D–term uplift, Moduli fixing and SVD in Heterotic–String Vacua

- Spinor-Vector Duality in the EFT limit
- Asymmetric free fermion – orbifold and string moduli
- D–term uplift in heterotic–string vacua

AEF, S Groot Nibbelink, M Hurtado Heredia NPB 969 (2021) 115473
AEF, S Groot Nibbelink, B Percival arXiv:2306.16443
A Diaz Avalos, AEF, B Percival, V Matyas, arXiv:2302:10075; 2306:16878

String Phenomenology 2023, Institute for Basic Science Daejeon, 7 July 2023

Fermionic $Z_2 \times Z_2$ orbifolds

'Phenomenology of the Standard Model and Unification'

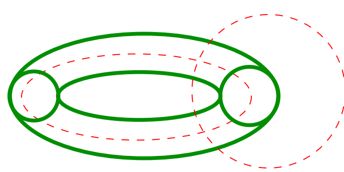
- Minimal Superstring Standard Model — NPB 335 (1990) 347 (with Nanopoulos & Yuan)
- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Moduli fixing — NPB 728 (2005) 83
- Classification — 2003 – $\cdots$ (with Kounnas, Rizos & ... Percival, Matyas)

Fermionic Construction

Left-Movers: $\psi^{\mu=1,2}$, $\chi_i,$ $y_i,$ ω_i $(i=1,\cdots,6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \; , \; \bar{\omega}_i & i=1,\cdots,6 \\ \\ \bar{\eta}_i & U(1)_i \quad i=1,2,3 \\ \bar{\psi}_{1,\cdots,5} & SO(10) \\ \bar{\phi}_{1,\cdots,8} & SO(16) \end{cases}$$



$$V \longrightarrow V \qquad\qquad f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} \, Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6, \qquad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $\quad X = 1 + s + \sum e_i + z_1 + z_2$

impose: $c\binom{z_1}{z_2} = -1$ & Gauge group $SO(10) \times U(1)^3 \times \text{hidden}$

Independent phases $c\left[{vi \atop v_j}\right] = \exp[i\pi(v_i|v_j)]$: upper block

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2
1	-1	-1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
S			-1	-1	-1	-1	-1	-1	-1	-1	1	1
e_1				$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_2					$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_3						$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_4							$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_5								$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_6									$\pm$	$\pm$	$\pm$	$\pm$
z_1										$\pm$	$\pm$	$\pm$
z_2											$\pm$	$\pm$
b_1												$\pm$
b_2												-1

A priori 55 independent coefficients $\rightarrow$ 2^{55} distinct vacua

PLB2021, Percival *et al* $\rightarrow$ Satisfiability Modulo Theories $\longrightarrow$ $t \times 10^{-3}$

Mirror symmetry

Enhance $SO(10) \to E_6$

from $X = 1 + S + \sum_i e_i + z_1 + z_2 = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}\}$

Euler characteristic $\chi = \#(\mathbf{27} - \overline{\mathbf{27}}) \longrightarrow -\chi$

Exchanges complex and Kähler structure moduli

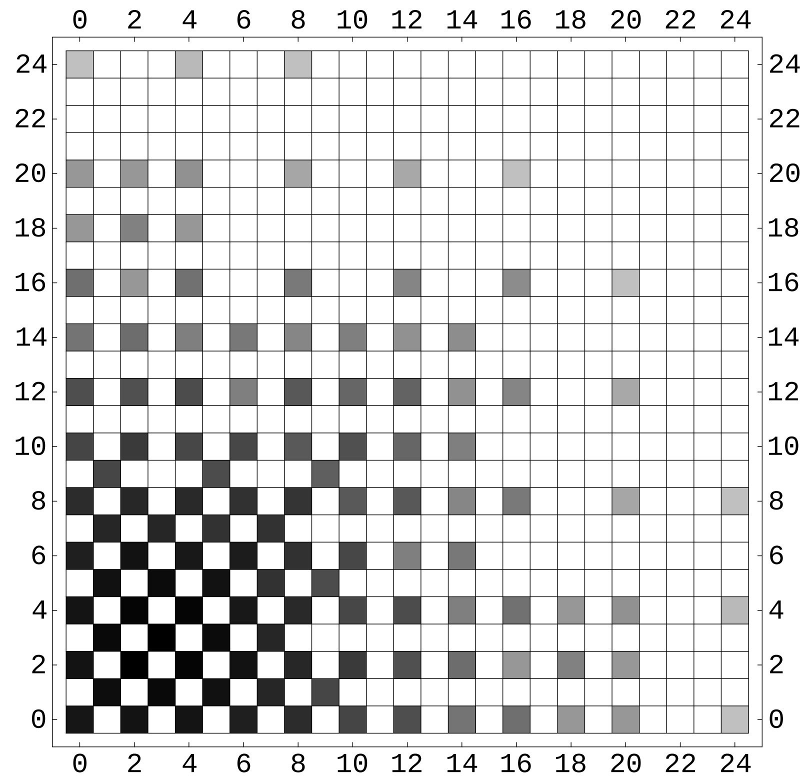
Moduli of the internal compactified space

Vafa–Witten 1994: Mirror symmetry in terms of discrete torsion

$$c\binom{b_1}{b_2} = +1 \to c\binom{b_1}{b_2} = -1$$

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \ <-> \ \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \ : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \ = \ \#(10)$ without E_6 symmetry

Spinor–Vector duality in Orbifolds:

Starting from: $$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} E_8 \times E_8 \,,$$

apply $\quad Z_2 \times Z_2' \;:\; g \times g'$

$$g \;:\; (0^7, 1|1, 0^7) \;\rightarrow\; \text{Wilson line} \;\rightarrow\; E_8 \times E_8 \rightarrow SO(16) \times SO(16)$$

$$g' \;:\; (x_4, x_5, x_6, x_7, x_8, x_9) \;\longrightarrow\; (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

Note: A single space twisting Z_2' $\quad \Rightarrow \quad N = 4 \rightarrow N = 2$

$$E_7 \rightarrow SO(12) \times SU(2)$$

• <u>sector b</u>

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s\overline{S}_{12}\overline{O}_4\overline{O}_{16}\,\right]+\right.$$

$$\left.\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{O}_{12}\overline{S}_4\overline{O}_{16}\,\right]\right\}\quad+\quad\text{massive}$$

where

$$P_\epsilon^+=\left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n}\qquad P_\epsilon^-=\left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_{m,n}$$

$$\epsilon\ =\ +1\quad\Rightarrow\quad P_\epsilon^+\ =\ \Lambda_{2m,n}\qquad P_\epsilon^-\ =\ \Lambda_{2m+1,n}$$

$$\epsilon\ =\ -1\quad\Rightarrow\quad P_\epsilon^+\ =\ \Lambda_{2m+1,n}\qquad P_\epsilon^-\ =\ \Lambda_{2m,n}$$

and $\quad 12\cdot 2\ +\ 4\cdot 2\ =\ 32$

Spinor-Vector duality on Calabi–Yau threefolds with vector bundles :

- From the "Land" to the "Swamp" w Groot–Nibellink & Hurtado-Heredia, arXiv:2103.13442, spinor–vector duality on a resolved orbifold. The role of the discrete torsion in the effective field theory limit

- Vafa–Witten 1994, the role of a discrete torsion in the $Z_2 \times Z_2$ orbifold in mirror symmetry

  FGNHH, 2211.01397 (2,0) GLSM $\longrightarrow$ discrete torsion in the EFT limit

- In similar spirit $\rightarrow$ the imprint of the worldsheet modular properties in the effective field theory limit

Mathematical implications

- On Calabi–Yau threefolds, the couplings correspond to intersections of curves $\longleftrightarrow$ rational curves on CY manifolds
- mirror symmetry is instrumental in counting of rational curves on CY 3–folds $\longleftrightarrow$ instrumental in enumerative geometry
- A tool developed for that purpose are the Gromov–Witten invariants

Questions

- Perform a similar analysis of correlators on spinor–vector dual vacua;
- What are the analogue of the Gromov–Witten invariants in the case of spinor–vector duality
- spinor–vector duality $\longrightarrow$ a tool to study CY manifolds with bundles
- moduli spaces of $(2,0)$ string compactifications
- Is it complete? Is it constraining the viable effective field limit of stringy quantum gravity.
- ...

A Swamplland Conjecture(?): AEF, EPJC 79 (2019) 703

- Every EFT $(2,0)$ heterotic–string compactification has to be connected to a (2,2) heterotic–string compactification by an orbifold or by continuous interpolation. If not $\rightarrow$ it is in the swampland

- Completeness?!

Moduli $\rightarrow$ WS Thirring interactions $(R-\frac{1}{R})J_L^i(z)\bar{J}_R^j(\bar{z}) = (R-\frac{1}{R})y^i\omega^i\bar{y}^j\bar{\omega}^j$

To identify the untwisted moduli in the free fermionic models

$\rightarrow$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z}) \quad \equiv \quad y^I\omega^I\bar{y}^J\bar{\omega}^J$$

that are allowed by the orbifold (fermionic) symmetry group

$$Z_2 \times Z_2 \qquad \{\,1\,,\;S\,,\;z_1\,,\;z_2\,\}\;+\;\{\,b_1\,,\;b_2\,\}$$

$$\rightarrow \quad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2}\bar{J}_R^{1,2} \quad ; \quad J_L^{3,4}\bar{J}_R^{3,4} \quad ; \quad J_L^{5,6}\bar{J}_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar{y}^{1,2}\bar{\omega}^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar{y}^{3,4}\bar{\omega}^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar{y}^{5,6}\bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds in realistic models

$$\{\,1\,,\;S\,,\;z_1\,,\;z_2\,\}\;\oplus\;\{\,b_1\,,\;b_2\,\}\;\oplus\;\{\,\alpha\,,\;\beta\,,\;\gamma\}$$

$$N=4 \qquad\qquad N=1$$

$$E_8\times E_8 \qquad\qquad Z_2\times Z_2$$

new feature Asymmetric orbifold $y^i\omega^i\bar{y}^i\bar{\omega}^i \;\rightarrow\; -\,y^i\omega^i\bar{y}^i\bar{\omega}^i$

the key focus: boundary conditions of the internal fermions

$$\{\;y\;,\;\omega\;\mid\;\bar{y}\;,\;\bar{\omega}\;\}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\rightarrow$ Ising model $\rightarrow$ symmetric real fermions

pairing of LL & RR fermions $\rightarrow$ complex fermions $\rightarrow$ asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Free Fermionic Webs of Heterotic T–folds :

- w Groot–Nibellink & Benjamin Percival, arXiv:2606.16443,

  free fermion $\longleftrightarrow$ asymmetric orbifolds via bosonisation

- the space of possible bosonisations via permutations.

  symmetric vs asymmetric is in the eye of the bosonisation

- enhanced T–duality group $\longleftrightarrow$ generalised T–folds

- intrinisically asymmetric $\longleftrightarrow$ all geometrical moduli are projected out

- moduli are projected in groups of four: 0, 4, 8, 12

(See Benjamin Percival parallel session talk)

Fayet–Iliopoulos D–term uplift (w Diaz-Avalos, Matyas, Percival)

“anomalous” $U(1)_A \quad \Rightarrow \quad$ FI–term in $N=1$ supersymmetric vacua

$$\mathrm{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle\phi_k\rangle|^2$$

$$D_j \;= 0 = \qquad \sum Q_k^j |\langle\phi_k\rangle|^2$$

$$\langle W\rangle = \langle \frac{\partial W_N}{\partial \eta_i}\rangle = 0 \;\; N = 3 \cdots\cdot$$

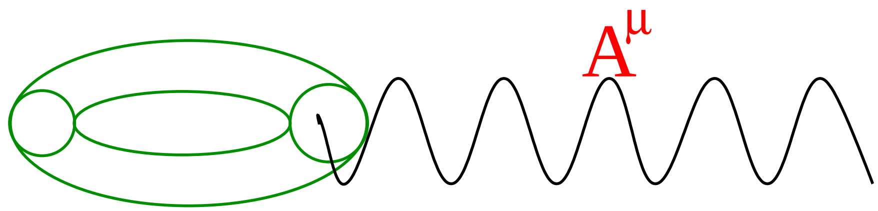
nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdots \cdot V_N^b\rangle$$

gauge & string invariant

Supersymmetric vacuum $\langle F\rangle = \langle D\rangle = 0.$

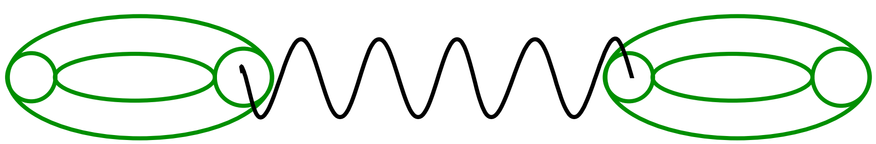
An anomalous $U(1)_A$ $\longleftrightarrow$ Recurring feature in $N = 0$ string vacua



The calculation of the "FI–term" follows through

$\rightarrow$ Alonzo Diaz-Avalos parallel session talk for details

$$V = \frac{1}{2} g_s^2 \zeta^2$$



Partition functions and the cosmological constant

Full Partition Function for Free Fermionic models:

$$Z_{ToT} \quad = \quad \int_{\mathfrak{F}} \frac{d^2\tau}{\tau_2^2} Z_B \, Z_F \quad \equiv \quad \Lambda$$

Integral over the inequivalent tori

- Fermionic contribution:

$$Z_F = \sum_{Sp.Str.} c\binom{\alpha}{\beta} \prod_f Z\begin{bmatrix}\alpha(f)\\ \beta(f)\end{bmatrix}$$

$$Z\begin{bmatrix}1\\1\end{bmatrix} = \sqrt{\frac{\theta_1}{\eta}}, \qquad Z\begin{bmatrix}1\\0\end{bmatrix} = \sqrt{\frac{\theta_2}{\eta}}, \qquad Z\begin{bmatrix}0\\0\end{bmatrix} = \sqrt{\frac{\theta_3}{\eta}}, \qquad Z\begin{bmatrix}0\\1\end{bmatrix} = \sqrt{\frac{\theta_4}{\eta}},$$

- Bosonic : $Z_B = \frac{1}{\tau_2}\frac{1}{\eta^2\bar{\eta}^2}$ from spacetime Bosons.

Evaluated using $q \equiv e^{2\pi i\tau}$ expansion

$$Z = \sum_{n.m} a_{mn} \int_{\mathfrak{F}} \frac{d^2\tau}{\tau_2^3} q^m \bar{q}^n$$

$$\begin{cases} d\tau_1 & \longrightarrow \textit{analytic} \\ d\tau_2 & \longrightarrow \textit{numeric} \end{cases}$$

$q -$ expansion of Z

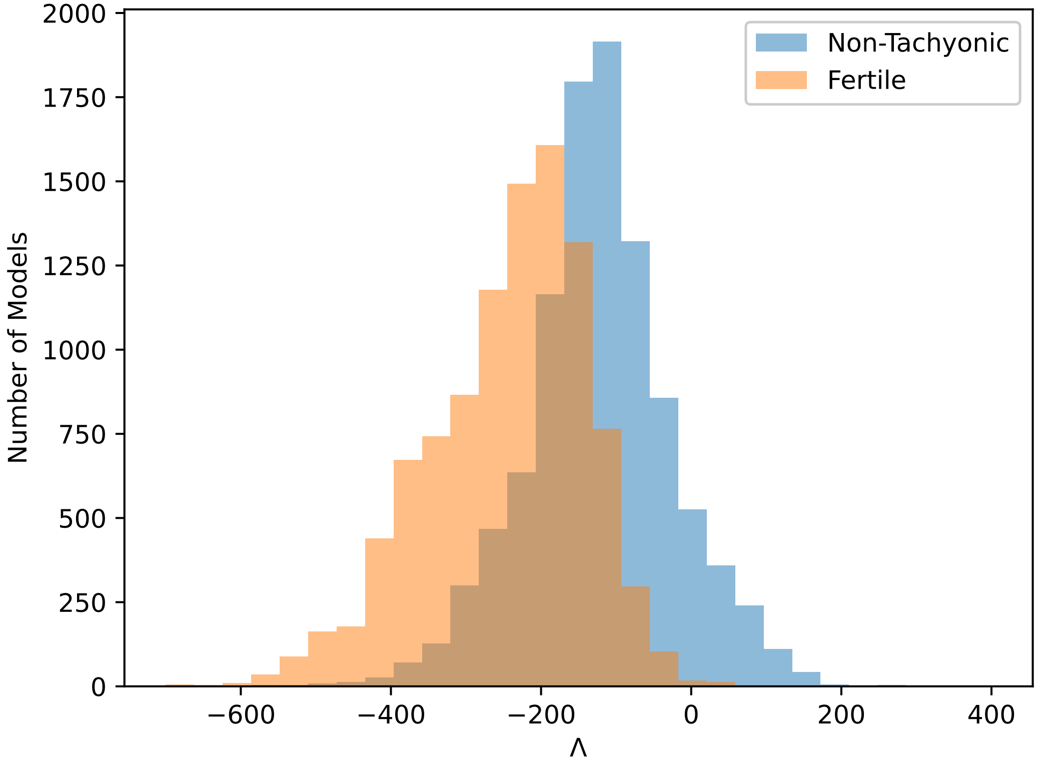
$$I_{mn} = \begin{cases} \infty & \text{if} \quad m+n<0 \ \wedge \ m-n \notin \mathbb{Z}\backslash\{0\} \\ \text{Finite} & \text{Otherwise.} \end{cases}$$

- On-Shell Tachyons cause divergence

- Off-Shell Tachyons allowed (necessary)

Modular invariance $\longrightarrow m-n \in \mathbb{Z}$.

Distribution of Λ



Away from the free fermionic point:

$$Z = \int \frac{d^2\tau}{\tau_2^2} \frac{\tau_2^{-1}}{\eta^{12}\bar\eta^{24}} \frac{1}{2^3} \left(\sum (-)^{a+b+ab} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix} \vartheta \begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix} \vartheta \begin{bmatrix} a+h_3 \\ b+g_3 \end{bmatrix} \right)_{\psi^\mu,\chi}$$

$$\times \left(\frac{1}{2} \sum_{\epsilon,\xi} \bar\vartheta \begin{bmatrix} \epsilon \\ \xi \end{bmatrix}^5 \bar\vartheta \begin{bmatrix} \epsilon+h_1 \\ \xi+g_1 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_2 \\ \xi+g_2 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_3 \\ \xi+g_3 \end{bmatrix} \right)_{\bar\psi^{1\dots5},\bar\eta^{1,2,3}}$$

$$\times \left(\frac{1}{2} \sum_{H_1,G_1} \frac{1}{2} \sum_{H_2,G_2} (-)^{H_1G_1+H_2G_2} \bar\vartheta \begin{bmatrix} \epsilon+H_1 \\ \xi+G_1 \end{bmatrix}^4 \bar\vartheta \begin{bmatrix} \epsilon+H_2 \\ \xi+G_2 \end{bmatrix}^4 \right)_{\bar\phi^{1\dots8}}$$

$$\times \left(\sum_{s_i,t_i} \Gamma_{6,6} \begin{bmatrix} h_i|s_i \\ g_i|t_i \end{bmatrix} \right)_{(y\omega\bar y\bar\omega)^{1\dots6}} \times e^{i\pi\Phi(\gamma,\delta,s_i,t_i,\epsilon,\xi,h_i,g_i,H_1,G_1,H_2,G_2)}$$

$$\Gamma_{1,1}[^h_g] = \frac{R}{\sqrt{\tau_2}} \sum_{\tilde m,n} \exp\left[-\frac{\pi R^2}{\tau_2} \left| (2\tilde m + g) + (2n+h)\,\tau \right|^2 \right]$$

SUSY broken by the choice of the phase Φ:

- explicit breaking *e.g.* $c\binom{S}{z_1} = +1$

  gravitino is projected out with mass $M \sim M_S$

- Shreck-Schwaz mechanism $\rightarrow$ Supersymmetry is broken sponateneously

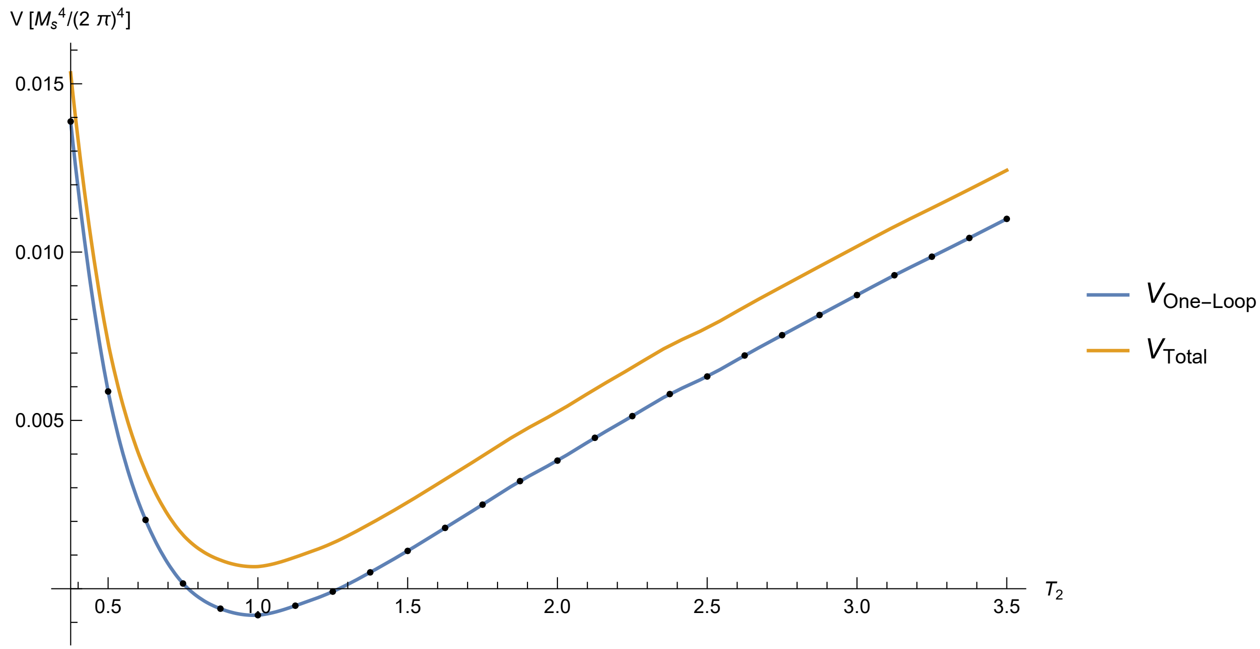
  $\Phi(X^5 + 2\pi R) = \mathrm{e}^{iQ}\Phi(X^5)$ Q fermion number

  $g = (-1)^F \delta \qquad \delta \rightarrow X = X + \pi R \qquad\qquad \longleftrightarrow \quad c\binom{S}{e_i} = +1$

$$M_{\frac{3}{2}} \sim \frac{1}{R}$$

Supersymmetry is restored in the $R \rightarrow \infty$ limit

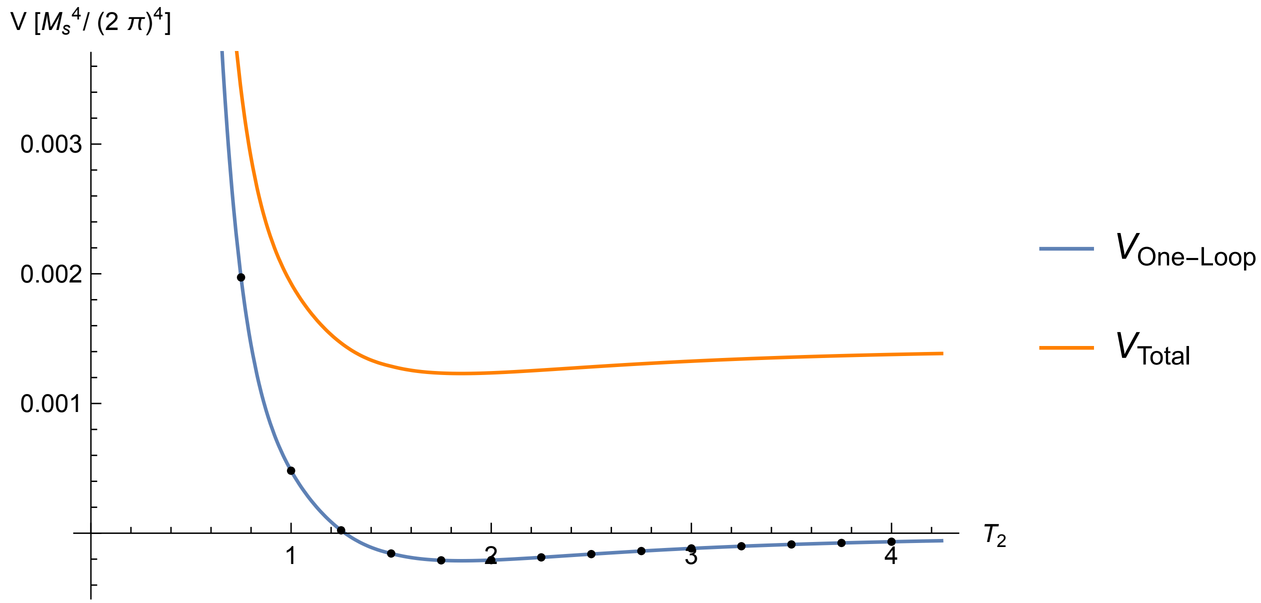
Uplift with explicit breaking $\quad (g_s \sim 1)$



$$\Lambda = \quad -0.000785598\mathcal{M}^4$$

$$V_D = \quad 0.00144365\mathcal{M}^4$$

Uplift with Scherk–Schwarz breaking



$$\Lambda = \quad -0.000215338\mathcal{M}^4$$

$$V_D = \quad 0.00144365\mathcal{M}^4$$

Conclusions

- Mirror symmetry, SVD $\longrightarrow$ How do they constrain the EFT limit ?

- Moduli spaces of (2,0) string compactifications

Symmetric $\longleftrightarrow$ Asymmetric

Classical geometry quantum geometry

Role of non–geometric backgrounds $\longleftrightarrow$ Moduli Fixing

- Non–SUSY string phenomenology $\cdots\cdots\cdots$

Role of the "Fayet–Iliopoulos" term in non–SUSY string vacua

D–term up lift $\rightarrow$ Towrad string dynamics and vacuum selection