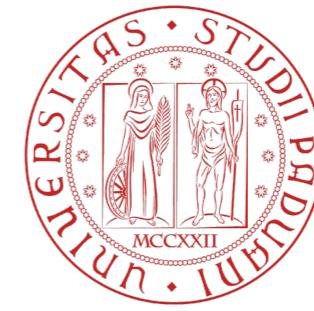




DIPARTIMENTO  
DI FISICA  
E ASTRONOMIA  
Galileo Galilei



# QG bounds, Gauss-Bonnet & wormholes

Luca Martucci

Padua University

based on: N. Risso, A. Valenti, L. Vecchi (in progress)  
N. Risso & T. Weigand 2210.10797

# Motivations

- EFTs generically contain a Gauss-Bonnet (GB) term

$$\gamma \int_{\mathcal{M}} E_{\text{GB}}$$

$$E_{\text{GB}} \equiv \frac{1}{32\pi^2} (R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2)$$

- Strong evidence of QG bound:  $\gamma > 0$

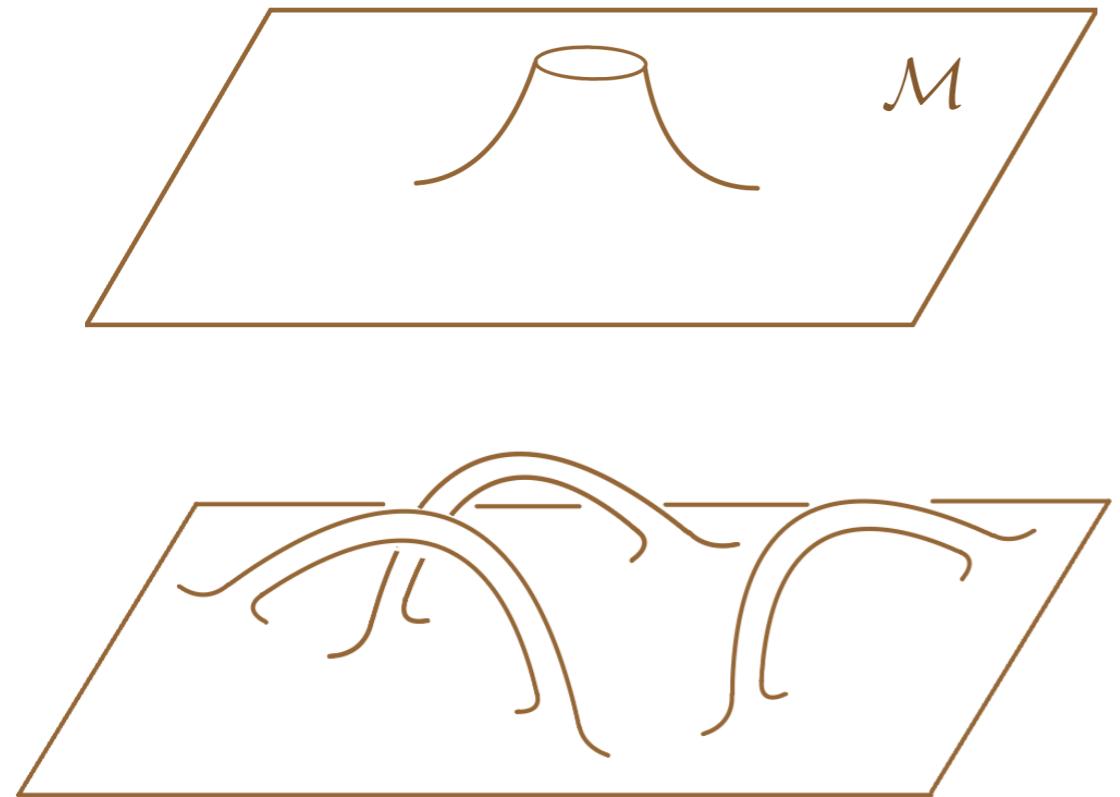
[..., Brigante-Liu-Myers-Shenker-Yaida '08,...  
Cheung-Remmen '16, GarcíaEtxebarria-Montero-Sousa-Valenzuela '20, Aalsma-Shiu '22, Chin Ong '22, LM-Risso-Weigand '23]

- Topological in 4d EFTs

$$\int_{\mathcal{M}} E_{\text{GB}} + \int_{\partial\mathcal{M}} Q_{\text{GB}} = \chi(\mathcal{M})$$

*physical implications? non-perturbative QG effects!*

- GB relevance to wormholes



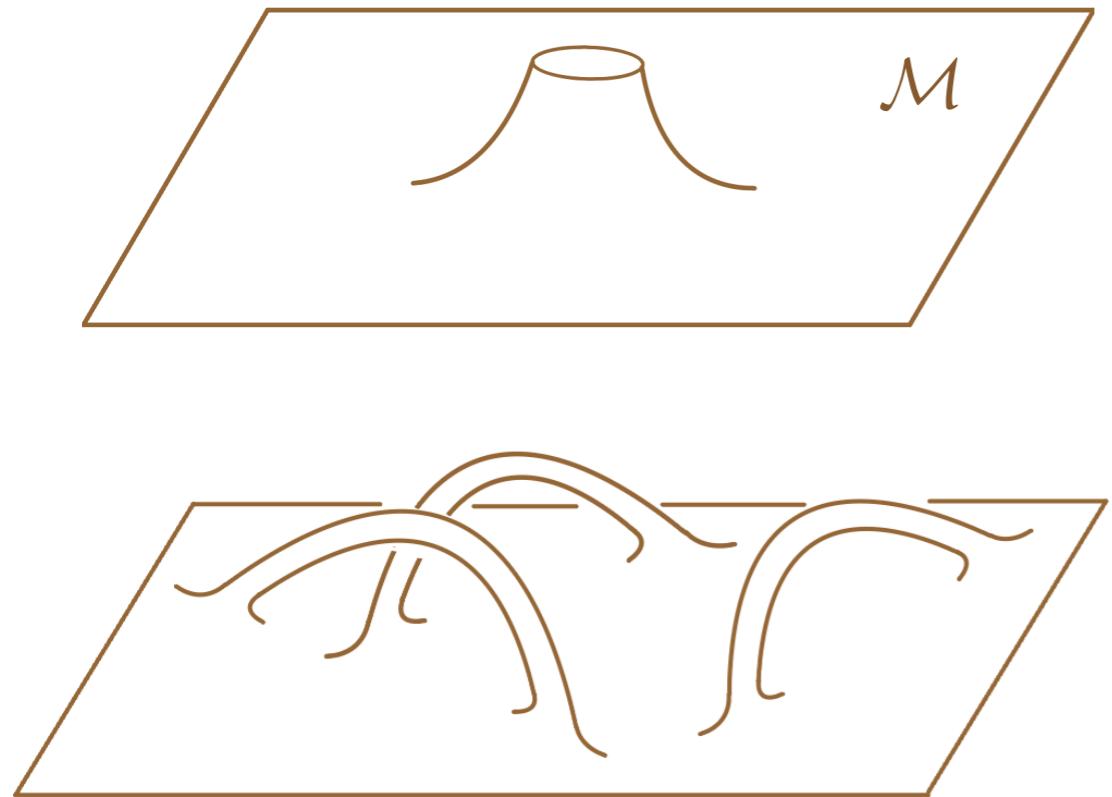
[Giddings-Strominger '88-'89, Coleman-Lee '88-'89, Preskill '89, ...  
Kallosh-Linde-Linde-Susskind '94,  
Alonso-Urbano '17, ...]

$$\Delta \mathcal{L}_{\text{eff}} = e^{-S_{\text{GB}}} \sum_i e^{-S_{\text{E}}^i} \mathcal{O}_i(x)$$

$$e^{-S_{\text{GB}}} = e^{\gamma[\chi(\mathcal{M})-1]} = e^{-\gamma}$$

$\gamma > 0 \quad \Rightarrow \quad \underline{\text{universal}} \\ \text{suppression !}$

- GB relevance to wormholes



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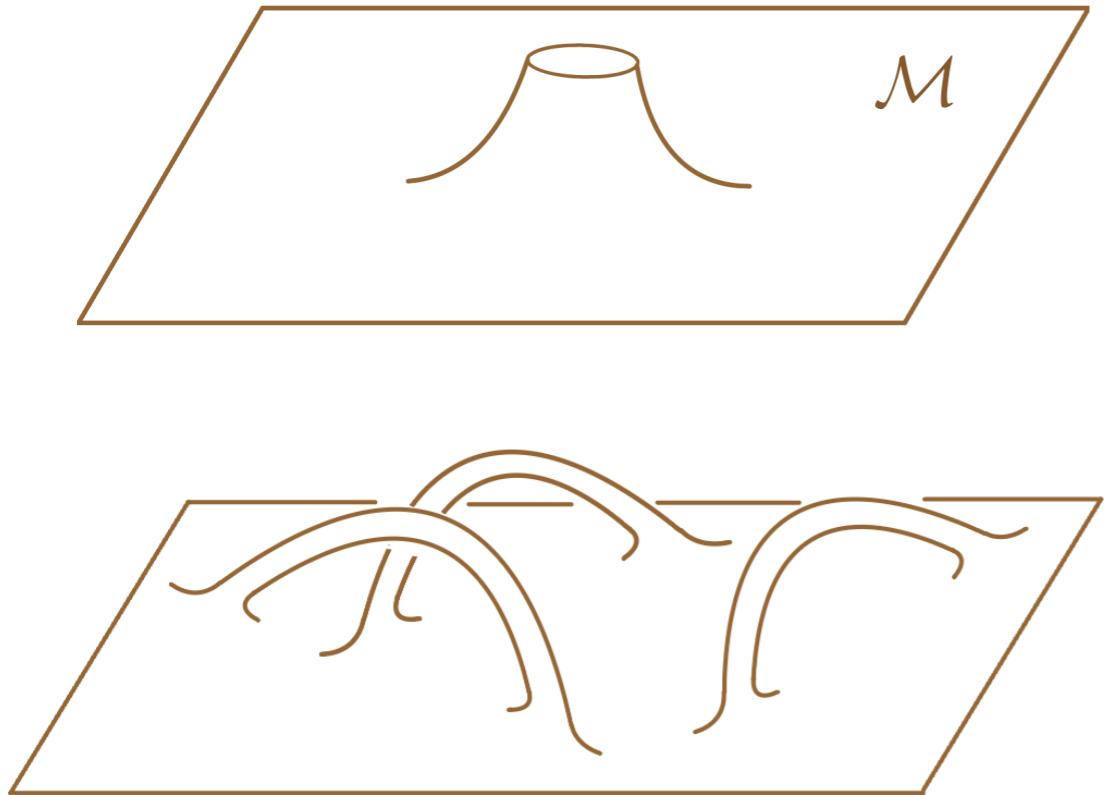
- Potentially important physical implications: axion physics

$$\Delta V(\text{axions}) \propto e^{-\gamma} \sum_m e^{-S_{\text{E}}^m} \cos(m\vartheta + \delta_m)$$

[Kim-Lee '88, Rey '89,...,  
Hebecker-Mikhail-Soler '18]

PQ quality problem  
axion inflation  
axion dark matter

- GB relevance to wormholes



[Giddings-Strominger '88-'89, Coleman-Lee '88-'89, Preskill '89, ...  
Kallosh-Linde-Linde-Susskind '94,  
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$$e^{-S_{\text{GB}}} = e^{\gamma[\chi(\mathcal{M})-1]} = e^{-\gamma}$$

$\gamma > 0 \quad \Rightarrow \quad \underline{\text{universal}} \\ \text{suppression !}$

- \* Additional QG-constraints on  $\gamma$  ?
- \* Conditions for  $\gamma \gg 1$  ?

4d setting

- I will assume (intermediate) 4d  $\mathcal{N} = 1$  UV completion

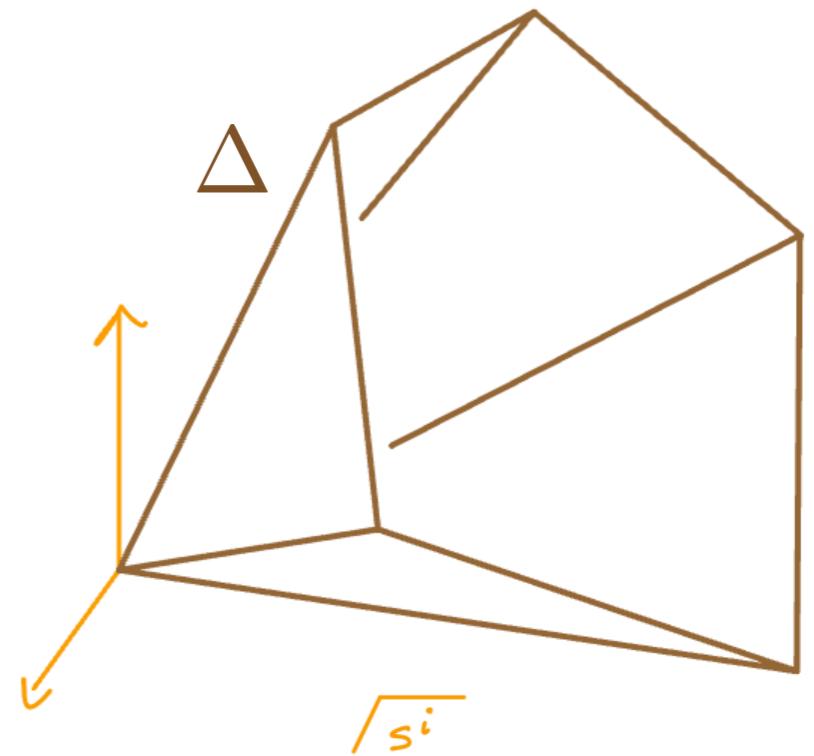
- perturbative regime specified by  $N$  (s)axions

$$a^i \simeq a^i + 1 \quad , \quad i = 1, \dots, N$$

$$\{s^i\} \in \Delta \equiv \{\text{saxionic cone}\}$$

[Lanza-Marchesano-LM-Valenzuela '20-'21]

e.g. volumes of effective divisors  
in F-Theory models



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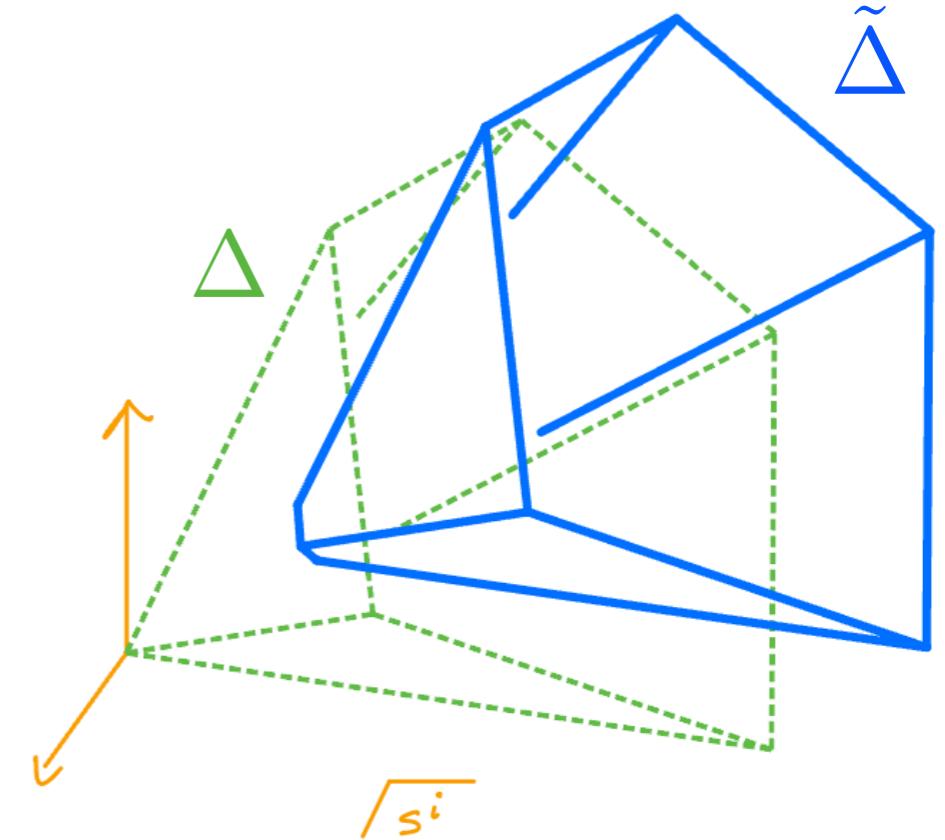
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[Lanza-Marchesano-LM-Valenzuela '20-'21]

e.g. volumes of effective divisors  
in F-Theory models



- Restrict to stretched saxionic convex hull:

$$s \in \tilde{\Delta} \quad \Rightarrow \quad \delta_{\text{F-inst.}} \mathcal{L} \lesssim e^{-2\pi} \simeq 10^{-3}$$

cf. [Demirtas-Long-McAllister-Stillman '18]

- I will assume (intermediate) 4d  $\mathcal{N} = 1$  UV completion

- perturbative regime specified by  $N$  (s)axions

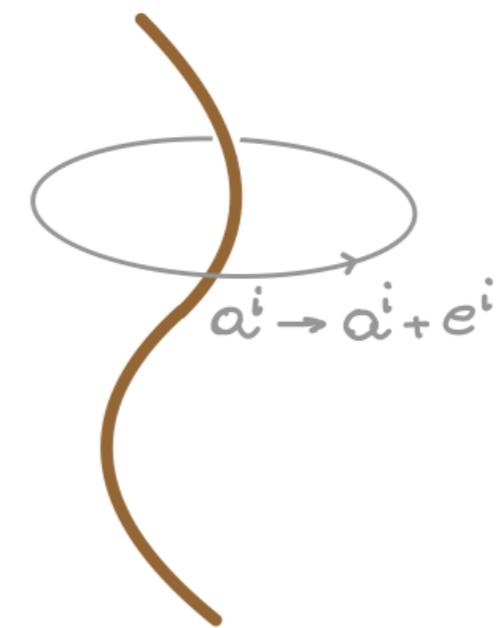
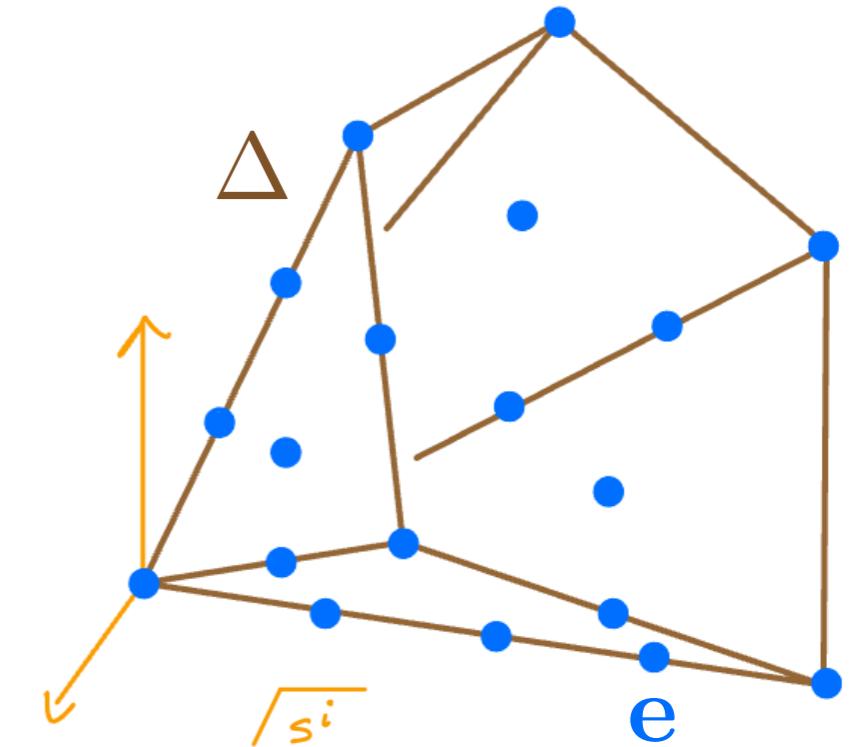
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 [Lanza-Marchesano-LM-Valenzuela '20-'21]  
e.g. volumes of effective divisors  
in F-Theory models

- populated spectrum of EFT string charges

$$\mathbf{e} \equiv \{e^i\} \in \mathcal{C}_S^{\text{EFT}} \equiv \{\text{saxionic cone}\}_{\mathbb{Z}}$$

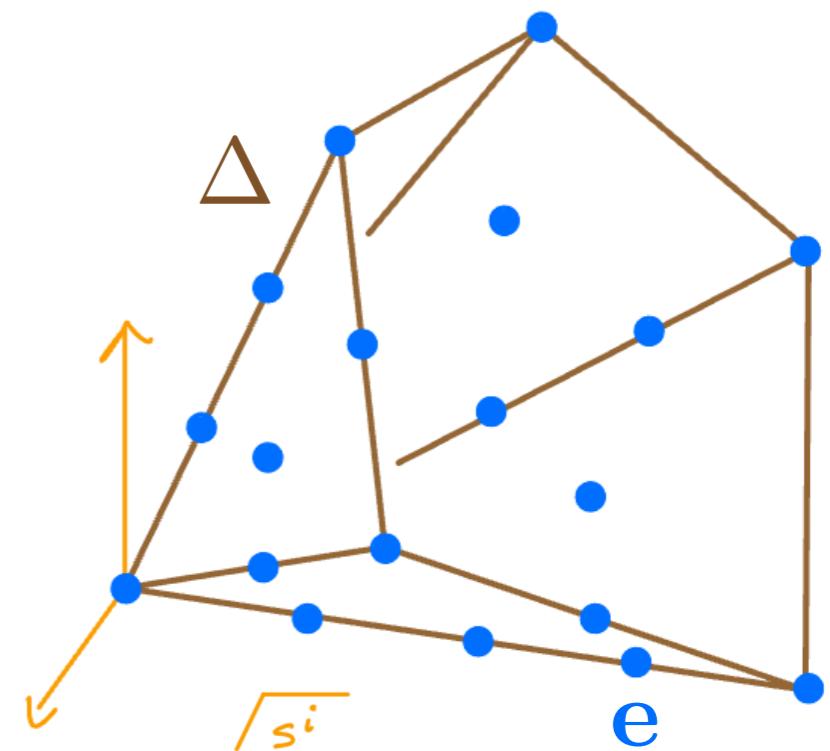


- $\mathcal{N} = 1$  GB-term:

$$\int_{\mathcal{M}} \gamma(s) E_{\text{GB}}$$

with

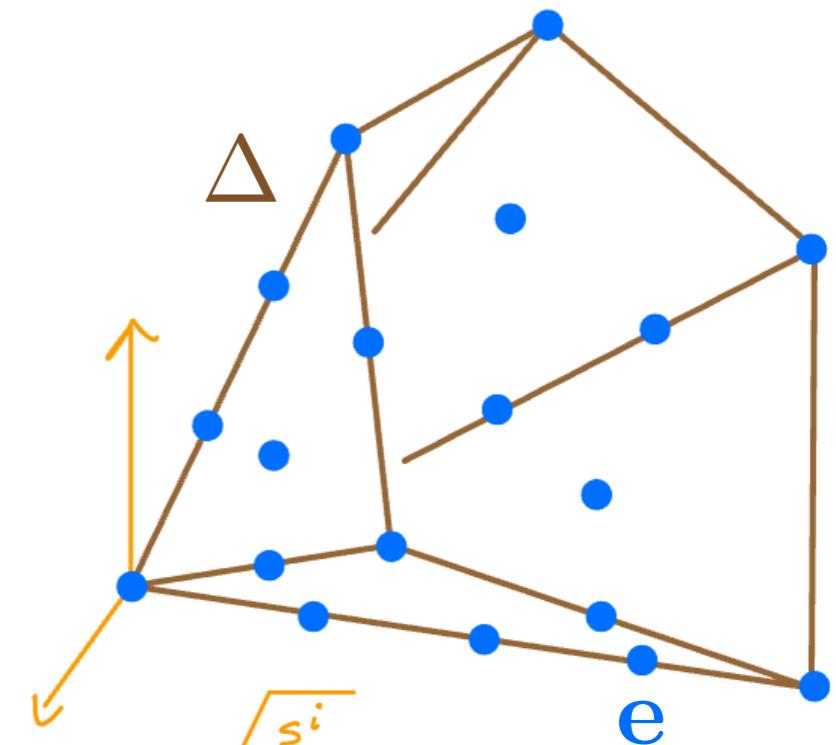
$$\gamma(s) \equiv \frac{\pi}{6} \tilde{C}_i s^i$$



- $\mathcal{N} = 1$  GB-term:  $\int_{\mathcal{M}} \gamma(s) E_{\text{GB}}$

with

$$\gamma(s) \equiv \frac{\pi}{6} \tilde{C}_i s^i$$



- Quantum consistency of EFT strings

[LM-Risso-Weigand '23]

$$\tilde{C}_i e^i \in 3\mathbb{Z}_{\geq 0}, \quad \forall \mathbf{e} \in \mathcal{C}_S^{\text{EFT}}$$



$$\gamma(s) > 0$$

positive GB coupling!

$$r(\mathbf{e}) \leq 2\tilde{C}_i e^i - 2, \quad \forall \mathbf{e} \in \mathcal{C}_S^{\text{EFT}}$$



bounds on ranks  
determined by GB!

rank of gauge group "detected" by e-string

- $\mathcal{N} = 1$  GB-term:

$$\int_{\mathcal{M}} \gamma(s) E_{\text{GB}}$$

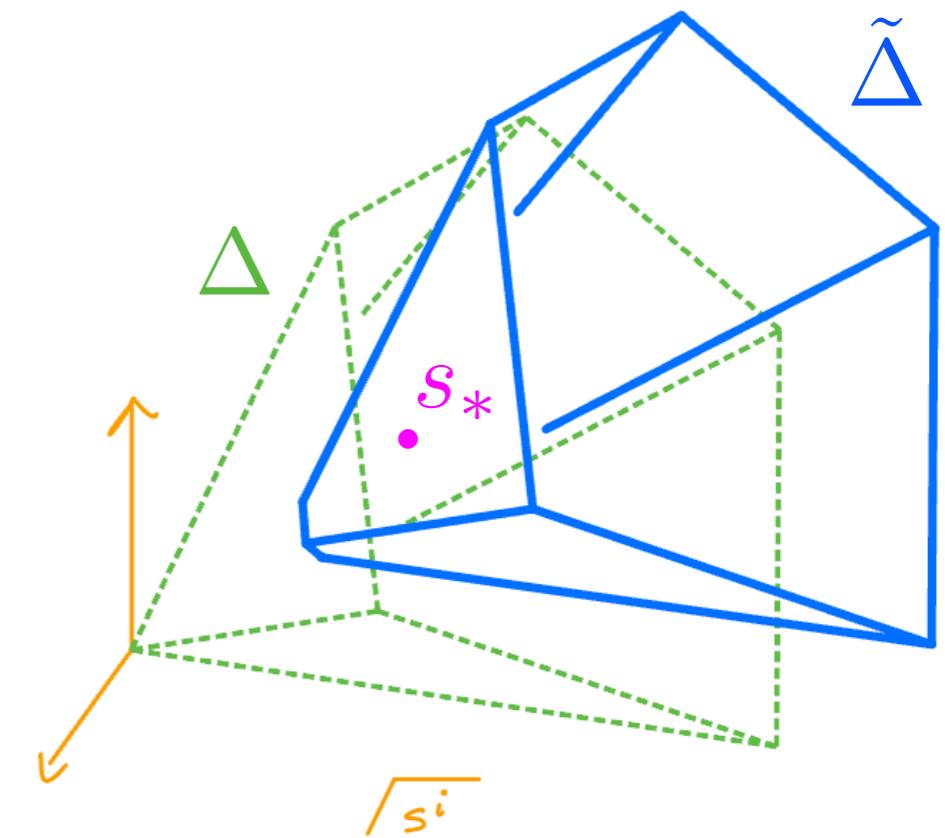
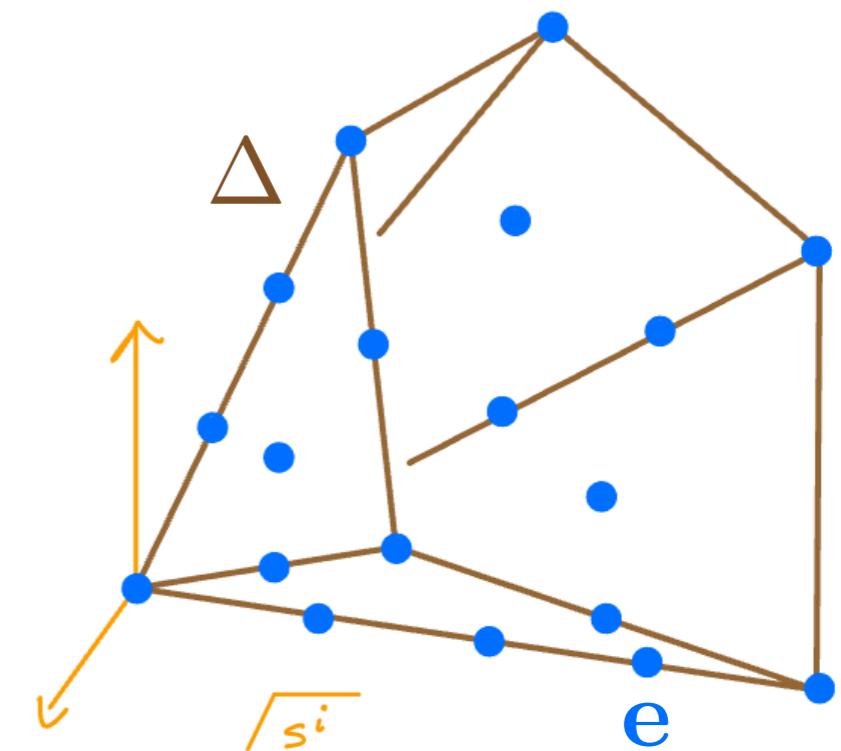
with

$$\gamma(s) \equiv \frac{\pi}{6} \tilde{C}_i s^i$$

$\downarrow$

*IR  
dynamics*

$$\gamma_{\text{IR}} \simeq \gamma_* = \frac{\pi}{6} \tilde{C}_i s_*^i$$

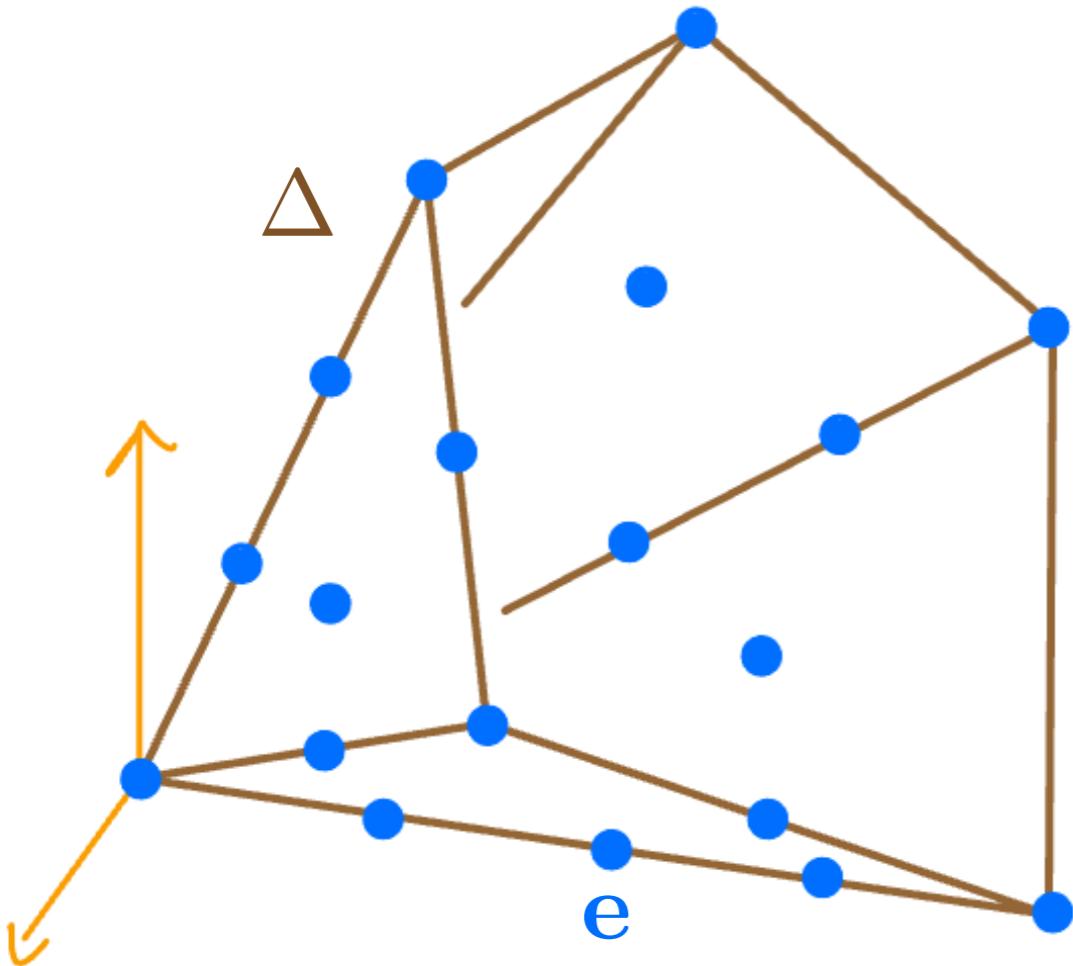


The  $\gamma_*$  bound

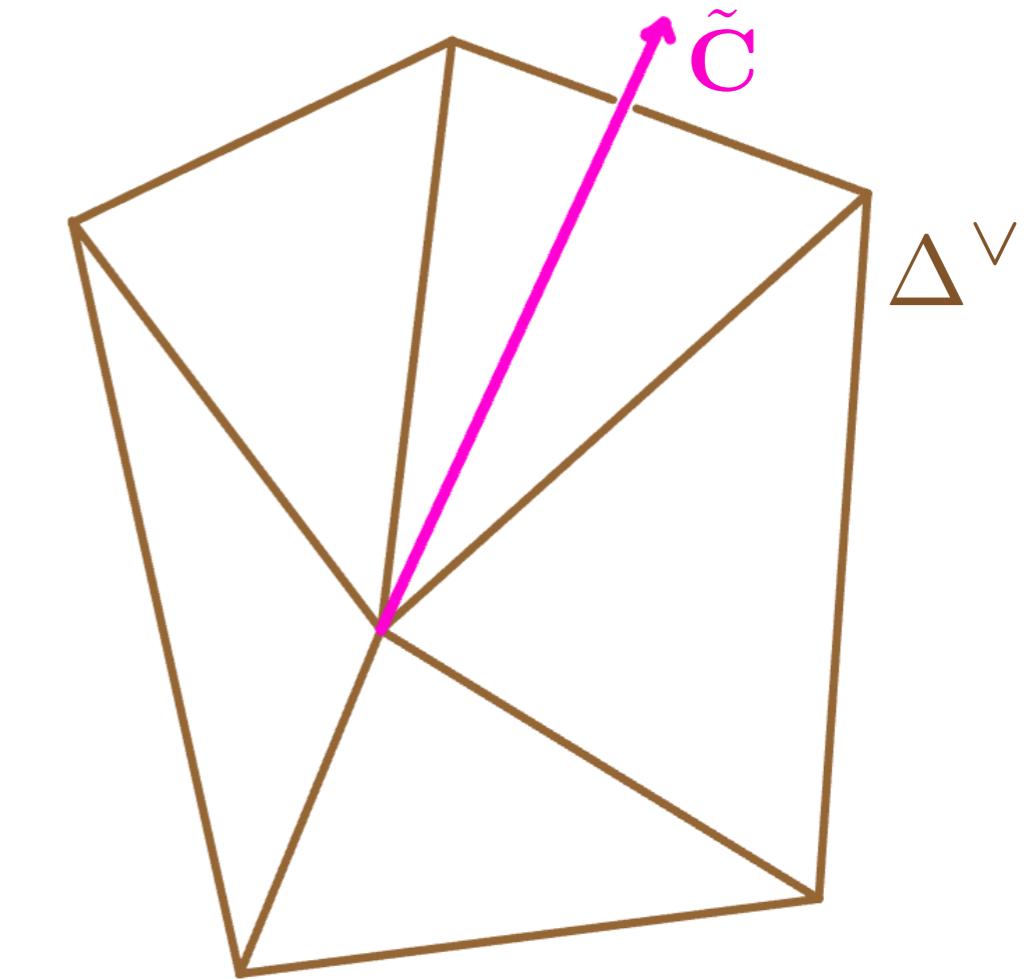
- In fact, generically:

$$\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$$

← chiral EFT string  
world-sheet



e



$\Delta^V$

- In fact, generically:

$$\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$$

← chiral EFT string  
world-sheet

- Take saxionic cone generated by basis of EFT string charges

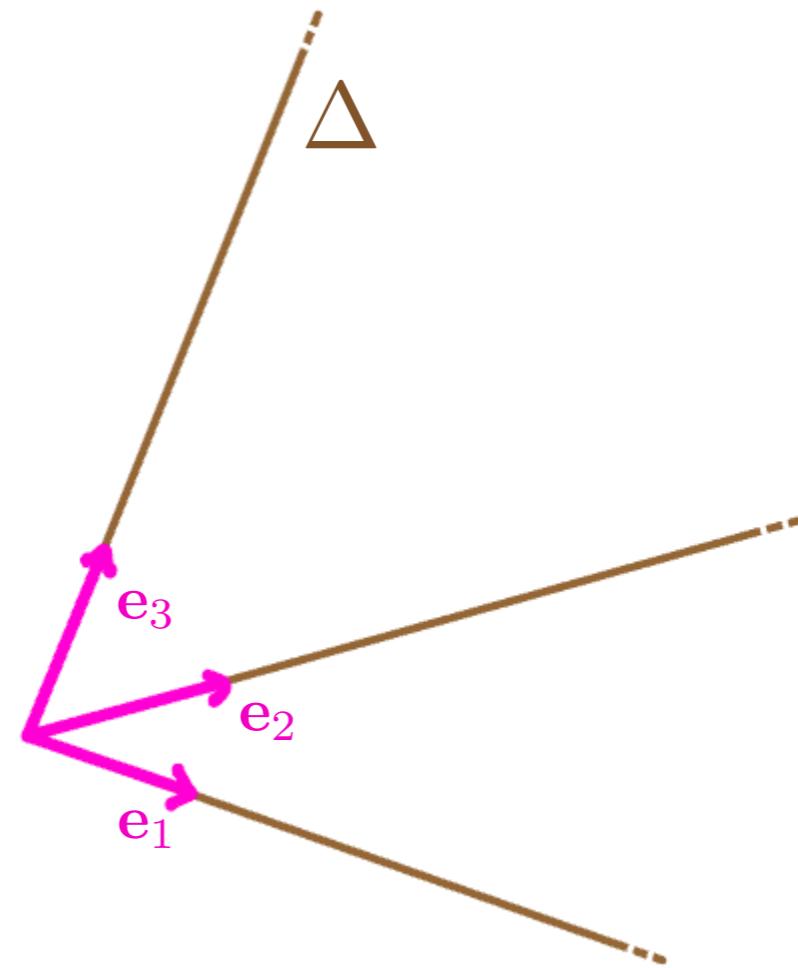
$$\mathbf{e}_{(i)} \in \mathcal{C}_S^{\text{EFT}}$$



$$\tilde{C}_i \in 3\mathbb{Z}_{>0}$$



$$\gamma_* = \frac{\pi}{6} \tilde{C}_i s_*^i \geq \frac{\pi}{2} \sum_{i=1}^N s_*^i$$



- In fact, generically:

$$\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$$

← chiral EFT string world-sheet

- Take saxionic cone generated by basis of EFT string charges

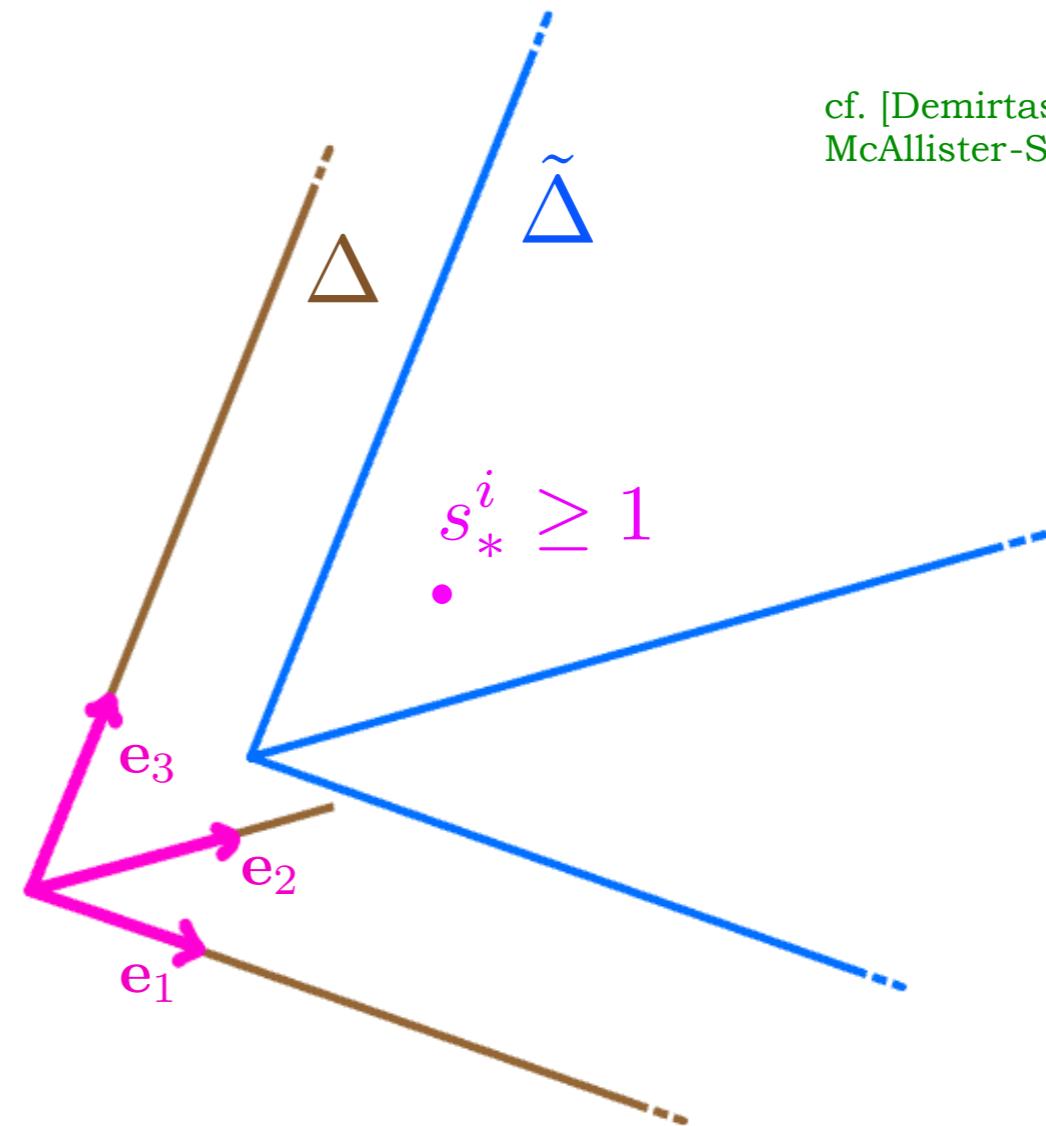
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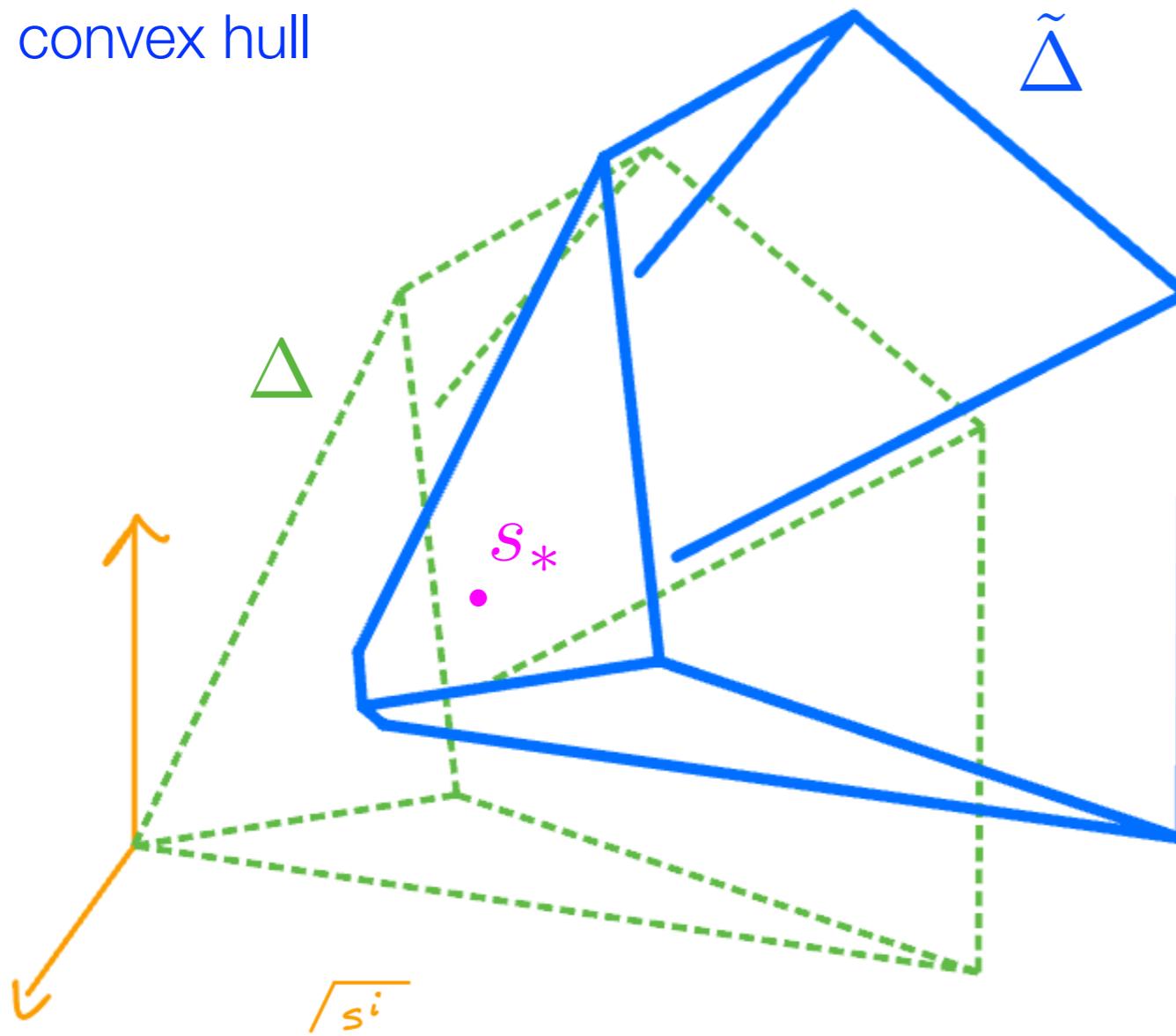
$$\gamma_* \geq \frac{\pi N}{2}$$

- In fact, generically:

$$\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$$

← chiral EFT string  
world-sheet

- More generically: stretched saxionic convex hull



$$\gamma_* \geq \frac{\pi N}{2}$$

# Implications on wormholes

• Purely axionic wormholes

[Giddings-Strominger '87, Lee '88,..., Montero-Uranga-Valenzuela '15, Bachenheimer-Long-McAllister '15, ...]

The diagram illustrates a wormhole geometry. It consists of two vertical brown lines forming a rectangle, with a central horizontal brown line connecting them. A magenta oval curve is drawn inside the rectangle, representing the boundary of the wormhole's cross-section. At the top center, there is a small circular protrusion. Two magenta lines extend from the right side of the oval towards the top and bottom boundaries of the rectangle, meeting at a point above the oval.

Mathematical expressions are overlaid on the diagram:

$$\frac{1}{2\pi} \oint \mathcal{H}_{3i} = m_i$$

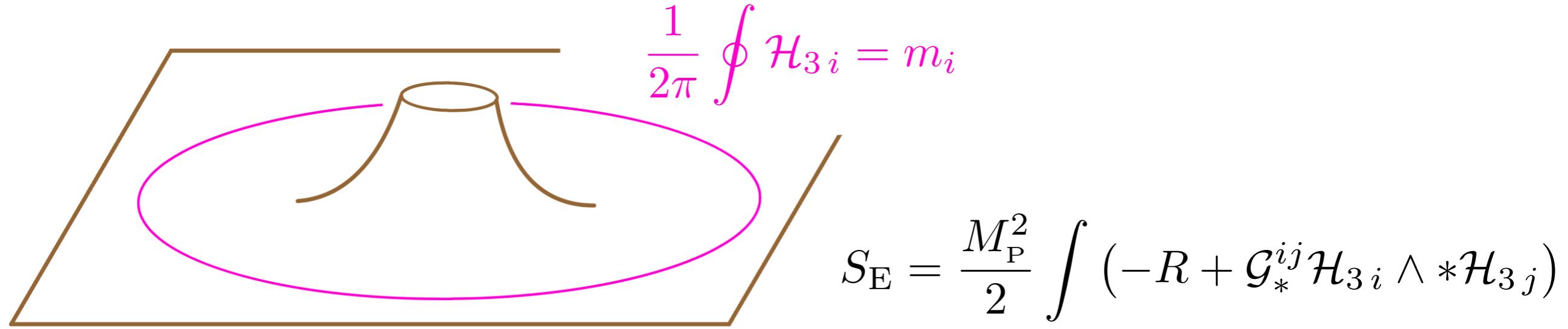
$$S_E = \frac{M_P^2}{2} \int (-R + \mathcal{G}_*^{ij} \mathcal{H}_{3i} \wedge * \mathcal{H}_{3j})$$

$$S_E^{\frac{1}{2} \text{wh}} = \frac{\sqrt{6} \pi^2}{2} \sqrt{\mathcal{G}_*^{ij} m_i m_j}$$

An orange arrow points from the expression  $S_E^{\frac{1}{2} \text{wh}}$  towards the central horizontal line of the rectangle.

• Purely axionic wormholes

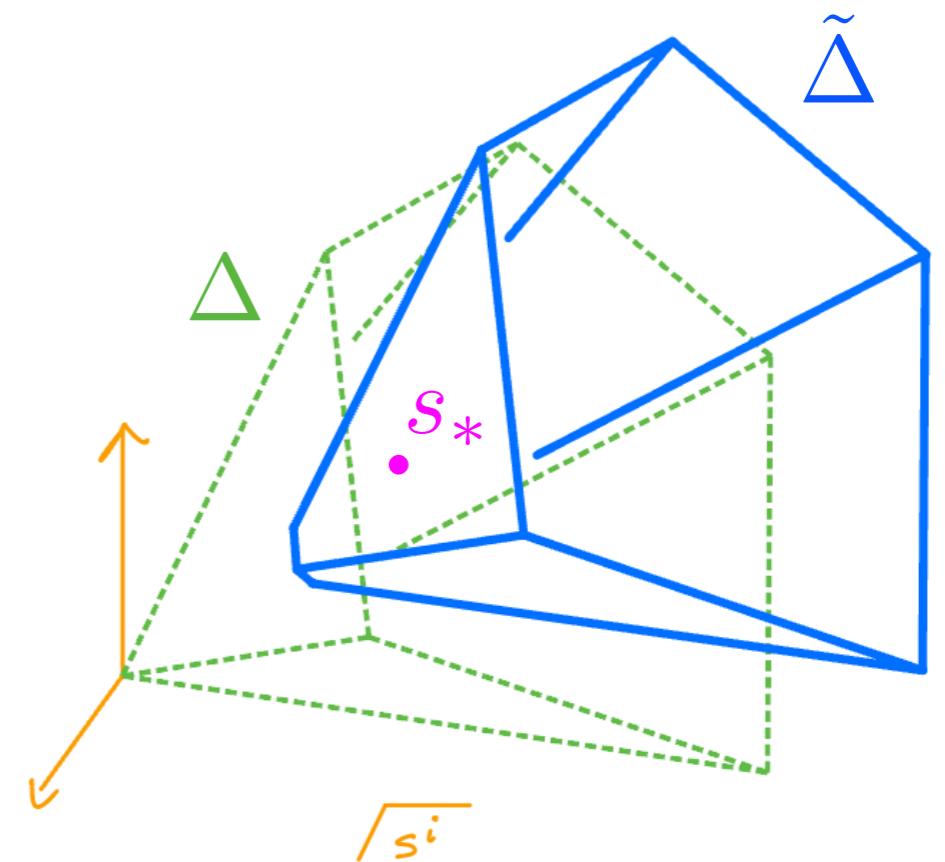
[Giddings-Strominger '87, Lee '88,..., Montero-Uranga-Valenzuela '15, Bachenheimer-Long-McAllister '15, ...]



$$S_E^{\frac{1}{2}\text{wh}} = \frac{\sqrt{6}\pi^2}{2} \sqrt{G_*^{ij} m_i m_j}$$

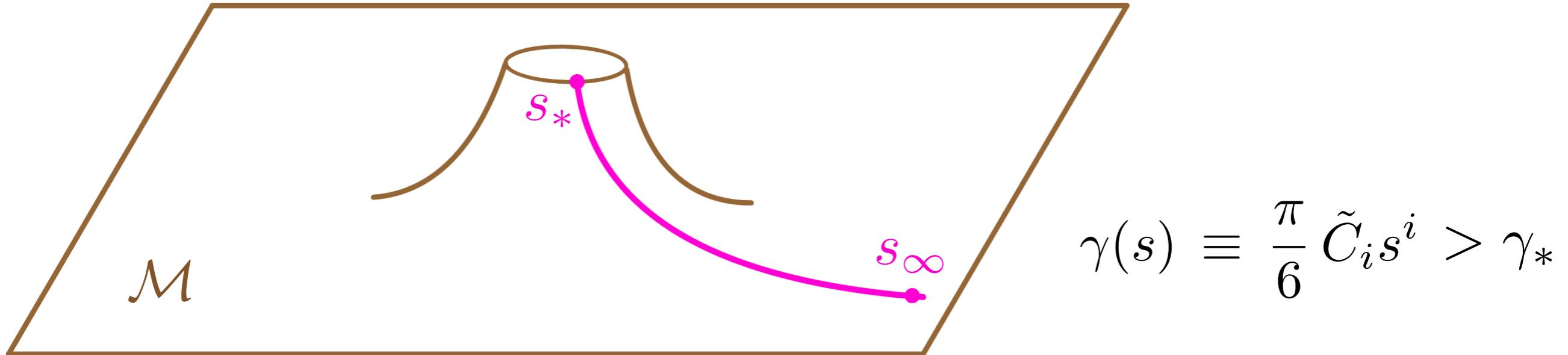
$$S_{\text{GB}} = \gamma_* \geq \frac{\pi N}{2}$$

dominates for  $N \gg \|\vec{m}\|!$



## Saxionic wormholes

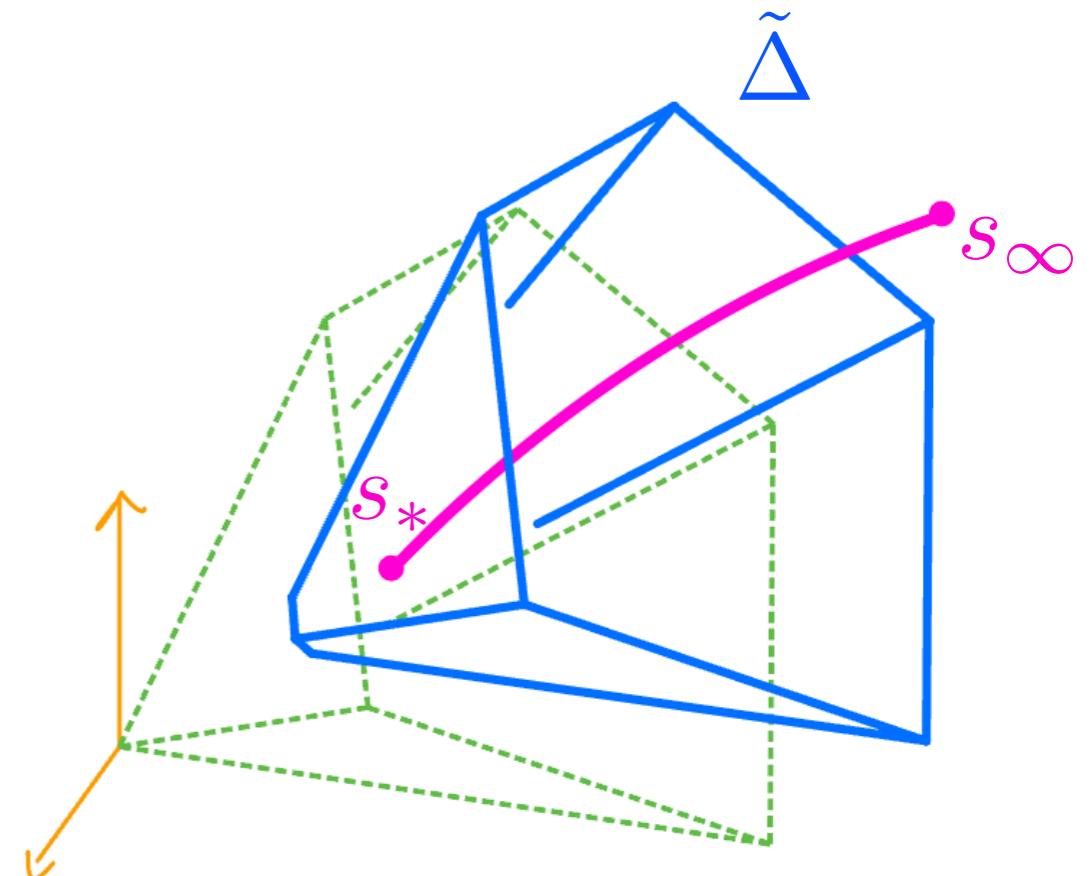
[Abbott-Deser '88, ..., Kallosh-Linde-Linde-Susskind '95, ..., Gutperle-Sabra '02, Bergshoeff-Collinucci-Gran-Roest-Vandoren '05, Arkani-Hamed, Orgera, Polchinski '07, ..., Hebecker-Mangat-Theisen-Witkowski '16, ... Alvey-Escudero '20, Andriolo-Shiu-Soler-Van Riet '22, LM-Risso-Valenti-Vecchi '23]



$$\begin{aligned} S_{\text{GB}} &= - \int_{\mathcal{M}} \gamma(s) E_{\text{GB}} \\ &\geq -\gamma_* \int_{\mathcal{M}} E_{\text{GB}} = \gamma_* \geq \frac{\pi N}{2} \end{aligned}$$

$\Rightarrow$

$$S_{\text{GB}} \geq \frac{\pi N}{2}$$



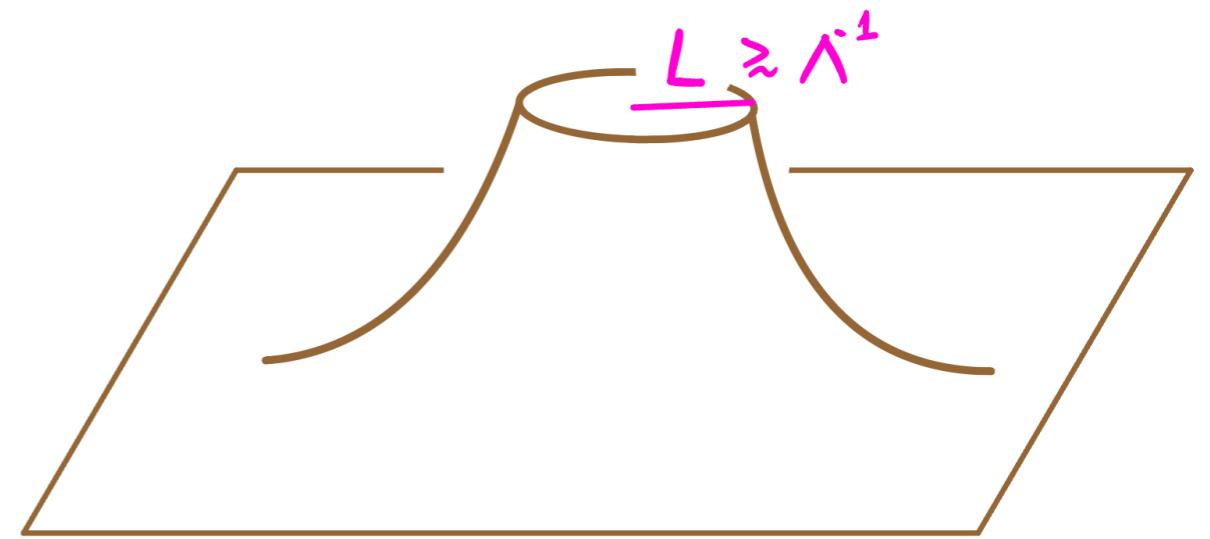
• So:  $S_{\text{tot}}^{\frac{1}{2} \text{wh}} = S_E + S_{\text{GB}}$  with

$$S_{\text{GB}} \geq \frac{\pi N}{2}$$

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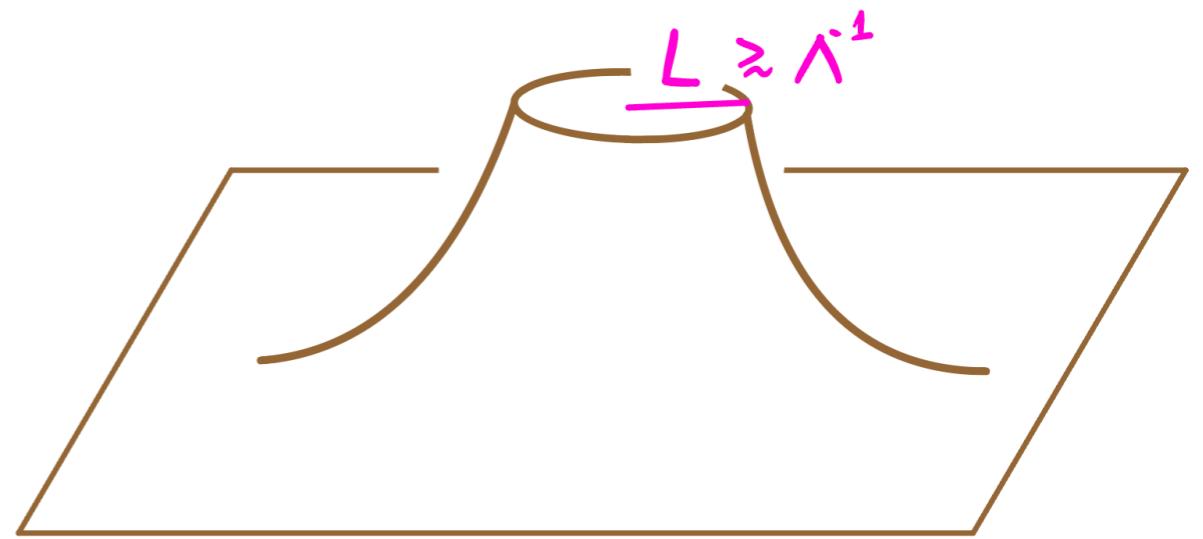
- EFT regime:  $S_E \simeq M_P^2 L^2 \geq \frac{M_P^2}{\Lambda^2}$



- So:  $S_{\text{tot}}^{\frac{1}{2} \text{wh}} = S_E + S_{\text{GB}}$  with

$$S_{\text{GB}} \geq \frac{\pi N}{2}$$

- EFT regime:  $S_E \simeq M_P^2 L^2 \geq \frac{M_P^2}{\Lambda^2}$



- $S_E \gtrsim S_{\text{GB}}$   $\iff \Lambda \lesssim \frac{M_P}{\sqrt{N}}$  species scale bound!

# Conclusions

- GB-term induces large- $N$  exponential suppression

$$\Delta_{\text{wh}} \mathcal{L} \sim e^{-S_{\text{GB}}} e^{-S_{\text{E}}} \leq \exp\left(-\frac{\pi N}{2}\right) e^{-S_{\text{E}}}$$

# Conclusions

- GB-term induces large- $N$  exponential suppression

$$\Delta_{\text{wh}} \mathcal{L} \sim e^{-S_{\text{GB}}} e^{-S_{\text{E}}} \leq \exp\left(-\frac{\pi N}{2}\right) e^{-S_{\text{E}}}$$

- E.g. QCD axion:

- \*  $\Delta V(\vartheta) \simeq \mu^4 e^{-\gamma_*} e^{-S_{\text{E}}} \cos(\vartheta + \delta)$

- \* no quality PQ-quality problem if  $\gamma_* > 170$

[Kallosh-Linde-Linde-Susskind '94  
Alonso-Urbano '17]



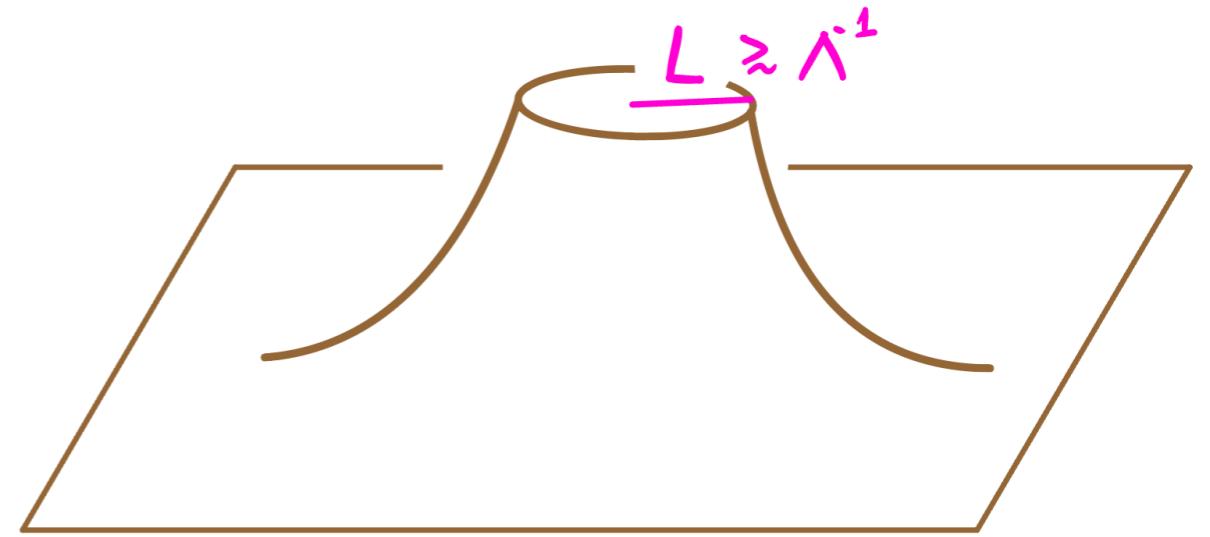
automatic if  $N > 120$  → cf.

[Demirtasa-Gendler-Long-McAllister-Moritz '22]

# Outlook

- Scale separation and 10/11d uplift?

[ ... Loges-Shiu-Van Riet '23]



- Gravitational path integral?

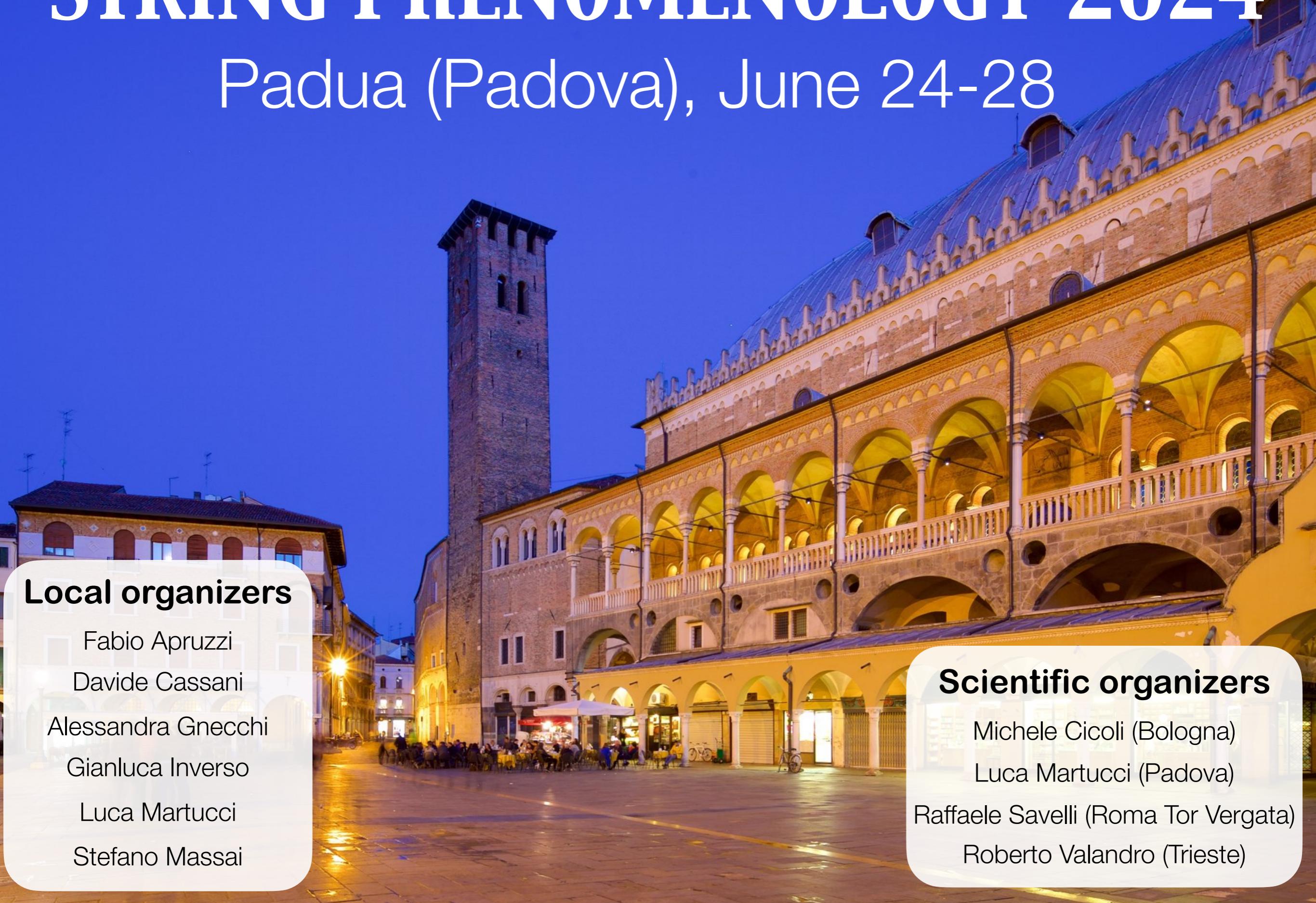
[ ... Loges-Shiu-Sudhir' 22, AguilarGutierrez-Hertog-Tielemans-van der Schaar-Van Riet 23]

- $\alpha$ -states and Baby Universe Hypothesis?

[ Coleman-Giddings-Strominger '88... Marolf-Maxfield '20, McNamara-Vafa '20...]

# STRING PHENOMENOLOGY 2024

Padua (Padova), June 24-28



## Local organizers

Fabio Apruzzi  
Davide Cassani  
Alessandra Gnechi  
Gianluca Inverso  
Luca Martucci  
Stefano Massai

## Scientific organizers

Michele Cicoli (Bologna)  
Luca Martucci (Padova)  
Raffaele Savelli (Roma Tor Vergata)  
Roberto Valandro (Trieste)



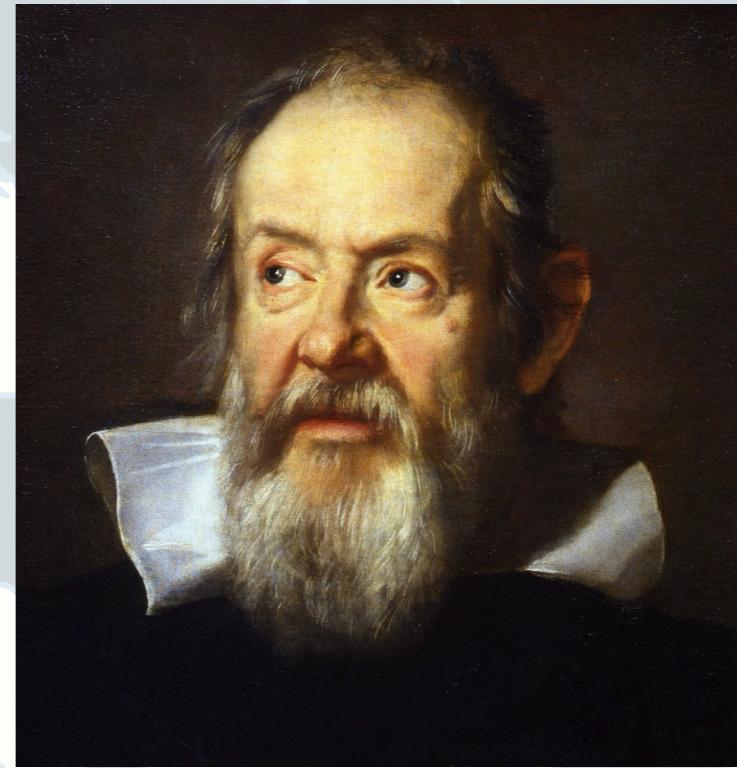
A grayscale map of Italy showing its regional boundaries. A single pushpin is pinned to the map, located in the northern part of the country, specifically in the region of Veneto, which corresponds to the location of Padua.

*Padua*

## University of Padua (1222-)



Padua



"the best eighteen years of my life" (1592-1610)

A grayscale map of Italy with two location markers. A purple pushpin is placed near the city of Padua in the Veneto region. An orange pushpin is placed near the city of Venice in the Veneto region. The word "Padua" is written in purple below the pin, and the word "Venice" is written in orange to the right of the pin.

Padua



*"The Landscape vs. the Swampland"*  
July 1- Aug. 9, 2024, VIENNA

Venice  
Padua





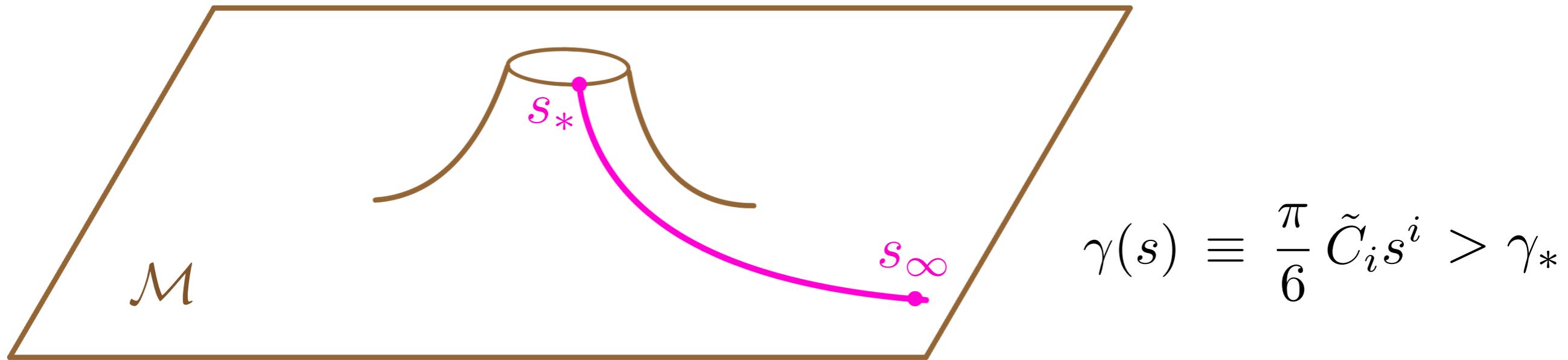
See you next year  
in Padova!

Thanks!

- Wormholes vs fundamental BPS instantons

$$S_E^{\frac{1}{2}\text{wh}} < S_E^{\text{BPS}}$$

[Gutperle-Sabra '02, Bergshoeff-Collinucci-Gran-Roest-Vandoren '05, Arkani-Hamed, Orgera, Polchinski '07, LM-Risso-Valenti-Vecchi '23]



but

$$S_E^{\frac{1}{2}\text{wh}} + S_{\text{GB}} > S_E^{\frac{1}{2}\text{wh}} + \gamma_* > S_E^{\text{BPS}}$$

[LM-Risso-Valenti-Vecchi '23]

implications for axion WGC?

[Arkani-Hamed-Motl-Nicolis-Vafa '07,  
Heidenreich-Reece-Rudelius '16  
Hebecker-Mangat-Theisen-Witkowski '16  
Hebecker-Mikhail-Soler '18,...  
Harlow-Heidenreich-Reece-Rudelius '22]

- In fact, generically:  $\tilde{C}_i e^i \in 3\mathbb{Z}_{>0}$   $\longleftrightarrow$  chiral EFT string world-sheet

- F-theory realization

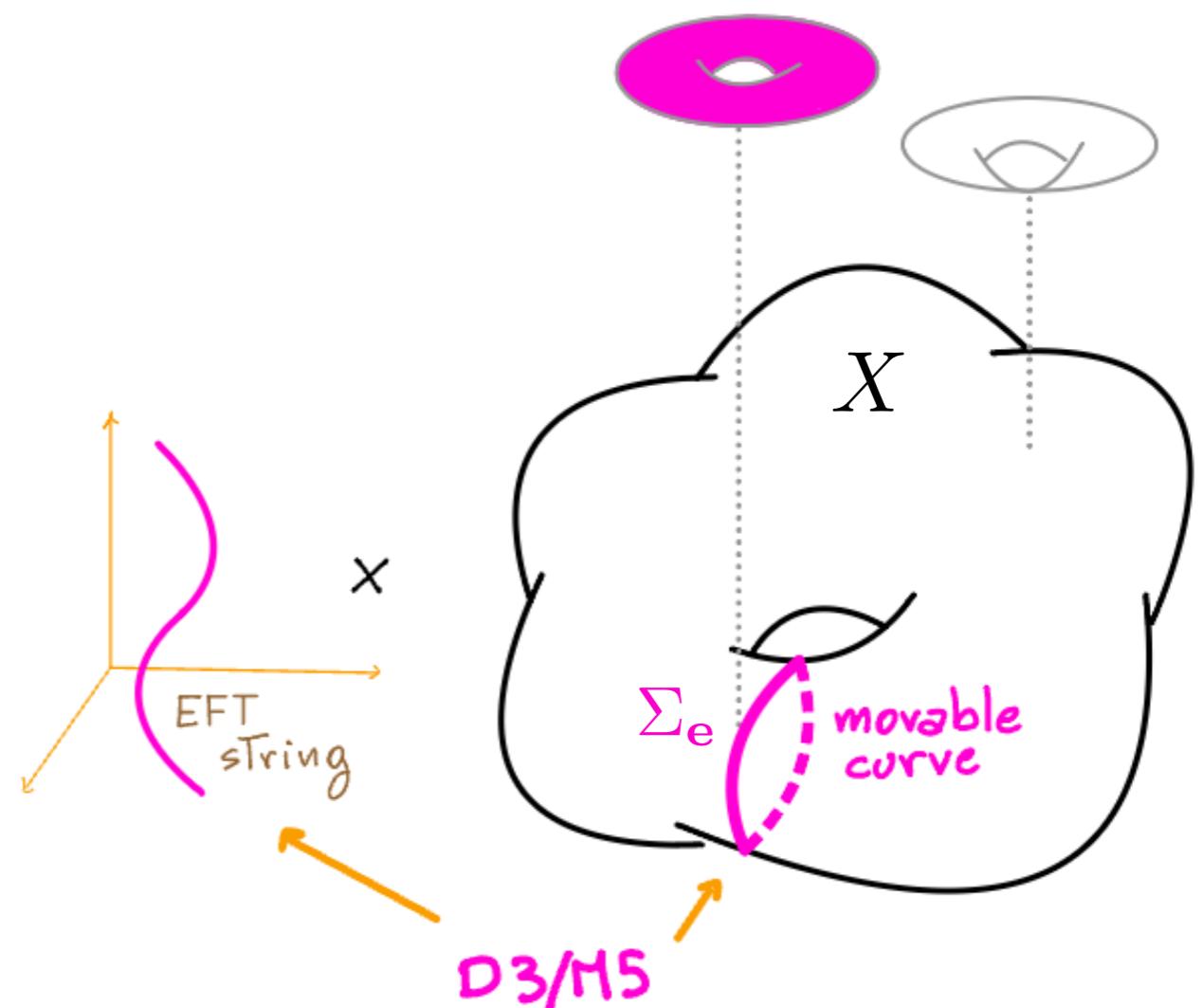
- \*  $\tilde{C}_i e^i = 6\Sigma_e \cdot \overline{K}_X$

- \*  $\overline{K}_X$  effective

- $\Sigma_e$  movable



$$\tilde{C}_i e^i \in 6\mathbb{Z}_{\geq 0}$$



- \* E.g. toric  $X \rightarrow \overline{K}_X = \sum_I D_I \rightarrow \Sigma_e \cdot \overline{K}_X > 0$   
 $I \in \{\text{toric div.}\}$