Aspects of potential matching across heterotic transitions

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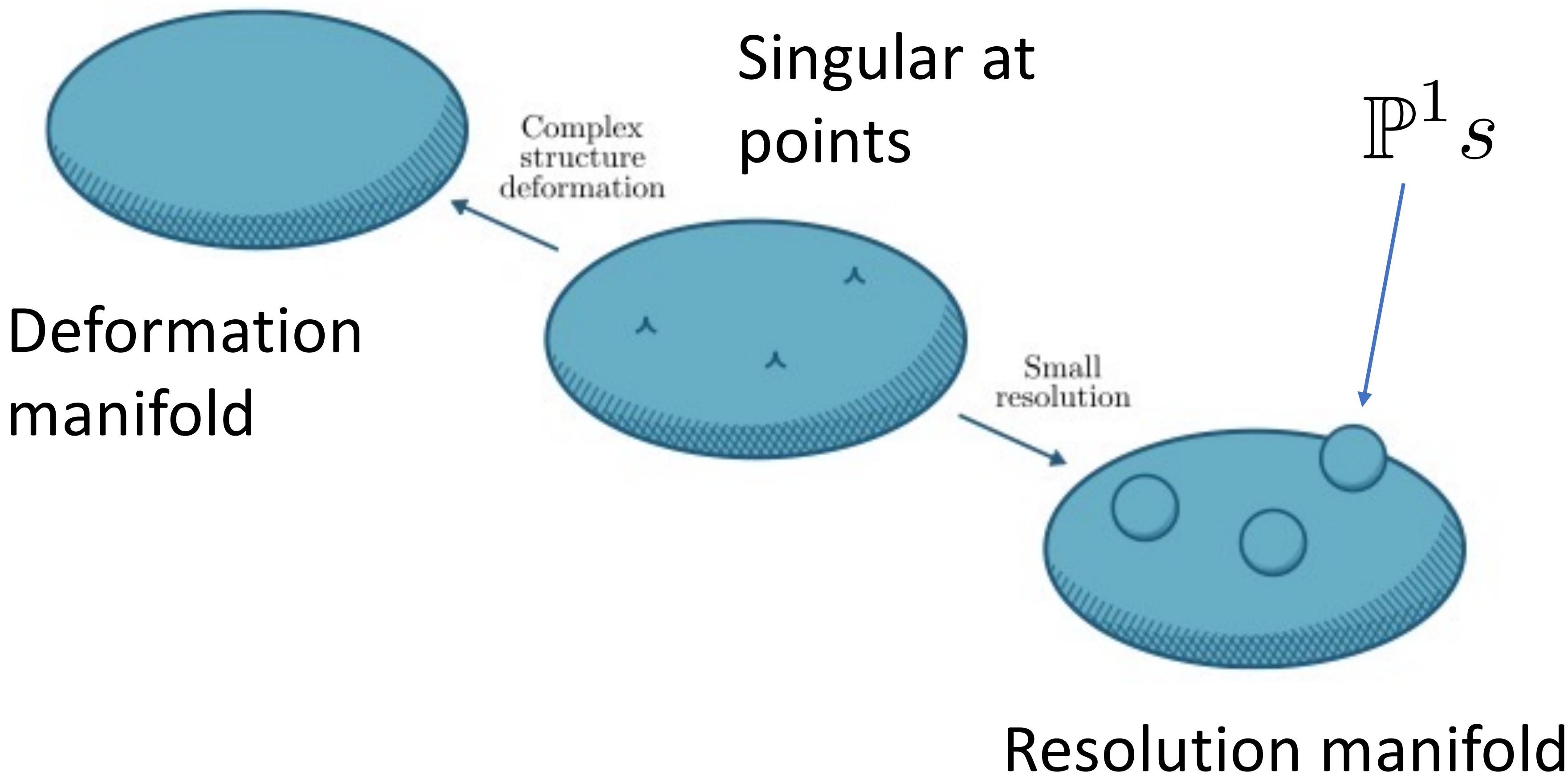
Based on work with:
Anderson and Brodie arXiv:2211.05804
Anderson and Brodie (appearing soon)
And Anderson, Brodie, Patel and Scanlon (in progress)

Conifold transitions

- At the level of geometry we have schematically:



- In an example:

Smooth quintic

$$X_D = \left[\ \mathbb{P}^4 \mid 5\ \right]$$

Deform complex structure

Nodal quintic

$$X_0 = \left[\ \mathbb{P}^4 \mid 5\ \right]$$

Blow up singular points of nodal quintic with (16) $\mathbb{P}^1 s$

$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array} \right]$$

- You can also think of this transition as a brane separating from the cotangent bundle (c.f. small instanton transitions where a brane separates from the gauge bundle):

$$0 \to f^*\Omega_{X_0} \to \Omega_{X_R} \to \mathcal{O}_{\mathbb{P}^1{}_s}(-2) \to 0$$

Deformation manifold cotangent bundle

Resolution manifold cotangent bundle

Sheaf localized on exceptional curves representing branes

Gauge bundles through conifolds

- Recall the anomaly cancelation condition in heterotic string-theory:

$$c_2(\Omega_{X_R}) = c_2(V_R)$$

- How does this change during the transition?

$$c_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = c_2(V_R) + [\mathbb{P}^1 s]$$

This is how the gravitational sector changes given the transition we have seen in the cotangent bundle.

The gauge sector must change in the same way.

- A natural guess is to add the same brane into the gauge bundle!
- The process turns out to be a complicated series of brane recombinations and small instanton transitions.
- Gives us the geometric description of the heterotic conifold transition.
- Example of some dual bundles on the example geometries:

Deformation side: $$0 \to \mathcal{O}(-5) \to \mathcal{O}(-1)^{\oplus 5} \to V_D \to 0$$

Resolution side:

$$0 \to \mathcal{O}(-1,-5) \oplus \mathcal{O}(0,-4) \to$$
$$\mathcal{O}(-1,0) \oplus \mathcal{O}(0,-5) \oplus \mathcal{O}(0,-1)^{\oplus 4} \to V_R \to 0$$

McNamara and Montero: The existence of such a process can be predicted from swampland considerations

- The pairs of compactifications that are linked in this manner are seemingly dual.
- In particular they have identical charged spectra and moduli counts.

→ Generalization of (0,2) target space duality studied in GLSM literature (includes M5 branes and arbitrary bundle constructions)

Distler and Kachru: hep-th/9501111, Blumenhagen hep-th/9707198, 9710021 and Blumenhagen and Rahn arXiv:1106.4998

Examples of applications:

- Indicates there is potentially a new, less redundant, way to formulate heterotic theories.
- May afford us computational control of some quantities (difficult on one side of the duality, easy on the other)
- Will allow us to reduce a large amount of redundancy in our scans for heterotic models etc. (a heterotic analogue of Wall's data would be particularly helpful here).

However, for the applications to be viable we need to know it is an actual duality!

- Currently just have a matching of some numbers (the spectrum)
- Would like to see more evidence for the duality by studying functions (such as potentials) in the effective theories.

Yukawa couplings

- To understand these we need information about the mapping on field space (for both moduli and charged matter).

- Some cases are easy to understand (if boring).

  Previous Example: SO(10) theory with no 10's
  – all couplings vanish.

- For more interesting cases we can simply compute on both sides of the duality and see if we can spot the structure we need.

For E6 theories: $H^1(V) \times H^1(V) \times H^1(V) \rightarrow H^3(\wedge^3 V)$

An E6 example:

Deformation side:

$$[\mathbb{P}^4 | 5]$$

$$0 \to V_D \to \mathcal{O}(1)^3 \oplus \mathcal{O}(2) \to \mathcal{O}(5) \to 0$$

Resolution side:

$$\left[\begin{array}{ccccccc|cc} y_0 & y_1 & y_2 & y_3 & y_4 & x_0 & x_1 & p_1 & p_2 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 & 4 & 4 \end{array}\right]$$

$$0 \to V_R \to \mathcal{O}(0,2)^2 \oplus \mathcal{O}(0,1) \oplus \mathcal{O}(1,0) \to \mathcal{O}(1,5) \to 0$$

Family structure and Yukawa Couplings

- Deformation side:

Equivalence class describing families ($H^1(V_D)$):

$$P_5 \sim P_5 + \alpha p_{(5)} + \sum_{a=1}^{3} \beta_a q^a_{(4)} + \gamma c_{(3)}$$

Equivalence class describing Yukawa coupling ($H^3(\wedge^3 V_D)$):

$$P_{15} \sim P_{15} + A p_{(5)} + \sum_{a=1}^{3} B_a q^a_{(4)} + D c_{(3)}$$

There is one equivalence class of degree 15 polynomials of this type. Multiply degree 5 polynomials together and compare to a degree 15 representative to obtain coupling.

- Resolution side:

Equivalence class describing families ($H^1(V_R)$):

$$P_{1,5} \sim P_{1,5} + \tilde{\alpha} p_{(0,5)} + \sum_{a=1}^{3} \beta_a q^a_{(1,4)} + \sum_{i=1}^{2} \gamma^i c^i_{(1,3)}$$

without loss of generality take one of the $c_{(1,3)}$ maps to be x_0 . Then this becomes equivalent to:

$$x_1 P_{0,5} \sim x_1 P_{0,5} + x_1 \alpha p_{(0,5)} + x_1 \sum_{a=1}^{3} \beta_a q^a_{(0,4)} + x_1 \gamma c_{(0,3)}$$

There is then an obvious mapping between these family equivalence classes and those for the quintic.

Equivalence class describing Yukawa coupling ($H^3(\wedge^3 V_R)$):

$$P_{3,15} \sim P_{3,15} + \tilde{A} p_{(0,5)} + \sum_{a=1}^{3} \tilde{B}_a q^a_{(1,4)} + \sum_{i=1}^{2} \tilde{D}_i c^i_{(1,3)}$$

Maps are same as for families so one of the $c_{(1,3)}$ maps has the same effect

$$x_1^3 P_{0,15} \sim x_1^3 P_{0,15} + x_1^3 A p_{(0,5)} + x_1^3 \sum_{a=1}^{3} B_a q^a_{(0,4)} + x_1^3 D c_{(0,3)}$$

At this stage we can see that the entire resolution side Yukawa computation is identical to the deformation side one, multiplied by spectator factors of x_1

- In this example we find that all 147,440 independent, non-vanishing, Yukawa couplings correctly match as holomorphic functions of the moduli on either side of the duality (there are 95 families in this case).
- This matching works for a class of monads where the rank of the final line bundle sum in the sequence is one.

- More generally we have been able to show that a more complex mapping of family structures is necessary, but have yet to find it.
- In some of these more complex examples, however, we know that the Yukawa couplings must match for other reasons… so work continues!

Summary

- Taking a gauge bundle through a conifold in heterotic theories naturally leads to seemingly dual compactifications.
- This duality generalizes (0,2) target space duality and has implications for our understanding of the heterotic moduli space and model building in string phenomenology.
- The duality seems to be holding correctly not just at the level of the spectrum, but also at the level of functions appearing in the superpotentials of the two theories.