Species and their Thermodynamics

DIETER LÜST (LMU, MPP)

Daejeon, 6th. July 2023

Species and their Thermodynamics

DIETER LÜST (LMU, MPP)

Joint work with

Niccolo Cribiori and Georgina Staudt, arXiv: 2212.10286,
Niccolo Cribiori and Carmine Montella, arXiv:2305.10489,
and with Niccolo Cribiori, arXiv:2306.08673

Daejeon, 6th. July 2023

Outline :

I) Introduction

II) Species Scale

III) Species Entropy

IV) Species Temperature

V) Species Thermodynamics

VI) Summary

Outline :

I) Introduction

II) Species Scale

III) Species Entropy

IV) Species Temperature

V) Species Thermodynamics

VI) Summary

Introduction:

Apparently Quantum Gravity is of holographic nature and also possesses thermodynamic and entropic properties.

Introduction:

Apparently Quantum Gravity is of holographic nature and also possesses thermodynamic and entropic properties.

- BH entropy of black hole geometries $\longrightarrow$ area law:

$$\mathcal{S}_{BH} \simeq (R_{BH})^{d-2} M_P^{d-2} \simeq (M_{BH})^{\frac{d-2}{d-3}} M_P^{\frac{2-d}{d-3}}$$

Introduction:

Apparently Quantum Gravity is of holographic nature and also possesses thermodynamic and entropic properties.

- BH entropy of black hole geometries ⟶ area law:

$$\mathcal{S}_{BH} \simeq (R_{BH})^{d-2} M_P^{d-2} \simeq (M_{BH})^{\frac{d-2}{d-3}} M_P^{\frac{2-d}{d-3}}$$

- Black hole decay ⟶ Hawking temperature:

$$T_{BH} \simeq (R_{BH})^{-1}$$

Introduction:

Apparently Quantum Gravity is of holographic nature and also possesses thermodynamic and entropic properties.

- BH entropy of black hole geometries ⟶ area law:

$$\mathcal{S}_{BH} \simeq (R_{BH})^{d-2} M_P^{d-2} \simeq (M_{BH})^{\frac{d-2}{d-3}} M_P^{\frac{2-d}{d-3}}$$

- Black hole decay ⟶ Hawking temperature:

$$T_{BH} \simeq (R_{BH})^{-1}$$

What is the entropy of particles species in quantum gravity ?

Is it extensive (volume law) or does it follow an area law ?

Outline :

I) Introduction

II) Species Scale

III) Species Entropy

IV) Species Temperature

V) Species Thermodynamics

VI) Summary

Species Scale :

UV cut-off $\longleftrightarrow$ „shortest“ possible length in EFT coupled to quantum gravity:

Species Scale :

UV cut-off $\longleftrightarrow$ „shortest“ possible length in EFT coupled to quantum gravity:

- Species scale: $$\Lambda_{sp} \simeq R_{sp}^{-1}$$

[G. Dvali (2007)]

$$\Lambda_{sp} \simeq \frac{M_P}{(N_{sp})^{\frac{1}{d-2}}} \Longleftrightarrow N_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$$

(Up to log-corrections)

Species Scale :

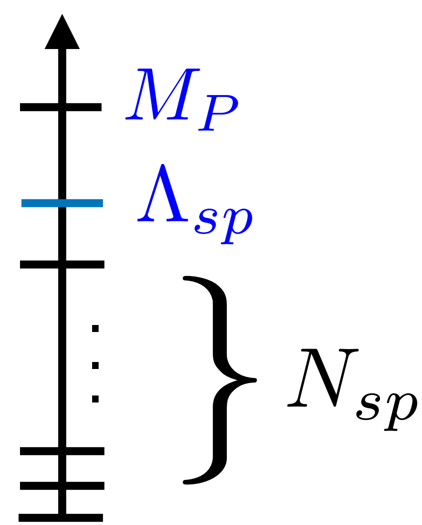
UV cut-off $\longleftrightarrow$ „shortest" possible length in EFT coupled to quantum gravity:

- Species scale: $\Lambda_{sp} \simeq R_{sp}^{-1}$

[G. Dvali (2007)]

$$\Lambda_{sp} \simeq \frac{M_P}{(N_{sp})^{\frac{1}{d-2}}} \Longleftrightarrow N_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$$

(Up to log-corrections)

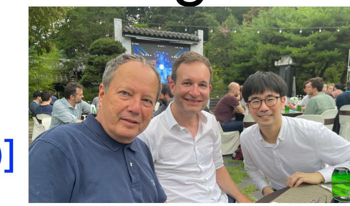


- N_{sp} : Number of particles below Λ_{sp} .

.... it depends on moduli fields: $N_{sp} = N_{sp}(\phi)$

Emergent String Conjecture :

The light tower of states, i.e. species, at large distances are given by either light string excitations or light KK modes.

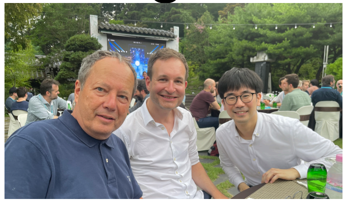


[S. Lee, W. Lerche, T. Weigand (2019)]

Emergent String Conjecture :

The light tower of states, i.e. species, at large distances are given by either light string excitations or light KK modes.

[S. Lee, W. Lerche, T. Weigand (2019)]



Geometric Species:
KK compactification with n large extra dimensions of radius R

- KK tower of species: $\Delta E = 1/R$

$$m_k = \frac{k}{R} \quad (k = 1, \ldots, N_{sp}) \qquad N_{sp} = (\Lambda_{sp} R)^n$$

- Species scale: $$\Lambda_{sp} \simeq \frac{M_P}{(M_P R)^{\frac{n}{d+n-2}}}$$

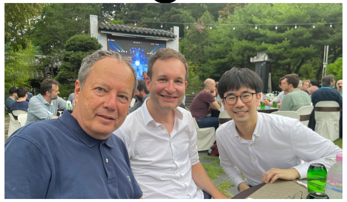
(Up to log-corrections)

- Agrees with the Planck mass/string scale in (4+n) dimensions.

Emergent String Conjecture :

The light tower of states, i.e. species, at large distances are given by either light string excitations or light KK modes.

[S. Lee, W. Lerche, T. Weigand (2019)]



Geometric Species:
KK compactification with n large extra dimensions of radius R

- KK tower of species: $\Delta E = 1/R$

$$m_k = \frac{k}{R} \quad (k = 1, \ldots, N_{sp}) \qquad N_{sp} = (\Lambda_{sp} R)^n \underset{R \to \infty}{\longrightarrow} \infty$$

- Species scale: $$\Lambda_{sp} \simeq \frac{M_P}{(M_P R)^{\frac{n}{d+n-2}}} \longrightarrow 0$$

(Up to log-corrections)

- Agrees with the Planck mass/string scale in (4+n) dimensions.

String Species :

- Regge tower: $m_k \simeq \sqrt{k} M_s$

- Effective number of species: $N_{sp} \simeq \frac{1}{g_s^2} \longrightarrow \infty$

$g_s \to 0$

- Species scale: $\Lambda_{sp} \simeq M_P \; g_s^{\frac{2}{d-2}} \longrightarrow 0$

(Up to log-corrections)

[G. Dvali, D.L. (2009);
G. Dvali, C. Gomez (2010)]

- Agrees with the string scale M_s in d dimensions.

Outline :

I) Introduction

II) Species Scale

III) Species Entropy

IV) Species Temperature

V) Species Thermodynamics

VI) Summary

Species Entropy:

Here we propose the following generalisation of the definition of the species scale:

Species Entropy:

Here we propose the following generalisation of the definition of the species scale:

- Replace the number of species N_{sp} by the species entropy $\mathcal{S}_{sp}$:

$$\Lambda_{sp} = \frac{M_P}{\mathcal{S}_{sp}^{\frac{1}{d-2}}}$$

Species Entropy:

Here we propose the following generalisation of the definition of the species scale:

- Replace the number of species N_{sp} by the species entropy $\mathcal{S}_{sp}$:

$$\Lambda_{sp} = \frac{M_P}{\mathcal{S}_{sp}^{\frac{1}{d-2}}}$$

- In other words: definition of species entropy:

$$\boxed{\mathcal{S}_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}}$$

Species Entropy:

Here we propose the following generalisation of the definition of the species scale:

- Replace the number of species N_{sp} by the species entropy $\mathcal{S}_{sp}$:

$$\Lambda_{sp} = \frac{M_P}{\mathcal{S}_{sp}^{\frac{1}{d-2}}}$$

- In other words: definition of species entropy:

$$\boxed{\mathcal{S}_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}}$$

- We will also introduce a species temperature ⟶

  Species thermodynamics:

  Determines the motion of species properties in moduli space.

Two arguments for the species entropy
A) Perturbative: one loop propagator

Consider N_0 massless (light) species:

Two arguments for the species entropy
A) Perturbative: one loop propagator

Consider N_0 massless (light) species:

- One loop graviton propagator:

$$G^{-1}(p^2) \simeq p^2 \left(1 - N_0 \left(\frac{p^2}{M_P^2}\right)^{\frac{d-2}{2}} \log\left(-\frac{p^2}{\mu^2}\right)\right)$$

[X. Calmet, R. Casadio, A. Kamenshchik, O. Teryaev (2017)]

Two arguments for the species entropy
A) Perturbative: one loop propagator

Consider N_0 massless (light) species:

- One loop graviton propagator:

$$G^{-1}(p^2) \simeq p^2 \left(1 - N_0 \left(\frac{p^2}{M_P^2}\right)^{\frac{d-2}{2}} \log\left(-\frac{p^2}{\mu^2}\right)\right)$$

[X. Calmet, R. Casadio, A. Kamenshchik, O. Teryaev (2017)]

- Species scale: Gravity becomes strong: $G^{-1}(p^2 = \Lambda_{sp}^2) = 0$

$$\Lambda_{sp}^{d-2} \simeq -(d-2)\frac{M_P^{d-2}}{N_0}\left[W_{-1}\left(-\frac{d-2}{N_0}\left(\frac{M_P}{\mu}\right)^{d-2}\right)\right]^{-1}$$

[A. Castellano, A. Herraez, L. Ibanez (2022)]

$$\underset{N_0 \to \infty}{\simeq} \frac{M_P^{d-2}}{N_0 \log N_0} \simeq \frac{M_P^{d-2}}{\log N_0!}$$

Two arguments for the species entropy
A) Perturbative: one loop propagator

Consider N_0 massless (light) species:

- One loop graviton propagator:

$$G^{-1}(p^2) \simeq p^2 \left(1 - N_0 \left(\frac{p^2}{M_P^2}\right)^{\frac{d-2}{2}} \log\left(-\frac{p^2}{\mu^2}\right)\right)$$

[X. Calmet, R. Casadio, A. Kamenshchik, O. Teryaev (2017)]

- Species scale: Gravity becomes strong: $G^{-1}(p^2 = \Lambda_{sp}^2) = 0$

$$\Lambda_{sp}^{d-2} \simeq -(d-2)\frac{M_P^{d-2}}{N_0}\left[W_{-1}\left(-\frac{d-2}{N_0}\left(\frac{M_P}{\mu}\right)^{d-2}\right)\right]^{-1}$$

[A. Castellano, A. Herraez, L. Ibanez (2022)]

$$\underset{N_0 \to \infty}{\simeq} \frac{M_P^{d-2}}{N_0 \log N_0} \simeq \frac{M_P^{d-2}}{\log N_0!}$$

- Reminiscent of entropy formula $\mathcal{S}_{sp} \simeq N_0 \log N_0 \simeq \log N_0!$

Two arguments for the species entropy
A) Perturbative: one loop propagator

Consider N_0 massless (light) species:

- One loop graviton propagator:

$$G^{-1}(p^2) \simeq p^2 \left(1 - N_0 \left(\frac{p^2}{M_P^2}\right)^{\frac{d-2}{2}} \log\left(-\frac{p^2}{\mu^2}\right)\right)$$

[X. Calmet, R. Casadio, A. Kamenshchik, O. Teryaev (2017)]

- Species scale: Gravity becomes strong: $G^{-1}(p^2 = \Lambda_{sp}^2) = 0$

$$\Lambda_{sp}^{d-2} \simeq -(d-2)\frac{M_P^{d-2}}{N_0}\left[W_{-1}\left(-\frac{d-2}{N_0}\left(\frac{M_P}{\mu}\right)^{d-2}\right)\right]^{-1}$$

[A. Castellano, A. Herraez, L. Ibanez (2022)]

$$\underset{N_0 \to \infty}{\simeq} \frac{M_P^{d-2}}{N_0 \log N_0} \simeq \frac{M_P^{d-2}}{\log N_0!}$$

- Reminiscent of entropy formula $\mathcal{S}_{sp} \simeq N_0 \log N_0 \simeq \log N_0!$

Note that there are no multiplicative log corrections for towers with increasing masses, like KK tower !

B) Non-perturbative argument: minimal BHs

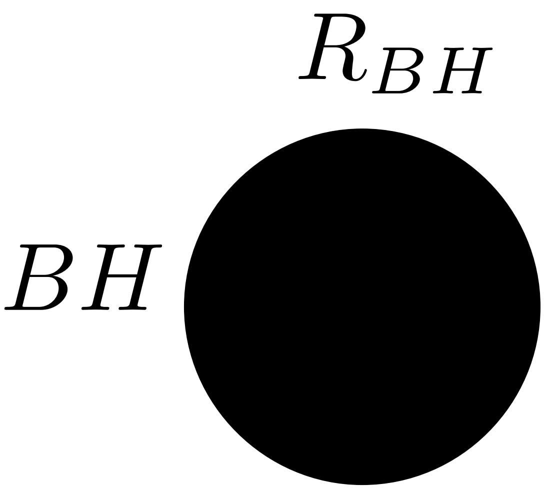
Recall: $$\mathcal{S}_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$$

Species entropy is identical to the BH-entropy of a minimal BH of size R_{sp}

B) Non-perturbative argument: minimal BHs

Recall: $$\mathcal{S}_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$$

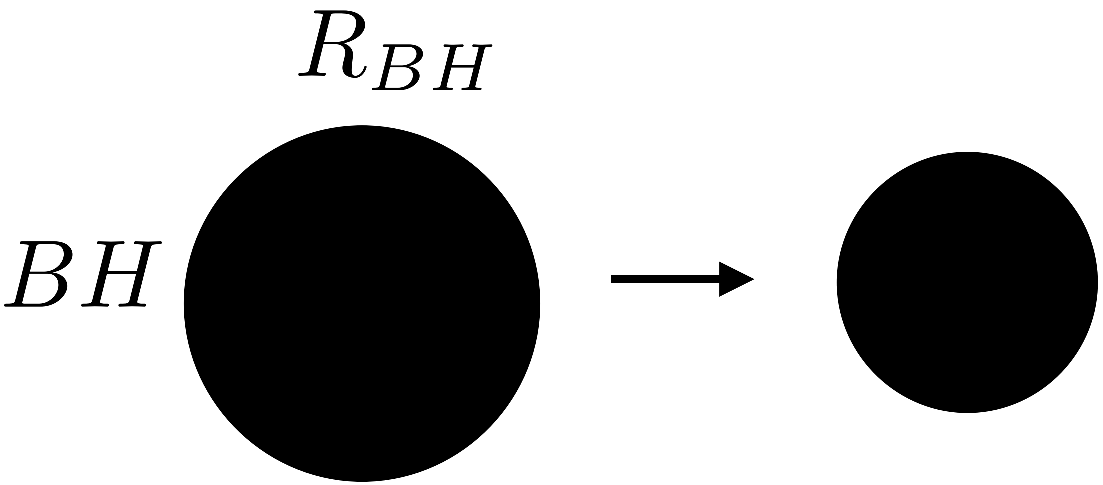
Species entropy is identical to the BH-entropy of a minimal BH of size R_{sp}



B) Non-perturbative argument: minimal BHs

Recall: $$\mathcal{S}_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$$

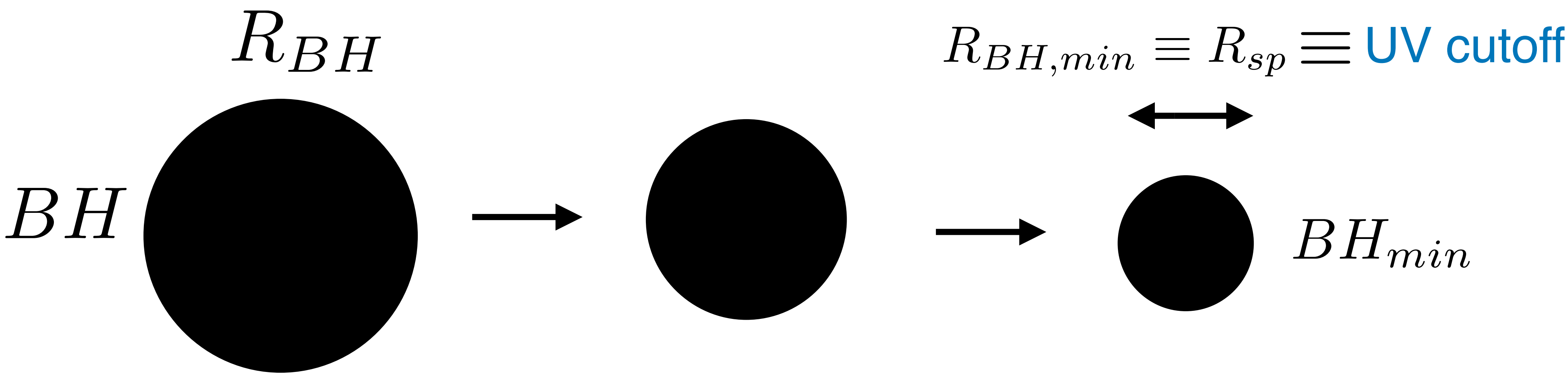
Species entropy is identical to the BH-entropy of a minimal BH of size R_{sp}



B) Non-perturbative argument: minimal BHs

Recall: $$S_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$$

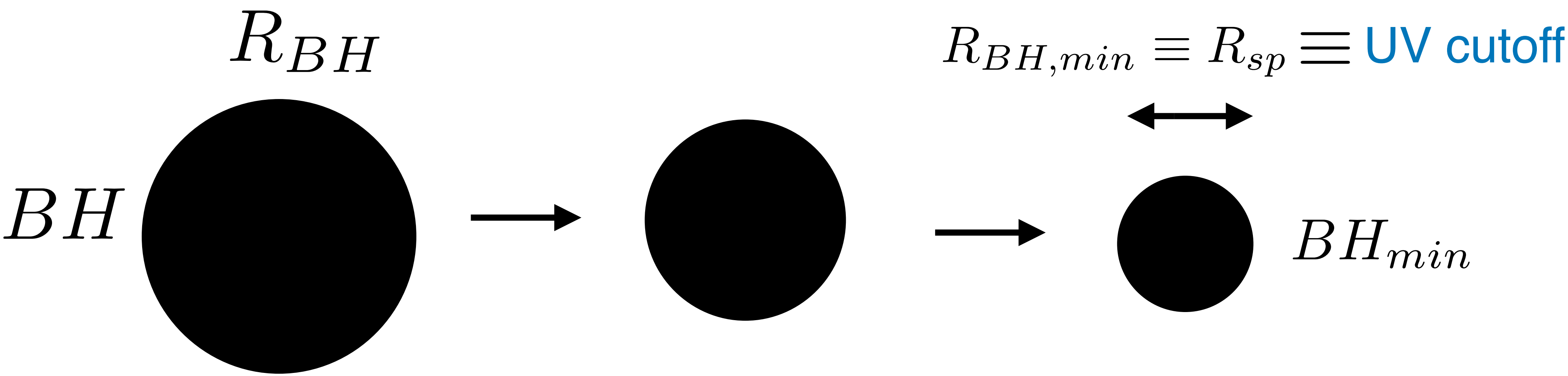
Species entropy is identical to the BH-entropy of a minimal BH of size R_{sp}



B) Non-perturbative argument: minimal BHs

Recall: $\mathcal{S}_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$

Species entropy is identical to the BH-entropy of a minimal BH of size R_{sp}



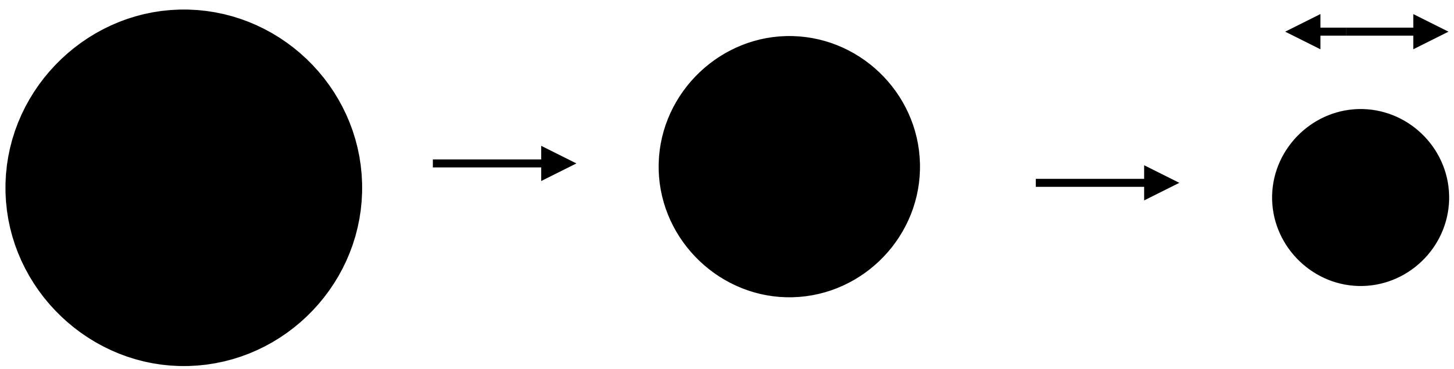
For any BH in the EFT: $R_{BH} \geq R_{sp}$ and $\mathcal{S}_{BH} \geq \mathcal{S}_{sp}$

B) Non-perturbative argument: minimal BHs

Recall: $\mathcal{S}_{sp} \simeq (R_{sp})^{d-2} M_P^{d-2}$

Species entropy is identical to the BH-entropy of a minimal BH of size R_{sp}

R_{BH} $\qquad$ $R_{BH,min} \equiv R_{sp} \equiv$ UV cutoff

BH  BH_{min}

For any BH in the EFT: $R_{BH} \geq R_{sp}$ and $\mathcal{S}_{BH} \geq \mathcal{S}_{sp}$

Dual description:	Particles	-	Geometry
	Species	-	Minimal BH as species bound state

This picture is closely related to the BH entropy conjecture:

In the limit $S_{BH} \to \infty$ there is a tower of light states with masses

$$m = \left(\frac{1}{S_{BH}}\right)^{\gamma}, \quad \gamma > 0$$

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

This picture is closely related to the BH entropy conjecture:

In the limit $\mathcal{S}_{BH} \to \infty$ there is a tower of light states with masses

$$m = \left(\frac{1}{\mathcal{S}_{BH}}\right)^{\gamma}, \quad \gamma > 0$$

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

Smallest possible BH with minimal entropy → tower is given in terms of species and the BH entropy becomes the species entropy $\mathcal{S}_{sp}$.

This picture is closely related to the BH entropy conjecture:

In the limit $\mathcal{S}_{BH} \to \infty$ there is a tower of light states with masses

$$m = \left(\frac{1}{\mathcal{S}_{BH}}\right)^{\gamma}, \quad \gamma > 0$$

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

Smallest possible BH with minimal entropy ⟶ tower is given in terms of species

and the BH entropy becomes the species entropy $\mathcal{S}_{sp}$.

Large species entropy limit:

$$\mathcal{S}_{sp} \to \infty \quad \Rightarrow \quad \mathcal{S}_{sp} \simeq N_{sp}$$

Construct minimal N=2 black holes:

[N. Cribiori, D.L., G. Staudt (2022)]

Construct minimal N=2 black holes:

[N. Cribiori, D.L., G. Staudt (2022)]

Charged N=2 Black holes that probe string states:

Heterotic dyonic black holes with charges p and q:

$$\mathcal{S} = 8\pi^2 pq \,, \qquad (g_s)^{-2} = \frac{q}{p}$$

[M. Cvetic, D. Youm(1995); M. Cvetic, A. Tseytlin (1995)]

String coupling at BH Horizon:

Construct minimal N=2 black holes:

[N. Cribiori, D.L., G. Staudt (2022)]

Charged N=2 Black holes that probe string states:

Heterotic dyonic black holes with charges p and q:

$$\mathcal{S} = 8\pi^2 pq\,, \qquad (g_s)^{-2} = \frac{q}{p}$$

[M. Cvetic, D. Youm(1995); M. Cvetic, A. Tseytlin (1995)]

String coupling at BH Horizon:

Smallest possible, i.e. minimal black hole:

$$p = 1 \quad \Longrightarrow \quad \mathcal{S}_{sp} = 8\pi^2 (g_s)^{-2} = N_{sp}$$

Construct minimal N=2 black holes:

[N. Cribiori, D.L., G. Staudt (2022)]

Charged N=2 Black holes that probe string states:

Heterotic dyonic black holes with charges p and q:

[M. Cvetic, D. Youm(1995); M. Cvetic, A. Tseytlin (1995)]

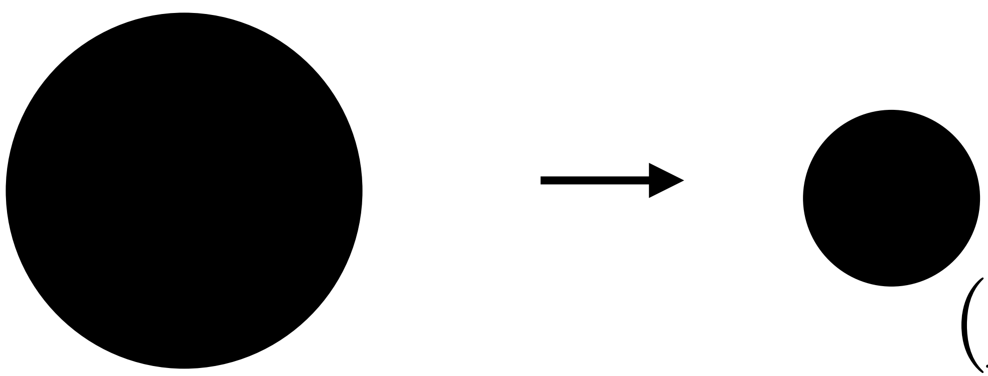
$$\mathcal{S} = 8\pi^2 pq\,, \qquad (g_s)^{-2} = \frac{q}{p}$$

String coupling at BH Horizon:

Smallest possible, i.e. minimal black hole:

$$p = 1 \quad \Longrightarrow \quad \mathcal{S}_{sp} = 8\pi^2 (g_s)^{-2} = N_{sp}$$

$BH:\ (p,q)$



$BH_{min}:$

$(p = 1, q = 1/g_s^2)$

Charged N=2 black holes that probes KK modes:

IIA, N=2 Calabi-Yau compactification (D0/D4/D4/D4 system):

$$\mathcal{S} = 2\pi\sqrt{\frac{q}{6}C_{ijk}p^ip^jp^k}$$

[K. Behrndt, G. Lopes Cardoso, B. De Wit, R. Kallosh, D.L. , T. Mohaupt (1996)]

Volume (at horizon): $$\mathcal{V} = 2\pi\sqrt{\frac{6q^3}{C_{ijk}p^ip^jp^k}}$$

Charged N=2 black holes that probes KK modes:

IIA, N=2 Calabi-Yau compactification (D0/D4/D4/D4 system):

$$\mathcal{S} = 2\pi\sqrt{\frac{q}{6}C_{ijk}p^i p^j p^k}$$

[K. Behrndt, G. Lopes Cardoso, B. De Wit, R. Kallosh, D.L. , T. Mohaupt (1996)]

Volume (at horizon): $\mathcal{V} = 2\pi\sqrt{\frac{6q^3}{C_{ijk}p^i p^j p^k}}$

Minimal BH:

$$\frac{1}{6}C_{ijk}p^i p^j p^k = 1 \quad \Rightarrow \quad \mathcal{S}_{sp} = 2\pi\mathcal{V}^{1/3} = N_{sp}$$

$\longrightarrow$ Minimal BH probes KK tower of two-cycle, minimal cycle on a CY.

Charged N=2 black holes that probes KK modes:

IIA, N=2 Calabi-Yau compactification (D0/D4/D4/D4 system):

$$\mathcal{S} = 2\pi\sqrt{\frac{q}{6}C_{ijk}p^i p^j p^k}$$

[K. Behrndt, G. Lopes Cardoso, B. De Wit, R. Kallosh, D.L. , T. Mohaupt (1996)]

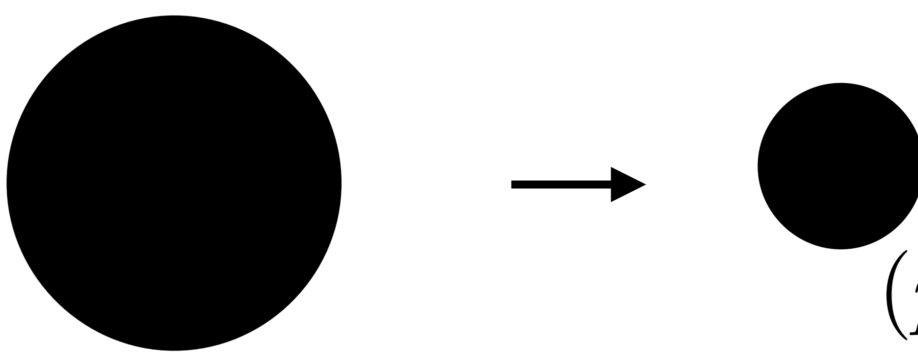
Volume (at horizon): $\mathcal{V} = 2\pi\sqrt{\frac{6q^3}{C_{ijk}p^i p^j p^k}}$

Minimal BH:

$$\frac{1}{6}C_{ijk}p^i p^j p^k = 1 \quad \Rightarrow \quad \mathcal{S}_{sp} = 2\pi\mathcal{V}^{1/3} = N_{sp}$$

→ Minimal BH probes KK tower of two-cycle, minimal cycle on a CY.

$BH: \ (p,q)$



$BH_{min}:$

$(p^i = 1, q = \mathcal{V}^{2/3})$

Higher derivative corrections - relation to topological string

Consider R^2 corrections to EFT, which are determined by second Chern class.

$$\mathcal{S} = 2\pi\sqrt{\frac{1}{6} q \left(C_{ijk} p^i p^j p^k + c_{2i} p^i\right)}$$

[J. Maldacena, A. Strominger, E. Witten (1997)]
[G. Cardoso, B. de Wit, T. Mohaupt (1999)]

Higher derivative corrections - relation to topological string

Consider R^2 corrections to EFT, which are determined by second Chern class.

$$\mathcal{S} = 2\pi\sqrt{\frac{1}{6}q\left(C_{ijk}p^ip^jp^k + c_{2i}p^i\right)}$$

[J. Maldacena, A. Strominger, E.Witten (1997)]
[G. Cardoso, B. de Wit, T. Mohaupt (1999)]

Minimal entropy: $C_{ijk}p^ip^jp^k = 0 \quad (p^j = 0\,,\, j \neq i)$

[N. Cribiori, D.L., G. Staudt (2022)]

$$\mathcal{S}_{sp} = 2\pi\sqrt{\frac{1}{6}c_{2i}p^i} = 2\pi F_1$$

This is the so-called topological free energy of the topological string and it agrees with the earlier result of D. van de Heisteeg, C.Vafa, M.Wiesner, D.Wu (2022)

Higher derivative corrections - relation to topological string

Consider R^2 corrections to EFT, which are determined by second Chern class.

$$\mathcal{S} = 2\pi\sqrt{\frac{1}{6}q\left(C_{ijk}p^ip^jp^k + c_{2i}p^i\right)}$$

[J. Maldacena, A. Strominger, E. Witten (1997)]
[G. Cardoso, B. de Wit, T. Mohaupt (1999)]

Minimal entropy: $C_{ijk}p^ip^jp^k = 0 \quad (p^j = 0\,,\; j \neq i)$

[N. Cribiori, D.L., G. Staudt (2022)]

$$\mathcal{S}_{sp} = 2\pi\sqrt{\frac{1}{6}c_{2i}p^i} = 2\pi F_1$$

This is the so-called topological free energy of the topological string and it agrees with the earlier result of D. van de Heisteeg, C. Vafa, M. Wiesner, D. Wu (2022)

Note that in the large volume limit one gets $\mathcal{S}_{sp} \simeq c_{2i}\mathrm{Im}\; t^i$

Modular invariant species entropy

[N. Cribiori, D.L. (2023)]

Consider heterotic string on $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ and include also the winding modes on the torus.

Use calculation of fermionic free energy

[S. Ferrara, C. Kounnas, D.L., F. Zwirner (1991)]

$$\mathcal{S}_{sp} \simeq F_1 \simeq \log\det M^\dagger M \simeq \sum_{i=1}^{h^{11}} \sum_{(m,n)\neq(0,0)} \log \frac{|m_i + n_i T_i|^2}{-i(T_i - \bar{T}_i)}$$

$$\mathcal{S}_{sp} \simeq -\log\left[(-i(T-\bar{T})^3 |\eta(T)|^{12}\right] \xrightarrow[\mathrm{Im}T \to \infty]{} \mathcal{V}_6^{\frac{1}{3}} - 3\log\mathcal{V}_6^{\frac{1}{3}}$$

Modular invariant species entropy

[N. Cribiori, D.L. (2023]

Consider heterotic string on $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ and include also the winding modes on the torus.

Use calculation of fermionic free energy

[S. Ferrara, C. Kounnas, D.L., F. Zwirner (1991]

$$\mathcal{S}_{sp} \simeq F_1 \simeq \log\det M^\dagger M \simeq \sum_{i=1}^{h^{11}} \sum_{(m,n)\neq(0,0)} \log \frac{|m_i + n_i T_i|^2}{-i(T_i - \bar{T}_i)}$$

$$\mathcal{S}_{sp} \simeq -\log\left[(-i(T-\bar{T})^3|\eta(T)|^{12}\right] \xrightarrow[\mathrm{Im}T \to \infty]{} \mathcal{V}_6^{\frac{1}{3}} - 3\log\mathcal{V}_6^{\frac{1}{3}}$$

Additive log-correction: $\Lambda_{sp} = \frac{M_P}{\sqrt{\mathcal{S}_{sp}}} > \frac{M_P}{\mathcal{V}_6^{\frac{1}{6}}} = M_s$

Modular invariant species entropy

[N. Cribiori, D.L. (2023)]

Consider heterotic string on $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ and include also the winding modes on the torus.

Use calculation of fermionic free energy

[S. Ferrara, C. Kounnas, D.L., F. Zwirner (1991]

$$\mathcal{S}_{sp} \simeq F_1 \simeq \log\det M^\dagger M \simeq \sum_{i=1}^{h^{11}} \sum_{(m,n)\neq(0,0)} \log \frac{|m_i + n_i T_i|^2}{-i(T_i - \bar{T}_i)}$$

$$\mathcal{S}_{sp} \simeq -\log\left[(-i(T-\bar{T})^3 |\eta(T)|^{12}\right] \xrightarrow[\mathrm{Im}T \to \infty]{} \mathcal{V}_6^{\frac{1}{3}} - 3\log \mathcal{V}_6^{\frac{1}{3}}$$

Additive log-correction:

$$\Lambda_{sp} = \frac{M_P}{\sqrt{\mathcal{S}_{sp}}} > \frac{M_P}{\mathcal{V}_6^{\frac{1}{6}}} = M_s$$

Most general expression:

[See also: M.Cvetic, A. Font, L. Ibanez, D.L., F. Quevedo (1991]

$$\mathcal{S}_{sp} \simeq -\log\left[(-i(T-\bar{T})^3 |\eta(T)|^{12} H(T)\right]$$

$$H(T) = \left(\frac{G_6(T)}{\eta(T)^{12}}\right)^m \left(\frac{G_4(T)}{\eta(T)^8}\right)^n \mathcal{P}(j) = (j - 12^3)^{\frac{m}{2}} j^{\frac{n}{3}} \mathcal{P}(j)$$

UV - IR mixing and entropy bound on species entropy:

[Recent related work by D. Andriot (2023); D. Van de Heisteg, C. Vafa, M. Wiesner (2023)]

UV $\longleftrightarrow$ IR

Mixing

$$\Lambda_{UV} \longleftrightarrow \Lambda_{IR}$$

[See also: A. Castellano, A. Herraez, L. Ibanez (2021); J. Calderon-Infante, A. Castellano, A. Herraez, L. Ibanez (2023)]

UV - IR mixing and entropy bound on species entropy:

[Recent related work by D.Andriot (2023); D.Van de Heisteg, C.Vafa, M.Wiesner (2023)]

UV $\longleftrightarrow$ IR

Mixing

$$\Lambda_{UV} \longleftrightarrow \Lambda_{IR}$$

[See also: A. Castellano, A. Herraez, L. Ibanez (2021); J. Calderon-Infante, A. Castellano, A. Herraez, L. Ibanez (2023)]

Λ_{UV} : species scale - UV entropy: species entropy $\mathcal{S}_{sp}$

UV - IR mixing and entropy bound on species entropy:

[Recent related work by D. Andriot (2023); D. Van de Heisteg, C. Vafa, M. Wiesner (2023)]

UV $\longleftrightarrow$ IR

Mixing

$$\Lambda_{UV} \longleftrightarrow \Lambda_{IR}$$

[See also: A. Castellano, A. Herraez, L. Ibanez (2021); J. Calderon-Infante, A. Castellano, A. Herraez, L. Ibanez (2023)]

Λ_{UV} : species scale - UV entropy: species entropy $\mathcal{S}_{sp}$

Λ_{IR} : cosmological constant - IR entropy: Gibbons-Hawking entropy

$$\mathcal{S}_{GH} = L_{cc}^{d-2} M_P^{d-2} = \Lambda_{cc}^{-\frac{d-2}{2}} M_P^{2(d-2)}$$

Hubble radius $\quad L_{cc}^2 = \Lambda_{cc}^{-1} M_P^2 \qquad \Lambda_{cc} \sim 10^{-122} M_P^4$

UV - IR mixing and entropy bound on species entropy:

[Recent related work by D.Andriot (2023); D.Van de Heisteg, C.Vafa, M.Wiesner (2023)]

UV ⟷ IR

Mixing

$$\Lambda_{UV} \longleftrightarrow \Lambda_{IR}$$

[See also: A. Castellano, A. Herraez, L. Ibanez (2021); J. Calderon-Infante, A. Castellano, A. Herraez, L. Ibanez (2023)]

Λ_{UV} : species scale - UV entropy: species entropy $\mathcal{S}_{sp}$

Λ_{IR} : cosmological constant - IR entropy: Gibbons-Hawking entropy

$$\mathcal{S}_{GH} = L_{cc}^{d-2} M_P^{d-2} = \Lambda_{cc}^{-\frac{d-2}{2}} M_P^{2(d-2)}$$

Hubble radius $L_{cc}^2 = \Lambda_{cc}^{-1} M_P^2$ $\quad \Lambda_{cc} \sim 10^{-122} M_P^4$

Covariant entropy bound: $\mathcal{S}_{sp} \leq \mathcal{S}_{GH}$

[R. Bousso (1999)]

Combine the Bousso bound with the anti-de Sitter conjecture:

In the limit of small cosmological constant there is a light tower of states with mass scale

$$m \simeq \Lambda_{cc}^{\alpha} M_P^{1-4\alpha}$$

[E. Palti, C.Vafa, D.L. (2019)]

Combine the Bousso bound with the anti-de Sitter conjecture:

In the limit of small cosmological constant there is a light tower of states with mass scale

$$m \simeq \Lambda_{cc}^{\alpha} M_P^{1-4\alpha}$$

[E. Palti, C. Vafa, D.L. (2019)]

For KK species tower with $m_{KK} = 1/R$

we get that

$$\frac{\mathcal{S}_{GH}}{\mathcal{S}_{sp}} = (RM_P)^{\frac{(D-2-2n\alpha)(d-2)}{(D-2)2\alpha}} \geq 1$$

$$\alpha \leq \frac{1}{2} + \frac{d-2}{2n}$$

If true for all d $\alpha \leq \frac{1}{2}$

This agrees with the Higuchi bound.

Outline :

I) Introduction

II) Species Scale

III) Species Entropy

IV) Species Temperature

V) Species Thermodynamics

VI) Summary

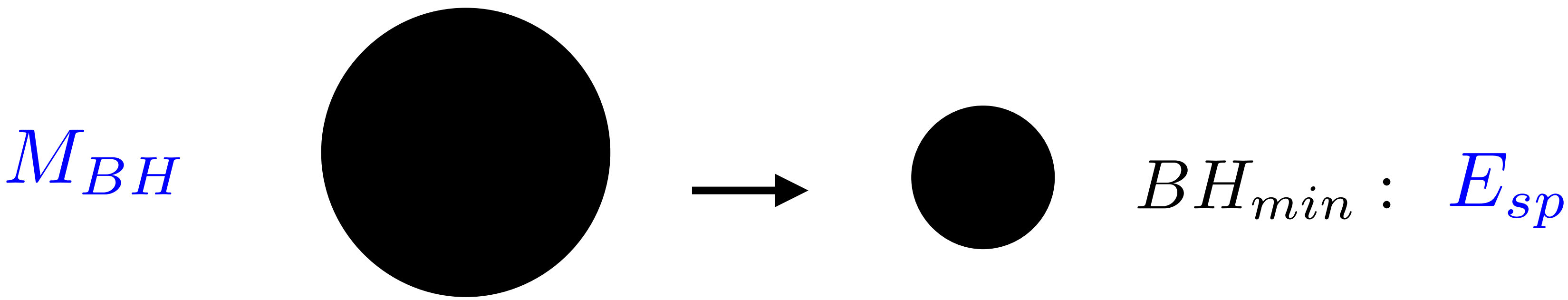
Species temperature:

General thermodynamic relation: $$\frac{1}{T} = \frac{\partial \mathcal{S}}{\partial E}$$

Species temperature:

General thermodynamic relation: $$\frac{1}{T} = \frac{\partial \mathcal{S}}{\partial E}$$

Energy of species: mass of minimal black hole.

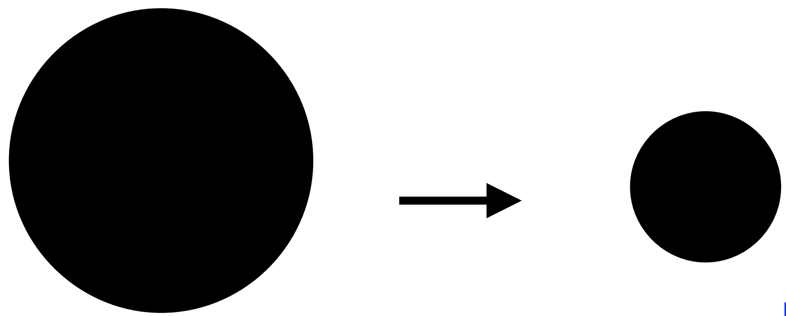


Species temperature:

General thermodynamic relation: $$\frac{1}{T} = \frac{\partial \mathcal{S}}{\partial E}$$

Energy of species: mass of minimal black hole.

M_{BH}



$BH_{min} : \; E_{sp}$

[Compare with G. Dvali, C. Gomez (2011)]

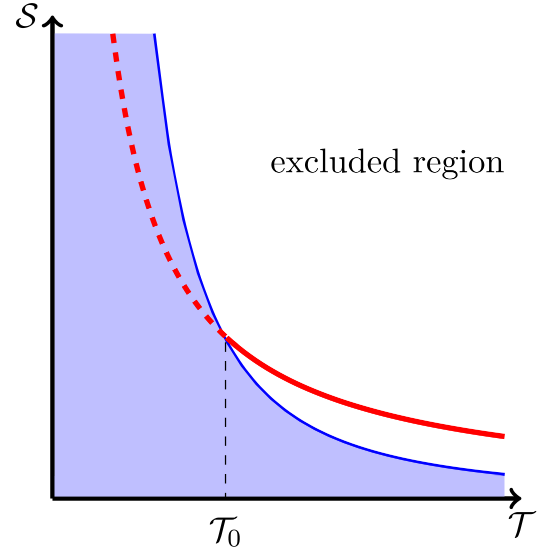
KK tower: Minimal BH can be considered as bound state of KK modes

$$E_k = \frac{k}{R} \qquad E_{sp} = \sum_{k=1}^{N_s} E_k \simeq \left(\mathcal{S}_{sp}\right)^{\frac{d-3}{d-2}}$$

$$\boxed{T_{sp} = \frac{1}{\mathcal{S}_{sp}^{\frac{1}{d-2}}} \equiv \Lambda_s}$$

This indeed agrees with the temperature of a minimal Schwarzschild BH.

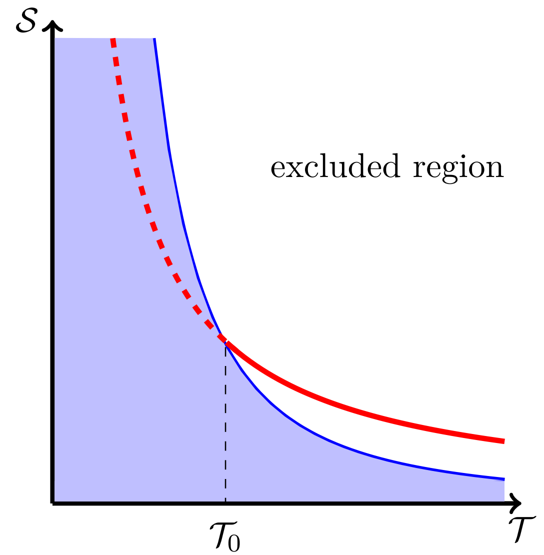
Temperature of (p,q) charged N=2, non-BPS BHs:



$$\mathcal{S}_{BH}^3 \simeq \left(\sqrt{\mathcal{V}^{1/3} p + \mathcal{S}_{BH}^3 T_{BH}^2} + \mathcal{S}_{BH} T_{BH}\right)^6$$

[N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

Temperature of (p,q) charged N=2, non-BPS BHs:



$$\mathcal{S}_{BH}^3 \simeq \left(\sqrt{\mathcal{V}^{1/3}p + \mathcal{S}_{BH}^3 T_{BH}^2} + \mathcal{S}_{BH} T_{BH}\right)^6$$

[N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

Minimal BH: $p = 1$

$$\mathcal{S}_{sp}^3 \simeq \left(\sqrt{\mathcal{V}^{1/3} + \mathcal{S}_{sp}^3 T_{sp}^2} + \mathcal{S}_{sp} T_{sp}\right)^6$$

$\mathcal{S}_{sp} \to \infty$ Again $T_{sp} = \frac{1}{\mathcal{S}_{sp}^{\frac{1}{d-2}}} \equiv \Lambda_s$

Outline :

I) Introduction

II) Species Scale

III) Species Entropy

IV) Species Temperature

V) Species Thermodynamics

VI) Summary

Species thermodynamics :

Time corresponds to a modulus ϕ that is moving adiabatically along a geodesics in moduli space:

$$\delta_\phi \mathcal{S}_{sp}(\phi)\,,\ \delta_\phi T_{sp}(\phi)$$

Cosmological string backgrounds: $t \equiv \phi$

Species thermodynamics :

Time corresponds to a modulus ϕ that is moving adiabatically along a geodesics in moduli space:

$$\delta_\phi \mathcal{S}_{sp}(\phi)\,,\ \delta_\phi T_{sp}(\phi)$$

Cosmological string backgrounds: $t \equiv \phi$

- Zeroth law of species thermodynamics:

Points in the moduli space with the same cutoff Λ_{sp} have the same temperature T_{sp} .

Species thermodynamics :

Time corresponds to a modulus ϕ that is moving adiabatically along a geodesics in moduli space:

$$\delta_\phi \mathcal{S}_{sp}(\phi)\,,\ \delta_\phi T_{sp}(\phi)$$

Cosmological string backgrounds: $t \equiv \phi$

- Zeroth law of species thermodynamics:

Points in the moduli space with the same cutoff Λ_{sp} have the same temperature T_{sp} .

- First law of species thermodynamics:

Any two neighbouring species towers are related by

$$\delta E_{sp} = T_{sp}\delta\mathcal{S}_{sp} + \ldots$$

Laws of species thermodynamics :

- Second law of species thermodynamics:

The species entropy does not increase when moving adiabatically towards the boundary of the moduli space:

$$\delta\Lambda_{sp}(\phi) \leq 0\,, \qquad \delta\mathcal{S}_{sp}(\phi) \geq 0$$

Laws of species thermodynamics :

- Second law of species thermodynamics:

The species entropy does not increase when moving adiabatically towards the boundary of the moduli space:

$$\delta\Lambda_{sp}(\phi) \leq 0\,, \qquad \delta\mathcal{S}_{sp}(\phi) \geq 0$$

It provides a preferred direction in moduli space: Movement towards its boundary

.. towards large volume, towards weak coupling

Laws of species thermodynamics :

- Second law of species thermodynamics:

The species entropy does not increase when moving adiabatically towards the boundary of the moduli space:

$$\delta\Lambda_{sp}(\phi) \leq 0\,, \qquad \delta\mathcal{S}_{sp}(\phi) \geq 0$$

It provides a preferred direction in moduli space: Movement towards its boundary

.. towards large volume, towards weak coupling

Two towers are coalescing:

$$\Lambda_{s_1+s_2} < \min(\Lambda_{s_1}, \Lambda_{s_2})$$

Laws of species thermodynamics :

- Third law of species thermodynamics:

It is impossible to reach

$$T_{sp}(\phi) = 0 \quad \text{resp.} \quad \Lambda_{sp}(\phi) = 0$$

in moduli space by a finite number of steps.

Laws of species thermodynamics :

- Third law of species thermodynamics:

> It is impossible to reach
>
> $$T_{sp}(\phi) = 0 \quad \text{resp.} \quad \Lambda_{sp}(\phi) = 0$$
>
> in moduli space by a finite number of steps.

It is impossible to reach the boundary of the moduli space by a finite number of steps,

…. it is at infinite distance.

Outline :

I) Introduction

II) Species Scale

III) Species Entropy

IV) Species Temperature

V) Species Thermodynamics

VI) Summary

Summary :

Summary :

- The laws of species thermodynamics shed new light on swampland conjectures, like the distance conjectures.

Summary :

- The laws of species thermodynamics shed new light on swampland conjectures, like the distance conjectures.

- There is a preferred direction in moduli space towards larger number of species, i.e. decompactification and weak coupling and smaller cosmological constant.

This is also relevant for dark dimension scenario.

[M. Montero, C.Vafa, I.Valenzuela (2022)]

Summary :

- The laws of species thermodynamics shed new light on swampland conjectures, like the distance conjectures.
- There is a preferred direction in moduli space towards larger number of species, i.e. decompactification and weak coupling and smaller cosmological constant.

This is also relevant for dark dimension scenario.

[M. Montero, C. Vafa, I. Valenzuela (2022)]

- There is a dual picture: species as particles - species as minimal black hole

Species entropy and black hole entropy follow area law.

This duality is also relevant for dark matter as KK particles or primordial black holes

[E. Gonzalo, M. Montero, G. Obied, C. Vafa, (2022);
L. Anchordoqui, I. Antoniadis, D.L. (2022)]

This duality might be also related to the emergence proposal:
species built space-time geometry

[A. Castellano, A. Herraez, L. Ibanez (2022,2023);
R. Blumenhagen, A. Gligovic, A. Paraskevopoulo (2023)]

Summary :

- The laws of species thermodynamics shed new light on swampland conjectures, like the distance conjectures.
- There is a preferred direction in moduli space towards larger number of species, i.e. decompactification and weak coupling and smaller cosmological constant.

This is also relevant for dark dimension scenario.

[M. Montero, C. Vafa, I. Valenzuela (2022)]

- There is a dual picture: species as particles - species as minimal black hole

Species entropy and black hole entropy follow area law.

This duality is also relevant for dark matter as KK particles or primordial black holes

[E. Gonzalo, M. Montero, G. Obied, C. Vafa, (2022);
L. Anchordoqui, I. Antoniadis, D.L. (2022)]

This duality might be also related to the emergence proposal:
species built space-time geometry

[A. Castellano, A. Herraez, L. Ibanez (2022,2023);
R. Blumenhagen, A. Gligovic, A. Paraskevopoulo (2023)]

- Decay of species via Hawking radiation.

[I. Basile, N. Cribiori, D.L., C. Montella, work in progress]

Thank you !