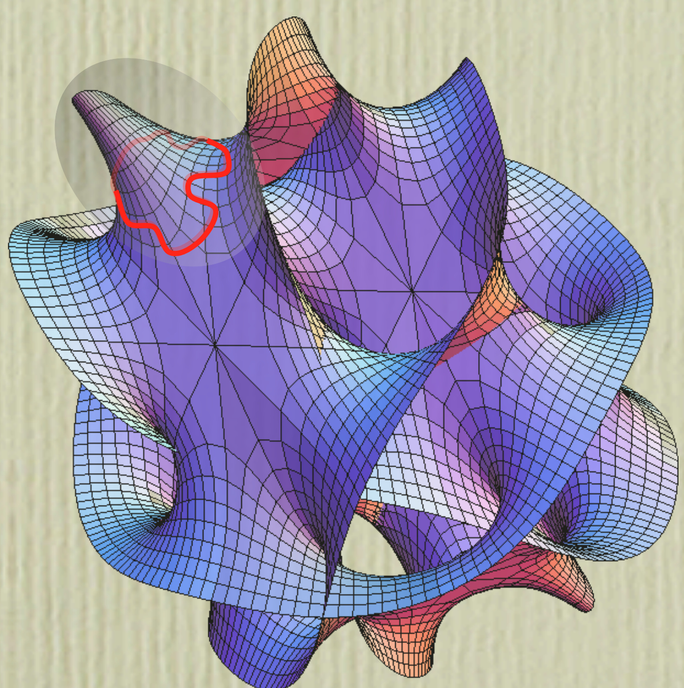
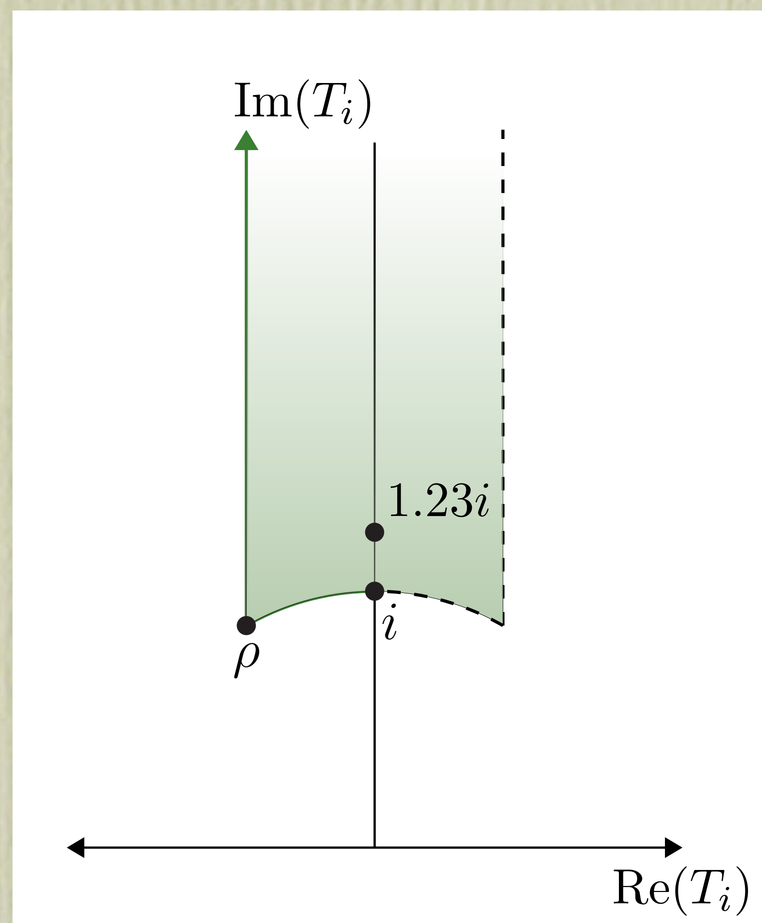
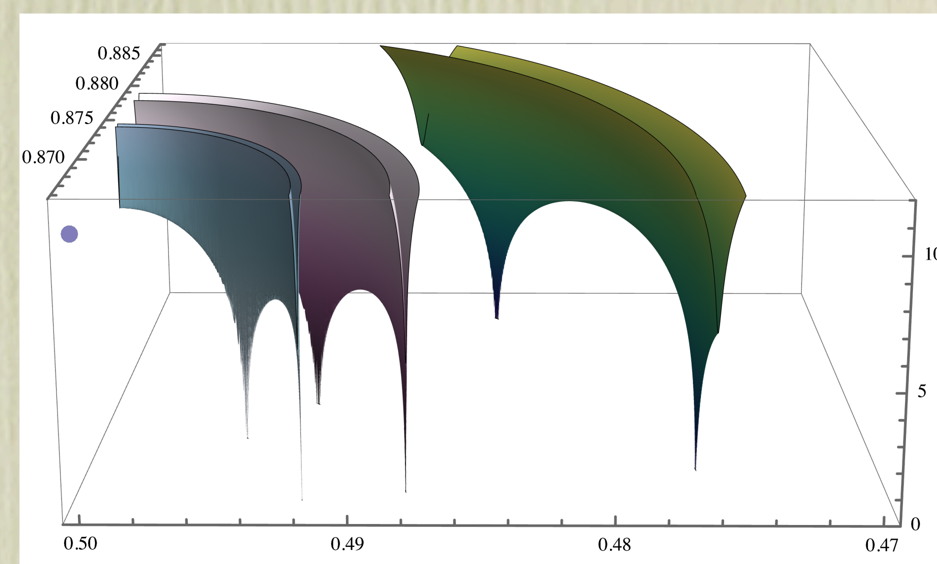
Heterotic de Sitter Beyond Modular Symmetry







Jacob Leedom, Nicole Righi & AW — arXiv:2212.03876
Abhiram Kidambi, Jacob Leedom, Nicole Righi & AW — work in progress
Alexander Westphal
(DESY)

de Sitter vacua in String Theory ...

- observation: $\rho_\Lambda \simeq 10^{-122} > 0 \qquad w_\Lambda = -0.961 \pm 0.077$

e.g. Planck 2018 + SNe + BAO

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- The Landscape: … from moduli stabilization!

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exponentially many meta-stable dS vacua, constructions of varying degrees of explicitness

[KKLT, LVS, Kähler Uplift, IIB on compact negatively curved spaces, …]

(in the interior of moduli space)

see e.g. Arthur's talk this morning!

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constructions of varying degrees of explicitness

[KKLT, LVS, Kähler Uplift, IIB on compact negatively curved spaces, …]

(in the interior of moduli space)

- The Swampland: see e.g. Arthur's talk this morning!

EFT constraints from quantum gravity/string theory

→ evidence against dS vacua in asymptotic regions of moduli space

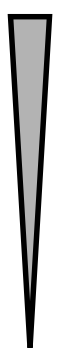
[Garg,Krishnan,'18]
[Ooguri, Palti, Shiu, Vafa, '18]
[Hebecker, Wrase, '18]

many recent works on acceleration
in asymptotic moduli regions … see e.g. Timm's talk !

de Sitter vacua in String Theory ...

starting point: partial no-go theorems - here heterotic

[Maldacena-Nunez]



Classical SUGRA?

No dS

AdS OK

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HE: $E_8 \times E_8$ HO: $Spin(32)/\mathbb{Z}_2$

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\,e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\left(\mathrm{Tr}|F|^2 - \mathrm{Tr}|R_+|^2\right)\right]$$

[Maldacena-Nunez]

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[Green+,'11]

Leading α'?

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[Maldacena-Nunez]

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No dS

AdS OK

[Gautason+,'12]

Infinite α' tower?

No dS

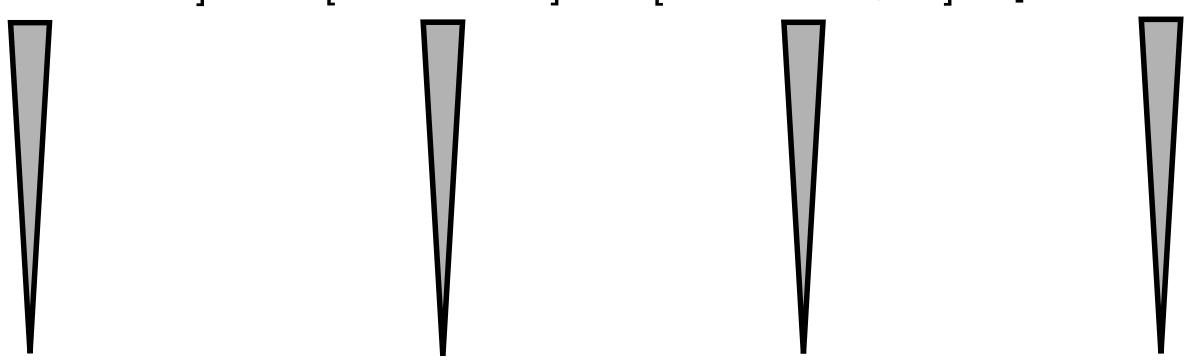
No AdS

de Sitter vacua in String Theory ...

starting point: partial no-go theorems - here heterotic

Includes worldsheet instantons & high curvature solutions

[Maldacena-Nunez]	[Green+,'11]	[Gautason+,'12]	[Kutasov+, '15]



Classical SUGRA?	Leading α'?	Infinite α' tower?	Nonperturbative α'?
No dS	No dS	No dS	No dS
AdS OK	AdS OK	No AdS	AdS OK

de Sitter vacua in String Theory ...

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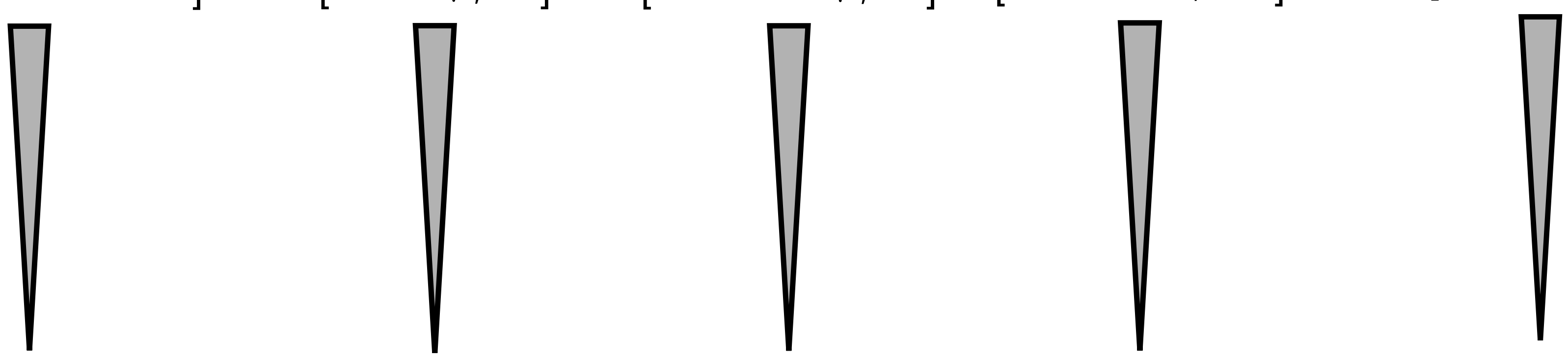
$$W(S) \sim e^{-S} \to \delta\mathcal{L} \sim \exp[-1/g_s^2]$$



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[Quigley,'15]
Nonperturbative g_s, Gaugino Condensation?
No dS*
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see also
[Brustein & de Alwis '04]

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Our goal is to extend the above results: → effects stronger than gaugino condensation

heterotic strings on torus orbifolds ...

[Font, Ibanez, Lust & Quevedo '90]
[Cvetic, Font, Ibanez, Lust & Quevedo '91]
[Gonzalo, Ibanez & Uranga '18]

- Overall Kähler Modulus T and Dilaton S
  - T has a $PSL(2,\mathbb{Z})$ symmetry from T-Duality:

$$T \to \gamma \cdot T = \frac{aT+b}{cT+d} \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$

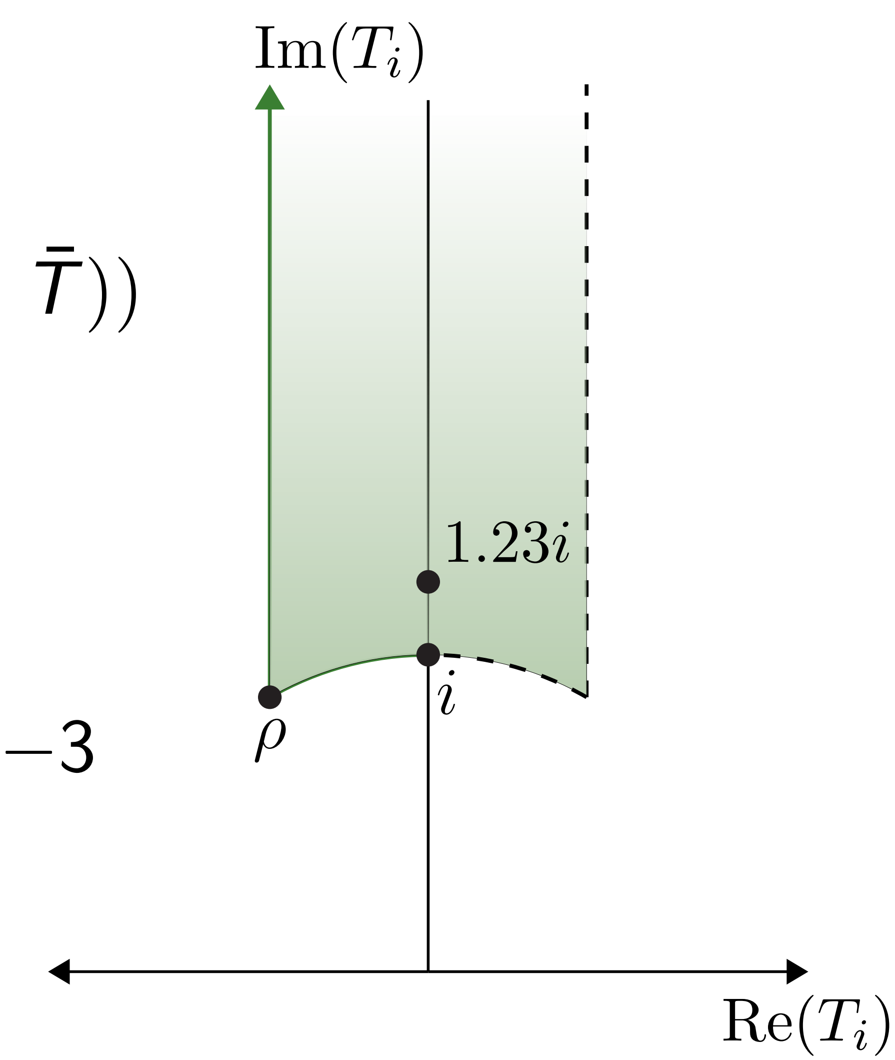
  - Kähler potential

$$\mathcal{K} = -\ln(S+\bar{S}) - 3\ln(-i(T-\bar{T}))$$

  - For action to be invariant under $PSL(2,\mathbb{Z})$,

$$\mathcal{G} = \mathcal{K} + \ln|W|^2$$

must be invariant $\Rightarrow$ W has modular weight -3



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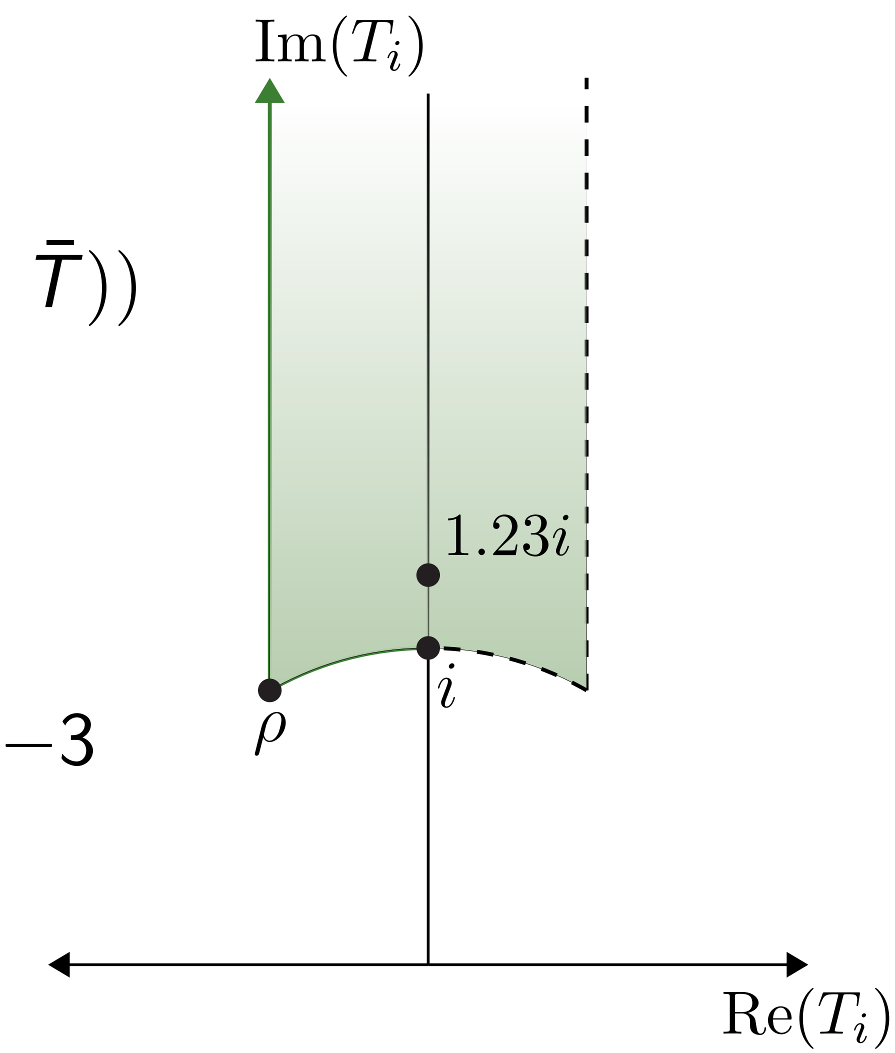

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- gaugino condensation:

$$\langle\lambda\lambda\rangle \sim \Lambda^3 \sim e^{-f_a/b_a} \sim e^{-S/b_a}$$

- modular invariance from thresholds:

$$\delta f_a \simeq b_a \ \ln[\eta^6(T)] + \cdots$$

$\Rightarrow$ Here be moonshine [Wrase,'14]

- $\rightarrow$ W:

$$W = \frac{H(T)e^{-S/b_a}}{\eta^6(T)}$$

$$H(T) = \left(\frac{G_4(T)}{\eta^8(T)}\right)^n \left(\frac{G_6(T)}{\eta^{12}(T)}\right)^m \mathcal{P}(j(T))$$

[Rademacher,Zuckerman,'38]
[Lehner]

infinite sum of $e^{2\pi i T}$ - terms — like WS instantons

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$\Omega(S)$

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heterotic strings on torus orbifolds ...

- scalar potential:

$$V(S,\bar{S},T,\bar{T}) = e^{\mathcal{K}}\left(\mathcal{K}^{S\bar{S}}F_S\bar{F}_{\bar{S}} + \mathcal{K}^{T\bar{T}}F_T\bar{F}_{\bar{T}} - 3|W|^2\right)$$

$$= e^{k(S,\bar{S})}Z(T,\bar{T})|\Omega(S)|^2\left\{|H(T)|^2\left(A(S,\bar{S})-3\right) + \hat{V}(T,\bar{T})\right\}$$

Conjectures [Gonzalo,Ibanez,Uranga,'18 - GIU]: no dS for tree-level $k(S,\bar{S})$

- 2 classes of S extrema:

Class A: $\Omega_S(S) + K_S\Omega(S) = 0 \rightarrow F_S = 0$

Class B: $F_S \neq 0$

heterotic strings on torus orbifolds ...

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class A no-go

[Leedom, Righi & AW '22]

- establish 2 no-go theorems — proving the GIU conjecture & extending it:

Class A

- **Theorem 1:** At a point (T_0, S_0), the scalar potential $V(T,S)$ can not simultaneously satisfy:
  1. $V(T_0, S_0) > 0$
  2. $\partial_S V(T_0, S_0) = 0 \quad \& \quad \partial_T V(T_0, S_0) = 0$
  3. $(\Omega_S + k_S \Omega)|_{S=S_{\mathbf{0}}} = 0$
  4. Eigenvalues of the Hessian of $V(T,S)$ at (T_0, S_0) are all $\geqslant 0$.

Proves several conjectures from GIU

class B no-go ?

[Leedom, Righi & AW '22]

- What about Class B extrema?
  In general Hessian doesn't factorize – much more complicated
  Enter the power of modular symmetry
- $V(T,S)$ is a non-holomorphic modular function in T, so $\partial_T V$ is a weight 2 modular form and vanishes at $T = i, \rho$
- All mixed derivatives of T & S are weight 2 modular forms
  $\Rightarrow$ Hessian is block diagonal
- Self dual points always extremum - when are they minima in T-sector?

class B no-go

[Leedom, Righi & AW '22]

Class B

- even SUSY-breaking S extrema cannot give dS minima if S has tree-level Kähler potential …

- **Theorem 2:** At a point (T_0, S_0), the scalar potential $V(T,S)$ with $k(S,\bar{S}) = -\ln(S+\bar{S})$ can not simultaneously satisfy:
  1. $V(T_0, S_0) > 0$
  2. $\partial_S V(T_0, S_0) = 0 \quad \& \quad \partial_T V(T_0, S_0) = 0$
  3. $\widetilde{F}_T(T_0) = 0$
  4. Eigenvalues of the Hessian of $V(T,S)$ at (T_0, S_0) are all $\geqslant 0$

a look into the modular landscape ...

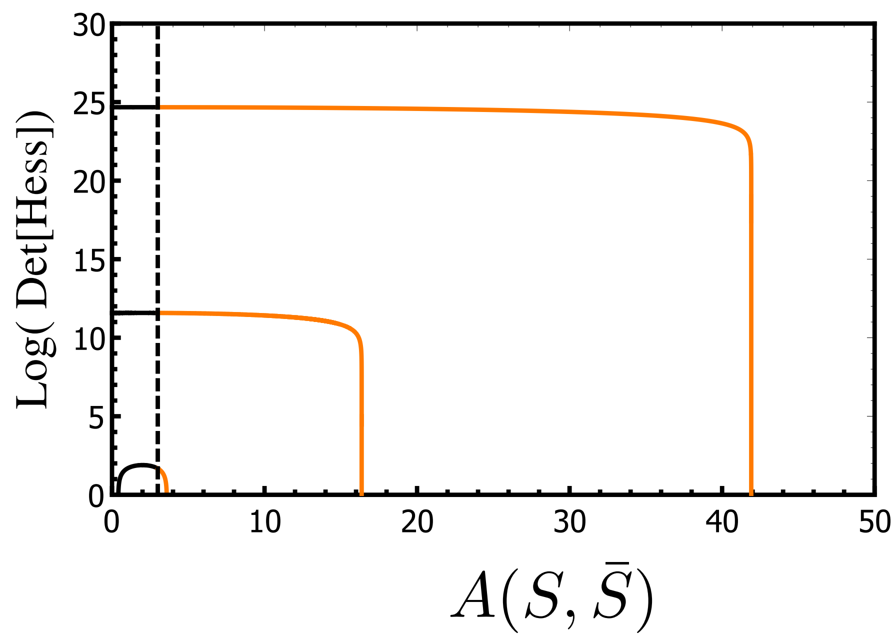
- $T = i$:

[Leedom, Righi & AW '22]

$$V(S,\bar{S},i,-i) = \frac{2^{4n+9}\pi^{8n+9}}{\Gamma^{12}(1/4)}|\Omega(S)|^2|\mathcal{P}(1728)|^2 e^{k(S,\bar{S})}\left(A(S,\bar{S})-3\right)$$

- Set $m = 0$ or else extremum is Minkowski
- dS extremum at $T = i$ if dilaton is stabilized with $\langle A(S,\bar{S})\rangle > 3$
- If we set $\mathcal{P}(j(T)) = 1$, then this point is stable in T sector if

$$2 - \frac{(1+8n)\Gamma^8(1/4)}{192\pi^4} < A(S,\bar{S}) < 2 + \frac{(1+8n)\Gamma^8(1/4)}{192\pi^4}$$



into the bulk ...

[Leedom, Righi & AW '22]

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- [Novichkov+ '22]: counter-examples   links to flavor symmetries:

  [Baur, Kade, Nilles, Ramos-Sanchez & Vaudrevange '20]

  [Knapp-Perez, Liu, Nilles, Ramos-Sanchez & Ratz '23]

  for certain (n,m) extrema near $T = \rho$ **off boundary**

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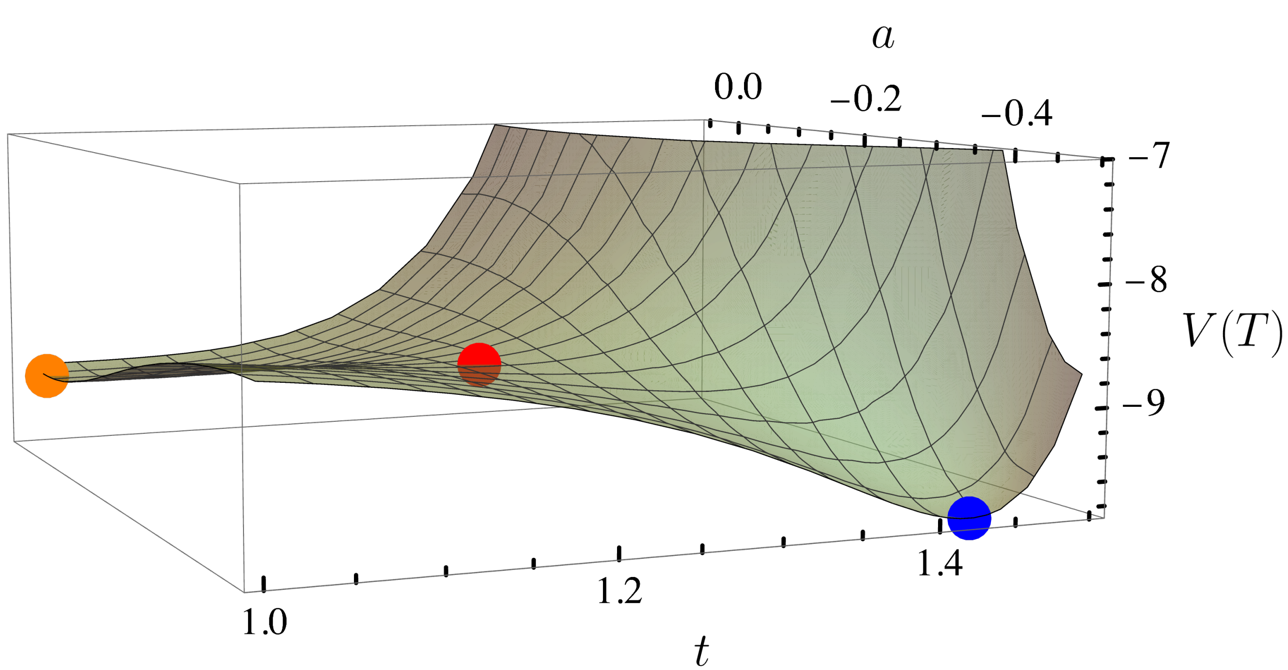
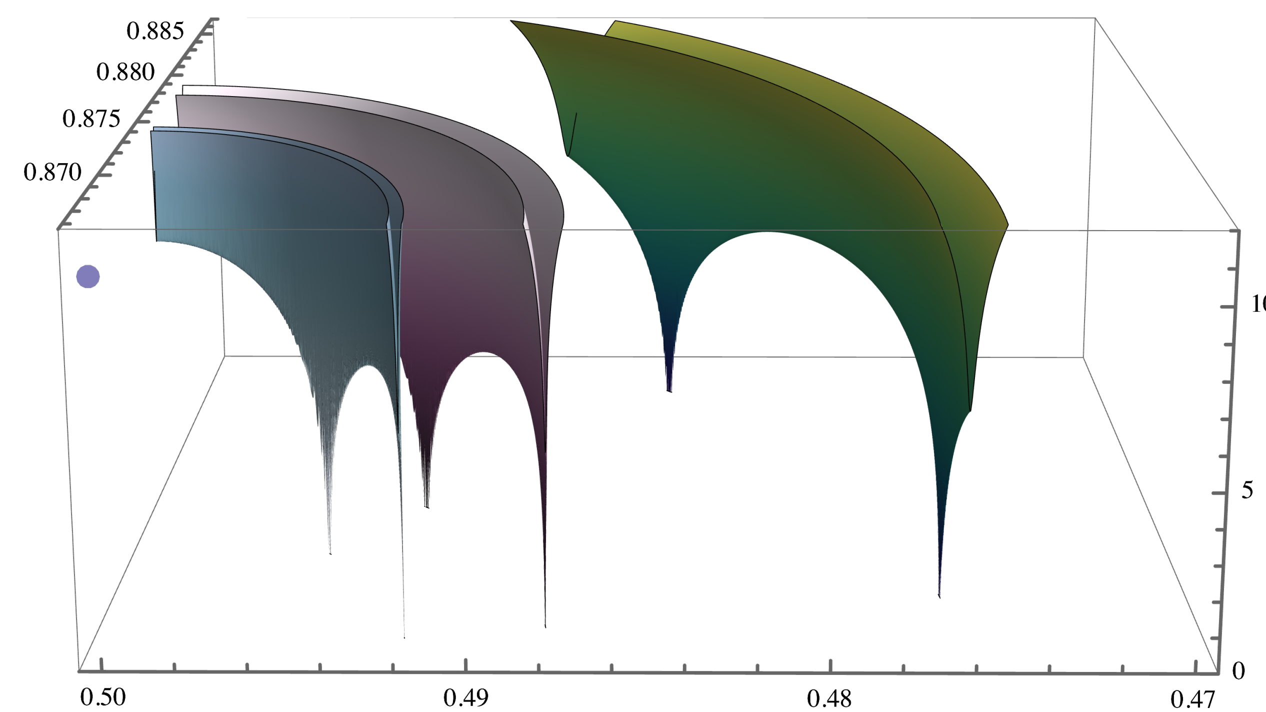
- verify & find more:

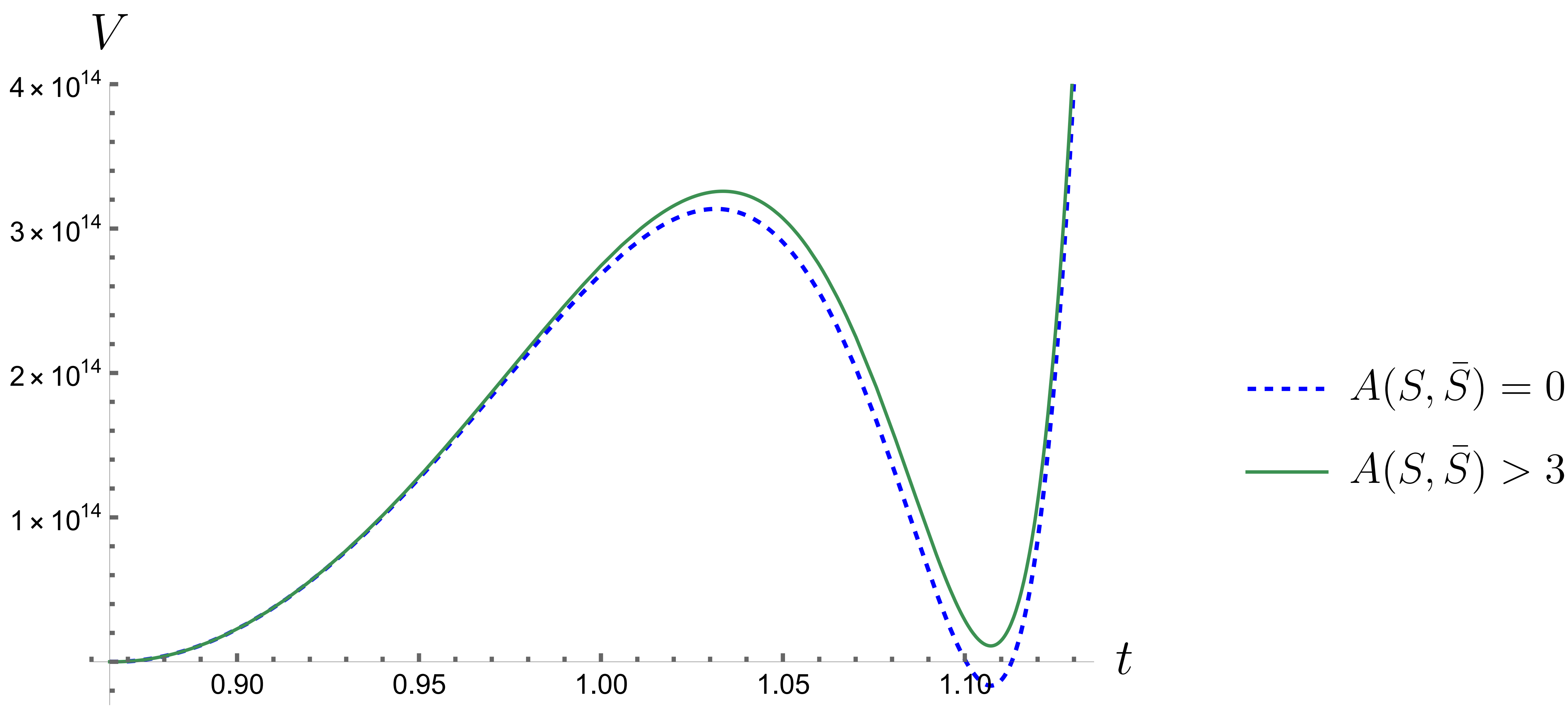




is there dS ?

[Leedom, Righi & AW '22]

- outcome: dS must come from class B …



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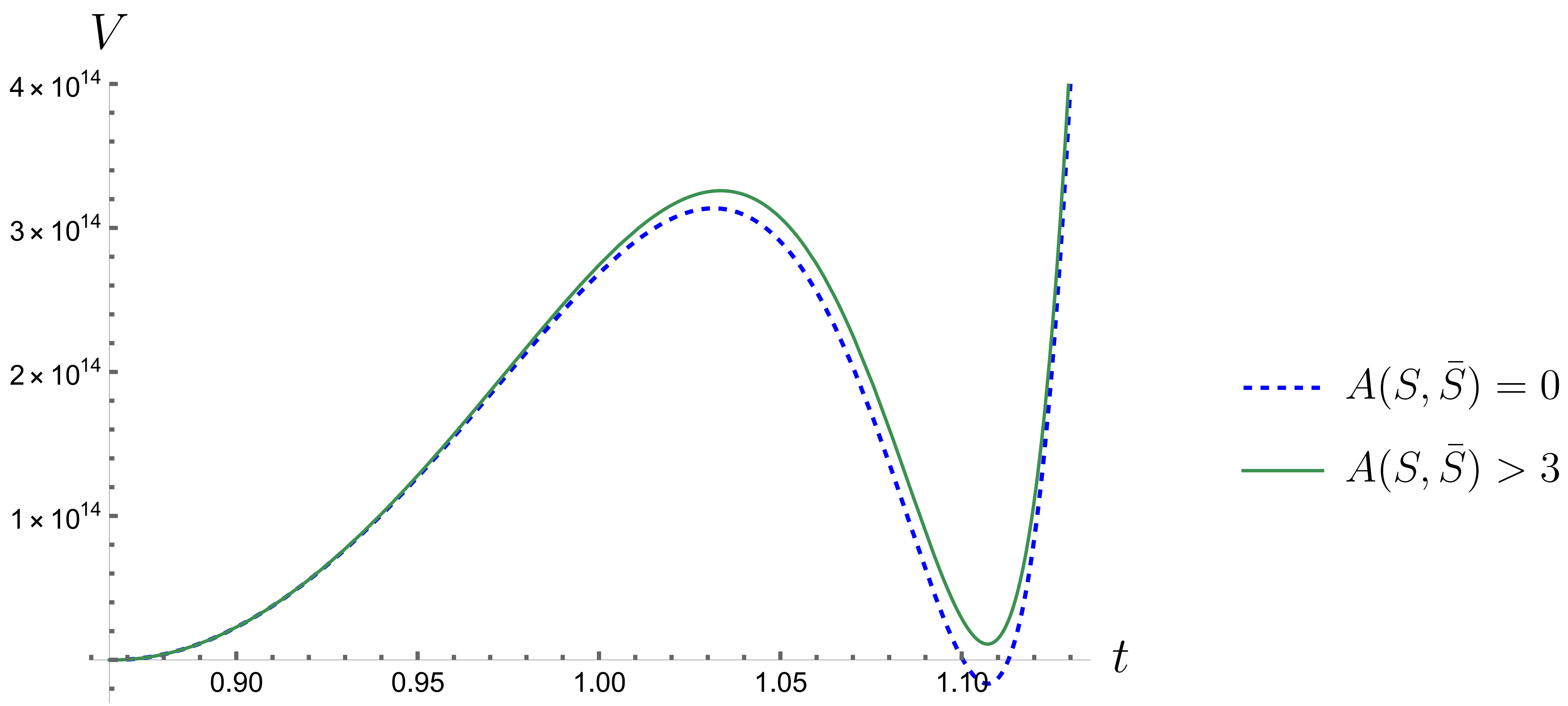

But impossible with tree level dilaton Kähler potential!

beyond the no-go ...

[Leedom, Righi & AW '22]

- [Shenker '90]: All closed string theories have effects of strength $\mathcal{O}(e^{-1/g_s})$

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  [Silverstein '96]: arguments from type I-heterotic & IIA-heterotic duality

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We want to utilize Shenker-like effects in Heterotic vacua.

We should be sure that they exist (… backup slides …)

evade the no-go ...

[Leedom, Righi & AW '22]

- Linear Multiplet Formalism: $L \supset \{\ell, \psi, B_2\}$

$$\mathcal{L}_{KE} = \int d^4\theta E\Big(-2 + f(L)\Big)$$

$$\left\langle \frac{\ell}{1+f(\ell)} \right\rangle = \frac{g_s^2}{2}$$

$$k(L) = \ln(L) + g(L)$$

evade the no-go ...

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- Parametrize Shenker-like effects [Gaillard & Nelson,'07]+:

$$f(\ell) = \sum_{n=0} A_n \ell^{-q_n} e^{-B/\sqrt{\ell}} \qquad L\frac{df}{dL} = -L\frac{dg}{dL} + f$$

evade the no-go ...

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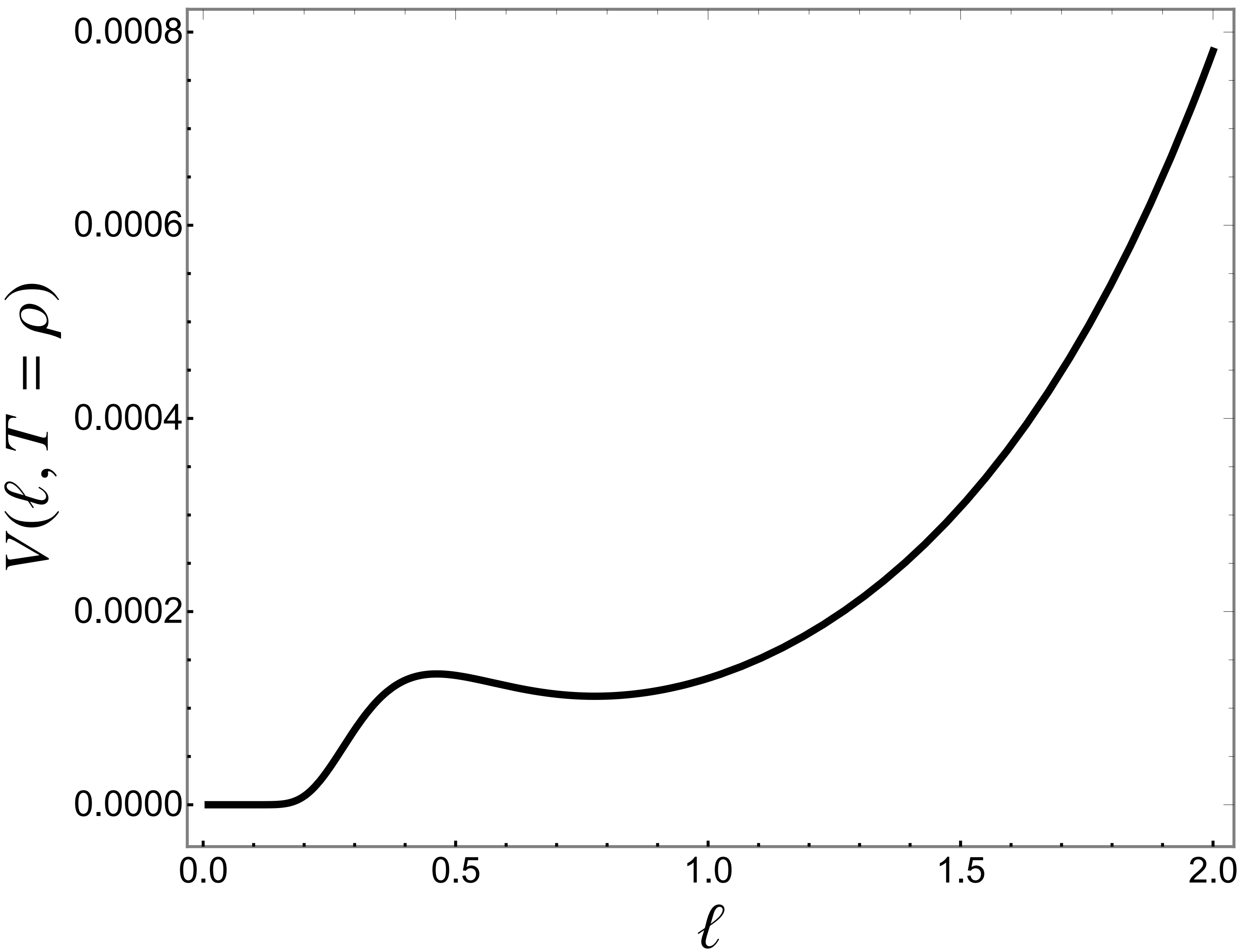
- Scalar potential for single gaugino condensate:

$$V(\ell) = \frac{\mathcal{T}}{\ell}\left[(1+\ell g')(1+b\ell)^2 - 3b^2\ell^2\right]e^{g-(f+1)/b\ell} \qquad \langle T\rangle = \rho$$

a road to heterotic dS - an example

[Leedom, Righi & AW '22]



$$f(\ell) = (A_0 + A_1 \ell^{\frac{1}{2}}) e^{-B/\sqrt{\ell}}$$

$$A_0 = 10 \qquad A_1 = 9$$

$$B = 0.6\pi \qquad b_{E_8} = \frac{30}{8\pi^2}$$

$$g_4 \simeq 0.70$$

Metastable dS

$$\langle e^{-B/\sqrt{\ell}} \rangle \simeq 0.11$$

all T^2 - moduli -> modular symmetry at genus-2

[Kidambi, Leedom, Righi & AW - WiP]

- full Kähler potential

$$K(T,\bar{T}) \to K(M,M^\dagger) = -\ln\left(-i\,\mathrm{det}(M - M^\dagger)\right)$$

$$M = \begin{pmatrix} T & Z \\ Z & U \end{pmatrix}$$

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Wilson line

$$M = \begin{pmatrix} T & Z \\ Z & U \end{pmatrix}$$

all T^2 - moduli -> modular symmetry at genus-2

[Kidambi, Leedom, Righi & AW - WiP]

- full Kähler potential

$$K(T,\bar{T}) \to K(M,M^\dagger) = -\ln\left(-i\det(M - M^\dagger)\right)$$

Wilson line

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T^2 complex structure modulus

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T^2 complex structure modulus

- does Kähler transformation under $Sp(4,Z)$

$$M \to \gamma(M) = (AM+B)(CM+D)^{-1} \quad , \quad \gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(4,\mathbb{Z})$$

$$K \to K + \ln\det(CM^\dagger + D) + \ln\det(CM + D)$$

all T^2 - moduli -> modular symmetry at genus-2

[Kidambi, Leedom, Righi & AW - WiP]

- F-term scalar potential from *Sp*(4,*Z*)-invariant *G*:

$$G = K + \ln W\bar{W}$$

forces *W* to be holomorphic Siegel modular form of definite weight!

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1-loop threshold: cusp forms

[Mayr & Stieberger '95]

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[Igusa '62 & '64]
[Freitag "Siegel Modular Forms"]

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1-loop threshold: cusp forms
[Mayr & Stieberger '95]

rational polynomial built from ring of Siegel modular forms

$$F_4, F_6, \mathcal{C}_{10}, \mathcal{C}_{12}$$

all T^2 - moduli -> modular symmetry at genus-2

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[Igusa '62 & '64]

[Freitag "Siegel Modular Forms"]

rational polynomial built from ring of Siegel modular forms

$$F_4, F_6, C_{10}, C_{12}$$

for much more — see N. Righi's talk !

Summary

[Maldacena-Nunez]	[Green+,'11]	[Gautason+,'12]	[Kutasov+, '15]	[Quigley,'15]	[Gonzalo+,'18]
Classical SUGRA?	Leading α'?	Infinite α' tower?	Nonperturbative α'?	Nonperturbative g_s, Gaugino Condensation?	Instantons,Condesates, Threshold Corrections*?
No dS	No dS	No dS	No dS	No dS*	No dS (numerically)
AdS OK	AdS OK	No AdS	AdS OK	No AdS*	AdS OK

Summary

[Maldacena-Nunez]	[Green+,'11]	[Gautason+,'12]	[Kutasov+, '15]	[Quigley,'15]	**[This Work]**
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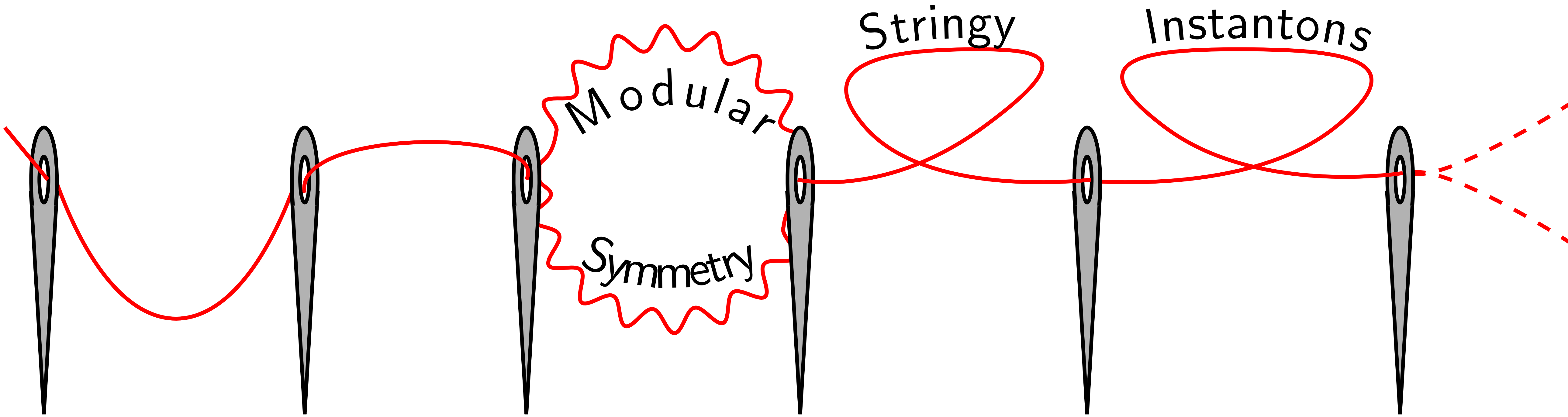
AdS OK	AdS OK	No AdS	AdS OK	No AdS*	& Some Class B

Summary



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Number Theory:**[CFC,JML,NR,AW]** & **[AK,JML,NR,AW]**,...

Shenker Effects:**[RAG,CFC,JML,NR]**

backup slides

evidence for heterotic Shenker-like effects

- [Silverstein,'96]: Can find Heterotic Shenker-like effects via duality arguments. They correct the Kähler potential
- Type I-Heterotic: $g^H_{MN} = \lambda_H g^I_{MN}$ & $\lambda_H = \lambda_I^{-1}$

$$\text{Type I Worldsheet Instantons:} \quad \delta\mathcal{L}_I \sim e^{-A^I/\alpha'} \leftrightarrow \delta\mathcal{L}_H \sim e^{-\frac{A^H}{\alpha'\lambda}}$$

- Type IIA-Heterotic: **If** $S_H \leftrightarrow T_{IIA}$ in 4d and **if** there is a non-trivial π_1:

$$\text{Type IIA Worldline Instantons :} \quad \delta\mathcal{L}_{IIA} \sim \sum_m e^{-mR^{IIA}} \leftrightarrow \delta\mathcal{L}_H \sim \sum_m e^{-m/\lambda}$$

- Does not explain the fundamental origins of these effects within the Heterotic frame
- Very schematic – no explicit calculations

Can do a bit better in M-Theory

evidence for heterotic Shenker-like effects

Low-Energy Limit: 11D Supergravity

$$S_{11D} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-G}\left(R - \frac{1}{2}|G_4|^2\right) - \frac{1}{6}\int C_3 \wedge G_4 \wedge G_4$$

evidence for heterotic Shenker-like effects

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$$\begin{tikzcd}
\text{Type IIA} & \text{M-Theory} \arrow[l, "\text{on } S^1"', "g_{IIA} \sim R^{\frac{3}{2}}"] \arrow[r, "\text{on } T^2", "U = \tau"'] \arrow[d, "\text{on } S^1/\mathbb{Z}_2"'] & \text{Type IIB} \\
 & \text{HE} &
\end{tikzcd}$$

$g_{HE} \sim R^{\frac{3}{2}}$

evidence for heterotic Shenker-like effects

Low-Energy Limit: 11D Supergravity

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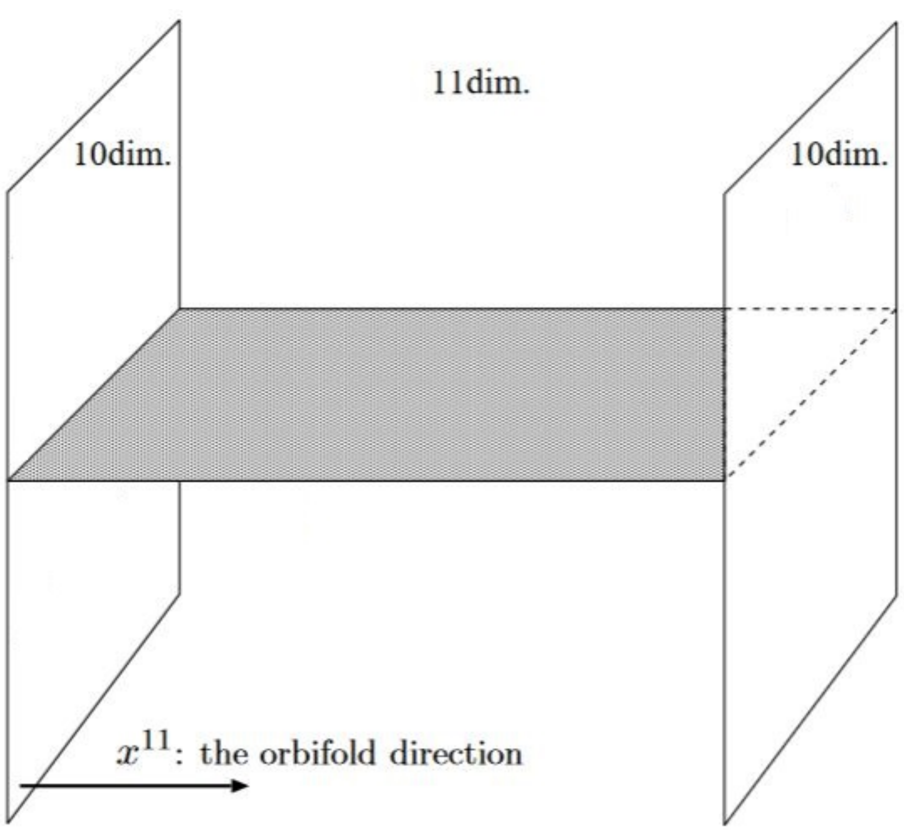
$$\text{Type IIA} \xleftarrow[g_{IIA} \sim R^{\frac{3}{2}}]{\text{on } S^1} \text{M-Theory} \xrightarrow[U = \tau]{\text{on } T^2} \text{Type IIB}$$

$$\text{M-Theory} \xrightarrow{\text{on } S^1/\mathbb{Z}_2} \text{HE}, \quad g_{HE} \sim R^{\frac{3}{2}}$$

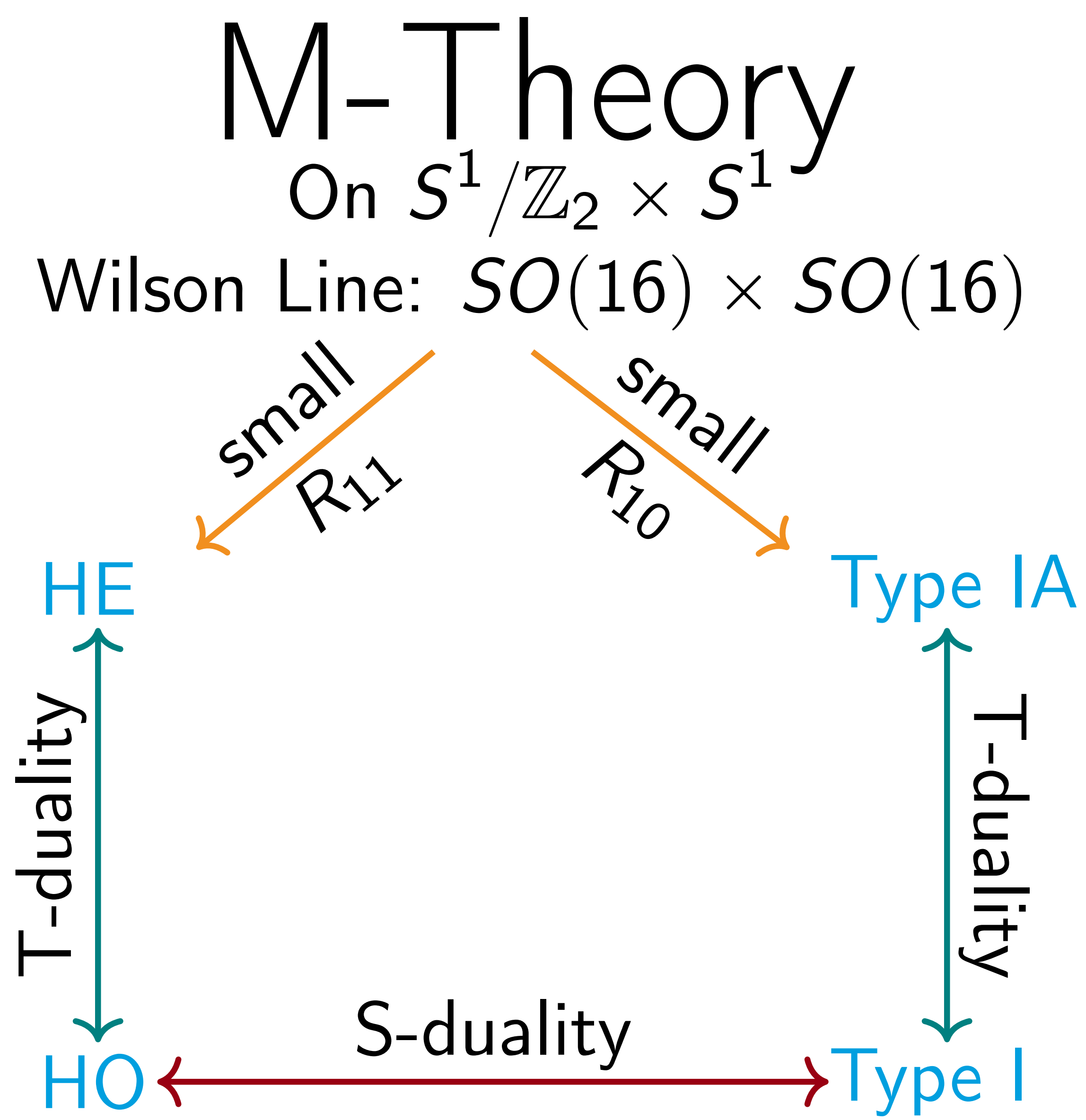
$$S_{HW} = S_{11D} + S_{YM} + S_B$$



evidence for heterotic Shenker-like effects

$S^1/\mathbb{Z}_2 : \ell_{11} R_{11}$

$S^1 : \ell_{11} R_{10}$



evidence for heterotic Shenker-like effects

$S^1/\mathbb{Z}_2 : \ell_{11} R_{11}$

$S^1 : \ell_{11} R_{10}$

$$g_{he} = R_{11}^{3/2}$$
$$r_{he} = R_{10}\sqrt{R_{11}}$$

$$\begin{tikzcd}
& \begin{array}{c}\text{M-Theory}\\ \text{On } S^1/\mathbb{Z}_2 \times S^1 \\ \text{Wilson Line: } SO(16)\times SO(16)\end{array} \arrow[dl, "\text{small } R_{11}"'] \arrow[dr, "\text{small } R_{10}"] & \\
\text{HE} \arrow[d, leftrightarrow, "\text{T-duality}"'] & & \text{Type IA} \arrow[d, leftrightarrow, "\text{T-duality}"] \\
\text{HO} \arrow[rr, leftrightarrow, "\text{S-duality}"] & & \text{Type I}
\end{tikzcd}$$

evidence for heterotic Shenker-like effects

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$$g_{he} = R_{11}^{3/2}$$
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$$g_{ho} = R_{11}/R_{10}$$
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evidence for heterotic Shenker-like effects

$S^1/\mathbb{Z}_2 : \ell_{11} R_{11}$

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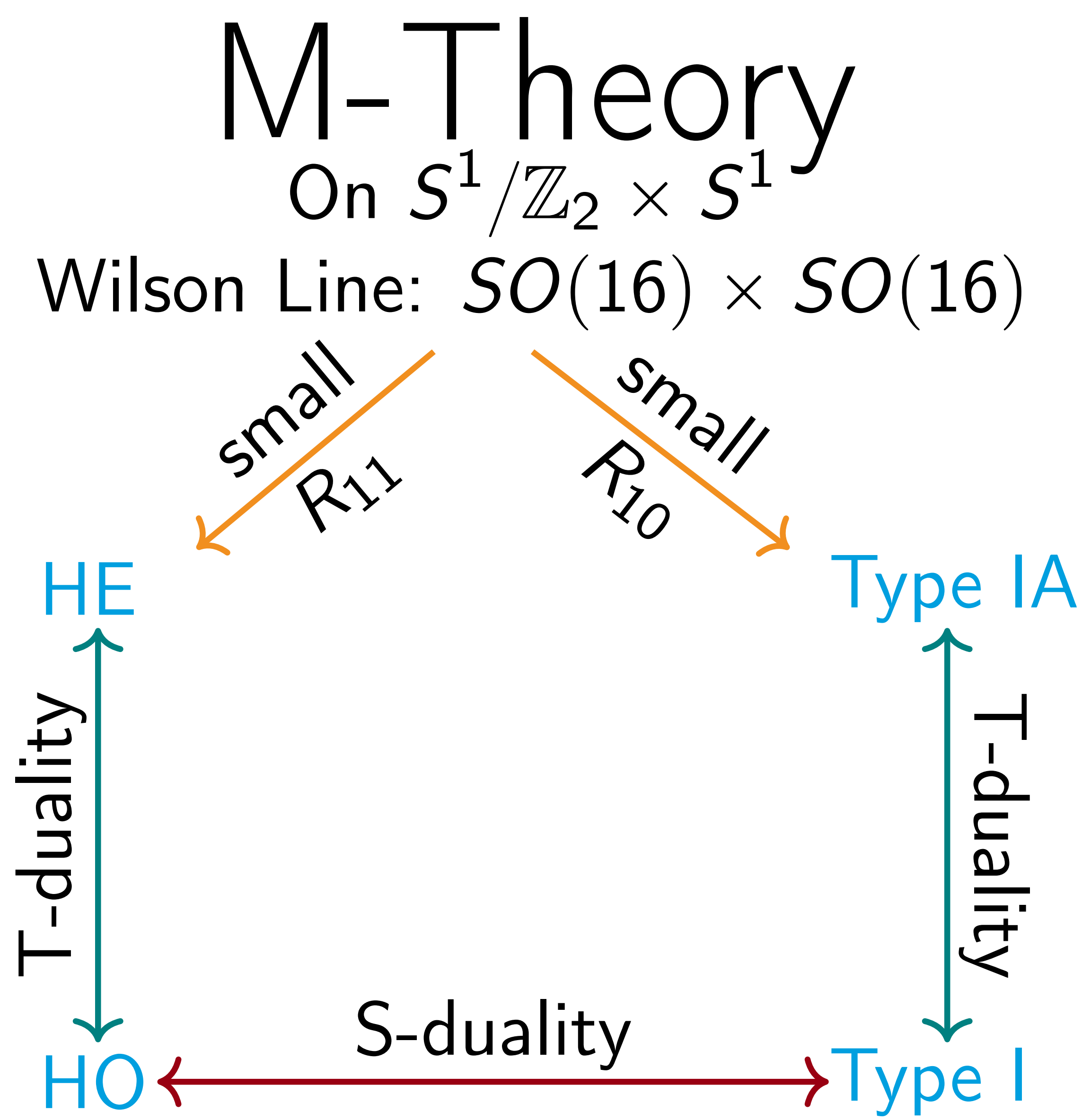

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evidence for heterotic Shenker-like effects

$S^1/\mathbb{Z}_2 : \ell_{11} R_{11}$

$S^1 : \ell_{11} R_{10}$

M-Theory

On $S^1/\mathbb{Z}_2 \times S^1$

Wilson Line: $SO(16) \times SO(16)$

$$
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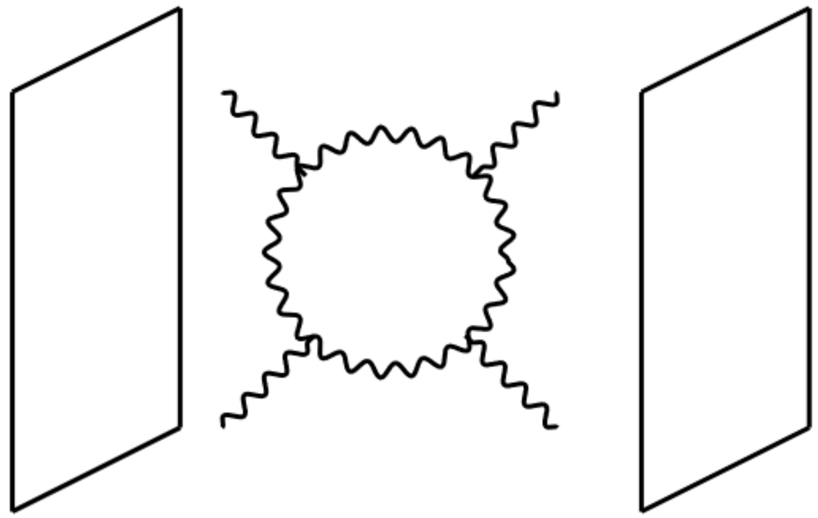
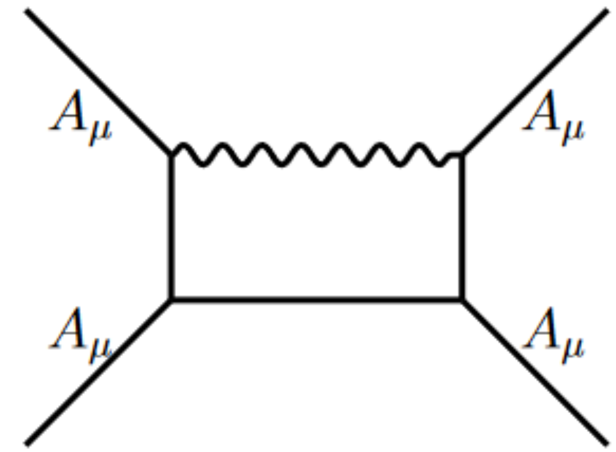
$$g_{ho} = R_{11}/R_{10}$$
$$r_{ho} = \frac{1}{R_{10}\sqrt{R_{11}}}$$

$$g_{I} = R_{10}/R_{11}$$
$$r_{I} = \frac{1}{R_{11}\sqrt{R_{10}}}$$

evidence for heterotic Shenker-like effects

Calculations from [Green, Rudra, '16]





In 10D HO:

$$S_{10D}^{HO} \supset \frac{g_{ho}^{-1/2}}{2^9(2\pi)^7 4! \ell_H^2} \int_{\mathcal{M}_{10}} d^{10}x \sqrt{-G}\, t_8 t_8 R^4 E_{3/2}(i g_{ho}^{-1})$$

evidence for heterotic Shenker-like effects

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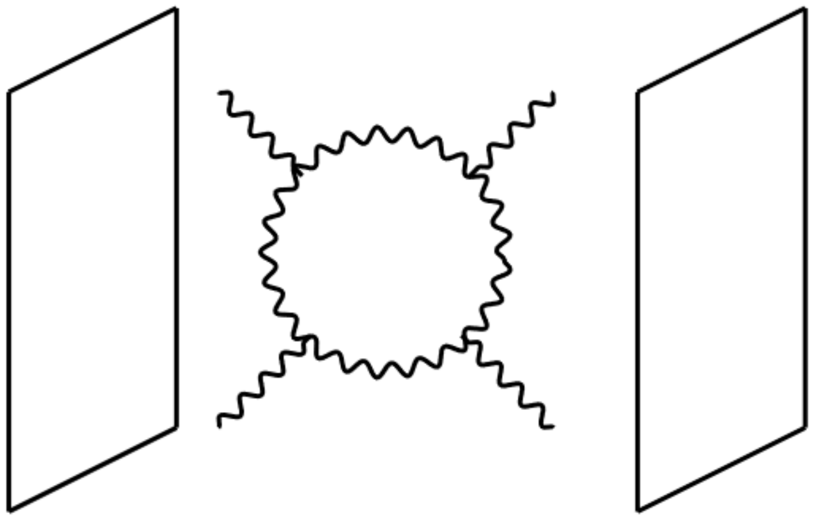

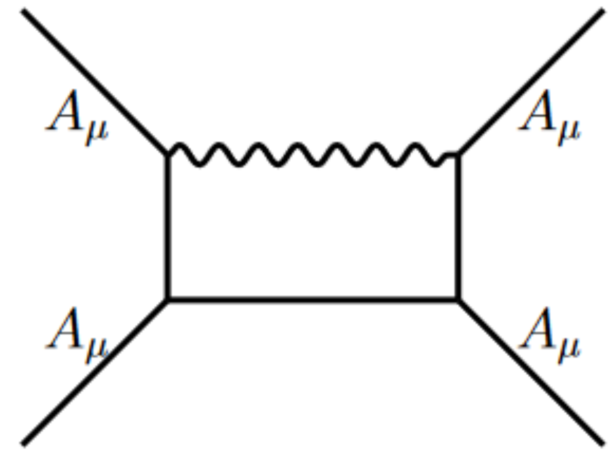

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$$E_s(\tau) = \sum_{(c,d)}{}' \frac{y^s}{|c\tau + d|^{2s}}$$

$$\tau = x + iy$$
$$E_s(\gamma \cdot \tau) = E_s(\tau)$$

evidence for heterotic Shenker-like effects

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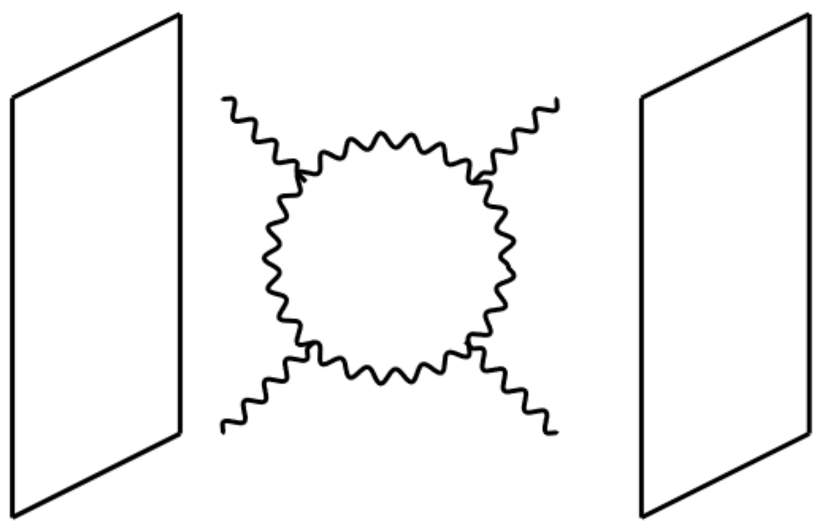

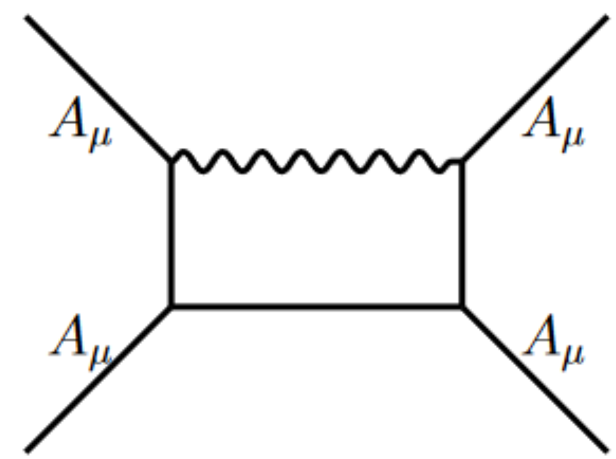

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evidence for heterotic Shenker-like effects

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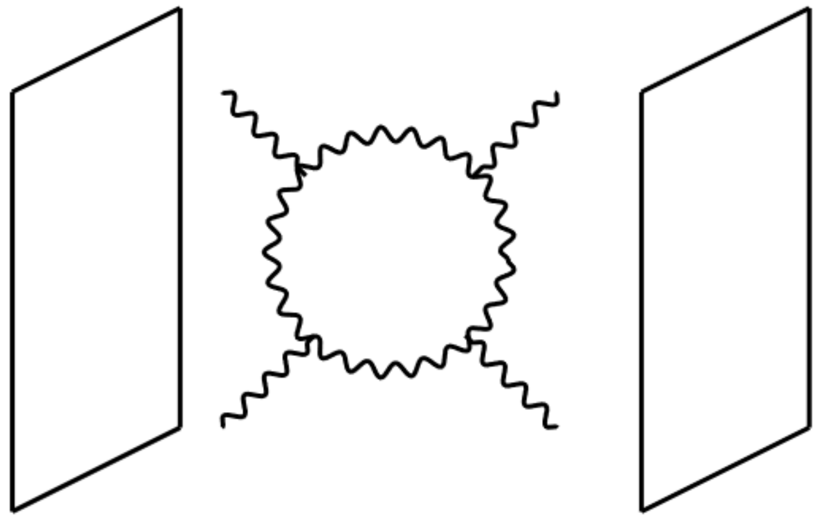

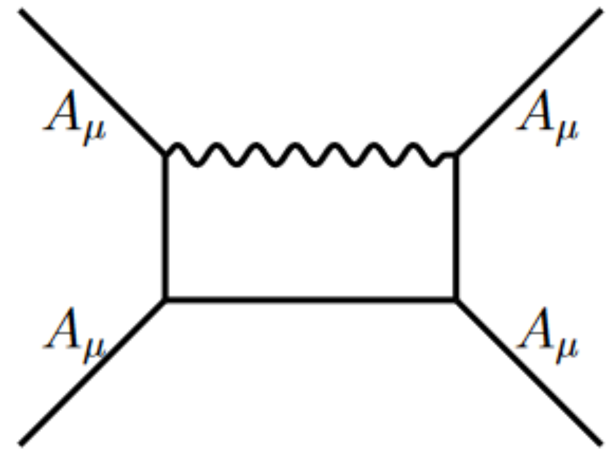

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Shenker-like Terms

evidence for heterotic Shenker-like effects

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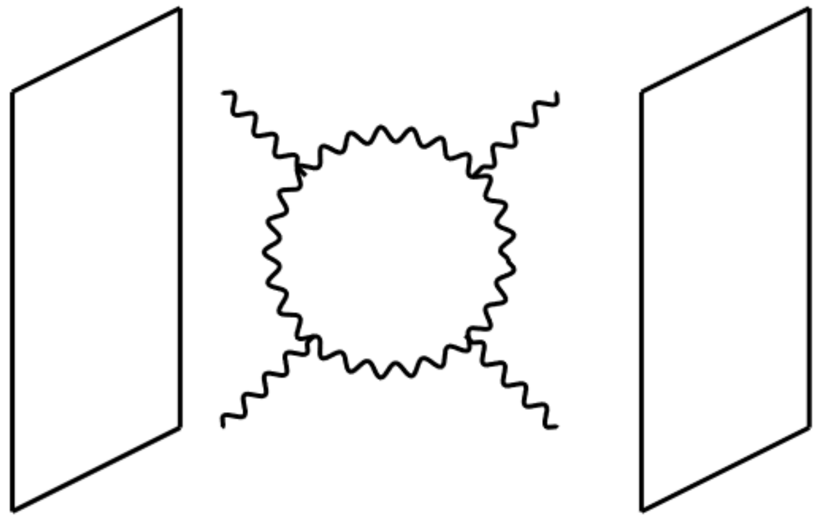

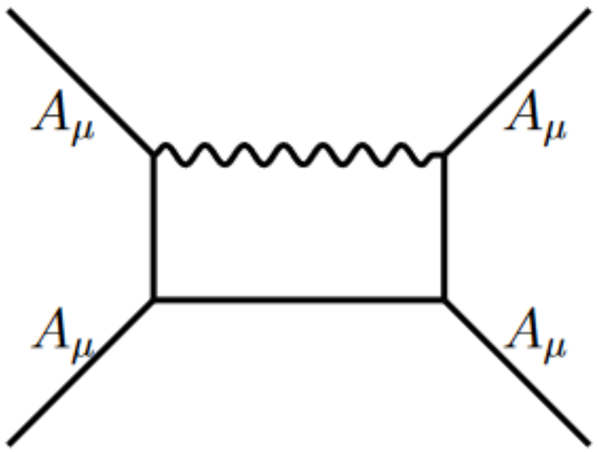

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Shenker-like Terms

Note: Similar terms vanish in 10D HE

evidence for heterotic Shenker-like effects

- But Why? Dualities

  Back to 9D: SO & Type I are S-Dual via $g_{ho} \leftrightarrow g_I^{-1}$

evidence for heterotic Shenker-like effects

- But Why? Dualities

  Back to 9D: SO & Type I are S-Dual via $g_{ho} \leftrightarrow g_I^{-1}$

$$S_{9D}^{HO} \supset \frac{r_{ho}}{2^9(2\pi)^6 4! \ell_H} \int d^9x \sqrt{-G} \left(\frac{2\zeta(3)}{g_{ho}^2}\right) t_8 t_8 R^4 + \cdots \qquad S_{9D}^{I} \supset \frac{r_I}{2^9(2\pi)^6 4! \ell_I} \int d^9x \sqrt{-G} \left(\frac{2\zeta(3)}{g_I^2}\right) t_8 t_8 R^4 + \cdots$$

evidence for heterotic Shenker-like effects

- But Why? Dualities

  Back to 9D: SO & Type I are S-Dual via $g_{ho} \leftrightarrow g_I^{-1}$

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$$\Longrightarrow \frac{r_{ho}}{\ell_H}2\zeta(3)g_{ho}t_8t_8R^4$$

evidence for heterotic Shenker-like effects

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$$\Longrightarrow \frac{r_{ho}}{\ell_H} 2\zeta(3) g_{ho} t_8 t_8 R^4$$

The $\frac{r_i}{\ell_i \sqrt{g_i}} f(g_i) t_8 t_8 R^4$ term requires a coefficient such that

$$f(ig_{ho}^{-1}) = f(ig_I^{-1})$$

evidence for heterotic Shenker-like effects

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The $\frac{r_i}{\ell_i\sqrt{g_i}} f(g_i) t_8 t_8 R^4$ term requires a coefficient such that

$$f(ig_{ho}^{-1}) = f(ig_I^{-1})$$

satisfied by the real-analytic Eisenstein series $E_s(\tau)$

s is determined by matching the perturbative part

evidence for heterotic Shenker-like effects

- And Self-Duality

  A 9D Theory we have left behind: M-theory on T^2 $\leftrightarrow$ IIB on S^1

$$S^{IIB}_{10D} \supset \frac{1}{\ell^2_{II}} \int d^{10}x \sqrt{-G}\, g_{IIB}^{-\frac{1}{2}} E_{\frac{3}{2}}(S)\, t_8 t_8 R^4$$

$S = C_0 + i g_{IIB}^{-1}$ is a complex scalar.
Orientifolding to Type I projects out C_0,
leaving only g_s^{-1} in the other 9D theories

- From What?

In Type I:

Non-BPS type I D-instantons. Responsible for O(32) $\Rightarrow$ SO(32) [Witten, '98]

In T-dual IIA frame, these are D-particles winding around the orbifolded x^{11} direction [Dasgupta, Gaberdiel, Green, '00]

In Heterotic: Unclear

sketch of proof of theorem 1 (& similarly, 2)

Proof: The proof by contradiction – assume 1 - 4 are true at (T_0, S_0)

$$\partial_S V(T,S) = \frac{F_S}{W} V(T,S) + \left\{ e^{k(S,\bar{S})} |\Omega(S)|^2 |H(T)|^2 Z(T,\bar{T}) \right\} \partial_S A(S,\bar{S})$$ $\Rightarrow$ vanishes by 3

$\Rightarrow$ $\partial_T^k \partial_{\bar{T}}^l \partial_S V(T_0, S_0) = 0$ $\quad \Rightarrow$ Hessian is block diagonal

To satisfy 1 , introduce $\Lambda > 0$ such that

$$V(T_0, S_0) = e^{k_0} |\Omega_0|^2 Z_0 \Lambda^4$$

which yields an expression for $H_T(T_0)$:

$$H_T(T_0) = \frac{3i}{2\pi} H_0 \widehat{G}_2(T_0, \bar{T}_0) \pm \frac{\sqrt{3}i}{T_0 - \bar{T}_0} \left(\Lambda^2 \pm i \sqrt{|H_0|^2 \left(3 - A(S_0, \bar{S}_0)\right)} \right)$$

$A(S_0, \bar{S}_0) = 0$ by (iii)

The 2nd condition in 2 gives a (long) expression for $H_{TT}(T_0)$

Plug these into the T-modulus sector of the Hessian:

$$(\partial_T \partial_{\bar{T}} V)_0 \propto -2\Lambda^4 < 0 \Rightarrow \quad \begin{aligned} \partial_t^2 V &= 2\partial_T \partial_{\bar{T}} V - 2\mathrm{Re}(\partial_T^2 V) \\ \partial_a^2 V &= 2\partial_T \partial_{\bar{T}} V + 2\mathrm{Re}(\partial_T^2 V) \\ \partial_t \partial_a V &= -2\mathrm{Im}(\partial_T^2 V) \end{aligned}$$

Cannot both be positive

$\Updownarrow$

dS minima not possible