The Taste of (heterotic) String Theory Flavor

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Outline

- The flavor structure of the Standard Model
- Traditional flavor symmetries
- Modular flavor symmetries
- Eclectic Flavor Group
- Local Flavor Unification
- Lessons from top-down model building
- Origin of hierarchies for masses and mixing angles
- Moduli stabilization in vicinity of boundary of fundamental domain of modular group

Importance of localized structures in extra dimensions

(Work with Baur, Knapp-Perez, Liu, Ratz, Ramos-Sanchez, Trautner, Vaudrevange, 2019-23)

The flavor structure of SM

Most of the parameters of the SM concern the flavor sector

- Quark sector: 6 masses, 3 angles and one phase
- Lepton sector: 6 masses, 3 angles, one phase and additional parameters from Majorana neutrino masses

The pattern of parameters

- Quarks: hierarchical masses und small mixing angles
- Leptons: two large and one small mixing angle, hierarchical mass pattern and extremely small neutrino masses

The Flavor structure of quarks and leptons is very different!

Bottom-up approach

Discrete symmetries have been used extensively in the discussion of flavor structures

- Many fits from bottom-up perspective with discrete symmetries ($S_3, A_4, S_4, A_5, \Delta(27), \Delta(54)$ etc.)
- Flavor symmetries seem to require different models for quark and lepton sector (small mixing angles for quarks versus large mixing in lepton sector)
- Flavor symmetries are spontaneously broken. This requires the introduction of so-called flavon fields and additional parameters
- bottom-up model building leads to many reasonable fits for various choices of groups and representations

But we are still missing a top-down explanation of flavor

Traditional vs Modular Symmetries

So far the flavor symmetries had specific properties and we refer to them as traditional flavor symmetries

- they are linearly realised
- need flavon fields for symmetry breakdown

Another type of flavor symmetries are modular symmetries

- motivated by string theory dualities

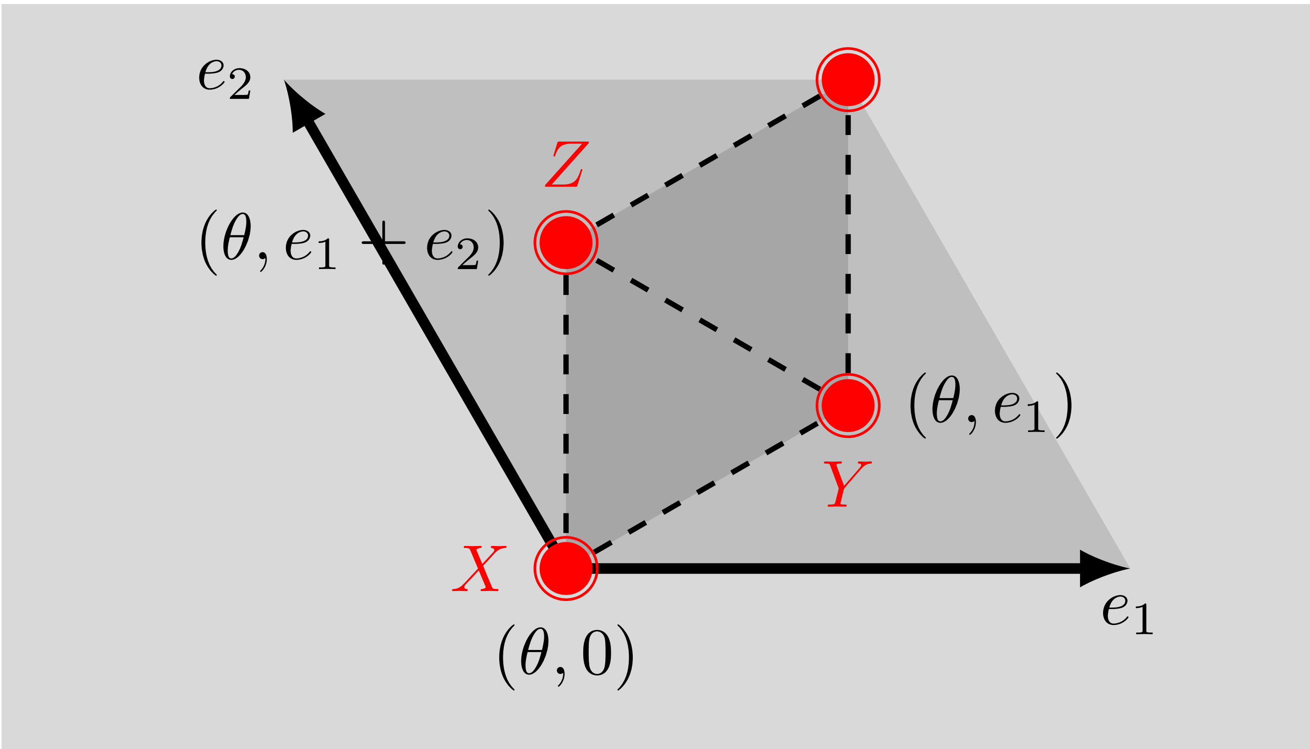
  (Lauer, Mas, Nilles, 1989; Lerche, Lüst, Warner, 1989)
- applied recently to lepton sector (Feruglio, 2017)
- nonlinearly realised (no flavon fields needed)

Combine with traditional flavor symmetries to the so-called "eclectic flavor group" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

Traditional Flavor Symmetries

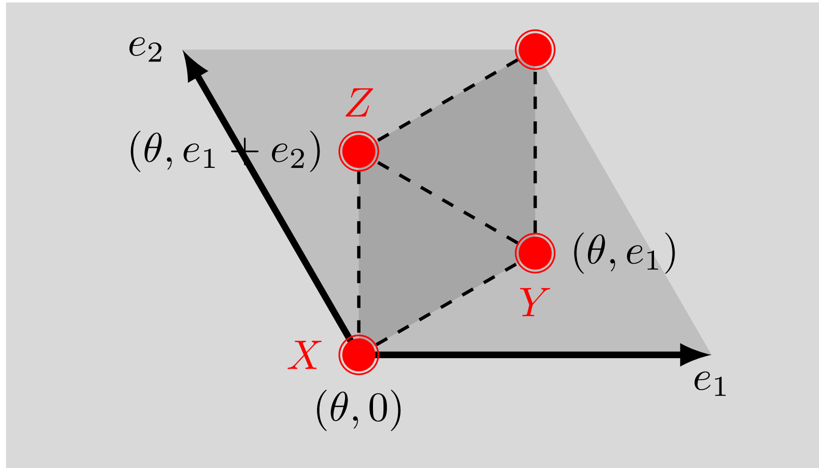
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\text{flavor}}$
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$)

(Kobayashi, Nilles, Ploger, Raby, Ratz, 2006)

Search for a general method

Did we find the complete flavor symmetry in these cases?

In reality we have six compact dimensions

- variety of complicated group structures possible
- dependence on localisation of fields in extra dimensions

We shall concentrate here on the heterotic string represented by the $(6, 6+16)$-dimensional Narain lattice

A general mechanism to find the complete set of flavor symmetries is based on the

outer automorphisms of the Narain space group.

(Baur, Nilles, Trautner, Vaudrevange, 2019)

Other top-down approaches are investigating Type IIB orientifolds or models with magnetized branes

(Talk of Tatsuo Kobayashi)

The Narain Lattice

In the string there are D right- and D left-moving degrees of freedom $Y = (y_R, y_L)$. Y compactified on a $2D$ torus

$$Y = \begin{pmatrix} y_R \\ y_L \end{pmatrix} \sim Y + E\hat{N} = \begin{pmatrix} y_R \\ y_L \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}$$

defines the $2D$ Narain lattice (4-dim for 2-torus ($D = 2$)) with

- the string's winding and Kaluza-Klein quantum numbers n and m (respectively 2-dim. for 2-torus)
- the Narain vielbein matrix E that depends on the moduli of the torus: radii, angles and anti-symmetric tensor fields B.

The Narain Space Group

A Z_K orbifold with twist Θ leads to the identification

$$Y \sim \Theta^k Y + E\hat{N} \quad \text{where} \quad \Theta = \begin{pmatrix} \theta_R & 0 \\ 0 & \theta_L \end{pmatrix} \quad \text{and} \quad \Theta^K = 1$$

with θ_L, θ_R elements of $SO(D)$. For a symmetric orbifold $\theta_L = \theta_R$ (we do not include roto-translations here).

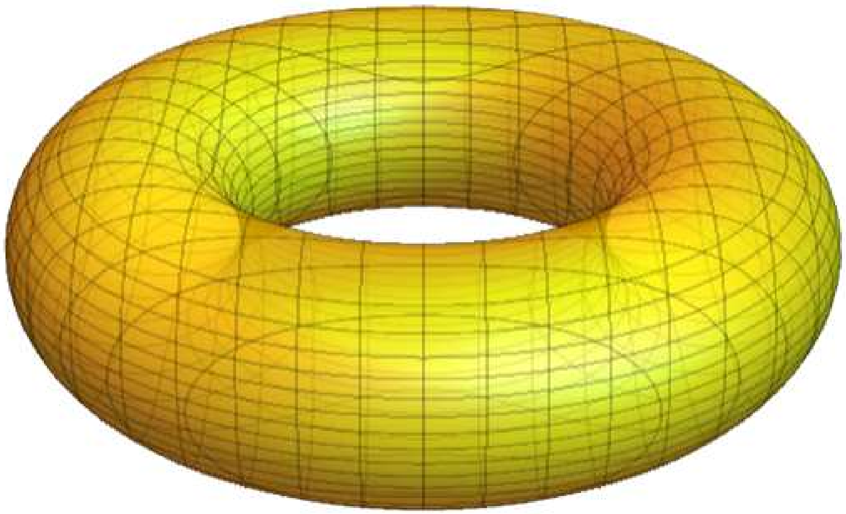
The Narain space group $g = (\Theta^k, E\hat{N})$ is then generated by

$$\text{twists} \quad (\Theta, 0) \quad \text{and shifts} \quad (1, E_i) \quad \text{for} \quad i = 1 \ldots 2D$$

Outer automorphisms map the group to itself but are not elements of the group (include modular transformations).

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

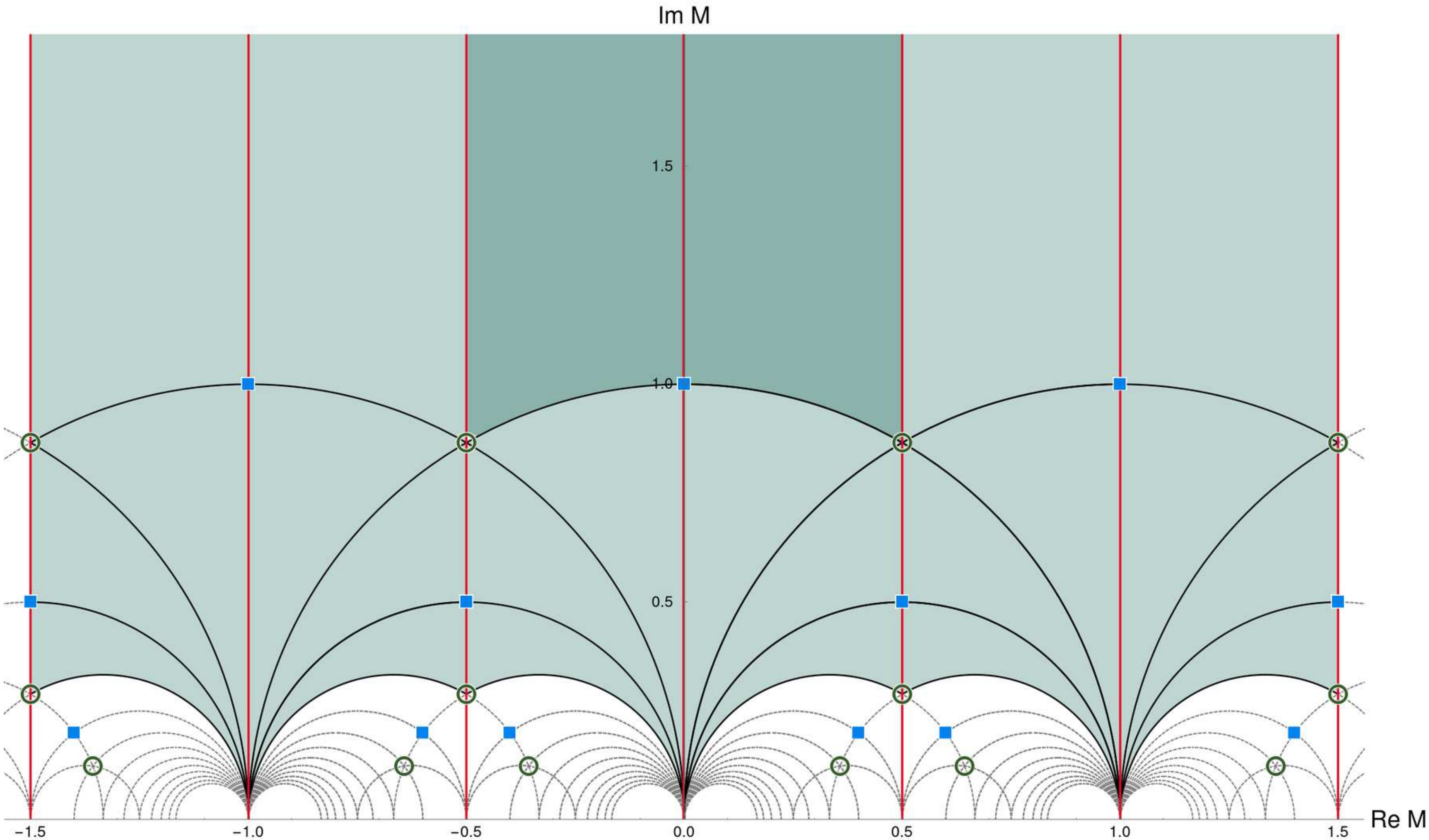
$$\mathrm{S}\ ,\ \mathrm{T}: \quad \text{with}\ \ \mathrm{S}^4 = 1\ \ \text{and}\ \ \mathrm{S}^2 = (\mathrm{ST})^3$$

A modulus M transforms as

$$\mathrm{S}: \quad M \to -\frac{1}{M} \quad \text{and} \quad \mathrm{T}: M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Three fixed points at M$= i$, $\omega = \exp(2\pi i/3)$ and $i\infty$

Modular Forms

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma : \quad M \rightarrow \frac{aM+b}{cM+d}$$

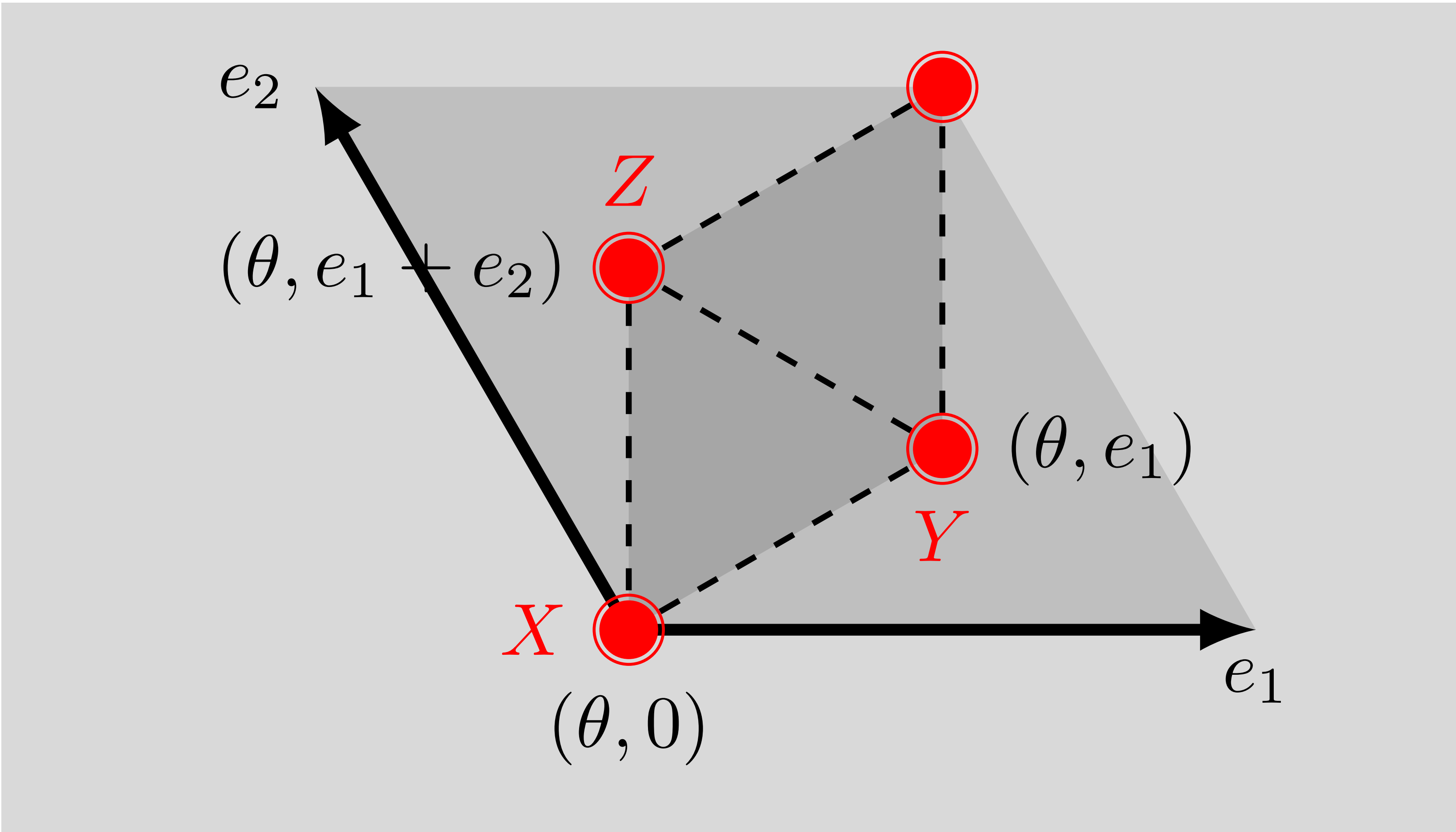
with $ad - bc = 1$ and integer a, b, c, d.

Matter fields transform as representations $\rho(\gamma)$ and modular functions of weight k

$$\gamma : \quad \phi \rightarrow (cM+d)^k \rho(\gamma) \phi \, .$$

Yukawa-couplings transform as modular functions as well. $G = K + \log |W|^2$ must be invariant under T-duality

Towards Modular Flavor Symmetry



Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen,

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2,Z)$

- $\Gamma(3) = SL(2,3Z)$ as a mod(3) subgroup of $SL(2,Z)$
- discrete modular flavor group $\Gamma'_3 = SL(2,Z)/\Gamma(3)$
- the discrete modular group is $\Gamma'_3 = T' \sim SL(2,3)$ (which acts nontrivially on twisted fields); the double cover of $\Gamma_3 \sim A_4$ (which acts only on the modulus).
- the CP transformation $M \to -\overline{M}$ completes the picture.

Full discrete modular group is $GL(2,3)$.

Eclectic Flavor Groups

We have thus two types of flavor groups

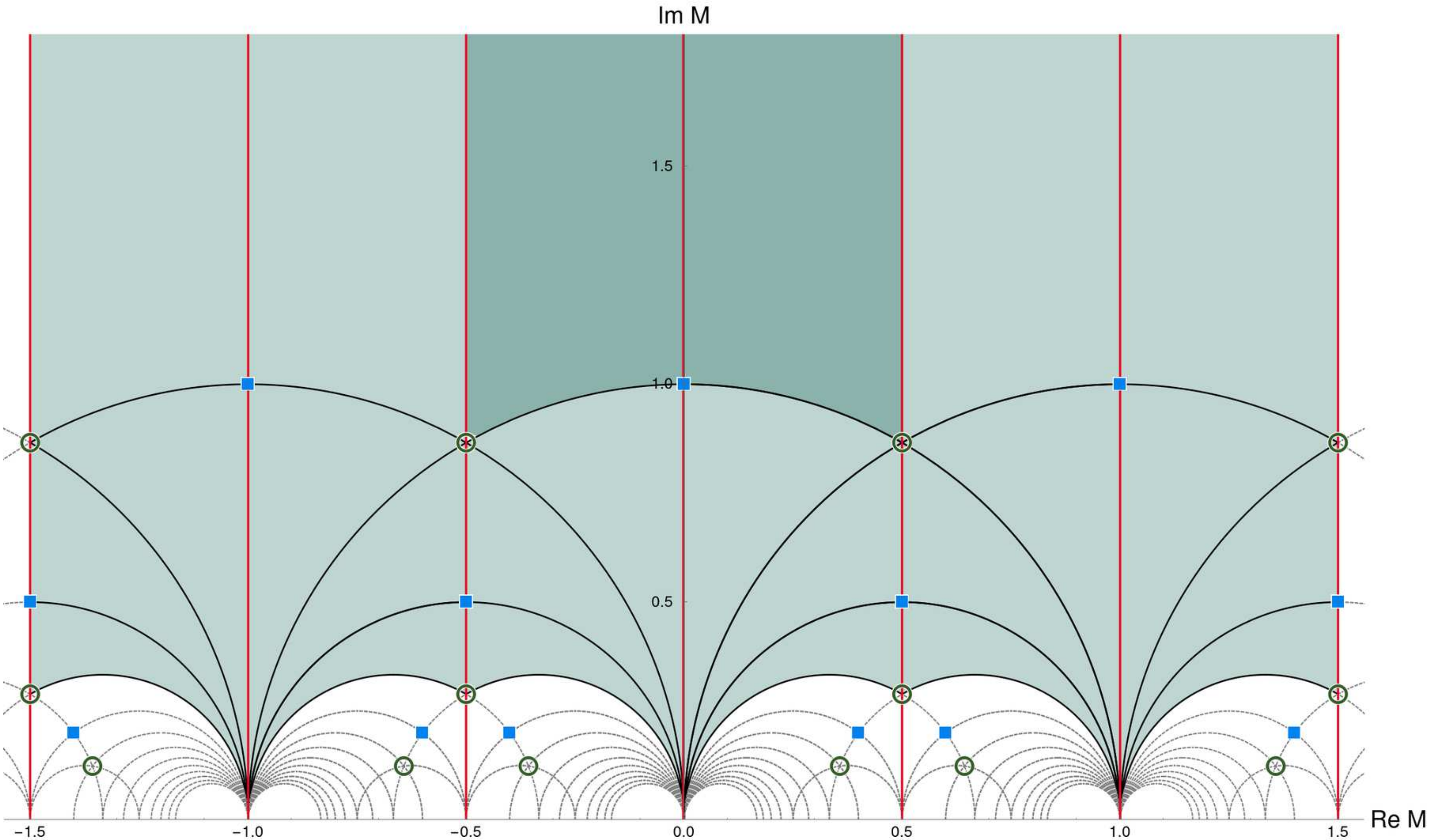
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2, 3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2, 3)$ including CP

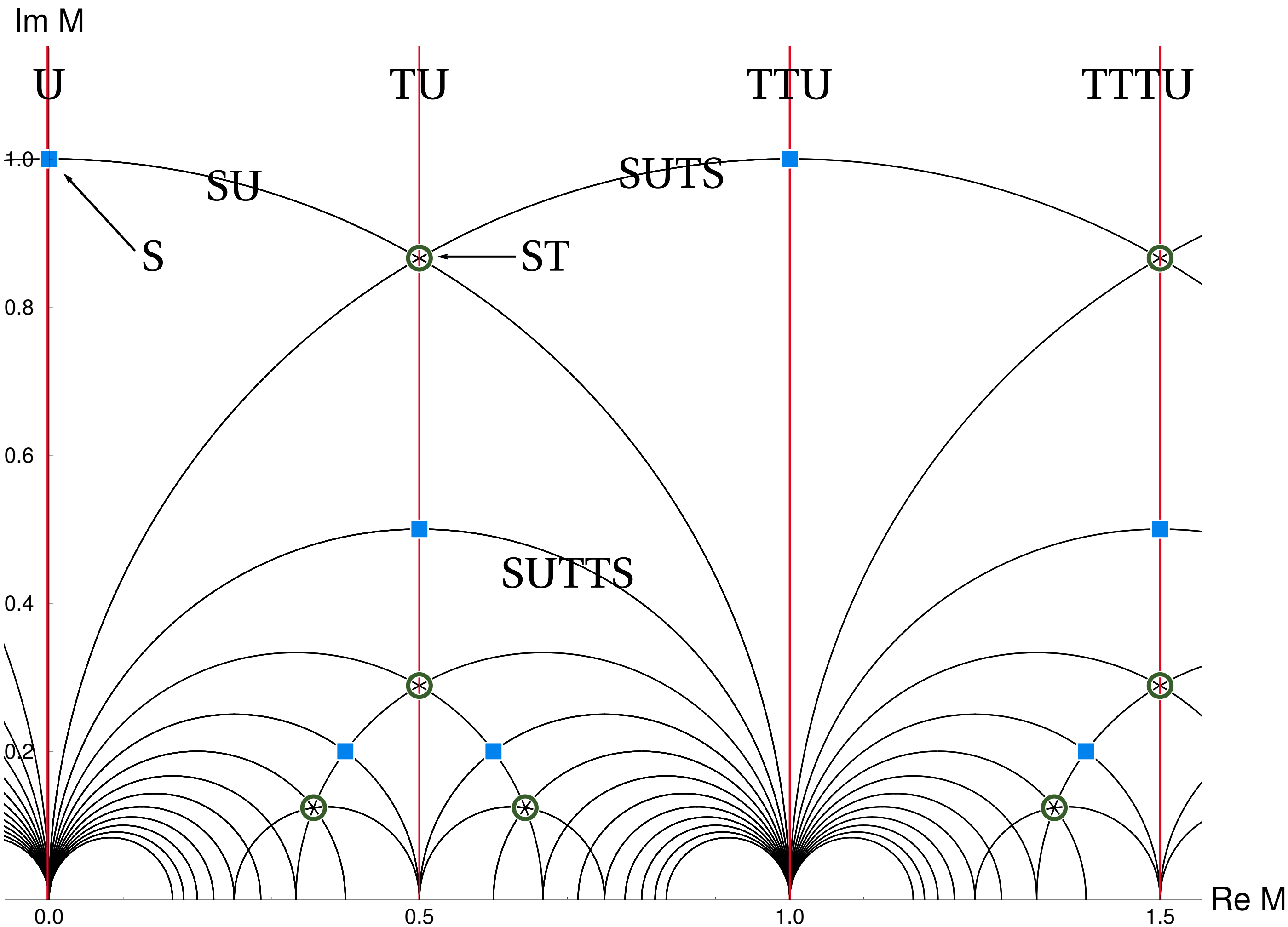
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



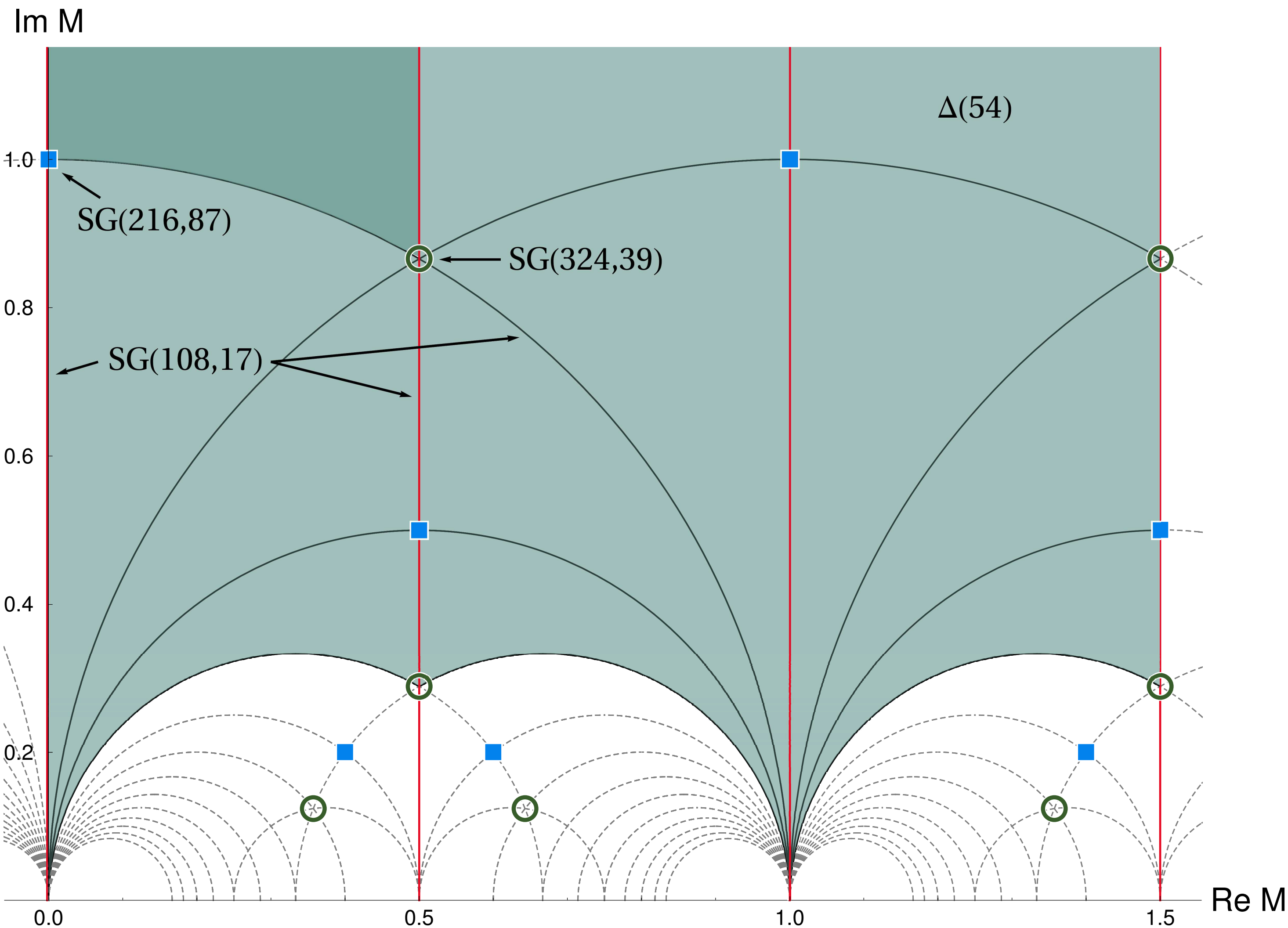
Moduli space of $\Gamma(3)$

Fixed lines and points



$$\mathrm{S}: M \to -\frac{1}{M}, \quad \mathrm{T}: M \to M+1 \quad \text{and} \quad \mathrm{U}: M \to -\overline{M}$$

Moduli space of flavour groups



"Local Flavor Unification"

Unification of Flavor and CP

Summary of predictions of the string picture:

- traditional flavor symmetries (universal in moduli space)
- modular flavor symmetries and CP are non-universal in moduli space

They unify in the eclectic picture of flavor symmetry.
You cannot just have one without the other.

The non-universality in moduli space leads to

- local flavor unification at specific points in moduli space
- hierarchical structures of masses, mixing angles and phases in vicinity of fixed points or lines
- potentially different pictures for quarks and leptons

Classification

- Modular symmetry $SL(2, Z)$ is intrinsically related to the 2-torus T^2 with two moduli T and U
- Chiral fermions require a twist Z_k of T^2, embedded in 6-dimensiponal compact space.
- relevant Z_k are $k = 2, 3, 4, 6$
- for $k = 3, 4, 6$ the T-modulus is fixed to allow for the twist
- higher k lead to larger modular symmetries Γ_k
- at the expense of smaller traditional flavor symmetries
- $k = 3$: $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2, 3)$
- embedding in 6-dimensional space give additional R-symmetries

Z_4 and Z_6

The 2-dimensional Z_3 orbifold gives already a promising result. What about the others?

- Z_6 gives modular group $\Gamma_2 \times \Gamma'_3 = S_3 \times T' = [144, 128]$
- traditional flavor group is abelian (one fixed point)
- Z_6 eclectic group is $[144, 128] \times Z_{36}^R$ (5184 elements)
- Z_4 gives S'_4 modular group with an intrinsic relation to the traditional flavor and R-symmetry
- traditional flavor symmetry is $[64, 185]$ (including the R-symmetry) and Z_4 eclectic group is $[384, 5614]$

Interpretation of quark- and lepton-multipets is less clear than in the Z_3 case

(Work in progress)

Z_2 orbifold: two moduli

Here the twist does not constrain the moduli T and U

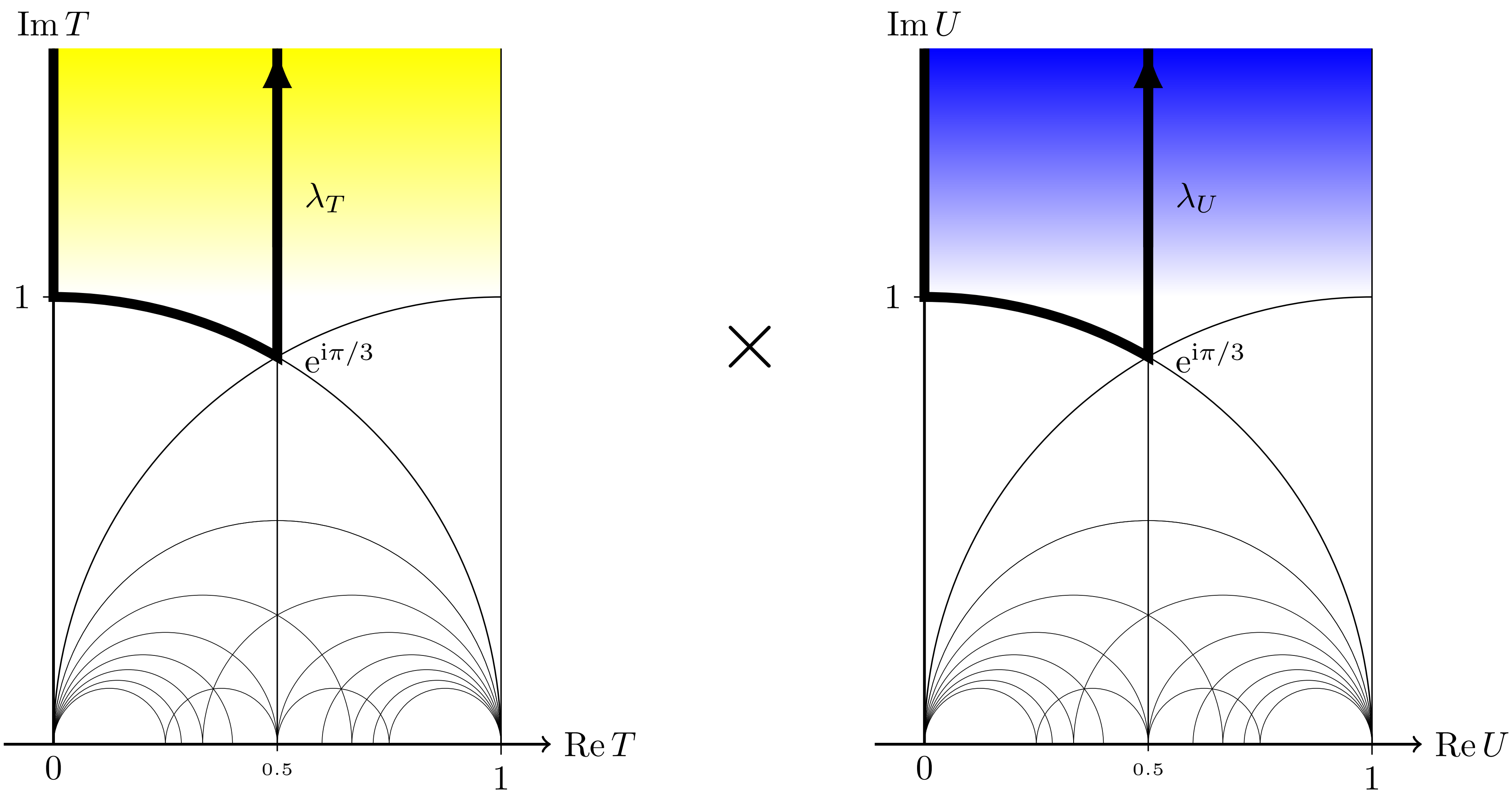
- and we have the full $SL(2,Z)_T \times SL(2,Z)_U$.
- The discrete modular group is $\Gamma_2 \times \Gamma_2 \times Z_2$,
- where $\Gamma_2 = S_3$ and
- Z_2 interchanges T and U (known as mirror symmetry).
- The traditional flavor group is the product of $(D_8 \times D_8)/Z_2$ and a Z_4 R-symmetry.

This leads to an

- eclectic group with 2304 elements (excluding CP)
- or 4608 elements (including CP)

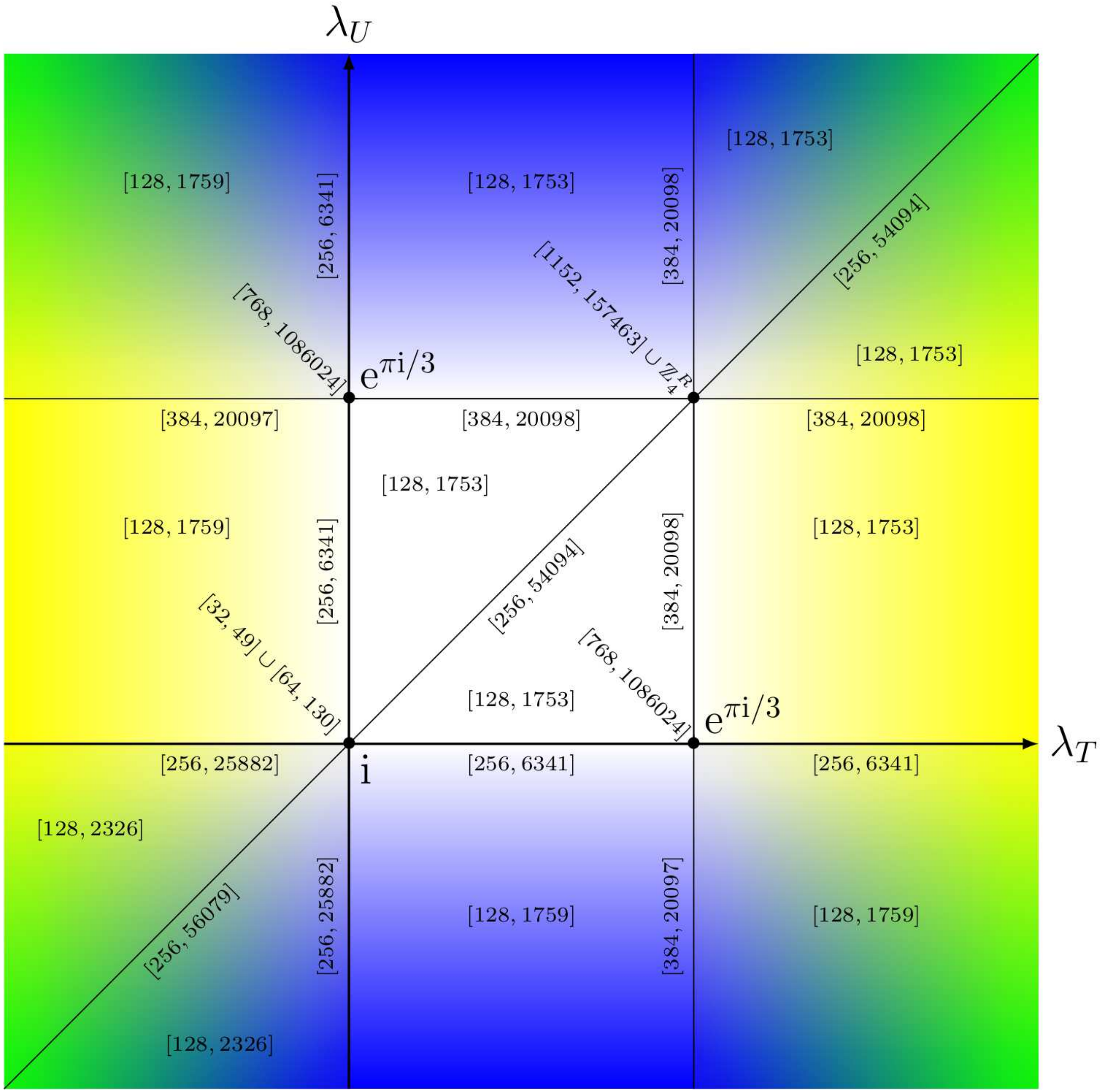
with a rich pattern of local flavor group enhancements.

Z_2-orbifold

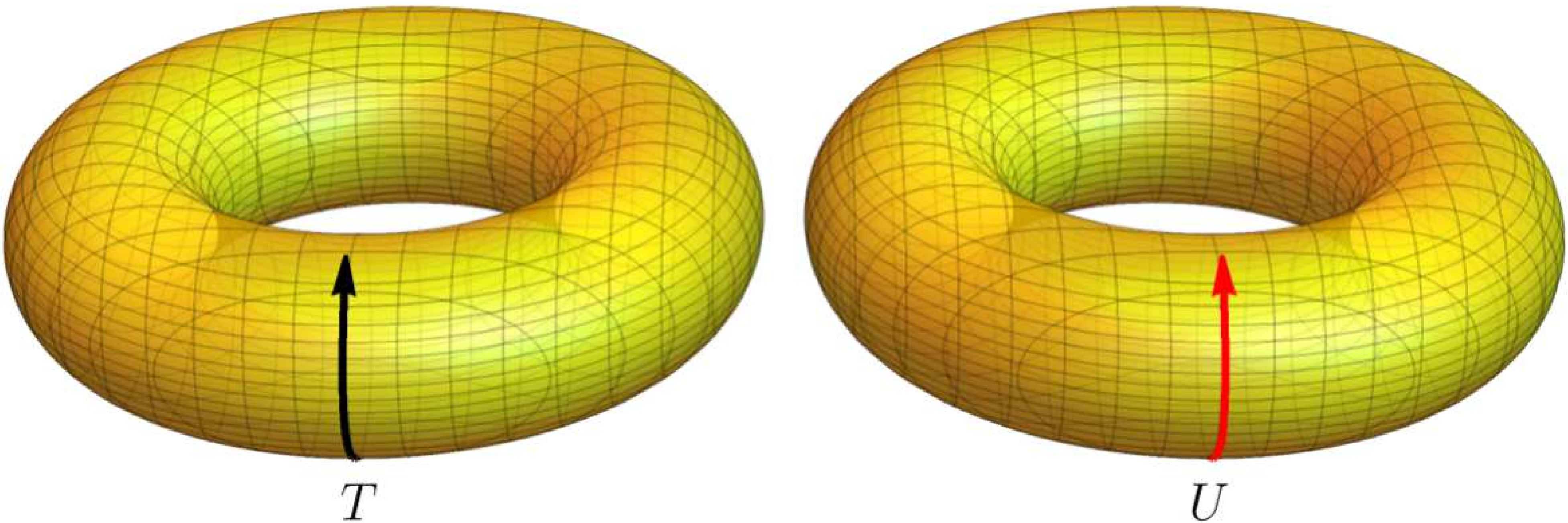


Here we have two unconstrained moduli: T and U

Enhancement

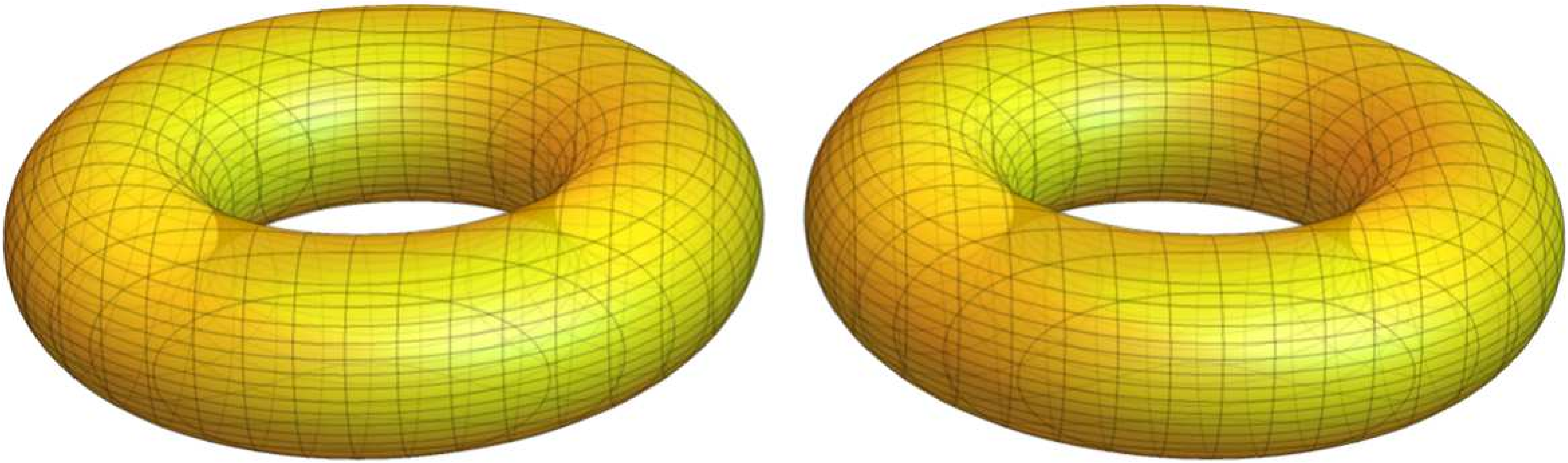


Auxiliary Surface: Double Torus



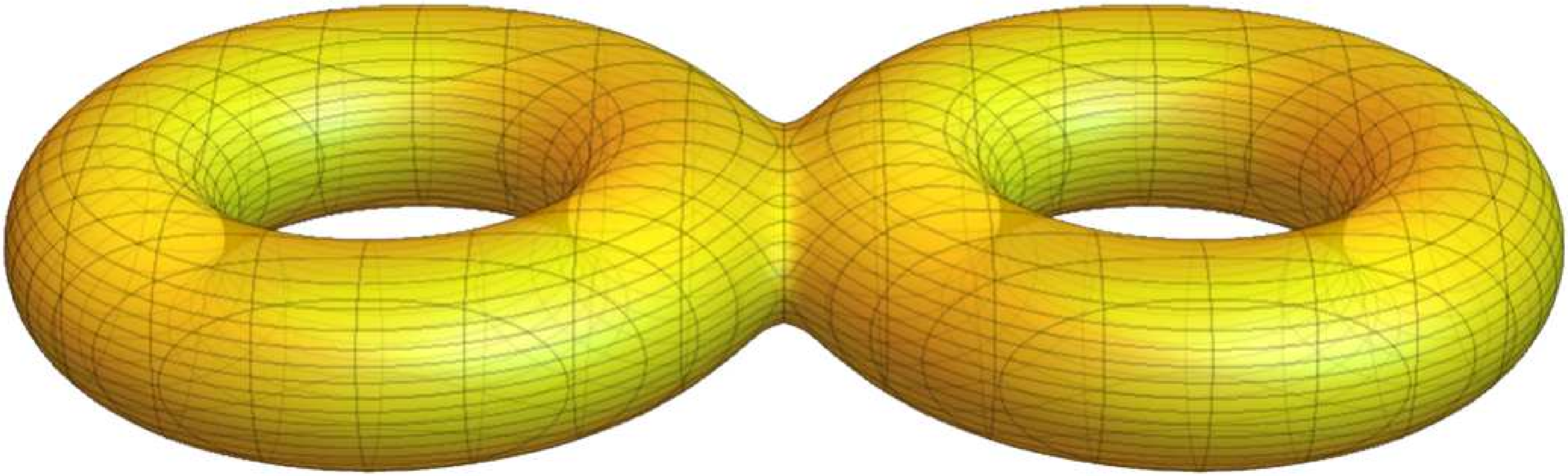
Auxiliary surface for two moduli: $SL(2,Z)_T \times SL(2,Z)_U$

Riemann surface of genus 2



Auxiliary surface for two moduli: T and U

Riemann Surface of Genus 2



Auxiliary surface with three moduli: T + U + Wilson line

Siegel Modular Forms

This leads to a generalization of the modular group to larger groups $Sp(2g, Z)$ characterized through Riemann surfaces of higher genus g: (Ding, Feruglio, Liu, 2021; Ishiguro, Kobayashi, Otsuka, 2022)

- for $g = 2$ the Siegel modular group $Sp(4, Z)$
- includes $SL(2, Z)_{U,T}$ and describes three moduli.
- Fundamental domain (6 points, 5 lines, 2 surfaces)
- Orbifold twists are connected to fixed loci in fundamental domain
- Discrete modular group $\Gamma_{g,k}$ ($\Gamma_{1,k} = \Gamma_k$)
- $\Gamma_{2,2} = S_6$ includes $S_3 \times S_3$ and mirror symmetry
- $\Gamma_{2,3}$ has already 51840 elements (work in progress)

Messages

The top-down approach to flavor symmetries leads to a

- unification of traditional (discrete) flavor, CP and modular symmetries within an eclectic flavor scheme
- modular flavor symmetry is a prediction of string theory
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a natural consequence of the symmetries of the underlying string theory
- the potential flavor groups are large and non-universal
- spontaneous breakdown as motion in moduli space

Top-Down versus Bottom-Up

This opens up a new arena for flavor model building:

- so far $\Delta(54) \times T'$ is the favourite "top-down" model
- need more explicit string constructions
- but it is not only the groups but also the representations and modular weights of matter fields that are relevant (top-down models very restrictive)
- there is still a huge gap between "top-down" and "bottom-up" constructions
- modular flavour group from outer automorphisms of traditional flavor group (Nilles, Ramos-Sanchez, Vaudrevange, 2020)
- recently applied in "bottom-up" constructions for $\Delta(27)$ as traditional flavor symmetry (Ding, King, Li, Liu, Lu, 2023)

Moduli Stabilization

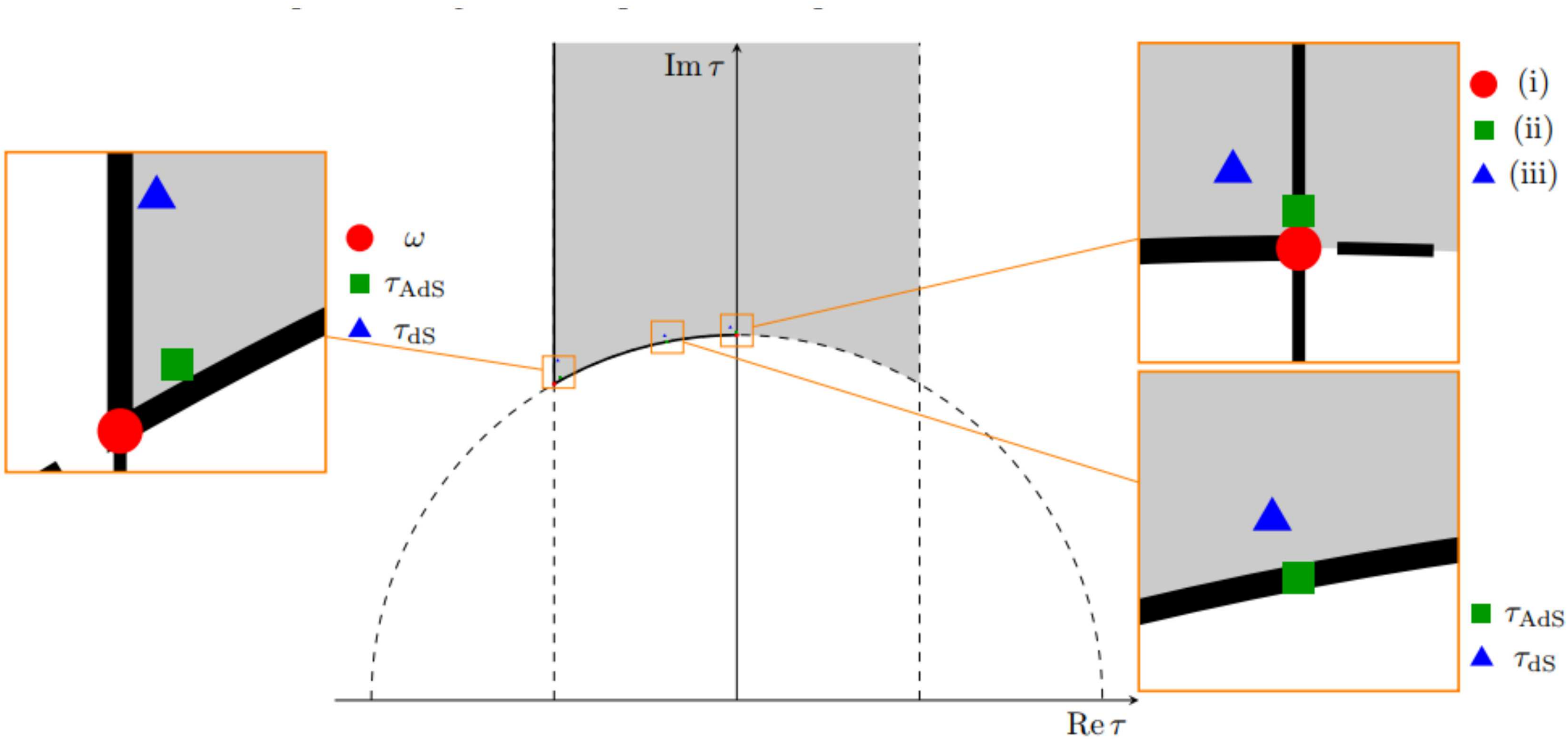
It has been stressed recently that many of the successful fits are in the vicinity of fixed points and lines

(Feruglio, 2022-23; Petcov, Tanimoto, 2022; Abe, Higaki, Kawamura, Kobayashi, 2023)

- moduli stabilization favours boundary of moduli space (Cvetic, Font, Ibanez, Lüst, Quevedo, 1991; Dent, 2002; Novichkov, Penedo, Petcov, 2022; Leedom, Righi, Westphal, 2022)
- but at the boundary we have some unbroken flavor or CP symmetries
- moreover, these minima at the boundary are AdS
- uplift moves them slightly away from the boundary and creates flavor hierarchies

(Knapp-Perez, Liu, Nilles, Ramos-Sanchez, Ratz, 2023)

Moduli fixing



Summary

String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- allows a different flavor structure for quarks and leptons