DIPARTIMENTO DI FISICA E ASTRONOMIA Galileo Galilei

QG bounds, Gauss-Bonnet & wormholes

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based on: N. Risso, A. Valenti, L. Vecchi (in progress)

N. Risso & T. Weigand 2210.10797

Motivations

- EFTs generically contain a Gauss-Bonnet (GB) term

$$\gamma \int_{\mathcal{M}} E_{\rm GB} \qquad\qquad E_{\rm GB} \equiv \frac{1}{32\pi^2}\left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2\right)$$

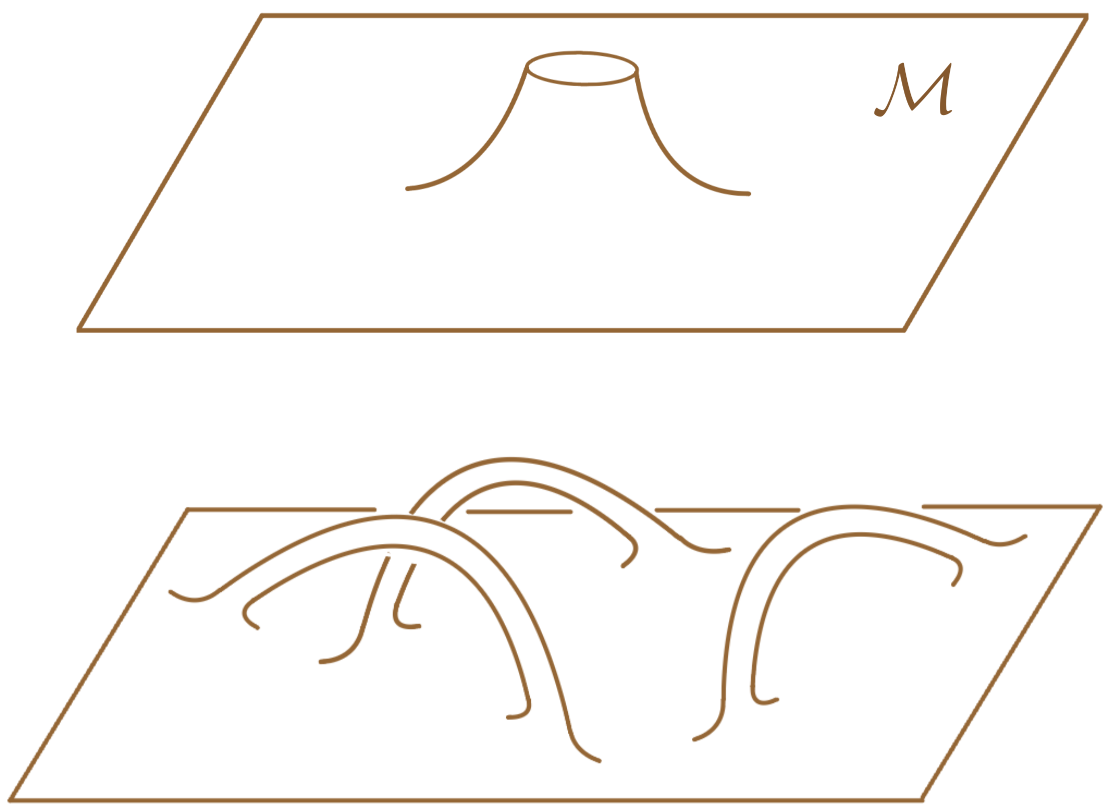
- Strong evidence of QG bound: $\gamma > 0$

[..., Brigante-Liu-Myers-Shenker-Yaida '08,... Cheung-Remmen '16, GarcíaEtxebarria-Montero-Sousa-Valenzuela '20, Aalsma-Shiu '22, Chin Ong '22, LM-Risso-Weigand '23]

- Topological in 4d EFTs $\quad \int_{\mathcal{M}} E_{\rm GB} + \int_{\partial\mathcal{M}} Q_{\rm GB} = \chi(\mathcal{M})$

physical implications? non-perturbative QG effects!

GB relevance to wormholes



[Giddings-Strominger ’88-‘89, Coleman-Lee ’88-’89, Preskill ’89, … Kallosh-Linde-Linde-Susskind ‘94, Alonso-Urbano ’17, …]

$$\Delta\mathcal{L}_{\text{eff}} = e^{-S_{\text{GB}}} \sum_i e^{-S^i_{\text{E}}} \mathcal{O}_i(x)$$

$$e^{-S_{\text{GB}}} = e^{\gamma[\chi(\mathcal{M})-1]} = e^{-\gamma}$$

$\gamma > 0 \Rightarrow$ universal suppression!

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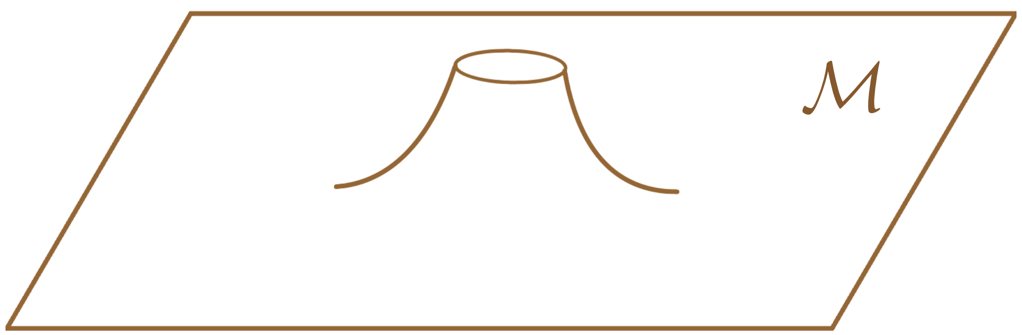

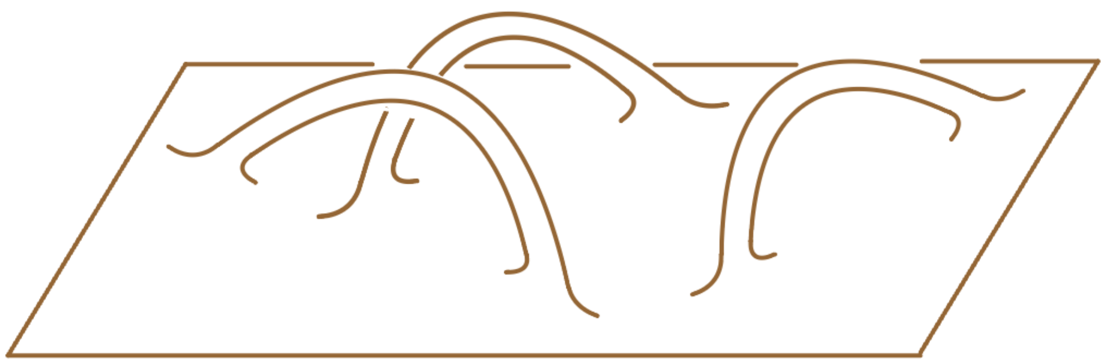

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$$e^{-S_{\text{GB}}} = e^{\gamma[\chi(\mathcal{M})-1]} = e^{-\gamma}$$

$\gamma > 0$ ⇒ universal suppression!

Potentially important physical implications: axion physics

$$\Delta V(\text{axions}) \propto e^{-\gamma} \sum_m e^{-S^m_{\text{E}}} \cos(m\vartheta + \delta_m)$$

[Kim-Lee ’88, Rey ’89,…, Hebecker-Mikhail-Soler `18]

PQ quality problem

axion inflation

axion dark matter

GB relevance to wormholes

[Giddings-Strominger '88-'89, Coleman-Lee '88-'89, Preskill '89, ...
Kallosh-Linde-Linde-Susskind '94,
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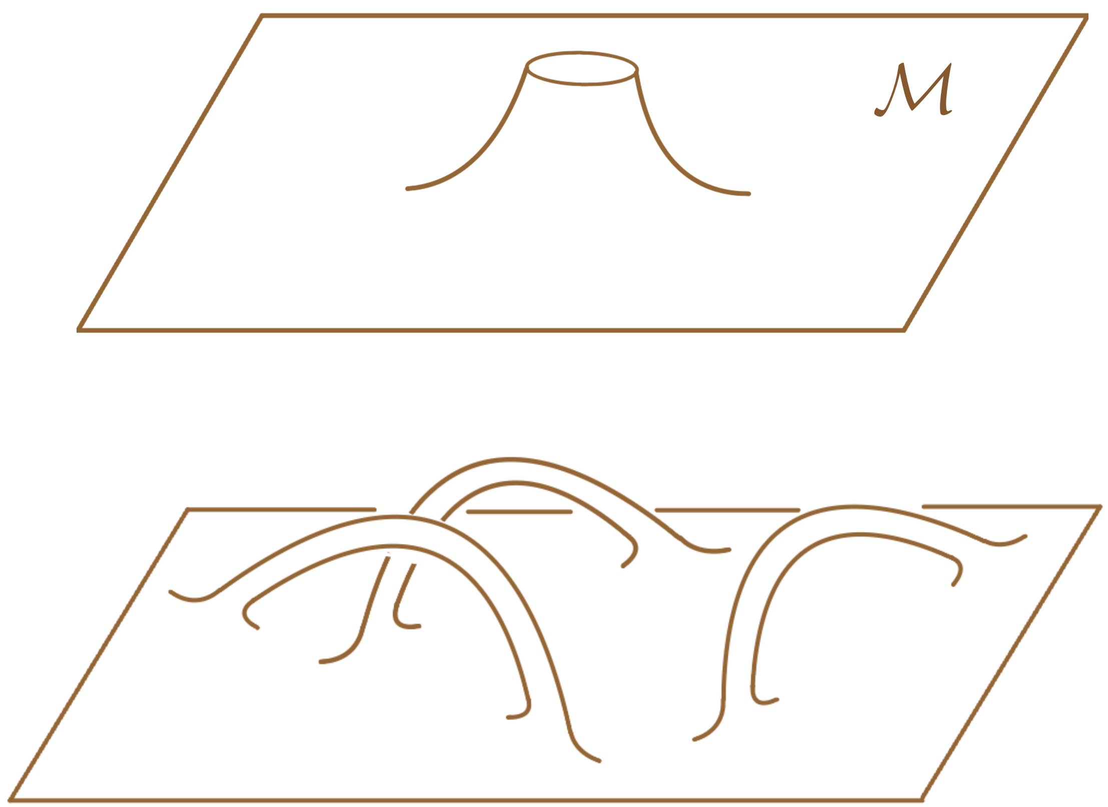

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$\gamma > 0 \Rightarrow$ universal suppression!

* Additional QG-constraints on γ ?

* Conditions for $\gamma \gg 1$?

4d setting

- I will assume (intermediate) 4d $\mathcal{N}=1$ UV completion

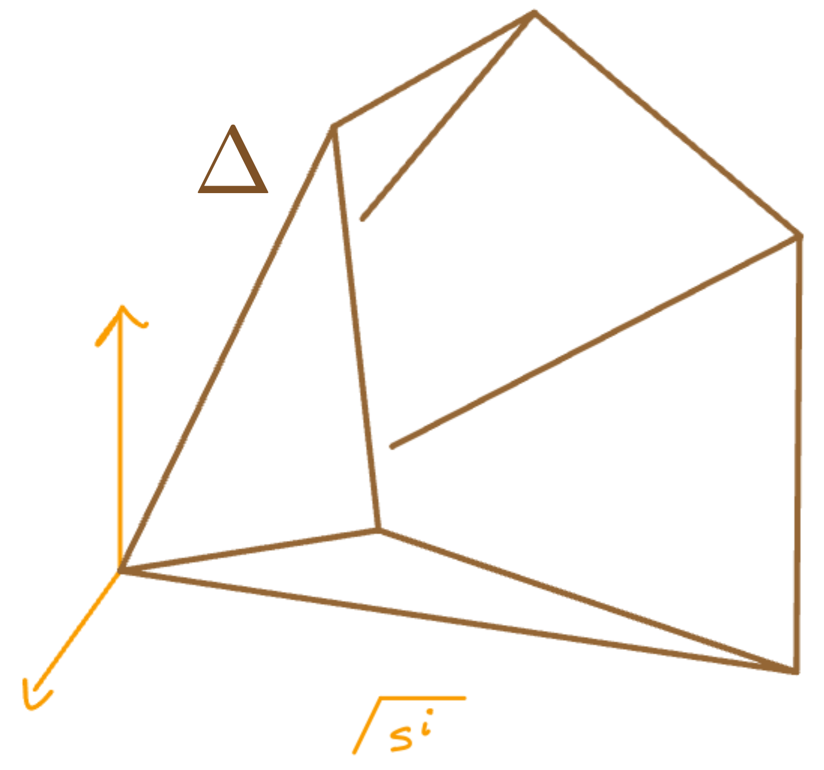
- perturbative regime specified by N (s)axions

$$a^i \simeq a^i + 1 \quad , \quad i = 1, \ldots, N$$

$$\{s^i\} \in \Delta \equiv \{\text{saxionic cone}\}$$

[Lanza-Marchesano-LM-Valenzuela '20-'21]

↳ e.g. volumes of effective divisors in F-Theory models



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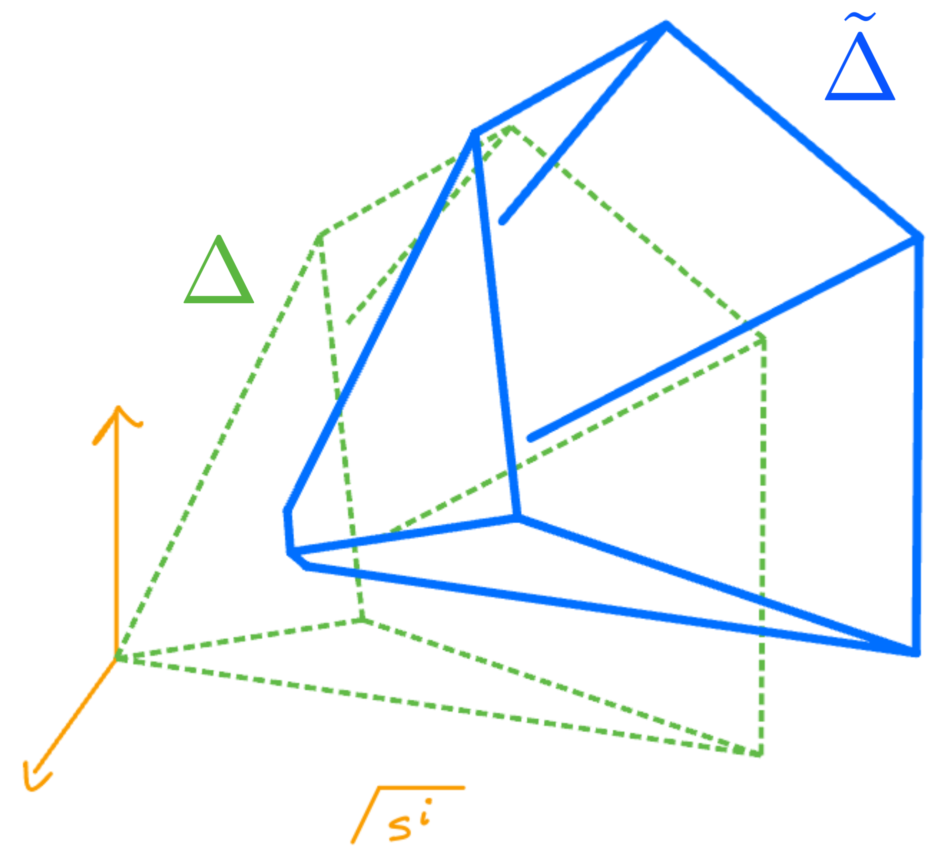

- Restrict to stretched saxionic convex hull:

$$s \in \tilde{\Delta} \quad \Rightarrow \quad \delta_{\text{F-inst.}}\mathcal{L} \lesssim e^{-2\pi} \simeq 10^{-3}$$

cf. [Demirtas-Long-McAllister-Stillman '18]

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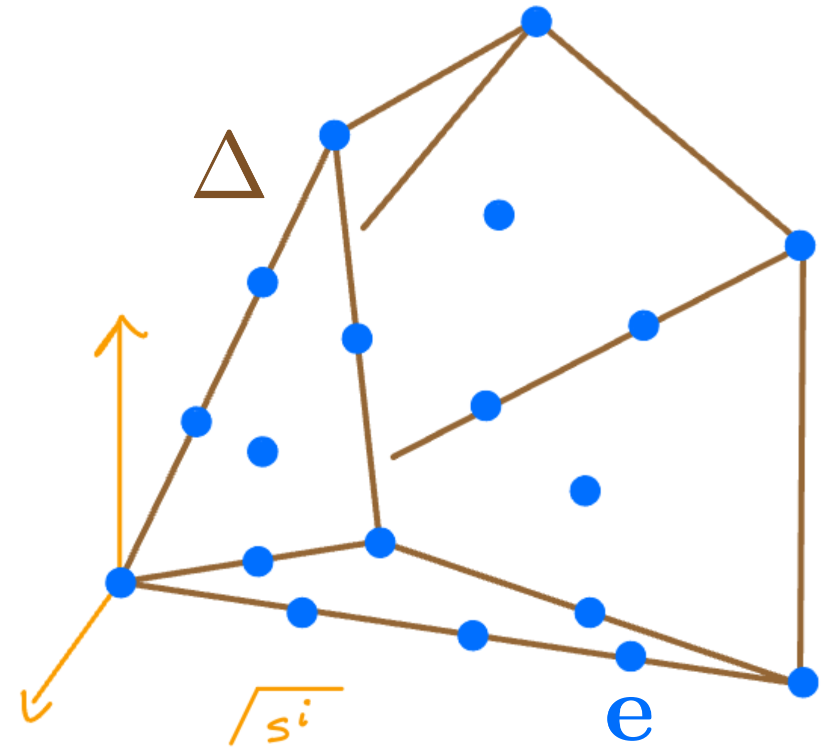
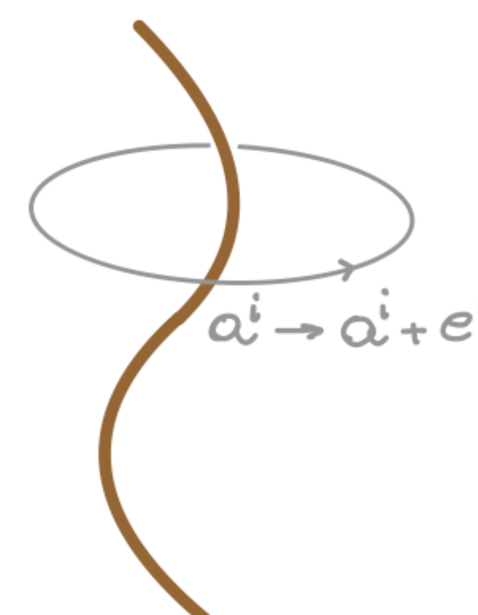


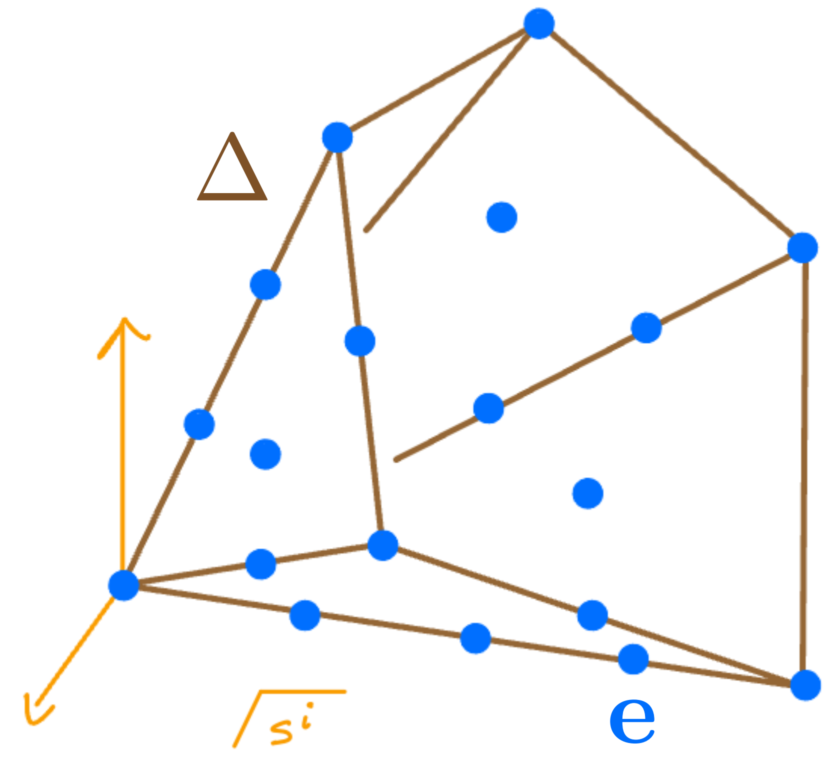
- populated spectrum of EFT string charges

$$\mathbf{e} \equiv \{e^i\} \in \mathcal{C}_{\rm S}^{\rm EFT} \equiv \{\text{saxionic cone}\}_{\mathbb{Z}}$$



$\mathcal{N} = 1$ GB-term: $$\int_{\mathcal{M}} \gamma(s)\, E_{\mathrm{GB}}$$

with $$\gamma(s) \equiv \frac{\pi}{6}\,\tilde{C}_i s^i$$



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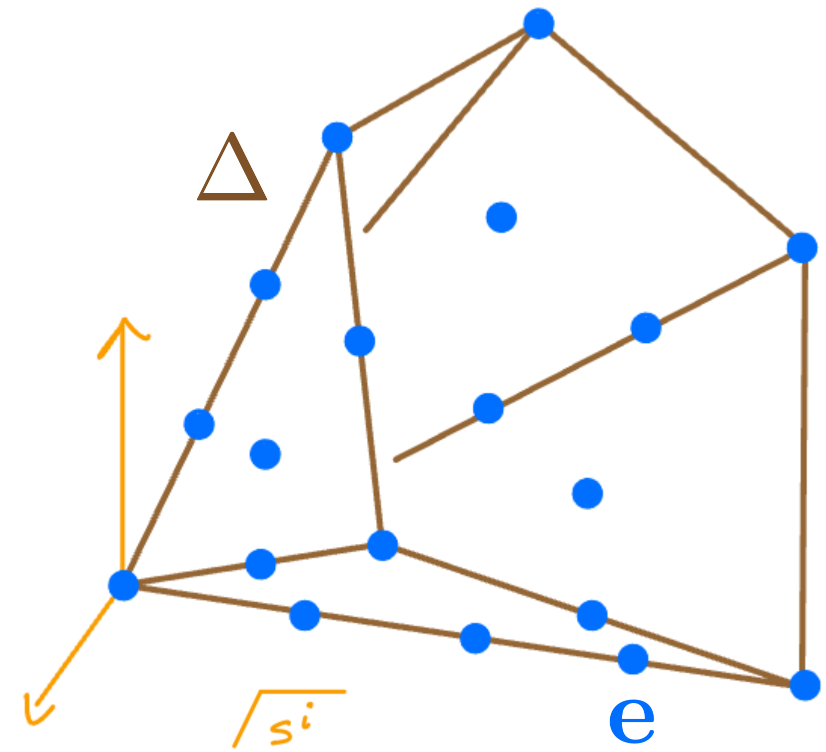


with $$\gamma(s) \equiv \frac{\pi}{6}\,\tilde{C}_i s^i$$

- Quantum consistency of EFT strings [LM-Risso-Weigand '23]

$$\tilde{C}_i e^i \in 3\mathbb{Z}_{\geq 0}\,, \qquad \forall \mathbf{e} \in \mathcal{C}_{\rm S}^{\rm EFT}$$ ⟶ $$\gamma(s) > 0$$ positive GB coupling!

$$r(\mathbf{e}) \leq 2\tilde{C}_i e^i - 2\,, \quad \forall \mathbf{e} \in \mathcal{C}_{\rm S}^{\rm EFT}$$ bounds on ranks determined by GB!

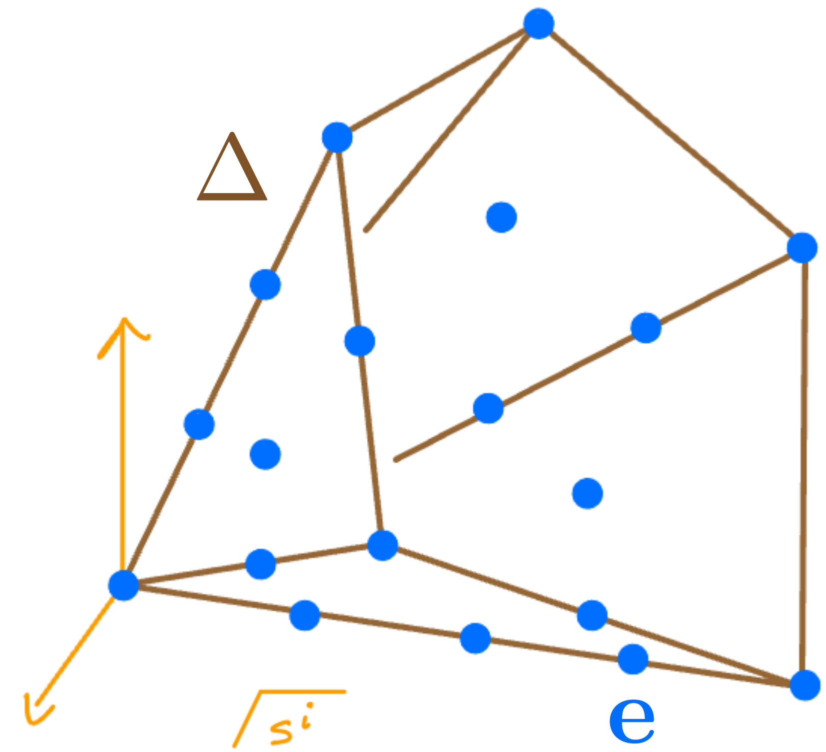
rank of gauge group "detected" by **e**-string

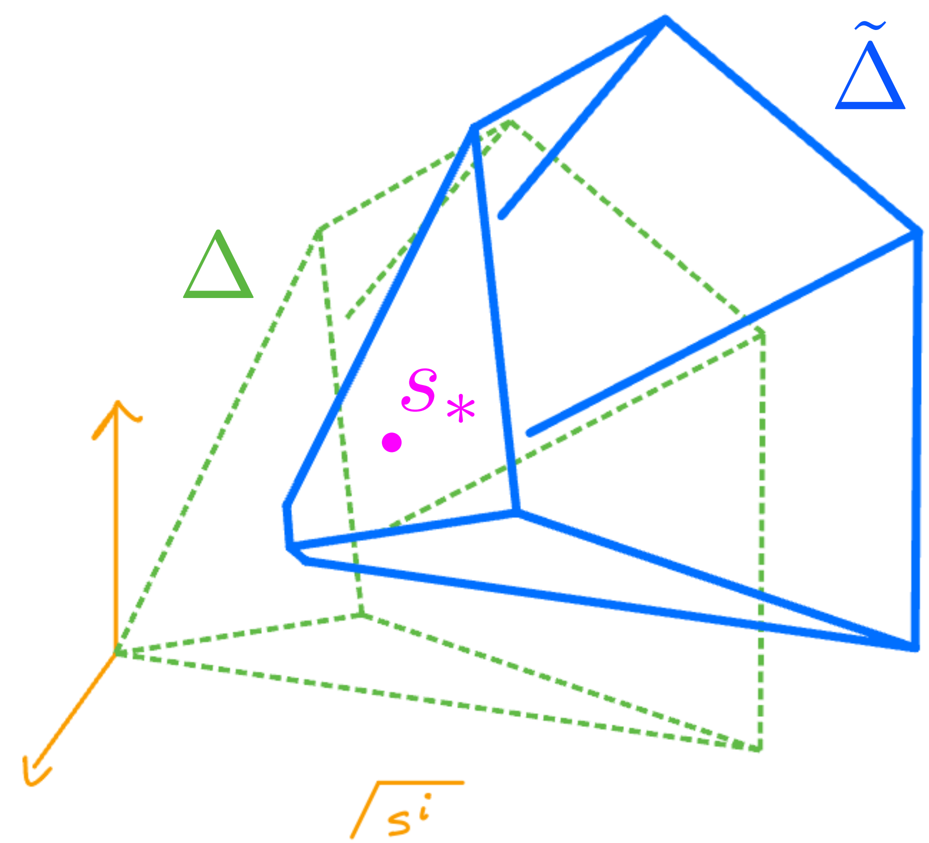
- $\mathcal{N}=1$ GB-term: $\int_{\mathcal{M}} \gamma(s)\, E_{\rm GB}$

with $$\gamma(s) \equiv \frac{\pi}{6}\,\tilde{C}_i s^i$$

↓ IR dynamics

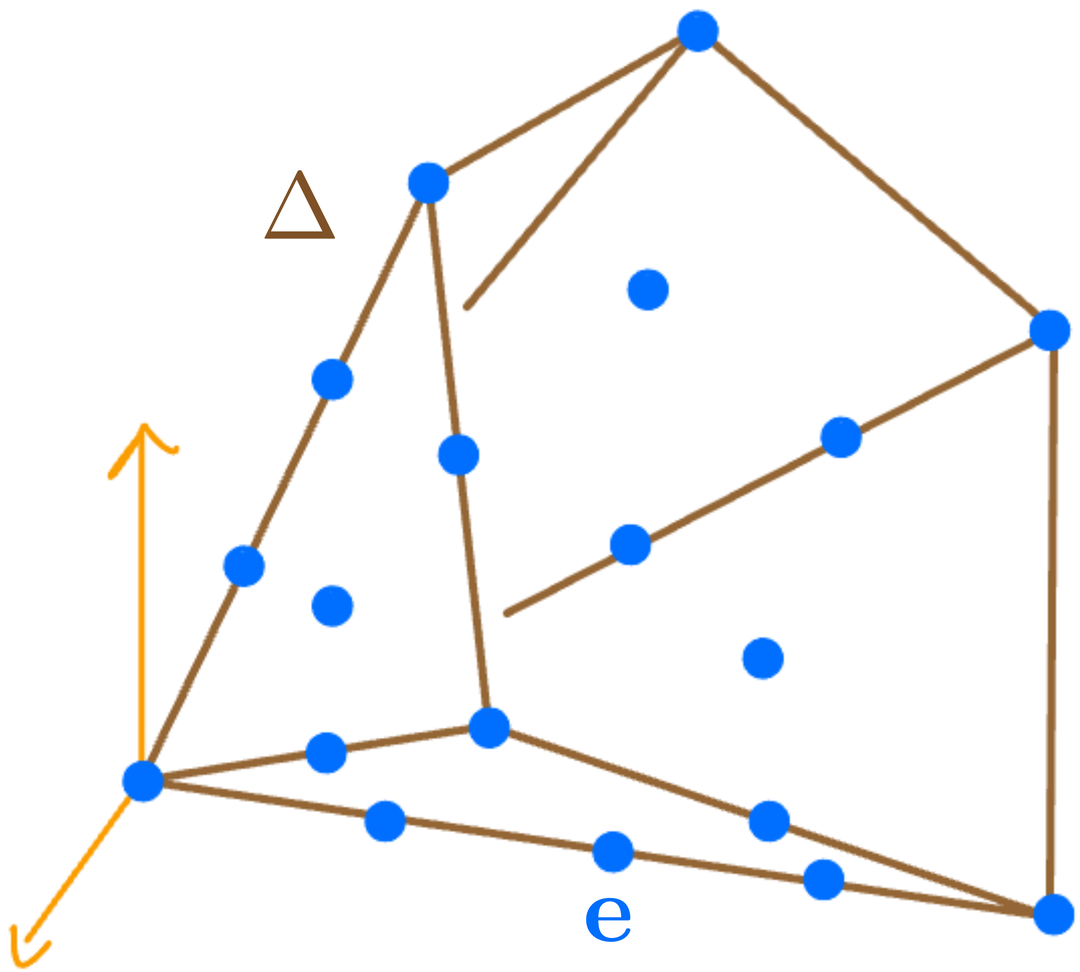
$$\gamma_{\rm IR} \simeq \gamma_* = \frac{\pi}{6}\,\tilde{C}_i s^i_*$$

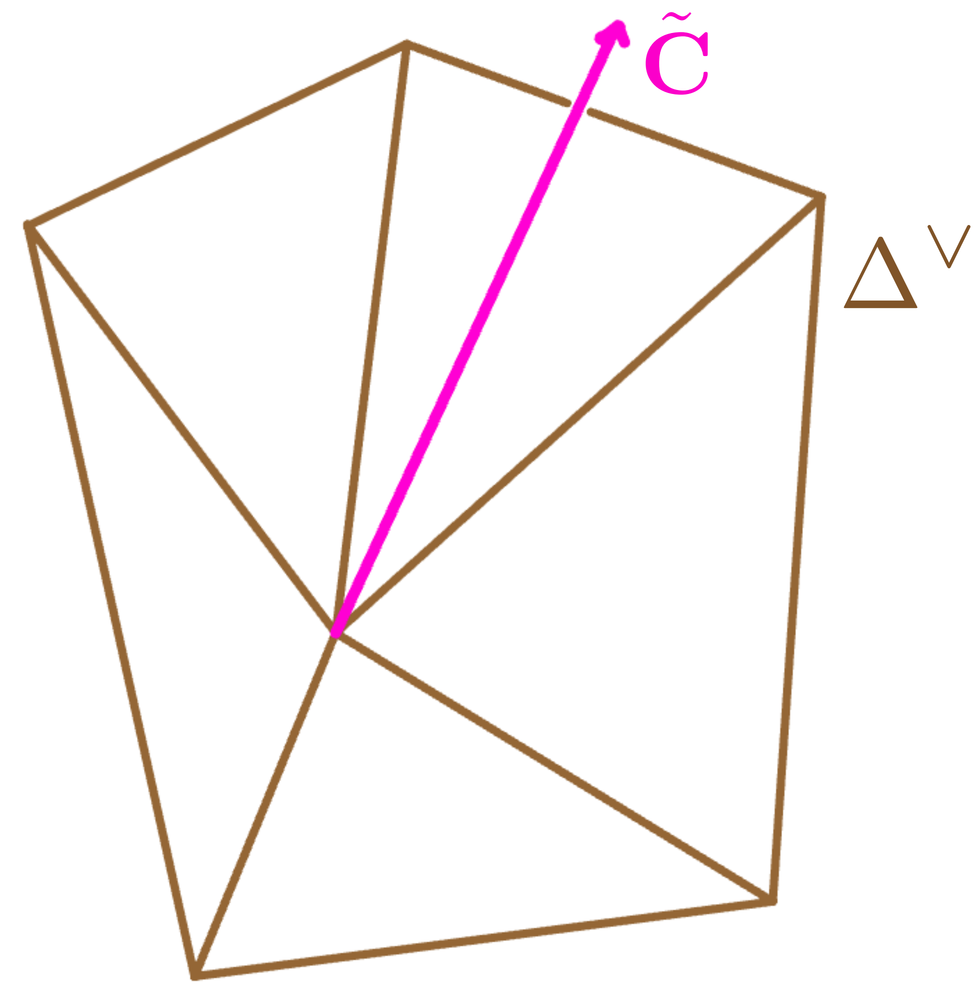




The γ_* bound

- In fact, generically: $\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$ ← chiral EFT string world-sheet





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- Take saxionic cone generated by basis of EFT string charges

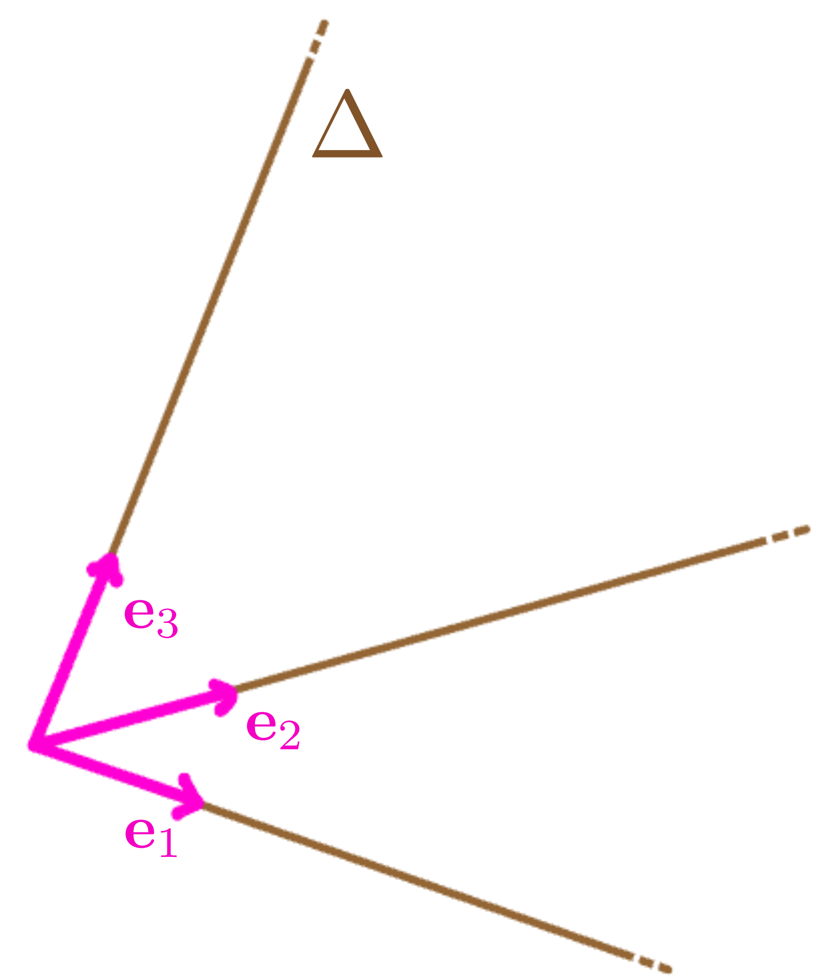
$$\mathbf{e}_{(i)} \in \mathcal{C}_{\rm S}^{\rm EFT}$$

$$\Downarrow$$

$$\tilde{C}_i \in 3\mathbb{Z}_{>0}$$

$$\Downarrow$$

$$\gamma_* = \frac{\pi}{6}\tilde{C}_i s^i_* \geq \frac{\pi}{2}\sum_{i=1}^{N} s^i_*$$



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cf. [Demirtas-Long-McAllister-Stillman '18]

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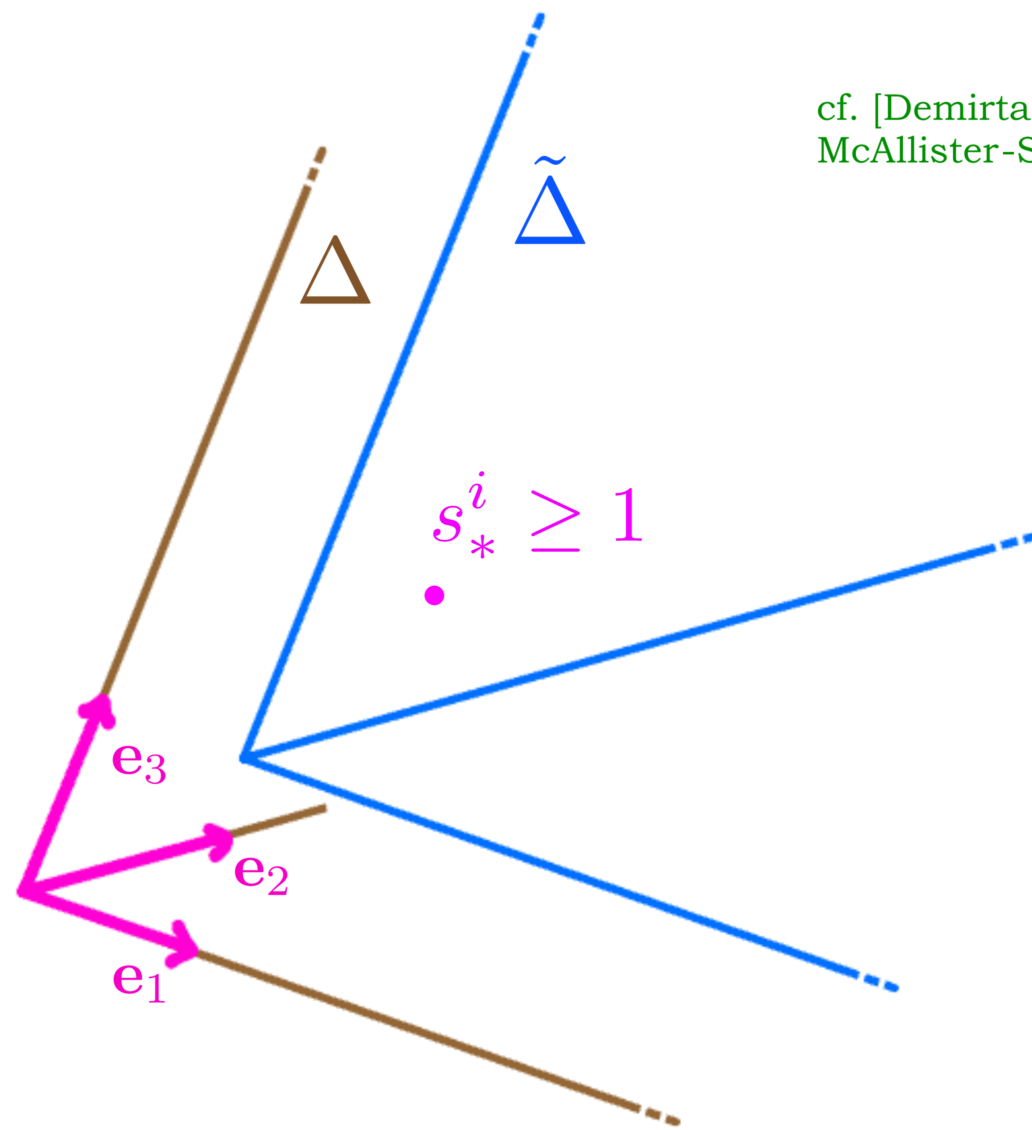


$$\mathbf{e}_{(i)} \in \mathcal{C}^{\rm EFT}_{\rm S}$$

$$\Downarrow$$

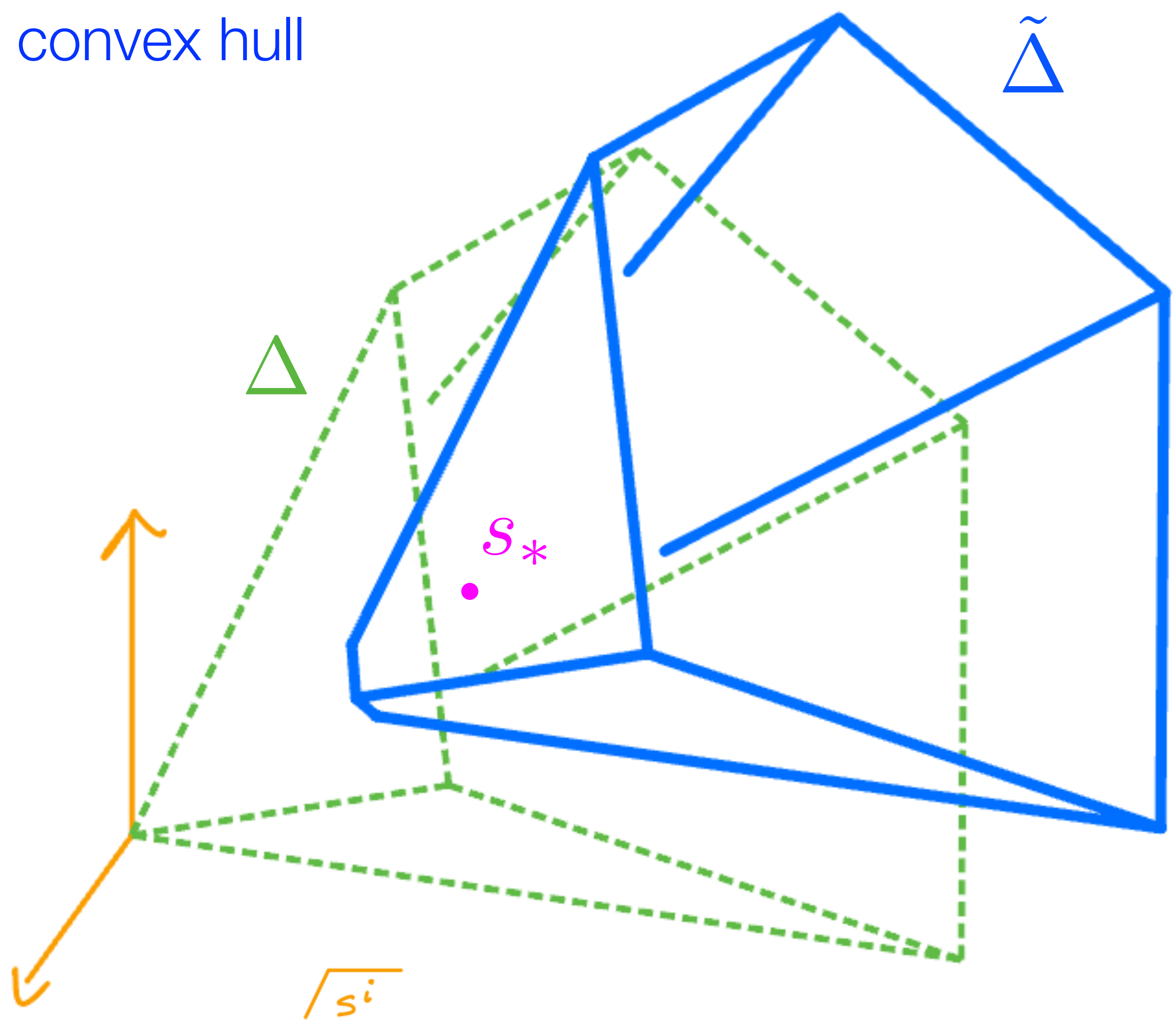
$$\tilde{C}_i \in 3\mathbb{Z}_{>0}$$

$$\Downarrow$$

$$\gamma_* = \frac{\pi}{6}\tilde{C}_i s^i_* \geq \frac{\pi}{2}\sum_{i=1}^{N} s^i_* \quad \Longrightarrow \quad \gamma_* \geq \frac{\pi N}{2}$$

- In fact, generically: $\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$ ← chiral EFT string world-sheet

- More generically: stretched saxionic convex hull



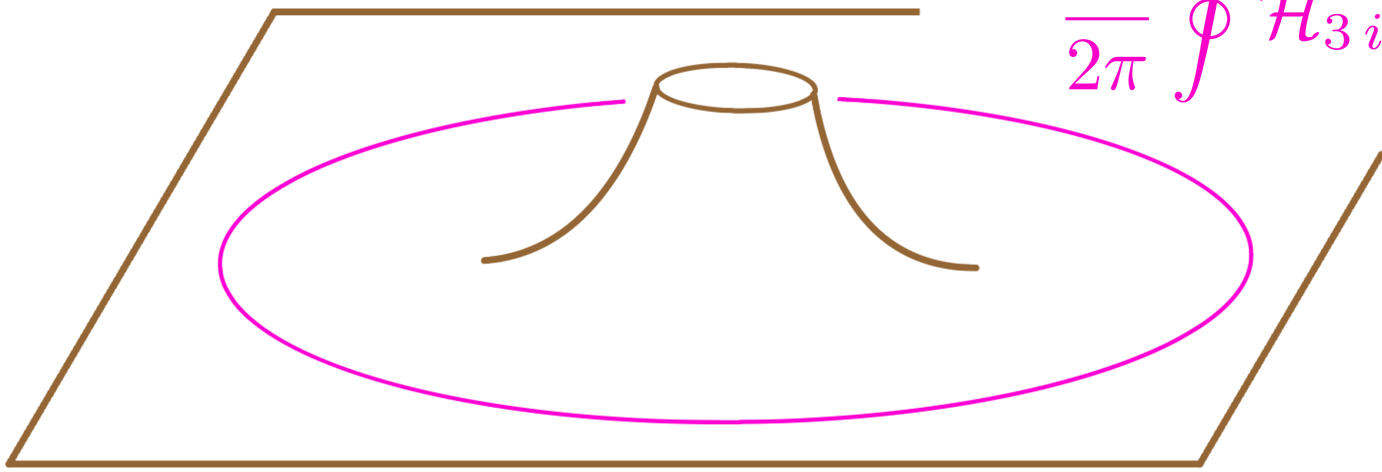
$$\gamma_* \geq \frac{\pi N}{2}$$

Implications on wormholes

Purely axionic wormholes

[Giddings-Strominger ’87, Lee ’88,..., Montero-Uranga-Valenzuela ’15, Bachlenchner-Long-McAllister ’15, ...]

$$\frac{1}{2\pi}\oint \mathcal{H}_{3\,i} = m_i$$



$$S_{\rm E} = \frac{M_{\rm P}^2}{2}\int \left(-R + \mathcal{G}_*^{ij}\mathcal{H}_{3\,i}\wedge *\mathcal{H}_{3\,j}\right)$$

$$S_{\rm E}^{\frac{1}{2}\,\rm wh} = \frac{\sqrt{6}\,\pi^2}{2}\sqrt{\mathcal{G}_*^{ij} m_i m_j}$$

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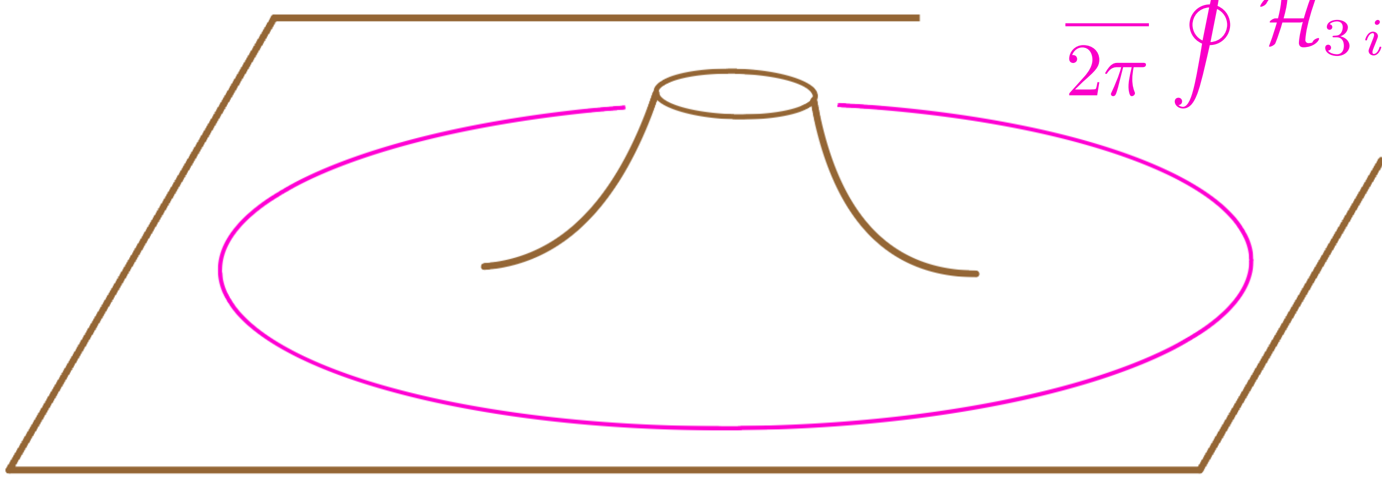

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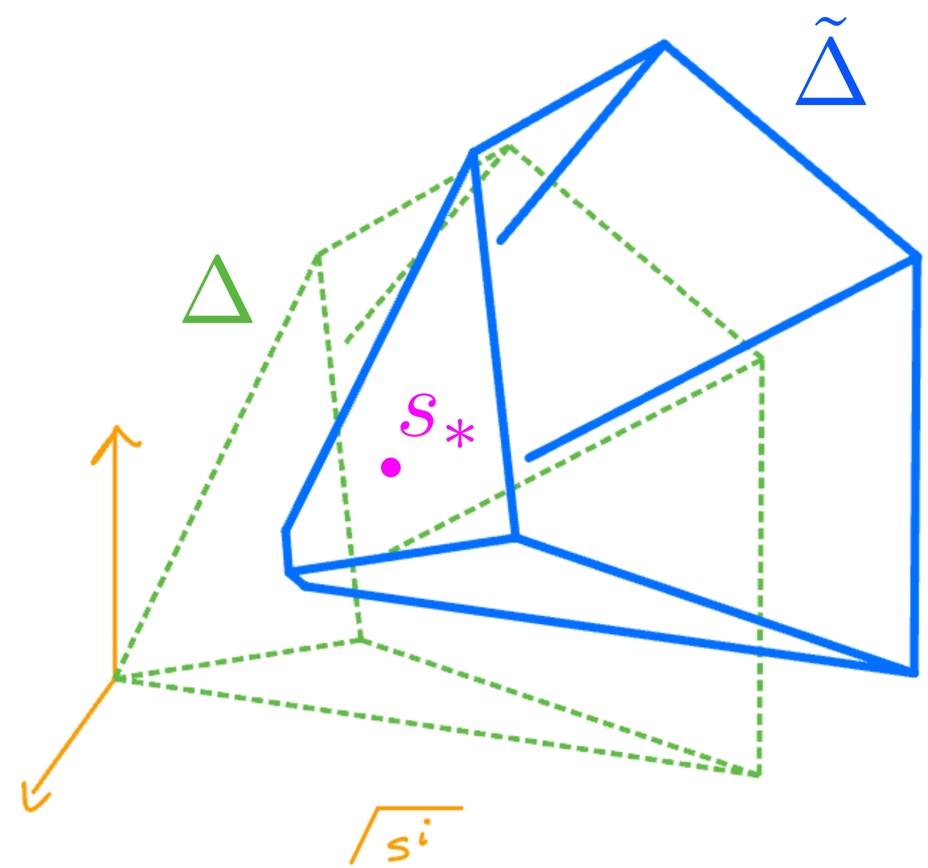
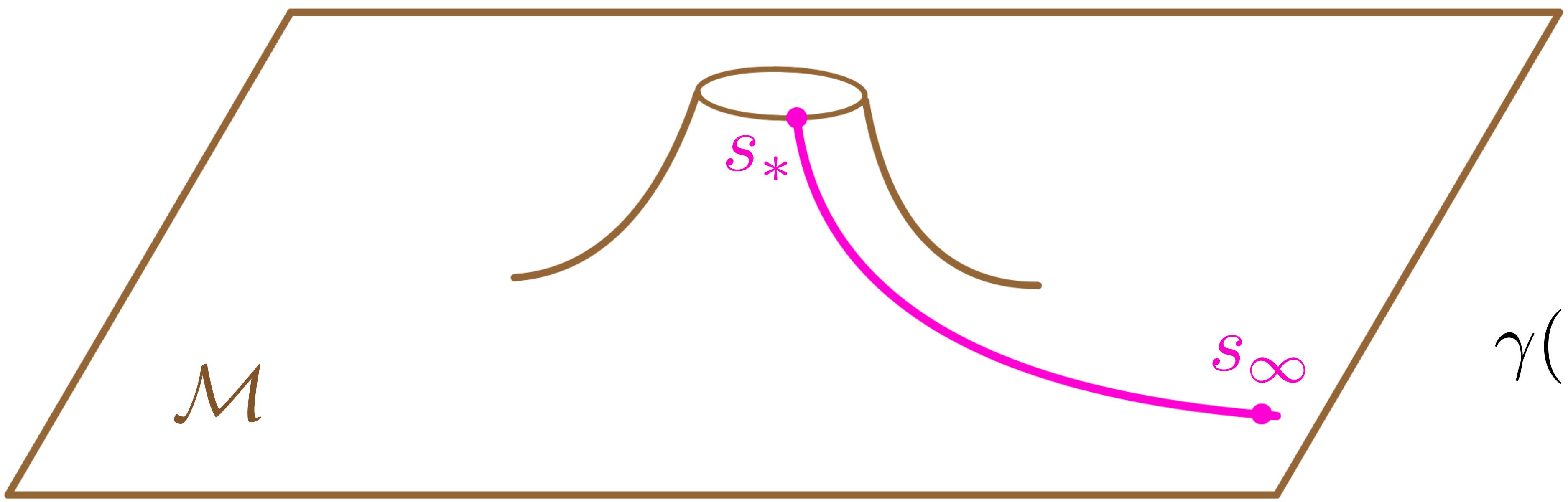
$$S_{\rm GB} = \gamma_* \geq \frac{\pi N}{2}$$

dominates for $N \gg \|\vec{m}\|$!



Saxionic wormholes

[Abbott-Deser '88, … , Kallosh-Linde-Linde-Susskind '95, …, Gutperle-Sabra '02, Bergshoeff-Collinucci-Gran-Roest-Vandoren '05, Arkani-Hamed, Orgera, Polchinski '07, … , Hebecker-Mangat-Theisen-Witkowski '16, … Alvey-Escudero '20, Andriolo-Shiu-Soler-Van Riet '22, LM-Risso-Valenti-Vecchi '23]

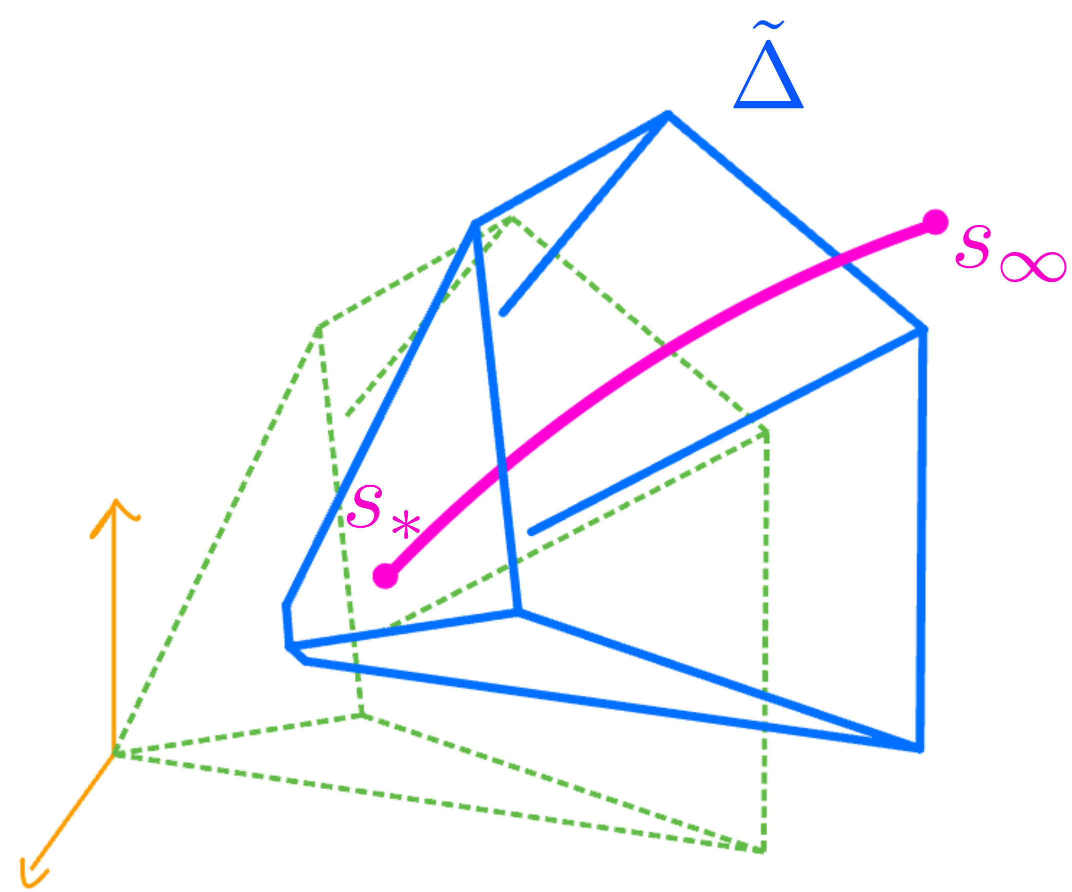


$$\gamma(s) \equiv \frac{\pi}{6}\tilde{C}_i s^i > \gamma_*$$

$$S_{\rm GB} = -\int_{\mathcal{M}} \gamma(s) E_{\rm GB}$$

$$\geq -\gamma_* \int_{\mathcal{M}} E_{\rm GB} = \gamma_* \geq \frac{\pi N}{2}$$

$$\Rightarrow \quad S_{\rm GB} \geq \frac{\pi N}{2}$$



- So: $S_{\rm tot}^{\frac{1}{2}\,\rm wh} = S_{\rm E} + S_{\rm GB}$ with $$S_{\rm GB} \geq \frac{\pi N}{2}$$

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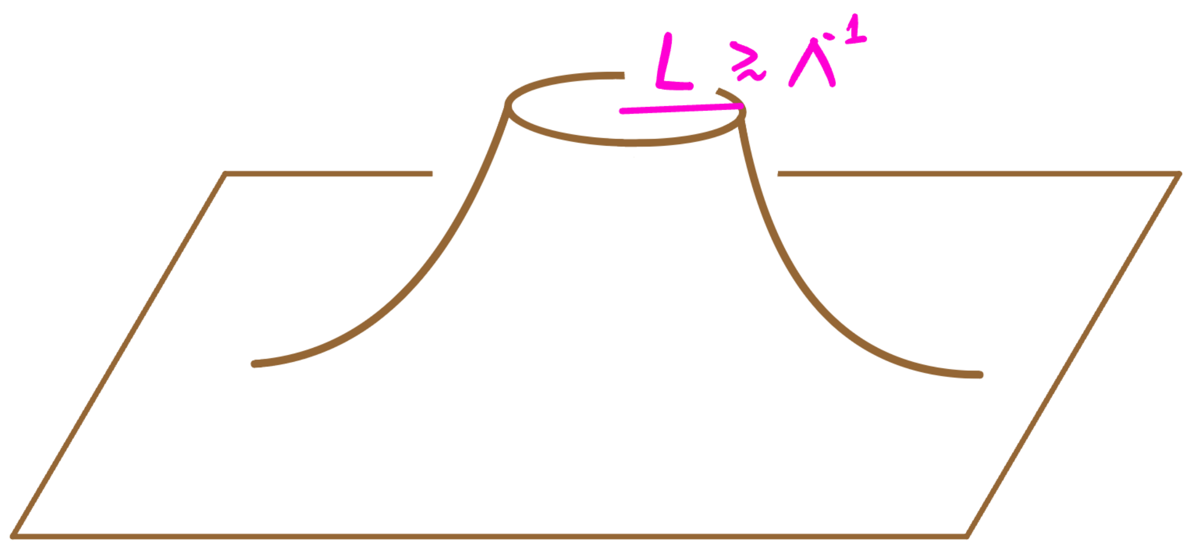
- EFT regime: $S_{\rm E} \simeq M_{\rm P}^2 L^2 \; \geq \dfrac{M_{\rm P}^2}{\Lambda^2}$



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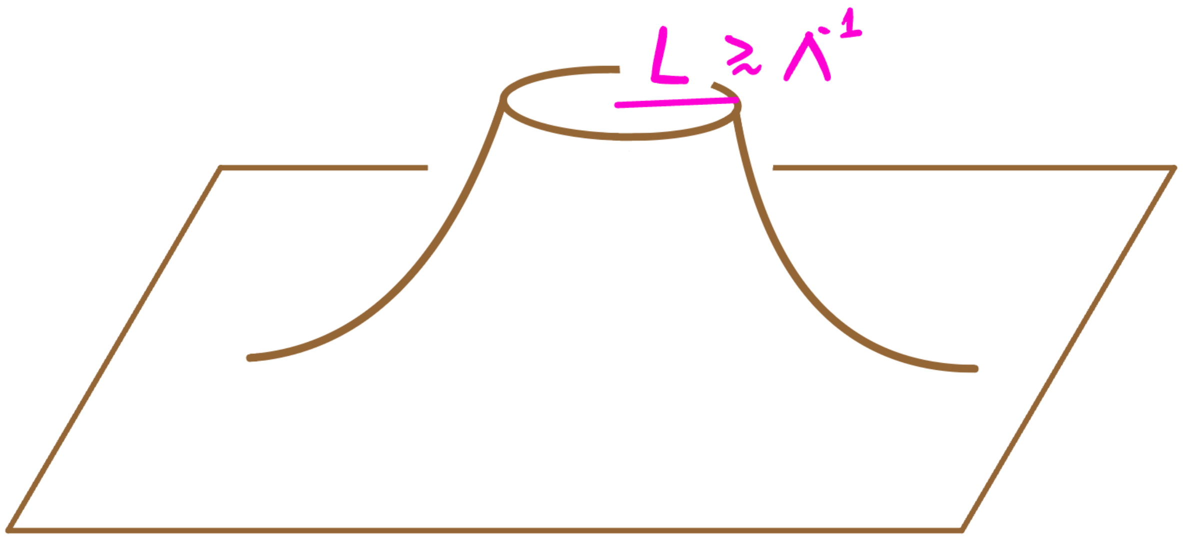


- $S_{\text{E}} \gtrsim S_{\text{GB}} \Longleftarrow \Lambda \lesssim \frac{M_{\text{P}}}{\sqrt{N}}$

species scale bound!

Conclusions

- GB-term induces large-N exponential suppression

$$\Delta_{\rm wh}\mathcal{L} \sim e^{-S_{\rm GB}} e^{-S_{\rm E}} \leq \exp\left(-\frac{\pi N}{2}\right) e^{-S_{\rm E}}$$

Conclusions

- GB-term induces large-N exponential suppression

$$\Delta_{\rm wh}\mathcal{L} \sim e^{-S_{\rm GB}} e^{-S_{\rm E}} \leq \exp\left(-\frac{\pi N}{2}\right) e^{-S_{\rm E}}$$

- E.g. QCD axion:

  * $\Delta V(\vartheta) \simeq \mu^4 \, e^{-\gamma_*} e^{-S_{\rm E}} \cos(\vartheta + \delta)$

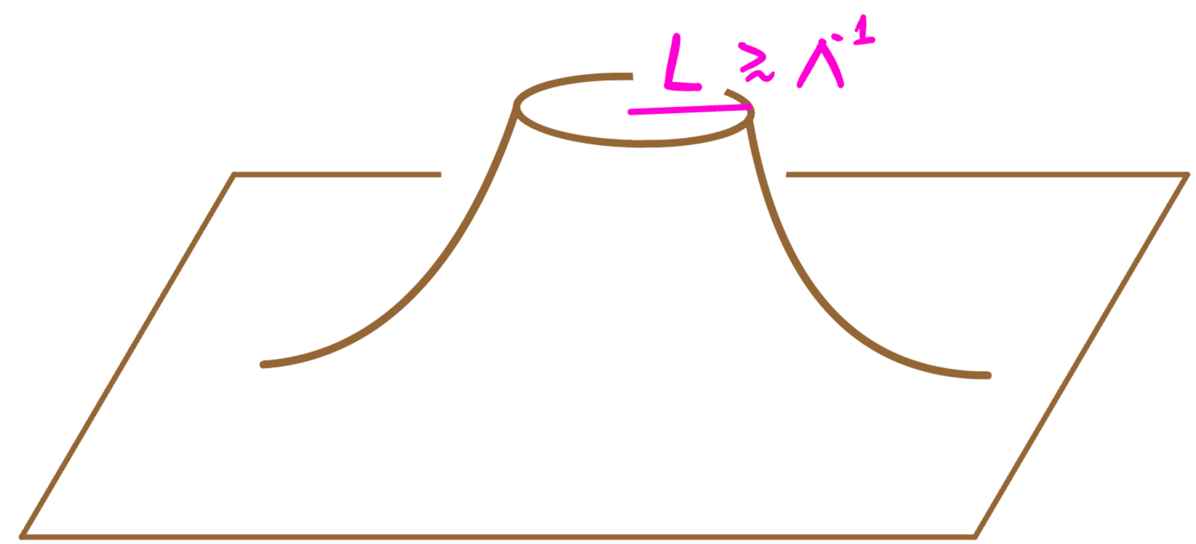
  * no quality PQ-quality problem if $\gamma_* > 170$ [Kallosh-Linde-Linde-Susskind '94 Alonso-Urbano '17]

automatic if $N > 120$ → cf. [Demirtasa-Gendler-Long-McAllisterc-Moritz '22]

Outlook

- Scale separation and 10/11d uplift?

  [… Loges-Shiu-Van Riet ’23]



- Gravitational path integral?

  [… Loges-Shiu-Sudhir’ 22, AguilarGutierrez-Hertog-Tielemans-van der Schaar-Van Riet 23]

- α-states and Baby Universe Hypothesis?

  [Coleman-Giddings-Strominger ’88… Marolf-Maxfield ’20, McNamara-Vafa ’20…]

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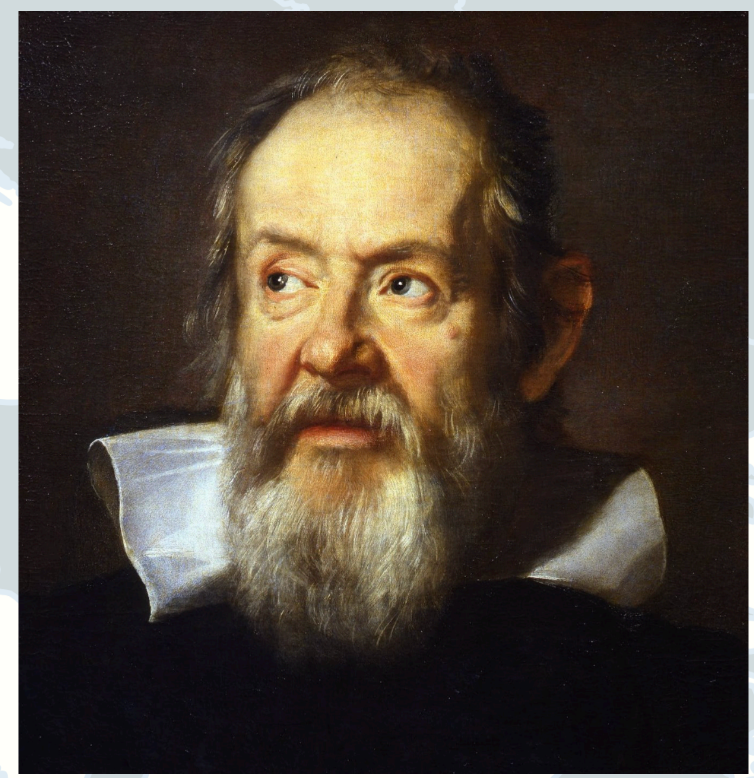

"**the best eighteen years of my life**" (1592-1610)


Venice
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"The Landscape vs. the Swampland"
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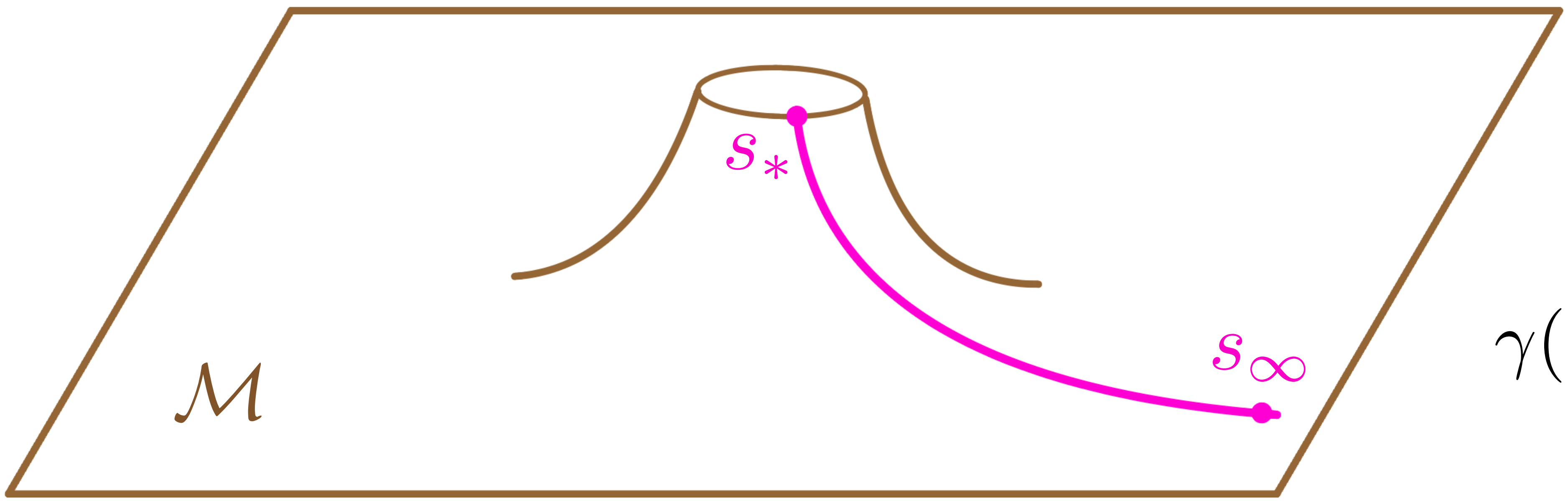
See you next year
in Padova!

Thanks!

Wormholes vs fundamental BPS instantons

$$S_{\rm E}^{\frac{1}{2}{\rm wh}} < S_{\rm E}^{\rm BPS}$$

[Gutperle-Sabra ’02, Bergshoeff-Collinucci-Gran-Roest-Vandoren ‘05, Arkani-Hamed, Orgera, Polchinski ’07, LM-Risso-Valenti-Vecchi ’23]



$$\gamma(s) \equiv \frac{\pi}{6}\tilde{C}_i s^i > \gamma_*$$

but

$$S_{\rm E}^{\frac{1}{2}{\rm wh}} + S_{\rm GB} > S_{\rm E}^{\frac{1}{2}{\rm wh}} + \gamma_* > S_{\rm E}^{\rm BPS}$$

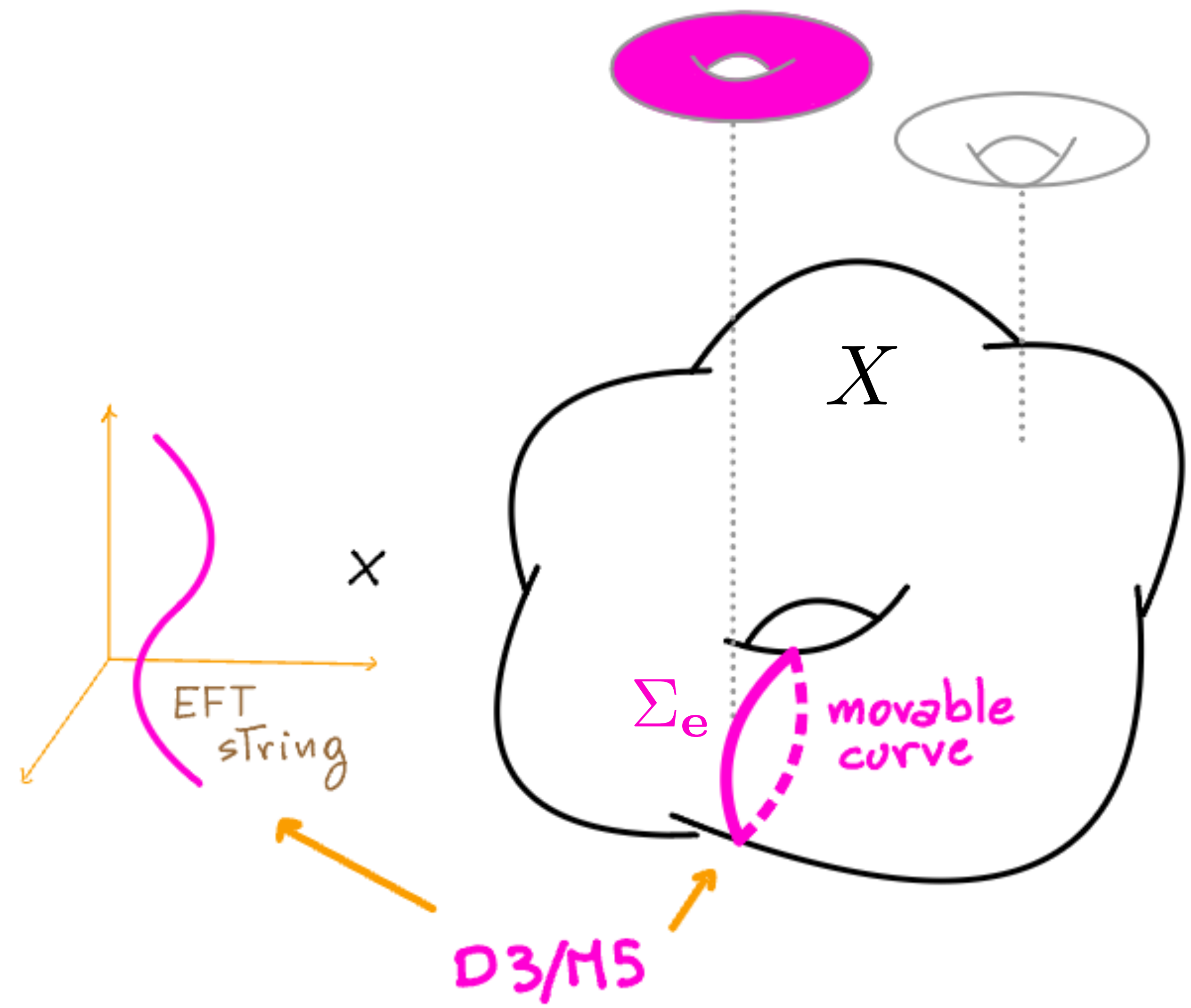
[LM-Risso-Valenti-Vecchi ’23]

implications for axion WGC?

[Arkani-Hamed-Motl-Nicolis-Vafa ’07,
Heidenreich-Reece-Rudelius ’16
Hebecker-Mangat-Theisen-Witkowski ’16
Hebecker-Mikhail-Soler ’18,…
Harlow-Heidenreich-Reece-Rudelius ’22]

- In fact, generically: $\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$ $\longleftrightarrow$ chiral EFT string world-sheet

- F-theory realization



* $\tilde{C}_i e^i = 6\Sigma_{\mathbf{e}} \cdot \overline{K}_X$

* $\overline{K}_X$ effective
  $\Sigma_{\mathbf{e}}$ movable

$$\Downarrow$$

$$\tilde{C}_i e^i \in 6\mathbb{Z}_{\geq 0}$$

* E.g. toric X $\longrightarrow$ $\overline{K}_X = \sum_{I \in \{\text{toric div.}\}} D_I$ $\longrightarrow$ $\Sigma_{\mathbf{e}} \cdot \overline{K}_X > 0$