Bounds on the Species Scale

Max Wiesner
Harvard University

Center for the Fundamental Laws of Nature
&
Center of Mathematical Sciences and Applications

HARVARD UNIVERSITY
CENTER OF MATHEMATICAL
SCIENCES AND APPLICATIONS

based on:

Damian van de Heisteeg, Cumrun Vafa, MW, David H. Wu
2212.06841, 2305.07701

Damian van de Heisteeg, Cumrun Vafa, MW
2303.13580

String Phone 2023
July 6th, 2023

Introduction

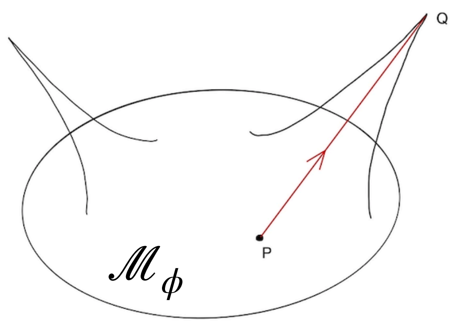
Common feature of many theories of Quantum Gravity:

1. light scalar fields ϕ^i determining couplings, masses, etc.
2. towers of massive states beyond EFT.

Distance Conjecture [Ooguri, Vafa '06]

Along paths in scalar field space traversing distances $d \gg l_p$ *an infinite tower of states becomes light in Planck units as*

$$\frac{M(Q)}{M_{pl}} \sim e^{-Ad(P,Q)}$$



Tested in string theory in great detail: Cf. reviews [Palti '19, van Beast, Calderon-Infante, Mirfendereski, Valenzuela '21; Agmon, Bedroya, Kang, Vafa '22]

Introduction

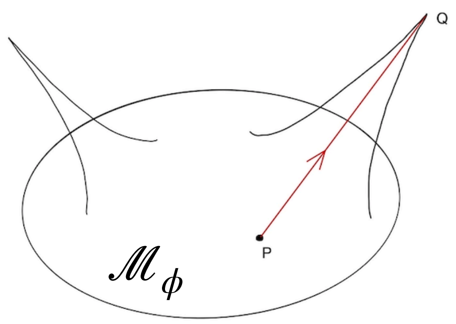
Common feature of many theories of Quantum Gravity:

1. light scalar fields ϕ^i determining couplings, masses, etc.
2. towers of massive states beyond EFT.

Distance Conjecture [Ooguri, Vafa '06]

Along paths in scalar field space traversing distances $d \gg l_p$ an infinite tower of states becomes light in Planck units as

$$\frac{M(Q)}{M_{pl}} \sim e^{-A d(P,Q)}$$



Tested in string theory in great detail: Cf. reviews [Palti '19, van Beast, Calderon-Infante, Mirfendereski, Valenzuela '21; Agmon, Bedroya, Kang, Vafa '22]

Question: What about light towers of states away from asymptotic regions?

(For more on infinite distance regions, see Timo and Ben's talk)

Introduction

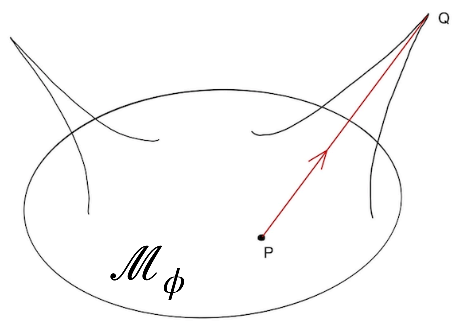
Common feature of many theories of Quantum Gravity:

1. light scalar fields ϕ^i determining couplings, masses, etc.
2. towers of massive states beyond EFT.

Distance Conjecture [Ooguri, Vafa '06]

Along paths in scalar field space traversing distances $d \gg l_p$ an infinite tower of states becomes light in Planck units as

$$\frac{M(Q)}{M_{pl}} \sim e^{-Ad(P,Q)}$$



Tested in string theory in great detail: Cf. reviews [Palti '19, van Beast, Calderon-Infante, Mirfendereski, Valenzuela '21; Agmon, Bedroya, Kang, Vafa '22]

Question: What about light towers of states away from asymptotic regions?

(For more on infinite distance regions, see Timo and Ben's talk)

Problem: Would need to know the exact spectrum of the theory at any point in field space.

Introduction

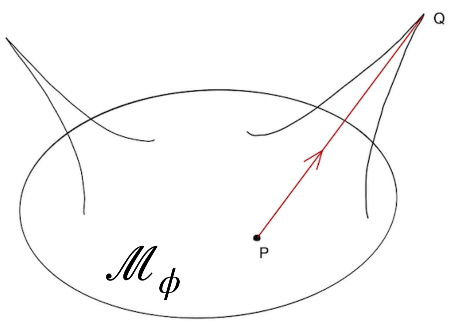
Common feature of many theories of Quantum Gravity:

1. light scalar fields ϕ^i determining couplings, masses, etc.
2. towers of massive states beyond EFT.

Distance Conjecture [Ooguri, Vafa '06]

Along paths in scalar field space traversing distances $d \gg l_p$ an infinite tower of states becomes light in Planck units as

$$\frac{M(Q)}{M_{pl}} \sim e^{-Ad(P,Q)}$$



Tested in string theory in great detail: Cf. reviews [Palti '19, van Beast, Calderon-Infante, Mirfendereski, Valenzuela '21; Agmon, Bedroya, Kang, Vafa '22]

Question: What about light towers of states away from asymptotic regions?

(For more on infinite distance regions, see Timo and Ben's talk)

Problem: Would need to know the exact spectrum of the theory at any point in field space.

Way out: Instead of exact spectrum of states study the variation of the Quantum Gravity cut-off Λ over field space $\mathscr{M}_\phi$ → **Species Scale!**

Introduction

[Dvali '07]

What is the species scale?

(See talks by David Andriot, Alvaro Herraez, Alberto Castellano, Niccolo Cribiori, Dieter Lüst, Marco Scalisi, Irene Valenzuela, ...)

- In the presence of N effective light degrees of freedom: entropy of any black hole needs to satisfy: $S > N$
- Compare to Schwarzschild black hole $S_{\text{Schwarzschild}} = \left(r_H M_{\text{pl}}\right)^{d-2}$

  → radius of minimal black hole describable within EFT

$$r_{\text{H,min}} M_{\text{pl}} \sim N^{1/(d-2)}$$

- Interpretation: Higher derivative corrections break down for curvatures $R \sim \mathcal{O}(r_{\text{H,min}}^{-2})$

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\text{pl}}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}} + \dots \right) \right].$$

$$\Lambda_s = \frac{M_{\text{pl}}}{N^{\frac{1}{d-2}}}$$

- Distance Conjecture: N varies over field space ⇒ $\Lambda_s = \Lambda_s(\phi)$

Moduli Dependent Species Scale

[van de Heisteeg, Vafa, MW, (Wu) '22, '23]

Questions:

1. From a pure EFT perspective: Can we give a bound on the variation of $\Lambda_s(\phi)$?
2. Can we give a closed expression for $\Lambda_s(\phi)$?
3. Can we use explicit form for $\Lambda_s(\phi)$ to bound potentials?

Moduli Dependent Species Scale

[van de Heisteeg, Vafa, MW, (Wu) '22, '23]

Questions:

1. From a pure EFT perspective: Can we give a bound on the variation of $\Lambda_s(\phi)$?
2. Can we give a closed expression for $\Lambda_s(\phi)$?
3. Can we use explicit form for $\Lambda_s(\phi)$ to bound potentials?

Answers:

1. Consistency of the perturbative expansion of a theory of Einstein gravity + higher derivative corrections requires: $$\frac{|\nabla \Lambda_s|}{\Lambda_s} \lesssim \mathcal{O}(1)$$
2. In theories with Type II compactifications the moduli dependence of Λ_s is captured by the topological genus-one free energy F_1.
3. Slowly varying positive potentials bounded by $$V(\phi) < A \exp\left(\frac{2}{\sqrt{(d-1)(d-2)}}\Delta\phi\right)$$

Can be fixed in explicit examples

→ can bound the maximal field range — including interior parts of field space)
cf. [Scalisi, Valenzuela '19]
(See Marco's talk on Monday)

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consider general Einstein theory of gravity + scalar field:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} R - \frac{1}{2} (\partial\phi)^2 + \ldots \right] .$$

- Step 1: Integrate out heavy states → generate higher-derivative terms suppressed by Λ_s.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) - \frac{1}{2} (\partial\phi)^2 + \ldots \right] .$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.

- Consistency requirement: Integrating out small-distance modes of a single field ϕ should not considerably change the perturbative expansion!

A single field does not significantly change the entropy of the smallest black hole and therefore does not significantly affect the species scale!

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consider general Einstein theory of gravity + scalar field:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} R - \frac{1}{2} (\partial \phi)^2 + \dots \right] .$$

- Step 1: Integrate out heavy states → generate higher-derivative terms suppressed by Λ_s.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \dots \right) - \frac{1}{2} (\partial \phi)^2 + \dots \right] .$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.

- Consistency requirement: Integrating out small-distance modes of a single field ϕ should not considerably change the perturbative expansion!

A single field does not significantly change the entropy of the smallest black hole and therefore does not significantly affect the species scale!

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consider general Einstein theory of gravity + scalar field:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} R - \frac{1}{2} (\partial \phi)^2 + \ldots \right] .$$

- Step 1: Integrate out heavy states → generate higher-derivative terms suppressed by Λ_s.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) - \frac{1}{2} (\partial \phi)^2 + \ldots \right] .$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.

- Consistency requirement: Integrating out small-distance modes of a single field ϕ should not considerably change the perturbative expansion!

A single field does not significantly change the entropy of the smallest black hole and therefore does not significantly affect the species scale!

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consider general Einstein theory of gravity + scalar field:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} R - \frac{1}{2} (\partial \phi)^2 + \ldots \right] .$$

- Step 1: Integrate out heavy states → generate higher-derivative terms suppressed by Λ_s.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) - \frac{1}{2} (\partial \phi)^2 + \ldots \right] .$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.

- Consistency requirement: Integrating out small-distance modes of a single field ϕ should not considerably change the perturbative expansion!

A single field does not significantly change the entropy of the smallest black hole and therefore does not significantly affect the species scale!

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) - \frac{1}{2}(\partial \phi)^2 + \ldots \right].$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.
- Expand the species scale: $\Lambda_s(\phi_0 + \delta\phi) = \Lambda_s(\phi_0) + \delta\phi\, \Lambda_s'(\phi_0) + \mathcal{O}\left(\delta\phi^2\right)$
- Get effective interaction through operator:

$$\frac{\mathcal{O}_n(R)}{\Lambda_s(\phi)^{n-2}} \to \frac{\mathcal{O}_n(R)}{\Lambda_s(\phi_0)^{n-2}} - \frac{(n-2)\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} \delta\phi\, \mathcal{O}_n(R)$$

- Two operators $\mathcal{O}_m$ and $\mathcal{O}_n$ give rise to effective term in action:

$$\sim \int d^d x d^d y\, M_{\rm pl}^{2(d-2)} \mathcal{O}_n(x) \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} G_\phi(x,y) \mathcal{O}_m(y) \frac{\Lambda_s'(\phi_0)}{\Lambda(\phi_0)_s^{m-1}}$$

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

$$S = \int d^d x \sqrt{-g}\left[\frac{M_{\rm pl}^{d-2}}{2}\left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots\right) - \frac{1}{2}(\partial\phi)^2 + \ldots\right].$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.
- Expand the species scale: $\Lambda_{\rm s}(\phi_0 + \delta\phi) = \Lambda_s(\phi_0) + \delta\phi\, \Lambda_{\rm s}'(\phi_0) + \mathcal{O}\left(\delta\phi^2\right)$
- Get effective interaction through operator:

$$\frac{\mathcal{O}_n(R)}{\Lambda_s(\phi)^{n-2}} \to \frac{\mathcal{O}_n(R)}{\Lambda_s(\phi_0)^{n-2}} - \frac{(n-2)\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}}\delta\phi\, \mathcal{O}_n(R)$$

- Two operators $\mathcal{O}_m$ and $\mathcal{O}_n$ give rise to effective term in action:

$$\sim \int d^d x d^d y\, M_{\rm pl}^{2(d-2)} \mathcal{O}_n(x) \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} G_\phi(x,y) \mathcal{O}_m(y) \frac{\Lambda_s'(\phi_0)}{\Lambda(\phi_0)_s^{m-1}}$$

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) - \frac{1}{2}(\partial\phi)^2 + \ldots \right].$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.
- Expand the species scale: $\Lambda_s(\phi_0 + \delta\phi) = \Lambda_s(\phi_0) + \delta\phi\, \Lambda_s'(\phi_0) + \mathcal{O}\left(\delta\phi^2\right)$
- Get effective interaction through operator:

$$\frac{\mathcal{O}_n(R)}{\Lambda_s(\phi)^{n-2}} \to \frac{\mathcal{O}_n(R)}{\Lambda_s(\phi_0)^{n-2}} - \frac{(n-2)\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} \delta\phi\, \mathcal{O}_n(R)$$

- Two operators $\mathcal{O}_m$ and $\mathcal{O}_n$ give rise to effective term in action:

$$\sim \int d^d x d^d y\, M_{\rm pl}^{2(d-2)} \mathcal{O}_n(x) \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} G_\phi(x,y) \mathcal{O}_m(y) \frac{\Lambda_s'(\phi_0)}{\Lambda(\phi_0)_s^{m-1}}$$

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) - \frac{1}{2}(\partial\phi)^2 + \ldots \right].$$

- Step 2: Integrate out small-distance modes of ϕ above Λ_s.

- Expand the species scale: $\Lambda_{\rm s}(\phi_0 + \delta\phi) = \Lambda_s(\phi_0) + \delta\phi\, \Lambda_{\rm s}'(\phi_0) + \mathcal{O}\left(\delta\phi^2\right)$

- Get effective interaction through operator:

$$\frac{\mathcal{O}_n(R)}{\Lambda_s(\phi)^{n-2}} \to \frac{\mathcal{O}_n(R)}{\Lambda_s(\phi_0)^{n-2}} - \frac{(n-2)\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}}\, \delta\phi\, \mathcal{O}_n(R)$$

- Two operators $\mathcal{O}_m$ and $\mathcal{O}_n$ give rise to effective term in action:

$$\sim \int d^d x d^d y\; M_{\rm pl}^{2(d-2)} \mathcal{O}_n(x) \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} G_\phi(x,y) \mathcal{O}_m(y) \frac{\Lambda_s'(\phi_0)}{\Lambda(\phi_0)_s^{m-1}}$$

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) - \frac{1}{2}(\partial\phi)^2 + \ldots \right].$$

- Two operators $\mathcal{O}_m$ and $\mathcal{O}_n$ give rise to effective term in action:

$$\sim \int d^d x d^d y \, M_{\rm pl}^{2(d-2)} \mathcal{O}_n(x) \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} G_\phi(x,y) \mathcal{O}_m(y) \frac{\Lambda_s'(\phi_0)}{\Lambda(\phi_0)_s^{m-1}}$$

- Integrating out small-distance modes of $\phi \Longleftrightarrow$ smearing the interaction over ball of radius Λ_s^{-1}

- Generate effective point-interaction:

$$S \supset M_{\rm pl}^{d-2} \int d^d x \sqrt{-g} \, M_{\rm pl}^{d-2} \frac{(\Lambda_s'(\phi_0))^2}{\Lambda_s(\phi_0)^{n+m}} \tilde{\mathcal{O}}_{m+n}(R),$$

- Consistency of effective higher-derivative expansion leads:

$$\left| \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)} \right|^2 \leq \frac{c}{M_{\rm pl}^{(d-2)}},$$

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

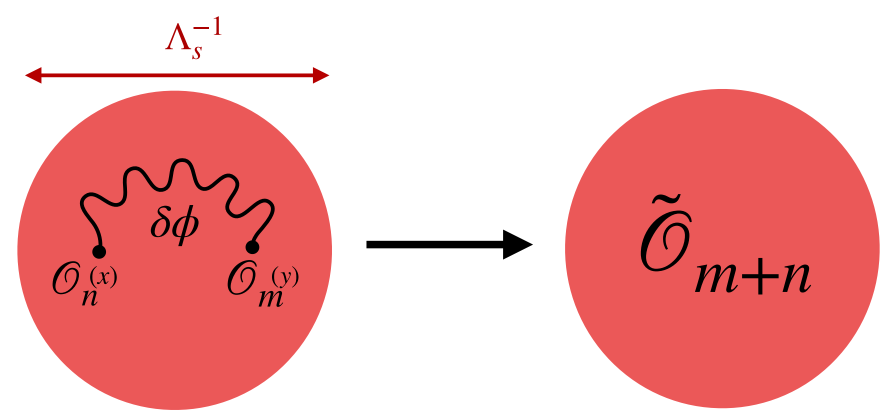
First: Bound on the slope of the species scale as a function of moduli.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \dots \right) - \frac{1}{2}(\partial\phi)^2 + \dots \right] .$$

- Two operators $\mathcal{O}_m$ and $\mathcal{O}_n$ give rise to effective term in action:

$$\sim \int d^d x d^d y \, M_{\rm pl}^{2(d-2)} \mathcal{O}_n(x) \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} G_\phi(x,y) \mathcal{O}_m(y) \frac{\Lambda_s'(\phi_0)}{\Lambda(\phi_0)_s^{m-1}}$$

- Integrating out small-distance modes of $\phi \Longleftrightarrow$ smearing the interaction over ball of radius Λ_s^{-1}



- Generate effective point-interaction:

$$S \supset M_{\rm pl}^{d-2} \int d^d x \sqrt{-g} \, M_{\rm pl}^{d-2} \frac{(\Lambda_s'(\phi_0))^2}{\Lambda_s(\phi_0)^{n+m}} \tilde{\mathcal{O}}_{m+n}(R) \,,$$

- Consistency of effective higher-derivative expansion leads:

$$\left| \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)} \right|^2 \leq \frac{c}{M_{\rm pl}^{(d-2)}} \,,$$

Species Scale and EFT Consistency

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \dots \right) - \frac{1}{2}(\partial\phi)^2 + \dots \right].$$

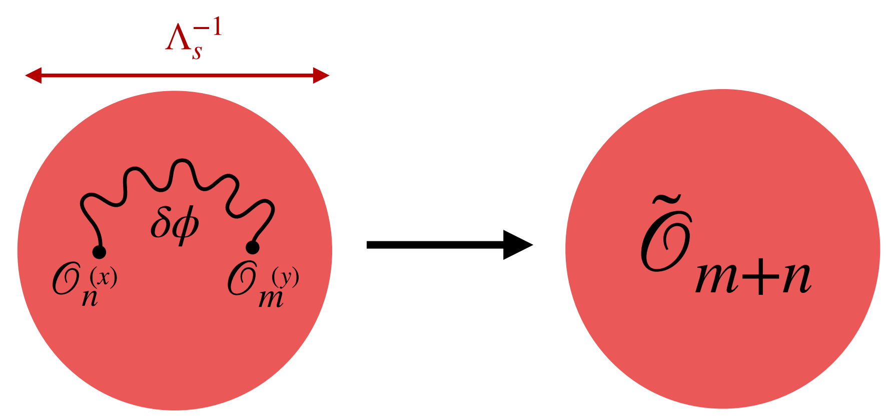
- Two operators $\mathcal{O}_m$ and $\mathcal{O}_n$ give rise to effective term in action:

$$\sim \int d^d x d^d y \, M_{\rm pl}^{2(d-2)} \mathcal{O}_n(x) \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)^{n-1}} G_\phi(x,y) \mathcal{O}_m(y) \frac{\Lambda_s'(\phi_0)}{\Lambda(\phi_0)_s^{m-1}}$$

- Integrating out small-distance modes of $\phi \Longleftrightarrow$ smearing the interaction over ball of radius Λ_s^{-1}

- Generate effective point-interaction:

$$S \supset M_{\rm pl}^{d-2} \int d^d x \sqrt{-g} \, M_{\rm pl}^{d-2} \frac{(\Lambda_s'(\phi_0))^2}{\Lambda_s(\phi_0)^{n+m}} \tilde{\mathcal{O}}_{m+n}(R)\,,$$



- Consistency of effective higher-derivative expansion leads:

$$\left| \frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)} \right|^2 \leq \frac{c}{M_{\rm pl}^{(d-2)}},$$

Asymptotic Behavior

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consistency of effective higher-derivative expansion leads:

$$\left|\frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)}\right|^2 \leq \frac{c}{M_{\rm pl}^{(d-2)}}$$

$\rightarrow$ exponential behavior $\Lambda_s \sim e^{-c\phi}$ is limiting case!

Asymptotic Behavior

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consistency of effective higher-derivative expansion leads:

$$\left|\frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)}\right|^2 \leq \frac{c}{M_{\rm pl}^{(d-2)}}$$

→ exponential behavior $\Lambda_s \sim e^{-c\phi}$ is limiting case!

- Asymptotic regions in field space seem to saturate the bound.

- Since $\Lambda_s \sim m_{\rm tower}^{\gamma}$: **masses of towers predicted by distance conjecture should not decay *faster* than exponential!!**

Similar results in [Calderon-Infante, Castellano, Herraez, Ibanez '23]

→ see also Alberto and Alvaro's talks

Asymptotic Behavior

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consistency of effective higher-derivative expansion leads:

$$\left|\frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)}\right|^2 \leq \frac{c}{M_{\rm pl}^{(d-2)}}$$

→ exponential behavior $\Lambda_s \sim e^{-c\phi}$ is limiting case!

- Asymptotic regions in field space seem to saturate the bound.

- Since $\Lambda_s \sim m_{\rm tower}^{\gamma}$: **masses of towers predicted by distance conjecture should not decay *faster* than exponential!!**

Similar results in [Calderon-Infante, Castellano, Herraez, Ibanez '23]

→ see also Alberto and Alvaro's talks

- According to Emergent string conjecture can have: [Lee, Lerche, Weigand '19]

Emergent string limits in d dimensions:

$$\frac{|\Lambda_s'|}{\Lambda_s} \to \frac{1}{\sqrt{d-2}}$$

Decompactification limits from $d \to D$ dimensions:

$$\frac{|\Lambda_s'|}{\Lambda_s} \to \sqrt{\frac{D-d}{(d-2)(D-2)}}$$

cf. also [Etheredge, Heidenreich, Kaya, Qiu, Rudelius'22]

Asymptotic Behavior

[van de Heisteeg, Vafa, MW '23]

First: Bound on the slope of the species scale as a function of moduli.

- Consistency of effective higher-derivative expansion leads:

$$\left|\frac{\Lambda_s'(\phi_0)}{\Lambda_s(\phi_0)}\right|^2 \leq \frac{c}{M_{\rm pl}^{(d-2)}}$$

→ exponential behavior $\Lambda_s \sim e^{-c\phi}$ is limiting case!

- Asymptotic regions in field space seem to saturate the bound.

- Since $\Lambda_s \sim m_{\rm tower}^{\gamma}$: **masses of towers predicted by distance conjecture should not decay *faster* than exponential!!**

Similar results in [Calderon-Infante, Castellano, Herraez, Ibanez '23]

→ see also Alberto and Alvaro's talks

- According to Emergent string conjecture can have: [Lee, Lerche, Weigand '19]

Emergent string limits in d dimensions:

$$\frac{|\Lambda_s'|}{\Lambda_s} \to \frac{1}{\sqrt{d-2}}$$

Decompactification limits from $d \to D$ dimensions:

$$\frac{|\Lambda_s'|}{\Lambda_s} \to \sqrt{\frac{D-d}{(d-2)(D-2)}}$$

cf. also [Etheredge, Heidenreich, Kaya, Qiu, Rudelius'22]

→ **Universal bound?** $c \stackrel{?}{=} \frac{1}{\sqrt{d-2}}$ → *need to have an explicit expression for* Λ_s

Explicit Expression for Λ_s?

[van de Heisteeg, Vafa, MW '23]

Question: Can we determine the constant c?

→ *need to have an explicit expression for* Λ_s

Consider again effective action:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) \right].$$

Explicit Expression for Λ_s?

[van de Heisteeg, Vafa, MW '23]

Question: Can we determine the constant c?

$\rightarrow$ *need to have an explicit expression for* Λ_s

Consider again effective action:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \dots \right) \right] .$$

In $d=4$ focus on term: $$S = \int d^d x \sqrt{-g} \, \frac{M_{\rm pl}^2}{\Lambda_s^2} R^2 .$$

How can we calculate this term? $\longrightarrow$ *can* compute it, e.g., in string perturbation theory but what about corrections?

$\longrightarrow$ cannot be trusted in at strong coupling!

Explicit Expression for Λ_s?

[van de Heisteeg, Vafa, MW '23]

Question: Can we determine the constant c?

$\rightarrow$ *need to have an explicit expression for* Λ_s

Consider again effective action:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) \right].$$

In $d = 4$ focus on term: $$S = \int d^d x \sqrt{-g} \frac{M_{\rm pl}^2}{\Lambda_s^2} R^2 .$$

How can we calculate this term? $\longrightarrow$ *can* compute it, e.g., in string perturbation theory but what about corrections?

$\longrightarrow$ cannot be trusted in at strong coupling!

However … in Type II string compactifications there exists an R^2 correction that can be computed in perturbation theory and that does not get corrected!

Moduli dependent species scale

[van de Heisteeg, Vafa, MW, Wu '23]

Consider topological string with target space CY threefold:

- Effective action contains term: $S \supset \int F_1 \, (R^-)^2$

  Topological genus-1 free energy

- Worldsheet theory described by CFT with $\mathcal{N} = (2,2)$ supersymmetry in two dimensions.

- For such a CFT can define moduli-dependent index

[Cecotti, Vafa '92, Bershadsky, Cecotti, Ooguri, Vafa '93]

$$F_1 = \frac{1}{2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \mathrm{Tr} \left[(-1)^F F_L F_R q^{H_0} \bar{q}^{\bar{H}_0} \right]$$

Moduli dependent species scale

[van de Heisteeg, Vafa, MW, Wu '23]

Consider topological string with target space CY threefold:

- Effective action contains term: $S \supset \int F_1 (R^-)^2$

  Topological genus-1 free energy

- Worldsheet theory described by CFT with $\mathcal{N} = (2,2)$ supersymmetry in two dimensions.

- For such a CFT can define moduli-dependent index

[Cecotti, Vafa '92, Bershadsky, Cecotti, Ooguri, Vafa '93]

$$F_1 = \frac{1}{2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \mathrm{Tr}\left[(-1)^F F_L F_R q^{H_0} \bar{q}^{\bar{H}_0}\right]$$

  - Satisfied holomorphic anomaly equation:

$$\partial_{\bar{j}} \partial_i F_1 = \mathrm{Tr}(-1)^F C_i \bar{C}_{\bar{j}} - \frac{1}{12} G_{i\bar{j}} \mathrm{Tr}(-1)^F ,$$

[Cecotti, Vafa '92]

Chiral ring structure constants — Zamolodchikov metric on field space

  - Holomorphic anomaly equation can be integrated to

$$F_1 = \frac{1}{2}\left(3 + h^{1,1} - \frac{\chi}{12}\right) K - \frac{1}{2} \log\det G_{i\bar{j}} + \log|f|^2 ,$$

Euler characteristic of mirror Y_3 — Kähler potential — Holomorphic ambiguity

  - Holomorphic ambiguity can be fixed by matching the behavior of F_1 at boundary of moduli space.

F_1 and the number of species

[van de Heisteeg, Vafa, MW, Wu '22]

Question: Why can F_1 be used to estimate the number of light species?

<u>A first look:</u>

- $N_{\rm sp}$ measures the accumulation of light modes in the spectrum of the theory.
- Spectrum of Laplacian Δ gives a measure for number of light states.
  $\rightarrow$ growth of $|\log(\det \Delta)|$ would measure accumulation of light states.
- Problem: spectrum of Δ difficult to compute!

F_1 and the number of species

[van de Heisteeg, Vafa, MW, Wu '22]

Question: Why can F_1 be used to estimate the number of light species?

A first look:

- N_{sp} measures the accumulation of light modes in the spectrum of the theory.
- Spectrum of Laplacian Δ gives a measure for number of light states.
  $\rightarrow$ growth of $|\log(\det \Delta)|$ would measure accumulation of light states.
- Problem: spectrum of Δ difficult to compute!
- However: exists combination of Laplacian $\Delta_{p,q}$ acting on (p,q)-forms that is computable since it is an index-like quantity.

$$\frac{1}{2}\sum_{p,q}(-1)^{p+q}\left(p-\frac{3}{2}\right)\left(q-\frac{3}{2}\right)\log(\det \Delta_{(p,q)})$$

[Bershadsky, Cecotti, Ooguri, Vafa '93]

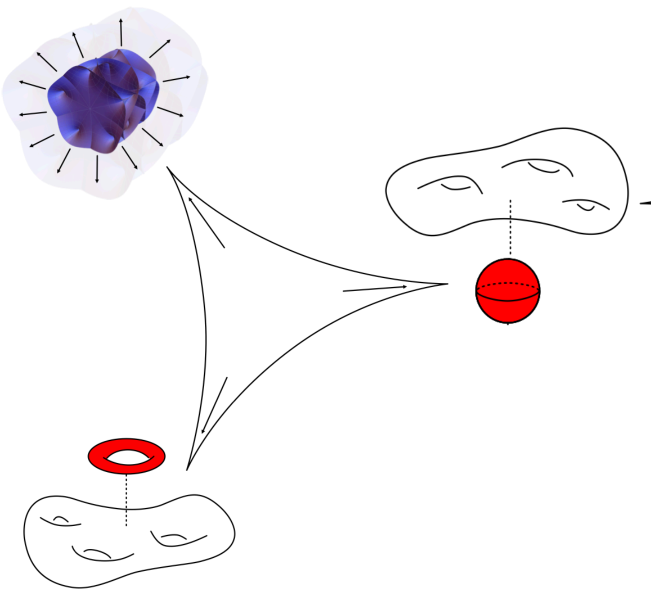
- This combination can be identified with F_1!
- General expectation: $\frac{\Lambda_s}{M_{\text{pl}}} \sim F_1^{-1/2}$ $\rightarrow$ *reproduces right behavior in asymptotic regions*

black hole perspective in [Cribiori, Lust, Staudt '22]

Asymptotic behavior of F_1

[van de Heisteeg, Vafa, MW '23]

How does F_1 behave asymptotically?



Three kinds of infinite distance limits: [Lee, Lerche, Weigand '19]

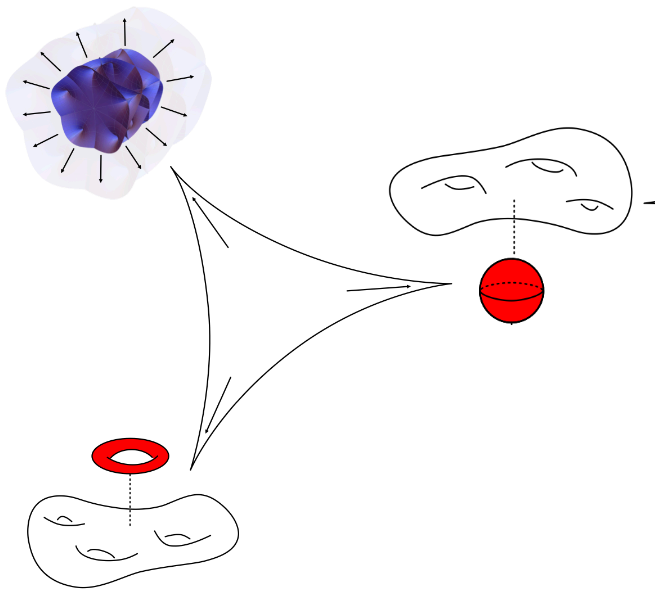
1. Large volume limit (decompactification to 5d M-theory)
2. Emergent String limit (requires K3-fibration for CY)
3. Decompactification to 6d (requires T^2-fibration for CY)

Asymptotic behavior of F_1

[van de Heisteeg, Vafa, MW '23]

How does F_1 behave asymptotically?

Three kinds of infinite distance limits: [Lee, Lerche, Weigand '19]



1. Large volume limit (decompactification to 5d M-theory)
2. Emergent String limit (requires K3-fibration for CY)
3. Decompactification to 6d (requires T^2-fibration for CY)

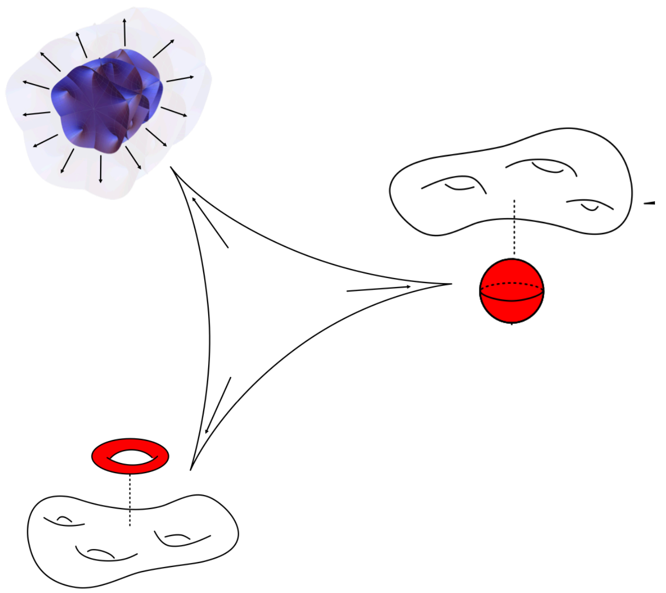
Parametrize all limits by $s \to \infty$: F_1 has asymptotic form $F_1 = c_2 s - \beta \log s$

$\Longrightarrow$ slope of Λ_s given by $\frac{|\nabla \Lambda_s|}{\Lambda_s} \to \alpha + \frac{\beta}{s} \log s$

Asymptotic behavior of F_1

[van de Heisteeg, Vafa, MW '23]

How does F_1 behave asymptotically?



Three kinds of infinite distance limits: [Lee, Lerche, Weigand '19]

1. Large volume limit (decompactification to 5d M-theory)
2. Emergent String limit (requires K3-fibration for CY)
3. Decompactification to 6d (requires T^2-fibration for CY)

Parametrize all limits by $s \to \infty$: F_1 has asymptotic form $F_1 = c_2 s - \beta \log s$

$\Longrightarrow$ slope of Λ_s given by $\frac{|\nabla \Lambda_s|}{\Lambda_s} \to \alpha + \frac{\beta}{s} \log s$

$$\alpha_{\text{dec}}^{d \to D} = \sqrt{\frac{(D-d)}{(D-2)(d-2)}}$$

$$\alpha_{\text{str.}}^{d} = \sqrt{\frac{1}{d-2}}$$

Limit	α	β
Large volume	$\sqrt{\frac{1}{6}}$	$\frac{1}{4}(18 + h^{1,1} + h^{2,1})$
Emergent String	$\sqrt{\frac{1}{2}}$	$\frac{1}{12}(6 + 5h^{1,1} + h^{2,1} - 6h^{1,1}_{\text{vert}})$
6d decompactifcation	$\frac{1}{2}$	$\frac{1}{12}\left(42 + 4h^{1,1} + 2h^{2,1} - 6h^{1,1}(B_2)\right)$

Asymptotic behavior of F_1

[van de Heisteeg, Vafa, MW '23]

$\Longrightarrow$ slope of Λ_s given by $\frac{|\nabla \Lambda_s|}{\Lambda_s} \to \alpha + \frac{\beta}{s} \log s$

Limit	α	β
Large volume	$\sqrt{\frac{1}{6}}$	$\frac{1}{4}(18 + h^{1,1} + h^{2,1})$
Emergent String	$\sqrt{\frac{1}{2}}$	$\frac{1}{12}(6 + 5h^{1,1} + h^{2,1} - 6h^{1,1}_{\text{vert}})$
6d decompactifcation	$\frac{1}{2}$	$\frac{1}{12}\left(42 + 4h^{1,1} + 2h^{2,1} - 6h^{1,1}(B_2)\right)$

$\frac{|\nabla \Lambda_s|}{\Lambda_s}$ typically approaches asymptotic value from **above!**

(see also David's talk on Monday)

Asymptotic behavior of F_1

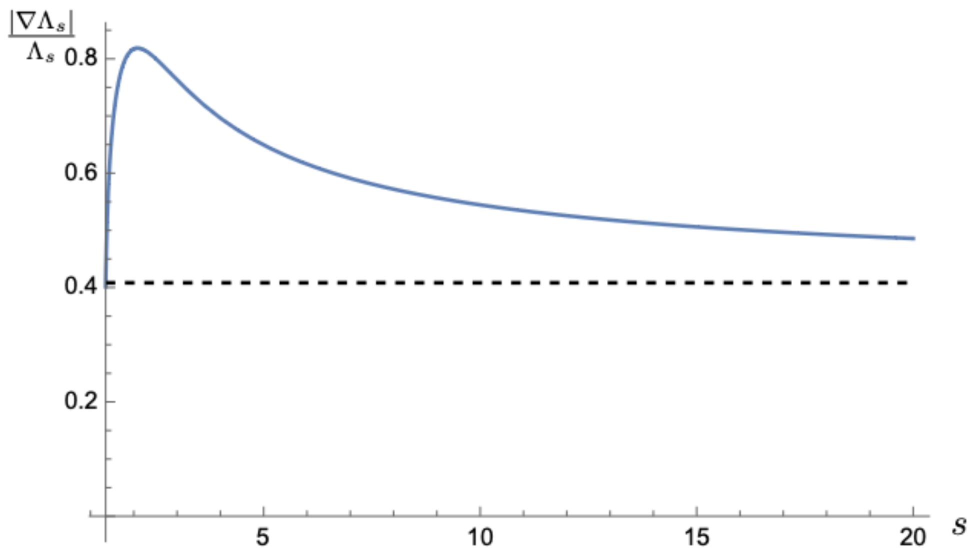
[van de Heisteeg, Vafa, MW '23]

$\Longrightarrow$ slope of Λ_s given by $\frac{|\nabla \Lambda_s|}{\Lambda_s} \to \alpha + \frac{\beta}{s} \log s$

Limit	α	β
Large volume	$\sqrt{\frac{1}{6}}$	$\frac{1}{4}(18 + h^{1,1} + h^{2,1})$
Emergent String	$\sqrt{\frac{1}{2}}$	$\frac{1}{12}(6 + 5h^{1,1} + h^{2,1} - 6h^{1,1}_{\text{vert}})$
6d decompactifcation	$\frac{1}{2}$	$\frac{1}{12}\left(42 + 4h^{1,1} + 2h^{2,1} - 6h^{1,1}(B_2)\right)$

$\frac{|\nabla \Lambda_s|}{\Lambda_s}$ typically approaches asymptotic value from **above!**

(see also David's talk on Monday)



11

Asymptotic behavior of F_1

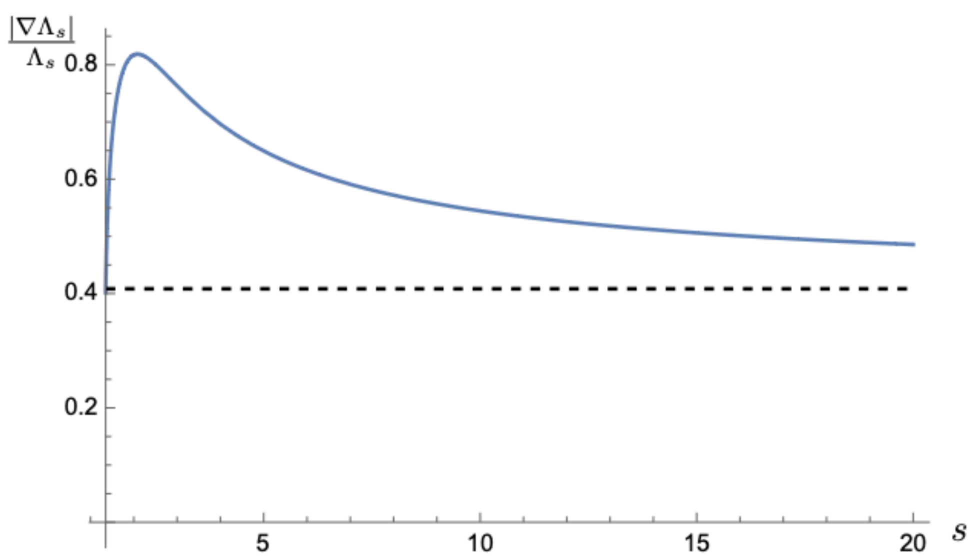
[van de Heisteeg, Vafa, MW '23]

$\Longrightarrow$ slope of Λ_s given by $\quad \frac{|\nabla \Lambda_s|}{\Lambda_s} \to \alpha + \frac{\beta}{s} \log s$

Limit	α	β
Large volume	$\sqrt{\frac{1}{6}}$	$\frac{1}{4}(18 + h^{1,1} + h^{2,1})$
Emergent String	$\sqrt{\frac{1}{2}}$	$\frac{1}{12}(6 + 5h^{1,1} + h^{2,1} - 6h^{1,1}_{\text{vert}})$
6d decompactifcation	$\frac{1}{2}$	$\frac{1}{12}\left(42 + 4h^{1,1} + 2h^{2,1} - 6h^{1,1}(B_2)\right)$

$\frac{|\nabla \Lambda_s|}{\Lambda_s}$ typically approaches asymptotic value from **above!**

(see also David's talk on Monday)



$\Longrightarrow$ universal bound $c = \frac{1}{\sqrt{d-2}}$

not correct!

$\Longrightarrow$ physical interpretation of **sign of** log-correction?

$\Longrightarrow$ Deviation from asymptotic behavior away from strict asymptotic regime!

Species Scale and Potentials

[van de Heisteeg, Vafa, MW, Wu '23]

Use the **species scale** to bound slowly varying potentials with $V > 0$!

$\rightarrow$ For slowly rolling field bound $V(\phi) \leq \Lambda_s^2$ cf. [Hebecker, Wrase '18, Scalisi, Valenzuela '18]

(see Marco's talk)

Species Scale and Potentials

[van de Heisteeg, Vafa, MW, Wu '23]

Use the **species scale** to bound slowly varying potentials with $V > 0$!

→ For slowly rolling field bound $V(\phi) \leq \Lambda_s^2$ cf. [Hebecker, Wrase '18, Scalisi, Valenzuela '18]

(see Marco's talk)

Motivation:

→ dS entropy $S_{\text{dS}} \sim H^{-2}$ should *at least* account for light species!

→ $H^2 < \Lambda_s$!

(alternatively: smallest black hole should 'fit' into dS space!

Species Scale and Potentials

[van de Heisteeg, Vafa, MW, Wu ’23]

Use the **species scale** to bound slowly varying potentials with $V > 0$!

$\rightarrow$ For slowly rolling field bound $V(\phi) \leq \Lambda_s^2$ cf. [Hebecker, Wrase ’18, Scalisi, Valenzuela ’18]

(see Marco’s talk)

Motivation:

$\rightarrow$ dS entropy $S_{\mathrm{dS}} \sim H^{-2}$ should *at least* account for light species!

$\rightarrow$ $H^2 < \Lambda_s$!

(alternatively: smallest black hole should ‘fit’ into dS space!

Asymptotically:
$\Lambda_s \sim A e^{-\lambda \Delta\phi}$ $\longrightarrow$ $V(\phi) \leq \Lambda_s^2$ implies: $V(\phi) < A \exp\left(\frac{2}{\sqrt{(d-1)(d-2)}}\Delta\phi\right)$

Emergent String Conjecture: $\lambda \geq \frac{1}{\sqrt{(d-1)(d-2)}}$

(Decompactification to $D = d+1$)

Species Scale and Potentials

[van de Heisteeg, Vafa, MW, Wu '23]

Use the **species scale** to bound slowly varying potentials with $V > 0$!

$\rightarrow$ For slowly rolling field bound $V(\phi) \leq \Lambda_s^2$ cf. [Hebecker, Wrase '18, Scalisi, Valenzuela '18]

(see Marco's talk)

Motivation:

$\rightarrow$ dS entropy $S_{\text{dS}} \sim H^{-2}$ should *at least* account for light species!

$\rightarrow H^2 < \Lambda_s$!

(alternatively: smallest black hole should 'fit' into dS space!

Asymptotically:
$\Lambda_s \sim Ae^{-\lambda \Delta\phi}$

$\rightarrow$ $V(\phi) \leq \Lambda_s^2$ implies: $V(\phi) < A \exp\left(\frac{2}{\sqrt{(d-1)(d-2)}}\Delta\phi\right)$

Emergent String Conjecture: $\lambda \geq \frac{1}{\sqrt{(d-1)(d-2)}}$

(Decompactification to $D = d+1$)

Same bound as TCC!!! *(But independent reasoning)*
[Bedroya, Vafa '19]

Species Scale and Potentials

[van de Heisteeg, Vafa, MW, Wu '23]

Use the **species scale** to bound slowly varying potentials with $V > 0$!

$\rightarrow$ For slowly rolling field bound $V(\phi) \leq \Lambda_s^2$ cf. [Hebecker, Wrase '18, Scalisi, Valenzuela '18]

(see Marco's talk)

Motivation:

$\rightarrow$ dS entropy $S_{\mathrm{dS}} \sim H^{-2}$ should *at least* account for light species!

$\rightarrow H^2 < \Lambda_s$!

(alternatively: smallest black hole should 'fit' into dS space!

Asymptotically:
$\Lambda_s \sim A e^{-\lambda \Delta\phi}$

$V(\phi) \leq \Lambda_s^2$ implies: $V(\phi) < A \exp\left(\frac{2}{\sqrt{(d-1)(d-2)}}\Delta\phi\right)$

Emergent String Conjecture: $\lambda \geq \frac{1}{\sqrt{(d-1)(d-2)}}$

(Decompactification to $D = d+1$)

Same bound as TCC!!! *(But independent reasoning)*
[Bedroya, Vafa '19]

Unlike TCC: species scale argument can be used to fix A!

(At least in cases where species scale is known everywhere and supersymmetry is broken mildly)

$\rightarrow$ again in CY compactifications of Type II string theory!

Species Scale and Potentials

[van de Heisteeg, Vafa, MW, Wu '23]

Simple application: Type II CY compactifications with fluxes!

- Supersymmetry broken to $\mathcal{N}=1 \rightarrow$ species scale still given by F_1??
- Yes! Since fluxes do not affect the topological string amplitudes! [Vafa '00]
- Effective action contains two terms: [Ooguri, Vafa '03]

$$S_{\mathcal{N}=1} \supset \int d^4x\, d^2\theta \mathscr{F}^{2g} N_i \frac{\partial F_g}{\partial S_i}\,, \qquad S_{\mathcal{N}=1} \supset g \int d^4x\, d^2\theta\ \mathscr{W}^2 \mathscr{F}^{2(g-1)} F_g\,,$$

For g=1 is just the same term as in N=2!

Species Scale and Potentials

[van de Heisteeg, Vafa, MW, Wu '23]

Simple application: Type II CY compactifications with fluxes!

- Supersymmetry broken to $\mathcal{N} = 1 \rightarrow$ species scale still given by F_1??
- Yes! Since fluxes do not affect the topological string amplitudes! [Vafa '00]
- Effective action contains two terms: [Ooguri, Vafa '03]

$$S_{\mathcal{N}=1} \supset \int d^4x\, d^2\theta \mathcal{F}^{2g} N_i \frac{\partial F_g}{\partial S_i}\,, \qquad S_{\mathcal{N}=1} \supset g \int d^4x\, d^2\theta\; \mathcal{W}^2 \mathcal{F}^{2(g-1)} F_g\,,$$

For g=1 is just the same term as in N=2!

We can constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

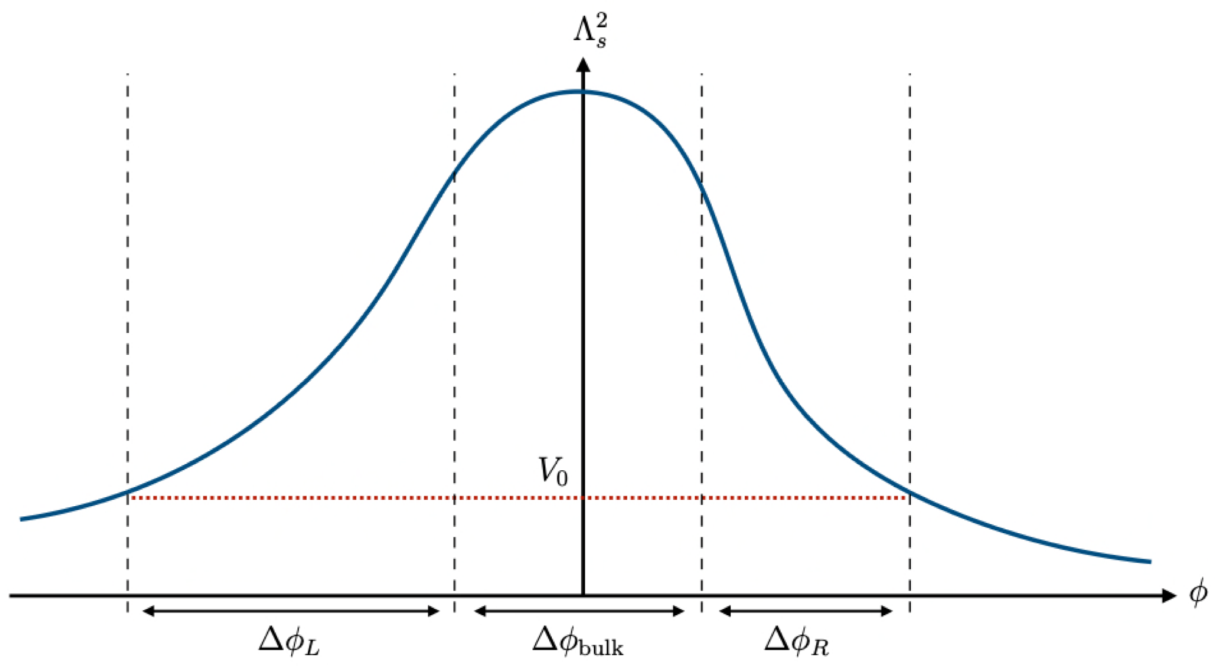
$$\Delta\phi \leq -\sqrt{6}\log\frac{V_0}{M_{\rm pl}^4} + b$$

Can we get large field distances in the interior?

Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

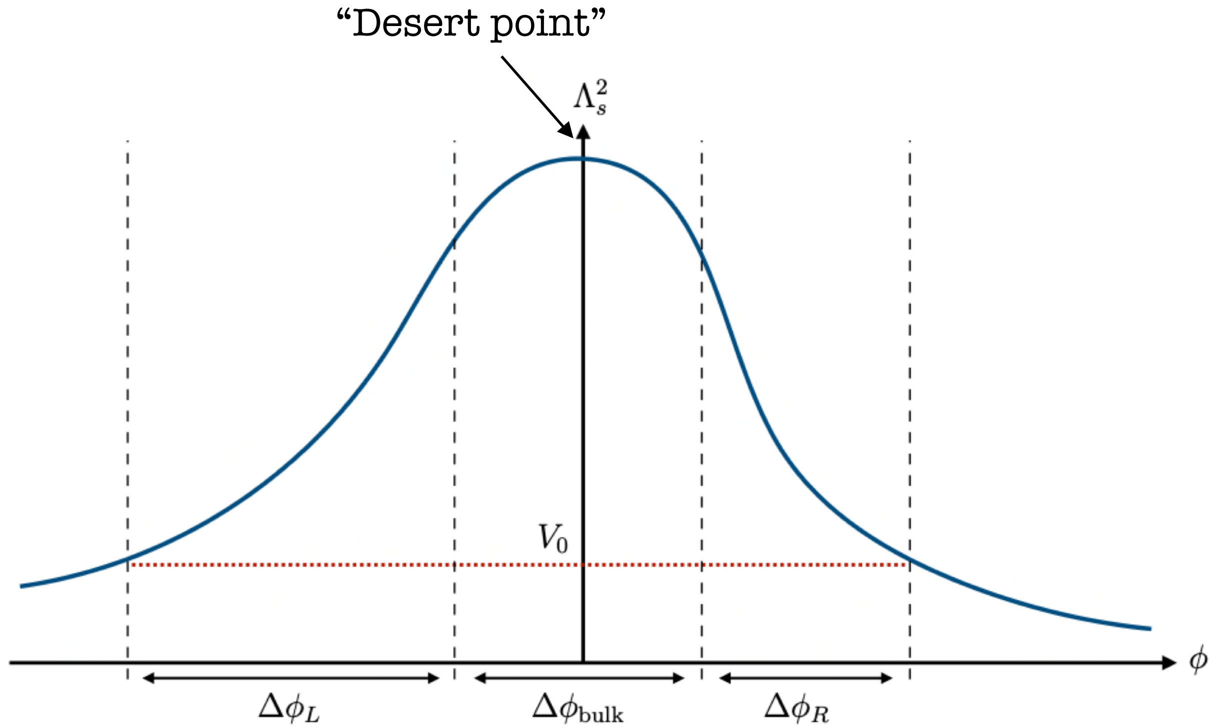


Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

Cf. [Long, Montero, Vafa, Valenzuela '21]



Split the field range in three parts *(including the interior)*

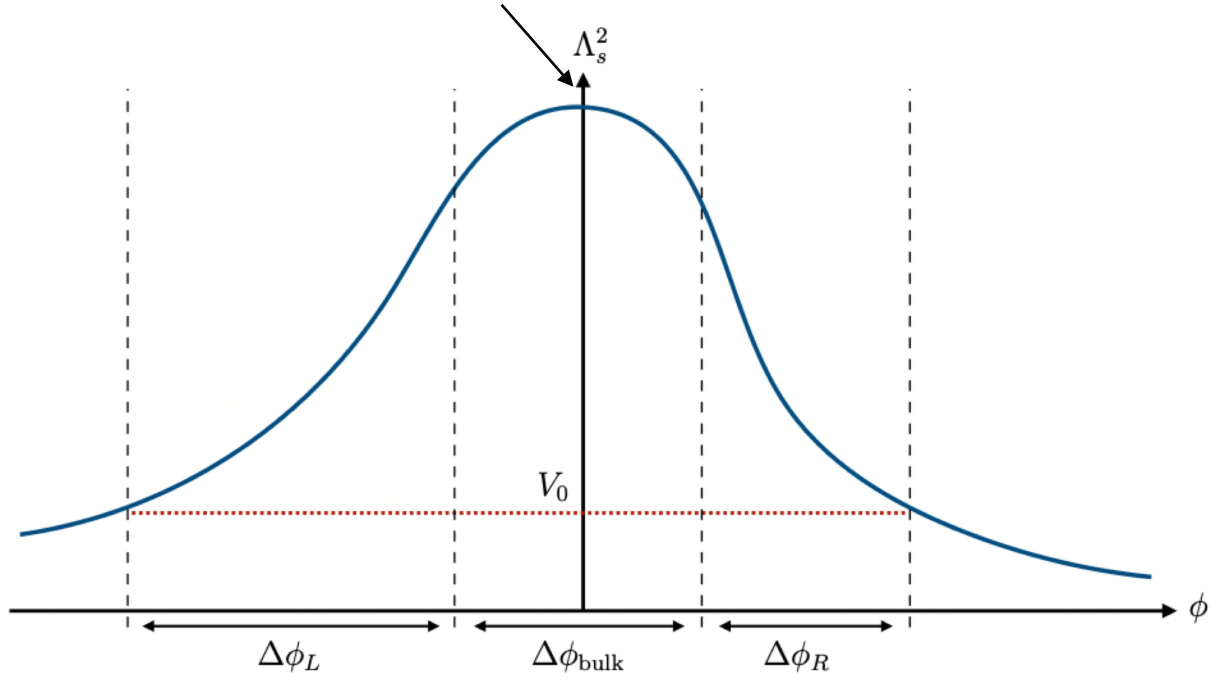
$$\Delta\phi(V_0) = \Delta\phi_L(V_0) + \Delta\phi_{\text{bulk}} + \Delta\phi_R(V_0)$$

Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

Cf. [Long, Montero, Vafa, Valenzuela '21]



Split the field range in three parts *(including the interior)*

$$\Delta\phi(V_0) = \Delta\phi_L(V_0) + \Delta\phi_{\text{bulk}} + \Delta\phi_R(V_0)$$

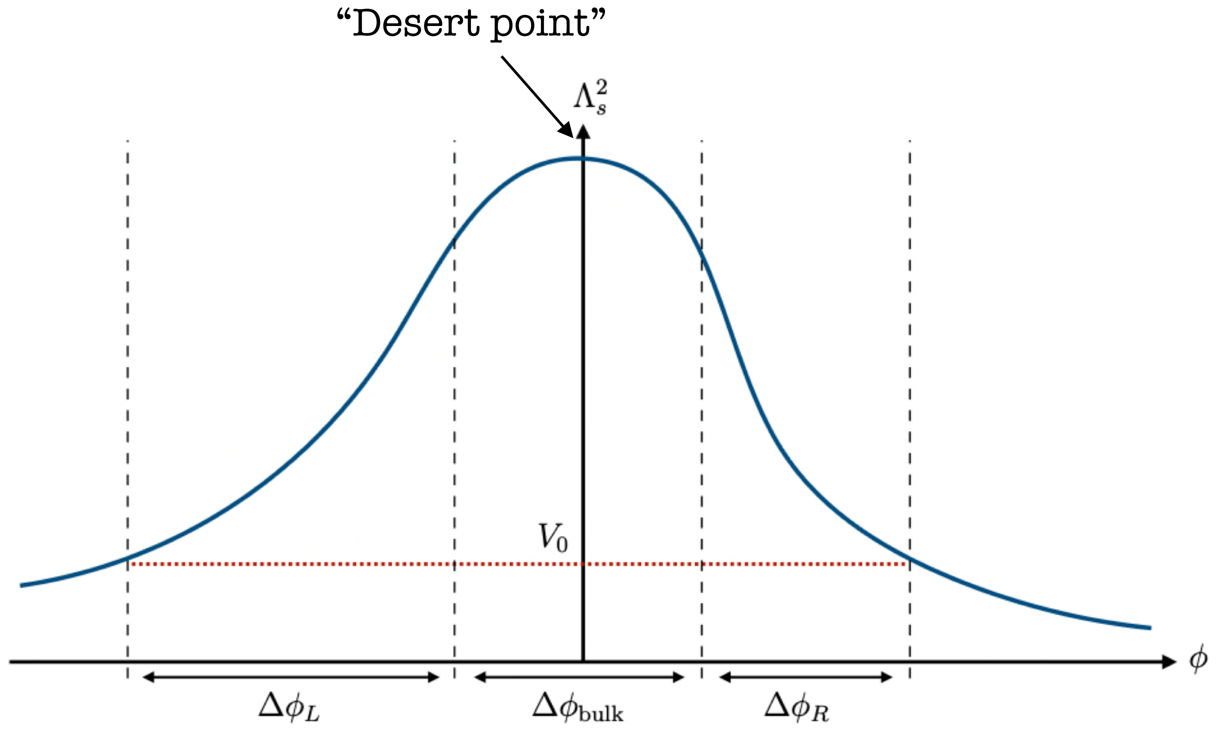
Can we get large field distances in the interior?

Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

Cf. [Long, Montero, Vafa, Valenzuela '21]



Split the field range in three parts *(including the interior)*

$$\Delta\phi(V_0) = \Delta\phi_L(V_0) + \Delta\phi_{\text{bulk}} + \Delta\phi_R(V_0)$$

Can we get large field distances in the interior?

From asymptotic expression: $\Lambda_s^2 = Ae^{-2\lambda\Delta\phi}$

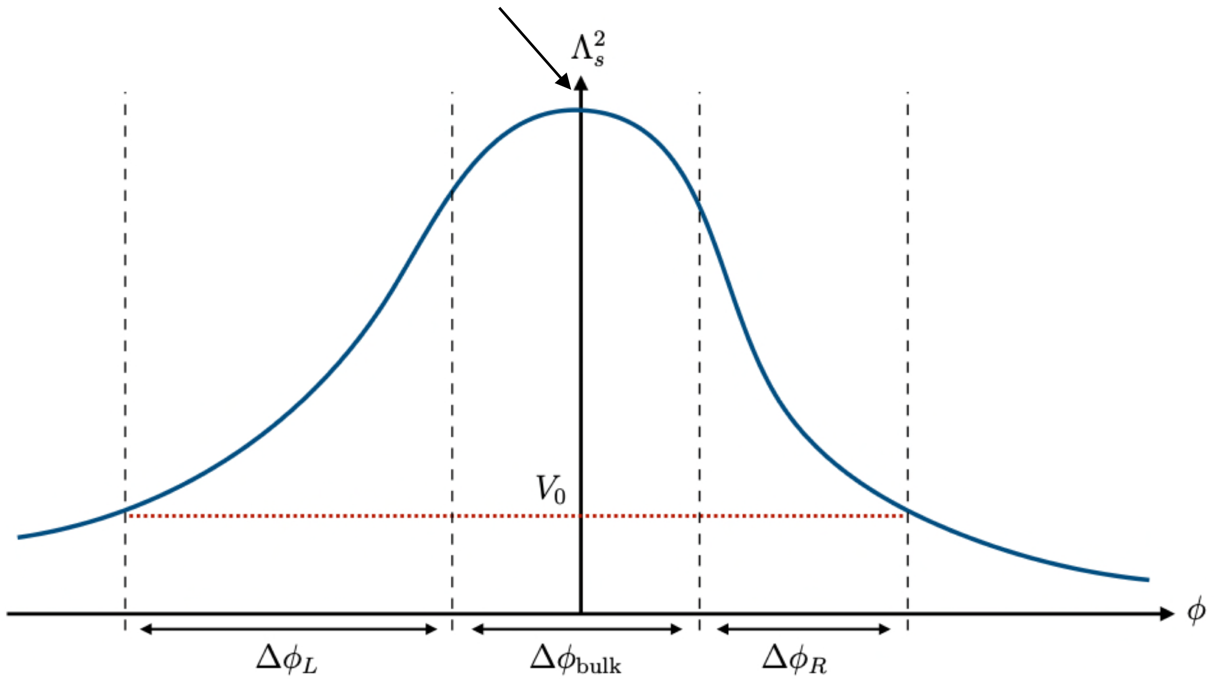
$$\longrightarrow \quad \Delta\phi_R(V_0) = -\frac{1}{2\lambda_R}\log[V_0] + \frac{1}{2\lambda_R}\log[A_R]$$

Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

Cf. [Long, Montero, Vafa, Valenzuela '21]



Split the field range in three parts *(including the interior)*

$$\Delta\phi(V_0) = \Delta\phi_L(V_0) + \Delta\phi_{\text{bulk}} + \Delta\phi_R(V_0)$$

Can we get large field distances in the interior?

From asymptotic expression: $\Lambda_s^2 = Ae^{-2\lambda\Delta\phi}$

$$\longrightarrow \quad \Delta\phi_R(V_0) = -\frac{1}{2\lambda_R}\log[V_0] + \frac{1}{2\lambda_R}\log[A_R]$$

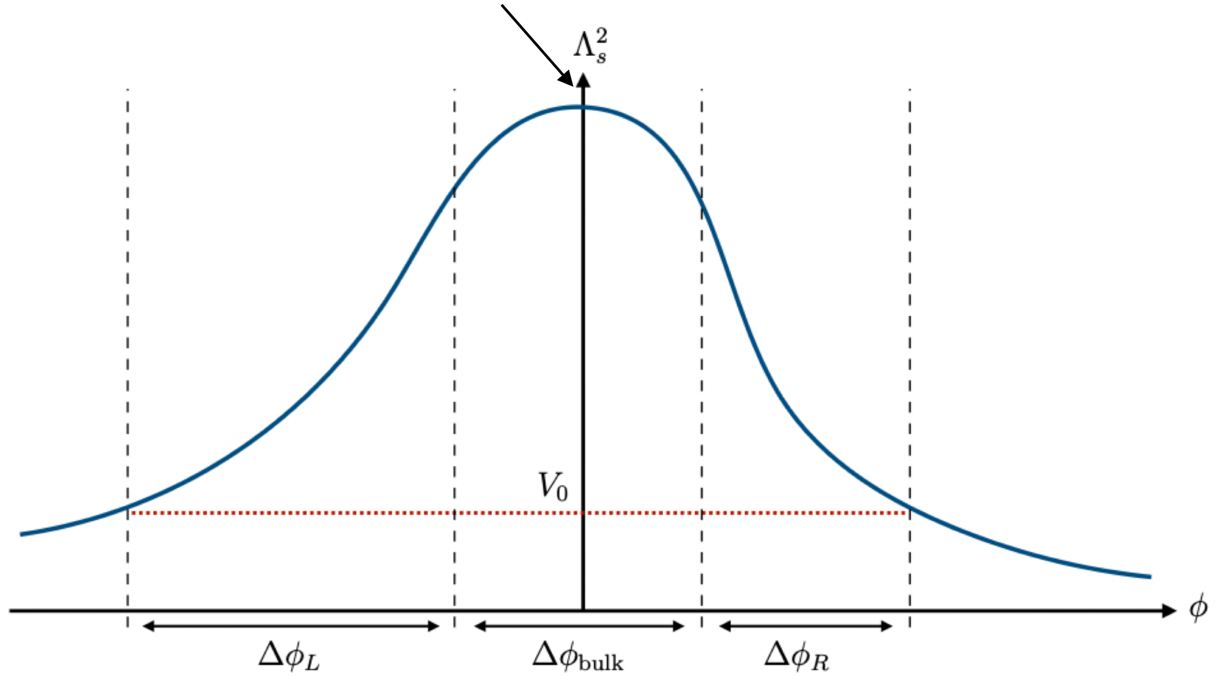
Since at large distance $F_1 \to \frac{1}{12}c_{2,i}\,\mathrm{Re}t^i$ we can determine $A = \frac{12}{2\pi\sum_i c_{2,i}} \lesssim \mathcal{O}(1)$

Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

Cf. [Long, Montero, Vafa, Valenzuela '21]



Split the field range in three parts *(including the interior)*

$$\Delta\phi(V_0) = \Delta\phi_L(V_0) + \Delta\phi_{\text{bulk}} + \Delta\phi_R(V_0)$$

Can we get large field distances in the interior?

From asymptotic expression: $\Lambda_s^2 = Ae^{-2\lambda\Delta\phi}$

$$\rightarrow \quad \Delta\phi_R(V_0) = -\frac{1}{2\lambda_R}\log[V_0] + \frac{1}{2\lambda_R}\log[A_R]$$

Since at large distance $F_1 \rightarrow \frac{1}{12}c_{2,i}\,\text{Re}t^i$ we can determine $A = \frac{12}{2\pi\sum_i c_{2,i}} \lesssim \mathcal{O}(1)$

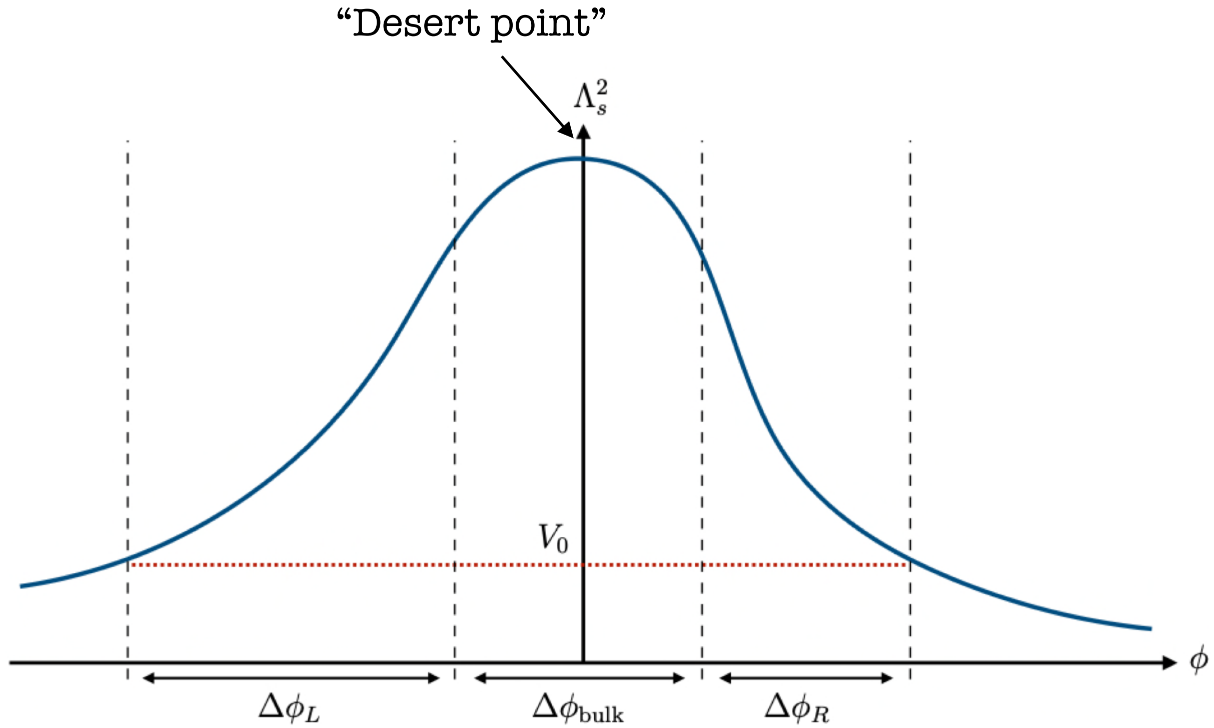
$$\Delta\phi(V_0) = -\left(\frac{1}{2\lambda_L} + \frac{1}{2\lambda_R}\right)\log[V_0] + \underbrace{\frac{1}{2\lambda_L}\log[A_L] + \frac{1}{2\lambda_R}\log[A_R] + \Delta\phi_{\text{bulk}}}_{=:b},$$

Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

Cf. [Long, Montero, Vafa, Valenzuela '21]



Split the field range in three parts *(including the interior)*

$$\Delta\phi(V_0) = \Delta\phi_L(V_0) + \Delta\phi_{\rm bulk} + \Delta\phi_R(V_0)$$

Can we get large field distances in the interior?

From asymptotic expression: $\Lambda_s^2 = Ae^{-2\lambda\Delta\phi}$

$$\longrightarrow \quad \Delta\phi_R(V_0) = -\frac{1}{2\lambda_R}\log[V_0] + \frac{1}{2\lambda_R}\log[A_R]$$

Since at large distance $F_1 \to \frac{1}{12}c_{2,i}\,\mathrm{Re}t^i$ we can determine $A = \frac{12}{2\pi\sum_i c_{2,i}} \lesssim \mathcal{O}(1)$

$$\Delta\phi(V_0) = -\left(\frac{1}{2\lambda_L} + \frac{1}{2\lambda_R}\right)\log[V_0] + \underbrace{\frac{1}{2\lambda_L}\log[A_L] + \frac{1}{2\lambda_R}\log[A_R] + \Delta\phi_{\rm bulk}}_{=:b},$$

Can check examples:

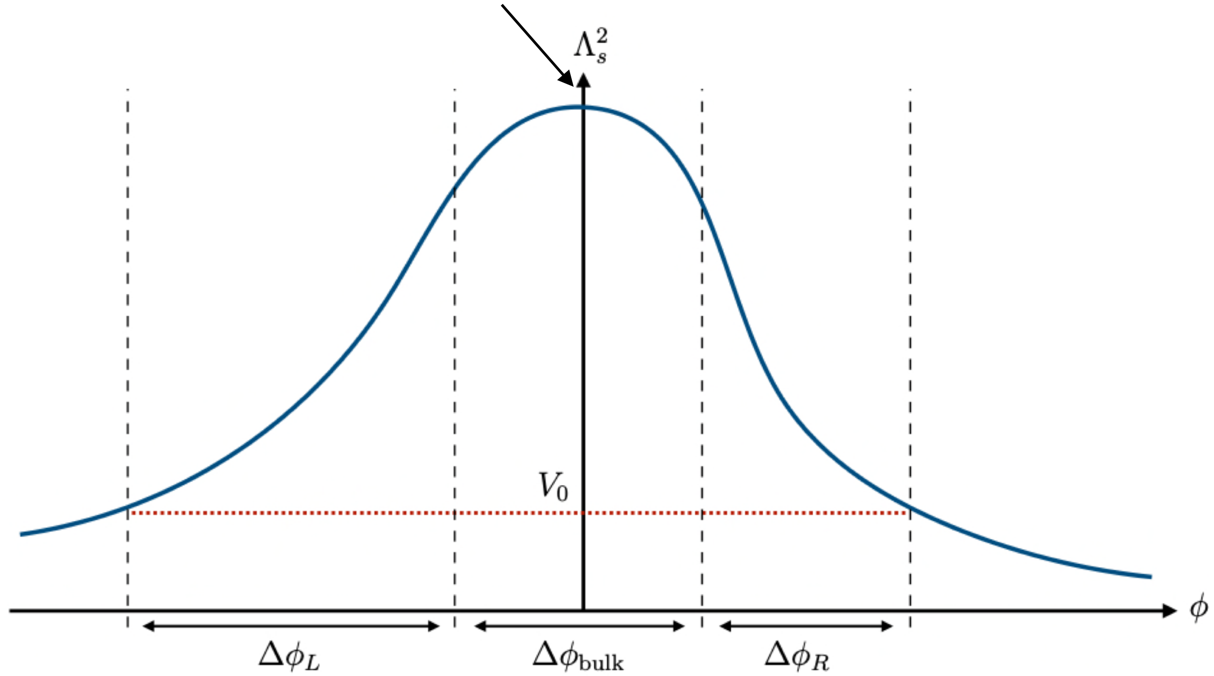
	$K3 \times T^2$	Mirror quintic	Mirror bi-cubic
$b =$	-1.198	-3.798	-4.815

Species Scale and Field Ranges

[van de Heisteeg, Vafa, MW, Wu '23]

We can further constrain the field range over which the potential is approximately flat $V \simeq V_0$ as:

Cf. [Long, Montero, Vafa, Valenzuela '21]



Split the field range in three parts *(including the interior)*

$$\Delta\phi(V_0) = \Delta\phi_L(V_0) + \Delta\phi_{\text{bulk}} + \Delta\phi_R(V_0)$$

Can we get large field distances in the interior?

From asymptotic expression: $\Lambda_s^2 = Ae^{-2\lambda\Delta\phi}$

$$\longrightarrow \quad \Delta\phi_R(V_0) = -\frac{1}{2\lambda_R}\log[V_0] + \frac{1}{2\lambda_R}\log[A_R]$$

Since at large distance $F_1 \to \frac{1}{12}c_{2,i}\,\mathrm{Re}t^i$ we can determine $A = \frac{12}{2\pi\sum_i c_{2,i}} \lesssim \mathcal{O}(1)$

$$\Delta\phi(V_0) = -\left(\frac{1}{2\lambda_L} + \frac{1}{2\lambda_R}\right)\log[V_0] + \underbrace{\frac{1}{2\lambda_L}\log[A_L] + \frac{1}{2\lambda_R}\log[A_R] + \Delta\phi_{\text{bulk}}}_{=:b},$$

Can check examples:

	$K3 \times T^2$	Mirror quintic	Mirror bi-cubic
$b =$	-1.198	-3.798	-4.815

$\rightarrow$ negative correction to asymptotic result

$\Rightarrow$ no large field ranges hidden in interior!

Conclusions

In this talk:

- Discussed the moduli-dependence of the effective quantum gravity cut-off, i.e., the species scale beyond asymptotic regimes.
- Showed a universal bound $\frac{|\nabla \Lambda_s|}{\Lambda_s} \leq \mathcal{O}(1)$ just from consistency of the effective action!
- Argued that in Type II CY compactifications the topological genus-1 free energy reliably computes the species scale everywhere in (vectormultiplet) moduli space.
- Allows to explicitly verify EFT bound in examples → naive asymptotic bound not correct!
- Species scale can be used to bound scalar potentials → leads to same constraint as TCC!
- In Type II setup: F_1 robust enough to still give species scale even in $\mathcal{N} = 1$
  → allows to fix $\mathcal{O}(1)$ coefficients and to bound the range for approx. flat, positive potentials (including the interior)

Thank you!!