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Geometry of Higher Symmetries On (non-)Compact F/M-Theory

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Motivation

Studying M/string theory on special holonomy spaces X:

- Non-compact spaces X →
  Geometric engineering of supersymmetric quantum field theories (SQFTs):

  Build dictionary:

  {operators, symmetries} ⟺ {geometry, topology}

  Focus: higher-form global symmetries
  (associated with ``flavor'' branes) ⟺ topology

- Compact spaces X →
  Quantum field theory (QFT) w/ gravity →
  Higher-form symmetries gauged or broken
  [Physical consistency conditions: swampland program]

Higher-form symmetries in (S)QFT - active field of research

[Gaiotto, Kapustin, Seiberg, Willet, 2014],…

Higher-form symmetries & geometric engineering

[Del Zotto, Heckman, Park, Rudelius, 2015],…

[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],…

[M.C., Dierigl, Lin, Zhang, 2020],..

[Apruzzi,Bhardwaj, Oh, Schäfer-Nameki, 2021],…

[M.C., Dierigl, Lin, Zhang, 2021],…

[M.C., Heckman, Hübner, Torres, 2022],
[Del Zotto, Garcia Etxebarria, Schäfer-Nameki, 2022],…

[Hübner, Morrison, Schäfer-Nameki,Wang, 2022],…

[Heckman, Hübner, Torres, Zhang, 2023],…

[M.C.,Heckman, Hübner, Torres, Zhang, 2023],..

Higher-form symmetries & and compact geometry

[M.C., Dierigl, Lin, Zhang, 2022],…

[M.C., Heckman, Hübner, Torres, 2023],…

Goals

- Identify geometric origin of higher-form symmetries for M-/string theory non-compact special holonomy spaces X

  Punchline: 0-form,1-form and 2-group symmetries via cutting and gluing of singular boundary geometries

- Examples: M-theory on non-compact elliptically fibered Calabi-Yau n-folds

New results for compact geometries
[compact elliptic examples –
dual to F-theory compactification]

Based on

non-compact geometries:

- M. C., J. J. Heckman, M. Hübner and E. Torres:
  "0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds," 2203.10102

& compact geometries:

- "Fluxbranes, Generalized Symmetries, and Verlinde's Metastable Monopole," 2305.09665
- "Generalized Symmetries, Gravity, and the Swampland,"2307...

Related: Gauge symmetry topology constraints in D=8 N=1SG

M.C., M.Dierigl, L.Lin and H. Y. Zhang:
``String Universality and Non-Simply-Connected Gauge Groups in 8d,'' PRL, 2008.10605

- ``Higher-form Symmetries and Their Anomalies in M-/F-theory Duality,'' 2106.07654 - 8D/7D & 6D/5D
- ``Gauge group topology of 8D Chaudhuri-Hockney-Lykken vacua,'' 2107.04031
- ``One Loop to Rule Them All: Eight and Nine Dimensional String Vacua from Junctions,'' arXiv:2203.03644 [hep-th]– String junctions
- M.C., M. Dierigl,.Lin E. Torres and Zhang, ``Generalized Symmetries and Frozen Singularities ,'' 2307…..

Overlap with talks by Jonathan Heckman & Max Hübner…

Outline:

- Introduction: Defect Group
- Focus: Defect group and higher-form symmetries → Topology of flavor symmetry group
- 2-group symmetries
- Compact (elliptic) examples → fate of higher-form symmetries
- Concluding remarks

I. Introduction

Defect group for M-theory on non-compact X

- Defect Group for extended p-dim operators associated with M2 and M5 branes: $\mathcal{D}_p = \mathcal{D}_p^{\mathrm{M2}} \oplus \mathcal{D}_p^{\mathrm{M5}}$
- M2, M5 in X wrap relative cycles:

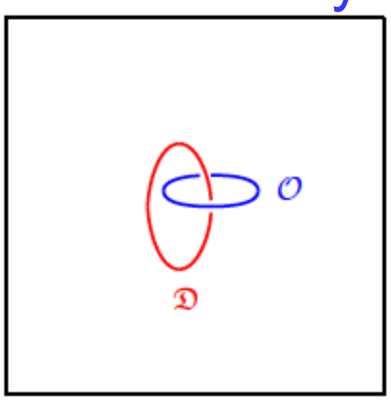
[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],
[Del Zotto, Heckman, Park, Rudelius, 2015]

$$\mathcal{D}_p^{\mathrm{M2}} = \frac{H_{3-p}(X, \partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\mathrm{triv}}$$ [p-dim el. operators in SCFT]

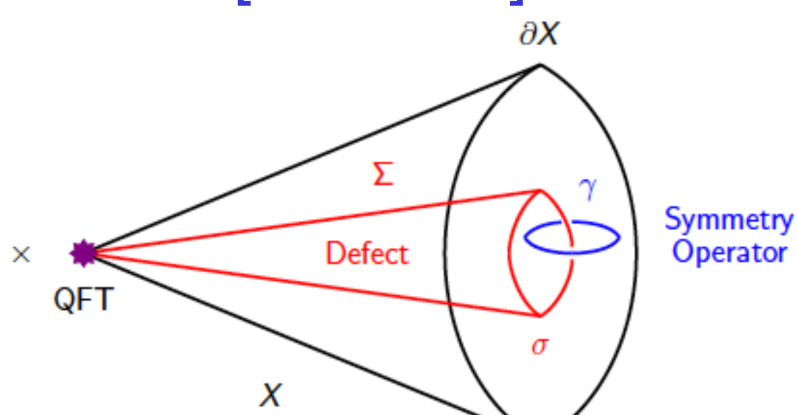
$$\mathcal{D}_p^{\mathrm{M5}} = \frac{H_{6-p}(X, \partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\mathrm{triv}}$$ [p-dim mag. operators in SCFT]

[torsional]

- Schematically:



Space-time

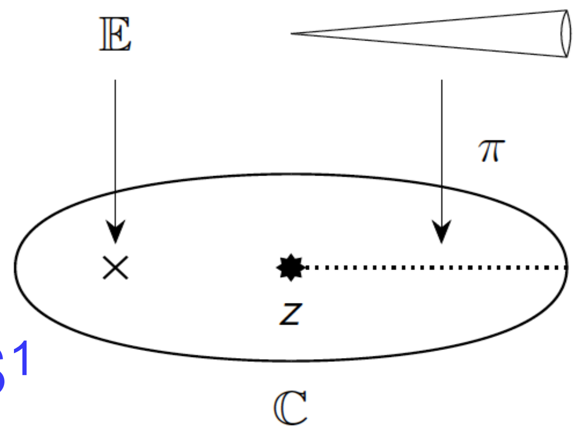


Focus on defect ops.;
Symmetry gen. ops.:
[Heckman, Hübner, Torres, Zhang 2022,..; M.C., Heckman, Hübner, Torres, Zhang 2023]

Example: non-compact *K3*

[M.C., Dierigl, Lin, Zhang, 2021, 2022]

- Local elliptically fibered *K3* $\mathbb{E} \hookrightarrow X \rightarrow \mathbb{C}$
- Singular fiber of Kodaira type ϕ at $z \in \mathbb{C}$ w/ monodromy M



$\mathfrak{T}$ - ``Kodaira Thimble''

[torsional two-cycle]

- Boundary $\mathbb{E} \hookrightarrow \partial X \rightarrow S^1$
- Exact sequence for spaces fibered over circles:

$$0 \rightarrow \operatorname{coker}(M_n - 1) \rightarrow H_n(X) \rightarrow \ker(M_{n-1} - 1) \rightarrow 0$$

M_n - monodromy in homology in degree n

$$\mathcal{D}_1^{\mathrm{M2}} = \mathcal{D}_4^{\mathrm{M5}} = H_2(X, \partial X)/H_2(X) \cong \operatorname{Tor} \operatorname{Coker}(M - 1) = \langle \mathfrak{T} \rangle$$

- X engineers 7D SYM with gauge algebra $\mathfrak{g}_\phi$ w/
  Defect group $\mathcal{D} = \langle \mathfrak{T} \rangle_1^{\mathrm{M2}} \oplus \langle \mathfrak{T} \rangle_4^{\mathrm{M5}}$

Example (continued):

- Non-trivial self-linking/intersection: $\ell(\partial\mathfrak{T},\partial\mathfrak{T}) = \mathfrak{T}\cdot\mathfrak{T} \neq 0$
- Elements of $\mathcal{D}_1^{M2}$, $\mathcal{D}_4^{M5}$ typically mutually non-local
- Choose electric polarization $\mathcal{D}_1^{M2}$ [main focus of the talk]
- Gauge group is simply connected G_ϕ w/algebra g_ϕ (ADE)
- Resulting 7D SYM theory w/ gauge group G_ϕ
- (Wilson) Line operators $\mathcal{D}_1^{M2}$ acted on by 1-form symmetry $Z_{G\phi}$ [Pontyagin dual to line operators] fixes group topology

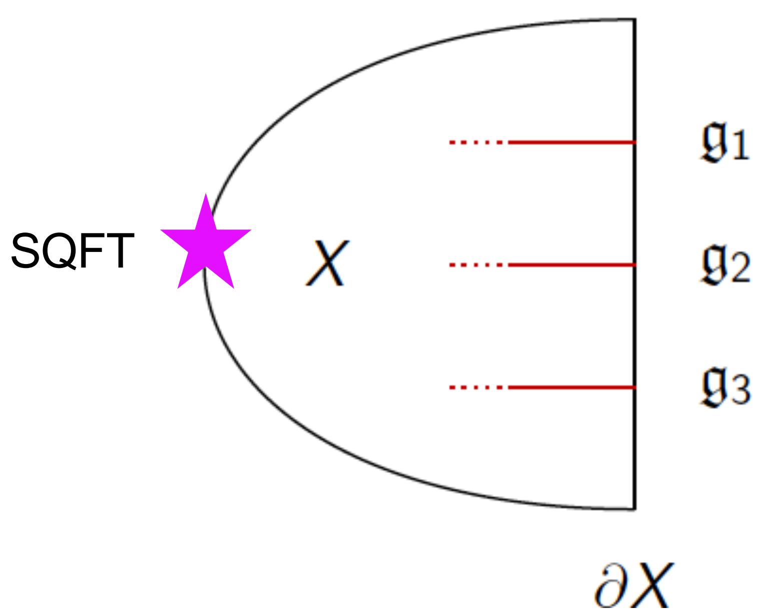
Now, turn to higher-form structures for non-compact spaces X in higher dimensions ($D \geq 6$)

→ leads to new phenomena

II. Geometrizing Topology of Flavor Group

Non-compact ADE loci ≡ flavor branes → flavor symmetries

Naïve flavor symmetry $\widetilde{G}_F$ (simply connected w/ Lie Algebra $\mathfrak{g}_i$)



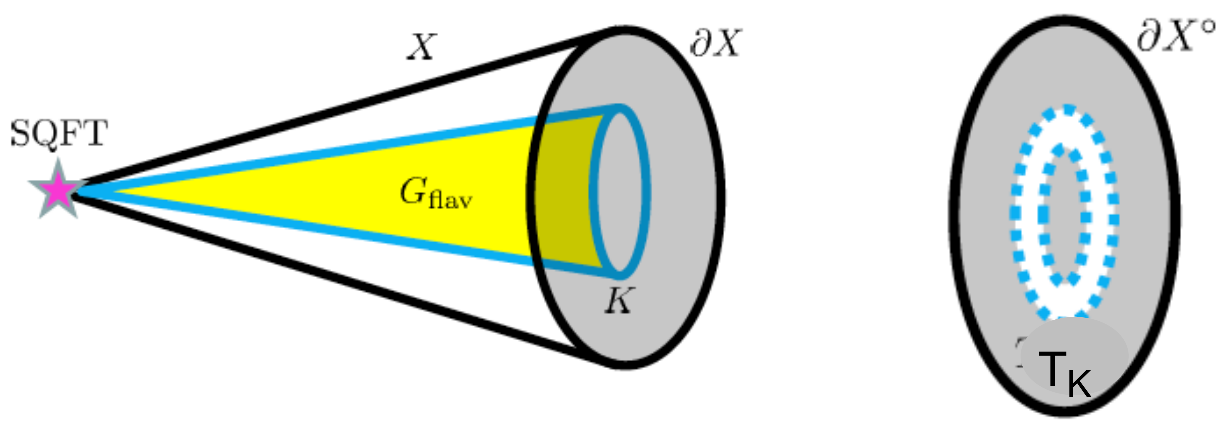
[From now on only ADE's in ∂X]

$$\widetilde{G}_F = \widetilde{G}_1 \times \widetilde{G}_2 \times \widetilde{G}_3 \times \ldots$$

(Flavor Wilson) lines → fix topology of favor symmetry G_F
from singular boundary topology

Boundary geometry of flavor branes:

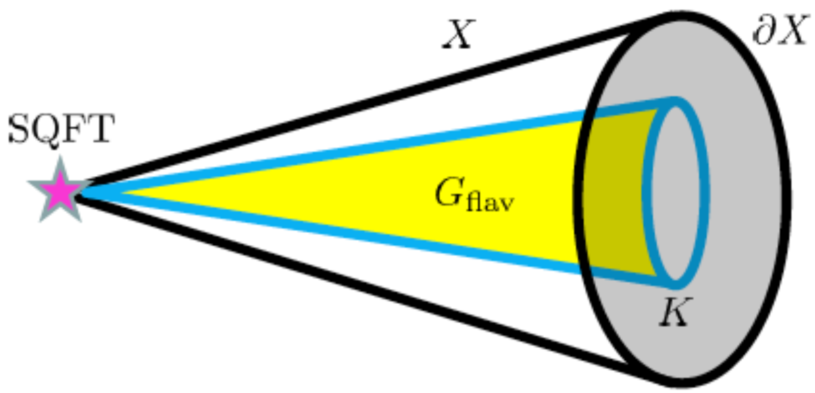
- Singular non-compact space X w/
  $K = \cup_i K_i$ - ADE loci (of flavor branes) in the boundary $\boldsymbol{\partial}$X
- Define a smooth boundary $\boldsymbol{\partial}X^o = \boldsymbol{\partial}X \setminus K$
  & a tubular region T_K (excise K)
- Locally $T_K \cap \partial X^\circ \cong \cup_i K_i \times S^3/\Gamma_i$

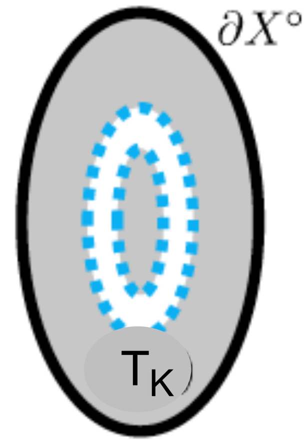


- Naïve flavor center symmetry:

$$Z_{\widetilde{G}_F} = \mathrm{Tor}\, H_1(T_K \cap \partial X^\circ) \cong Z_{\widetilde{G}_1} \oplus Z_{\widetilde{G}_2} \oplus Z_{\widetilde{G}_3} \oplus \ldots$$

Boundary geometry of true flavor center symmetry Z_{G_F}





[Mayer, 1929], [Vietoris, 1930]

- Key: Mayer-Vietoris sequence in homology for singular boundary $\partial X = \partial X^\circ \cup T_K$ → obtain:

$$0 \to \underbrace{\ker(\iota_1)}_{\text{flavor center}} \to \underbrace{H_1(\partial X^\circ \cap T_K)}_{\text{naive flavor center}} \xrightarrow{\iota_1} \underbrace{H_1(\partial X^\circ)}_{\text{(un)twisted sector}} \oplus H_1(T_K) \to \underbrace{H_1(\partial X)}_{\text{1-form symmetry}} \to 0$$

naïve 1-form symmetry

$$Z_{G_F} = \mathrm{Ker}\left(\iota_1 : Z_{\widetilde{G}_F} \cong H_1(\partial X^\circ \cap T_K) \to H_1(\partial X^\circ) \oplus H_1(T_K)\right)$$

- Motivated by orbifold homology: $H_1(\partial X^\circ) = H_1^{\mathrm{orb}}(\partial X)$

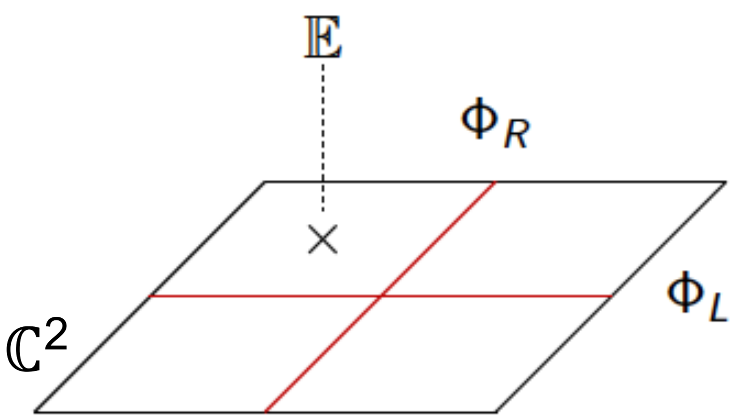
[Thurston, 1980], [Moerdijk, Pronk, 1997]

Example: Elliptically fibered threefold

$$\mathbb{E} \hookrightarrow X_3 \to B = \mathbb{C}^2$$ (non-compact base)

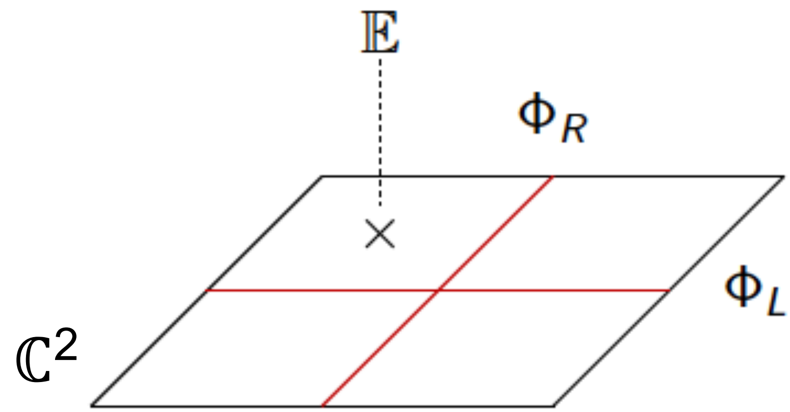
[(G_L; G_R)-Conformal Matter]
[Del Zotto, Heckman, Tomasiello, Vafa, 2014]

- Non-compact discriminant loci: ϕ_L on $\mathbb{C}$ x {0} & ϕ_R on {0} x $\mathbb{C}$



- Boundary five-manifold $\mathbb{E} \hookrightarrow \partial X_3 \to S^3$
- Discriminant locus consists of two linking circles S^1_L & S^1_R
- Excise singular fibers $\mathbb{E} \hookrightarrow \partial X^\circ \to S^1_L \times S^1_R$

Example (continued):



[$(G_L; G_R)$-Conformal Matter]

- Again, exact sequence for spaces fibered over circles $S^1_{L\ R}$

$$0 \to \operatorname{coker}(M_n - 1) \to H_n(X) \to \ker(M_{n-1} - 1) \to 0$$

M_n - monodromy in homology in degree n.

- From monodromies M_L & M_R about S^1_L & S^1_R:

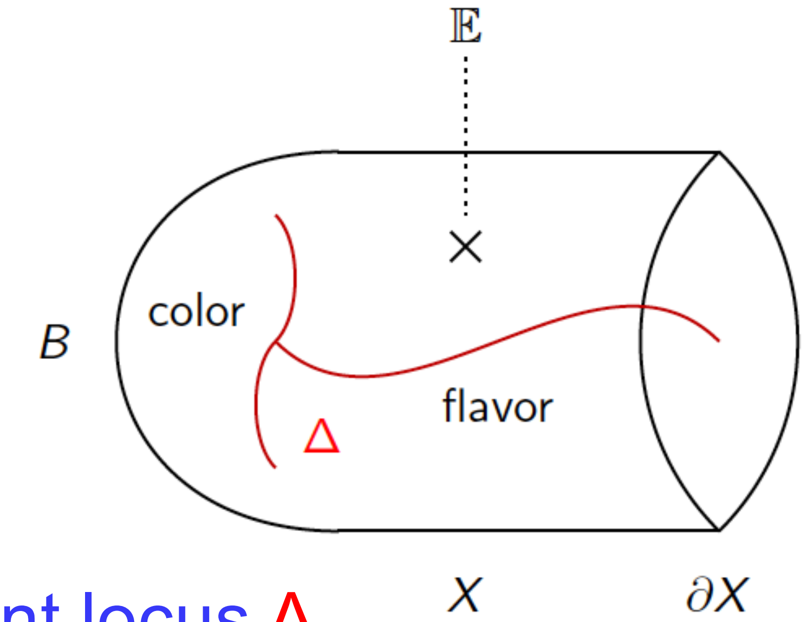
$$\operatorname{Tor} H_1(\partial X^\circ) = \operatorname{Tor} \frac{\mathbb{Z}^2}{\operatorname{Im}(M_{\Phi_L} - 1) \cup \operatorname{Im}(M_{\Phi_R} - 1)}$$

- (SU(n); SU(m)) Conformal Matter $G_F = \dfrac{SU(n) \times SU(m)}{\mathbb{Z}_{\gcd(n,m)}}$

- General: $G_F = \dfrac{G_L \times G_R}{Z_{\text{diag}}}$

General Elliptically fibered Calabi-Yau n-folds

- $\mathbb{E} \hookrightarrow X_n \to B_{n-1}$



- Focus on non-compact discriminant locus Δ

  [Hübner, Morrison, Schäfer-Nameki, Wang 2022]

- Homology groups of ∂X, ∂X° & $\partial X_F = \partial X \setminus \{\text{singular fibers}\}$
- Deformation retractions of $\partial B \setminus \Delta$ lift to ∂X_F
- Glue singular fibers back in

$$\mathrm{Tor}\, H_1\left(\partial X^\circ\right) = \mathrm{Tor}\, H_1\left(\partial X_F\right) \oplus \mathrm{Tor}\, H_1(\partial B)$$

No time

→ Further explicit elliptic examples; also, orbifolds, G_2,…

III. 2-Groups

[Benini, Cordova, Hsin, 2019],
[Lee, Ohmori, Tachikawa, 2021], . . .

- Two key short exact sequence

$$0 \to \mathcal{C} \to Z_{\widetilde{G}_F} \to Z_{G_F} \to 0$$

$$0 \to \mathcal{C}^\vee \to \widetilde{\mathcal{A}}^\vee \to \mathcal{A}^\vee \to 0$$

[& Postnikov class]

- Z_{G_F} : Center flavor symmetry
- $Z_{\widetilde{G}_F}$: Naïve center flavor symmetry
- $\mathcal{A}^\vee$: Line operators modulo screening by local operators
- $\widetilde{\mathcal{A}}^\vee$: (Naïve) Line operators modulo screening by local operators transforming in reps. of Z_{GF}
- $\mathcal{C}^\vee$: Line operators in the kernel of $\widetilde{\mathcal{A}}^\vee \to \mathcal{A}^\vee$

Pontryagin duals: $\mathcal{D}^\vee = \mathrm{Hom}(\mathcal{D},U(1))$; [Line operators ←→ One-forms]

- Taking the Pontryagin dual (one-form symmetries) of line- operators in the second sequence & reversing arrows:

Z$_{\text{0-form}}$: $0 \to \mathcal{C} \to Z_{\widetilde{G}} \to Z_G \to 0$

1-form: $0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to \mathcal{C} \to 0$

w/ Postnikov class $P \in H^3(BG, \mathcal{A})$

[obtained via first extension $w_2 \in H^2(BZ_{\widetilde{G}}, \mathcal{C})$ & Bockstein homomorphis $\beta : H^2(BZ_G, \mathcal{C}) \to H^3(BZ_G, \mathcal{A})$]

- Sufficient: when second sequence does not split, i.e., $\tilde{\mathcal{A}} \neq \mathcal{A} \oplus C$ → 2-group

- One can collapse the two sequences:

2-group: $0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to Z_{\widetilde{G}_F} \to Z_{G_F} \to 0$

2-groups and Mayer-Vietoris

- Derive parallel homology sequences (physically motivated):

$$0 \to \ker(\iota_1) \to H_1(\partial X^\circ \cap T_K) \xrightarrow{\iota_1} \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to 0\,,$$

$$0 \to \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to H_1(\partial X^\circ) \oplus H_1(T_K) \to H_1(\partial X) \to 0$$

- Precisely identified with previous two sequences!

- Again, when the bottom sequence does not split → 2-group mixing 0-form and 1-form (flavor) symmetries

- Found Elliptically fibered CY's and Orbifold examples w/ 2-group

IV. Compact Models

- Compact singular space X → theory that includes quantum gravity & global symmetries gauged or broken
- What is M-theory gauge group?
- Elliptically fibered geometries (via M/F-theory duality):

  - Non-Abelian group algebras – ADE Kodaira classification

  Group topology → Mordell-Weil torsion

  [Aspinwall, Morrison, 1998], [Mayrhofer, Morrison,Till, Weigand, 2014], [M.C., Lin, 2017]

  - Abelian groups → Mordell-Weil ``free'' part

  [Morrison, Park 2012], [M.C., Klevers, Piragua, 2013], [Borchmann, Mayrhofer, Palti, Weigand, 2013]…

  - Total group topology → Shoida map of Mordell-Weil

  [M.C., Lin, 2017]

$$\frac{U(1)^r \times G_{\text{non-ab}}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{t} \mathbb{Z}_{k_j}}$$

Digression: F-theory on elliptically fibered Calabi-Yau fourfolds
w/specific elliptic fibration (F_{11} polytope)
led to D=4 N=1effective theory

[M.C., Klevers, Peña, Oehlmann, Reuter, '15]

Standard Model gauge group $\dfrac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$

w/ gauge group topology
(geometric - encoded in Shioda Map of MW)

[M.C., Lin, '17]

w/ toric bases B_3 (3D polytopes)

[M.C., Halverson, Lin, Liu, Tian '19]

Quadrillion Standard Models (QSMs)
with 3-chiral families & gauge coupling unification

[gauge divisors – in class of *anti-canonical divisor* $\overline{K}_C$]

Current efforts: determination the exact matter spectra
(including vector pair & # of Higgs pairs)
→ Studies of (limit) root bundles on matter curves

[Bies, M.C., Donagi,(Liu), Ong, '21,'22,'23]

No time, c.f., Martin Bies' talk

How to relate these results, encoded in arithmetic structure of elliptic curve, due Mordell-Weil, to higher-form symmetries?

- Global symmetries, including higher-form ones should be gauged or broken [No Global Symmetry Hypothesis]

…[Harlow, Ooguri '18]

- In 8D N =1 Supergravity:
quantified conditions under which no anomalies
due to gauged 1-form symmetry
[magnetic version – preferred polarization!]

[M.C., Diriegl, Ling, Zhang, '20]

- True for all 8D N=1 string compactifications (beyond F-theory)
via (refined) string junction construction

[M.C., Diriegl, Ling, Zhang, '22]

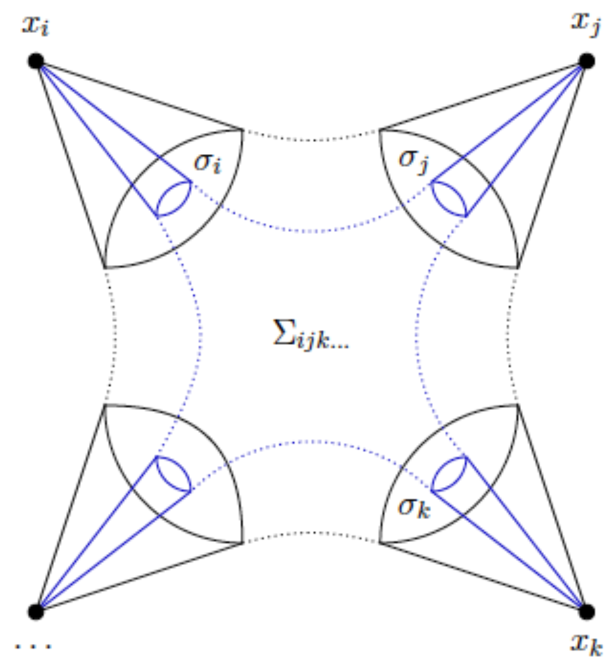
[M. C., Heckman, Hübner, Torres,
"Generalized Symmetries, Gravity, and the Swampland," 2307...]

Fate of higher-form structures in Compact Geometries

- Decompose $X = \cup_n X_n$ into local models X_n

  Converse: Glue $\{X_n\}$ to X ⬌ Couple $\{SQFT_n\}$ to a resulting QFT & include gravity

  Schematically:



Σ_{ijk} – two-chain glues defects

- Some relative Cycles in X_n survive & compactify → (Some) defects in $SQFT_n$ become dynamical - ``gauged''
  → determine the global gauge group in compact models

Fate of higher-form structures in Compact Geometries (continued):

- Mayer-Vietoris Long Exact Sequence for covering $\{X_n\}$ relates homologies of X_n to X - Subtleties No time

- Results in decomposition of compact two-cycles into a sum of relative cycles associated with each local model

$$\partial_2 : H_2(X) \rightarrow \bigoplus_n H_1(\partial X_n)$$

- Analysis for elliptically fibered geometries: specifically, torsional cycles associated w/ Mordell-Weil decompose into relative cycles of $\{X_n\}$ $\rightarrow$ complementary geometric results!

- Analysis beyond elliptically fibered models: all T^4/Γ_i , some T^6/Γ_i ,… [Abelian factors; fate of 2-group; non-invertible symmetries,…]

Further details: Jonathan Heckman's & Max Hübner's talks

Thank you!