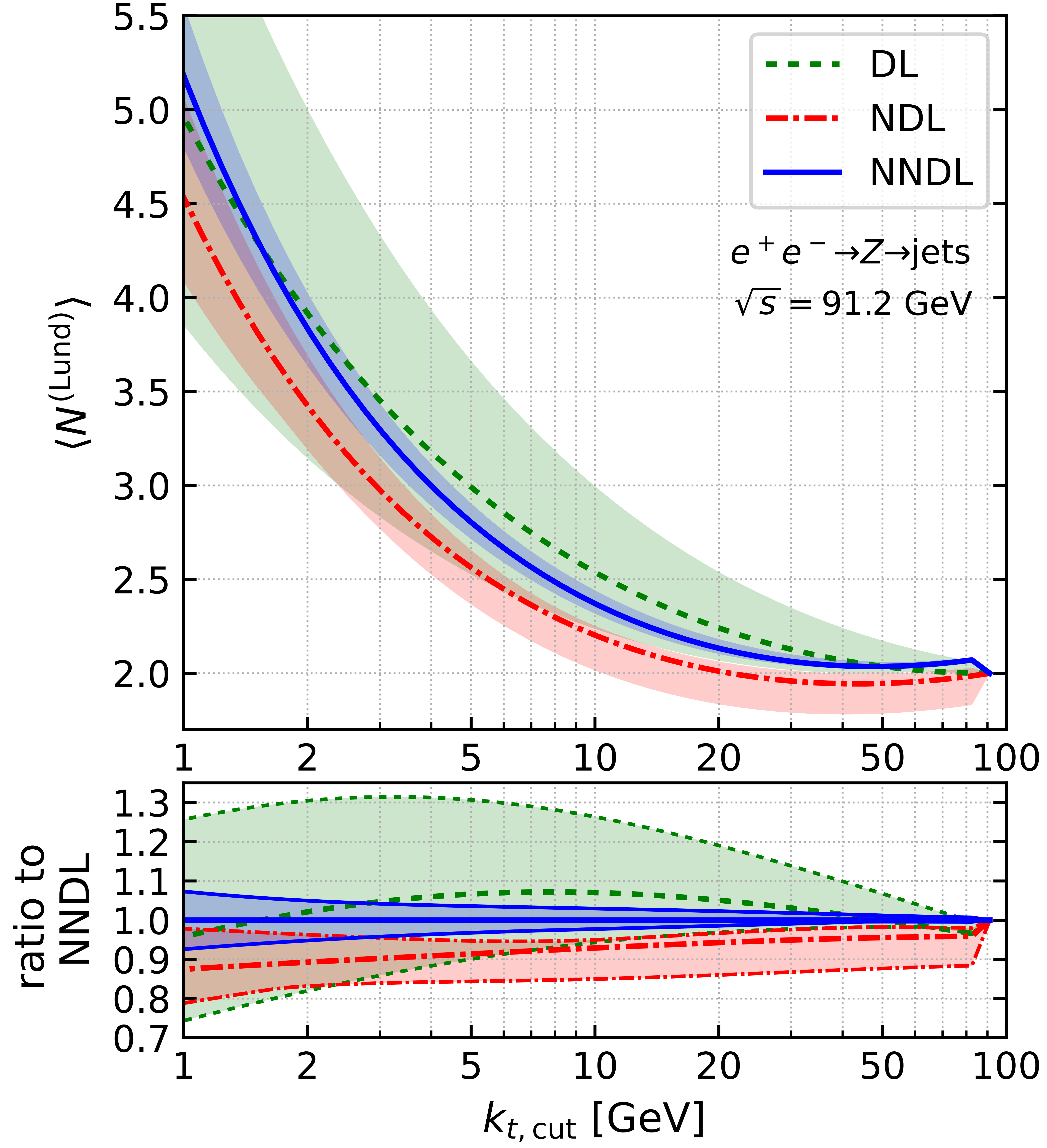
Universal vs large-angle contributions to multiplicity





Almost 90% of the Lund multiplicity has a collinear origin

The importance of higher logarithmic accuracy: e^+e^-

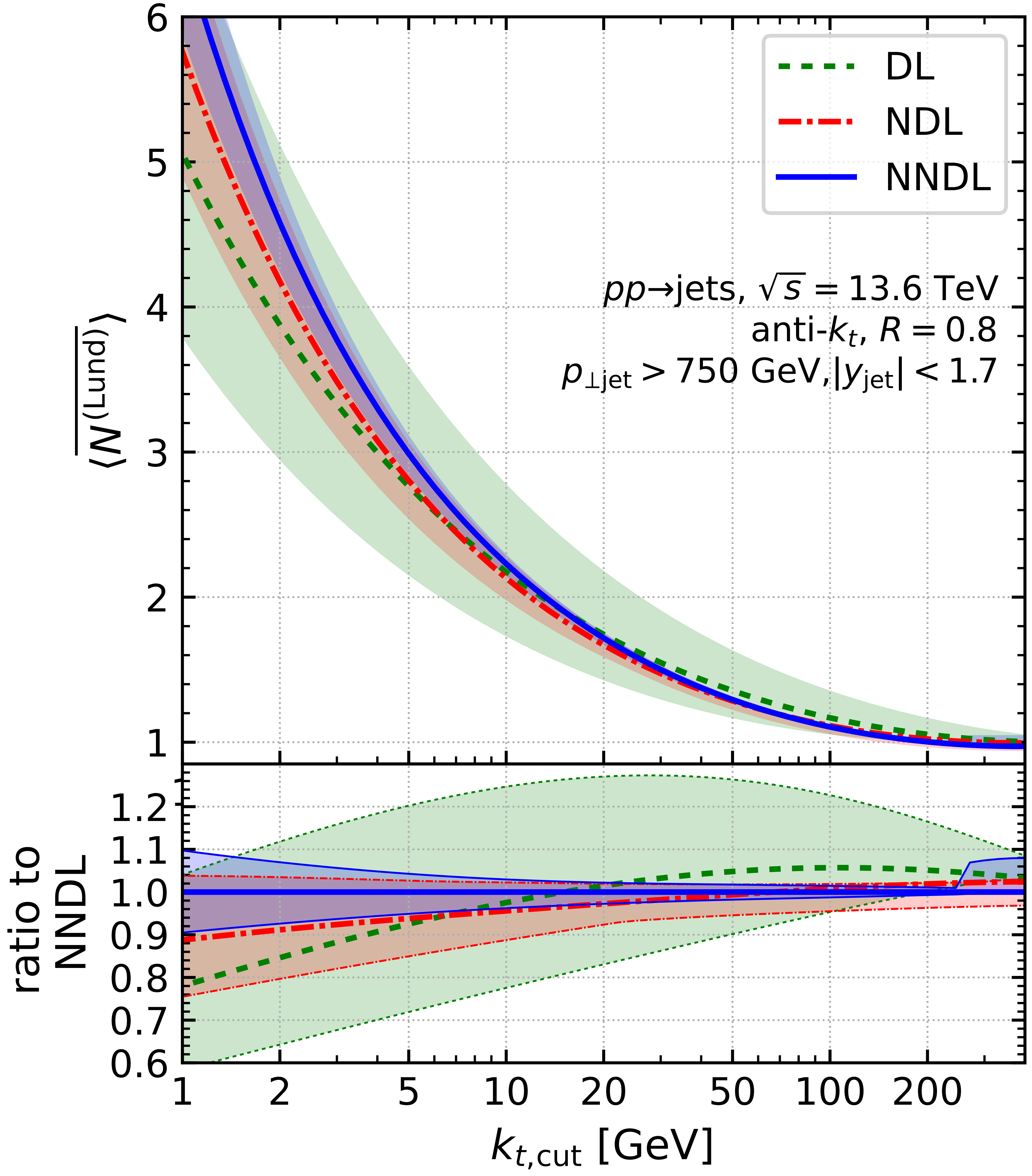


Band = $\alpha_s(x_R Q), \ln(x_L Q / k_{t,\mathrm{cut}})$

The uncertainty of the theoretical prediction at $k_{t,\mathrm{cut}} = 5$ GeV is

DL: $20\,\%$, NDL: $6\,\%$, NNDL: $3\,\%$

The importance of higher logarithmic accuracy: pp

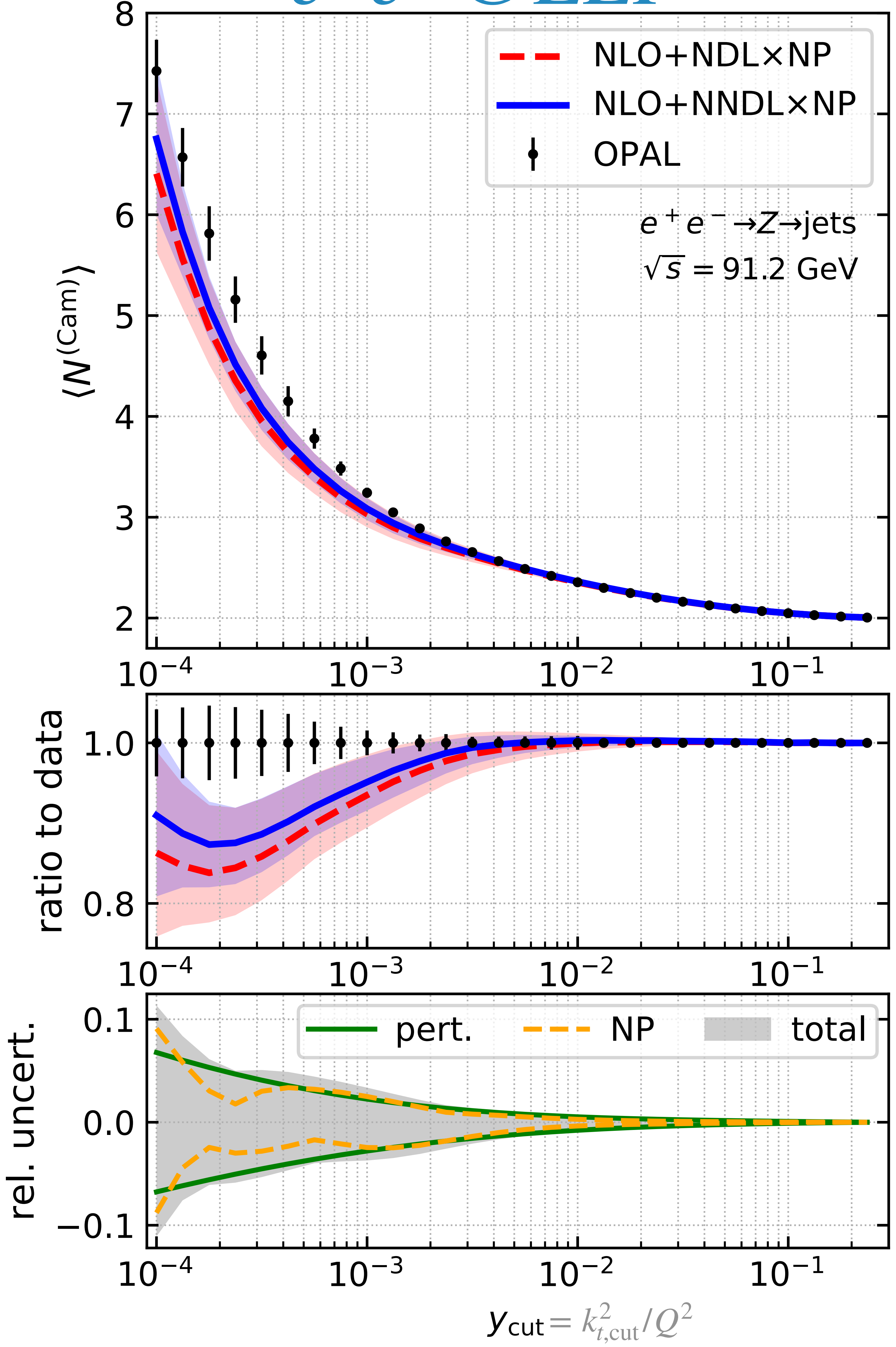


The uncertainty of the theoretical prediction at $k_{t,\mathrm{cut}} = 5$ GeV is

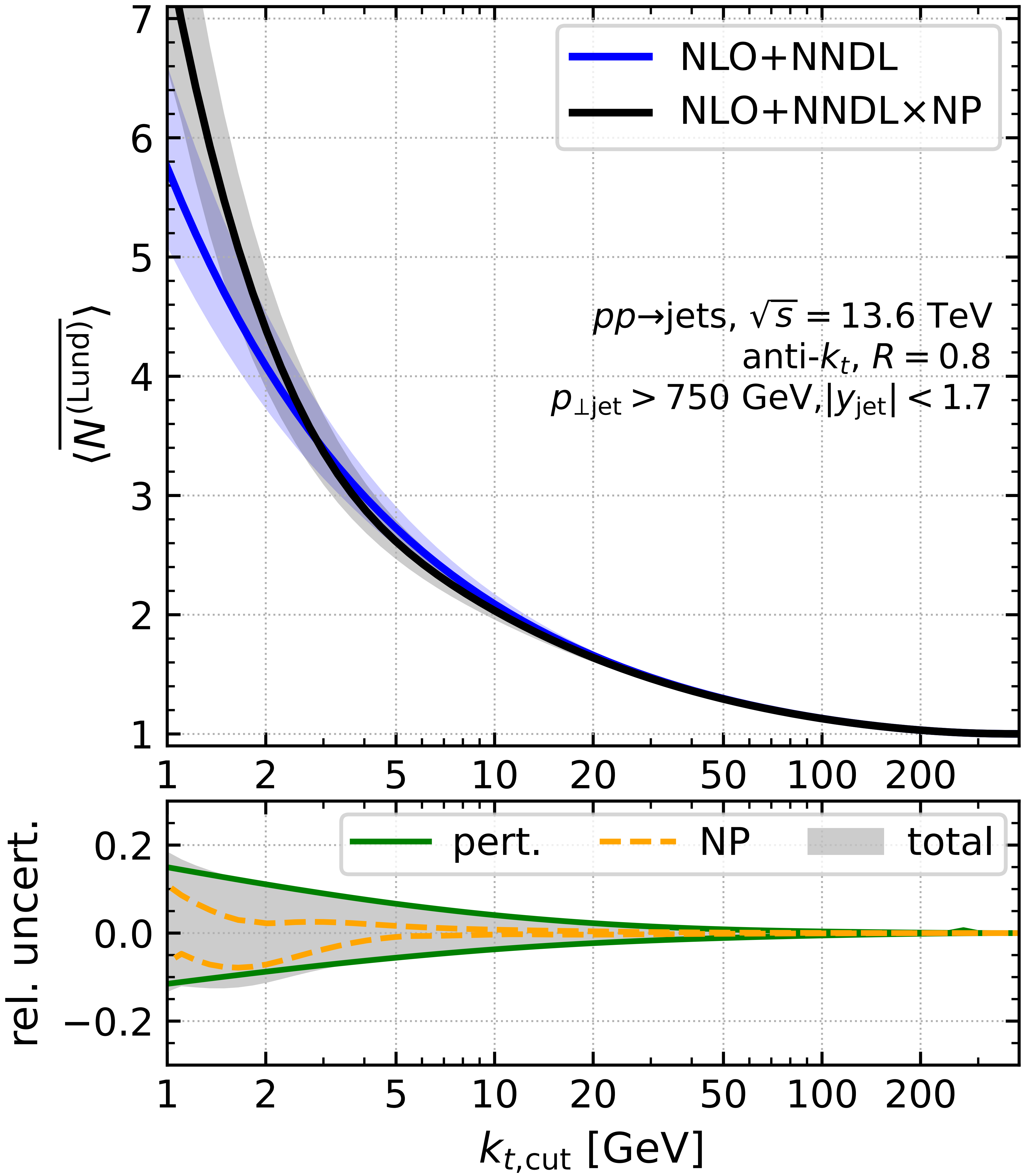
DL: $28\,\%$, NDL: $10\,\%$, NNDL: $5\,\%$

Phenomenological studies

e^+e^-@LEP



pp@LHC



Wrap up

- Jet multiplicity has played a historic role in the development of pQCD
- No progress in the analytic prediction since 1992
- New, Lund-based definition is amenable to high-precision calculations
- Predictions status: NLO ($\mathcal{O}(\alpha_s^2)$) exact + MC non-perturbative corrections +
    - e^+e^-: NNDL massless quarks, NDL massive quarks
    - pp: NDL in DY production, NNDL for jets

Stay tuned for new experimental data and parton shower tests