TADPOLES AND GAUGE SYMMETRIES

[2304.06751]

with A. Braun, B. Fraiman, M. Graña, H. Parra de Fraitas

Severin Lüst

CNRS, Laboratoire Charles Coulomb Montpellier

String Phenomenology 2023
Institute for Basic Science, Daejeon
July 5

STRING THEORY LANDSCAPE

- String theory (once compactified)

(too) many possibilities

→ String Phenomenology very difficult!

- Here: Landscape of (classical) flux vacua

constrained by integer quantization and tadpole bounds

lower-dim gravity ⟷ *arising from compactification*

FLUX COMPACTIFICATION OF IIB AND M-THEORY

- Calabi-Yau compactification with flux:

$$M_D = M_{D-2n} \times_w CY_n \qquad\qquad G_n \in H^n(\mathbb{Z})$$

(related via F-theory)

- Vacuum condition:

$$\star\, G_n = (-i)^n G_n \quad (\,*\,)$$

$\rightarrow$ *fixes complex-structure moduli!*

- GVW superpotential:

$$W = \int G_n \wedge \Omega \qquad\qquad D_i W = 0 \;\Leftrightarrow\; (\,*\,)$$

KÄHLER MODULI STABILIZATION

- For IIB and F-theory:

at the classical level: W independent of Kähler moduli

→ *Kähler moduli not stabilized by fluxes!*

- Non-perturbative quantum effects:

$$W = \int G_n \wedge \Omega + \sum_{\mathbf{k}} \mathscr{A}_{\mathbf{k}}\, e^{-2\pi k^\alpha T_\alpha}$$

→ *potential for complex structure moduli + Kähler moduli (KKLT)*

recent progress:
[Demirtas, Kim, McAllister, Moritz, Rios-Tascon '21]

But: tension with holographic interpretation!
[SL, Vafa, Wiesner, Xu '21]

This talk: agnostic with respect to Kähler moduli stabilization!

TADPOLE IN F-THEORY

- Fluxes: constrained by tadpole cancellation conditions!
- In M/F-theory: $$\frac{1}{2}\int G_4 \wedge G_4 = \frac{\chi(CY_4)}{24}$$
  $> 0 \ (G_4 = \star G_4)$
- Scaling with $h^{3,1}$:

  #moduli to be stabilized by G_4
  $(h^{3,1} \gg h^{1,1}, h^{2,1})$

$$\frac{\chi(CY_4)}{24} = \frac{1}{4}(8 + h^{1,1} + h^{3,1} - h^{2,1}) \propto \frac{1}{4}h^{3,1}$$

(see Arthur's talk)

→ Scaling of $\frac{1}{2}\int G_4 \wedge G_4$ with $h^{3,1}$?

Notice:
tadpole bound often control parameter for size of corrections
[Bena, Dudas, Graña, SL '18]
[Gao, Hebecker, Schreyer, Venken '22]
[Blumenhagen, Gligovic, Kaddachi '22]

THE TADPOLE CONJECTURE

[Bena, Blåbäck, Graña, SL '20]

F-theory on Calabi-Yau four-fold:

- large number of complex structure moduli: $h^{3,1} \gg 1$
- all stabilized by integer fluxes G_4
- at generic point in moduli space

The D3 charge of the fluxes satisfies:

$$Q_{D3}^{\text{flux}} = \frac{1}{2} \int G_4 \wedge G_4 \gtrsim \alpha \times h^{3,1}$$

(α: $\mathcal{O}(1)$-constant)

For $\alpha > \frac{1}{4}$: moduli stabilization at large $h^{3,1}$ generically not possible!

LARGE COMPLEX STRUCTURE VS. INTERIOR OF MODULI SPACE

- At Large Complex Structure (mirror dual to large volume):
  - *Evidence for the tadpole conjecture* [Marchesano, Prieto, Wiesner '21] [Plauschinn '21] [SL '21] [Coudarchet, Marchesano, Prieto, Urkiola '22, '23]
  - *Analytic arguments in the strong asymptotic limit* [Tsagkaris, Plauschinn '22] [Graña, Grimm, van de Heisteeg, Herraez, Plauschinn '22]

- Special, symmetric points in the deep interior of moduli space: [Giryavets, Kachru, Tripathy, Trivedi '03]

  Tadpole conjecture can be avoided! [SL, Wiesner '22]

  (stabilize 4932 moduli with $N_{\text{flux}} = 3$)

  see also: [Braun, Valandro '20] [Blanco-Pillado, Sousa, Urkiola, Wachter '20] [Becker, Gonzalo, Walcher, Wrase '22] [Cicoli, Licheri, Mahanta, Maharana '22]

- Interior of moduli space (away from symmetric points)

  very difficult to study! → Here: M-theory on K3xK3

M-THEORY ON K3 X K3

- Use the (special) four-fold

$$K3 \times K3$$

  as a laboratory for flux stabilization.

  (first studied in [Aspinwall, Kallosh '05])

- Benefit:
  - no explicit knowledge of period integrals necessary

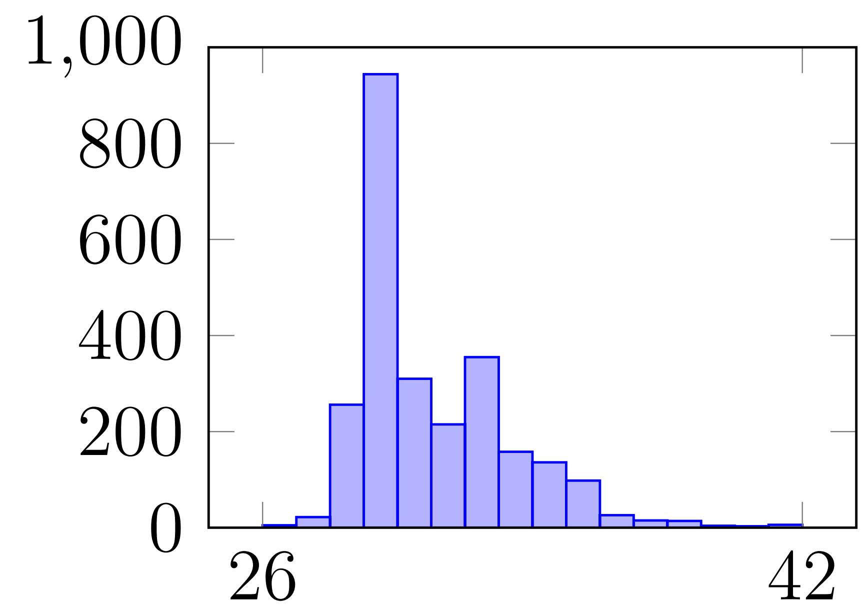
1. Generic, smooth K3s: using differential evolution
2. Attractive K3s: Systematic and exhaustive classification

DIFFERENTIAL EVOLUTION FOR K3 X K3

[Bena, Blåbäck, Graña, SL '21]

(see Andre's talk)

- Find fluxes with differential evolutionary algorithms that
  - ★ stabilize all moduli
  - ★ at a generic point (no gauge enhancement)
  - ★ with as small charge $N_{\text{flux}} = \frac{1}{2}\int G_4 \wedge G_4$ as possible
- Results:
  - $\mathcal{O}(10^5)$ flux solutions with $Q_{D3}^{\text{flux}} = 25$
  - 0 solutions with $Q_{D3}^{\text{flux}} \leq 24$



- Remember: $\frac{\chi(K3 \times K3)}{24} = 24$

No such solutions within tadpole bound found!

MODULI STABILIZATION ON ATTRACTIVE K3 X K3

- Differential Evolution suggests:

Moduli stabilization at generic (smooth) point in moduli space not possible!

But: could be statistical effect…?

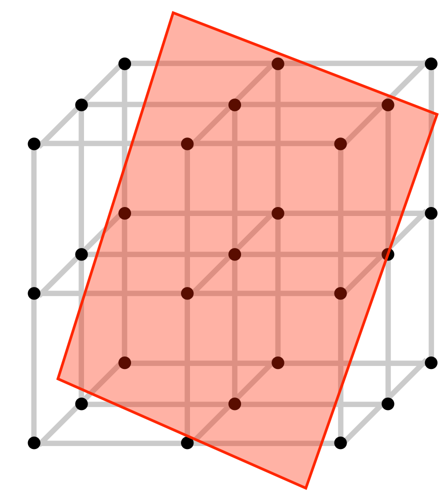
- If we assume that
  - both K3s are attractive
  - existence of F-theory uplift (elliptic fibration)

→ *complete survey of flux vacua possible!*

→ lower bound on the rank of the gauge group for each $N_{flux} \leq 30$

THE MODULI SPACE OF K3

- Middle cohomology of K3:



$$H^2(K3,\mathbb{Z}) \cong (-E_8) \oplus (-E_8) \oplus U \oplus U \oplus U$$

even, self-dual lattice of sign. (3,19)

- Point in moduli space:

$H^2_+ = \text{span}\{\omega_i\}$

choice of *three* self-dual 3-forms $\omega_i \in H^2(K3,\mathbb{R})\ , \quad i = 1,2,3$

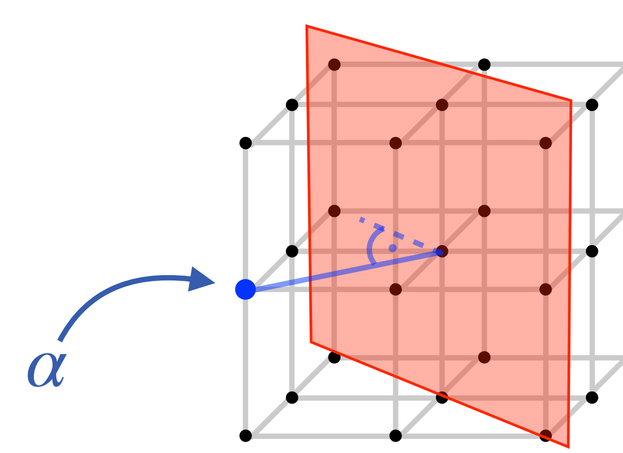
$$\rightarrow \quad \Omega = \omega_1 + i\omega_2\ , \ J \sim \omega_3$$

57 hyperkähler moduli!

- Orbifold singularity:

root $\alpha \in H^2(K3,\mathbb{Z})$ $(\|\alpha\|^2 = -2)$

s.t. $(\alpha, \omega_i) = 0 \quad \forall i$



PICARD LATTICE AND ATTRACTIVE K3

- Picard lattice of a K3-surface S:

$$\mathrm{Pic}(K3) = H^{1,1}(K3) \cap H^2(K3,\mathbb{Z})$$

attractive K3: $\mathrm{rank}\big(\ \mathrm{Pic}(K3)\ \big) = 20$

- characterise $\mathrm{Pic}(S)$ by its orthogonal complement:

$$T = \mathrm{Pic}(K3)^{\perp}$$

attractive K3:

two generators of T: (p, q) $\longrightarrow$ intersection pairings: $\begin{pmatrix} p\cdot p & p\cdot q \\ q\cdot p & q\cdot q \end{pmatrix} = \begin{pmatrix} 2a & b \\ b & 2c \end{pmatrix}$

- attractive K3s characterised in terms of 3 numbers [a, b, c]!

FLUX VACUA ON ATTRACTIVE K3 X K3

- simplest possible flux: [Aspinwall, Kallosh '05]

$$G_4 = Re(\gamma \Omega_1 \wedge \overline{\Omega}_2)$$

⟶ *both K3s are attractive!*

- express Ω_i in terms of $[a_i, b_i, c_i]$: *intersection data of $T_i = Pic(K3_i)^\perp$*

$$\Omega_i = p_i + \tau_i q_i \qquad \textit{with} \qquad \tau_i = \frac{-b_i + i\sqrt{4a_i c_i - b_i^2}}{2c_i}$$

- integrality of G_4 and tadpole bound:

simple conditions on $[a_i, b_i, c_i]$: *complete classification possible!*

CLASSIFICATION OF ATTRACTIVE K3 X K3 FLUX VACUA:

N_{flux}	$[a_1 b_1 c_1]$	$[a_2 b_2 c_2]$	γ
24	[8 8 8]	[1 1 1]	$\gamma^{(6)}$
	[6 0 6]	[1 0 1]	$\gamma^{(4)}$
	[6 0 3]	[2 0 1]	$\pm i/\sqrt{2}$
	[6 0 2]	[3 0 1]	$\pm i/\sqrt{3}$
	[6 0 2]	[1 1 1]	$\gamma^{(6)}$
	[6 0 1]	[6 0 1]	$\pm i/\sqrt{6}$
	[4 4 4]	[2 2 2]	$\gamma^{(6)}$
	[3 0 3]	[2 0 2]	$\gamma^{(4)}$
	[3 0 3]	[1 0 1]	$(1+i)\gamma^{(4)}$
	[3 0 2]	[3 0 2]	$\pm i\sqrt{2/3}$
	[3 0 1]	[2 2 2]	$\gamma^{(6)}$
	[2 2 2]	[1 1 1]	$2\gamma^{(6)}$
	[2 0 1]	[2 0 1]	$\pm 1 \pm i/\sqrt{2}$
23	[6 1 1]	[6 1 1]	$\pm 2i/\sqrt{23}$
	[3 1 2]	[3 1 2]	$\pm 4i/\sqrt{23}$
22	[6 2 2]	[3 1 1]	$\pm 2i/\sqrt{11}$
21	[7 7 7]	[1 1 1]	$\gamma^{(6)}$
	[6 3 3]	[2 1 1]	$\pm 2i/\sqrt{7}$
	[1 1 1]	[1 1 1]	$(2 \pm \sqrt{3}i)\gamma^{(6)}$

N_{flux}	$[a_1 b_1 c_1]$	$[a_2 b_2 c_2]$	γ
20	[5 0 5]	[1 0 1]	$\gamma^{(4)}$
	[5 0 1]	[5 0 1]	$\pm i/\sqrt{5}$
	[3 2 2]	[3 2 2]	$\pm 2i/\sqrt{5}$
	[1 0 1]	[1 0 1]	$(1 \pm 2i)\gamma^{(4)}$
19	[5 1 1]	[5 1 1]	$\pm 2i/\sqrt{19}$
18	[6 6 6]	[1 1 1]	$\gamma^{(6)}$
	[3 3 3]	[2 2 2]	$\gamma^{(6)}$
16	[4 0 4]	[1 0 1]	$\gamma^{(4)}$
	[4 0 2]	[2 0 1]	$\pm i/\sqrt{2}$
	[4 0 1]	[4 0 1]	$\pm i/2$
	[4 0 1]	[1 0 1]	$\gamma^{(4)}$
	[2 0 2]	[2 0 2]	$\gamma^{(4)}$
	[2 0 2]	[1 0 1]	$(1+i)\gamma^{(4)}$
	[2 0 1]	[2 0 1]	± 1
	[1 0 1]	[1 0 1]	$2\gamma^{(4)}$
15	[5 5 5]	[1 1 1]	$\gamma^{(6)}$
	[4 1 1]	[4 1 1]	$\pm 2i/\sqrt{15}$
	[2 1 2]	[2 1 2]	$\pm 4i/\sqrt{15}$
14	[4 2 2]	[2 1 1]	$\pm 2i/\sqrt{7}$
	[2 1 1]	[2 1 1]	$\pm 1 \pm i/\sqrt{7}$

N_{flux}	$[a_1 b_1 c_1]$	$[a_2 b_2 c_2]$	γ
12	[4 4 4]	[1 1 1]	$\gamma^{(6)}$
	[3 0 3]	[1 0 1]	$\gamma^{(4)}$
	[3 0 1]	[3 0 1]	$\pm i/\sqrt{3}$
	[3 0 1]	[1 1 1]	$\gamma^{(6)}$
	[2 2 2]	[2 2 2]	$\gamma^{(6)}$
	[1 1 1]	[1 1 1]	$2\gamma^{(6)}$
11	[3 1 1]	[3 1 1]	$\pm 2i/\sqrt{11}$
9	[3 3 3]	[1 1 1]	$\gamma^{(6)}$
	[1 1 1]	[1 1 1]	$\sqrt{3}i\gamma^{(6)}$
8	[2 0 2]	[1 0 1]	$\gamma^{(4)}$
	[2 0 1]	[2 0 1]	$\pm i/\sqrt{2}$
	[1 0 1]	[1 0 1]	$(1+i)\gamma^{(4)}$
7	[2 1 1]	[2 1 1]	$\pm 2i/\sqrt{7}$
6	[2 2 2]	[1 1 1]	$\gamma^{(6)}$
4	[1 0 1]	[1 0 1]	$\gamma^{(4)}$
3	[1 1 1]	[1 1 1]	$\gamma^{(6)}$

[Braun, Kimura, Watari ’14]
[Braun, Fraiman, Graña, SL, Parra de Fraitas ’23]

F-THEORY AND FRAME-LATTICE

- F-theory: elliptically fibered K3

$$\begin{array}{ccc} E & \hookrightarrow & K3 \\ & & \downarrow \\ & & B \end{array}$$

- corresponding lattice data:

two algebraic curves (fiber and section) with intersection:

$$\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \simeq U$$

- Frame lattice W:

$$\mathrm{Pic}(S) = U \oplus W$$

root sublattice $W_{\mathrm{root}} \subset W$:

→ *F-theory gauge group*

- Goal: For a given M-theory flux vacuum (in terms of $[a_i, b_i, c_i]$):

find all possible embeddings $U \hookrightarrow \mathrm{Pic}(S)$

KNESER NICHIYAMA-METHOD

[Nishiyama '96, '97]
[Braun, Kimura, Watari, '13]

- For $T = \mathrm{Pic}(K3)^\perp$: Find an embedding $T \hookrightarrow E_8$ and determine the orthogonal complement in E_8:

$$T_0 = T^\perp \subset E_8$$

lattice of signature (6,0) → T_0

Theorem:

$$Pic(S) = U \oplus W \quad \Leftrightarrow$$

there exists an even self-dual lattice N of signature (24,0) such that $T_0 \hookrightarrow N$, $W \hookrightarrow N$ primitively and mutually orthogonal

- Find all embeddings $T_0 \hookrightarrow N$ for all possible N and determine the orthogonal complements in N:

⟶ *all possible frame lattices W!*

KNESER NICHIYAMA-METHOD

[Braun, Fraiman, Graña, SL, Parra de Fraitas ’23]

- Even self-dual lattices of signature (24,0):

24 Niemeier lattices N_I

- Extensive computer search:

  1. *Given T (in terms of [a,b,c]) find* $T_0 = T^{\perp} \subset E_8$
  2. *Determine all embeddings* $T_0 \hookrightarrow N_I$ *(for all* $I = 1,\ldots,24$*)*
  3. *For each embedding:*
     Check for roots in the orthogonal complement $W = T_0^{\perp} \subset N_I$
     $\rightarrow$ *F-theory gauge group*

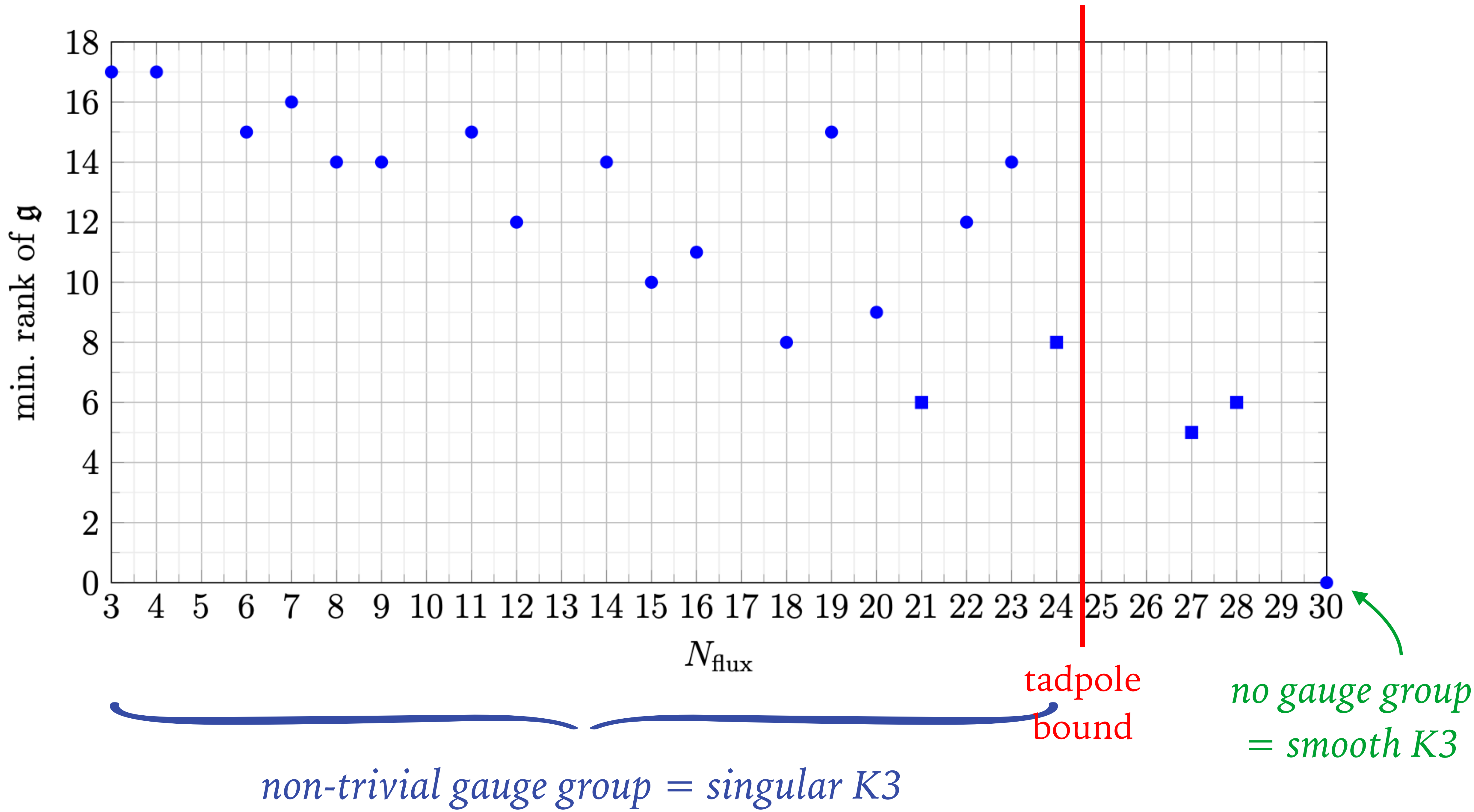
- Result:

Every flux vacuum (within the tadpole bound) has a gauge group!

KNESER–NISHIYAMA METHODS: RESULTS

[Braun, Fraiman, Graña, SL, Parra de Fraitas ’23]

Rank of the minimal gauge group as a function of the tadpole charge:



CONCLUSION

[Bena, Blåbäck, Graña, SL ’20, ’21]
[Braun, Fraiman, Graña, SL, Parra de Fraitas ’23]

M-theory on K3 x K3:

- stabilization of all moduli
- generic point in moduli space (no orbifold singularity)
- fluxes with arbitrary small M2/D3-charge ($Q \lesssim 24$)

→ cannot have all three!

Generalization:

fluxes with small tadpole	$\overset{?}{\longleftrightarrow}$	*“special” points in moduli space*

THANK YOU!