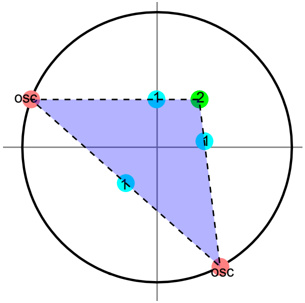
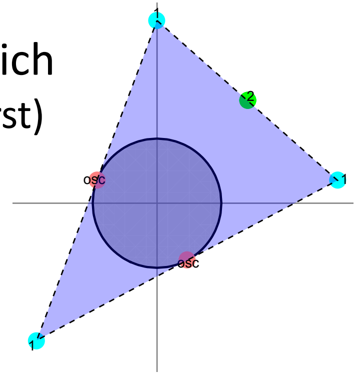
# Taxonomy of Infinite Distance Limits




Ben Heidenreich
(UMass Amherst)




Etheredge, BH, Kaya, Qiu, Rudelius, 2206.04063
Etheredge, BH, McNamara, Rudelius, Ruiz, Valenzuela, 2306.16440
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String Phenomenology 2023 — July 5, 2023

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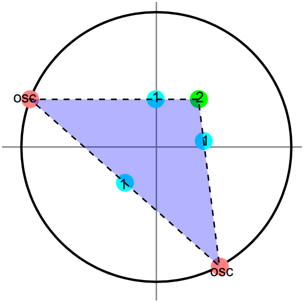



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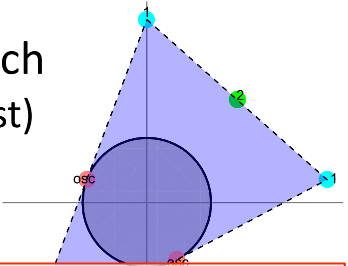


Irene's talk

Ignacio Ruiz's parallel talk

Etheredge, BH, Kaya, Qiu, Rudelius, 2206.04063

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# Goal: Classify infinite distance limits in moduli space

**1. What towers become light?**

**2. How quickly?**

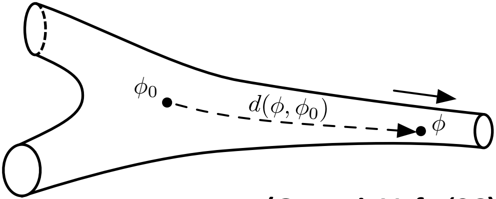
**3. How do various infinite-distance limits fit together in one moduli space?**

# Reminder:

Distance Conjecture (DC): in each infinite distance limit, an infinite tower of particles becomes light exponentially quickly:

$$m_{tower}(\phi) \lesssim e^{-\alpha\, d(\phi,\phi_0)}$$

(in Planck units, $\kappa_d = 1$)




for some O(1) constant $\alpha$

**(Ooguri, Vafa '06)**

# 1. What towers become light?

Emergent String Conjecture (ESC): every infinite distance limit is either

1. A decompactification limit
   (in which the lightest tower is a KK tower)

OR

2. An "emergent string limit"
   (in which the lightest tower consists of the oscillator modes of a fundamental string)

**(Lee, Lerche, Weigand '19)**

# 2. How quickly?

Sharpened Distance Conjecture: The DC holds with

$$\alpha \geq \frac{1}{\sqrt{d-2}}$$

(i.e., in every infinite distance limit, the mass of the **lightest** tower decreases at least this quickly)

**(Etheredge, BH, Kaya, Qiu, Rudelius '22)**

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(i.e., in every infinite distance limit, the mass of the **lightest** tower decreases at least this quickly)

Related to Emergent String Conjecture, because *typically* $\alpha_{\mathrm{osc}} = \frac{1}{\sqrt{d-2}}$ and $\alpha_{\mathrm{KK}} > \frac{1}{\sqrt{d-2}}$.

**(Etheredge, BH, Kaya, Qiu, Rudelius '22)**

# 3. How do various infinite-distance limits fit together in one moduli space?

Sharpened Scalar Weak Gravity Conjecture:

For each modulus $\phi$ there is a particle satisfying

$$-\frac{1}{\sqrt{G_{\phi\phi}}}\frac{\partial m}{\partial \phi} \geq \frac{1}{\sqrt{d-2}}\, m$$

**(Etheredge, BH, Kaya, Qiu, Rudelius ’22, building on Palti ’17; Calderon-Infante, Uranga, Valenzuela ’20)**

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- $\sqrt{G_{\phi\phi}}$: scalar coupling (metric on moduli space)
- $\frac{\partial m}{\partial \phi}$: scalar charge
- $\frac{1}{\sqrt{d-2}}$: sharpened O(1) factor
- $m$: mass

**(Etheredge, BH, Kaya, Qiu, Rudelius ’22, building on Palti ’17; Calderon-Infante, Uranga, Valenzuela ’20)**

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scalar coupling (metric on moduli space)

scalar charge

sharpened O(1) factor

mass

**Analogous to WGC (but unrelated!)**

**(Etheredge, BH, Kaya, Qiu, Rudelius ’22, building on Palti ’17; Calderon-Infante, Uranga, Valenzuela ’20)**

# 3. How do various infinite-distance limits fit together in one moduli space?

<u>Sharpened Scalar Weak Gravity Conjecture:</u>

With $n > 1$ moduli, same as convex hull condition:

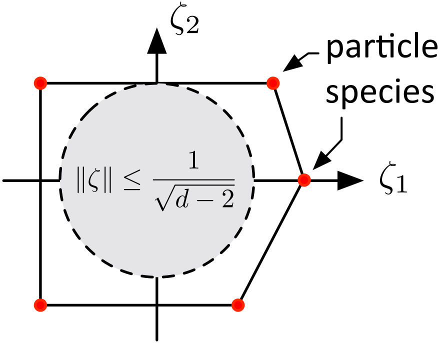
Define:

$$\zeta_i \equiv -\frac{\partial \log m}{\partial \phi^i}$$

$$\|\zeta\|^2 \equiv G^{ij}\zeta_i\zeta_j$$

C.H.C.

$\Longrightarrow$

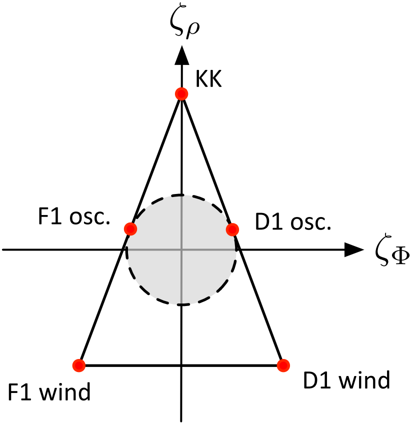



**(Etheredge, BH, Kaya, Qiu, Rudelius '22,**

**building on Palti '17; Calderon-Infante, Uranga, Valenzuela '20)**

# 3. How do various infinite-distance limits fit together in one moduli space?

Example: Type IIB string theory on $S^1$



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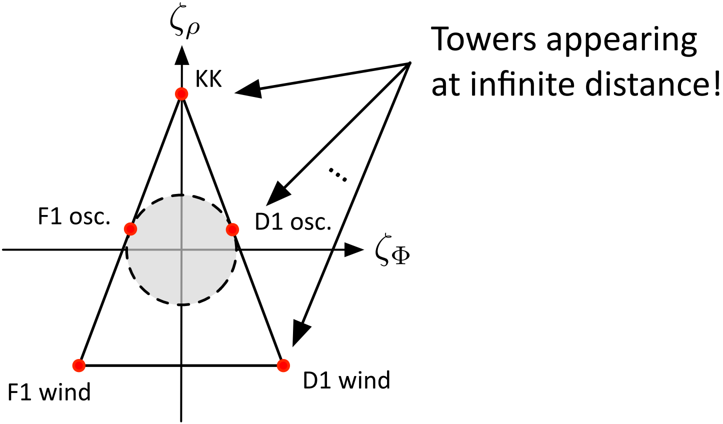

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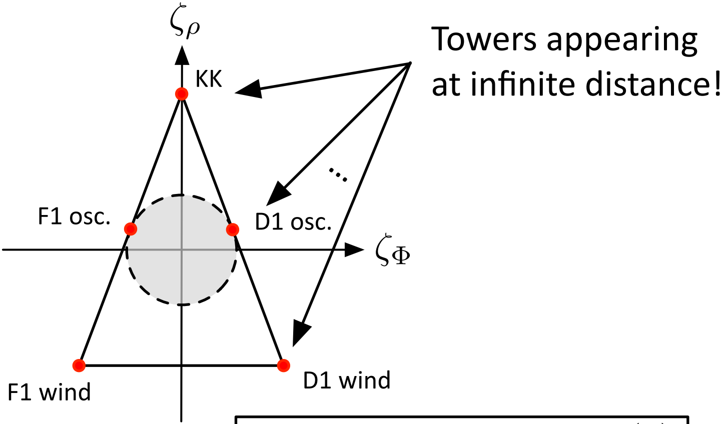


Convex hull determines $\alpha(\theta)$!

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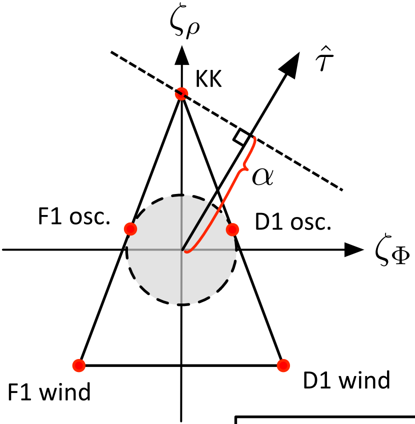


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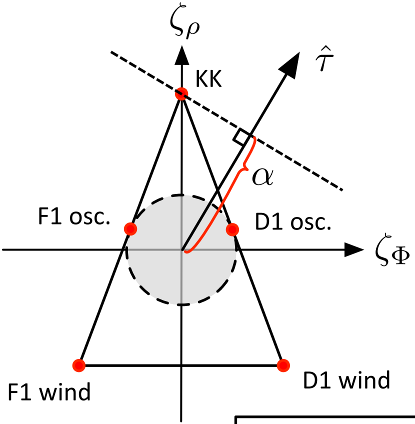


$$m_{\text{tower}} \propto e^{-\alpha\tau}$$ ← geodesic distance

$$\hat{\tau}^i = \frac{d\phi^i}{d\tau}$$ tangent to geodesic $\left(\|\hat{\tau}\| = 1\right)$

$$\alpha = -\frac{d\log m}{d\tau} = \vec{\zeta}\cdot\hat{\tau}$$

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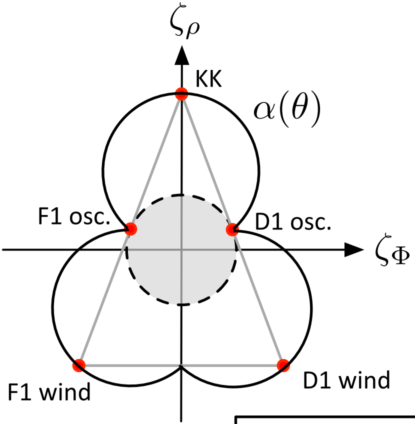


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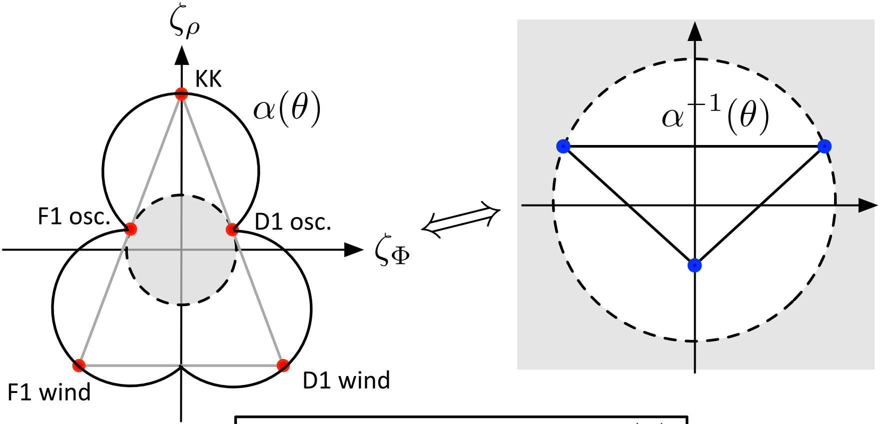


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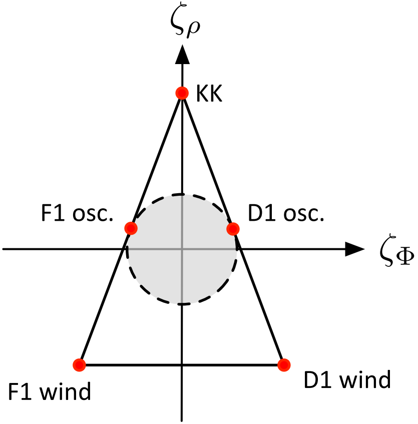
# 3. How do various infinite-distance limits fit together in one moduli space?

Tower hull related to dualities!



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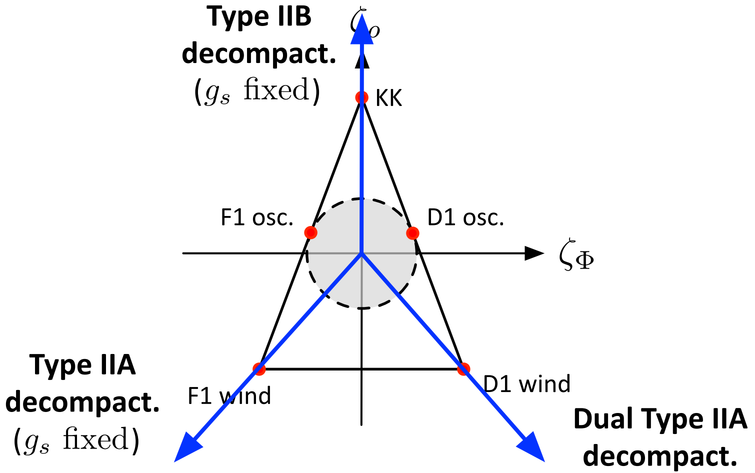

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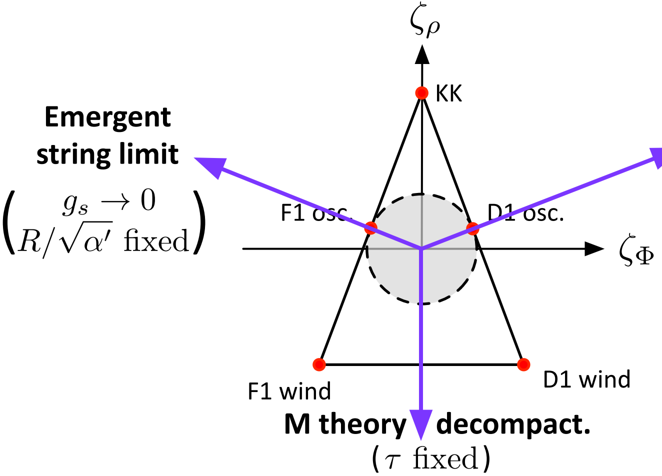

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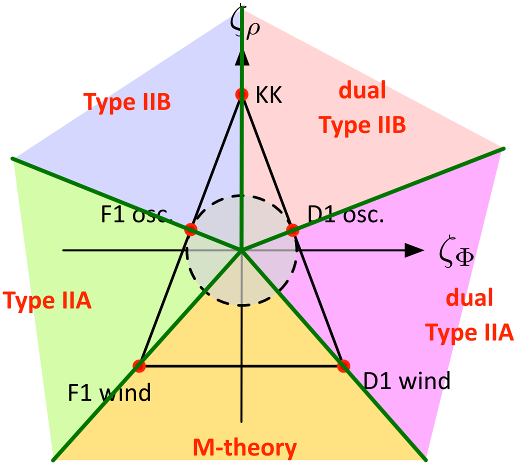

# 3. How do various infinite-distance limits fit together in one moduli space?

In general, assuming that

i. The moduli space $\mathcal{M}$ is flat
ii. The tower convex hull is same for all $\phi \in \mathcal{M}$
(the generating towers do not move)
iii. Sharpened SWGC is satisfied by infinite towers
(the "tower SWGC")

…then the sharpened Distance Conjecture follows.

In such cases, the tower convex hull concisely summarizes all infinite-distance limits.

# i. The moduli space is flat?

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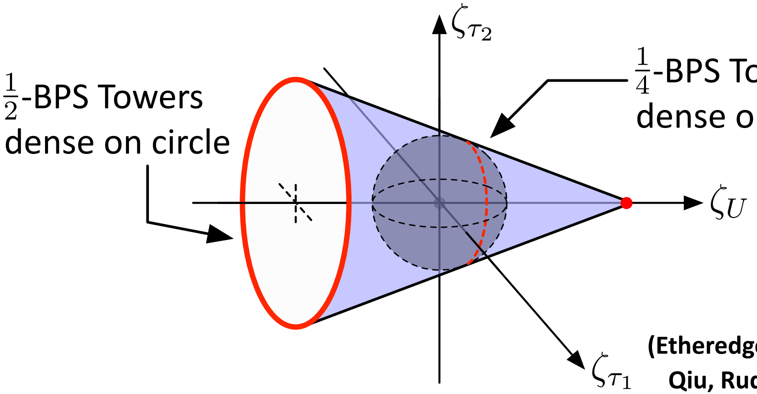
Ignored axions so far. Actual tower hull is:

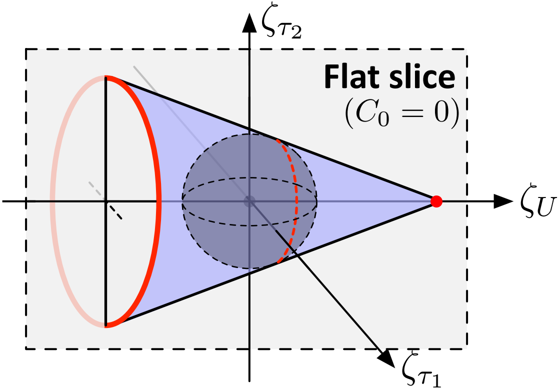



**(Etheredge, BH, Kaya, Qiu, Rudelius '22)**

Connecting the (sharpened) SWGC and DC becomes more subtle, see Etheredge 2307.xxxxx

# ✗ i. The moduli space is flat? $\mathcal{M} = \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SL}(2,\mathbb{Z})\times\mathrm{SO}(2)}\times\mathbb{R}$

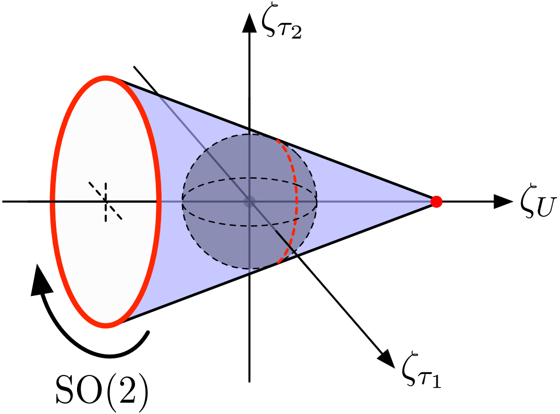
We recover previous result upon taking a flat slice:




Every geodesic (straight line) in slice goes to infinite dist. and every infinite dist. limit is **dual** to one in slice

# ✗ i. The moduli space is flat? $\mathcal{M} = \frac{SL(2,\mathbb{R})}{SL(2,\mathbb{Z})\times SO(2)} \times \mathbb{R}$

Different flat slices are related by dualities



## ii. Towers generating hull don’t move?

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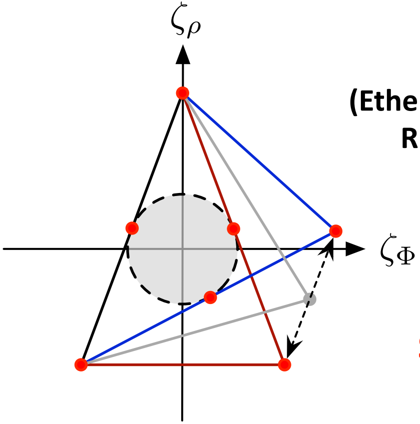
In many examples, they do!




**(Etheredge, BH, McNamara, Rudelius, Ruiz, Valenzuela, 2306.16440)**

**See Irene & Ignacio's talks**

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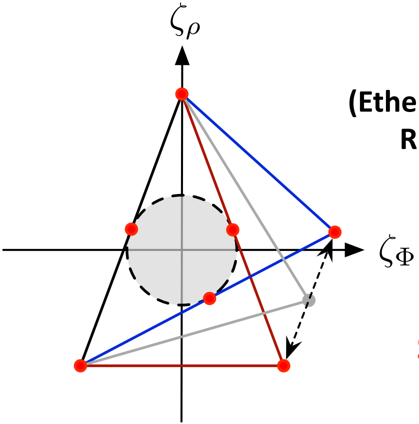


**(Etheredge, BH, McNamara, Rudelius, Ruiz, Valenzuela, 2306.16440)**

**See Irene & Ignacio's talks**

**This** talk: focus on exs where this **doesn't** happen

# iii. Sharpened (tower) SWGC satisfied?

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- Etheredge, BH, Kaya, Qiu, Rudelius ’22:

  - **True** in maximal SUGRA (at least $d \geq 4$)
  - Highly non-trivial bottom-up evidence in vector multiplet moduli space in $d \geq 5$ (related to TWGC)

- Etheredge, BH, McNamara, Rudelius, Ruiz, Valenzuela ’22:

  - **True** in 9d $\mathcal{N} = 1$ theories (in highly nontrivial way)

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# SWGC in maximal SUGRA

Etheredge, BH, Kaya, Qiu, Rudelius '22

U duality plays essential role

$$\mathcal{M} = \frac{G}{G_{\mathbb{Z}} \times H}$$

| $d$ | $G$ | $H$ | $R_{\text{part}}^{(G)}$ | $R_{\text{str}}^{(G)}$ |
|---|---|---|---|---|
| 9 | $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SO}(1,1)$ | $\mathrm{SO}(2)$ | $\square \oplus \mathbf{1}$ | $\square$ |
| 8 | $\mathrm{SL}(3,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ | $\mathrm{SO}(3) \times \mathrm{SO}(2)$ | $(\square, \square)$ | $(\square, \mathbf{1})$ |
| 7 | $\mathrm{SL}(5,\mathbb{R})$ | $\mathrm{SO}(5)$ | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | $\square$ |
| 6 | $\mathrm{Spin}(5,5)$ | $\mathrm{Spin}(5) \times \mathrm{Spin}(5)$ | $S$ | $\square$ |
| 5 | $E_{6(6)}$ | $\mathrm{USp}(8)$ | $\mathbf{27}$ | $\mathbf{27}'$ |
| 4 | $E_{7(7)}$ | $\mathrm{SU}(8)$ | $\mathbf{56}$ | $\mathrm{Adj} = \mathbf{133}$ |

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branching to stabilizer *H*

| $d$ | $H$ | $R_{\text{mod}}$ | $R_{\text{part}}$ | $R_{\text{str}}$ |
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# SWGC in maximal SUGRA

Etheredge, BH, Kaya, Qiu, Rudelius '22

Use rep. thy. to fix form of $\frac{1}{2}$-BPS bound, shortening condition, and scalar charge, e.g., in $d = 7$

$H = \mathrm{SO}(5)$ $\quad \zeta \in \square\!\square$ $\quad Z \in \begin{smallmatrix}\square\\\square\end{smallmatrix}$ $\quad$ short cond. $\in \square$

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$$M^2 = \frac{1}{2} Z^{ab} Z_{ab} \qquad \Downarrow \qquad \varepsilon^{abcde} Z_{ab} Z_{cd} = 0$$

$$\zeta_{ab} = \frac{Z_{ac} Z_b{}^c}{M^2} - \frac{2}{5}\delta_{ab} \longleftarrow$$ normalized by KK modes: $\zeta^2 \equiv \zeta_{ab}\zeta^{ab} = \frac{6}{5}$

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**Dual** to tower hull is set $P \in \square\!\square$ s.t. $P \cdot \zeta_{\frac{1}{2}\text{-BPS}} \leq 1$

$\Rightarrow$ largest two eigenvals satisfy $\lambda_1 + \lambda_2 \leq 1$

Find $(P^2)_{\max} = 5$

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Find $(P^2)_{\max} = 5 \implies (\zeta^2)_{\min} = \dfrac{1}{5} = \dfrac{1}{d-2}$ ✓

# SWGC in maximal SUGRA

Etheredge, BH, Kaya, Qiu, Rudelius ’22

Continue to lower dimensions…

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| 6 | $\mathrm{Spin}(5) \times \mathrm{Spin}(5)$ | $(\square, \square)$ | $(S, S)$ | $(\square, 1) + (1, \square)$ |
| 5 | $\mathrm{USp}(8)$ | $\begin{array}{c}\square\\\square\\\square\\\square\end{array}$ | $\begin{array}{c}\square\\\square\end{array}$ | $\begin{array}{c}\square\\\square\end{array}$ |
| 4 | $\mathrm{SU}(8)$ | $\begin{array}{c}\square\\\square\\\square\\\square\end{array}$ | $\begin{array}{c}\square\\\square\end{array}_{\mathbb{C}}$ | $\mathrm{Adj} \oplus \begin{array}{c}\square\\\square\\\square\\\square\end{array}$ |

Calculations are highly non-trivial!

…but everything works out.

# Taxonomy of tower hulls

(Etheredge, BH, McNamara, Rudelius, Ruiz, Valenzuela, to appear)

A. Consider a flat slice of the moduli space, within which every line goes to infinite distance

B. Assume the generating towers remain fixed across this slice

C. Assume the sharpened DC / Emergent String Conjecture

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**Classify the possible tower hulls within each slice!***

*Technical assumption: tangent space of slice cuts orthogonally through faces of SWGC hull as with, e.g., fixed plane of a symmetry

# Types of towers

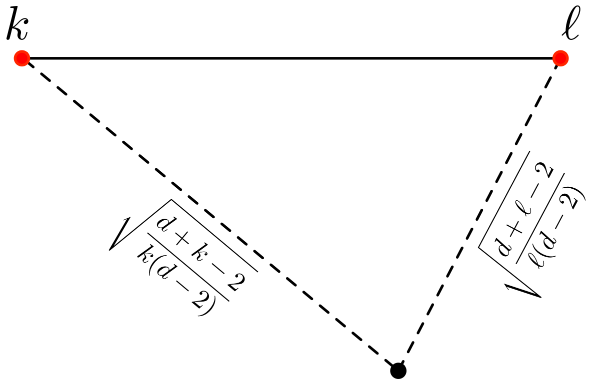
Per Emergent String Conjecture, generating towers should be either (a) KK towers or
(b) string oscillator towers

(a) $\|\zeta_{\mathrm{KK}}^{(k)}\| = \sqrt{\frac{d+k-2}{k(d-2)}}$ (decompact. to a $d+k$ dimensional **vacuum***)

(b) $\|\zeta_{\mathrm{osc}}\| = \frac{1}{\sqrt{d-2}}$ (at least for **perturbative** emergent string limits)

(*See Irene and Ignacio's talks for caveats)

# Angle between two KK towers



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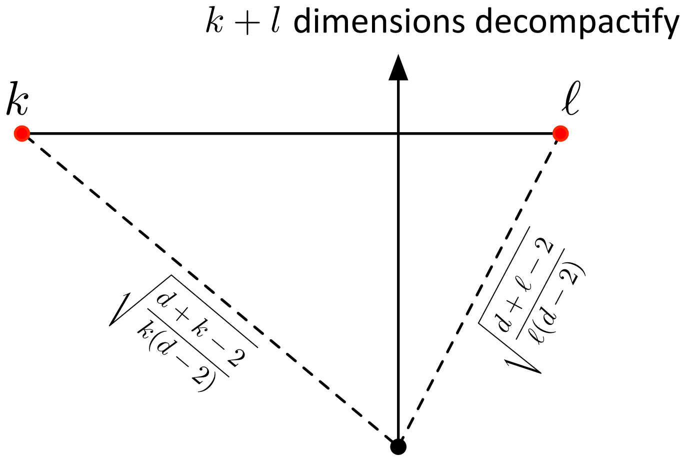

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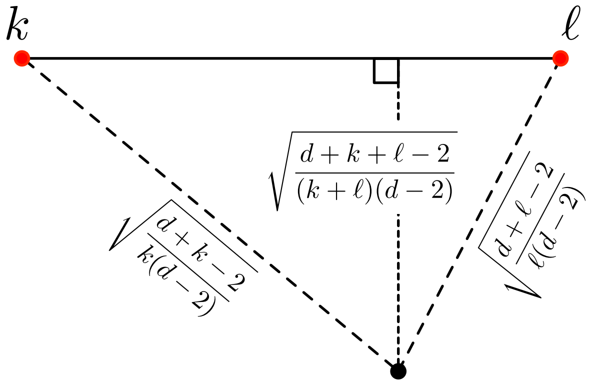

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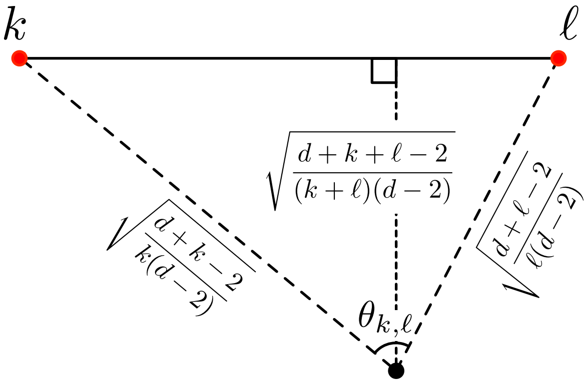
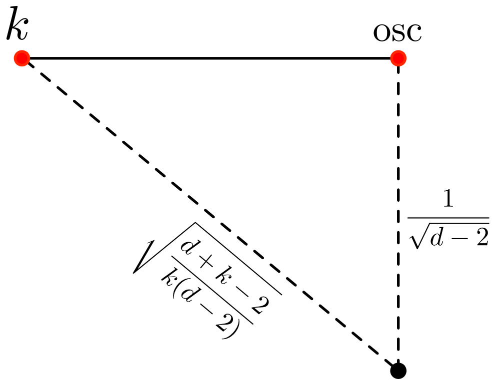


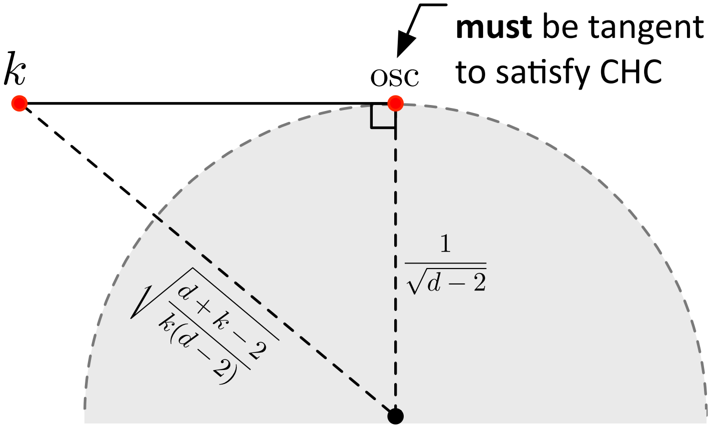

$$\cos\theta_{k,\ell} = \sqrt{\frac{k\ell}{(d+k-2)(d+\ell-2)}}$$

# Angle between KK, emergent string towers



# Angle between KK, emergent string towers

line between towers **must** be tangent to circle to satisfy CHC



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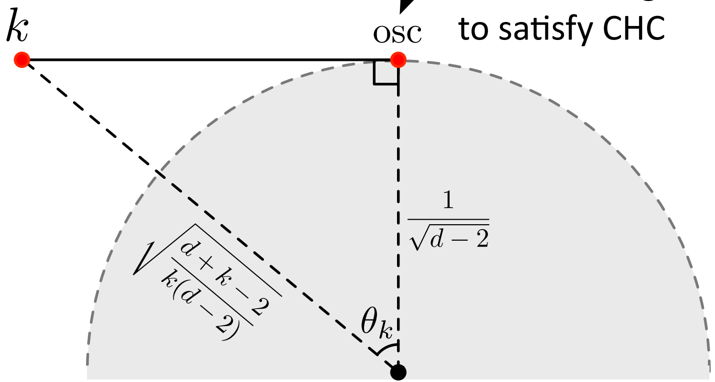
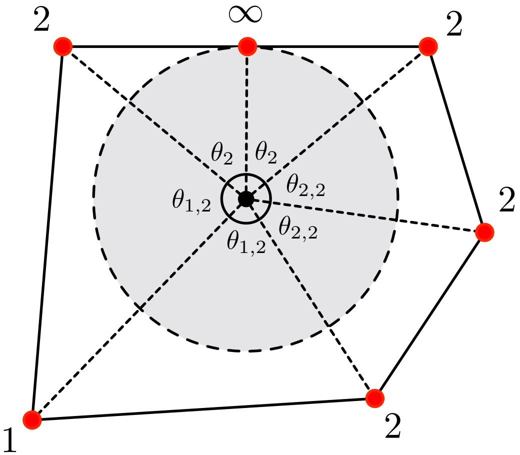



line between towers **must** be tangent to circle to satisfy CHC

$$\cos\theta_k = \sqrt{\frac{k}{d+k-2}}$$

(formally $\ell \to \infty$ limit of prev result)

# Classifying 2d slices – angle constraint




would need $2\theta_{1,2} + 2\theta_{2,2} + 2\theta_2 = 2\pi$

# Classifying 2d slices – setup

Assume $D = d + k \leq 11$ for every decompact. limit (due to supersymmetry)

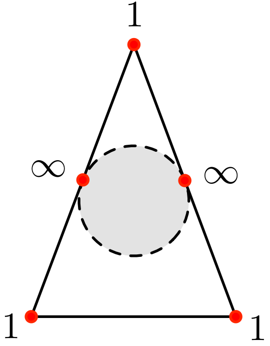
Assume $D \leq 10$ for decompact. limits adjacent to emergent string limits (no 11d strings)

(Implies flat slice has dimension $n \leq 11 - d$)

Due to angle constraint and $D$ bound, finitely many $n = 2$ flat slices possible; tabulate with a computer

# Classifying 2d slices – 9d results

In $d = 9$, only **one** option:

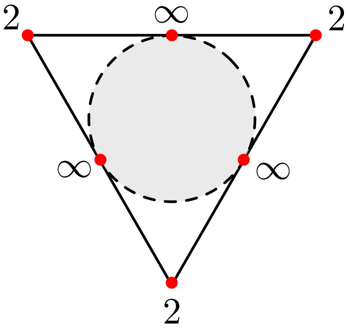



$(1, 1, \infty, 1, \infty)$

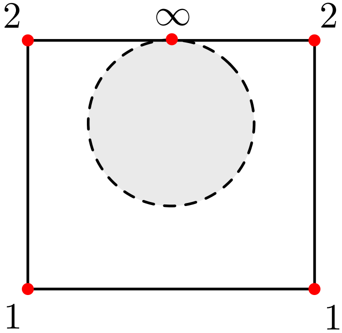
Same as maximal SUGRA!

# Classifying 2d slices – 8d results

In $d = 8$, **two** options:



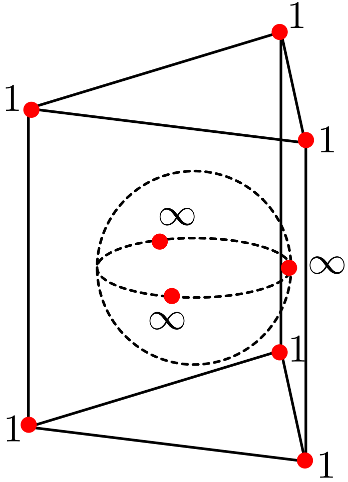

$(2, \infty, 2, \infty, 2, \infty)$




$(1, 1, 2, \infty, 2)$

# Classifying 2d slices – 8d results

Both are slices of the 8d maximal SUGRA tower hull



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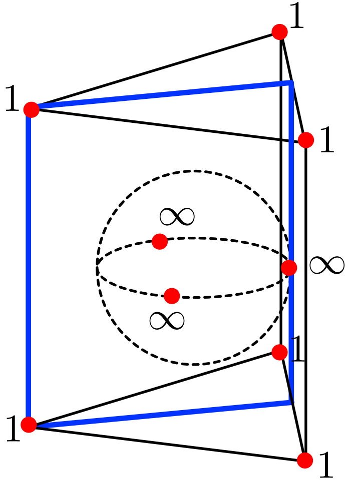

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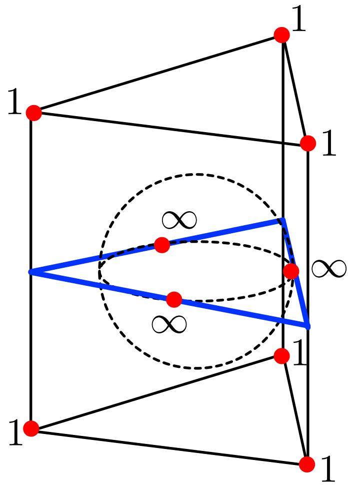
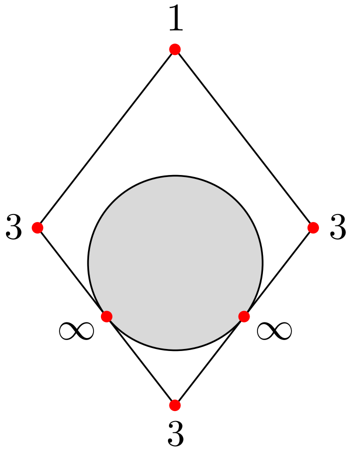
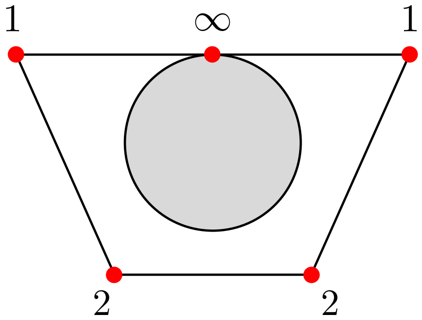


# Classifying 2d slices – 7d results

In $d = 7$, **two** options:



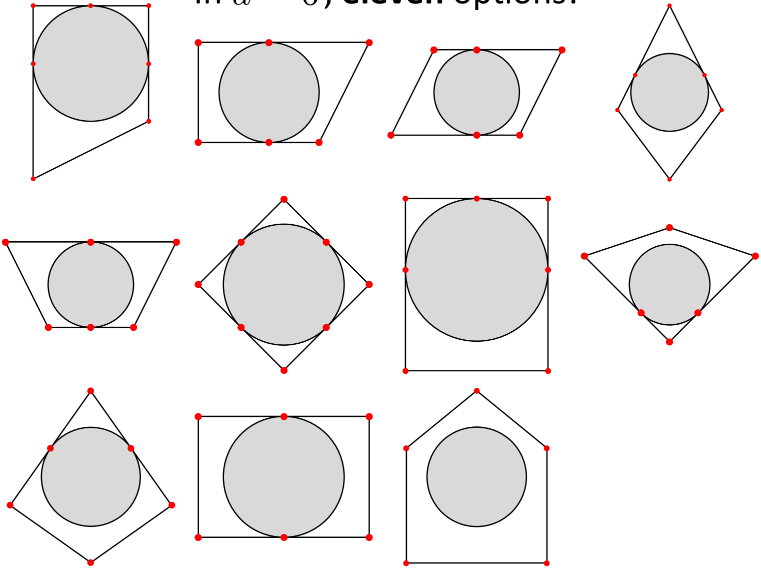

$(1, 3, \infty, 3, \infty, 3)$




$(1, 2, 2, 1, \infty)$

# Classifying 2d slices – 6d results

In $d = 6$, **eleven** options!



# Assessment

In cases we've checked, these are all orthogonal slices of maximal SUGRA hull (but could still be realized by different theories)

Need to consider $n > 2$ slices; have some results but not shown here.

There seems to be a **unique** answer for $n = 11 - d$ (except when $d = 10$), i.e., the maximal SUGRA hull

Trying to relax our assumptions and obtain more comprehensive results…

Thank you for listening!